Workshop on Tensions in Cosmology
Corfu Summer Institute
6-13 September - 2023

Addressing the cosmological tensions with a majoron model

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New physics?

Even ignoring:
☐ (more or less) compelling theoretical motivations (quantum gravity theory, flavour problem, hierarchy and naturalness problems,…) and
☐ Experimental anomalies (e.g., $(g-2)_\mu$, R_K, R_K^*,…)

Standard physics (SM+GR) cannot explain:

- *Cosmological Puzzles* :

1. Dark matter
2. Matter - antimatter asymmetry
3. Inflation
4. Accelerating Universe at present

- *Neutrino masses and mixing*

problem of the origin of matter in the universe

Cosmological tensions offer an exciting opportunity to understand the nature of cosmological puzzles and shed light on the origin of neutrino masses and mixing

Minimal seesaw mechanism (type I)

- Dirac + (right-right) Majorana mass term

(Minkowski '77; Gell-mann,Ramond,Slansky; Yanagida; Mohapatra,Senjanovic '79)

Dirac Majorana

$$-\mathcal{L}^{\nu}_{mass} = \underbrace{\overline{\nu}_L m_D \nu_R}_{\text{Dirac}} + \underbrace{\frac{1}{2}\overline{\nu^c_R}\, M\, \nu_R}_{\text{Majorana}} + h.c. = -\frac{1}{2}\left(\overline{\nu_L}\ \overline{\nu^c_R}\right)\begin{pmatrix} 0 & m_D^T \\ m_D & M \end{pmatrix}\begin{pmatrix} \nu_L \\ \nu^c_R \end{pmatrix} + h.c.$$

In the *see-saw limit* (M >> m_D) the mass spectrum splits into 2 sets:

- 3 light **Majorana neutrinos** with masses (seesaw formula):

$$m_\nu = -m_D M^{-1} m_D^T \Rightarrow \mathrm{diag}(m_1, m_2, m_3) = -U^\dagger m_\nu U^*$$

- 3(?) heavier "seesaw" neutrinos **N_1, N_2, N_3** with **$M_3 > M_2 > M_1$**

- LH-RH (active-sterile) neutrino mixing

$$\nu_{1L} \simeq U^\dagger_{1\alpha}\left(\nu_{L\alpha} - \frac{m_{D\alpha 1}}{M_1}\nu^c_{R1}\right)$$

$$N_{1R} \simeq \nu_{1R} + \frac{m_{D\alpha 1}}{M_1}\nu^c_{L\alpha}$$

→ lightest seesaw neutrino

This active-sterile neutrino mixing has different phenomenological applications

Extra (or dark) Radiation

$$\varrho_R(T) = g_\rho(T)\frac{\pi^2}{30}T^4$$

$$g_\rho(T) = g_\rho^{SM}(T) + \Delta g_\varrho(T)$$

extra (or dark) radiation degrees of freedom

$$\Delta g_\rho(T) \equiv \frac{7}{4}\Delta N_\nu(T)\left(\frac{T_\nu}{T}\right)^4$$

effective number of (extra-)neutrino species

$$\Delta N_\nu(T) \equiv N_\nu(T) - N_\nu^{SM}(T)$$

$$N_\nu^{SM}(T \gg m_e) = 3 \qquad N_\nu^{SM}(T \ll m_e) = 3.043$$

(Cielo,Escudero,Mangano,Pisanti 2306.05460)

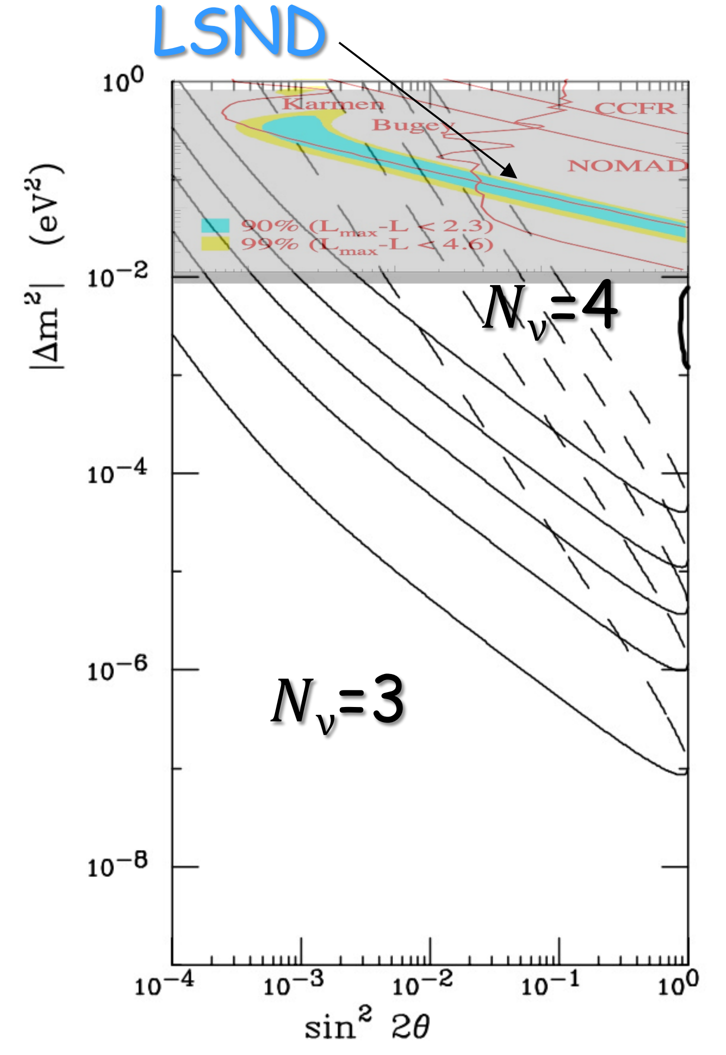
three traditional ways to get information on ΔN_ν:

- BBN + $Y_p \Rightarrow \Delta N_\nu\,(T_{fr})$
- BBN + D/H $\Rightarrow \Delta N_\nu\,(T_{nuc})$
- CMB anisotropies $\Rightarrow \Delta N_\nu\,(T_{rec})$

Example: active-sterile neutrino oscillations in the early universe

(Dolgov '81; Enqvist,Kainulainen '90; Barbieri Dolgov '90; Cline '92)

- In order to explain the LSND anomaly the sterile neutrino gets fully thermalized: N_ν=4
- WMAP7 data N_ν=4.34±0.85
- Combining with BBN data⇒ case for a sterile neutrino "friendly" cosmology
  (Hamann et al. 1006.5276)
- WMAP9 data were still compatible with ΔN_ν~1 (N_ν =3.84±0.40)
- though BBN less friendly, still caveats justifying ΔN_ν~1
- *Planck* data were eagerly awaited!



(from PDB et al. hep-ph/9907548)

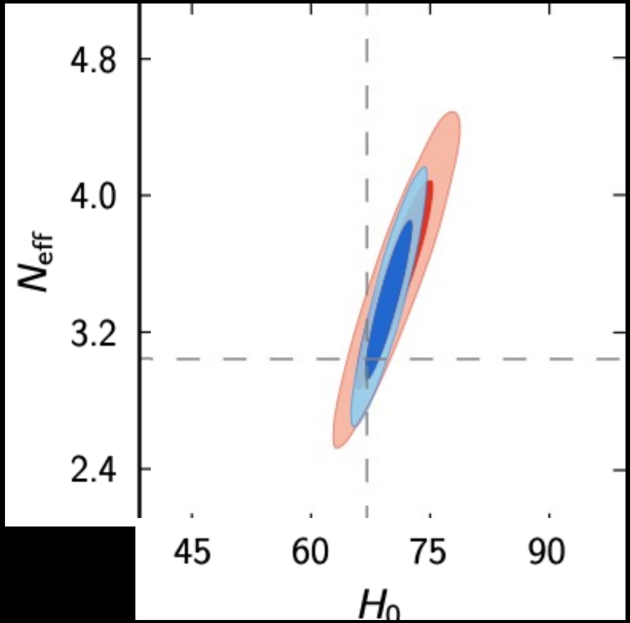
Rising of the Hubble tension and fractional N_ν

$$H_0^{(Planck13)} = 67.3 \pm 1.2 \text{ km s}^{-1}\text{Mpc}^{-1}$$

$$N_\nu^{(Planck13)} = 3.36 \pm 0.34$$

$$H_0^{(SNe)} = 73.8 \pm 2.4 \text{ km s}^{-1}\text{Mpc}^{-1}$$

$$N_\nu^{(Planck13+SNe)} = 3.62 \pm 0.25$$



(from Planck2013 1303.5076)

Many proposed models for $\Delta N_\nu(T_{rec}) \sim 0.5$:

- long-lived particle decays **(PDB, S.F. King, A. Merle 1303.6267)**
- **Axionic dark radiation (J.Conlon, M.C. David Marsh, 1304.1804)**
- Goldstone boson **(S. Weinberg 1305.1971)**
- …………

Cosmological tensions: beyond a fractional N_ν

Different cosmological tensions tension (talk by Leandros Perivolaropouros)

- Hubble tension:

$$H_0^{(P18)} = 67.66 \pm 0.42 \text{ km s}^{-1}\text{Mpc}^{-1} \xleftrightarrow{\sim 5\sigma\ \text{tension}} H_0^{(SH0ES)} = 73.30 \pm 1.04 \text{ km s}^{-1}\text{Mpc}^{-1}$$

- Growth tension
- Cosmic dipoles
- CMB anisotropy anomaly

A model should improve the ΛCDM baseline model rather than solve one tension in isolation.

