Lattice determination of the strong coupling constant from moments of quarkonium correlators

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Determination of the strong coupling constant and heavy quark masses from the 2+1 flavor lattice QCD calculations of the moments of pseudo-scalar quarkonium correlators

PP, J. Weber, PRD 100 (2019) 034519
PP, J. Weber, EPJC 82 (2022) 64

Extension of the previous work
Maezawa, PP, PRD PRD 94 (2016) 034504

Uncertainties of the lattice (r_1) scale

alphas-2024: Workshop on precision measurements of the QCD coupling constant, ECT*, Feb 5-9, 2024

Moments of quarkonium correlators

We use moments method pioneered by HPQCD and Karlsruhe group:

$$G_n = \sum_t t^n G(t), \ G(t) = a^6 \sum_{\mathbf{x}} (am_{h0})^2 \langle j_5(\mathbf{x},t) j_5(0,0) \rangle \qquad j_5 = \bar\psi \gamma_5 \psi$$

Calculated continuum perturbation theory to order α_s^3

$$G_n = \frac{g_n(\alpha_s(\mu), \mu/m_h)}{am_h^{n-4}(\mu_m)}$$

To cancel lattice effects, consider the reduced moments

$$R_n = \left(\frac{G_n}{G_n^0}\right)^{1/(n-4)}$$

Allison et al, PRD78 (2008) 054513

and similarly on the weak coupling side:

$$R_n = \begin{cases} r_4 & (n=4) \\ r_n \cdot (m_{h0}/m_h(\mu_m)) & (n \ge 6) \end{cases},$$

$$r_n = 1 + \sum_{j=1}^{3} r_{nj}(\mu/m_h) \left(\frac{\alpha_s(\mu)}{\pi}\right)^j$$

+ contribution from condensate

$$\sim \frac{1}{m_h^4} \langle \frac{\alpha_s}{\pi} G^2 \rangle$$

$R_4, \ R_6/R_8, R_8/R_{10} \Rightarrow \alpha_s(\mu)$

$R_6, \ R_8 \ R_{10} \Rightarrow m_h(\mu_m)$

Some lattice details

Highly improved Staggered Quark (HISQ) action and tree-level improved gauge action

HotQCD gauge configurations : 2+1 flavor QCD
physical m_s , $m_l=m_s/20$: $m_K=504$ MeV, $m_\pi=161$ MeV
Bazavov et al, PRD 90 (2014) 094503
Lattice spacing set by the r_1 scale

$$\left(r^2 \frac{dE_0(r)}{dr} \right)_{r=r_1} = 1$$

$r_1=0.3106(14)(8)(4)$ fm (pion decay constant)
MILC, PoS LATTICE2010,074

Temperature is varied by the lattice spacing a
$T = (1/N_\tau a)$ →

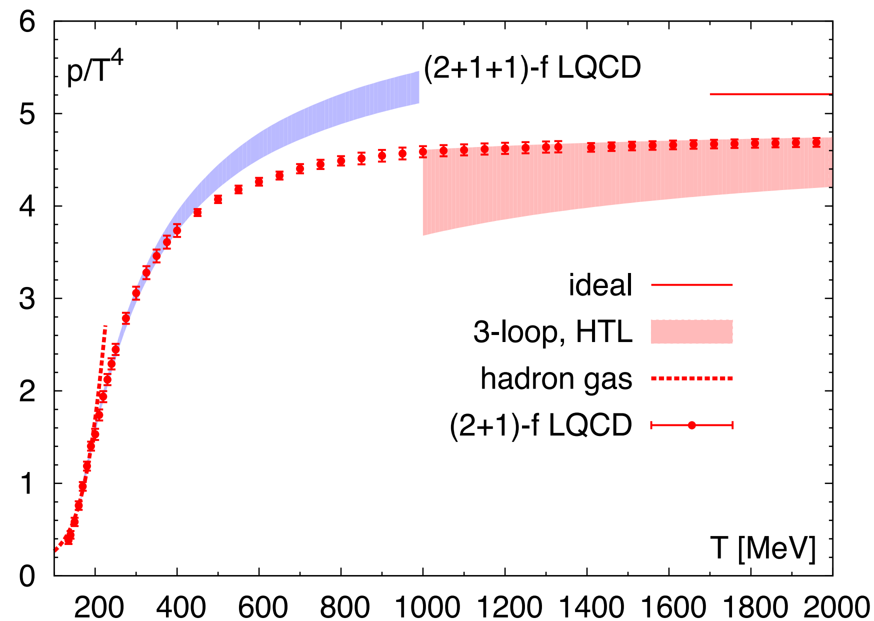
Many lattice spacings available, $a_{min}=0.041$fm

Additional gauge configurations for
$m_l=m_s/5$ on 64^4 lattices with $a=0.035$ fm,
0.029 fm and 0.025 fm to obtain the static
quark potential at shorter distances
Bazavov, PP, Weber, PRD 97 (2018)

statistics for the $T=0$ runs:

24^3x32:	4-8K TU
32^4, 32^3x64:	7-40K TU
48^4:	8-16K TU
48^3x64:	8-9K TU
64^4 :	9K TU

in molecular dynamic time units (TU)



am_c^0 from spin averaged 1S charmonium mass or from $\eta_c(1S)$ mass

Lattice results on the moments of quarkonium correlators

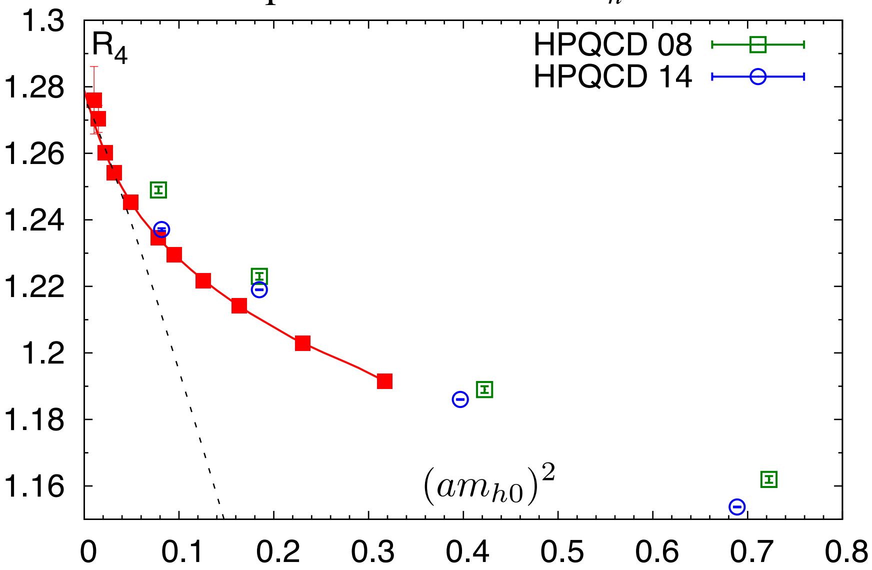
$$m_h = m_c,\ 1.5m_c,\ 2m_c,\ 3m_c,\ 4m_c,\ m_b$$

Random color wall sources => statistical errors are negligible; Dominant errors are the finite volume errors and errors due to mistuning of the heavy quark mass.

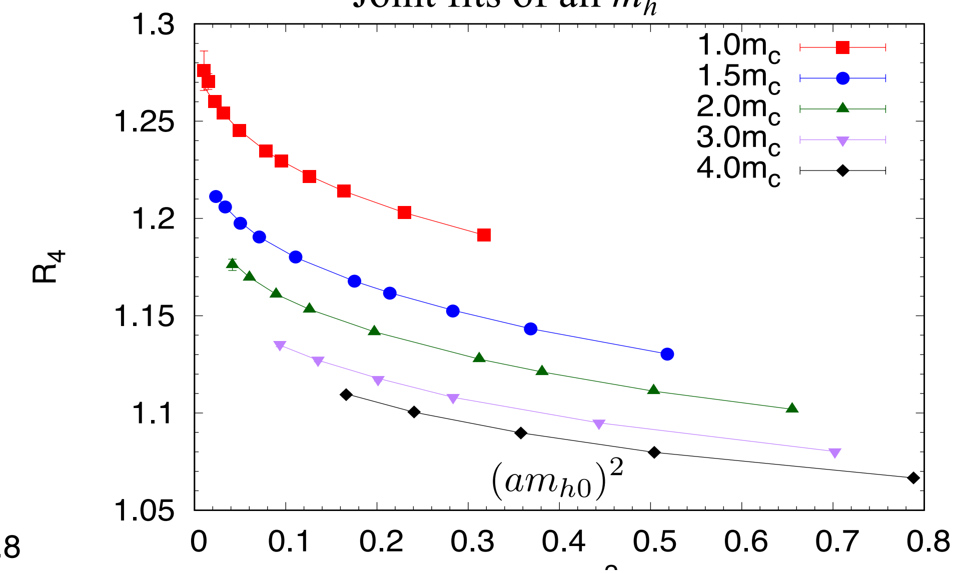
Volume errors are estimated from the free theory calculations (upper bound), are largest at small *a*

Sea quark mass effects are smaller than statistical errors $\Rightarrow$ combine $m_s/20$ and $m_s/5$ data

