

A brief review of Lepton Flavor Violation at the ILC



José I. Illana
CAFPE, U. Granada



-
1. Motivation
 2. Lepton flavor violating processes
 3. Lepton Flavor Violation from BSM Physics
 4. Probing LFV at the ILC
-

Motivation

Motivation

- Evidence of LFV: **neutrinos oscillate**

⇒ Neutrinos have a (small) mass and mix:

$$\nu_{L\alpha} \equiv \mathcal{U}_{\alpha i} \nu_i \quad \alpha = e, \mu, \tau \quad i = 1, 2, 3$$

⇒ LFV in the charged sector: **negligible** (one loop, GIM suppressed) in the SM

$$\ell_\alpha \rightarrow \ell_\beta : \quad \left| \frac{\alpha}{\pi} \sum_i \mathcal{U}_{\alpha i}^* \mathcal{U}_{\beta i} \frac{m_{\nu_i}^2}{M_W^2} \right|^2 \sim \left| \frac{\alpha}{\pi} \sin 2\theta \frac{\Delta m_\nu^2}{M_W^2} \right|^2 \sim 10^{-54}$$

Motivation

- Evidence of LFV: **neutrinos oscillate**

⇒ Neutrinos have a (small) mass and mix:

$$\nu_{L\alpha} \equiv \mathcal{U}_{\alpha i} \nu_i \quad \alpha = e, \mu, \tau \quad i = 1, 2, 3$$

⇒ LFV in the charged sector: **negligible** (one loop, GIM suppressed) in the SM

$$\ell_\alpha \rightarrow \ell_\beta : \quad \left| \frac{\alpha}{\pi} \sum_i \mathcal{U}_{\alpha i}^* \mathcal{U}_{\beta i} \frac{m_{\nu_i}^2}{M_W^2} \right|^2 \sim \left| \frac{\alpha}{\pi} \sin 2\theta \frac{\Delta m_\nu^2}{M_W^2} \right|^2 \sim 10^{-54}$$

- Opportunity for New Physics $[m_{\nu_i}^2 / M_W^2 \rightarrow f(m_{F_i}^2 / \Lambda_{NP}^2)]$

- Most NP models suffer the flavor problem: **new sources of flavor mixing**
- Lepton sector is more clean
- Examples: SUSY, Little Higgs, ...

LFV processes

- Where to look for LFV?
 - Rare lepton decays: [μ : MEGA/MEG, SINDRUM (PSI) | τ : Belle, BaBar, LHC]
 - radiative ($l_j \rightarrow l_i \gamma$),
 - leptonic ($l_j \rightarrow l_i l_k \bar{l}_k$, $l_j \rightarrow l_i l_i \bar{l}_k$),
 - semileptonic ($\tau \rightarrow \mu \eta$, $\tau \rightarrow \mu \rho$, $\tau \rightarrow \mu \pi \pi$, ...)
 - $\mu - e$ conversion in nuclei [SINDRUM2]
 - Decays of other standard particles:
 - $Z \rightarrow l_i \bar{l}_j$ [..., LEP, GigaZ]
 - $H \rightarrow l_i \bar{l}_j$ [..., LHC, ILC]
 - $D^0 \rightarrow l_i \bar{l}_j$, $B \rightarrow \mu e$, ...
 - Decays of non-standard particles: [LHC, ILC]
e.g. charged sleptons, sneutrinos, charginos, neutralinos

LFV processes

- Present and future limits:

LFV process	Present best UL (90% CL)	Future sensitivity
$\mathcal{B}(\mu \rightarrow e\gamma)$	1.2×10^{-11} [MEGA 1999] 2.8×10^{-11} [MEG 2010]	$10^{-13} - 10^{-14}$ [MEG]
$\mathcal{B}(\tau \rightarrow e\gamma)$	3.3×10^{-8} [BaBar 2010]	3×10^{-9} [SuperB]
$\mathcal{B}(\tau \rightarrow \mu\gamma)$	4.4×10^{-8} [BaBar 2010]	2.4×10^{-9} [SuperB]
$\mathcal{B}(\mu \rightarrow ee\bar{e})$	1×10^{-12} [SINDRUM 1988]	$10^{-13} - 10^{-14}$ [MEG]
$\mathcal{B}(\tau \rightarrow ee\bar{e})$	2.7×10^{-8} [Belle 2010]	$10^{-9} - 10^{-10}$ [SuperB]
$\mathcal{B}(\tau \rightarrow \mu\mu\bar{\mu})$	2.1×10^{-8} [Belle 2010]	$10^{-9} - 10^{-10}$ [SuperB]
$\mathcal{B}(\tau \rightarrow e\mu\bar{\mu})$	2.7×10^{-8} [Belle 2010]	$10^{-9} - 10^{-10}$ [SuperB]
$\mathcal{B}(\tau \rightarrow \mu e\bar{e})$	1.8×10^{-8} [Belle 2010]	$10^{-9} - 10^{-10}$ [SuperB]

