Strong Coupling Constant from NNPDF Fits

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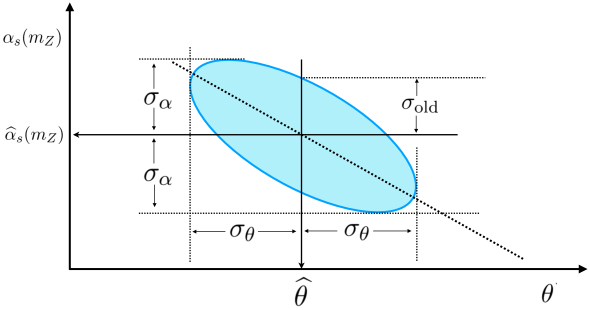
NNPDF

need for precision

- include all available data (with cuts), latest theory calculations
- robust error bars are mandatory
- control systematic errors
- **simultaneous** fits of α_s and PDFs
- NNPDF determination [nnpdf 18] and methodological issues [Forte & Kassabov 20]

correlated fit parameters

the usual story...

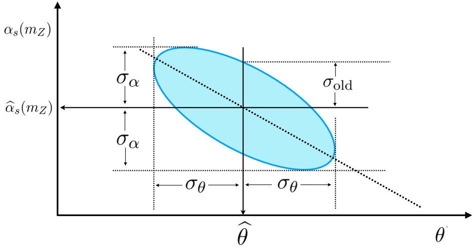


$(\hat{\alpha}_s, \hat{\theta})$ depends on Y

$$L(\alpha_s, \theta; Y) = p(Y|\alpha_s, \theta)\,, \quad (\hat{\alpha}_s, \hat{\theta}) = \arg\min_{\alpha_s,\theta} \left[\chi^2(\alpha_s, \theta; Y)\right]$$

$$\chi^2(\alpha_s, \theta; Y) = \chi^2_{\min}(Y) + \frac{1}{2}\,(\Delta\alpha_s, \Delta\theta)\, M_Y^{-1} \begin{pmatrix} \Delta\alpha_s \\ \Delta\theta \end{pmatrix}$$

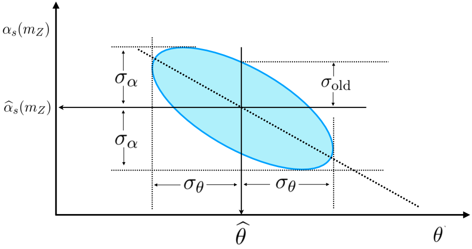
more of the usual story...



$$M = \begin{pmatrix} M_{\alpha\alpha} & M_{\alpha\theta} \\ M_{\theta\alpha} & M_{\theta\theta} \end{pmatrix} \quad \Rightarrow \quad \sigma_\alpha^2 = \text{Var}[\alpha] = M_{\alpha\alpha}$$

$$\sigma_{\text{old}}^2 = \sigma_\alpha^2 \left[1 + \sum_\mu \rho_{\alpha\theta_\mu}^2 \right] \leq \sigma_\alpha^2 \, ; \quad \rho_{f\alpha} = \frac{\sum_\mu \partial_\mu f(x;\hat{\theta})\, \rho_{\alpha\theta_\mu} \sigma_\mu}{\left(\sum_\mu |\partial_\mu f(x;\hat{\theta})|^2 \sigma_\mu \right)^{1/2}}$$

finding the minimum



minimization along orthogonal directions for any given set of data Y

$$(1) : \hat{\theta}(\alpha_s) = \arg\min_{\theta} \left[\chi^2(\alpha_s, \theta; Y)\right]$$

$$(2) : \hat{\alpha}_s = \arg\min_{\alpha_s} \left[\chi^2(\alpha_s, \hat{\theta}(\alpha_s); Y)\right] , \quad \hat{\theta} = \hat{\theta}(\hat{\alpha}_s)$$

if $Y = Y_g$ is the global dataset, $\hat{\theta}_g(\alpha_s)$ is called *best fit* line

Monte Carlo method

ensemble of *replicas* reproducing the statistical distribution of the data

$$\mathcal{Y} = \left\{ Y^{(k)}; k = 1, \ldots N_{\text{rep}} \right\} \longrightarrow \chi^2(\alpha_s, \theta; Y^{(k)}) = \chi^{2,(k)}(\alpha_s, \theta)$$

for each replica determine

$$(\hat{\alpha}_s^{(k)}, \hat{\theta}^{(k)}) = \arg \min_{\alpha_s, \theta} \left[\chi^{2,(k)}(\alpha_s, \theta) \right]$$

