The More Things Change...
do they really stay the same?

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Workshop on “Tensions in Cosmology"

$K\epsilon\rho\kappa\upsilon\rho\alpha$, $E\lambda\lambda\alpha\delta\alpha$

6- 12 September 2023

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Introduction: tensions in standard cosmology

- Modern cosmology based on a maximally symmetric spacetime (FLRW)
  - ✓ Homogeneous: all regions of space look alike, no preferred positions
  - ✓ Isotropic: no preferred directions
  - ✓ Perfect-fluid assumptions
- The isotropy assumption is valid only on very large scales, i.e. on scales bigger than galaxy clusters
- Recent cosmological observations have shown that the universe is currently undergoing accelerated expansion
- Not conclusively known what caused this acceleration, the prevailing argument being that dark energy caused it
- Among the most widely considered candidates of dark energy is the vacuum energy of the cosmological constant Λ
- Some serious problems (tensions)
  - ✓ Cosmological Constant Problem [1](vacuum catastrophe): measured energy density of the vacuum over 120 orders of magnitude less than the theoretical prediction
    - Worst prediction in the history of physics (and of science in general)
    - Casts doubt on dark energy being a cosmological constant
  - ✓ Cosmic Coincidence Problem [2]: dark matter and dark energy densities have the same order of magnitude at the present moment of cosmic history, while differing with many orders of magnitude in the past and the predicted future

[1] Weinberg, S. Rev. Mod. Phys, 61 (1), 1 (1989)

[2] Velten, H. E. et al. Eur. Phys. J. C, 74 (11), 1 (2014)

- Latest tensions vis-à-vis precise theoretical predictions and observational measurements:
  - ✓ H_0 CMB vs local measurements, more than 3σ discrepancy
    - Planck2018, ΛCDM model
    $$H_0 = 67.27 \pm 0.60 \; km/s/Mpc$$
    - Estimate using SNIa measurements (2016)
    $$H_0 = 73.24 \pm 1.74 \; km/s/Mpc$$
    - Parallax measurements of Milky Way Cepheids (2018)
    $$H_0 = 73.48 \pm 1.66 \; km/s/Mpc$$
  - ✓ S_8 vs cosmic shear data, more than 2.5σ discrepancy between Planck data and local measurements of
  $$S_8 = \sigma_8 \sqrt{\Omega_m/0.3}$$
  where σ_8 measures the amplitude of the linear power spectrum on the $8h^{-1}$Mpc scale
  - ✓ Ω_K, zero or not zero? ΛCDM assumes flat universe, but Planck temperature and polarisation power spectra give an above 3σ deviation:
  $$\Omega_K \approx -0.044^{+0.018}_{-0.015}$$
- Several alternatives proposed, such as:
  - ✓ Interacting vacuum, $\Lambda = \Lambda(t)$, $G = G(t)$
  - ✓ Interacting dark matter and dark energy $\rightarrow$ non-gravitational interactions
- Of particular interest for us here are those anisotropic models filled with viscous matter and with changing gravitational and cosmological 'constants' - $G(t)$ and $\Lambda(t)$

- Since the isotropy assumption is only an approximation on large scales, and not something explained from first principles, there is the possibility that the spatially homogeneous and anisotropic cosmological modes play a significant role in explaining the evolution of the universe at its early stages when it was full of anisotropies with a highly irregular mechanism that isotropized later
- In fact, there are several claims regarding some degree of anisotropy in the observed universe that necessitates the consideration of a non-FLRW geometry
- Bianchi models to the rescue: homogenous but not [necessarily] isotropic cosmological models - 9 possible cosmological solutions

Group Class	Group Type	n_1	n_2	n_3
	I	0	0	0
	II	+	0	0
	VI_0	0	+	−
A ($a_i = 0$)	VII_0	0	+	+
	IX	+	+	+
B ($a_i \neq 0$)	V	0	0	0
	IV	0	0	+
	VI_h	0	+	−
	VII_h	0	+	+

- Dirac's hypothesis [3] that the gravitational constant decreases with time has been a matter of scrutiny for some time, but recent attempts to consider both Λ and the universal gravitational constant G as dynamical quantities, and therefore not as constants, has gained more attention due to the aforementioned not-so-well-explained cosmic acceleration.
- Different forms of changing Λ and G assumptions exist in the literature, such as:

$$\Lambda = \frac{\alpha}{a^2} + \beta H^2 \, , \quad G = G_0 a^{\delta}$$

  ✓ Constants α , β , δ etc to be determined from both theoretical and observational considerations

[3] $G \propto \frac{1}{t}$; physical constants depend on the age of the universe t.