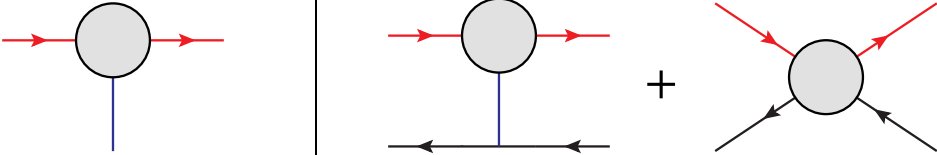
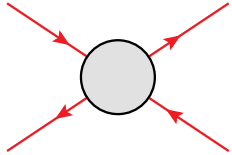
LFV processes

- Present and future limits:

LFV process	Present best UL (90% CL)	Future sensitivity
$\mathcal{R}(\mu \rightarrow e, \text{Au})$	7.0×10^{-13} [SINDRUM2 2004]	10^{-16} [Mu2E (Fermilab)]
$\mathcal{R}(\mu \rightarrow e, \text{Al})$		10^{-16} [COMET (J-PARC)]
$\mathcal{R}(\mu \rightarrow e, \text{Ti})$	4.3×10^{-12} [SINDRUM2 2004]	10^{-18} [PRISM/PRIME (J-PARC)]
$\mathcal{B}(Z \rightarrow \mu^\pm e^\mp)$	1.7×10^{-6} [LEP 1995]	2×10^{-9} [GigaZ]
$\mathcal{B}(Z \rightarrow \tau^\pm e^\mp)$	9.8×10^{-6} [LEP 1993]	$6.5\kappa \times 10^{-8}$ [GigaZ] $\kappa \in [0.2, 1]$
$\mathcal{B}(Z \rightarrow \tau^\pm \mu^\mp)$	1.2×10^{-5} [LEP 1997]	$2.2\kappa \times 10^{-8}$ [GigaZ]

LFV processes

- NP at 1-loop transitions:

	$\Delta F = 1$	$\Delta F = 2$
		
$\mu \rightarrow e\gamma$	$\mu \rightarrow ee\bar{e}$	
	$\mu - e$ conversion in nuclei	
$\tau \rightarrow e\gamma$ $\tau \rightarrow \mu\gamma$	$\tau \rightarrow ee\bar{e}$ $\tau \rightarrow e\mu\bar{\mu}$ $\tau \rightarrow \mu\mu\bar{\mu}$ $\tau \rightarrow \mu e\bar{e}$	$\tau \rightarrow ee\bar{\mu}$ $\tau \rightarrow \mu\mu\bar{e}$
$Z \rightarrow \mu\bar{e}, e\bar{\mu}$ $Z \rightarrow \tau\bar{e}, e\bar{\tau}$ $Z \rightarrow \tau\bar{\mu}, \mu\bar{\tau}$	\Uparrow	\Uparrow
$H \rightarrow \mu\bar{e}, e\bar{\mu}$ $H \rightarrow \tau\bar{e}, e\bar{\tau}$ $H \rightarrow \tau\bar{\mu}, \mu\bar{\tau}$	\Uparrow	\Uparrow
$e^+e^- \rightarrow l_i\bar{l}_j, \quad \gamma\gamma \rightarrow l_i\bar{l}_j$		

Lepton Flavor Violation from BSM Physics

- Origin of LFV: misalignment of fermion and sfermion mass matrices

– **Charged sleptons:** $\tilde{\ell}_{L_i}$ and $\tilde{\ell}_{R_i}$ superpartners of ℓ_{L_i} and ℓ_{R_i}

$$\mathbf{M}_{\tilde{\ell}}^2 = \begin{pmatrix} \mathbf{m}_{LL}^2 & \mathbf{m}_{LR}^{2+} \\ \mathbf{m}_{LR}^2 & \mathbf{m}_{RR}^2 \end{pmatrix} \quad \begin{aligned} (m_{LL}^2)_{ij} &= (m_{\tilde{L}}^2)_{ij} + \left[m_{\ell_i}^2 + \left(-\frac{1}{2} + \sin^2 \theta_W \right) M_Z^2 \cos 2\beta \right] \delta_{ij} \\ (m_{RR}^2)_{ij} &= (m_{\tilde{e}_R}^2)_{ij} + (m_{\ell_i}^2 - M_Z^2 \sin^2 \theta_W \cos 2\beta) \delta_{ij} \\ (m_{LR}^2)_{ij} &= (A_e)_{ij} v \cos \beta / \sqrt{2} - m_{\ell_i} \mu^* \tan \beta \delta_{ij} \end{aligned}$$