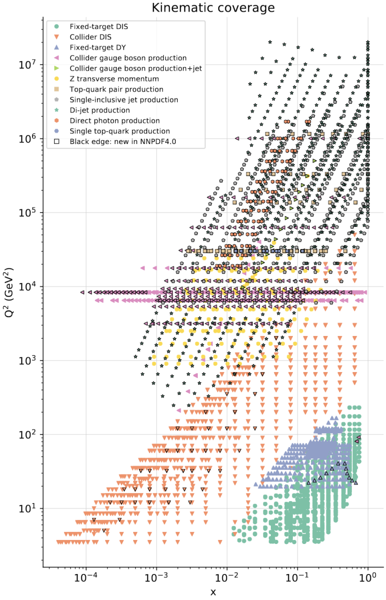
step (1): minimize $\chi^{2,(k)}$ for 21 values of α_s, $0.106 \leq \alpha_s \leq 0.130$

$\hookrightarrow$ determine $\hat{\theta}^{(k)}(\alpha_s)$, correlated PDFs for a given replica

step (2): discrete values of $\chi^{2,(k)}(\alpha_s, \hat{\theta}^{(k)}(\alpha_s))$ and interpolate by parabola

fitted data

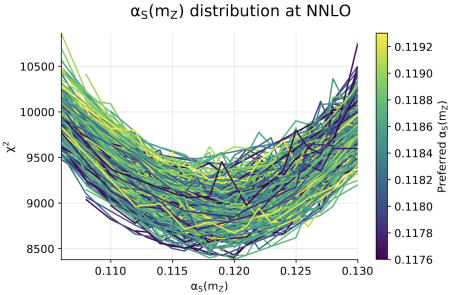
	NLO	NNLO
Fixed-target charged lepton DIS	973	973
Fixed-target neutrino DIS	908	908
Collider DIS (HERA)	1221	1211
Fixed Target Drell-Yan	189	189
Collider Drell-Yan	378	388
Inclusive jets	164	164
$Z\ p_T$	120	120
Top quark pair production	26	26
Total	3979	3979

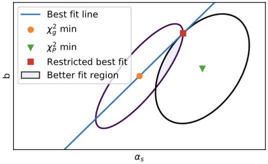


Kinematic coverage
Fixed-target DIS
Collider DIS
Fixed-target DY
Collider gauge boson production
Collider gauge boson production+jet
Z transverse momentum
Top-quark pair production
Single-inclusive jet production
Di-jet production
Direct photon production
Single top-quark production
Black edge: new in NNPDF4.0
Q^2 (GeV^2)
x
10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
10^{1} 10^{2} 10^{3} 10^{4} 10^{5} 10^{6} 10^{7}

Dataset	NLO	NNLO
DIS measurements	$W^2 \geq 12.5$ GeV^2; $Q^2 \geq 3.5$ GeV^2	$W^2 \geq 12.5$ GeV^2; $Q^2 \geq 3.5$ GeV^2
HERA I+II $\sigma^c_{\rm NC}$ (in addition to the above)	—	$Q^2 \geq 8$ GeV^2 (fitted charm)
E866/E605 σ^p	$\tau \leq 0.08$; $\|y/y_{\rm max}\| \leq 0.0663$	$\tau \leq 0.08$; $\|y/y_{\rm max}\| \leq 0.0663$
D0 W electron/muon asymmetry	—	$\|A_\ell\| \geq 0.03$
ATLAS low-mass DY 7 TeV	$m_{\ell\ell} > 22$ GeV	—
ATLAS high-mass DY 7 TeV	$m_{\ell\ell} < 210$ GeV	$m_{\ell\ell} < 210$ GeV
CMS DY 2D 7 TeV	$30 \leq m_{\ell\ell} \leq 200$ GeV; $\|y_{\ell\ell}\| \leq 2.2$	$m_{\ell\ell} \leq 200$ GeV; $\|y_{\ell\ell}\| \leq 2.2$
LHCb $W, Z \to \mu$ 7 TeV	—	$\|\eta_\mu\|/\|y_{\mu\bar{\mu}}\| \geq 2.25$
ATLAS low-mass DY 2D 8 TeV	$m_{\ell\ell} \leq 116$ GeV	$m_{\ell\ell} \leq 116$ GeV
LHCb $W, Z \to \mu$ 8 TeV	—	$\|\eta_\mu\|/\|y_{\mu\bar{\mu}}\| \geq 2.25$
LHCb $Z \to ee/Z \to \mu\mu$ 13 TeV	—	$\|y_{\ell\ell}\| \geq 2.25$
ATLAS $W^\pm$+jet 8 TeV	$p_T^W \geq 25$ GeV	$p_T^W \geq 25$ GeV
ATLAS Z p_T 8 TeV $(p_T, m_{\ell\ell})$	$p_T^Z \geq 30$ GeV	$p_T^Z \geq 30$ GeV
ATLAS Z p_T 8 TeV (p_T, y_Z)	$30 \leq p_T^Z \leq 150$ GeV	$30 \leq p_T^Z \leq 150$ GeV
CMS Z p_T 8 TeV	$30 \leq p_T^Z \leq 170$ GeV; $\|y_Z\| \leq 1.6$	$30 \leq p_T^Z \leq 170$ GeV; $\|y_Z\| \leq 1.6$
CMS incl. jets 8 TeV	$p_T^{\rm jet} \geq 74$ GeV	$p_T^{\rm jet} \geq 74$ GeV

