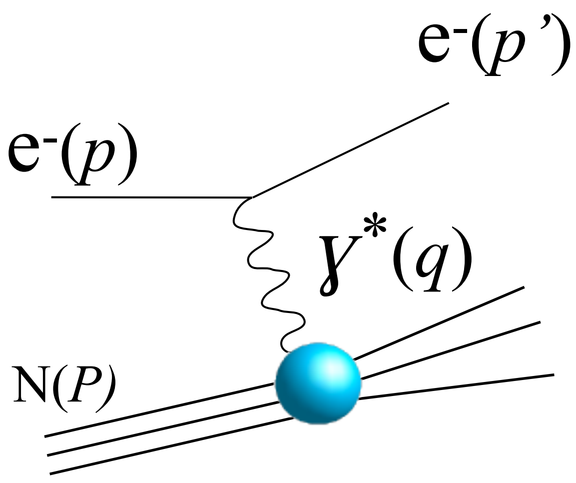
EIC and JLab perspectives on measurements of α_s from spin structure functions

A. Deur
Jefferson Lab

Part of this work done with: T. Kutz (MIT), J. R. Pybus (MIT), D. W. Upton (UVA, ODU), C. Cotton (UVa), A. Deshpande (CFNS, Stony Brook U.), W.B. Li (CFNS, Stony Brook U.), D. Nguyen (JLab, UTK), M. Nycz (UVa), X. Zheng (UVa), and the former ECCE Consortium (now part of the ePIC Collaboration).

Inclusive lepton-nucleon scattering



- $p=(E,\mathbf{p})$, $p'=(E-\nu,\mathbf{p}-\mathbf{q})$, $q=(\nu,\mathbf{q})$
- γ^* virtual photons: q^2
- Since $q^2 < 0$ here, we use $Q^2 = -q^2$.
- Inclusive experiments: only the scattered electrons are detected: target or target fragments are ignored.
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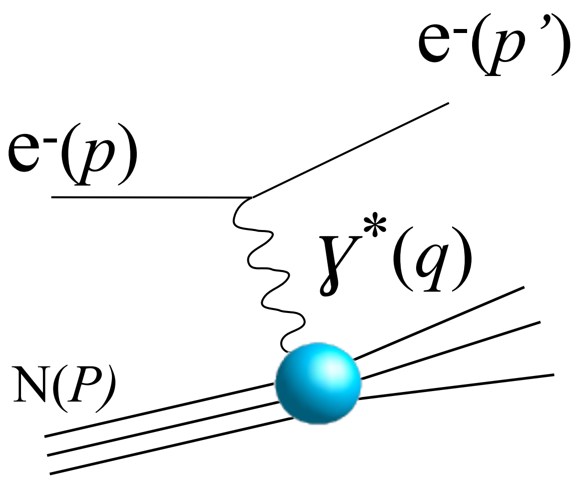

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Cross section: $\sigma = \sigma_{Mott}[\alpha F_1(x,Q^2)+\beta F_2(x,Q^2)+\gamma g_1(x,Q^2)+\varpi g_2(x,Q^2)]$

pointlike scattering×(spin independent + spin dependent)

F_1, F_2, g_1 and g_2: structure functions

F_1 and F_2, are obtained with unpolarized beam and target and varying kinematic factors α and β.
g_1 and g_2 are obtained with beam and target both polarized, measuring beam spin asymmetries and varying the target spin direction.

Considering the nucleon inclusive spin structure, α_s can be extracted from:

- Q^2-evolution of $g_1(x, Q^2)$. Complex task: involves DGLAP global fit, non-perturbative inputs: quark and gluon distributions, possibly higher-twists for low-Q^2 / large-x data.
- Q^2-evolution of moment $\int_0^1 g_1(x, Q^2)dx$. Simpler: no x-dependence, non-perturbative inputs: more-or-less well measured axial charges a_0, a_3 and a_8 (+ possibly higher-twists for low-Q^2 data). Issues: unmeasurable low-x contribution, a_0 is Q^2-dependent and may have contribution from gluon ΔG pdf (but not the case in $\overline{MS}$).
- Q^2-evolution of isovector moment $\int_0^1 g_1^{p-n}(x, Q^2)dx$, i.e Bjorken sum. Simplest. Axial charge $a_3 = g_A$ precisely measured ($g_A = 1.2762 \pm 0.0005$). DGLAP-evolution known to higher order than single nucleon case (nowadays, this is often the limitation in extracting α_s). No gluon contribution. But low-x issue and demands measurement on polarized p and n.

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Bjorken sum rule

$$\Gamma_1^{p-n} \equiv \int g_1^{p-n} dx = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58 \left(\frac{\alpha_s}{\pi}\right)^2 - 20.21 \left(\frac{\alpha_s}{\pi}\right)^3 - 175.7 \left(\frac{\alpha_s}{\pi}\right)^4 - \sim 893 \left(\frac{\alpha_s}{\pi}\right)^5 \right]$$

$$+ \frac{M^2}{Q^2} \left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s) \right] + \ldots$$

Nucleon's First spin structure function

Nucleon axial charge. (Value of $\Gamma_1^{p-n}(Q^2)$ in the $Q^2 \to \infty$ limit)

pQCD radiative corrections ($\overline{MS}$ Scheme.)

Non-perturbative $1/Q^{2n}$ power corrections. (+rad. corr.)

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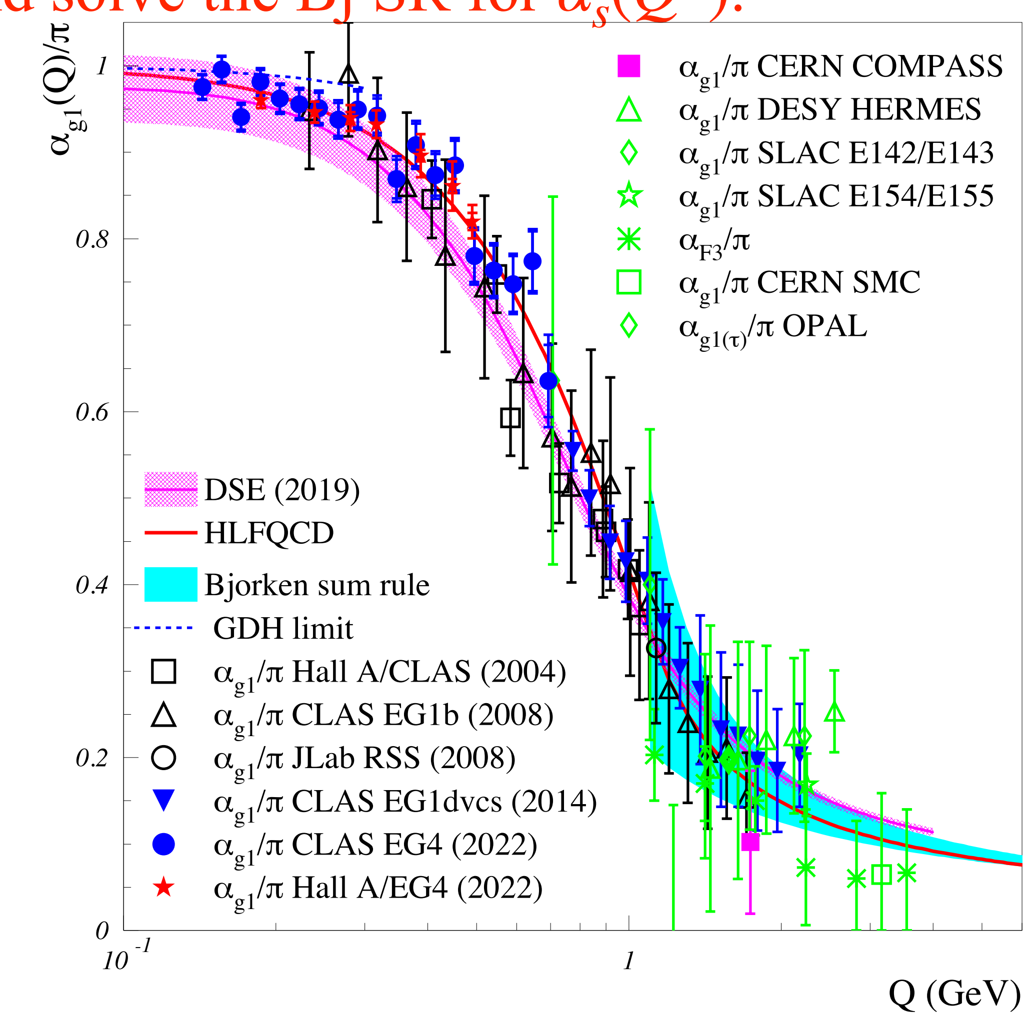

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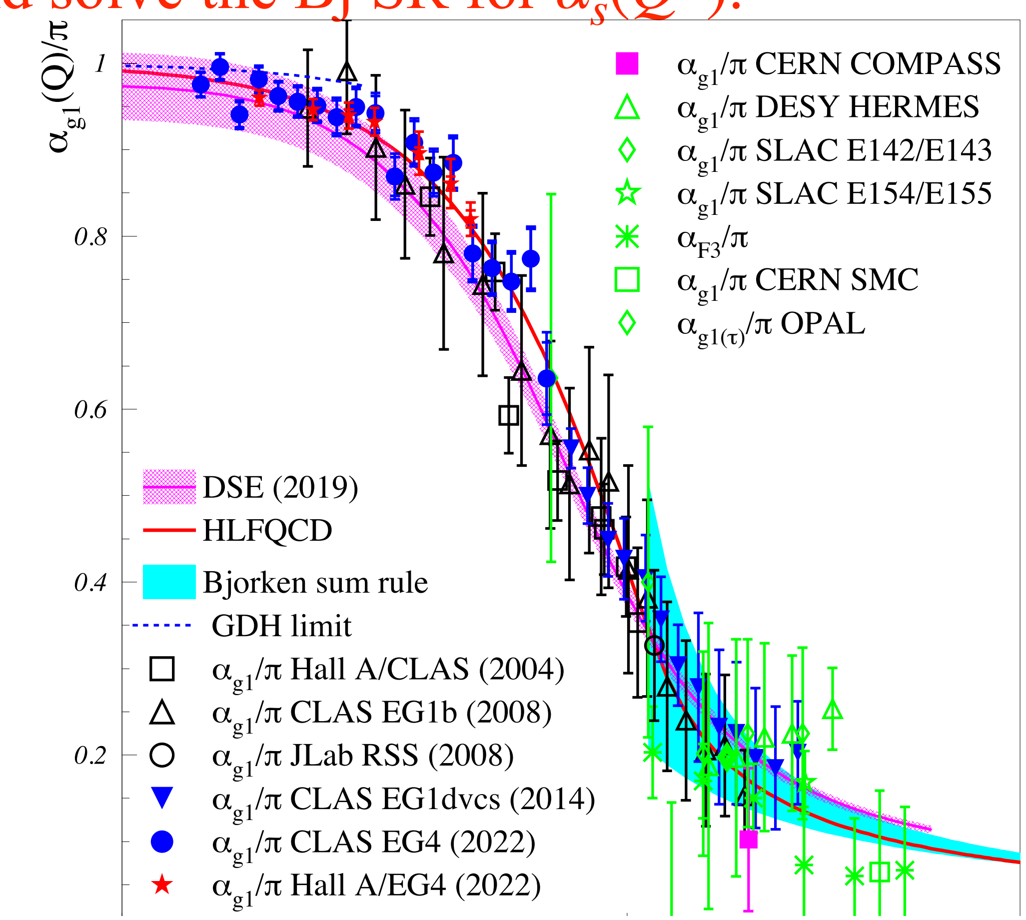

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  - Good accuracy: 1990's CERN/SLAC data yielded: $\alpha_s(M_Z)=0.120\pm0.009$

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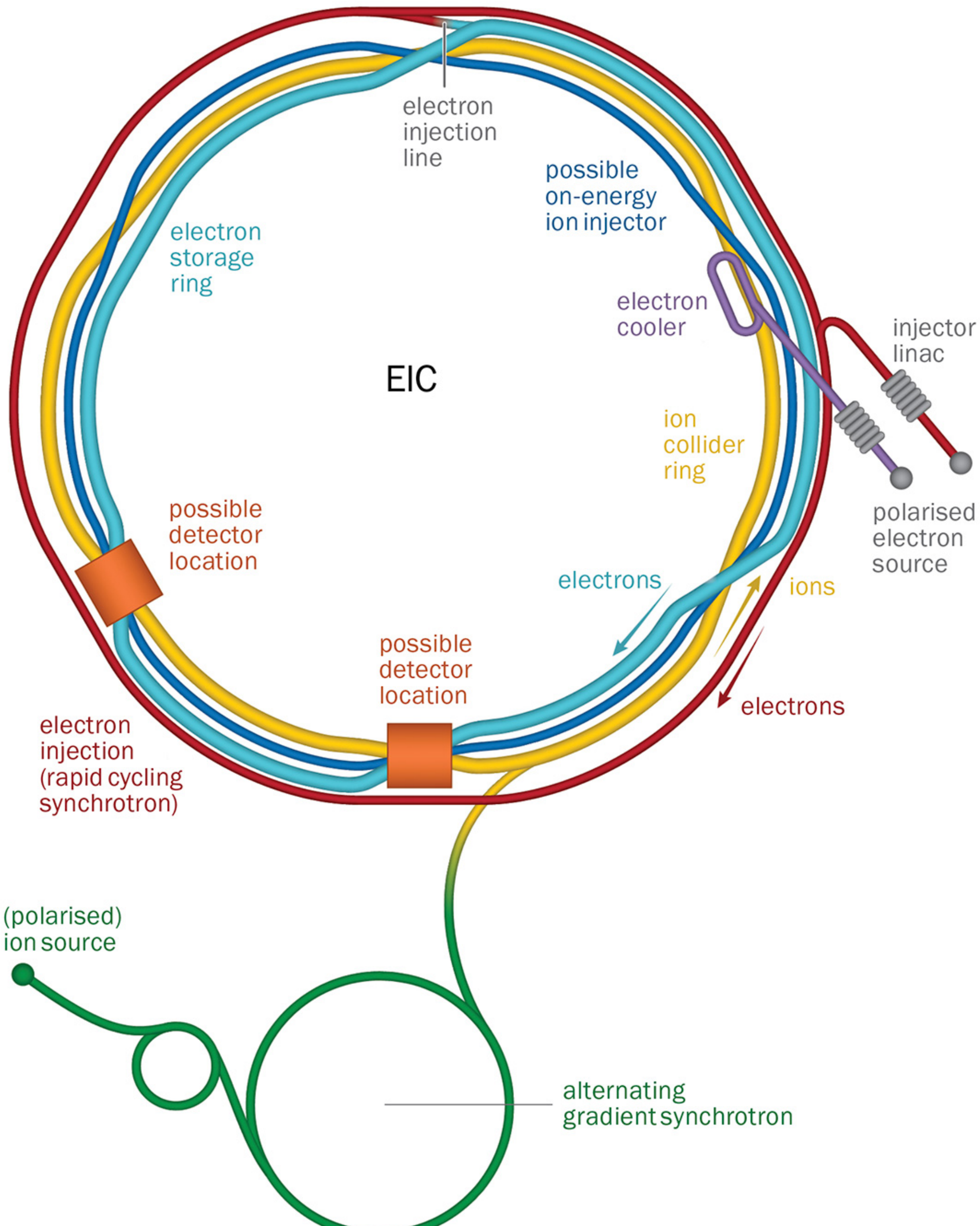
α_s from $\Gamma_1^{p-n}(Q^2)$ measured at the EIC

