FLAG α_s Working Group

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FLAG
Flavour Lattice Averaging Group

prepared for

alphas-24: Workshop on the strong coupling constant

ECT* Trento, 5 February 2024

Outline

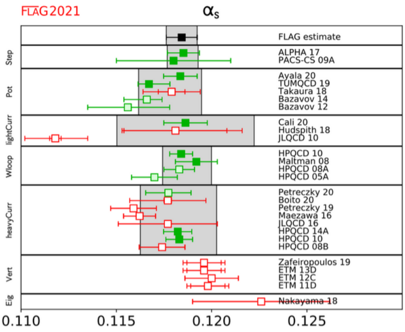
- FLAG and its policies
- Reminders on determinations of $\alpha_s(m_Z)$
- Parametric uncertainties of Λ-parameter, perturbative behaviour and FLAG criteria
- New results & developments since FLAG 21
- α_s from the decoupling strategy
- Situation for $N_\mathrm{f} = 0$, some selected results
- Conclusions & points under consideration for FLAG 2024

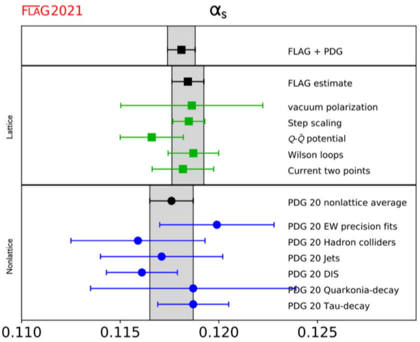
FLAG = FLavour Lattice Averaging Group

- FLAG is an effort by the international lattice QCD community to provide the wider high energy physics community with lattice results for quantities of phenomenological interest, satisfying clearly defined quality criteria
- Original focus was on flavour physics, but now FLAG includes also sections on α_s, nucleon matrix elements and scale setting.
- FLAG website: `http://flag.unibe.ch`
- Besides the quality criteria FLAG requires acceptance by or publication in a peer reviewed journal.
- **Cutoff date for FLAG 2024: 30 April 2024**
- Collaborations may be contacted by us for comments/clarifications/feedback
- Timeline: draft FLAG report is due in early June, review over the summer, aim for publication in October 2024
- Time between successive reports 2-3 years, intermediate web updates by individual WG's are possible.

Nota Bene: FLAG requires that anyone using FLAG results cites the original references which enter the averages. A bibtex entry containing these references can be obtained from the FLAG website (go to the relevant plot and click on bib link next to it)

FLAG 2021 plots





Some reminders on determinations of $\alpha_s(m_Z)$

- FLAG 21 average: $\alpha_s(m_Z) = 0.1184(8)$, the uncertainty is 0.7%
- All but one determinations: $N_\mathrm{f} = 2+1$, combined with 4-loop matching across charm and bottom thresholds
- A 1% error on α_s requires $\Delta\Lambda_{\overline{\mathrm{MS}}}^{(N_\mathrm{f}=3)} < 5\%$

$\Rightarrow$ isospin breaking due to electromagnetism + mass differences is not yet relevant for α_s

Majority of determinations affected predominatly by systematics, in particular:

- Perturbative truncation errors: requires $\mu \gg \Lambda_{\overline{\mathrm{MS}}}$
- continuum limit: requires, $\mu \ll 1/a$

Note: given the very good quantitative perturbative description of decoupling across charm and bottom threshold, the determination of α_s is equivalent to a non-perturbative result for the Λ-parameter with $N_\mathrm{f} = 3, 4$

Reminder: parametric uncertainty of Λ-parameter & perturbative criteria

Λ-parameter in mass-independent renormalization scheme:

$$\Lambda_{\overline{\rm MS}} = \mu\varphi\left(\bar{g}(\mu)\right)$$

$$\varphi\left(\bar{g}\right) = \left[b_0\bar{g}^2\right]^{-\frac{b_1}{2b_0^2}} \mathrm{e}^{-\frac{1}{2b_0\bar{g}^2}} \exp\Big\{\underbrace{-\int_0^{\bar{g}} dg\left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]}_{=I[\bar{g};\beta]}\Big\}$$