Slepton mass eigenstates:

$$\mathbf{S}^{\tilde{\ell}} \mathbf{M}_{\tilde{\ell}}^2 \mathbf{S}^{\tilde{\ell}^\dagger} = \mathbf{diag}(m_{\tilde{\ell}_X}^2) \quad X = 1, \dots, 6$$

$$\tilde{\ell}_X = S_{Xi}^{\tilde{\ell}} \tilde{\ell}_{L_i} + S_{X,i+3}^{\tilde{\ell}} \tilde{\ell}_{R_i} \quad X = 1, \dots, 6 \quad i = 1, 2, 3$$

- Origin of LFV: misalignment of fermion and sfermion mass matrices
 - **Sneutrinos**: $\tilde{\nu}_{L_\alpha}$ superpartners of ν_{L_α}

$$(M_{\tilde{\nu}}^2)_{\alpha\beta} = (m_{\tilde{L}}^2)_{\alpha\beta} + \frac{1}{2}M_Z^2 \cos 2\beta \delta_{\alpha\beta}$$

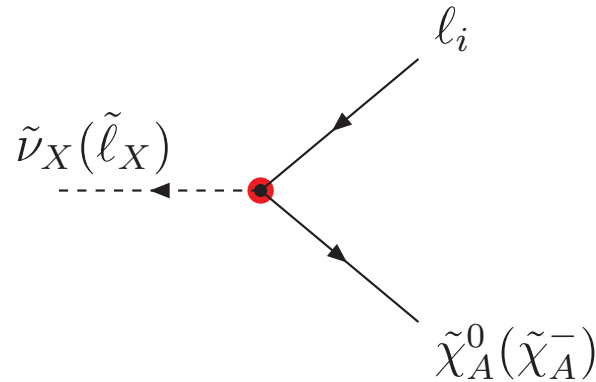
Sneutrino mass eigenstates:

$$\mathbf{S}^{\tilde{\nu}} \mathbf{M}_{\tilde{\nu}}^2 \mathbf{S}^{\tilde{\nu}\dagger} = \mathbf{diag}(m_{\tilde{\nu}_X}^2) \quad X = 1, 2, 3$$

$$\tilde{\nu}_X = S_{X\alpha}^{\tilde{\nu}} \tilde{\nu}_{L_\alpha} = S_{X\alpha}^{\tilde{\nu}} \mathcal{U}_{\alpha i} \tilde{\nu}_i \quad X = 1, 2, 3 \quad \alpha = e, \mu, \tau \quad i = 1, 2, 3$$

⇒ Off-diagonal terms are responsible for LFV

- LFV vertices $ig (c_{iAX}^L P_L + c_{iAX}^R P_R)$ where iAX label mass eigenstates



$$\begin{aligned}
 \tilde{\nu}_X^+ \tilde{\chi}_A^- \ell_i : \quad & c_{iAX}^{L[C]} = -V_{A1}^* S_{X\alpha}^{\tilde{\nu}} U_{\alpha i} \\
 & c_{iAX}^{R[C]} = \frac{m_{\ell_i}}{\sqrt{2} M_W \cos \beta} U_{A2} S_{X\alpha}^{\tilde{\nu}} U_{\alpha i} \\
 \tilde{\ell}_X^+ \tilde{\chi}_A^0 \ell_i : \quad & c_{iAX}^{L[N]} = \frac{1}{\sqrt{2}} (\tan \theta_W N_{A1}^* + N_{A2}^*) S_{Xi}^{\tilde{\ell}} - \frac{m_{\ell_i}}{\sqrt{2} M_W \cos \beta} N_{A3}^* S_{X,i+3}^{\tilde{\ell}} \\
 & c_{iAX}^{R[N]} = -\sqrt{2} \tan \theta_W N_{A1} S_{X,i+3}^{\tilde{\ell}} - \frac{m_{\ell_i}}{\sqrt{2} M_W \cos \beta} N_{A3} S_{Xi}^{\tilde{\ell}}
 \end{aligned}$$