The majoron model is quite well motivated in particle physics: it explains the generation of the neutrino Majorana masses in the seesaw model, as the Higgs mechanism explains the Dirac masses

Majorana mass generation in the Majoron model

(Y. Chikashige, R. Mohapatra, R. Peccei 1981)

$$-\mathcal{L}_{N_I+\sigma} = \overline{L_\alpha}\, h_{\alpha I}\, N_I\, \tilde{\Phi} + \frac{\lambda_I}{2}\sigma\, \overline{N_I^c}\, N_I + V_0(\sigma) + h.c. \quad (respecting\, U_L(1)\, symmetry)$$

$$\sigma = \frac{1}{\sqrt{2}}(\sigma_1 + i\sigma_2)\,, \qquad <\sigma> = \frac{v_T}{\sqrt{2}}$$

Typically one assumes that the σ-phase transition occurs before EWSB

At the end of the σ-phase transition, after SSB, L is violated and

$$\sigma = \frac{e^{i\theta}}{\sqrt{2}}(v_0 + S + iJ) \qquad M_I = \lambda_I \frac{v_0}{\sqrt{2}} \sim M \;\; (seesaw\ scale)$$

Dirac neutrino mass matrix $m_D = v_{ew}\, h/\sqrt{2}$ generated after EWSB

After both symmetry breakings: $m_\nu = -\frac{v_{ew}^2}{2}\frac{h_{\alpha I} h_{\beta I}}{M_I}$

S is a massive boson while J is a massless (Goldstone) boson referred to as **majoron**

DARK SECTOR ≡ N_I 's + J + S   VISIBLE SECTOR ≡ SM particles

Majoron model at low energies and neutrino rethermalisation

(Chacko,Hall,Okui,Oliver hep-ph 0312267, PDB, Rahat 2307.03184)

- Let us now assume that the temperature of the σ-phase transition T_* occurs not only after the EWSB but even after neutrino decoupling ($T_* \lesssim 1$ MeV)
- This low energy phase transition generates Majorana masses for N′ light RH neutrinos (minimal case N′ = 1)
- At these temperatures ordinary neutrinos interact with the Majoron and η, that can be regarded as another majoron that was produced during a high energy phase transition:

$$-\mathcal{L}_{\nu-\mathrm{dark}} = \frac{i}{2}\sum_{i=2,3}\lambda_i\,\overline{\nu_i}\,\gamma^5\,\nu_i\,\eta + \frac{i}{2}\,\lambda_1\overline{\nu_1}\,\gamma^5\,\nu_1\,J + \mathrm{h.c.}\,,$$

- These interactions couple neutrinos to majorons, so that the dark sector thermalises prior to the phase transition to a common temperature T_D:

$$r_{\nu-\mathrm{D}} \equiv \frac{T_{\mathrm{D}}}{T_\nu} = \left(\frac{3.043}{3.043 + N' + 12/7 + 4\,\Delta g/7}\right)^{\frac{1}{4}}$$

contribution from η and σ

- Minimal case: N′ = 1 and $\Delta g=0 \Rightarrow r_{\nu\text{-}D}=0.815$
- Notice that T_ν denotes the standard neutrino temperature

Constraint from BBN + Deuterium abundance

(PDB, Rahat 2307.03184)

$$g_\rho(T) = g_\rho^{\gamma+e^\pm+3\nu}(T) + \frac{7}{4}\Delta N_\nu(T)\left(\frac{T_\nu}{T}\right)^4$$

- Prior to neutrino rethermalisation, above neutrino decoupling, ΔN_ν is negligible
- After the phase transition and the decay of N_h massive particles (S + N′ right-handed neutrinos):

$$\Delta N_\nu \simeq 3.043\left[\left(\frac{3.043+N'+12/7+4\Delta g/7}{3.043+N'+12/7+4\Delta g/7-N_\mathrm{h}}\right)^{\frac{1}{3}}-1\right]$$

- For Δg = 0, 1, 2, 3 ⇒ ΔN_ν= 0.46, 0.41, 0.37, 0.33

For $T_* > T_{nuc} \simeq 65$ keV one has to confront BBN+D/H constraint. There are 2 different results:

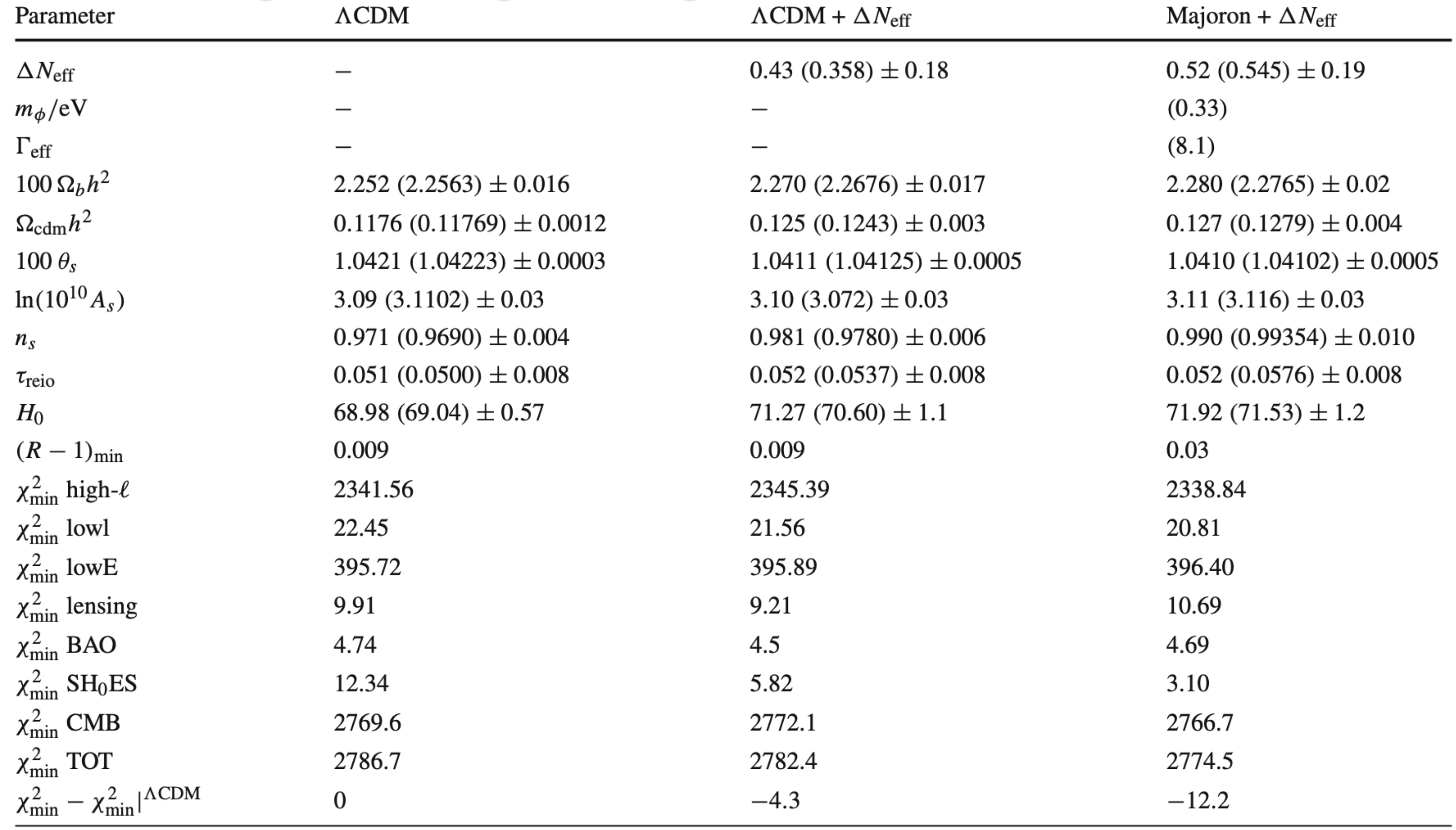
- $\Delta N_\nu(T_{nuc}) = -0.05 \pm 0.22 \Rightarrow \Delta N_\nu(t_{nuc}) \lesssim 0.4$ (95% C.L.) **(Pisanti et al. 2011.11537)**
- $\Delta N_\nu(T_{nuc}) = 0.3 \pm 0.15$ **(Pitrou et al. 2011.11320)**

The model can nicely address this potential *Deuterium problem*

Confronting the cosmological tensions

(M.Escudero, S. Whitte 1909.04044)

In addition to extra radiation, it also couples the majoron background to neutrinos reducing r_s allowing for larger H_0



Parameter	ΛCDM	ΛCDM + $\Delta N_{\rm eff}$	Majoron + $\Delta N_{\rm eff}$
$\Delta N_{\rm eff}$	–	0.43 (0.358) ± 0.18	0.52 (0.545) ± 0.19
m_ϕ/eV	–	–	(0.33)
$\Gamma_{\rm eff}$	–	–	(8.1)
$100\,\Omega_b h^2$	2.252 (2.2563) ± 0.016	2.270 (2.2676) ± 0.017	2.280 (2.2765) ± 0.02
$\Omega_{\rm cdm} h^2$	0.1176 (0.11769) ± 0.0012	0.125 (0.1243) ± 0.003	0.127 (0.1279) ± 0.004
$100\,\theta_s$	1.0421 (1.04223) ± 0.0003	1.0411 (1.04125) ± 0.0005	1.0410 (1.04102) ± 0.0005
$\ln(10^{10} A_s)$	3.09 (3.1102) ± 0.03	3.10 (3.072) ± 0.03	3.11 (3.116) ± 0.03
n_s	0.971 (0.9690) ± 0.004	0.981 (0.9780) ± 0.006	0.990 (0.99354) ± 0.010
$\tau_{\rm reio}$	0.051 (0.0500) ± 0.008	0.052 (0.0537) ± 0.008	0.052 (0.0576) ± 0.008
H_0	68.98 (69.04) ± 0.57	71.27 (70.60) ± 1.1	71.92 (71.53) ± 1.2
$(R-1)_{\rm min}$	0.009	0.009	0.03
$\chi^2_{\rm min}$ high-ℓ	2341.56	2345.39	2338.84
$\chi^2_{\rm min}$ lowl	22.45	21.56	20.81
$\chi^2_{\rm min}$ lowE	395.72	395.89	396.40
$\chi^2_{\rm min}$ lensing	9.91	9.21	10.69
$\chi^2_{\rm min}$ BAO	4.74	4.5	4.69
$\chi^2_{\rm min}$ SH$_0$ES	12.34	5.82	3.10
$\chi^2_{\rm min}$ CMB	2769.6	2772.1	2766.7
$\chi^2_{\rm min}$ TOT	2786.7	2782.4	2774.5
$\chi^2_{\rm min} - \chi^2_{\rm min}\|^{\Lambda\rm CDM}$	0	−4.3	−12.2