A non-perturbatively defined coupling $\bar{g}^2(\mu)$ implies a non-perturbative β-function:

$$\beta(\bar{g}) \stackrel{\rm def}{=} \mu\frac{\partial\bar{g}(\mu)}{\partial\mu}, \qquad \beta(g) = -b_0 g^3 - b_1 g^5 + \ldots$$

with universal 1- and 2-loop coefficients b_0, b_1:

$$b_0 = (11 - \tfrac{2}{3}N_{\rm f})/(4\pi)^2, \qquad b_1 = (102 - \tfrac{38}{3}N_{\rm f})/(4\pi)^4\,.$$

At large μ, use perturbative knowledge $\leq n_l + 1$-loops for $\beta \to \beta^{(n_l+1)}$ $(\alpha = g^2/(4\pi))$

$$I[g;\beta] \quad \stackrel{g\to 0}{\simeq} \quad I[g,\beta^{(n_l+1)}] + O\left(g^{2n_l}\right)$$

$\Rightarrow$ For large μ expect remaining μ-dependence $\Lambda_{\overline{\rm MS}}^{\rm estimated}/\Lambda_{\overline{\rm MS}} = 1 + O\left(\alpha^{n_l}(\mu)\right)$

Reminder: definition of effective coupling, α_{eff}

Starting point for all α_s determinations: Euclidean short distance quantity Q, that

- can be measured in a lattice simulation
- has a perturbative expansion, $Q = c_0 + c_1\alpha + c_2\alpha^2 + \ldots$.

We associate an effective coupling to Q, by normalizing

$$\alpha_{\text{eff}} = (Q - c_0)/c_1$$

- Advantage: no need to refer to a particular scale, α_{eff} is measured, possibly after chiral and continuum extrapolations (exception: couplings at $\mu = 1/a$, e.g. from small Wilson loops).
- Loop counting: Relate to the $\overline{\text{MS}}$ scheme:

$$\alpha_{\text{eff}} = \alpha_{\overline{\text{MS}}} + d_1\alpha_{\overline{\text{MS}}}^2 + d_2\alpha_{\overline{\text{MS}}}^3 + d_3\alpha_{\overline{\text{MS}}}^4 + \ldots$$

  If d_k are known up to $k = n_l$ the loop order is n_l. Currently best cases have $n_l = 3$ (plus partial information on $n_l = 4$ for static potential/force)

Reminder FLAG 19/21 criteria

Renormalization scale

- ★ all points in the analysis have $\alpha_{\text{eff}} < 0.2$
- ○ all points have $\alpha_{\text{eff}} < 0.4$ and at least 1 with $\alpha_{\text{eff}} < 0.25$
- ■ otherwise

Perturbative behaviour

- ★ verified over a range of a factor 4 change in $\alpha_{\text{eff}}^{n_l}$ without power corrections or alternatively $\alpha_{\text{eff}}^{n_l} < \frac{1}{2}\Delta\alpha_{\text{eff}}/(8\pi b_0 \alpha_{\text{eff}}^2)$ is reached.
- ○ verified over a range of a factor $(3/2)^2$ change in $\alpha_{\text{eff}}^{n_l}$ possibly fitting with power corrections or alternatively $\alpha_{\text{eff}}^{n_l} < \Delta\alpha_{\text{eff}}/(8\pi b_0 \alpha_{\text{eff}}^2)$ is reached.
- ■ otherwise

Continuum limit: at a reference point of $\alpha_{\text{eff}} = 0.3$ (or less) require

- ★ three lattice spacings with $\mu a < 1/2$ and full O(a) improvement, or three lattice spacings with $\mu a \leq 1/4$ and 2-loop O(a) improvement, or $\mu a \leq 1/8$ and 1-loop O(a) improvement
- ○ three lattice spacings with $\mu a < 3/2$ reaching down to $\mu a = 1$ and full O(a) improvement, or three lattice spacings with $\mu a \leq 1/4$ and 1-loop O(a) improvement
- ■ otherwise

plus convention for μ in different quantities (e.g. $\mu = q$ in momentum space observables, or $\mu = 1/L$ for step-scaling)

News since FLAG 21

$N_\mathrm{f} = 4$: TUMQCD, no publication yet (?), cf. talk by V. Leino at alphas-24
$N_\mathrm{f} = 3$:

- Petreczky and Weber '22: update on moments of heavy quark 2-point functions: $\Lambda^{(3)}_{\overline{\mathrm{MS}}} = 332(17)(2)_{\mathsf{scale}}$ MeV (published) (cf. talk by P. Petreczky)
- ALPHA '22 Decoupling method, relates $N_\mathrm{f} = 3$ to $N_\mathrm{f} = 0$ (cf. talk by A. Ramos)
- further lattice talks at alphas-24 by C. Ayala and Hai-Tao Shu (no publications?)