Viscous fluids

- One normally assumes the universe is filled with a perfect-fluid medium described by a divergence-free total EMT:

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p g_{\mu\nu} \; , \qquad T^{\mu\nu}{}_{;\mu} = 0$$

  where ρ and p are the energy density and isotropic pressure of matter, respectively, often related by the barotropic equation of state $p = w\rho$
- Increased interest of late in viscous fluids for the many obvious reasons (dark energy, dark matter, tensions. . .)
- Viscous effects are quite ubiquitous in ordinary hydrodynamics, cosmic fluids no exception [4]
- Two viscosity coefficients most commonly considered, corresponding to first-order deviation from thermal equilibrium: the shear viscosity η and the bulk viscosity ξ
- Generalised EMT for imperfect fluids [5]

$$T_{\mu\nu} = \rho u_\mu u_\nu + (p - 3H\xi)(g_{\mu\nu} + u_\mu u_\nu) + \Sigma_{\mu\nu} \tag{1}$$

- Shear viscosity often omitted due to assumption of spatial isotropy of fluid

$$\Sigma_{\mu\nu} = -\eta\sigma_{\mu\nu}$$

[4] Brevik,I and Normann, B.D. *Symmetry* **12**, 1085 (2020)

[5] Baumann, D., Nicolis, A., Senatore, L., and Zaldarriaga, M. *JCAP*, **07**, 051 (2012)

- Here we consider the Bianchi type$-V$ with spacetime metric of the form[6]

$$ds^2 = dt^2 - A^2 dx^2 - e^{2mx}[B^2 dy^2 + C^2 dz^2]$$

where m is constant

- We assume that the universe is filled by a viscous fluid whose distribution in space is represented by the following energy momentum tensor:

$$T_{\mu\nu} = (\rho + \bar{p})u_\mu u_\nu + \bar{p} g_{\mu\nu} - 2\eta\sigma_{\mu\nu} \ , \quad \bar{p} = p - (3\xi - 2\eta)\, H \qquad (2)$$

where η and ξ are coefficients of shear and bulk viscosity respectively, σ_{ij} is the shear and $\bar{p}$ is the effective pressure

- Assume a linear equation of state

$$p = w\rho \ , \qquad -1 \leq w \leq 1$$

- The shear tensor is given by

$$\sigma_{\mu\nu} = \left(u_{\mu;\lambda} h^\lambda_\nu + \dot{u}_{\nu;\lambda} h^\lambda_\mu\right) - \frac{1}{3}\theta h_{\mu\nu} \text{ , where } h_{\mu\nu} \equiv g_{\mu\nu} + u_\mu u_\nu$$

[6] Tiwari, R.K., Alfedeel, A. H., Sofuoğlu, D., AA, Eltagani, I.H., & Shukla, B. H. *Front. Astron. Space Sci.* **9** 965652 (2022)

- The Einstein field equations with time-varying cosmological parameter $\Lambda(t)$ in geometrical units where $c = 1$ are given by [7]

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\kappa G T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{3}$$

- For the Bianchi-V metric, the EFEs in (3) for a viscous fluid distribution reduce to the following set of pdes:

$$\begin{aligned}
&\frac{m^2}{A^2} - \frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} + 2\eta\frac{\dot{A}}{A} = \kappa G\left[p - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - \Lambda \\
&\frac{m^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{\dot{A}}{A}\frac{\dot{C}}{C} + 2\eta\frac{\dot{B}}{B} = \kappa G\left[p - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - \Lambda \\
&\frac{m^2}{A^2} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} + 2\eta\frac{\dot{C}}{C} = \kappa G\left[p - \left(\xi - \frac{2}{3}\eta\right)\theta\right] - \Lambda \\
&-\frac{3m^2}{A^2} + \frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = \kappa G\rho + \Lambda \\
&\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - 2\frac{\dot{A}}{A} = 0
\end{aligned} \tag{4}$$