Lepton Flavor Violation

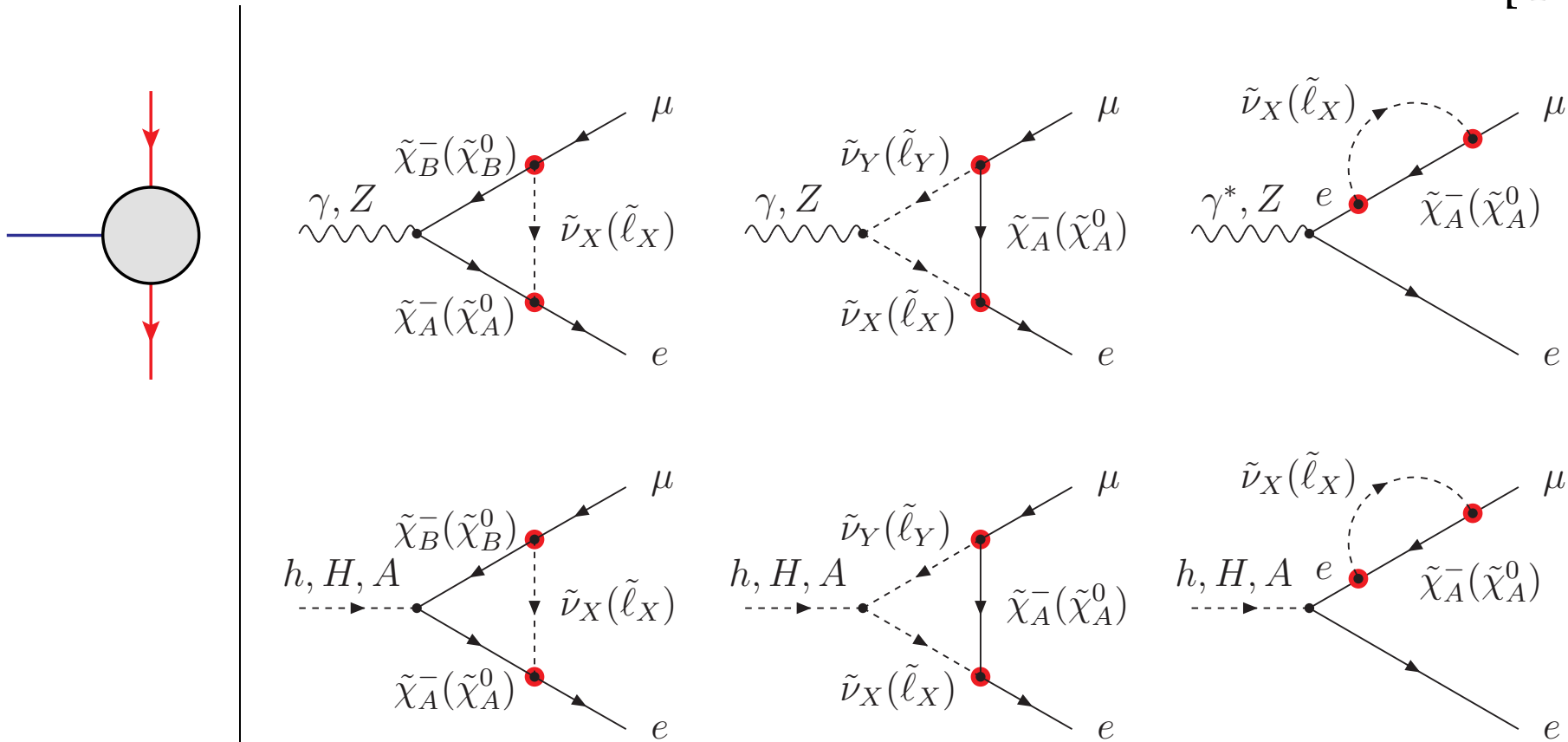
Supersymmetry

- One-loop transitions: bubbles and triangles ($\Delta F = 1$)

$[\mu \rightarrow e\gamma]$

$[Z \rightarrow e\bar{\mu}]$

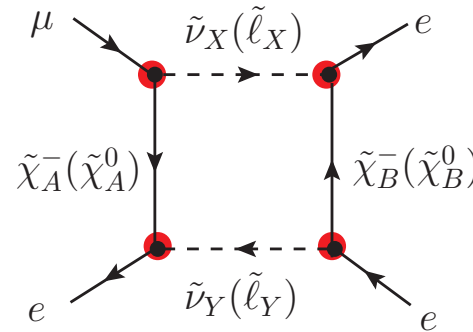
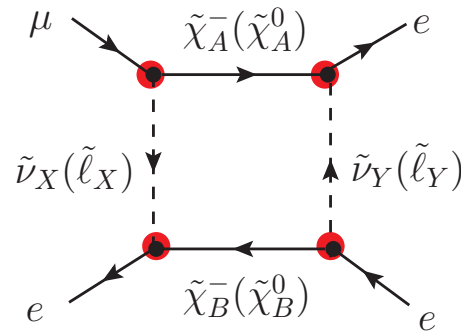
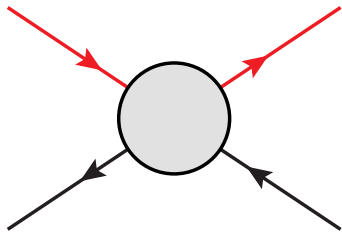
$[\Phi \rightarrow e\bar{\mu}]$



