

The complete EM Euclidean Adler function and its interplay with α_s and α_{QED}

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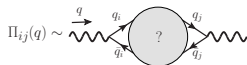
JHEP 04 (2023) 067



The Euclidean Adler function, $D(Q^2)$

- Two point correlation functions of neutral quark current

$$\Pi_{ij}^{\mu\nu}(q) \equiv i \int d^4x e^{-iqx} \langle 0 | T (\bar{q}_i(x) \gamma^\mu q_i(x) \bar{q}_j(0) \gamma^\nu q_j(0)) | 0 \rangle.$$



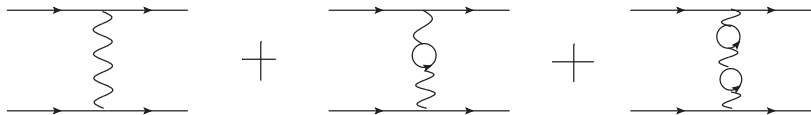
- Asymptotic limit, $Q^2 = -q^2 \gg \Lambda_{\text{QCD}}^2$

$$\Pi_{ij}^{\text{part}}(Q^2) \sim \frac{N_C}{\pi^2} \delta_{ij} [\log(Q^2) + C_{\text{subs}}] \rightarrow D_{ij}(Q^2) \sim \pi^2 Q^2 \frac{d\Pi_{ij}(Q)}{dQ^2} \sim N_C \delta_{ij}$$

- EM Adler function, $D_{\text{EM}} = \sum_{i,j} Q_i Q_j D_{ij}$
- Separation from the asymptotic regime, $\delta D_{ij} \sim N_C \delta_{ij} \alpha_s(Q^2)$
- Perturbative QCD (pQCD) breaks down at low Q . $D_{ij}(Q^2 \rightarrow 0) = 0$

Interplay with α_{QED} and low-energy data

- Hadronic running of α_{QED} . $\Delta\alpha_{\text{had}}(Q^2) \sim (\Pi(Q^2) - \Pi(0)) \sim \int_0^{Q^2} dQ'^2 \frac{D(Q'^2)}{Q'^2}$

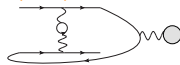


$$F(q^2) = \frac{1}{q^2} \left[\frac{\alpha_0}{1 - \Pi_0(q^2)} \right] = \frac{1}{q^2} \left[\frac{\alpha_R}{1 - \Pi_R(q^2)} \right]$$

$$q^2 F(0) = \alpha_{OS} \rightarrow F(q^2) = \frac{1}{q^2} \frac{\alpha_{OS}}{1 - (\Pi(q^2) - \Pi(0))} = \frac{\alpha_{\text{QED}}(q^2)}{q^2}.$$

- Precise lattice data results at low Q [BMW, Nature 593 \(2021\) 51](#).

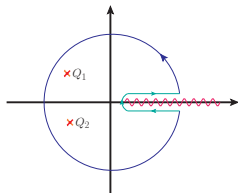
[Mainz, JHEP 08 \(2022\) 220](#). Driven by muon $g-2$ program



- Connection to experimental data (optical theorem)

$$R(q^2) \sim q^2 \sigma(e^+ e^- \rightarrow \text{hadrons}) \sim \text{Im}\Pi(q^2)$$

Interplay with α_{QED} and low-energy data



$$W(Q^2; Q_1^2, Q_2^2) = \frac{1}{(Q^2 - Q_1^2)(Q^2 - Q_2^2)}$$

$$\begin{aligned} & \frac{1}{2\pi i} \oint_{|Q^2|=Q_0^2} dQ^2 \frac{\Pi_{ij}(Q^2)}{(Q^2 - Q_1^2)(Q^2 - Q_2^2)} + \frac{1}{2\pi i} \left(\int_{|Q_{th}^2|e^{-i\pi}}^{|Q_0^2|e^{-i\pi}} - \int_{|Q_{th}^2|e^{i\pi}}^{|Q_0^2|e^{i\pi}} \right) dQ^2 \frac{\Pi_{ij}(Q^2)}{(Q^2 - Q_1^2)(Q^2 - Q_2^2)} \\ &= \frac{\Pi_{ij}(Q_2^2)}{Q_2^2 - Q_1^2} + \frac{\Pi_{ij}(Q_1^2)}{Q_1^2 - Q_2^2} \end{aligned}$$