$N_\mathrm{f} = 0$

- Bribian et al. (2021, published): step-scaling for gradient flow coupling with twisted periodic boundary conditions
- Hasenfratz et al. (2023, published) and Wong et al (2023, Lattice '22 proceedings): use flow time as renormalization scale (infinite volume), 2-loop conversion to $\overline{\mathrm{MS}}$ by Harlander & Neumann (2016)
- Chimirri (2022, Lattice '22 proceedings) study of moments of heavy quark currents (cf. talk by R. Sommer)
- TUMQCD (December '23, preprint): static force with insertion of chromo-electric field (cf. talk by J. Steudte) and gradient flow time as intermediate regulator.

Please notify the FLAG α_s working group of any work relevant for the FLAG report!

Notification:

- Email one of us directly (Stefan Sint, Luigi Del Debbio, Peter Petreczky)
- And/Or: Use the submission form on the FLAG web page at the university of Berne: `http://flag.unibe.ch`

α_s from the decoupling strategy

[ALPHA 2019-2022]:

- Decoupling well described by PT (available up to 4 loops) $\Rightarrow$ use it as a tool!
- Connection between QCD Λ-parameters for $N_\mathrm{f}=3$ and $N_\mathrm{f}=0$, by decoupling triplet of heavy mass degenerate quarks.

$$\bar{g}_s^{(3)}(\mu/\Lambda_s^{(3)}, M) = \bar{g}_s^{(0)}(\mu/\Lambda^{(0)}) + \mathrm{O}(\mu^2/M^2)\,, \qquad (1)$$

- in PT this leads to

$$[\bar{g}^{(0)}_{\overline{\mathrm{MS}}}(\mu)]^2 = C\left(\bar{g}^{(3)}_{\overline{\mathrm{MS}}}(m_\star)\right)[\bar{g}^{(3)}_{\overline{\mathrm{MS}}}(m_\star)]^2, \qquad m_\star = \overline{m}_{\overline{\mathrm{MS}}}(m_\star),$$

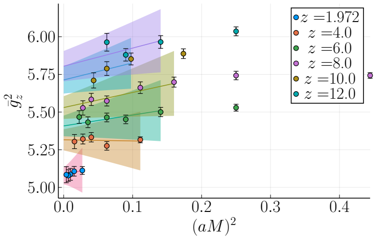
and for $\mu = m_\star$ one finds $C(x) = 1 + c_2 x^4 + c_3 x^6 + c_4 x^8 + \ldots$.

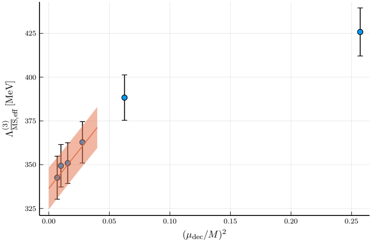
- Reformulation with $P = \varphi^{(0)}_{\overline{\mathrm{MS}}}\left(g_\star\sqrt{C(g_\star)}\right)/\varphi^{(3)}_{\overline{\mathrm{MS}}}(g_\star)$, $g_\star = g^{(3)}_{\overline{\mathrm{MS}}}(m_\star)$:

$$\frac{\Lambda^{(3)}_{\overline{\mathrm{MS}}}}{\mu_\mathrm{dec}} = \frac{\Lambda^{(0)}_{\overline{\mathrm{MS}}}}{\Lambda^{(0)}_s} \times \lim_{M/\mu_\mathrm{dec}\to\infty}\left[\frac{\varphi_s^{(0)}\left(\bar{g}_s^{(3)}(\mu_\mathrm{dec}, M)\right)}{P\left(\frac{M}{\mu_\mathrm{dec}}\Big/\frac{\Lambda^{(3)}_{\overline{\mathrm{MS}}}}{\mu_\mathrm{dec}}\right)}\right]$$