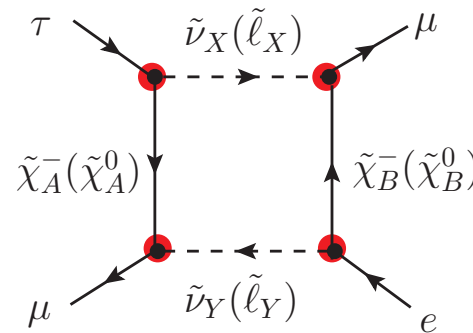
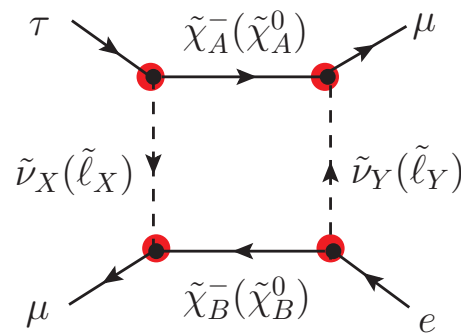
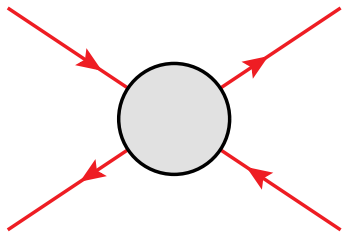
Lepton Flavor Violation

Supersymmetry

- One-loop transitions: boxes ($\Delta F = 1$ and $\Delta F = 2$)



$[\mu \rightarrow ee\bar{e}]$



$[\tau \rightarrow \mu\mu\bar{e}]$

- Tree level chain decays:

Production and decay of charged sleptons, sneutrinos, charginos and neutralinos

$$\begin{aligned}
 e^+e^- \rightarrow \tilde{\ell}_X^+ \tilde{\ell}_Y^- &\rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \tilde{\chi}_1^0 \\
 &\tilde{\nu}_X \tilde{\nu}_Y \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^+ \tilde{\chi}_1^- \\
 &\tilde{\chi}_A^0 \tilde{\chi}_B^0 \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \tilde{\chi}_1^0 \\
 &\tilde{\chi}_A^+ \tilde{\chi}_B^- \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp
 \end{aligned}$$

- The model(s)
 - The Higgs is a pseudo-Goldstone boson of an approximate global symmetry broken at a scale $f \sim \text{TeV}$
 - The global symmetry (partly gauged) gets explicitly broken by gauge and Yukawa interactions, giving the Higgs a mass and non-derivative couplings, still preserving the cancellation of one-loop quadratic corrections (*collective breaking*)
 - This introduces extra fermions and gauge fields (and extra Higgses): new source of flavor mixing

Assume the most popular model: **Littlest Higgs with T Parity**

The SM spectrum (T-even) is supplemented with:

★ T-odd fermion $SU(2)_L$ doublets $l_{HL} \equiv (\nu_{HL}, \ell_{HL})$ and $l_{HR} \equiv (\nu_{HR}, \ell_{HR})$

★ T-odd gauge bosons W_H, A_H, Z_H

- Origin of LFV: **missalignment of T-even and T-odd fermion mass matrices**

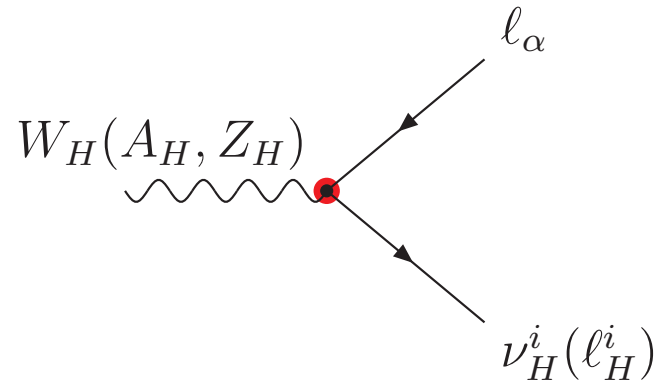
\mathcal{L}_{Y_H} generates heavy masses inducing **heavy-light mixings**:

$$\sqrt{2}f \bar{l}_{HL}^0 \mathbf{Y}_H l_{HR}^0 \xrightarrow{\text{diag}} l_{HL}^0 = V_H l_{HL} \quad l_{HR}^0 = U_H l_{HR}$$

$$\Rightarrow \mathcal{L}_{\text{LHT}} \sim g \bar{l}_{HL}^0 \mathcal{G}_H^+ l_L^0 = g (\bar{\nu}_{HL} V_H^+, \bar{\ell}_{HL} V_H^+) \mathcal{G}_H^+ \begin{pmatrix} \mathcal{U} \nu_L \\ \ell_L \end{pmatrix} \quad \begin{aligned} V_H^+ \mathcal{U} &\equiv V_{H\nu} \\ V_H^+ &\equiv V_{H\ell} \end{aligned}$$

$$\Rightarrow \boxed{V_{H\ell}^{i\alpha} \bar{\nu}_{HL}^i W_H^+ \ell_L^\alpha} \text{ CC and } \boxed{V_{H\ell}^{i\alpha} \bar{\ell}_{HL}^i \{A_H, Z_H\} \ell_L^\alpha} \text{ NC}$$