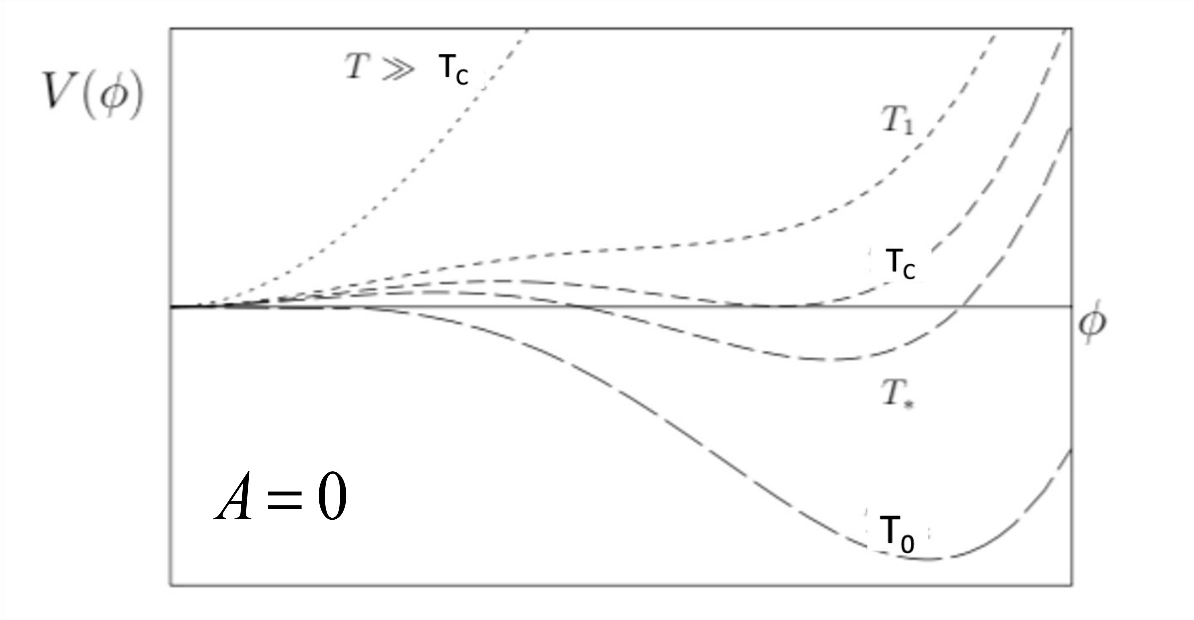
Significant improvement compared to the ΛCDM model but new calculations neutrino-majoron interaction rate seems to reduce the statistical significance **(S. Sandner, M.Escudero, S. Whitte 2305.01692)**

First order phase transition in the early universe

(Kirzhnits,Linde '72; Dolan, Jackiw '74; Anderson, Hall '92; Dine et al. '92; Quiros '98, Curtin et al. 2016)

effective potential
$$V(\phi,T)=V_{\text{tree}}(\phi)+\sum_i \overset{\nearrow \text{1-loop zero T}}{V^i_{\text{CW}}}(\phi)+\sum_i V^i_{\text{T}}(\phi,T) \quad \nearrow \text{1 loop thermal potential}$$

$$\simeq D(T-T_0)^2\phi^2-(ET+A)\,\phi^3+\frac{\lambda(T)}{4}\phi^4+\ldots$$



This picture relies on the validity of perturbative expansion and in the SM, at the EWSB, this would imply $M_H < M_W$. With the large M_H measured value, there is not even a PT in the SM, just a smooth crossover.

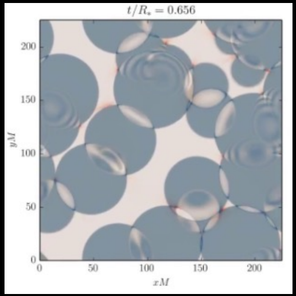
From the Euclidean action to the GW spectrum

(Kamionkowski,Kosowsky,Turner '93;Apreda et al 2001; Grogejan,Servant 2006; Ellis,Lewicki,No 2020)

time and temperature of nucleation

$$\int_o^{t_*} \frac{dt\,\Gamma}{H^3} \sim 1 \Rightarrow \int_{T_*}^{\infty} \frac{dT}{T}\left(\frac{90}{8\pi^3 g_*}\right)^2 \left(\frac{T}{M_{\mathrm{P}}}\right)^4 e^{-S_3/T} = 1 \Rightarrow \frac{S_3(T_*)}{T_*} \approx -4\ln\left(\frac{T_*}{M_{\mathrm{P}}}\right) \Rightarrow T_*$$