[7] But $8\pi \equiv \kappa$ in this work

- The $\nabla^\mu T_{\mu\nu} = 0$ equation leads to the fluid continuity equation:

$$\kappa G \left[\dot{\rho} + (\overline{p} + \rho) \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \right] + \kappa \rho \dot{G} + \dot{\Lambda} - 4\kappa G \eta \sigma^2 = 0$$

- Split this into the following two equations (total matter content conserved):

$$\begin{aligned} &\dot{\rho} + 3H \left[p + \rho - (3\xi - 2\eta) H \right] - 4\eta\sigma^2 = 0 \\ &\kappa \rho \dot{G} + \dot{\Lambda} = 0 \end{aligned}$$

- Meanwhile, the EFEs can be written in terms of H, σ and q as

$$\kappa G \overline{p} - \Lambda = H^2(2q - 1) - \sigma^2 + \frac{m^2}{A^2}$$

$$\kappa G \rho + \Lambda = 3H^2 - \sigma^2 - \frac{3m^2}{A^2}$$

with

$$a \equiv (ABC)^{1/3} \ , \quad H \equiv \frac{\dot{a}}{a} = \frac{1}{3} \left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \ , \quad q \equiv -\frac{a\ddot{a}}{\dot{a}^2}$$

$$\sigma^2 \equiv \frac{1}{6} \left[\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2 + \left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C} \right)^2 + \left(\frac{\dot{C}}{C} - \frac{\dot{A}}{A} \right)^2 \right]$$

- The generalized Raychaudhuri equation reads:

$$\dot{H} + 3H^2 - \frac{2m^2}{a^2} - \Lambda + \frac{\kappa G}{2}(p - \rho) - \kappa G \left(\frac{3\xi}{2} - \eta \right) H = 0$$

- To find the solution by introducing extra information in the form of initial conditions and a constraint, we consider the following form of the Friedmann Equation:

$$1 = \Omega_m + \Omega_\Lambda + \Omega_\sigma + \Omega_\chi$$

where the density parameters are defined as:

$$\Omega_m \equiv \frac{\kappa G \rho_m}{3H^2} , \qquad \Omega_\Lambda \equiv \frac{\kappa G \rho_\Lambda}{3H^2} , \qquad \Omega_\sigma \equiv \frac{\sigma^2}{3H^2} , \qquad \Omega_\chi \equiv \frac{K^2}{H^2 a^2}$$

- The current values of these dimensionless density parameters are given in terms of the current values of the quantities that describe them, as

$$\Omega_{m_0} = \frac{\kappa G_0 \rho_{m_0}}{3H_0^2} , \qquad \Omega_{\Lambda_0} = \frac{\kappa G_0 \rho_{\Lambda_0}}{3H_0^2} , \qquad \Omega_{\sigma_0} = \frac{\sigma_0^2}{3H_0^2} , \qquad \Omega_{\chi 0} = \frac{K^2}{H_0^2 a_0^2}$$

- We also define the following dimensionless parameters:

$$h \equiv \frac{H}{H_0} , \qquad a = \frac{1}{(1+z)} , \qquad \xi = \alpha H_0 (\rho_m/\rho_{m0})^n , \qquad \eta = \beta H$$