Separate fits at each m_h



Joint fits of all m_h



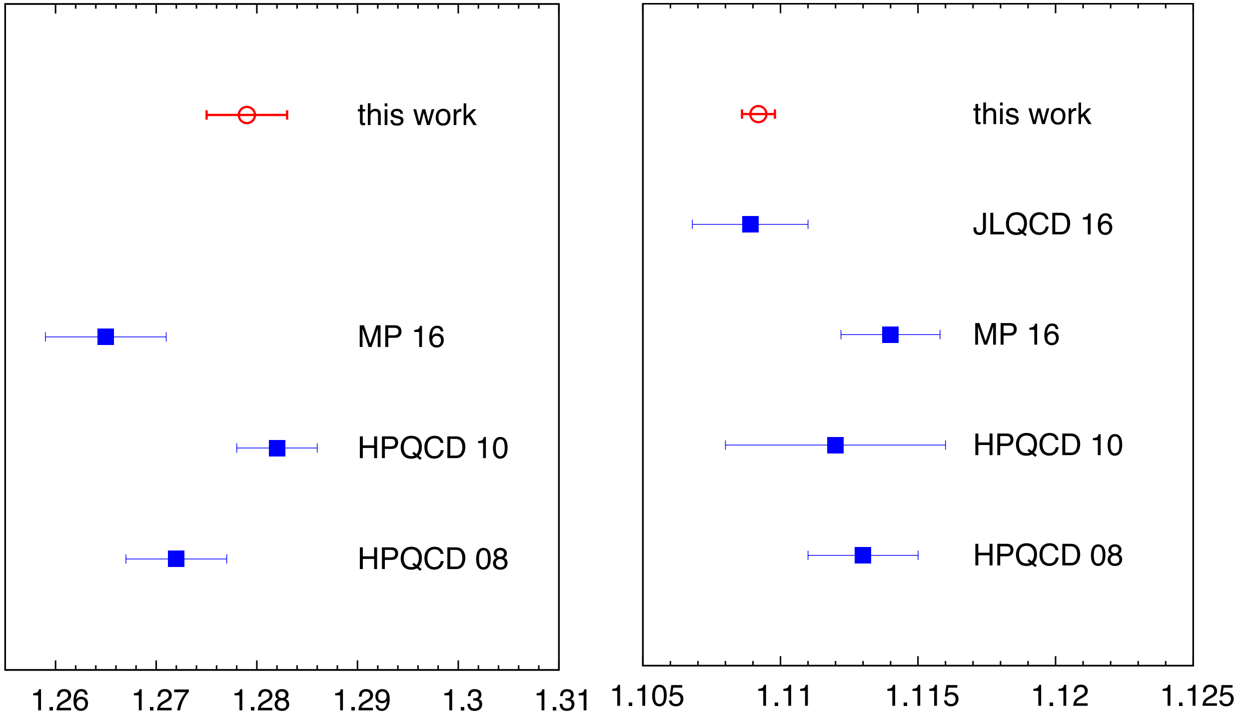
Continuum results are needed but there is a significant dependence on the lattice spacing

$$R_4(m_h) = R_4^{cont}(m_h) + \sum_{i=1}^{N}\sum_{j=1}^{M_i} b_{ij}(\alpha_s^b)^i \left[\sum_{k=0}^{I} d_{ijk} \ln^k(am_{h0})\right](am_{h0})^{2j}$$

$\Rightarrow$ fits with $N=2,\ J=0-1$ and various $M_i = 2-8,\ \alpha_s^b = \frac{g_0^2}{4\pi u_0^4}$

Continuum results on the moments @ m_c

$R_4,\ m_h = m_c$ $R_6/R_8,\ m_h = m_c$ $R_8/R_{10},\ m_h = m_c$



Finite volume effects are too large !

HPQCD: 2+1 flavor improved staggered (asqtad) sea + valence HISQ,
Allison et al, PRD 78 (2008) 054513; McNeile, PRD 82 (2010) 034512

JLQCD: 2+1 flavor Domain-Wall Fermions,
Nakayama, Fahy, Hashimoto, PRD 94 (2016) 054507

MP 16: 2+1 flavor HISQ (sea and valence sectors)
Maezawa, PP, PRD PRD 94 (2016) 034504