2030s:



EIC

Simulated data: $\vec{e}$-$\vec{p}$ and $\vec{e}$-$\overrightarrow{^3He}$ DIS events generated with DJANGOH event generator for 6 collision energies (5×41 GeV, 10×100 GeV & 18×275 GeV for p, 5×41 GeV/nucleon, 10×100 GeV/nucleon & 18×166 GeV/nucleon for ^3He) and longitudinal & transverse hadron polarizations settings.

Neutron information extracted from $\overrightarrow{^3He}$ ($\simeq \vec{n}$)

Tag two spectator protons from $\vec{e}$-$\overrightarrow{^3He}$: minimize nuclear corrections for neutron information.

Use 10 fb^{-1} luminosity (i.e., about 2×3 years of running at $\mathscr{L} = 5 \times 10^{33}\text{cm}^{-2}\text{s}^{-1}$).

Monte Carlo simulation of detector effects (resolution, efficiency, acceptance, radiative effects)

⇒Very realistic simulation

Longitudinal & transverse asymmetries, $A_{||}(x, Q^2)$, $A_{\perp}(x, Q^2)$ generated using world data parameterizations.

Then, $A_{||}$ & $A_{\perp} \rightarrow A_1 \simeq g_1/F_1 \rightarrow g_1 \rightarrow \Gamma_1$, the Bjorken sum.

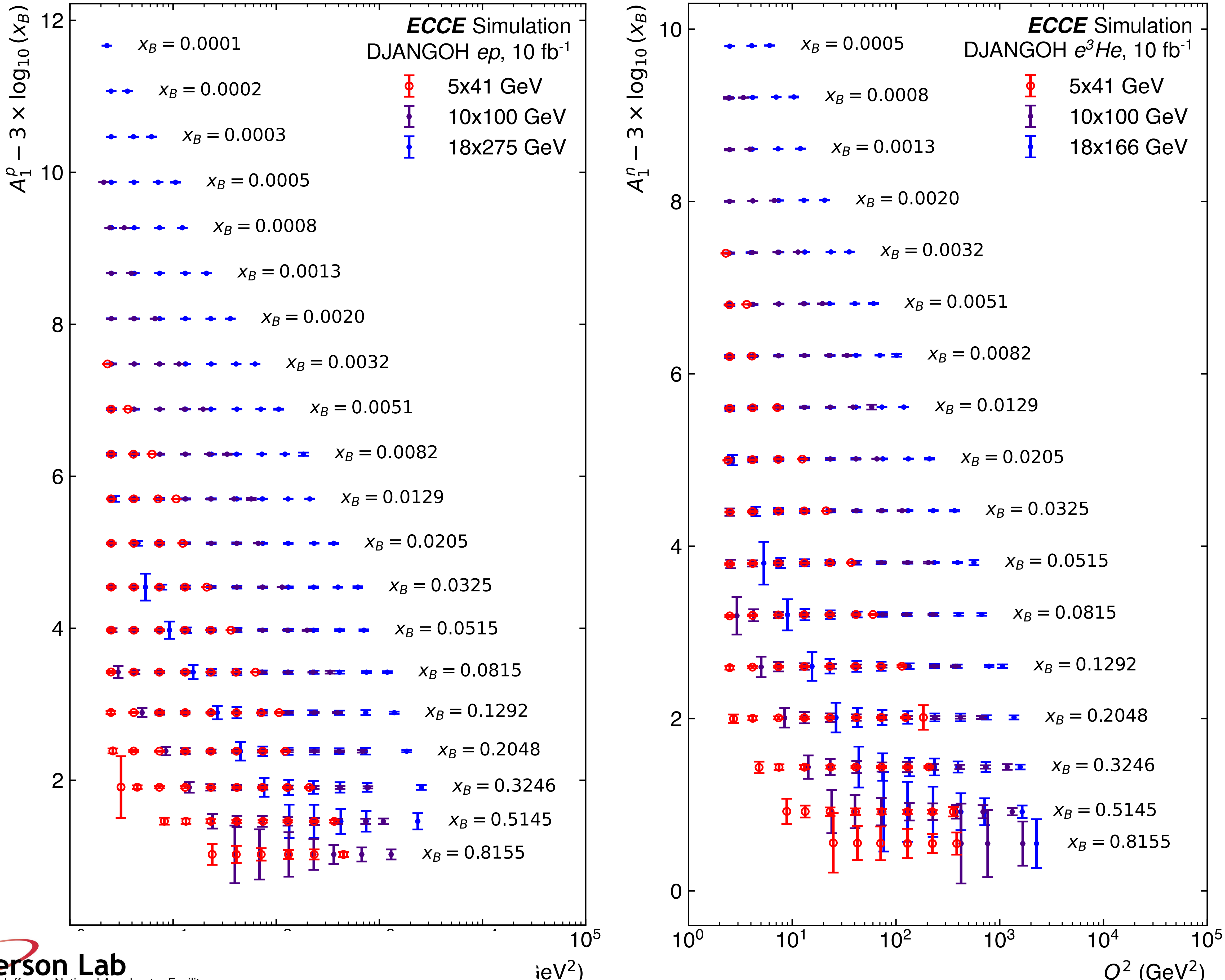
Uncertainties

Statistics;

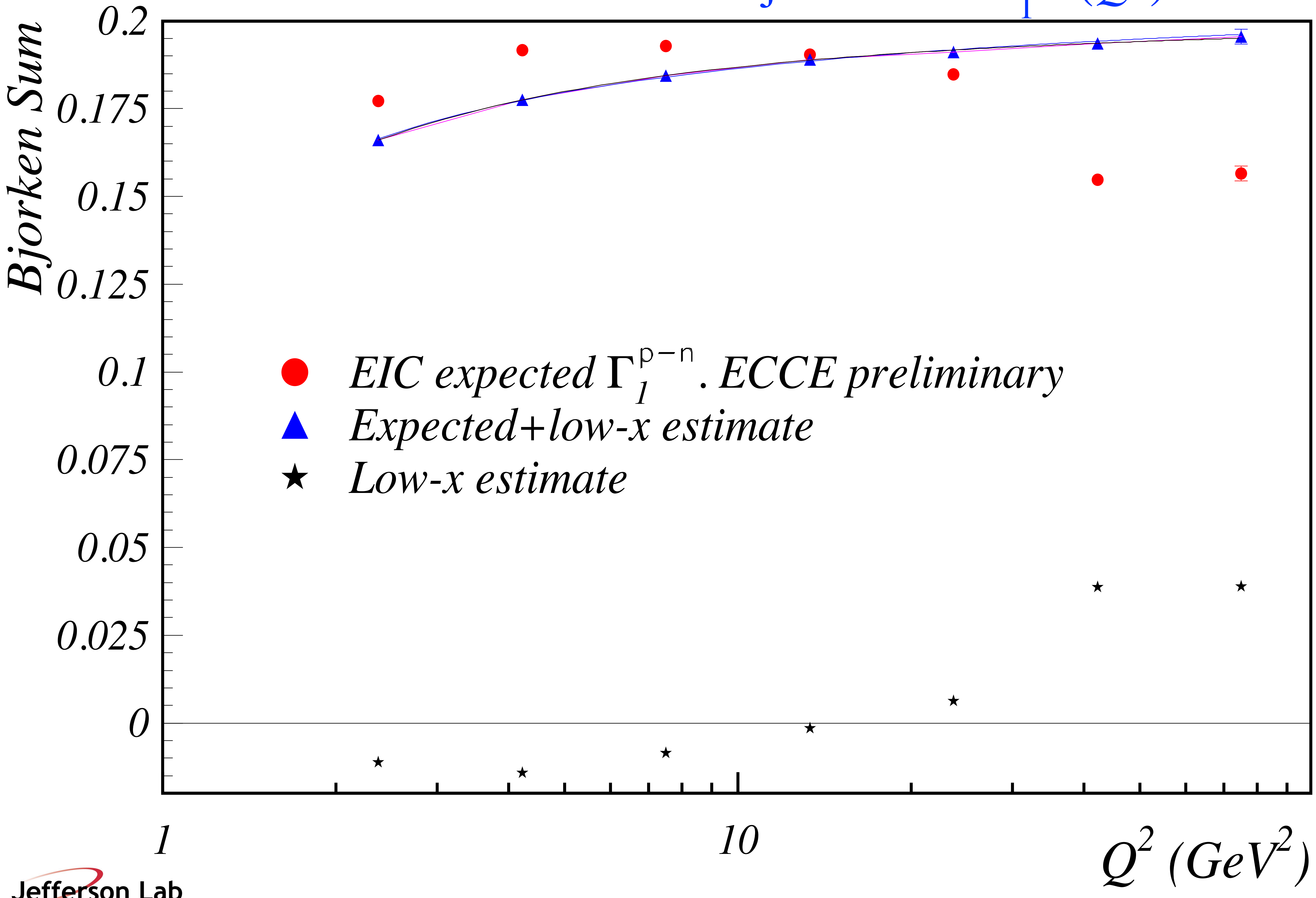
Systematics:

- detector effects,
- beam polarimetries,
- radiative corrections,
- missing high- and low-*x* part,
- PDF parameterizations;
- Negligible: neutron information extraction.

EIC: generated pseudo-data



Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



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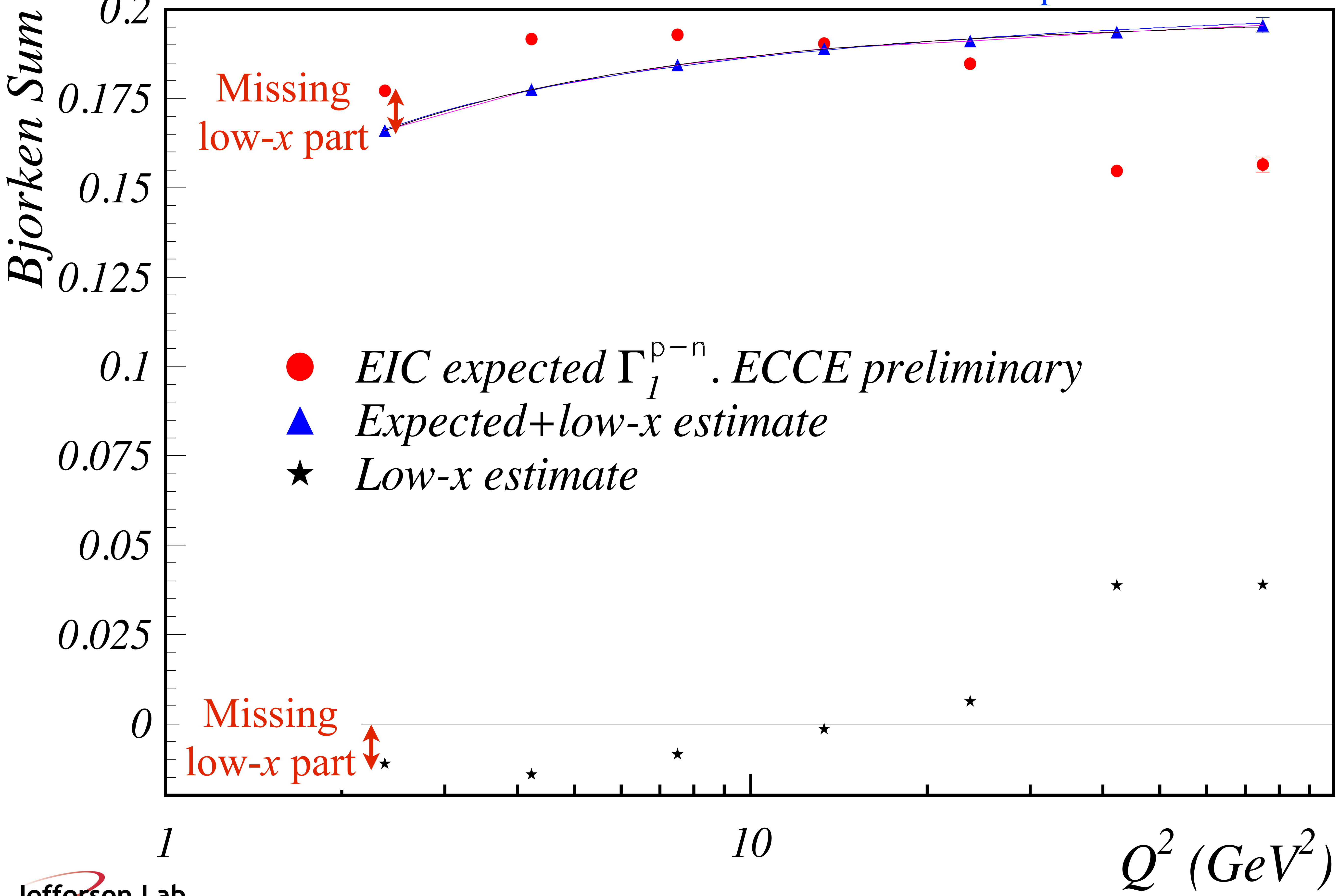