with α, β and $0 \leq n \leq \frac{1}{2}$ introduced as dimensionless constants

- We assume the ansatz, in accordance with Dirac's hypothesis

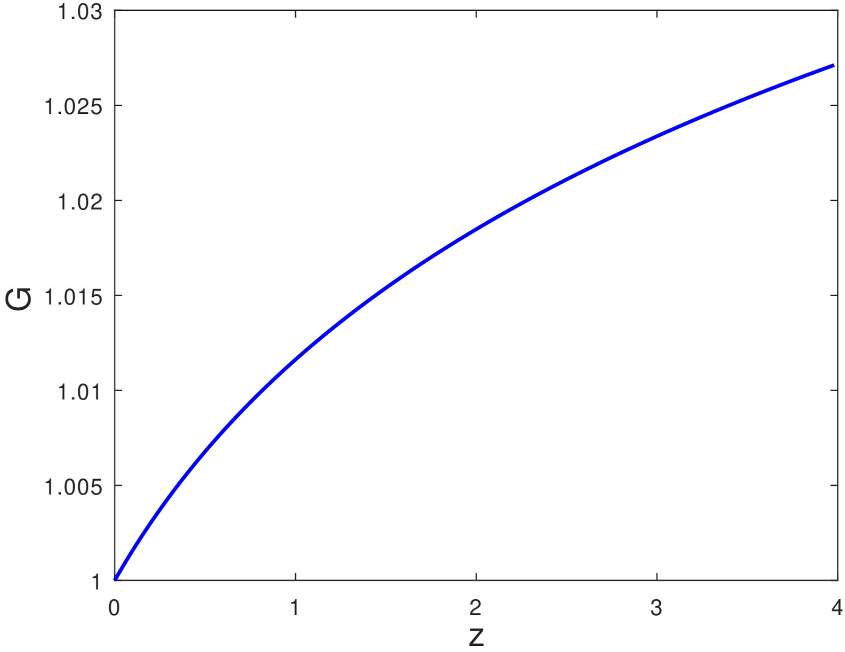
$$G(t) = G_0 a^{\delta}$$

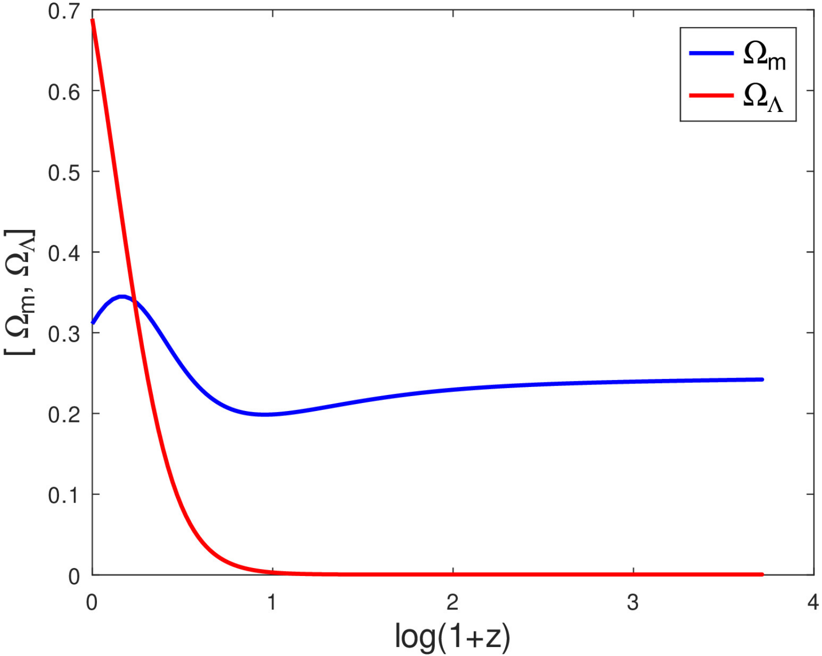
- Here, $\delta = -1/60$ is a constant obtained from observational constraints[8] [9]
- The conservation equations in terms of the dimensionless density parameters:

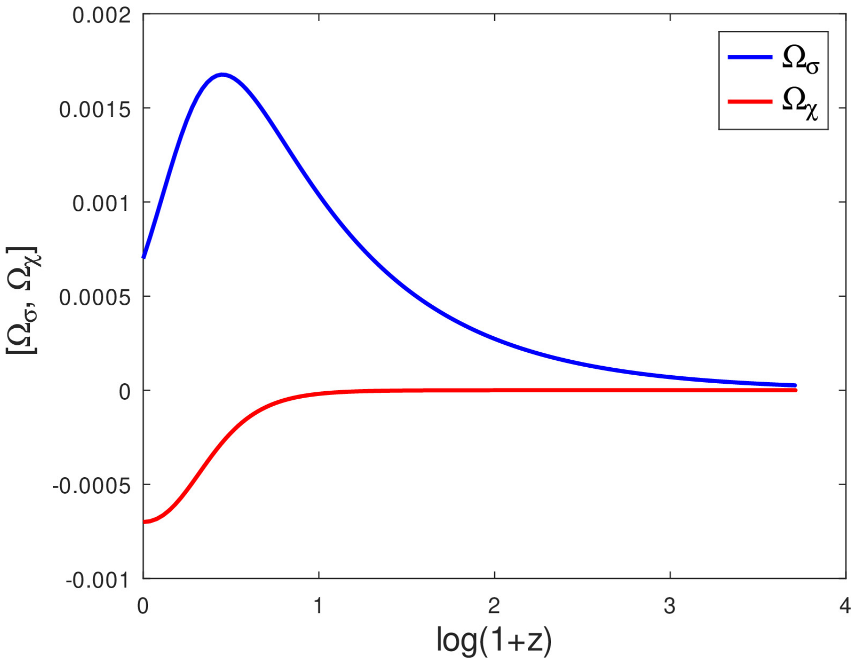
$$
\begin{aligned}
h' &= \frac{h}{(1+z)}\left[3 - 2\Omega_\chi - 3\Omega_\Lambda - \frac{3}{2}(1-w_m)\Omega_m\right] \\
&\quad - \frac{\kappa G_0}{(1+z)^{1+\delta}}\left[\frac{3\alpha}{2}\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \beta h\right] \\
\Omega_m' &= -\frac{2h'}{h}\Omega_m + \frac{1}{1+z}\left(-\delta + 3 + 3w_m\right)\Omega_m \\
&\quad - \frac{\kappa G_0}{(1+z)^{1+\delta}}\left[\frac{3\alpha}{h}\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - 2\beta + 4\beta\Omega_\sigma\right] \\
\Omega_\Lambda' &= -\frac{2h'}{h}\Omega_\Lambda - \frac{\delta}{1+z}\Omega_m \\
\Omega_\chi' &= -\frac{2h'}{h}\Omega_\chi + \frac{2\Omega_\chi}{1+z}\,, \\
\Omega_\sigma' &= -\frac{2h'}{h}\Omega_\sigma + \frac{6\Omega_\sigma}{1+z}
\end{aligned}
\tag{5}
$$

[8] Williams, J.G.; Turyshev, S.G.; Boggs, D.H. Int. J. Mod. Phys. D 2009, 18, 1129–1175

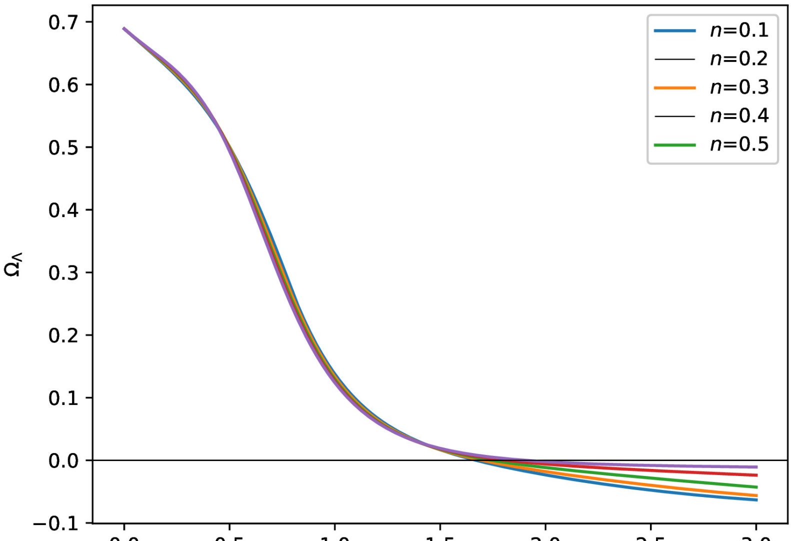
[9] Copi, C.J.; Davis, A.N.; Krauss, L.M. Phys. Rev. Lett. 2004, 92, 171301