Discrepancies are understood to be due simple $a^2 + a^4$ extrapolations

Extracting the strong coupling constant in 3f QCD

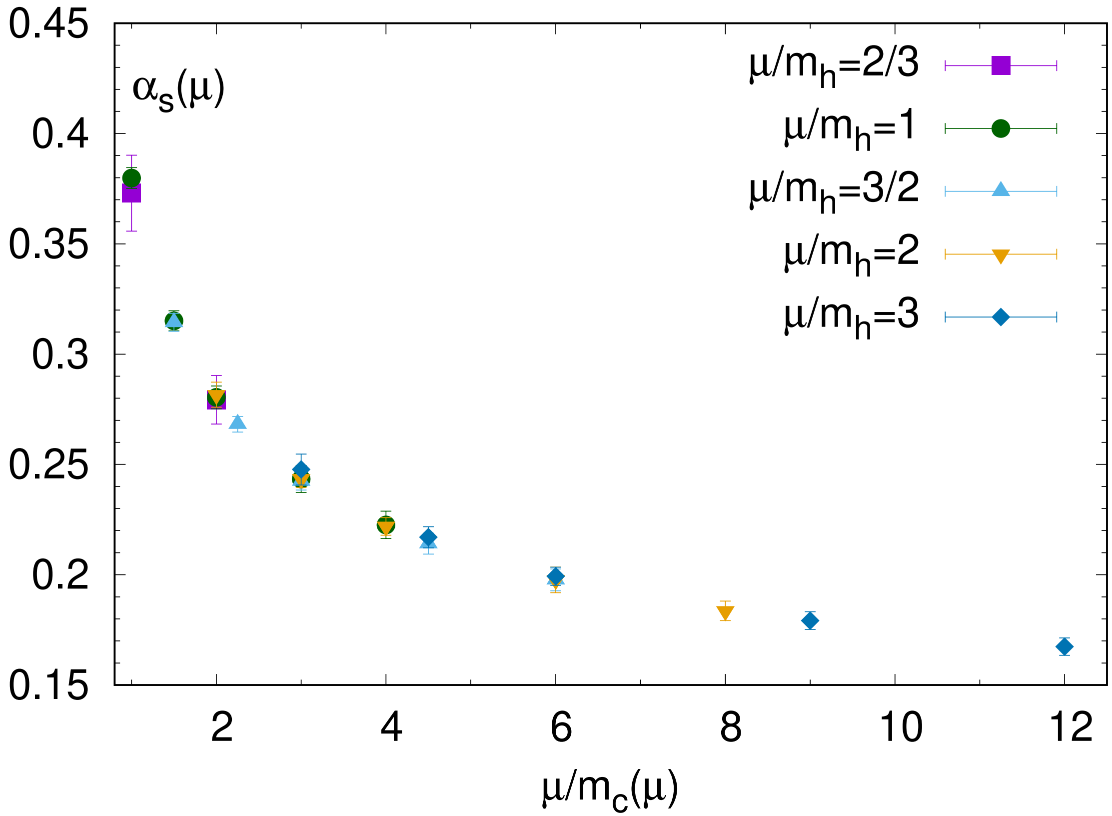
Natural choice: $\mu = \mu_m$

Use different renormalization scales: $\mu = 2/3 m_h,\ m_h,\ 3/2 m_h, 2m_h,\ 3m_h$

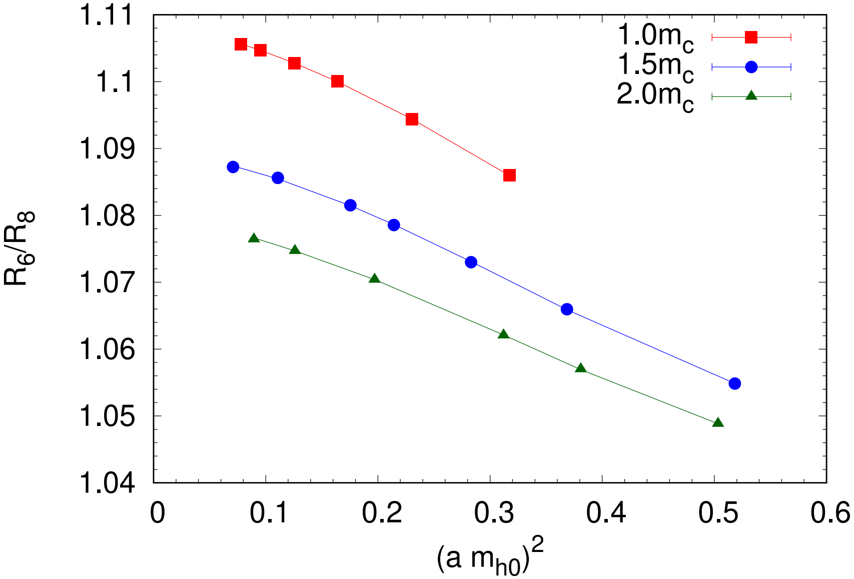
$R_4 \rightarrow \alpha_s(m_h)$

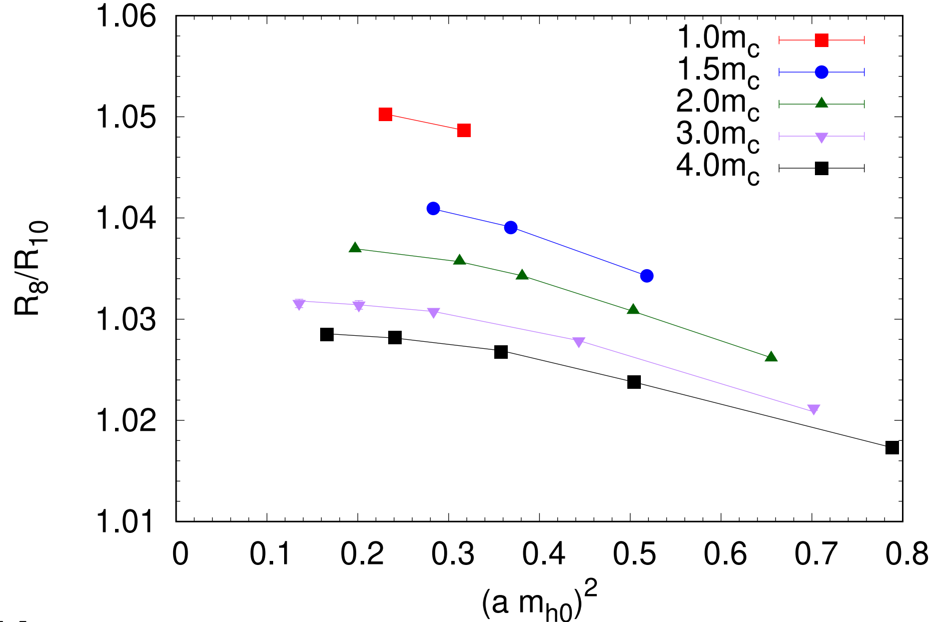
perturbative error: $\pm 5 \times r_{n3} \left(\frac{\alpha_s}{\pi}\right)^4$

condensate error: $\langle \frac{\alpha_s}{\pi} G^2 \rangle = (-0.006 \pm 0.012)\ \mathrm{GeV}^4$