- LFV vertices $ig (g_L P_L + g_R P_R)$



	$W_H^+ \bar{\nu}_H^i \ell^\alpha$	$A_H \bar{\ell}_H^i \ell^\alpha$	$Z_H \bar{\ell}_H^i \ell^\alpha$
g_L	$\frac{1}{\sqrt{2} s_W} V_{H\ell}^\alpha$	$\left(\frac{1}{10 c_W} - \frac{x_H v^2}{2 s_W f^2} \right) V_{H\ell}^\alpha$	$-\left(\frac{1}{2 s_W} + \frac{x_H v^2}{10 c_W f^2} \right) V_{H\ell}^\alpha$
g_R	0	0	0

↑↑

Identical to SM with heavy neutrinos, replacing $V_{H\ell}$ by \mathcal{U} and W_H by W

Lepton Flavor Violation

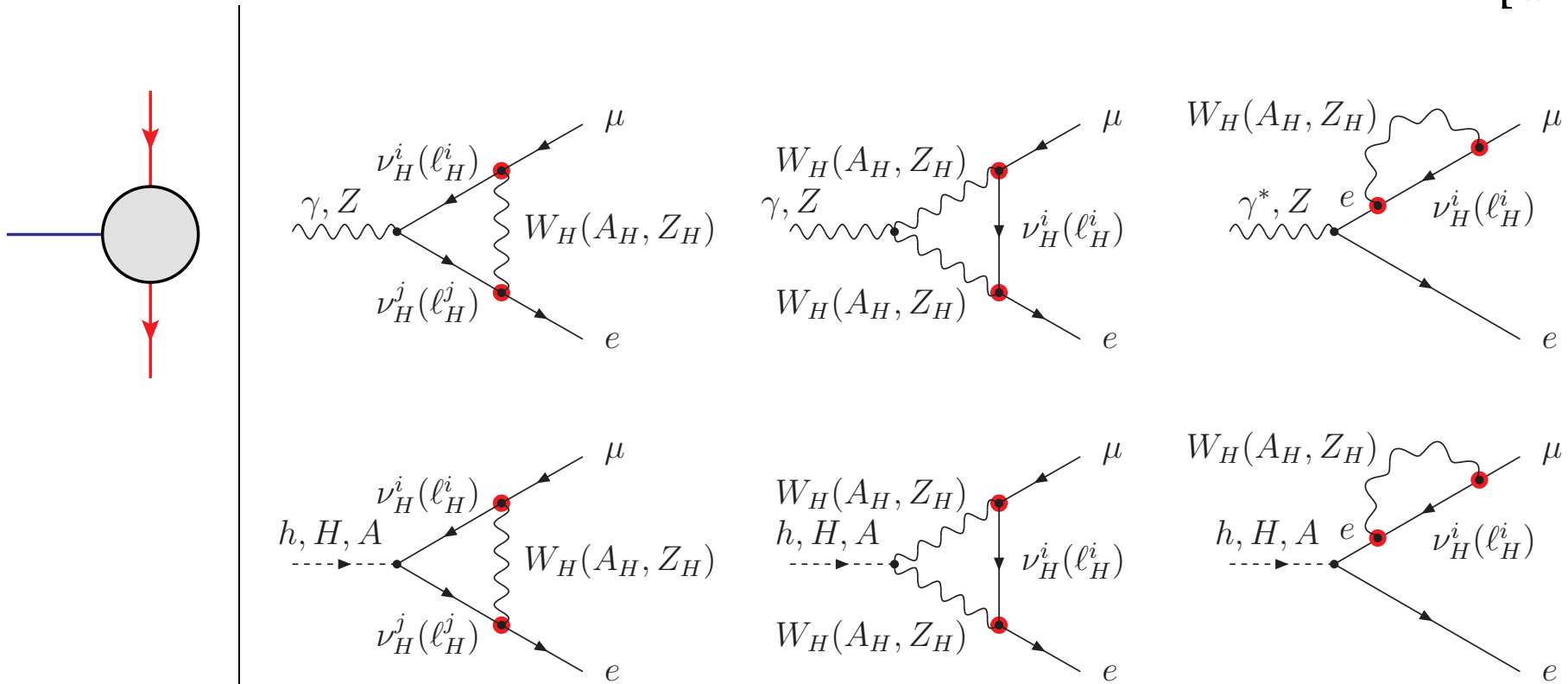
Littlest Higgs with T Parity

- One-loop transitions: bubbles and triangles ($\Delta F = 1$)