result by replica

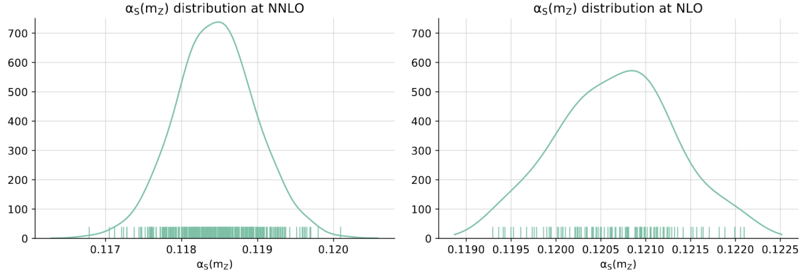




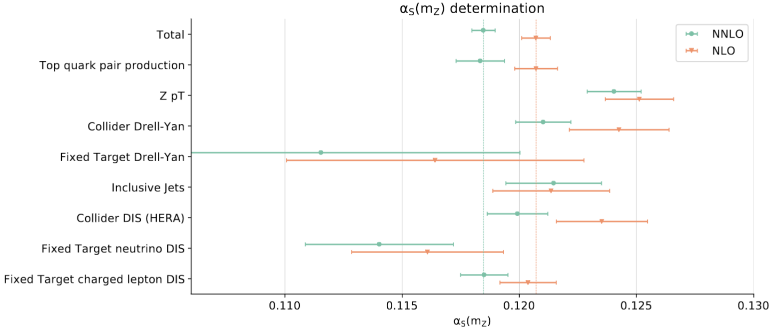
$$\chi_g^{2,(k)}(\alpha_s) = \chi^2\left(\alpha_s, \hat{\theta}_g^{(k)}(\alpha_s); Y_g^{(k)}\right)$$

minima of the interpolated curves yields $\hat{\alpha}_s^{(k)}$, if the fit has converged

distribution of α_s



processes - restricted χ^2



$$\chi_P^{r2,(k)}(\alpha_s) = \chi^2\left(\alpha_s, \hat{\theta}_g^{(k)}(\alpha_s), Y_P^{(k)}\right)$$

$\Longrightarrow$ for P *best-fit* values, we need $\chi_P^{2,(k)}(\alpha_s) = \chi^2\left(\alpha_s, \hat{\theta}_P^{(k)}(\alpha_s), Y_P^{(k)}\right)$

systematics I - replica selection

choice of $N_{\min}$

$N_{\min}$	$\alpha_s\,(m_Z)$	N_{rep}	Δ_{α_s}
18	$0.11842 \pm 0.00031\;(0.3\%)$	12	0.00009
15	$0.11844 \pm 0.00044\;(0.4\%)$	92	0.00005
6	$\mathbf{0.11845 \pm 0.00052\;(0.5\%)}$	**379**	**0.00003**
3	$0.11844 \pm 0.00056\;(0.5\%)$	400	0.00003

$$\Delta_{\text{sel}} = 0.00003(0.03\%)$$

systematics II - parametrization of χ^2

trimming the values of α_s in parabolic fits

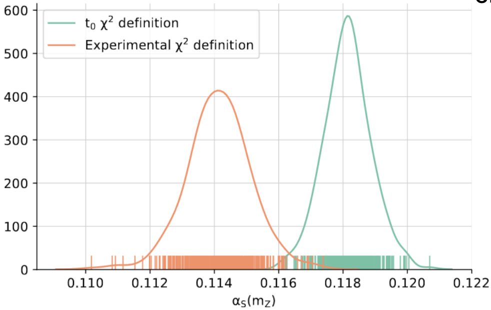
$N_{\rm trim}$	fitted $\alpha_s(m_Z)$ range	$\alpha_s(m_Z)$	$N_{\rm rep}$
0	$[\mathbf{0.106}, \mathbf{0.130}]$	$\mathbf{0.11845 \pm 0.00052}$ **(0.4%)**	**379**
2	$[0.108, 0.128]$	0.11846 ± 0.00045 (0.4%)	218
5	$[0.110, 0.126]$	0.11852 ± 0.00051 (0.4%)	290
10	$[0.114, 0.124]$	0.11869 ± 0.00046 (0.4%)	32
15	$[0.115, 0.120]$	0.11822 ± 0.00079 (0.7%)	10
4	$[0.113, 0.130]$	0.11850 ± 0.00058 (0.5%)	296
5	$[0.106, 0.124]$	0.11855 ± 0.00059 (0.5%)	197