$$\Delta\alpha_{\text{had}}(Q^2) = \frac{\alpha Q^2}{3\pi} \int_{s_{th}}^{\infty} ds \frac{R(s)}{s(s + Q^2)}$$

$$D(Q^2) \sim Q^2 \int_{s_{th}}^{\infty} ds \frac{R(s)}{(s + Q^2)^2}$$

pQCD EM Adler function: state-of-art and approach

- Previous assessments of the total D_{EM} made in MOM scheme [hep-ph/9812521](#), [CERN Yellow Reports: Monographs 3 \(2020\) 9](#)
- Decoupling scheme, but less known perturbative expansion (up to α_s^2)
- $D_{ij}^{n_f, m_q=0}$ known in the $\overline{\text{MS}}$ at 5 loops
- $m_{u,d} \ll m_s \ll \Lambda_{\text{QCD}} \ll m_c \ll m_b \ll m_t$. Interested in $Q < 2m_c$
- Start from $n_f = 3$ massless and systematically adds expansions in $\frac{Q^2}{(2m_c)^2}$.
Inputs from [FLAG](#)
- $D_{\text{EM}} = \sum_{i,j} Q_i Q_j D_{ij}$. Study the different two-point functions separately

pQCD Adler function. Light quarks, massless

$$D_{ii}^{L,(0)}(Q^2) = N_C \left\{ 1 + \sum_{n=1} \sum_{p=0}^{n-1} K_{n,p} \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^n \log^p(Q^2/\mu^2) \right\},$$

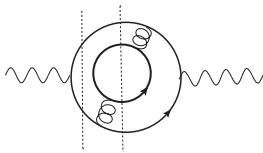
All coefficients known up to (and including) $K_{4,n}$

			$D_{ii}^{L,(0)}(Q^2)$					
$\alpha_s^{(n_f=5)}(M_Z^2)$	Q	$\alpha_s(Q^2)$	0	1	2	3	4	5*
0.115	1.0	0.4227	3	3.4036	3.4927	3.5392	3.5874	3.6238
	1.5	0.3197	3	3.3053	3.3562	3.3764	3.3921	3.4011
	2.0	0.2751	3	3.2627	3.3005	3.3133	3.3219	3.3262
0.120	1.0	0.5277	3	3.5039	3.6427	3.7332	3.8504	3.9606
	1.5	0.3681	3	3.3515	3.4191	3.4498	3.4776	3.4958
	2.0	0.3085	3	3.2946	3.3420	3.3601	3.3738	3.3813

Strange mass corrections also known (bad behaviour, but tiny)

$$\Delta_{m_s} D_{33}^L(Q^2) = -3N_C \frac{m_s^2(Q^2)}{Q^2} \sum_n (2c_n^{L+T} + e_n^{L+T} + f_n^{L+T}) \left(\frac{\alpha_s(Q^2)}{\pi} \right)^n + \mathcal{O}\left(\frac{m_s^4}{Q^4}\right)$$

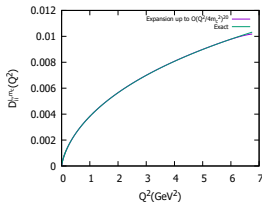
pQCD Adler function. Light quarks. Charm corrections



- Suppressed by $\sim \alpha_s^2 \frac{Q^2}{(2m_c)^2}$.
- Reconstructed from known contribution on δR_q [Phys.Lett.B 338 \(1994\) 330-335](#)

$$D_{ii}^{L,m_c}(Q^2) = Q^2 \int_{s_{th}}^{\infty} ds \frac{\delta R_q(s)}{(s + Q^2)^2}$$