$[\mu \rightarrow e\gamma]$

$[Z \rightarrow e\bar{\mu}]$

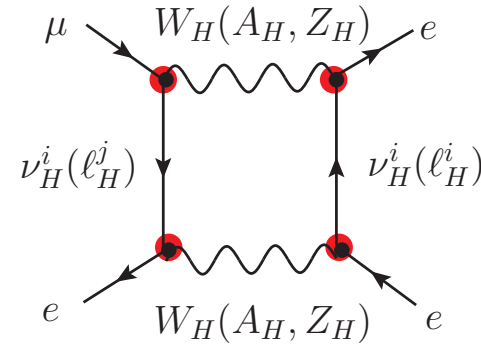
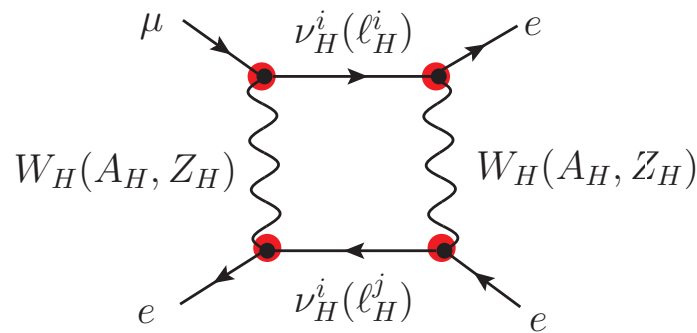
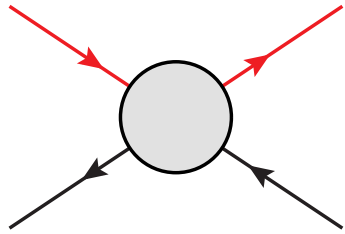
$[\Phi \rightarrow e\bar{\mu}]$



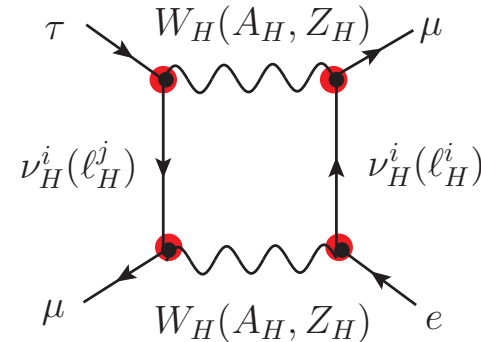
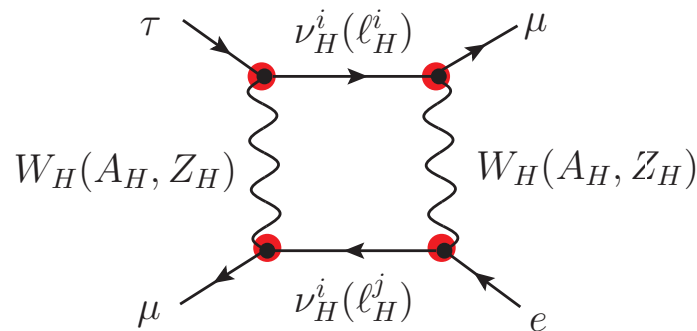
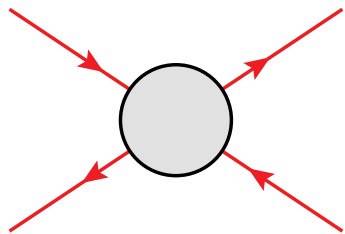
Lepton Flavor Violation

Littlest Higgs with T Parity

- One-loop transitions: boxes ($\Delta F = 1$ and $\Delta F = 2$)



$[\mu \rightarrow ee\bar{e}]$



$[\tau \rightarrow \mu\mu\bar{e}]$

Probing LFV at the ILC

- Guchait, Kalinowski, Roy [EPJC 21 (2001) 163]

Deppisch, Päs, Redelbach, Rückl, Shimizu [PRD 69 (2004) 054014]

Production and decay of sleptons, charginos and neutralinos (SUSY seesaw)

$$\begin{aligned}
 e^+e^- \rightarrow \tilde{l}_i^+ \tilde{l}_j^- &\rightarrow l_\alpha^+ l_\beta^- \tilde{\chi}_1^0 \tilde{\chi}_1^0 & \text{Signal} &= l_\alpha^+ l_\beta^- + \cancel{E}_T \\
 &\tilde{\nu}_i \tilde{\nu}_j^c &&\rightarrow l_\alpha^+ l_\beta^- \tilde{\chi}_1^+ \tilde{\chi}_1^- && l_\alpha^+ l_\beta^- + 4\text{jets} + \cancel{E}_T \\
 &\tilde{\chi}_a^0 \tilde{\chi}_b^0 &&\rightarrow l_\alpha^+ l_\beta^- \tilde{\chi}_1^0 \tilde{\chi}_1^0 && l_\alpha^+ l_\beta^- + l + 2\text{jets} + \cancel{E}_T \\
 &\tilde{\chi}_a^+ \tilde{\chi}_b^- &&\rightarrow l_\alpha^+ l_\beta^- \tilde{\chi}_1^\pm \tilde{\chi}_1^\mp && \text{(purely leptonic overwhelmed by bkgd)}
 \end{aligned}$$

(Background from $e^+e^- \rightarrow t\bar{t}g$ reduced by appropriate cuts)

- Hohenwater-Sodek, Kernreiter [JHEP 06 (2007) 071]