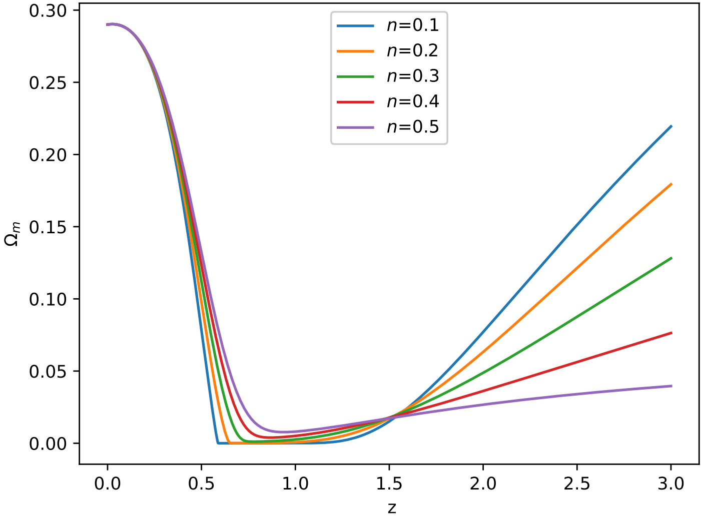

1.03
1.025
1.02
1.015
1.01
1.005
1
G
0
1
2
3
4
Z

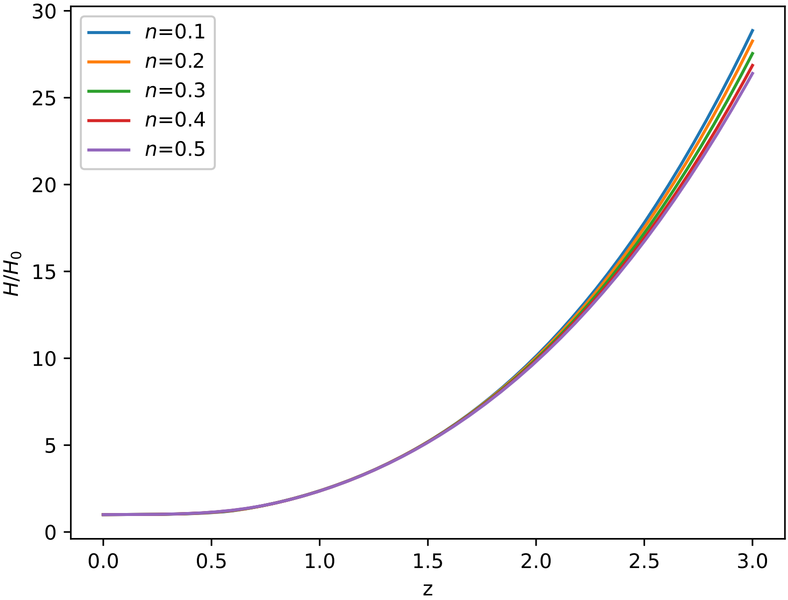

0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1
2
3
4
Ω_m
Ω_Λ
$[\, \Omega_m, \, \Omega_\Lambda]$
log(1+z)


0.002
0.0015
0.001
0.0005
0
-0.0005
-0.001
0
1
2
3
4
log(1+z)
$[\Omega_\sigma, \Omega_\chi]$
Ω_σ
Ω_χ

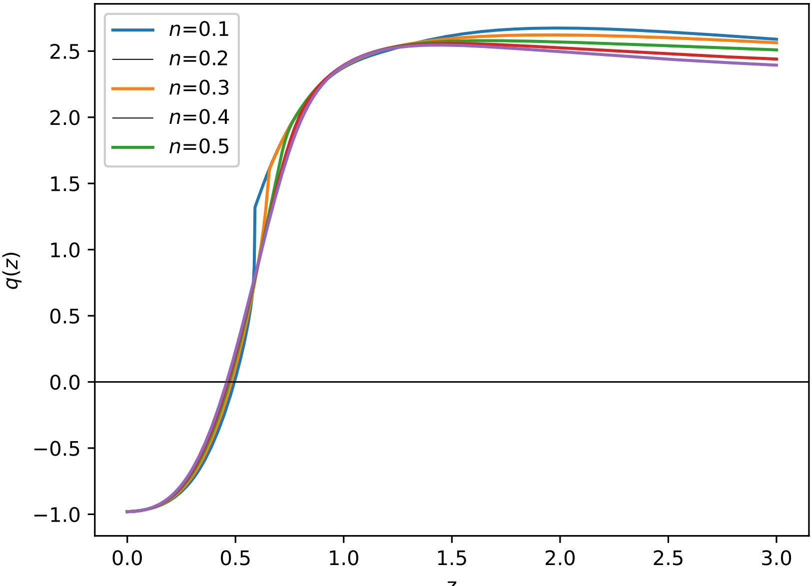
More results for different values of n...