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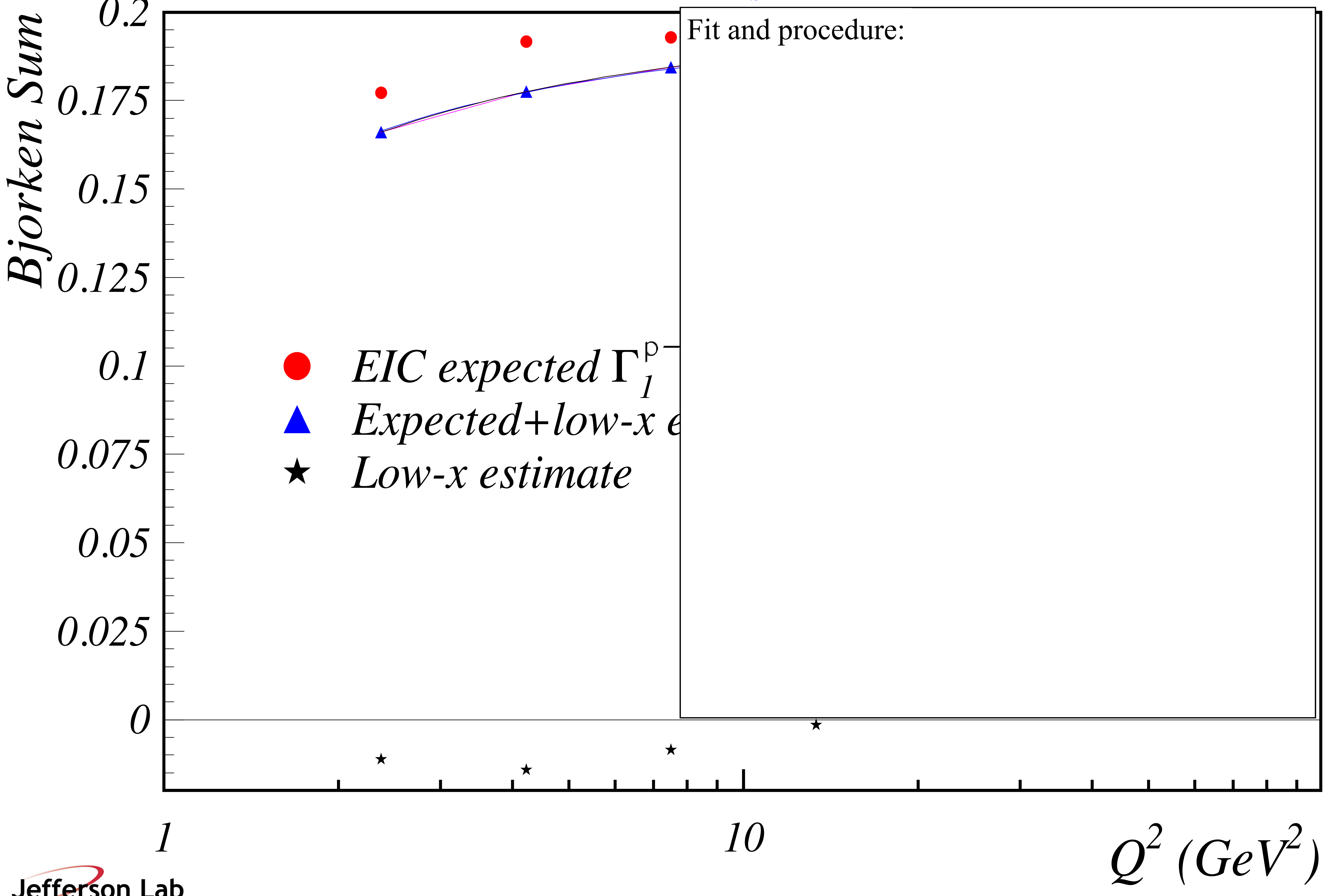

Fit and procedure:

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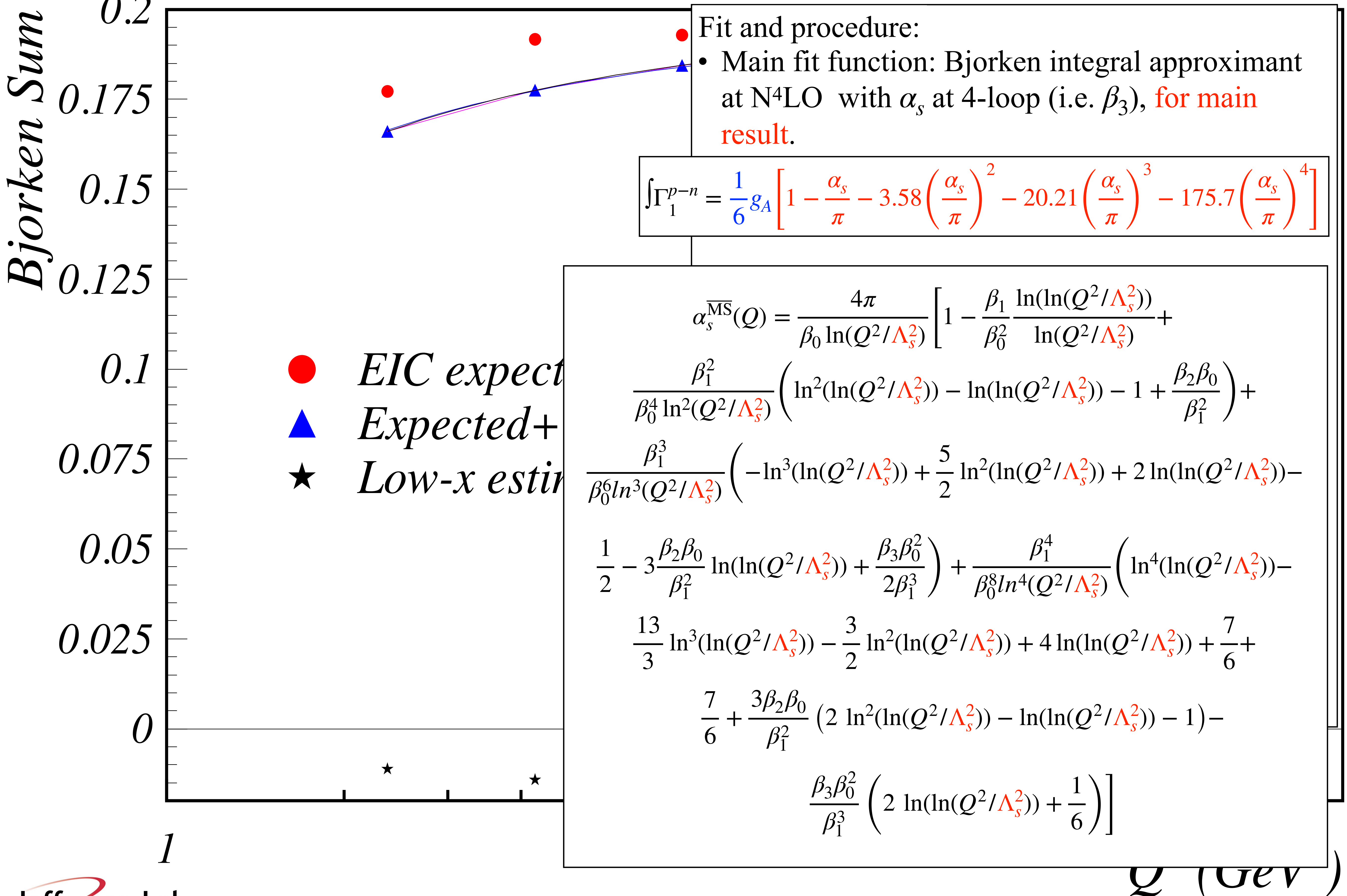

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$$\alpha_s^{\overline{\mathrm{MS}}}(Q) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda_s^2)} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln(\ln(Q^2/\Lambda_s^2))}{\ln(Q^2/\Lambda_s^2)} + \frac{\beta_1^2}{\beta_0^4 \ln^2(Q^2/\Lambda_s^2)} \left(\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1 + \frac{\beta_2\beta_0}{\beta_1^2} \right) + \frac{\beta_1^3}{\beta_0^6 ln^3(Q^2/\Lambda_s^2)} \left(-\ln^3(\ln(Q^2/\Lambda_s^2)) + \frac{5}{2} \ln^2(\ln(Q^2/\Lambda_s^2)) + 2\ln(\ln(Q^2/\Lambda_s^2)) - \frac{1}{2} - 3\frac{\beta_2\beta_0}{\beta_1^2} \ln(\ln(Q^2/\Lambda_s^2)) + \frac{\beta_3\beta_0^2}{2\beta_1^3} \right) + \frac{\beta_1^4}{\beta_0^8 ln^4(Q^2/\Lambda_s^2)} \left(\ln^4(\ln(Q^2/\Lambda_s^2)) - \frac{13}{3} \ln^3(\ln(Q^2/\Lambda_s^2)) - \frac{3}{2} \ln^2(\ln(Q^2/\Lambda_s^2)) + 4\ln(\ln(Q^2/\Lambda_s^2)) + \frac{7}{6} + \frac{7}{6} + \frac{3\beta_2\beta_0}{\beta_1^2} \left(2\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1\right) - \frac{\beta_3\beta_0^2}{\beta_1^3} \left(2\ln(\ln(Q^2/\Lambda_s^2)) + \frac{1}{6} \right) \right]$$

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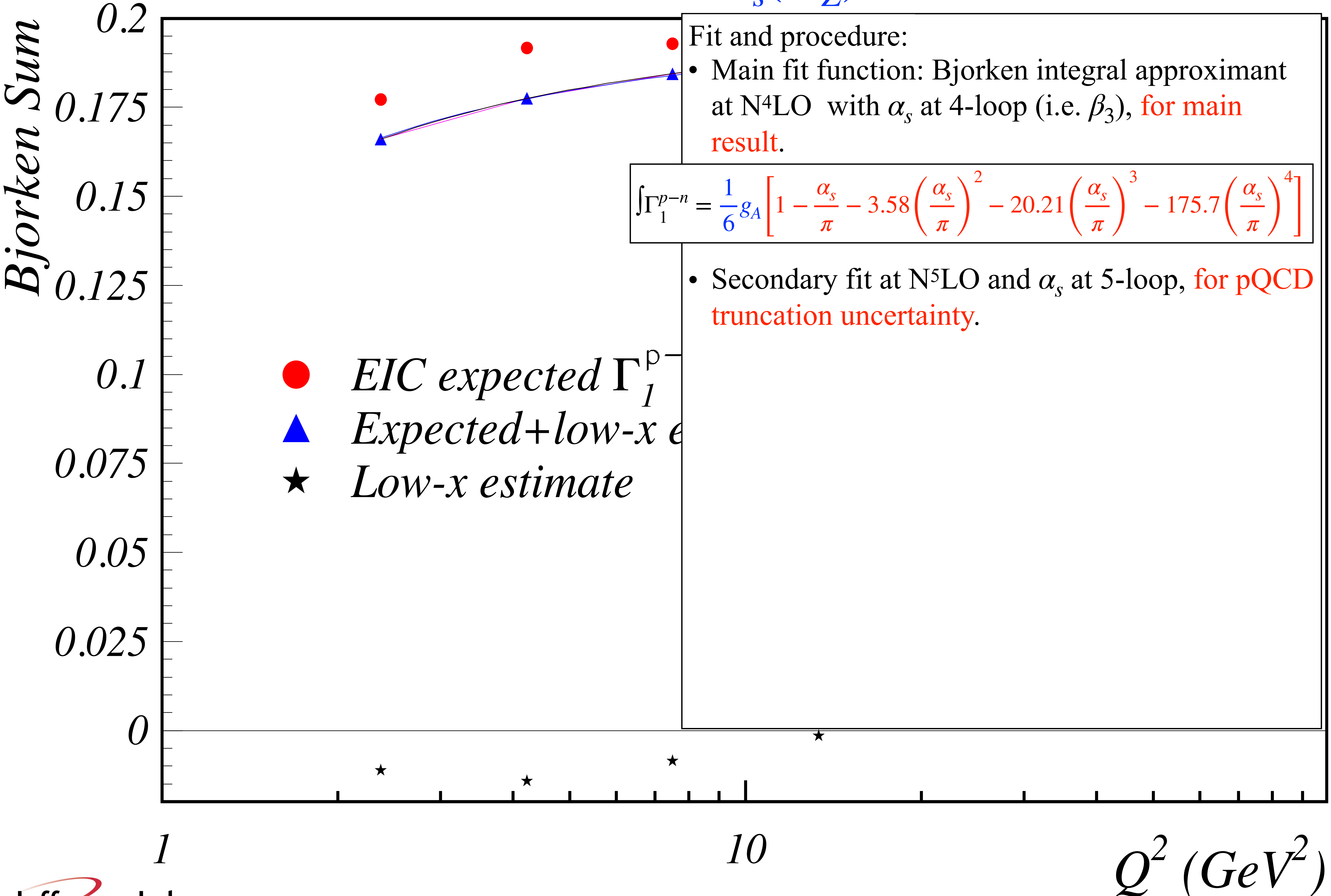