The strong coupling constant from ratio of moments





$$\frac{R_n(m_h)}{R_{n+2}(m_h)} = \left(\frac{R_n(m_h)}{R_{n+2}(m_h)}\right)^{cont} + \sum_{I=1}^{N}\sum_{j=1}^{M_i} f_{ij}^{(n)} (\alpha_s^b)^i (am_{h0})^{2j},\ n \geq 6$$

$$N = 2$$

$$M_1 = M_2 = 3, 4$$

R_6/R_8			R_8/R_{10}		
m_h/m_c	continuum	$\alpha_s(m_h)$	m_h/m_c	continuum	$\alpha_s(m_h)$
1.0	1.10895(32)	0.3826(14)(178)(39)	1.0	Large finite volume effects	
1.5	1.09100(25)	0.3137(10)(76)(8)	1.5	1.04310(45)	0.3166(34)(82)(17)
2.0	Reliable continuum extrapolation is not possible		2.0	1.03830(68)	0.2808(51)(50)(4)
3.0			3.0	1.03249(94)	0.2382(69)(24)(1)
4.0			4.0	1.02987(106)	0.2191(293)(17)(0)

The strong coupling constant from ratio of moments

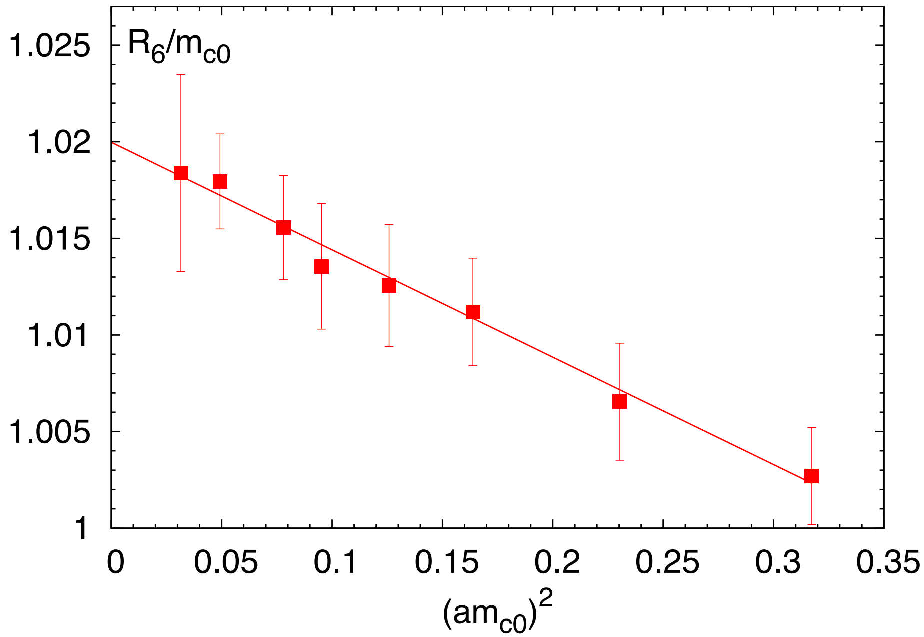
$\alpha_s(\mu = m_h)$:

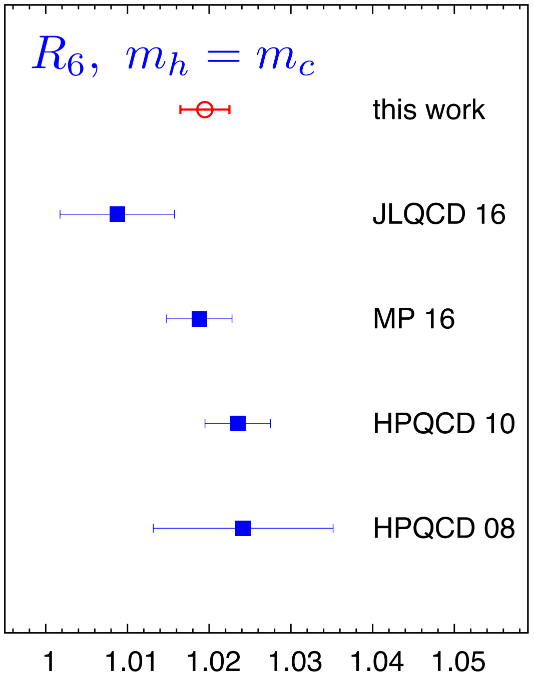
m_h/m_c	R_4	R_6/R_8	R_8/R_{10}
1.0	0.3798(28)(31)(22)	0.3826(14)(178)(39)	Large finite volume effects
1.5	0.3151(43)(14)(4)	0.3137(10)(76)(8)	0.3166(34)(82)(17)
2.0	0.2804(51)(9)(1)	Reliable continuum extrapolation is not possible	0.2808(51)(50)(4)
3.0	0.2434(61)(5)(0)		0.2382(69)(24)(1)
4.0	0.2226(62)(4)(0)		0.2191(293)(17)(0)