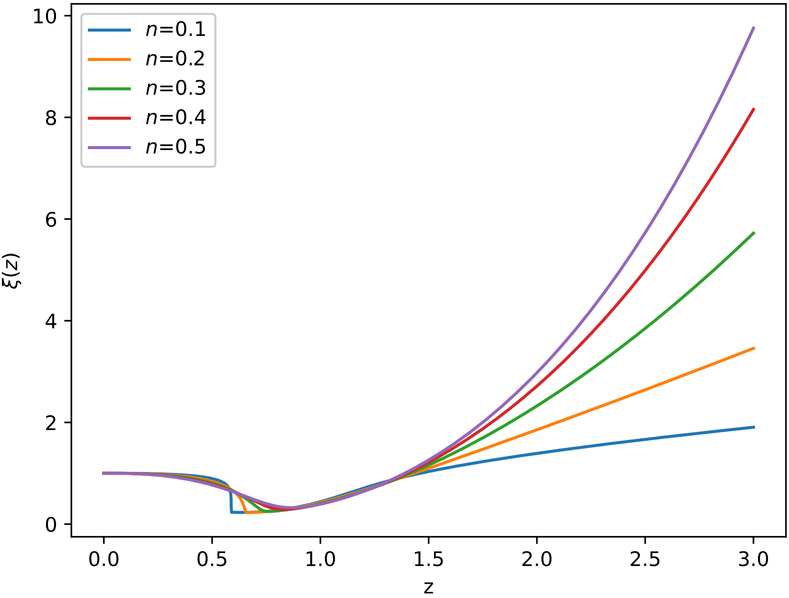



n=0.1
n=0.2
n=0.3
n=0.4
n=0.5
Ω_m
z
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.0
0.5
1.0
1.5
2.0
2.5
3.0

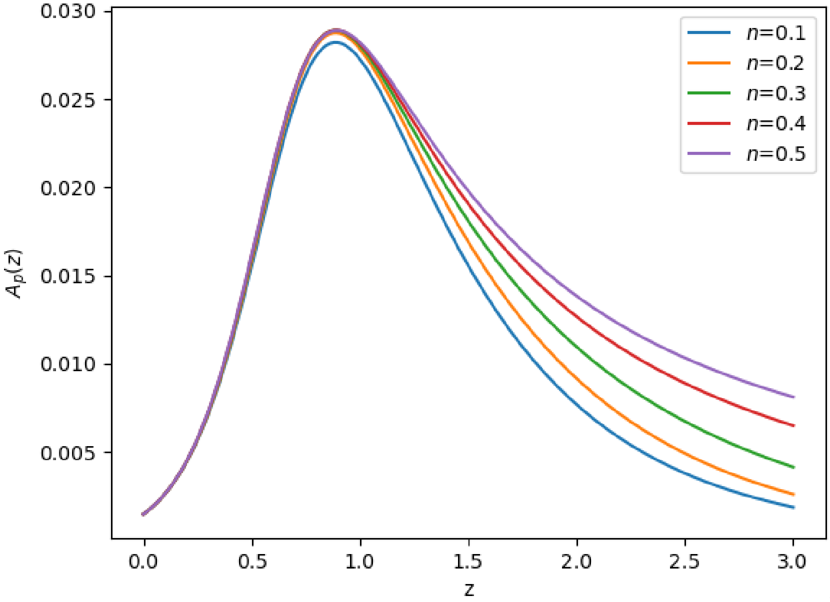


n=0.1
n=0.2
n=0.3
n=0.4
n=0.5
H/H_0
z
0
5
10
15
20
25
30
0.0
0.5
1.0
1.5
2.0
2.5
3.0


n=0.1
n=0.2
n=0.3
n=0.4
n=0.5
q(z)
z
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
0.0
0.5
1.0
1.5
2.0
2.5
3.0



n=0.1
n=0.2
n=0.3
n=0.4
n=0.5
ξ(z)
z
0
2
4
6
8
10
0.0
0.5
1.0
1.5
2.0
2.5
3.0



n=0.1
n=0.2
n=0.3
n=0.4
n=0.5
0.030
0.025
0.020
0.015
0.010
0.005
$A_p(z)$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
z