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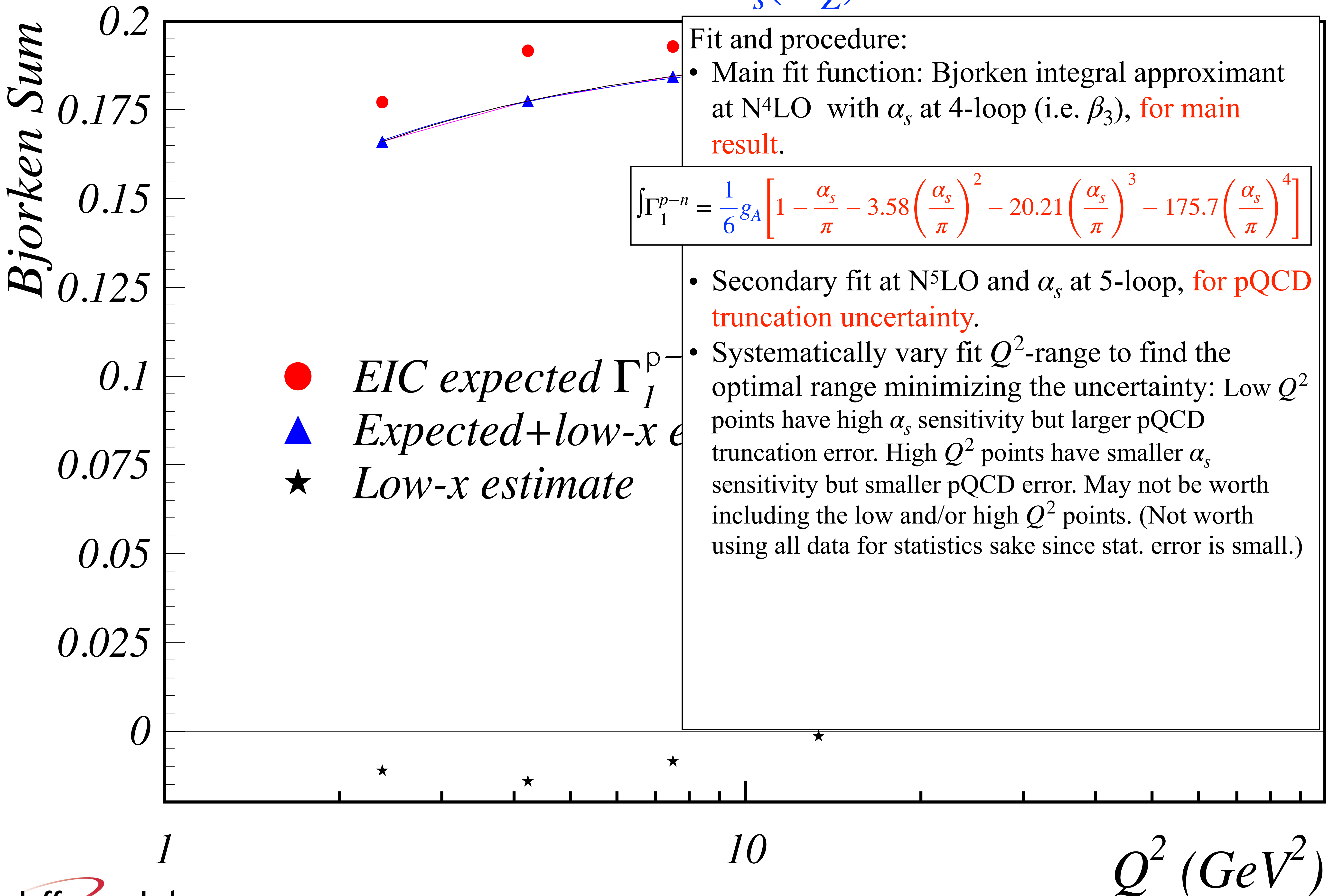

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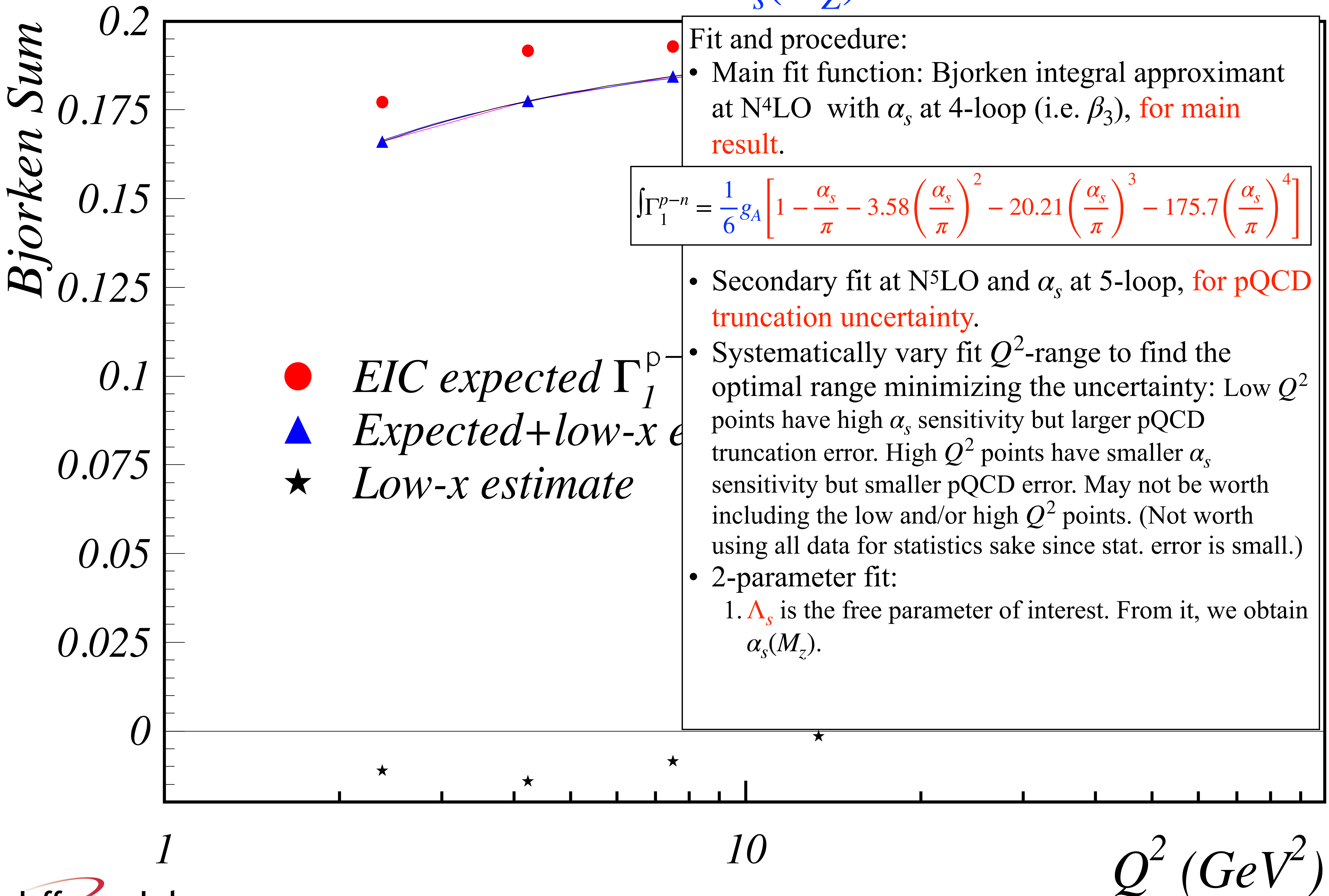

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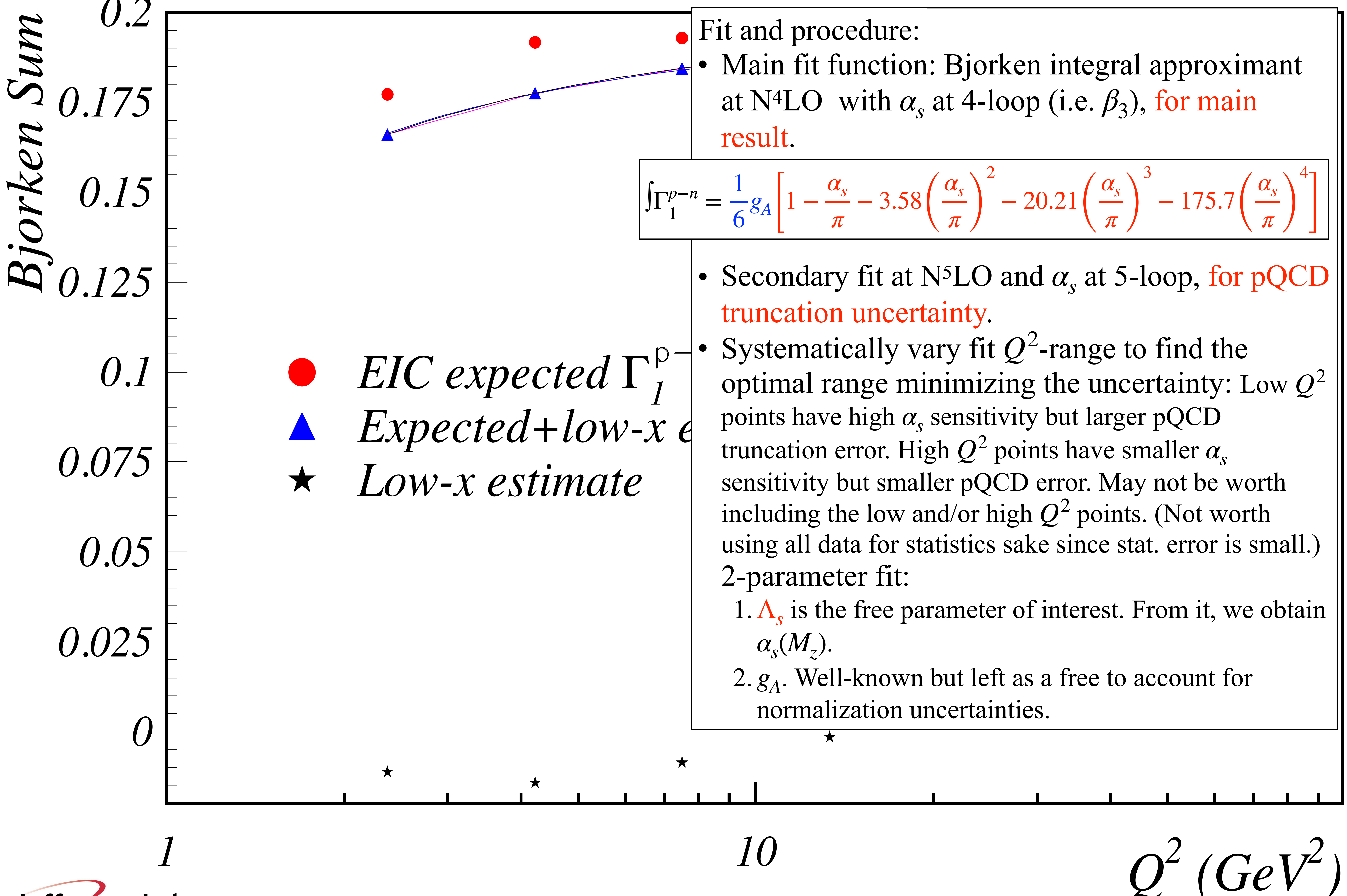

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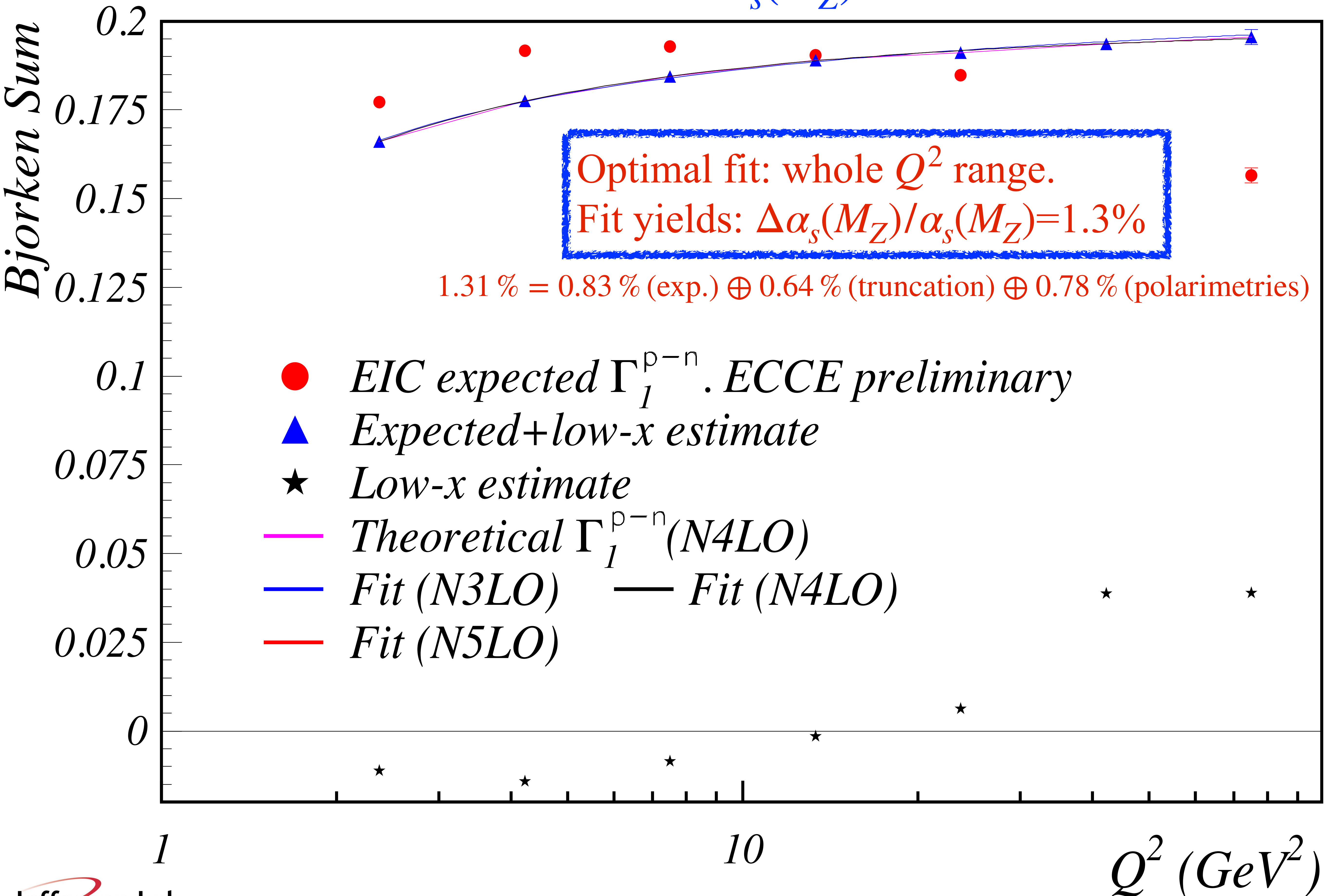