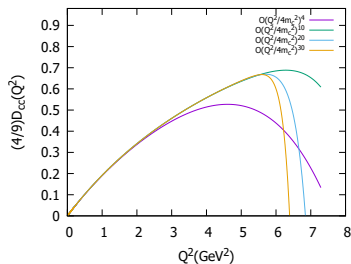
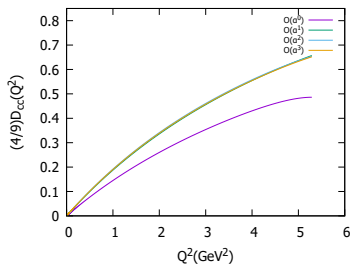
$$= N_C C_F T_F Q^2 \left(\int_0^{4m_c^2} ds \frac{\rho_V(s)}{(s + Q^2)^2} + \int_{4m_c^2}^{\infty} ds \frac{\rho_R(s) + \rho_V(s)}{(s + Q^2)^2} \right) \left(\frac{\alpha_s(\mu^2)}{\pi} \right)^2$$



pQCD Adler function. Heavy quarks. Charm corrections

$$D_{ii}(Q^2) = -\frac{9}{4} \sum_j (-1)^j j \bar{C}_j(\mu) \left(\frac{Q^2}{4m_i^2(\mu^2)} \right)^j$$

Excellent knowledge of coefficients thanks to cumulative efforts of many works
($\sim \mathcal{L}_{\text{EFT}}^{D=60}$ up to ~ 4 loops!)

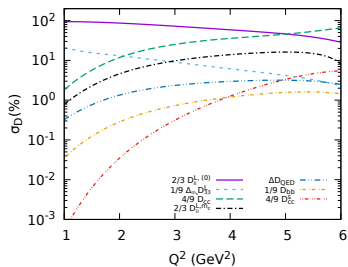
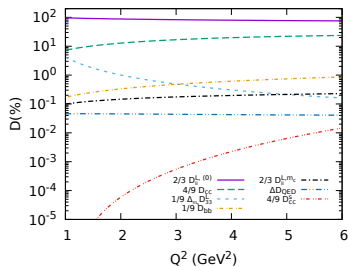
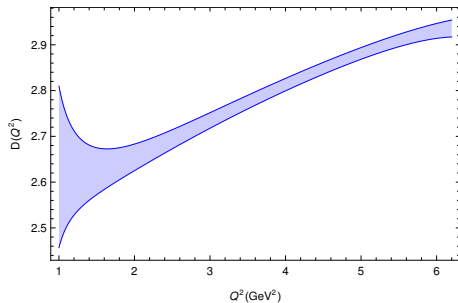


pQCD Adler function. Results

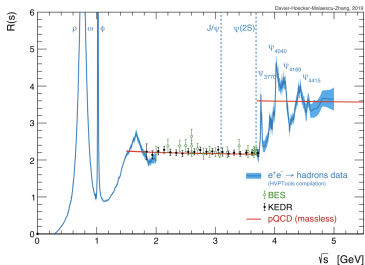
$$D(Q^2) = \sum_{i,j} Q_i Q_j D_{ij}(Q^2) = \frac{2}{3} D_{ii}^{L,(0)}(Q^2) + \frac{1}{9} \Delta_{m_s} D_{33}^L(Q^2) + \frac{2}{3} D_{ii}^{L,m_c}(Q^2) + \frac{4}{9} D_{cc}(Q^2) + \frac{4}{9} D_{cc}^s(Q^2) + \frac{1}{9} D_{bb}(Q^2) + \Delta D_{\text{QED}}(Q^2)$$