Ratio of the moments gives results on the strong coupling constant that are:

1) Compatible within errors to the ones extracted from the fourth moments
2) Have larger perturbative uncertainties
3) Have larger uncertainties from the condensate contribution

Determination of the quark masses

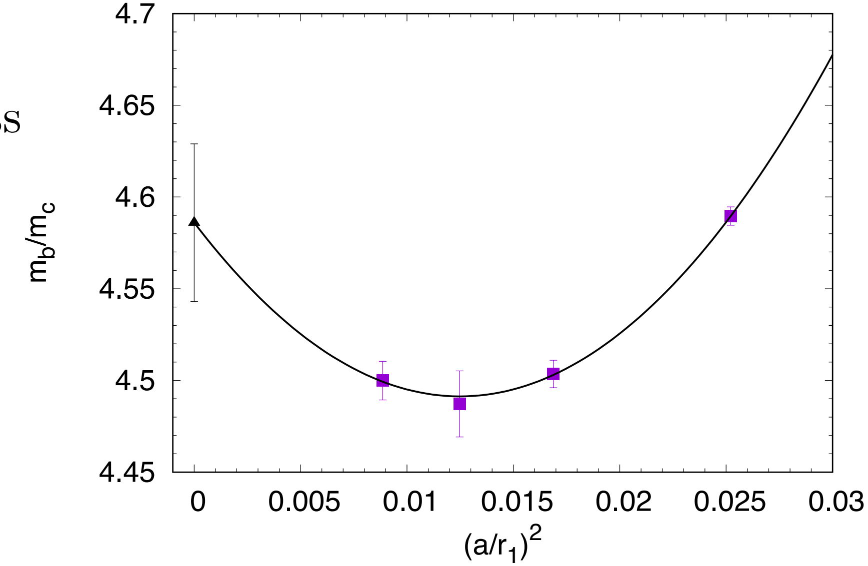




Determine am_b^0 by fixing the $\eta_b(1S)$ mass to its physical value from PDG

$$m_b/m_c = 4.586(43)$$

agrees with other lattice determinations



Determination of the quark masses

m_h	R_6	R_8	R_{10}	av.
$1.0m_c$	1.2740(25)(17)(11)(61)	1.2783(28)(23)(00)(43)	1.2700(72)(46)(13)(33)	1.2754(39)
$1.5m_c$	1.7147(83)(11)(03)(60)	1.7204(42)(14)(00)(40)	1.7192(35)(29)(04)(30)	1.7191(38)
$2.0m_c$	2.1412(134)(07)(01)(44)	2.1512(71)(10)(00)(29)	2.1531(74)(19)(02)(21)	2.1507(52)
$3.0m_c$	2.9788(175)(06)(00)(319)	2.9940(156)(08)(00)(201)	3.0016(170)(16)(00)(143)	2.9949(153)
$4.0m_c$	3.7770(284)(06)(00)(109)	3.7934(159)(08)(00)(68)	3.8025(152)(15)(00)(47)	3.7956(110)
m_b	4.1888(260)(05)(00)(111)	4.2045(280)(07)(00)(69)	4.2023(270)(14)(00)(47)	4.1985(163)

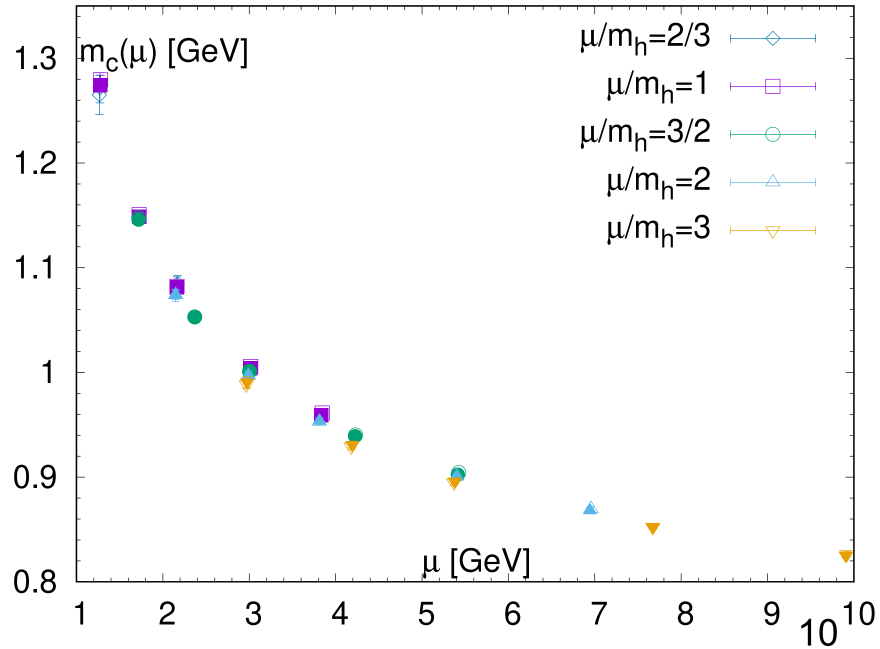
R_6, R_8 and R_{10} give consistent results for $m_h(m_h)$