different functional forms: $\chi^2(\alpha_s) \mapsto \chi^2\left(f(\alpha_s)\right)$

$$\Delta_{\rm par} = 0.00010(0.08\%)$$

systematics III - t_0 prior

using the t_0 covariance



choice of t_0 prior

t_0	$\alpha_s\,(m_Z)$	$N_{\rm rep}$
I	$0.11844 \pm 0.00052(0.4\%)$	379
II	$0.11845 \pm 0.00052(0.4\%)$	379
III	$0.11841 \pm 0.00051(0.4\%)$	356

$$\Delta_{t_0} = 0.00004(0.03\%)$$

summary

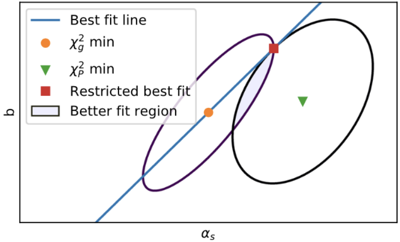
$$\alpha_s^{\mathrm{NNLO}}(m_Z) = 0.1180 \pm 0.0004 \ (0.3\%) \,,$$
$$\alpha_s^{\mathrm{NLO}}(m_Z) = 0.1203 \pm 0.0004 \ (0.3\%) \,.$$

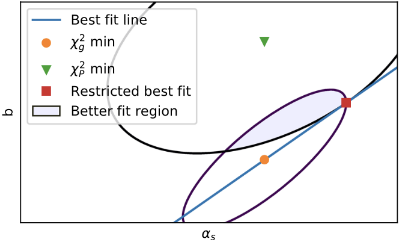
methodology uncertainty

$$\Delta_{\mathrm{meth}} = 0.00011(0.09\%)$$

MHOU: $\Delta_{\mathrm{pert}} = \left|\alpha_s^{\mathrm{NNLO}} - \alpha_s^{\mathrm{NLO}}\right| = 0.0022$

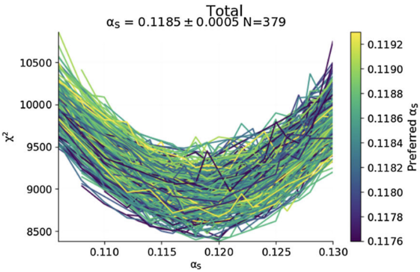
Forte & Kassabov (2020)

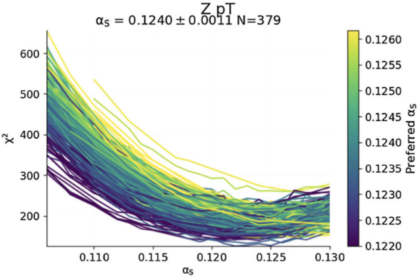


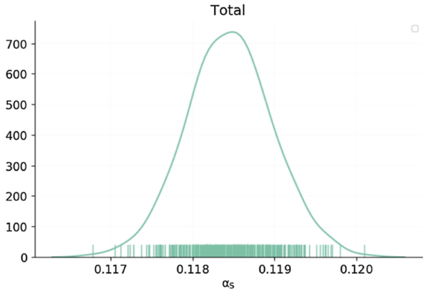


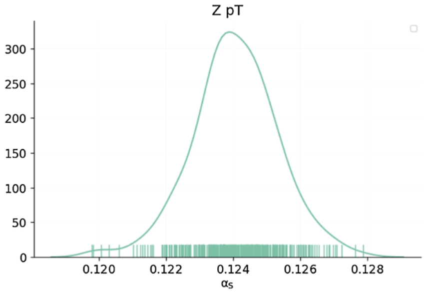
- the minimum of the restricted χ^{r^2} is not the best fit for P
- there are (α_s, θ) points that yield a better description of both P and the global dataset
- it is not even a 'compromise' between the best fits... (right plot)

results from restricted χ^{r^2}









conclusions

- PDFs and α_s need to be determined together
- latest global fit error dominated by MHOU [nnpdf 18]
- FK2020 consistent PDFs and α_s, no shortcut
- MHOU now incorporated in TH covariance [nnpdf 24]
- stay tuned for a global fit with MHOU
- analyses that do not refit the PDFs together with α_s are biased