$\alpha_s^{(n_F=5)}(M_Z^2)$	Q^2	$\frac{2}{3} D_{ii}^{L,(0)}$	$\frac{1}{9} \Delta_{m_s} D_{33}^L$	$\frac{2}{3} D_{ii}^{L,m_c}$	$\frac{4}{9} D_{cc}$	$\frac{4}{9} D_{cc}^s$	$\frac{1}{9} D_{bb}$	ΔD_{QED}	D
0.115	3	2.2395(77)	-0.0123(12)	0.0039(10)	0.4484(21)(24)	0.0000(00)	0.0130(01)	0.0012(04)	2.694(09)
	4	2.2175(52)	-0.0080(07)	0.0045(12)	0.5435(24)(26)	0.0000(01)	0.0171(02)	0.0012(04)	2.776(07)
	5	2.2033(39)	-0.0058(04)	0.0050(13)	0.6197(31)(26)	0.0001(03)	0.0212(02)	0.0012(04)	2.845(06)
0.120	3	2.2866(156)	-0.0141(17)	0.0053(21)	0.4649(47)(24)	0.0000(01)	0.0132(01)	0.0012(04)	2.757(17)
	4	2.2542(98)	-0.0089(09)	0.0062(24)	0.5629(53)(26)	0.0001(02)	0.0174(02)	0.0012(04)	2.833(12)
	5	2.2343(70)	-0.0063(06)	0.0069(26)	0.6429(65)(27)	0.0002(05)	0.0215(02)	0.0012(04)	2.900(11)
0.1184(8)	3	2.2699(124)	-0.0134(15)	0.0048(17)	0.4591(36)(24)	0.0000(01)	0.0131(01)	0.0012(04)	2.735(17)
	4	2.2414(79)	-0.0086(08)	0.0055(19)	0.5561(41)(26)	0.0001(02)	0.0173(02)	0.0012(04)	2.813(14)
	5	2.2236(58)	-0.0061(05)	0.0062(20)	0.6348(51)(27)	0.0002(04)	0.0214(02)	0.0012(04)	2.881(13)

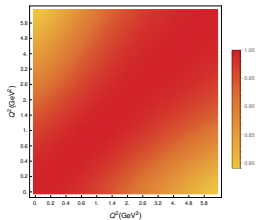
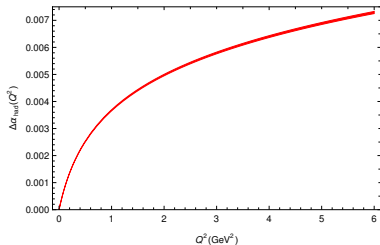
pQCD Adler function. Results



Experimental input for the EM Adler function

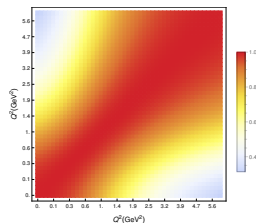
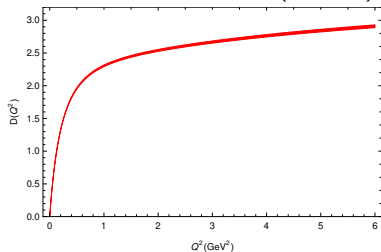


$$\Delta\alpha_{\text{had}}(Q^2) = \frac{\alpha Q^2}{3\pi} \int_{s_{\text{th}}}^{\infty} ds \frac{R(s)}{s(s+Q^2)}$$

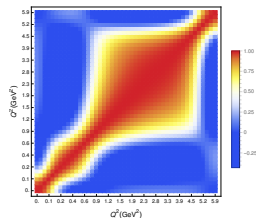
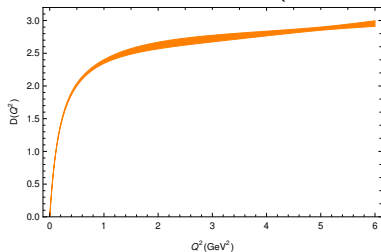


Alternative evaluations of the EM Adler function

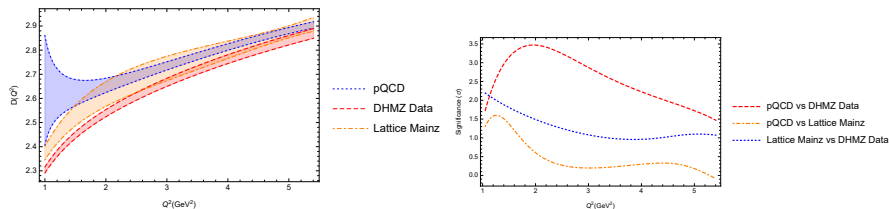
- R-ratio based evaluation (DHMZ)