- Corrections $\mathrm{O}(1/M^2)$ and $\mathrm{O}(\alpha^4(\mu=M))$, requires extrapolation

Continuum and large mass extrapolations





- Data for $z = M/\mu_{\rm dec} \in \{1.972, 4, 6, 8, 10, 12\}$, extrapolated to $a = 0$ using global fits with 2 cuts in $(aM)^2 < 0.16, 0.25$; fixed $\hat{\Gamma} \in [-1, 1]$ and $\hat{\Gamma}' \in [-1/9, 1]$.

$$\bar{g}^2(z_i, a) = C_i + p_1[\alpha_{\overline{\rm MS}}(a^{-1})]^{\hat{\Gamma}}(a\mu_{\rm dec})^2 + p_2[\alpha_{\overline{\rm MS}}(a^{-1})]^{\hat{\Gamma}'}(aM_i)^2 \,.$$

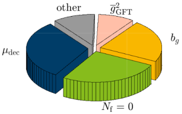
- $1/z^2$ extrapolation: solve equation for target ρ,

$$\rho \times \underbrace{P(z/\rho)}_{\text{PT} + \mathrm{O}\left(\alpha^4_{\overline{\rm MS}}(m_\star)\right)} = \frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\mu_{\rm dec}}, \qquad \rho = \frac{\Lambda^{(3)}_{\overline{\rm MS},\rm eff}}{\mu_{\rm dec}} = \frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{\rm dec}} + \mathrm{O}(1/z^2)$$

Combination with ALPHA 17 and prospects of further error reduction

$$\Lambda^{(3)}_{\overline{\rm MS}} = 336(10)(6)_{b_{\rm g}}(3)_{\hat{\Gamma}_m}\,{\rm MeV} = 336(12){\rm MeV} \quad \Rightarrow \quad \alpha_s(m_Z) = 0.1182(8)$$

- Total error is of the same size as in ALPHA '17 ($341(12)$MeV, $\alpha_s(m_Z) = 0.1185(8)$)



contributions to error^2

$\Rightarrow$ common (squared) error with ALPHA '17 (due the common scale setting) only 28%!

- Combine published results ALPHA 17 and ALPHA 22:

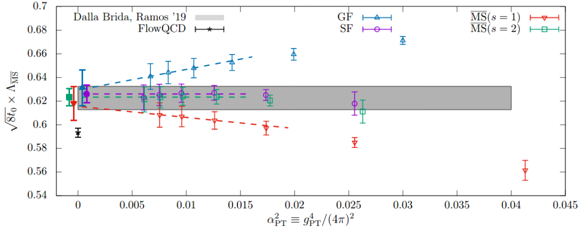
$$\Lambda^{(3)}_{\overline{\rm MS}} = 339.5(9.6) \qquad \Rightarrow \qquad \alpha_s(m_Z) = 0.1184(7)$$

- Clear path to further error reduction:
  - Improved determination of $\Lambda^{(0)}_{\overline{\rm MS}}/\mu_{\rm dec}$
  - Improved physical scale setting for $\mu_{\rm dec}$ from CLS ensembles
  - Non-perturbative determination of $b_{\rm g}$

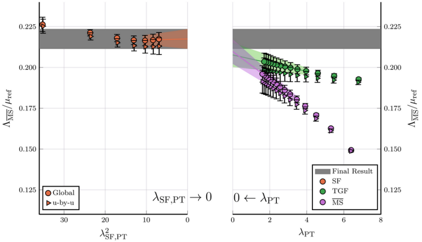
$\Rightarrow$ $b_{\rm g}$ now determined non-perturbatively (ALPHA '23); analysis is under way!

Situation for $N_f = 0$

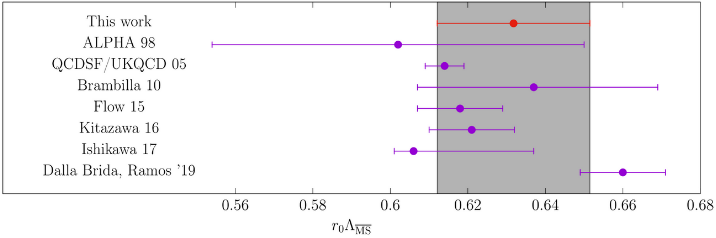
- Dalla Brida and Ramos '19, Nada and Ramos '21
  $\sqrt{8t_0}\Lambda^{(0)}_{\overline{\rm MS}} = 0.6227(98) \quad \Leftarrow \quad$ enters the ALPHA decoupling result