$m_h(m_h)/h$, $h = m_h/m_c$ is consistent with expected $m_c(\mu)$

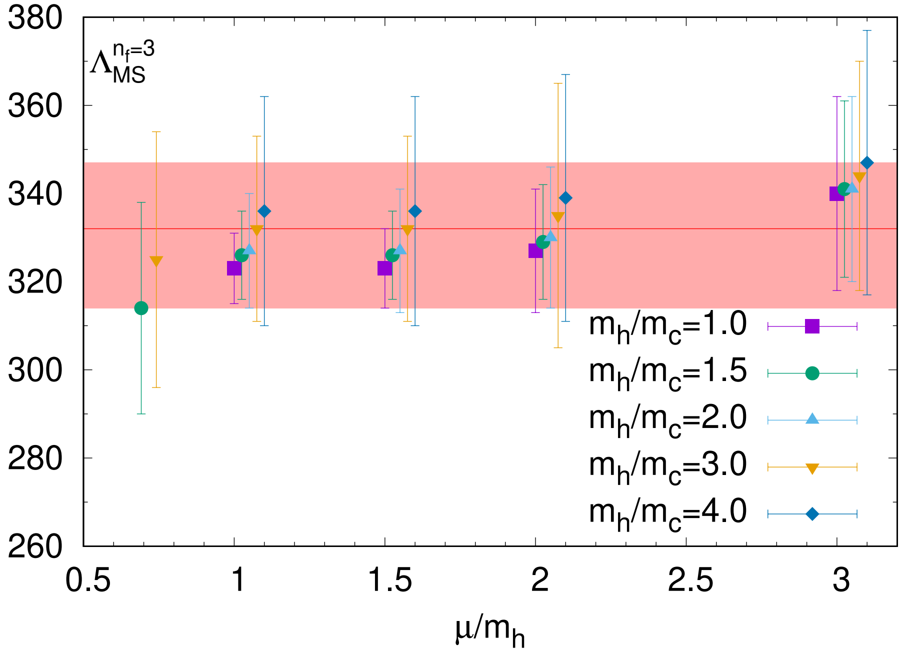
Final results after running with RunDeC

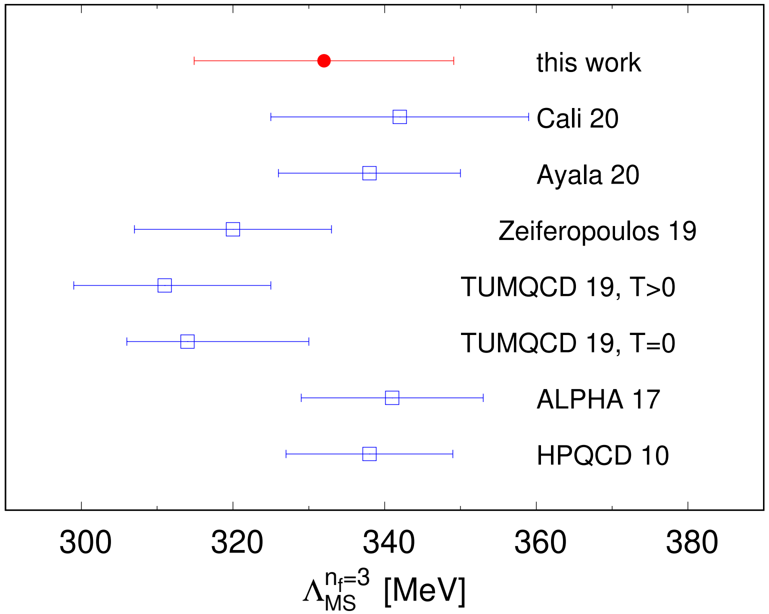
$m_c(\mu = m_c, n_f = 4) = 1.265(10)$ GeV

$m_b(\mu = m_b, n_f = 5) = 4.188(37)$ GeV



The Λ-parameter and strong coupling constant in 5f QCD





Average the results at different μ/m_h and m_h
use the spread as an error estimate

$$\Lambda_{\overline{MS}}^{n_f=3} = 332 \pm 17 \pm 2\ (scale)\ \mathrm{MeV}$$

$\Lambda_{\overline{MS}}^{n_f=3}$ is agreement with
other lattice determinations

Matching to 4 and 5 flavor theory using RunDeC
at $M_c = 1.5$ GeV and $M_b = 4.8$ GeV or
at $m_c(\mu)$, $\mu = m_c - 2$ GeV and $m_b(m_b)$

$$\alpha_s^{n_f=5}(\mu = M_Z) = 0.1177(12)$$

Error from running and matching: 0.0003

The Λ-parameter and lattice scale determination

In the above analysis r_1=0.3106(14)(8)(4) fm from pion decay constant used to set the lattice spacing
MILC Coll. , PoS LATTICE2010,074

TUMQCD Collaboration (PRD 107 (2023) 074503)
obtained the value r_1=0.3037 (25) fm using pseudo-scalar meson decay constant f_{p4s} (derived from the pion decay constant) MILC Coll., PRD 87 (2012) 054505