More precisely T* has to be identified with the *percolation temperature,* Slightly more involved definition than the nucleation temperature



$$\beta = \frac{\dot{\Gamma}}{\Gamma}, \quad \Gamma = \Gamma_0\, e^{-S(t)} \simeq \Gamma_0 e^{-S(t_*)}\, e^{-\left.\frac{dS}{dt}\right|_{t_*}(t-t_*)} \Rightarrow \beta \simeq -\left.\frac{dS}{dt}\right|_{t_*} \Rightarrow \frac{\beta}{H_*} = T_* \left.\frac{d(S_3/T)}{dT}\right|_{T_*}$$

Notice that $\beta/2\pi$ gives the characteristic frequency f* of the FOPT while β the time scale of its duration

Latent heat freed in the PT

$$\varepsilon = -\Delta V(\phi) - T\Delta s = V(\phi_{\mathrm{false}}) - V(\phi_{true}) + T\frac{\partial V}{\partial T} \Rightarrow \alpha = \frac{\varepsilon(T_*)}{\rho_R(T_*)}$$

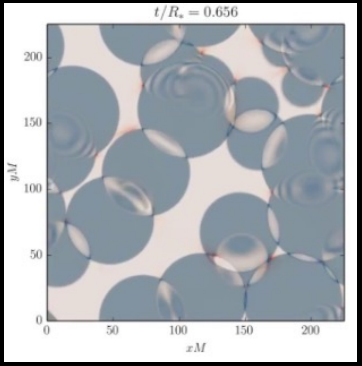
Strength of the PT

In our case we also need $\alpha_D = \varepsilon(T_*)/\rho_{R,D}(T_*) > \alpha$

From α and β/H* one can calculate the GW spectrum

Gravitational waves from first order phase transitions

(Hindmarsh et al. 1704.05871; D. Weir 1705.01783, PDB, Rahat 2307.03184)



GW spectrum

$$h^2\,\Omega_{\rm GW0}(f) = \frac{1}{\rho_{c0}h^{-2}}\,\frac{d\rho_{\rm GW0}}{d\ln f}$$

3 (known) contributions: bubble wall collision, sound waves and turbulence but in the case of a PTA in the dark sector the sound wave contribution is the dominant one, approximately:

$$h^2\Omega_{\rm sw0}(f) = 1.845{\times}10^{-6}\,\frac{\widetilde{\Omega}_{\rm gw}}{10^{-2}}\,\frac{v_{\rm w}(\alpha)}{\beta/H_\star}\left[\frac{\kappa(\alpha_{\rm D})\,\alpha}{1+\alpha}\right]^2\left(\frac{15.5}{g'_{s\star}}\right)^{4/3}\left(\frac{g'_{\rho\star}}{15.5}\right)\,S_{\rm sw}(f)\,\Upsilon(\alpha,\alpha_{\rm D},\beta/H_\star)\,.$$

Peak frequency

$$f_{\rm sw} = 8.9\,\mu{\rm Hz}\,\frac{1}{v_{\rm w}}\,\frac{\beta}{H_\star}\,\frac{T_\star}{100\,{\rm GeV}}\left(\frac{g^\star_\rho}{106.75}\right)^{1/6}$$

Spectral shape

$$S_{\rm sw}(f) = (f/f_{\rm sw})^3\left(\frac{7}{4+3\,(f/f_{\rm sw})^2}\right)^{7/2}$$

Bubble wall velocity (detonation)

$$v_w = \frac{\sqrt{1/3}+\sqrt{\alpha^2+2\alpha/3}}{1+\alpha} \geq c_s$$

Efficiency factor

$$\kappa(\alpha_{\rm D}) \simeq \frac{\alpha_{\rm D}}{0.73+0.083\sqrt{\alpha_{\rm D}}+\alpha_{\rm D}}$$

Approximately the GW spectrum depends on β/H_* , α and α_D . They can be derived from the effective potential for a given choice of the parameters of the model

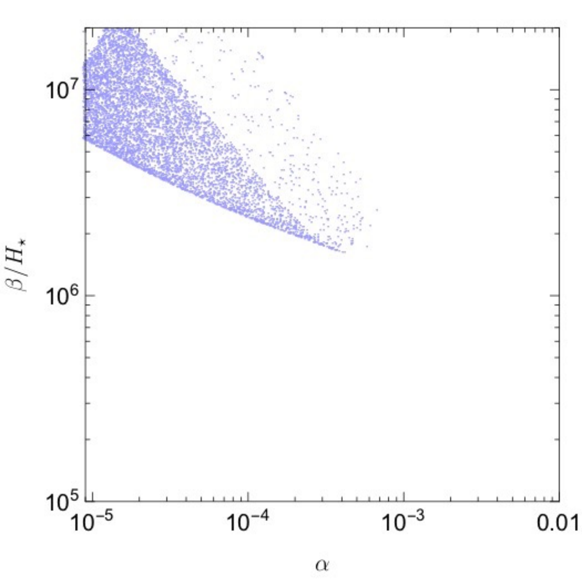
The minimal model

$$V_0(\sigma) = -\mu^2 |\sigma|^2 + \lambda |\sigma|^4 \;\Rightarrow\; v_0 = \sqrt{\mu^2/\lambda} \qquad (\lambda, \mu^2 > 0)$$