- Bribian, Dasilva, Garcia-Perez,Ramos '21: twisted b.c.'s, $\sqrt{8t_0}\Lambda^{(0)}_{\overline{\rm MS}} = 0.603(17)$



Situation for $N_\mathrm{f} = 0$



- Direct extraction of $\Lambda_{\overline{\mathrm{MS}}}$ difficult (only universal β-function), easier to pass via SF scheme.
- New results by Hasenfratz et al. (2023) and Wong et al (2023) using GF scheme in infinite volume

$\Rightarrow$ contact to PT seems to require very large scales!

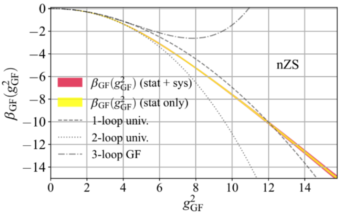
$\rightarrow$ requires infinite volume extrapolation, theory expectation?

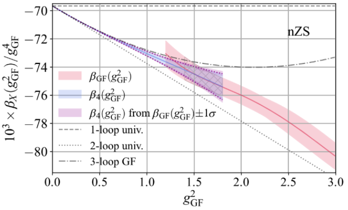
- Chimirri (2022) study of moments of heavy quark currents
- Need to re-evaluate method with very precise values by QCDSF-UKQCD-05 and Kitazawa 16 (s. below)

Hasenfratz et al. (2023, published) and Wong et al. (proceedings)

- renormalized coupling in infinite volume gradient flow scheme
- β-function known to 3-loops (Harlander and Neumann 2016)
- β-function from differentiating w.r.t. flow time
- Quote final result $\sqrt{8t_0}\Lambda^{(0)}_{\overline{\rm MS}} = 0.622(10)$ (perfectly agrees with Dalla Brida 19).

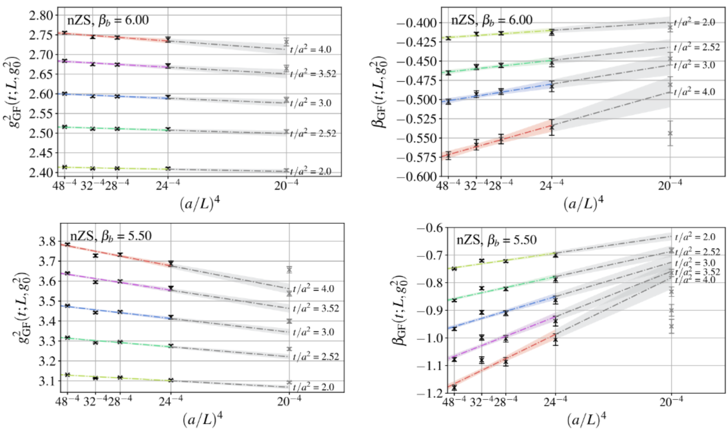
HOWEVER: Perturbative behaviour:





Hasenfratz et al. (2023, published) and Wong et al. (proceedings)

Infinite volume extrapolations:

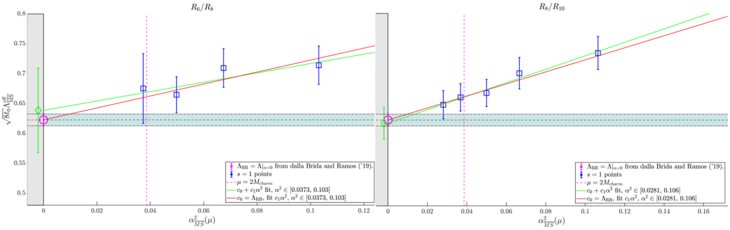


Chimirri, and Chimirri et al. (2 proceedings contributions for Lattice '22)

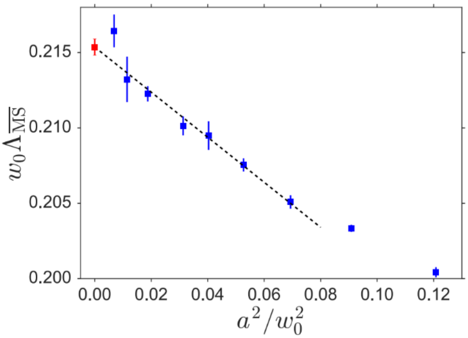
Moments of heavy quark 2-point functions:

- very large and fine lattices, down to $a = 0,01\,\mathrm{fm}$.
- Still there is a sizeable parametric uncertainty $\propto \alpha^2$.