Similar findings in 2+1 flavor QCD

2% decrease in the lattice spacing

See talks by Hai-Tao Shu and Viljami Leino

$$\Lambda_{\overline{MS}}^{n_f=3} = 340 \pm 17 \pm 7\ (scale)\ \text{MeV}$$

A more conservative error on the scale of 2.2%

↓

$$\alpha_s^{n_f=5}(\mu = M_Z) = 0.1182(12)(5)_{scale}$$

In complete agreement with the FLAG average

Summary

- Precise determination of the strong coupling constant from the moments of quarkonium correlators is challenging because:

1) Large lattice cutoff dependence of the lowest moment and the ratios of the moments
2) Uncertainties in the weak coupling expansion

- The most recent determination gives:

$$\alpha_s^{n_f=5}(\mu = M_Z) = 0.1177(12)$$

or

$$\alpha_s^{n_f=5}(\mu = M_Z) = 0.1182(12)(5)_{scale}$$

when considering new results on r_1 scale

- The heavy quark masses can be well determined from the moments of quarkonium correlators

$$m_c(\mu = m_c, n_f = 4) = 1.265(10)\ \text{GeV} \qquad m_b(\mu = m_b, n_f = 5) = 4.188(37)\ \text{GeV}$$

- There are some tension in the continuum extrapolated values of the moments from different groups, but the main cause of discrepancy could be due to the perturbative error

➡ 5-loop calculations will certainly help.

Back-up

2022:

m_h	R_4	R_6/m_{c0}	R_8/m_{c0}	R_{10}/m_{c0}	$\alpha_s(m_h)$
$1.0m_c$	1.2778(20)	1.0200(16)	0.9166(17)	0.8719(21)	0.3798(28)(31)(22)
$1.5m_c$	1.2303(30)	1.0792(20)	0.9860(20)	0.9462(23)	0.3151(43)(14)(4)
$2.0m_c$	1.2051(37)	1.1182(23)	1.0317(23)	0.9944(26)	0.2804(51)(9)(1)
$3.0m_c$	1.1782(44)	1.1729(27)	1.0923(26)	1.0574(31)	0.2434(61)(5)(0)
$4.0m_c$	1.1631(45)	1.2098(31)	1.1321(30)	1.0985(31)	0.2226(62)(4)(0)

2019:

m_h	R_4	R_6/R_8	R_8/R_{10}	av.	$\Lambda_{\overline{MS}}^{n_f=3}$ MeV
$1.0m_c$	0.3815(55)(30)(22)	0.3837(25)(180)(40)	0.3550(63)(140)(88)	0.3782(65)	314(10)
$1.5m_c$	0.3119(28)(4)(4)	0.3073(42)(63)(7)	0.2954(75)(60)(17)	0.3099(48)	310(10)
$2.0m_c$	0.2651(28)(7)(1)	0.2689(26)(35)(2)	0.2587(37)(34)(6)	0.2648(29)	284(8)
$3.0m_c$	0.2155(83)(3)(1)	0.2338(35)(19)(1)	0.2215(367)(17)(1)	0.2303(150)	284(48)

No proper description of cutoff effects
Because of the limited amount of data

$m_h = 2m_c$ result is an outlier; weighted average + spread : $\Lambda_{\overline{MS}}^{n_f=3} = 298 \pm 16$ MeV

m_h	R_4	R_6/R_8	R_8/R_{10}
$1.0m_c$	1.279(4)	1.1092(6)	1.0485(8)
$1.5m_c$	1.228(2)	1.0895(11)	1.0403(10)
$2.0m_c$	1.194(2)	1.0791(7)	1.0353(5)
$3.0m_c$	1.158(6)	1.0693(10)	1.0302(5)

Back-up

m_h/m_c	fit 1	fit 2	fit 3	fit 4	fit 5
1.0	1.2805(14)	1.2782(10)	1.2817(18)	1.2810(13)	1.2806(10)
1.5	1.2308(14)	1.2286(10)	1.2318(16)	1.2309(12)	1.2302(10)
2.0	1.2045(14)	1.2024(09)	1.2053(15)	1.2044(11)	1.2038(10)
3.0	1.1763(15)	1.1743(10)	1.1770(15)	1.1759(11)	1.1754(10)
4.0	1.1608(14)	1.1582(10)	1.1610(15)	1.1601(11)	1.1595(10)

- fit 1: $(am_{h0})^2_{max} = 0.4,\ M_1 = 3,\ M_2 = 2,\ d_{111} \neq 0$
- fit 2: $(am_{h0})^2_{max} = 0.6,\ M_1 = 3,\ M_2 = 2,\ d_{111} \neq 0$
- fit 3: $(am_{h0})^2_{max} = 0.8,\ M_1 = 4,\ M_2 = 2,\ d_{111} \neq 0$
- fit 4: $(am_{h0})^2_{max} = 1.0,\ M_1 = 3,\ M_2 = 2,\ d_{111} \neq 0,\ d_{121} \neq 0$
- fit 5: $(am_{h0})^2_{max} = 1.2,\ M_1 = 4,\ M_2 = 2,\ d_{111} \neq 0,\ d_{121} \neq 0$