- Lattice-based evaluation (From rational approximation in JHEP 08 (2022) 220)



Comparison



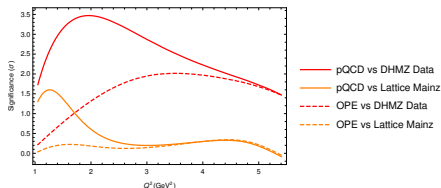
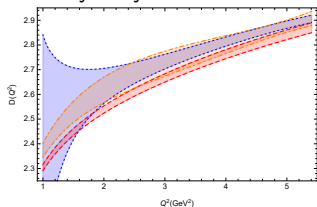
- Lattice curve above e^-e^+ data one, but with small significance.
- pQCD curve in good agreement with lattice but in tension with e^+e^- data

EM Adler function: estimates of power corrections

- Non-perturbative corrections scaling as $\Lambda_{\text{QCD}}^4/Q^4$. Existence well established beyond pQCD
- Hard to separate from pQCD series (partially in pQCD uncertainties)
- Estimate based on limited knowledge

$$\delta D_{\text{em}}^{L,D=4} \approx \frac{(0.10 \pm 0.18) \text{ GeV}^4}{Q^4} + \frac{-(0.36 \pm 0.36) \text{ GeV}^6}{Q^6}$$

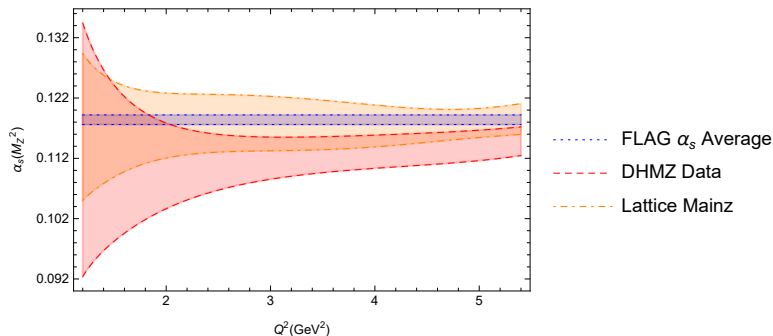
- They only become subleading source of uncertainties from $Q \gtrsim 2 \text{ GeV}$ s



- It may explain low-energy tension, but harder at high energies

EM Adler function: sensitivity to α_s

$$D^{\text{OPE}}(Q^2, \alpha_s(Q^2)) - D^{\text{data}}(Q^2) = 0 \rightarrow \alpha_s(Q^2)$$



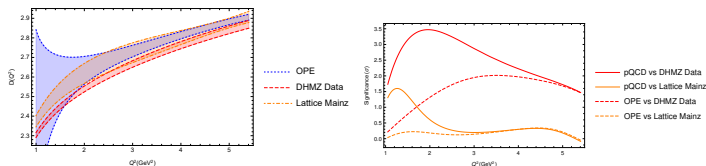
Different fitting strategies analysed (avoid small Q^2 , combine strongly correlated points) Fit to (pre-CMD-3) DHMZ compilation: $\alpha_s^{(n_f=5)}(M_Z^2) = 0.1136 \pm 0.0025$

Larger central value if lattice is used, $\alpha_s^{(n_f=5)}(M_Z^2) = 0.1179 \pm 0.0025$

RGE could be tested, but more data at large s is needed

Conclusions

- Comprehensive study of the perturbative EM Adler function at $\mathcal{O}(2 \text{ GeV})$
- Implementation within $\overline{\text{MS}}$ overcomes full-scheme limitations
- Comparison with e^+e^- and lattice data provides interesting insights



- Recent progress in understanding issues with inconsistent e^+e^- data sets
- Potential $\sim 1\%$ level $\alpha_s(M_Z)$ from data-based Euclidean Adler function
- Very important interplay to extract precise value of $\alpha_{\text{EM}}(M_Z)$