$\Rightarrow$ Lessons for $N_\mathrm{f} = 3$?



A closer look at Kitazawa 16

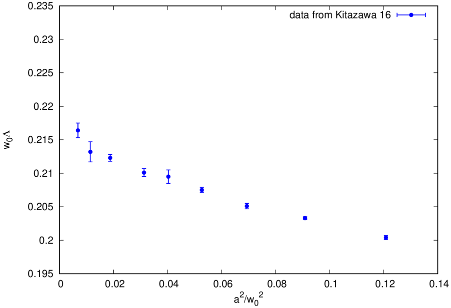


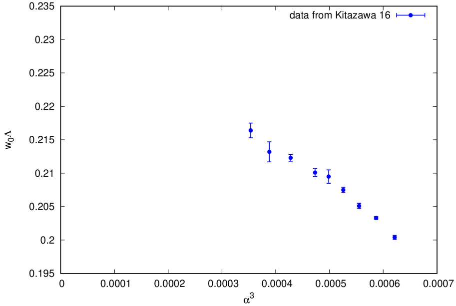
- Follow QCDSF-UKQCD-05: use boosted bare coupling $g_\square^2 = g_0^2/P^{1/4}$.
- Construct $a\Lambda_\square$ and multiply by w_0/a, then convert to $\overline{\rm MS}$ scheme.
- Extrapolate in a^2/w_0^2; very small error (plus 1% due to topology freezing):

$$w_0\Lambda^{(0)}_{\overline{\rm MS}} = 0.2154(5)_{\rm stat}(11)(22) = 0.2154(33) \qquad \text{linear error combination}$$

$\Rightarrow$ BUT: expect also α^3 parametric uncertainty!

A closer look at Kitazawa 16





Conclusions & points under consideration for FLAG 2024 (cutoff: publication by 30 April 2024)

- Lattice QCD completely by-passes problems with quark confinement (hadronisation, quark-hadron duality etc.)
- But α_s poses a hard problem for lattice QCD due to large scale difference between perturbative and hadronic regimes!
- Complete solution exists in terms of step-scaling procedure; requires dedicated effort and resources;

$\Rightarrow$ finite volume essential, most high order QCD PT results cannot be used!

- Most calculations avoid step-scaling: trade higher PT order for lower energy scale:

$\Rightarrow$ systematics from truncation of perturbative series and non-perturbative effects; can we improve control of such effects (cf. talk by A. Kronfeld)?

- Other possible compromise: extrapolation to infinite volume, systematics at high/intermediate/low energy scales?
- Significant progress in precision for $\alpha_s(m_Z)$, mostly driven by decoupling method (cf. talk by A. Ramos)

Conclusions & points under consideration for FLAG 2024 (cutoff: publication by 30 April 2024)

- Decoupling method means $N_\mathrm{f} = 0$ results are physically relevant; uptake in community effort since FLAG 2021 report
- New methods come with new systematics: extrapolations to
  - decoupling limit $M \to \infty$
  - infinite volume limit (Hasenfratz et al, Wong et al.)
  - $\alpha \to 0$ for parametric uncertainty in Λ-parameter desirable ($N_\mathrm{f} = 0$); requires a wide range for α^{n_l}!
  - zero flow time limit (TUMQCD 2023, cf. talk by J. Mayer-Steudte)

$\Rightarrow$ use case-by-case assessments, possibly data driven (i.e. error in relation to distance covered by extrapolation)

- Reference scales: switch from r_0 to $\sqrt{8t_0}$: Need ratios of r_0, r_1, w_0 (also for $N_\mathrm{f} = 0$!)
- Incorporate scale variation (cf. review by Del Debbio & Ramos) as optional additional measure for perturbative truncation errors.
- Set-up of FLAG criteria seems adequate in structure: in the future, numerical limits on α_eff, $a\mu$ could be tightened, but not yet for FLAG 2024