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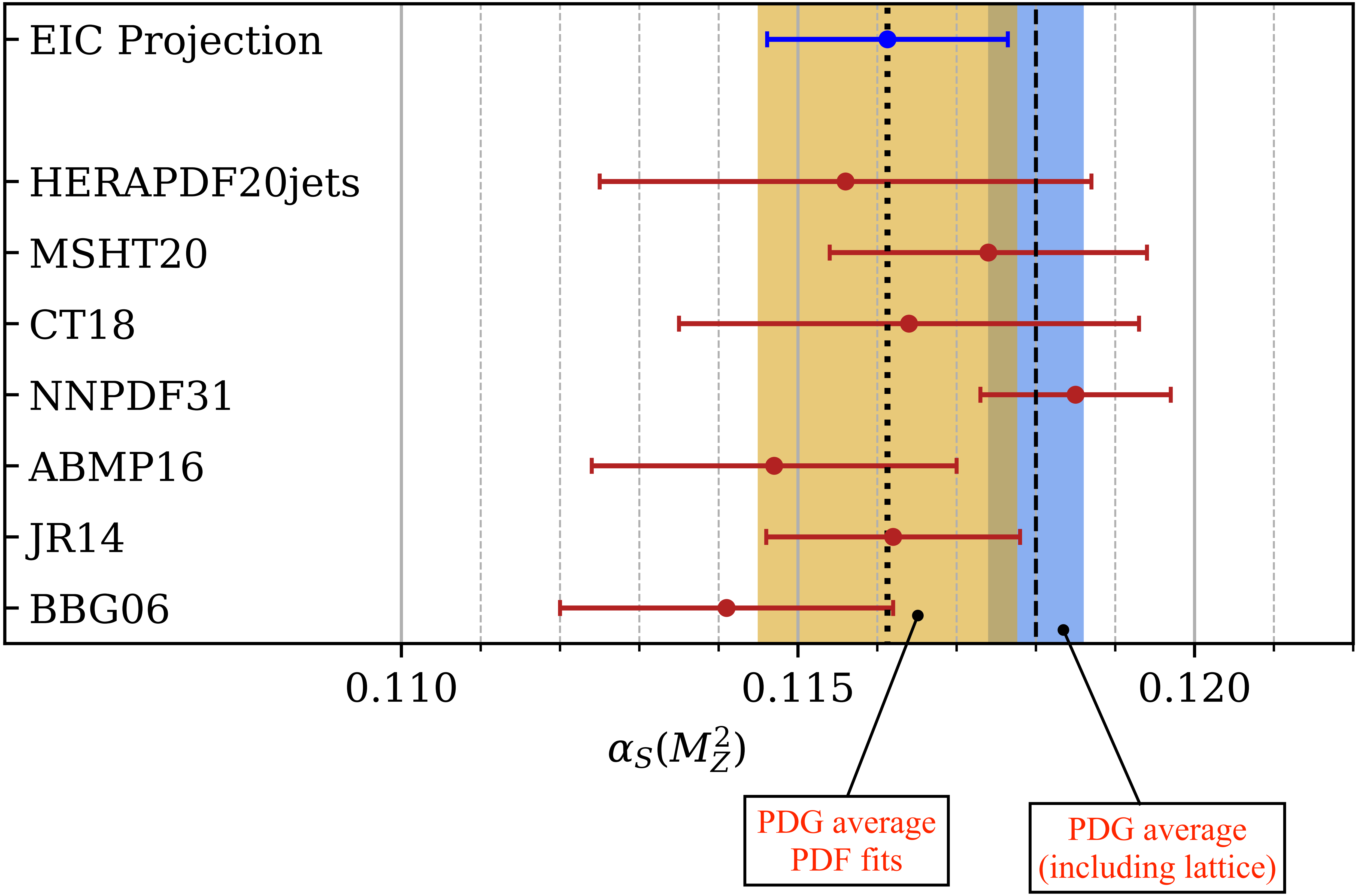

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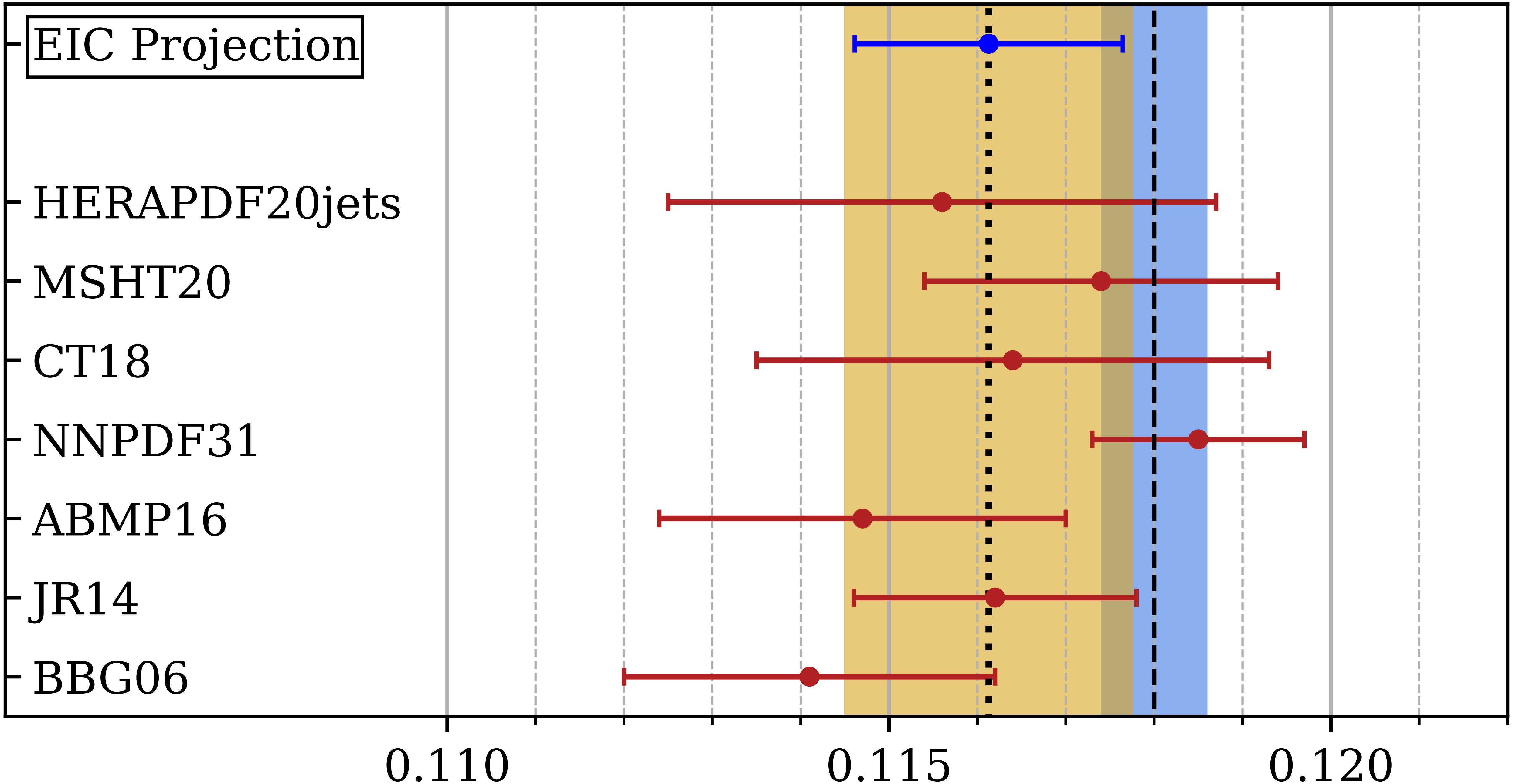

Conclusion:

- Realistic simulation shows that EIC can yield a competitive measurement.
- Just one method. Other extractions will be available, e.g.:
  - Global fits (unpolarized and polarized)
  - Inclusive neutral current reactions (EIC+HERA). S. Cerci, *et al.* EPJC, 83(11):1011, 2023: $\Delta\alpha_s(M_Z)/\alpha_s(M_Z)$=0.4%

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Bjorken sum rule at JLab@22 GeV

- Statistical uncertainties are expected to be negligible:
  - JLab is a high-luminosity facility;
  - A JLab@22 GeV program would include polarized DVCS and TMD experiments. Those imply long running times compared to those needed for inclusive data gathering;
  - High precision data already available from 6 GeV and 12 GeV for the lower Q^2 bins and moderate x.