Effects of LFV on chargino production

- $\sigma(e^+e^- \rightarrow \chi_a^+ \chi_b^-)$ at $\sqrt{s} = 500$ GeV and assuming beam polarization enhanced up to a factor 2 within limits from $\tau \rightarrow \mu\gamma$ due to LFV $\tilde{\nu}_X$ exchange (t -channel)

- Ibarra, Roy [JHEP 05 (2007) 059]

Sensitivity of future e^+e^- and e^-e^- colliders to LFV in scenarios with a stau NLSP

- **Signal:** two heavily ionizing tracks due to the long-lived staus accompanied by two (LFV in production) or four (LFV in selectron decay) charged leptons
- Sensitivity (better e^-e^- than e^+e^-) competitive with the present and projected sensitivities to LFV from rare tau decays, although not from $\mu \rightarrow e\gamma$

- Kanemura et al [PLB 599 (2004) 2004]

Search for LFV in Higgs boson decay at a Linear Collider

$$e^+e^- \rightarrow Z^* \rightarrow Zh^0 \rightarrow Z\tau^\pm\mu^\mp$$

- After taking appropriate kinematic cuts, the number of background events is considerably reduced, so that the signal can be visible when the branching ratio of $h^0 \rightarrow \tau^\pm\mu^\mp$ is larger than about 10^{-4} (10^{-3} in the 2HDM)
(Energy tuned to optimize Higgsstrahlung ($\sqrt{s} = M_Z + 2m_h$) and $\mathcal{L} = 1 \text{ ab}^{-1}$)

- *Canoni, Panella, Kolb [LCWS 04]*

Majorana neutrino and light sneutrino contributions to $e^-e^- \rightarrow \mu e, \tau e$

Majorana $\nu \lesssim 2 - 3$ TeV feasible, and $\sigma \sim 10^{-2}$ fb in SUSY reachable (polariz.)

Probing LFV at the ILC

Supersymmetry

- [JI, Masip \[PRD 67 \(2003\) 035004\]](#)

LFV Z decays from SUSY

- GigaZ: $Z \rightarrow \mu e$ invisible due to $\mu \rightarrow e\gamma$ constraint, but $Z \rightarrow \mu e$ at reach

Probing LFV at the ILC

Heavy neutrinos

- [JI, Riemann \[PRD 63 \(2001\) 053004\]](#)

LFV Z decays from heavy neutrinos

- GigaZ: $Z \rightarrow \ell_i^\pm \ell_j^\mp$ visible if heavy neutrinos are lighter than the TeV and mix enough to light neutrinos

- Wei Ma, Chong-Xing Yue, Jiao Zhang, and Yan-Bin Sun [1010.0052]

Jinzhong Han, Xuelei Wang, Bingfang Yang [1101.3598]

$e^+e^- \rightarrow \ell_i^\pm \ell_j^\mp$ and $\gamma\gamma \rightarrow \ell_i^\pm \ell_j^\mp$ and in the LHT at the ILC

(mediated by LFV in $\gamma\ell_i\ell_j$ and $Z\ell_i\ell_j$ and constrained by rare lepton decays)

– $\sigma(e^+e^- \rightarrow \ell_i^\pm \ell_j^\mp)$ can reach several fb

– $\sigma(\gamma\gamma \rightarrow \ell_i^\pm \ell_j^\mp)$ can reach 0.1 – 1 fb

- Chong-Xing Yue and Shuang Zhao [EPJ C50 (2007) 897]

Signals of the LHT doubly charged scalar triplet Φ from $e^+e^- \rightarrow \ell_i^\pm \ell_j^\mp$

(mediated by the tree level subprocess $e^-e^- \rightarrow \Phi^{--} \rightarrow \ell_i^- \ell_j^-$)

- At $\sqrt{s} = 500$ GeV the LFV contribution is small
but Φ can give a significant contribution to $e^+e^- \rightarrow \ell_i^\pm \ell_i^\mp$

- Ferreira, Guedes, Santos [PRD 75 (2007) 055015]

Different LFV decay widths and the cross sections $\sigma(e^+e^- \rightarrow \ell_i^\pm \ell_j^\mp)$ are compared using dimension-six effective operators

- ILC ($\sqrt{s} = 1 \text{ TeV}$, $\mathcal{L} = 1 \text{ ab}^{-1}$) explores $\Lambda \lesssim 100 \text{ TeV}$.
- Limits from $Z \rightarrow \ell_i^\pm \ell_j^\mp$ allow enough events in $e^+e^- \rightarrow \ell_i^\pm \ell_j^\mp$ even from future bounds, but if the four-fermion coupling is dominant $\mathcal{B}(\mu \rightarrow ee\bar{e}) < 10^{-12}$ already kills $e^+e^- \rightarrow e^\pm \mu^\mp$
- $\sigma(\gamma\gamma \rightarrow \ell_i^\pm \ell_j^\mp)$ exceedingly small from constraints on $\ell_j \rightarrow \ell_i \gamma$
- GigaZ will improve the limits on the LFV Z decays but is insensitive to the four-fermion operators