J is a massless Majoron and S has a mass $m_S=(2\lambda)^{1/2}v_0$

For the one-loop finite temperature effective potential one finds a polynomial

$$V_{\rm eff}^T(\sigma_1) \simeq D\,(T^2 - T_0^2)\sigma_1^2 - A\,T\,\sigma_1^3 + \frac{1}{4}\lambda_T\,\sigma_1^4\,,$$



The GW signal turns out to be a few many order of magnitude below the experimental sensitivity of any experiment

Split majoron model

(PDB, Marfatia,Zhou 2106.00025; PDB, Rahat 2307.03184)

$$V_0(\eta,\sigma) = V_0(\sigma) + V_{\eta\sigma}(\eta,\sigma) + V_\eta(\eta)$$

The most important term is contained in

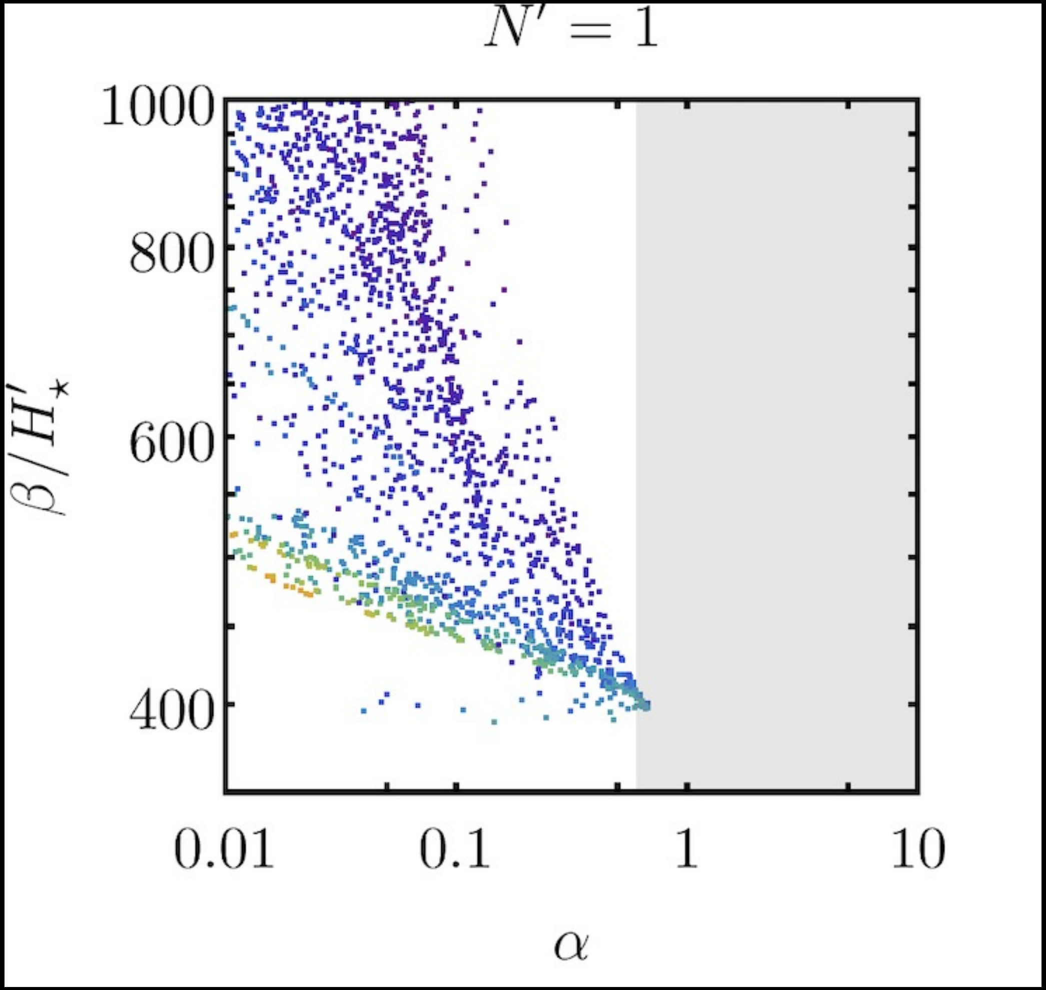
$$V_{\eta\sigma}(\eta,\sigma) = \frac{\delta_1}{2}|\sigma|^2\eta + \frac{\delta_2}{2}|\sigma|^2\eta^2$$

The scalar field η can be regarded as a Majoron from a high energy phase transition that generated usual seesaw Majorana masses

$$\Rightarrow V^T_{eff}(\sigma_1) = \frac{1}{2}\tilde{M}_T^2\sigma_1^2 - (A\,T + \tilde{\mu})\sigma_1^3 + \frac{1}{4}\lambda_T\sigma_1^4 \quad with\ \tilde{\mu} \propto \delta_2$$