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- Use 6% for experimental systematics (i.e. not including the uncertainty on unmeasured low-x).
  - Nuclear corrections:
    - D: negligible assuming we can tag the ~spectator proton
    - ^{3}He: 2% (5% on n, which contribute to 1/3 to the Bjorken sum: 5%/3≃2%)
  - Polarimetries: Assume ΔP_e-$\Delta P_N = 3\%$.
  - Radiative corrections: 1%
  - F_1 to form g_1 from A_1: 2%
  - g_2 contribution to longitudinal asym: Negligible, assuming it will be measured.
  - Dilution/purity:
    - Bjorken sum from P & D: 4%
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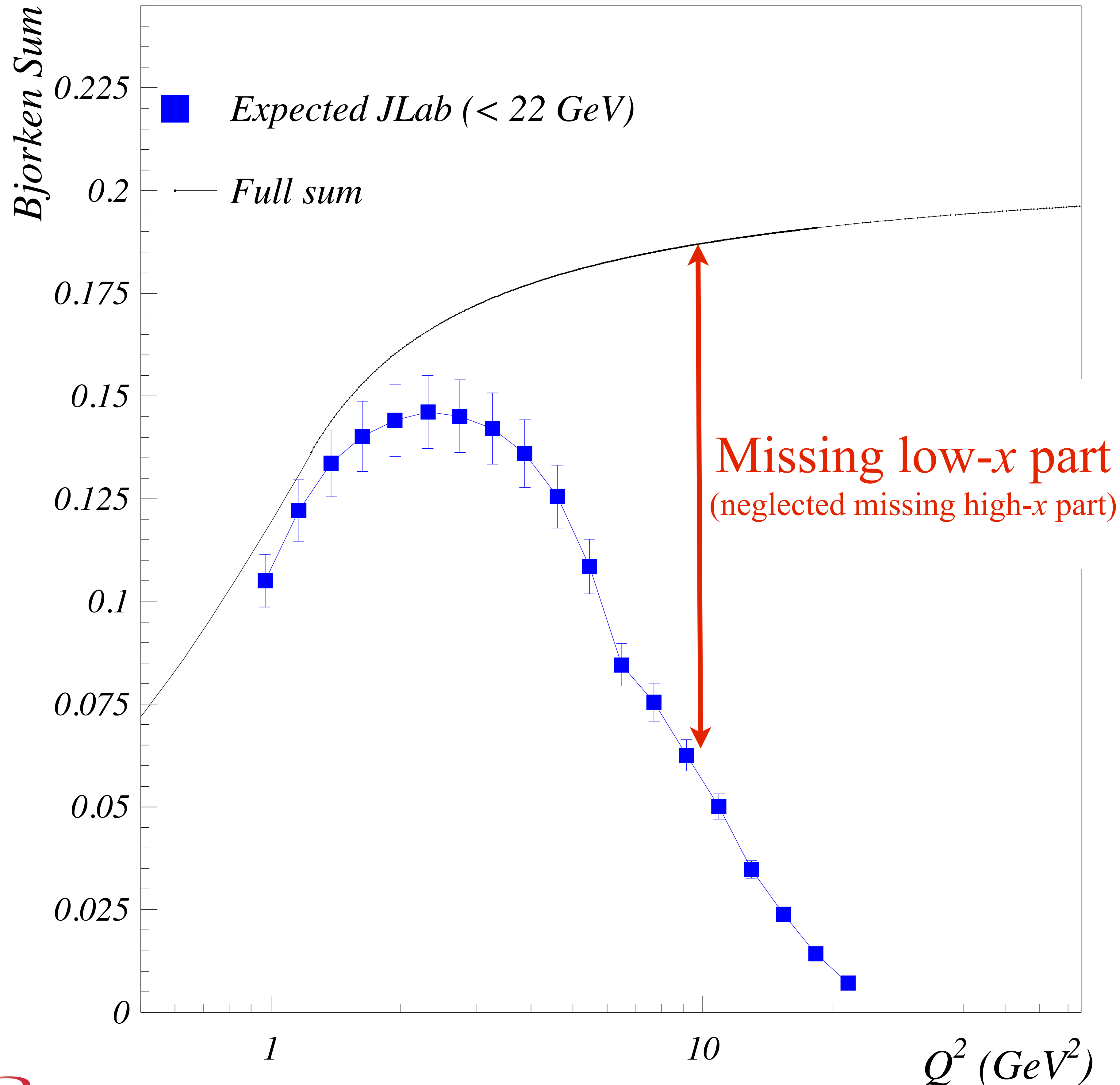
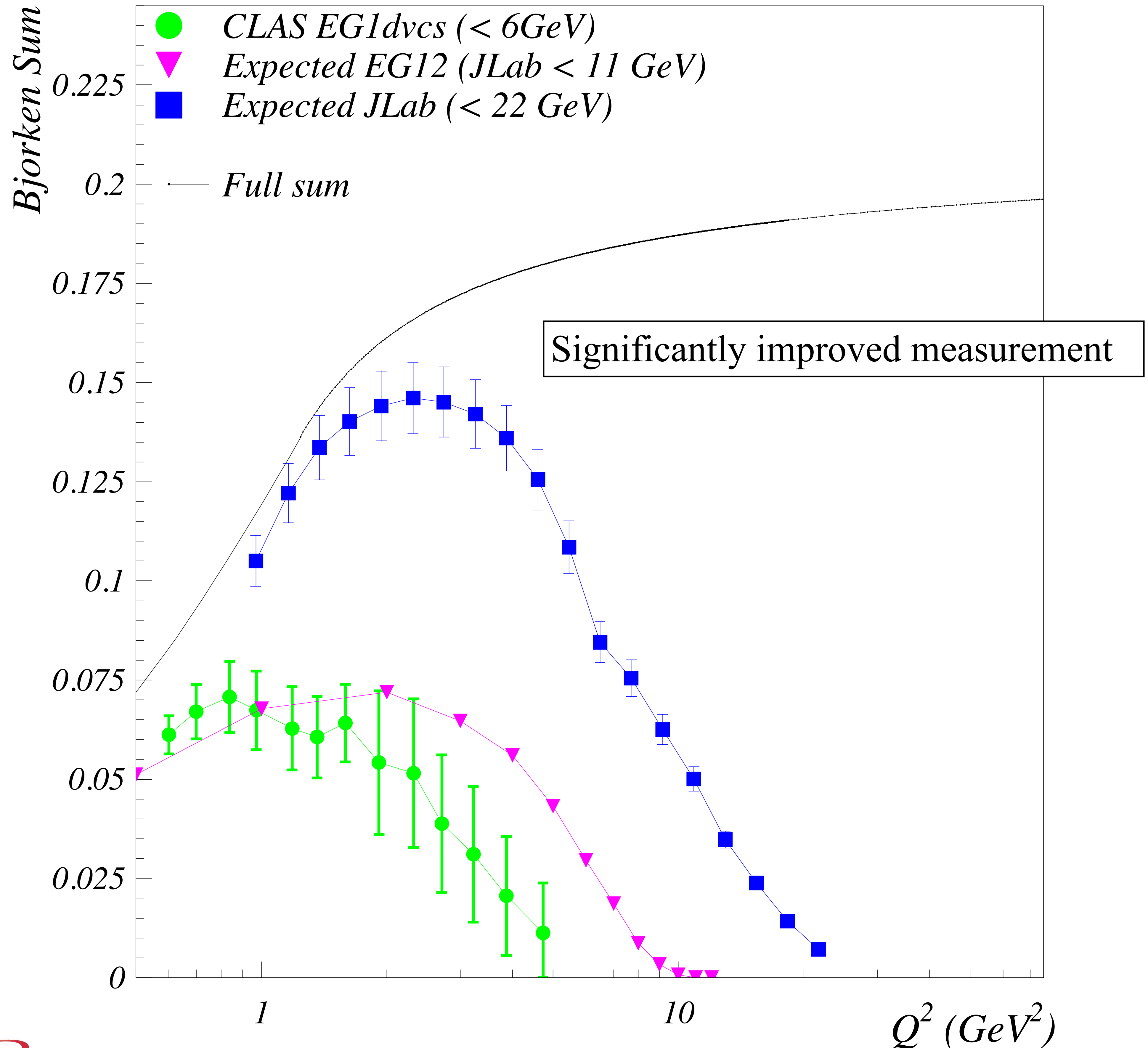
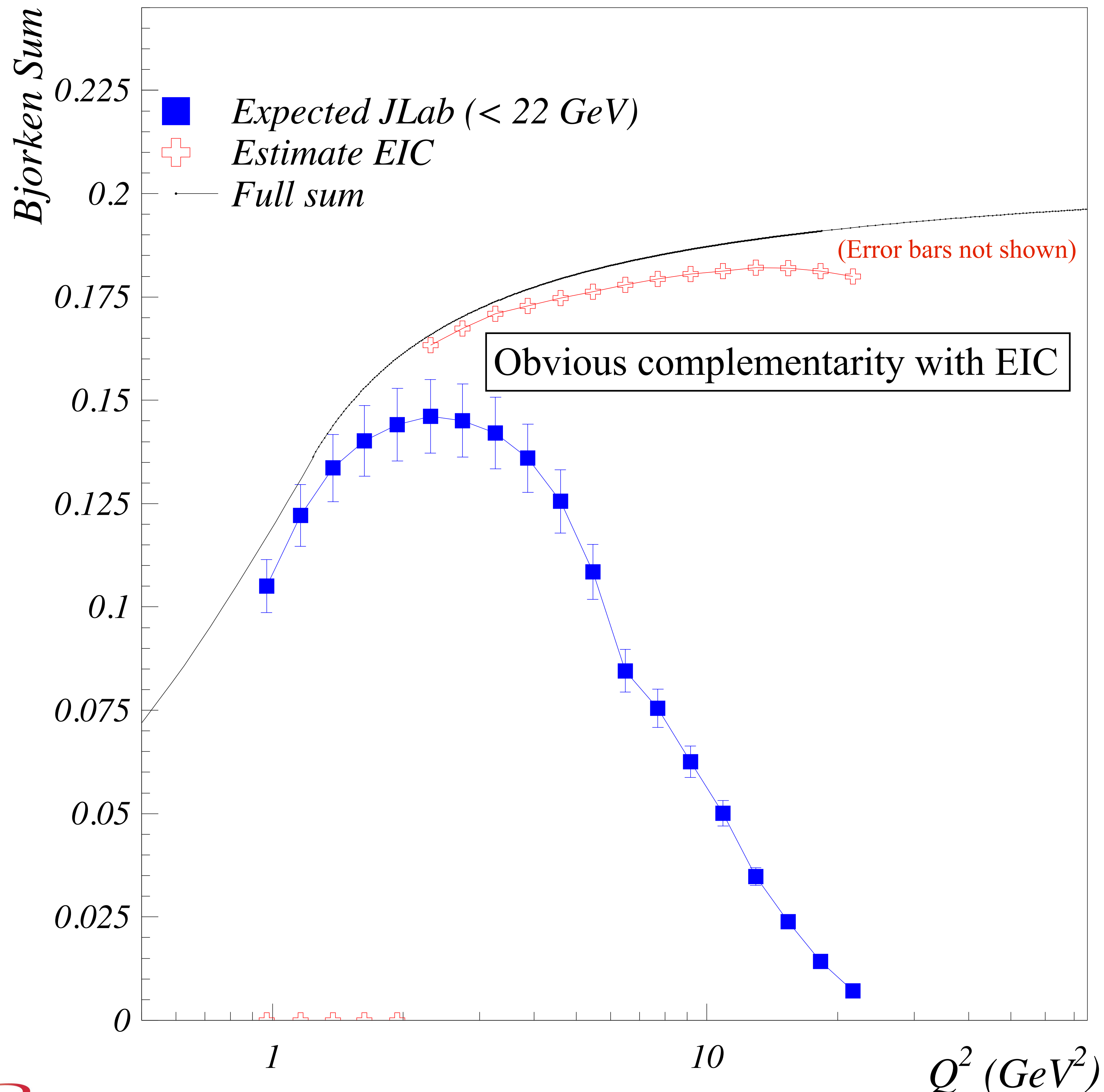
Under these assumptions:



Comparison with JLab at 6 and 11 GeV



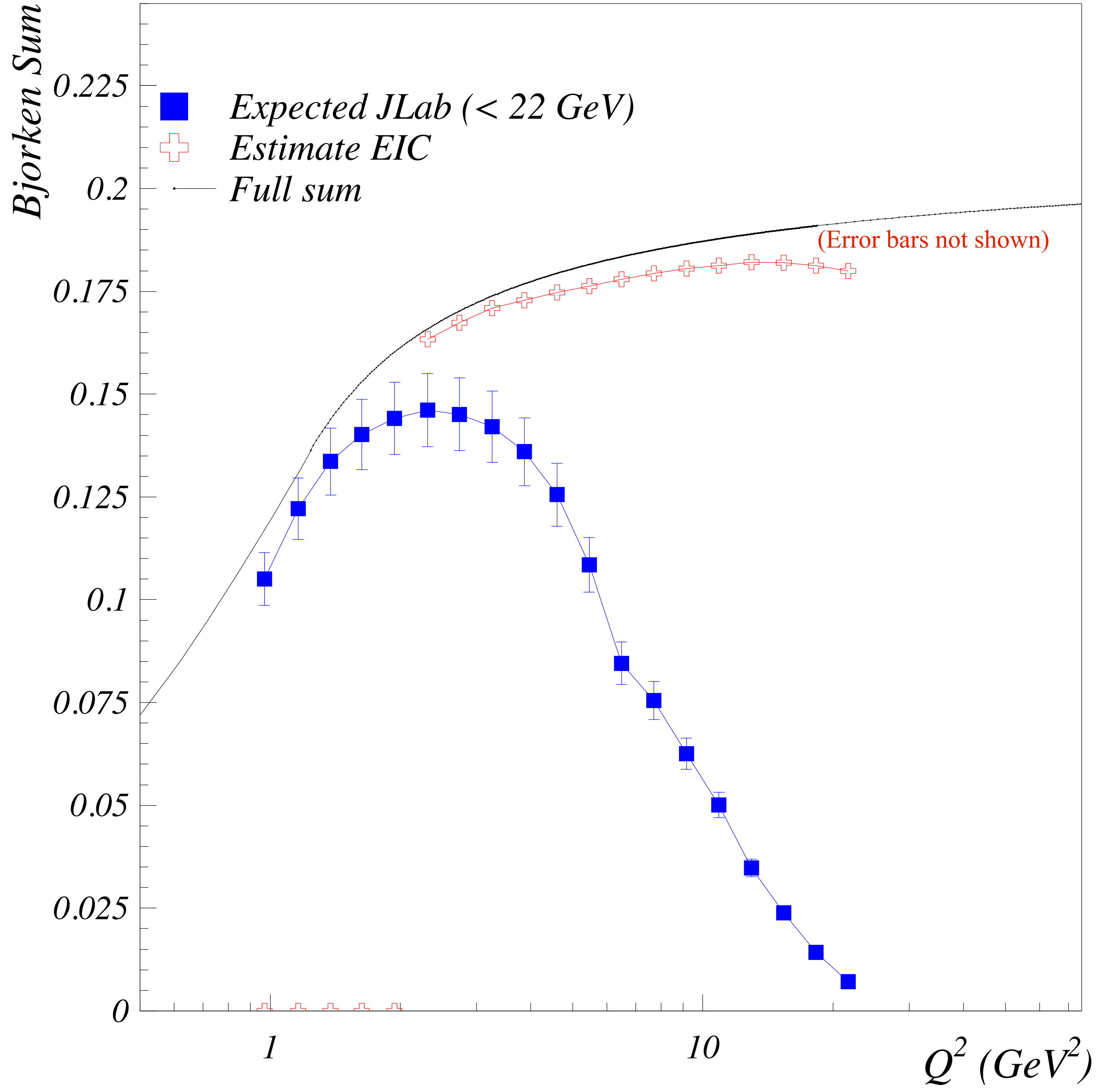
Comparison with EIC



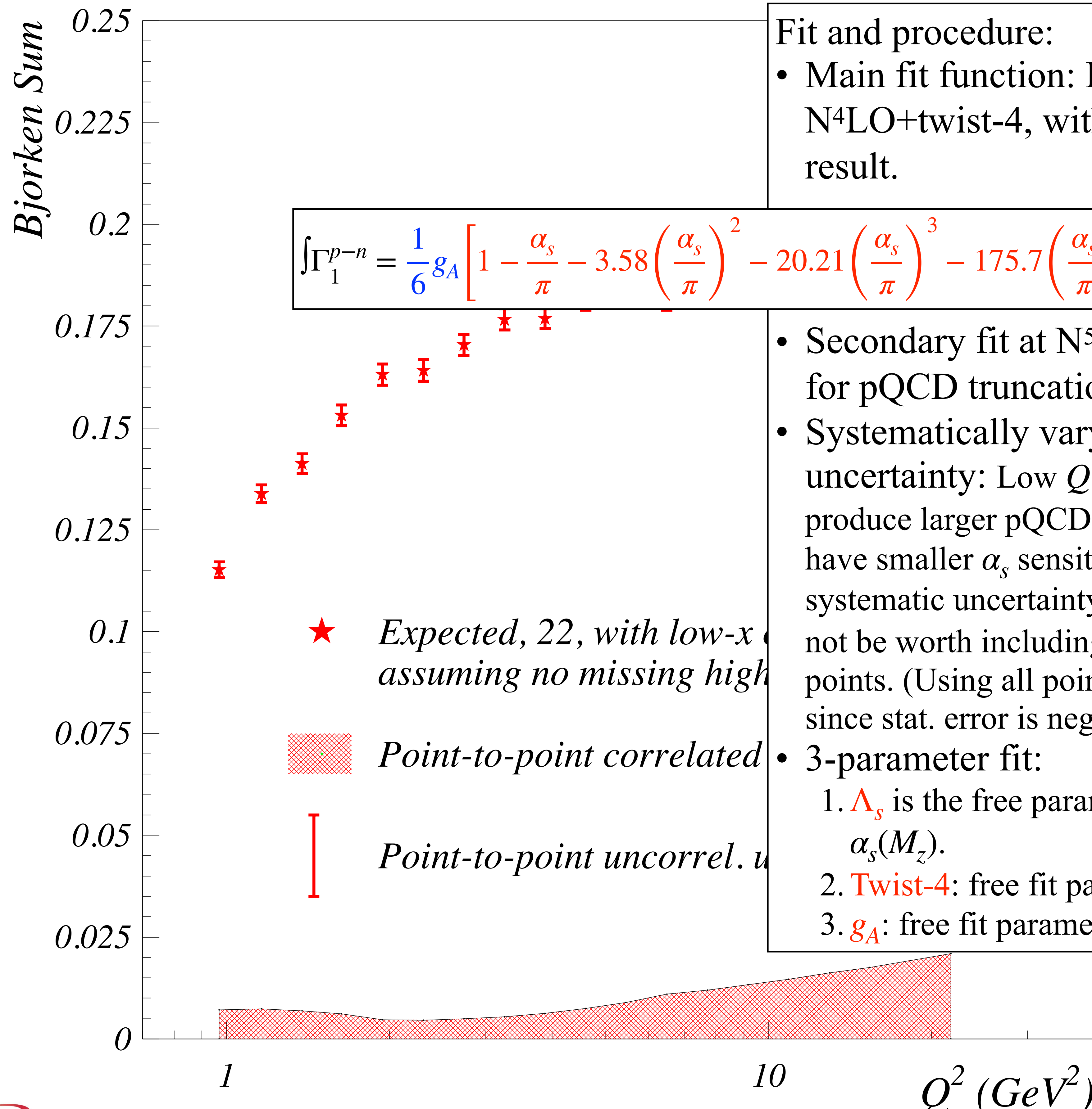
Low-x uncertainty

- For the Q^2 bins covered by EIC, global fits will be available up to the lowest x covered by EIC.
  $\Rightarrow$ assume 10% uncertainty on that missing (for the JLab measurement) low-x part.
  Assume 100% for the very small-x contribution not covered by EIC.