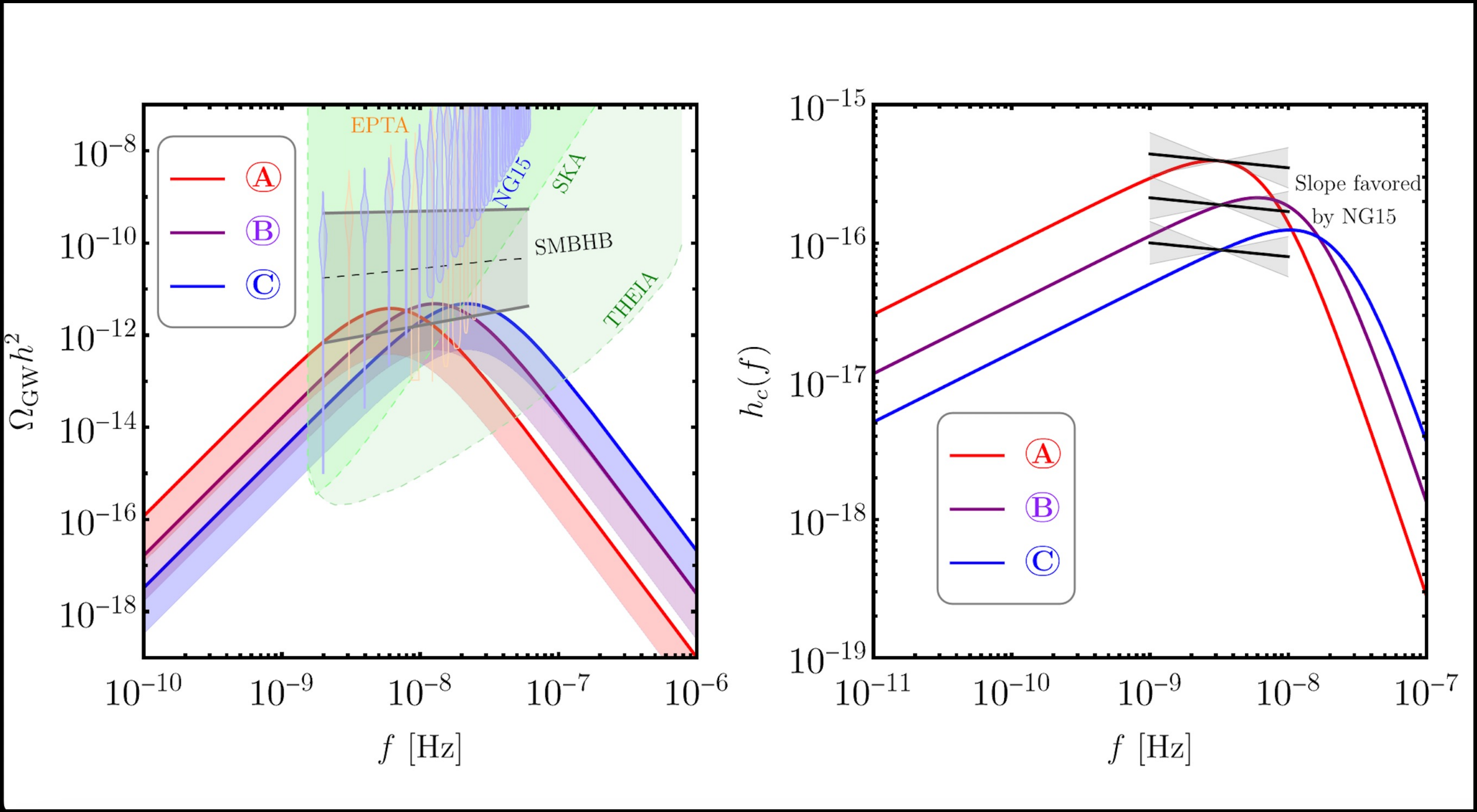
Split majoron model

(PDB, Rahat 2307.03184)



The split majoron model confronts the NANOGrav signal

(PDB, Rahat 2307.03184)



B.P.	N'	λ_1	v_1 [keV]	M [keV]	C [keV]	α	α_D	κ_D	β / H_*	T_* [keV]
A	1	0.001	71.0	20.0	0.75	0.52	2.40	0.74	424.0	240.58
B	1	0.001	83.0	23.0	1.70	0.60	2.62	0.75	399.73	515.11
C	1	0.001	144.0	40.0	3.0	0.59	2.56	0.75	393.63	888.35

Conclusions

- The majoron model at low energies can motivate a modification of pre-recombination era and be related to the generation of a light Majorana mass

- It can alleviate cosmological tensions and might solve a potential Deuterium problem that might be regarded as a kind of signature of the model.

- At the phase transition GWs can be generated with a spectrum that can peak in the NANOGrav frequencies

- It cannot explain the whole signal but it might contribute marginally in addition to SMBH binaries