- For the 5 lowest Q^2 bins not covered by EIC:
  - Bin #5 close to the EIC coverage $\Rightarrow$ Constrained extrapolation, assume 20% uncertainty on missing low-x part.
  - Bin #4, assume 40% uncertainty, Bin #3, assume 60%, Bin #2, assume 80%, Bin #1, assume 100%.



Extraction of $\alpha_s(M_Z)$



Fit and procedure:

- Main fit function: Bjorken sum approximant at N⁴LO+twist-4, with α_s at 4-loop (i.e. β_3), for main result.

$$\int \Gamma_1^{p-n} = \frac{1}{6} g_A \left[1 - \frac{\alpha_s}{\pi} - 3.58\left(\frac{\alpha_s}{\pi}\right)^2 - 20.21\left(\frac{\alpha_s}{\pi}\right)^3 - 175.7\left(\frac{\alpha_s}{\pi}\right)^4 \right] + \frac{M^2}{Q^2}\left[a_2(\alpha_s) + 4d_2(\alpha_s) + 4f_2(\alpha_s) \right]$$

- Secondary fit at N⁵LO+twist-4 and α_s at 5-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2 range to minimize total uncertainty: Low Q^2 points have high α_s sensitivity but produce larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity and larger experimental systematic uncertainty but smaller pQCD error. ⇒May not be worth including the lowest and/or highest Q^2 points. (Using all points for statistics sake is not worth it, since stat. error is negligible.)
- 3-parameter fit:
  1. Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
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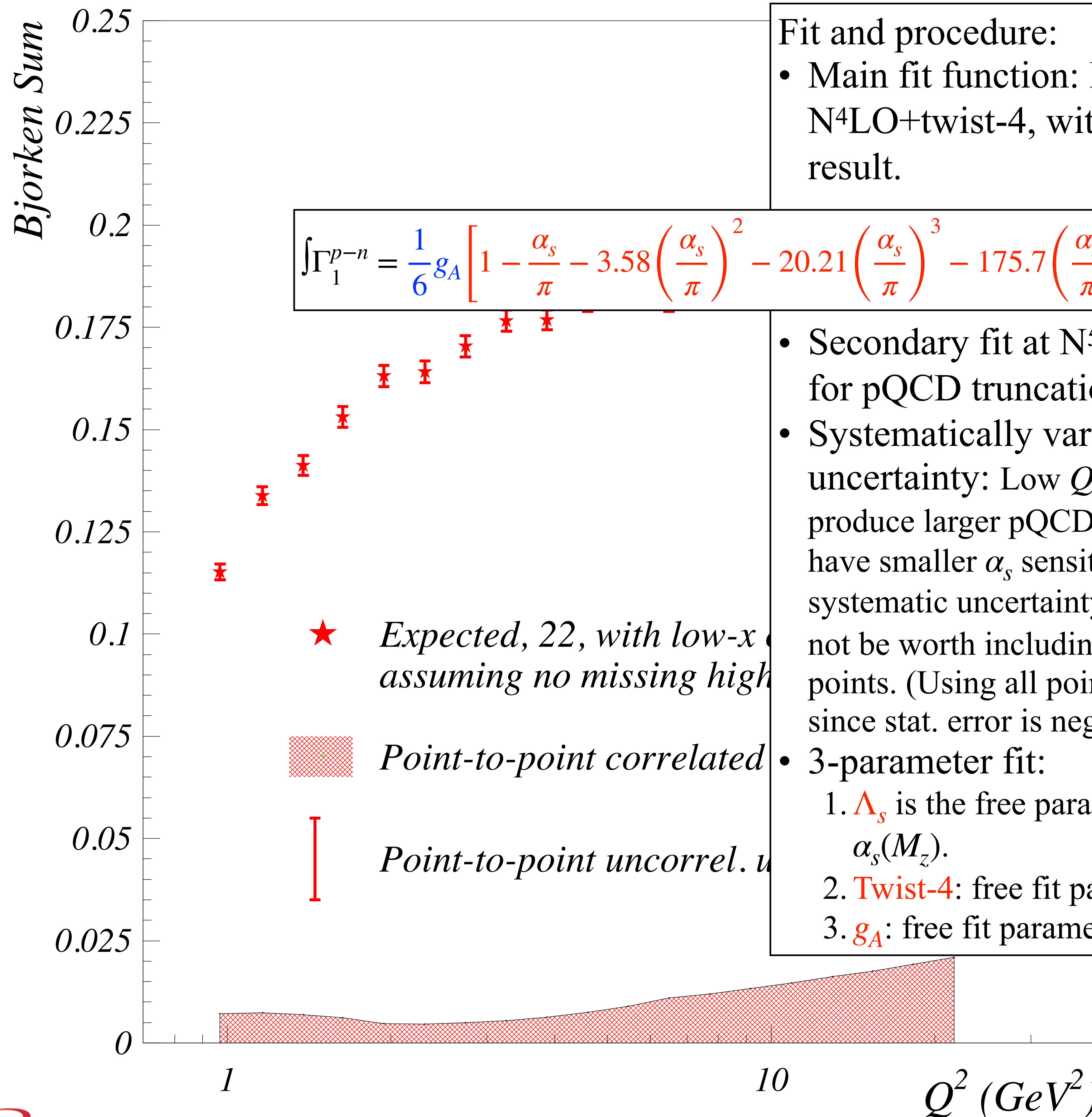

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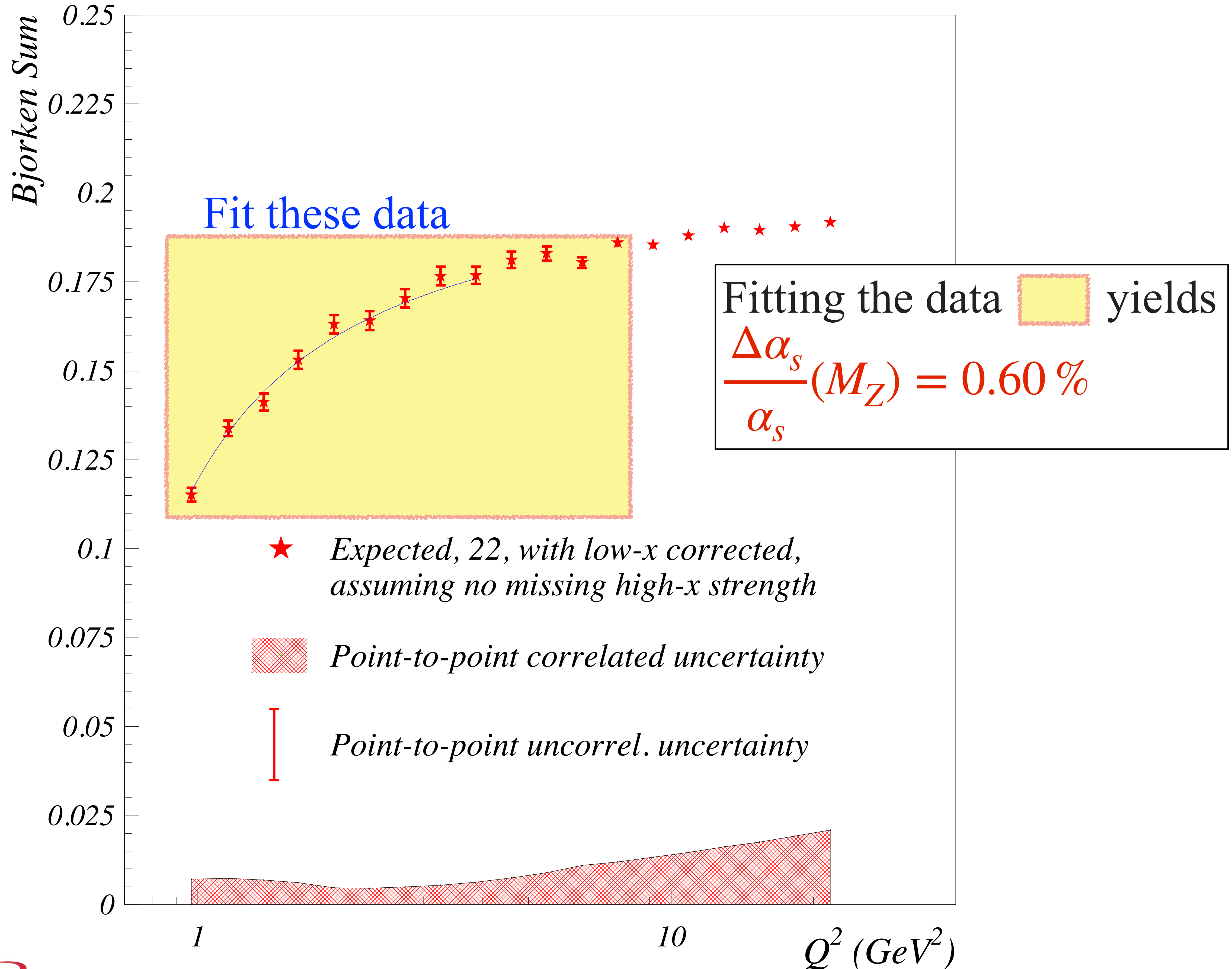

Comparison JLab@22 GeV and EIC

EIC

- Best low-x coverage.
- No Higher-Twist uncertainties
- Smaller pQCD uncertainties.

JLab@22 GeV

- Covers region with strong Q^2-dependence: best sensitivity to α_s. (Up to 50 time more sensitive.)
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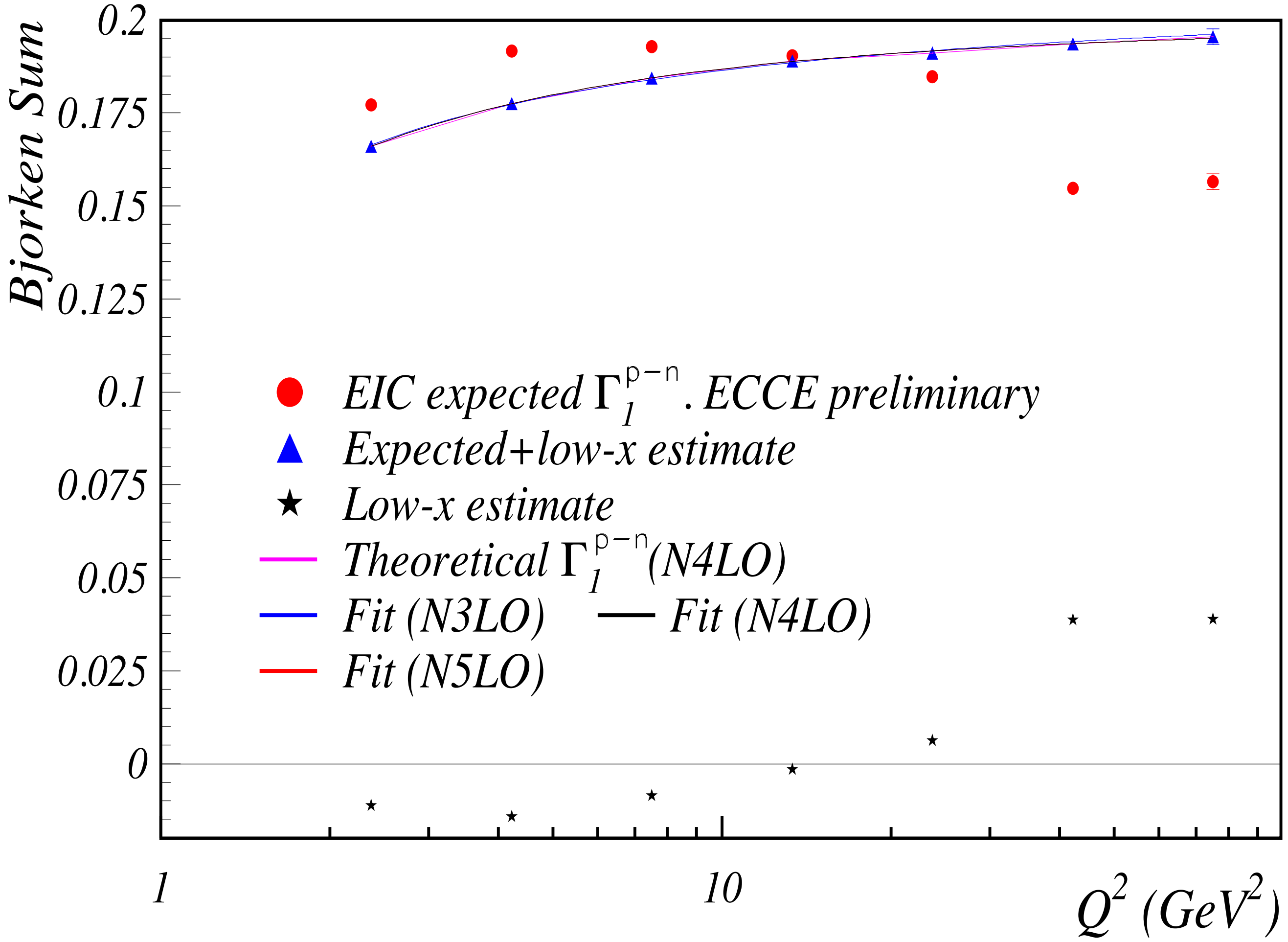

$\frac{\Delta\alpha_s}{\alpha_s} \simeq 1.3\ \%$ EIC alone.

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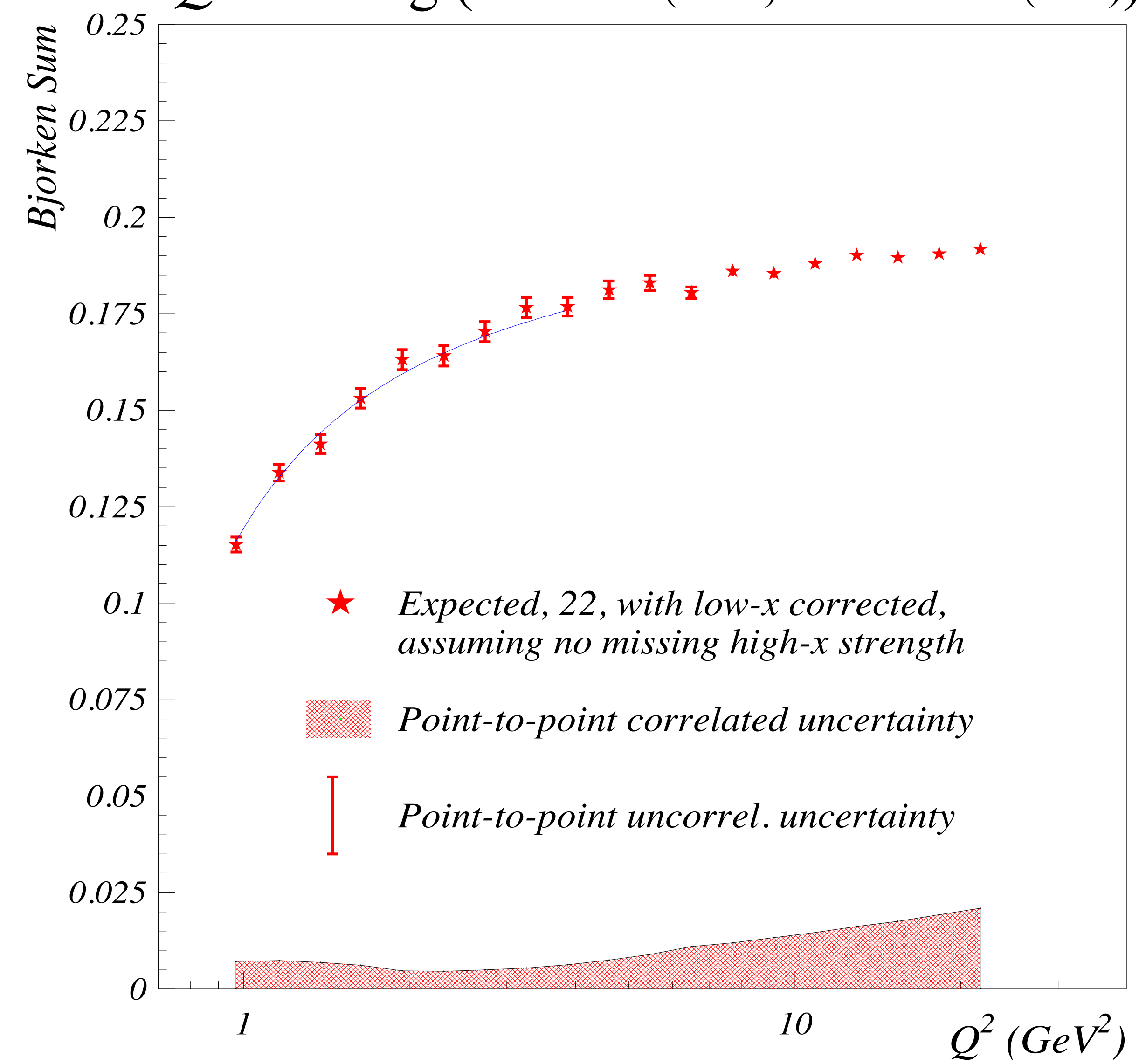
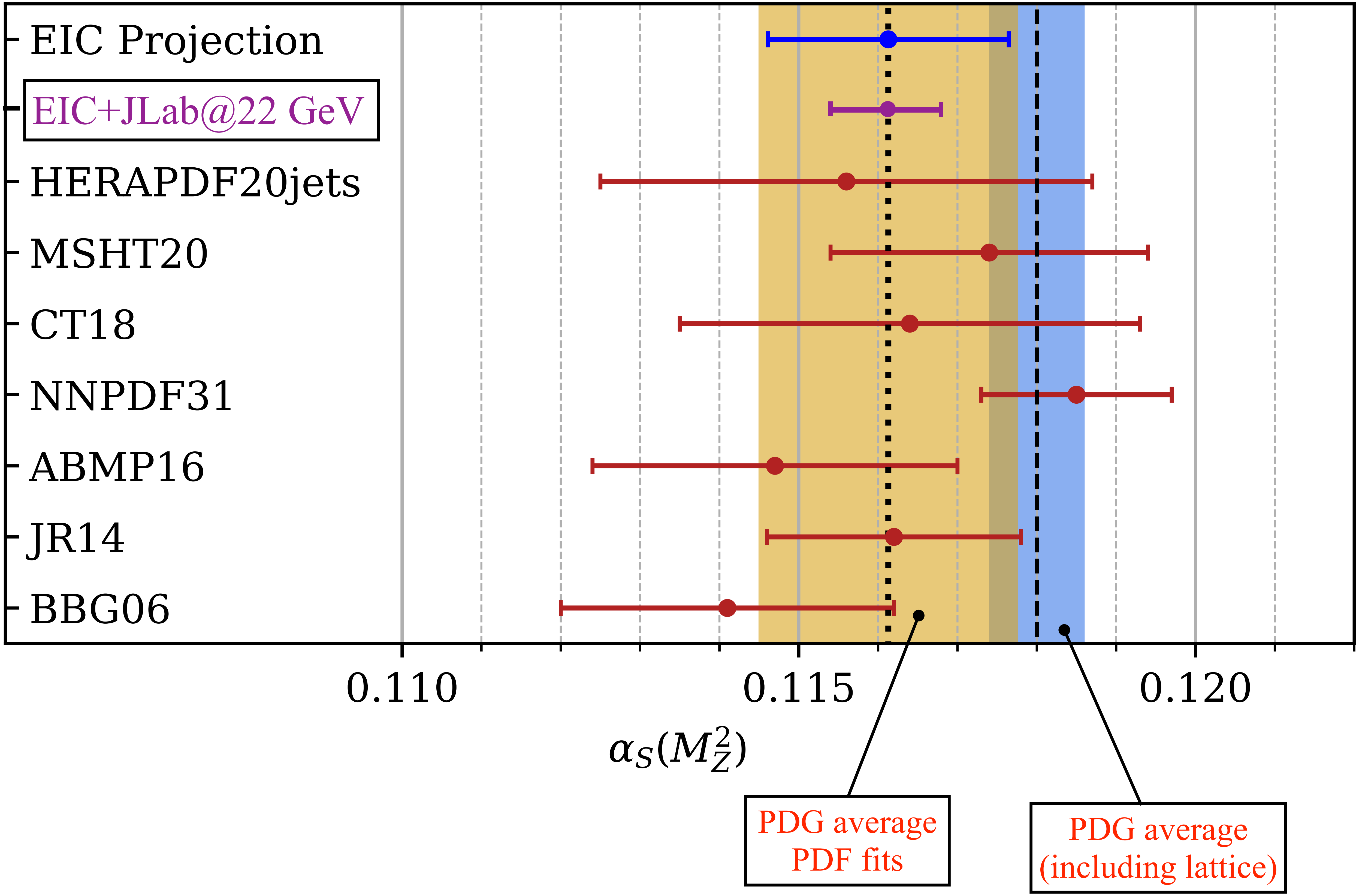


$\frac{\Delta\alpha_s}{\alpha_s} \simeq 0.60\ \%$. EIC+JLab

Compared to other DIS results and world average (from PDG)



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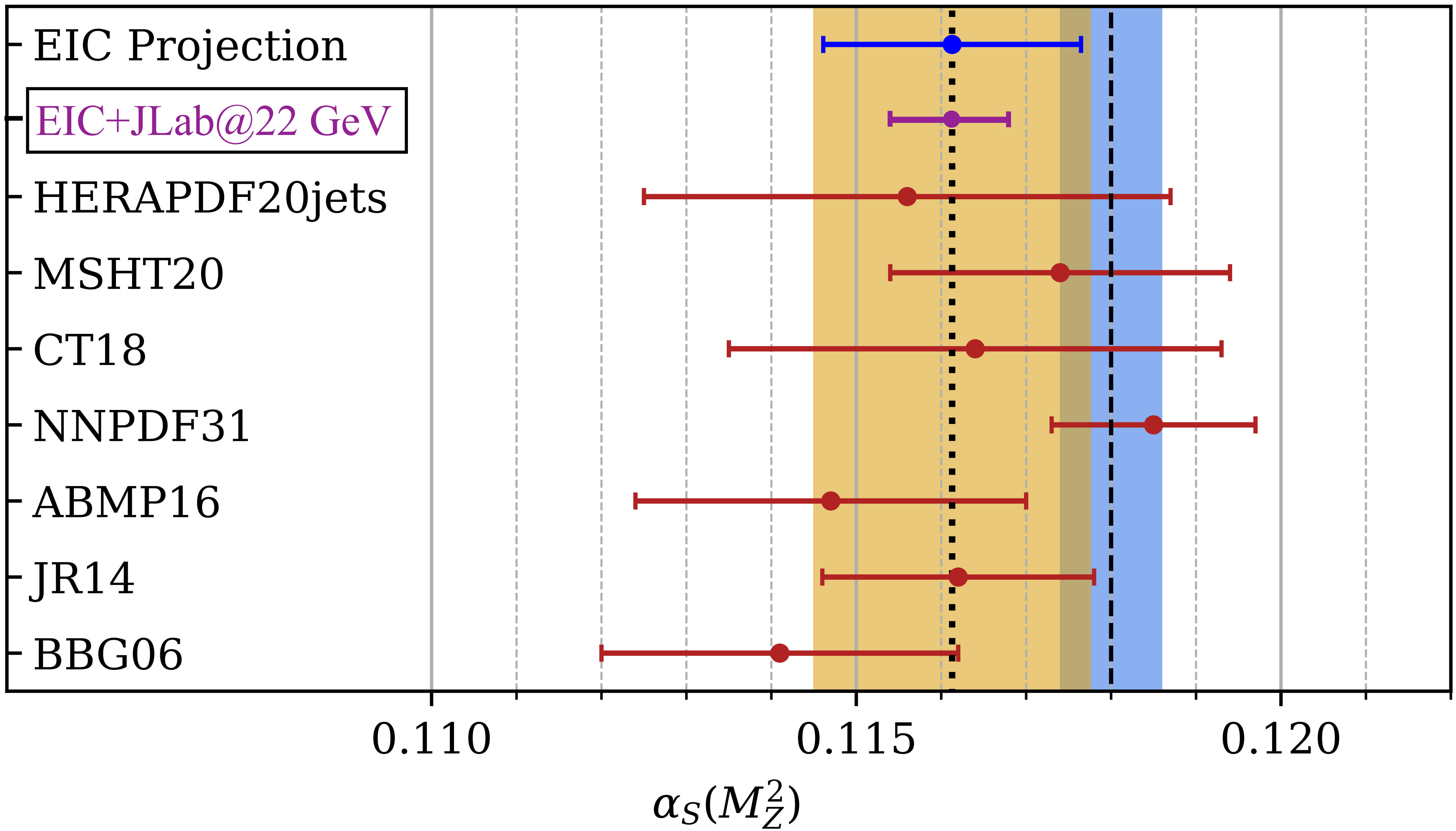

Under reasonable assumptions, EIC+JLab@22 GeV can yield a compelling 0.6% measurement of $\alpha_s(M_Z)$ from the Bjorken sum rule.

Summary

- The Bjorken sum $\Gamma_1^{p-n}(Q^2)=\int g_1^{p-n}(x, Q^2)dx$ offers a simple and competitive method to determine α_s.
- Realistic simulation shows that EIC can yield a measurement with 1.3% precision.
  - Use only g_1 from inclusive polarized DIS reaction.
- Preliminary study shows that a JLab@22 GeV upgrade would lower this result to ~0.6% using the same method.
- Very different data (polarized DIS), simple (⇒clean) extraction, competitive accuracy: valuable comparison of α_s extracted from different processes.
- Possibilities of further improvement:
  1. Improved knowledge of pQCD series: $\alpha_s(M_Z)$ at β_4 already available. Estimate for N^5LO results for Γ_1^{p-n} available.
  2. Improved perturbative methods minimizing pQCD truncation. Some have already been worked out for Γ_1^{p-n}.
- This is but one of several ways to determine α_s at EIC or JLab. Others, e.g., global fits of (un)polarized PDFs or inclusive neutral current reactions would also provide competitive measurements.