Covariant Gauge-invariant Perturbations [10]

- We define covariant and gauge-invariant gradient variables that describe perturbations in the matter energy density, expansion and shear, as per the $1+3$ covariant perturbation formalism:

$$D_a \equiv \frac{a\tilde{\nabla}_a \rho}{\rho}\,, \quad Z_a \equiv a\tilde{\nabla}_a \Theta\,, \quad \Sigma_a \equiv a\tilde{\nabla}_a \sigma$$

- Generally believed that large-scale structure formation follows spherical clustering
- Take the spherically symmetric components of the gradient vectors by writing:

$$\Delta \equiv a\tilde{\nabla}^a D_a\,, \quad Z \equiv a\tilde{\nabla}^a Z_a\,, \quad \Sigma \equiv a\tilde{\nabla}^a \Sigma_a$$

- Dimensionless quantities:

$$\gamma \equiv \frac{k^2}{H_0^2}\,, \qquad \mathcal{Z} \equiv \frac{Z}{H_0}\,, \qquad \mathcal{S} \equiv \frac{\Sigma}{H_0}$$

with wavenumber $k \equiv \frac{2\pi a}{\lambda}$, λ is the comoving wavelength of the perturbations

[10] AA, Alfedeel, A. H., Sofuoğlu, D., Eltagani, I.H., & Tiwari, R.K. Universe **9**, 61 (2023)

Evolution of the matter perturbations:

$$\Delta' + \frac{3}{(1+z)}\left\{ w - \frac{\kappa G_0}{\Omega_m h(1+z)^\delta}\left[\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right] - \frac{4\beta\kappa G_0}{3(1+z)^\delta}\frac{\Omega_\sigma}{\Omega_m} \right.$$
$$\left. + \frac{4\beta\Omega_\sigma}{3\Omega_m}\frac{\kappa G_0}{(1+z)^\delta}\left[\frac{\alpha n\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{wh\Omega_m}{\kappa G_0(1+z)^{-\delta}}}{\frac{(1+w)h\Omega_m}{\kappa G_0(1+z)^{-\delta}} - \left[\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right]}\right]\right\}\Delta$$
$$-\frac{1}{h(1+z)}\left\{1 + w - \frac{\kappa G_0}{\Omega_m h(1+z)^\delta}\left[\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right] - \frac{4\beta\kappa G_0}{3(1+z)^\delta}\frac{\Omega_\sigma}{\Omega_m}\right.$$
$$\left. + \frac{4\beta\Omega_\sigma}{3\Omega_m}\frac{\kappa G_0}{h(1+z)^\delta}\left[\frac{\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{4\beta}{3}h}{\frac{(1+w)h\Omega_m}{\kappa G_0(1+z)^{-\delta}} - \left[\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right]}\right]\right\}\mathcal{Z}$$
$$-\frac{8\beta\kappa G_0}{3h(1+z)^{\delta+1}}\frac{\sqrt{3\Omega_\sigma}}{\Omega_m}\mathcal{S} = 0 \tag{6}$$

Evolution of the perturbations in the expansion:

$$\mathcal{Z}' - \frac{1}{(1+z)\ h}\Bigg\{2h - \frac{3}{2}\frac{\kappa G_0}{(1+z)^{\delta}}\left[\alpha\left(\frac{h^2\Omega_m(1+z)^{\delta}}{\Omega_{m0}}\right)^n - \frac{4\beta}{3}h\right]$$

$$- \frac{\alpha\left(\frac{h^2\Omega_m(1+z)^{\delta}}{\Omega_{m0}}\right)^n - \frac{4\beta}{3}h}{\frac{(1+w)h^2\Omega_m}{\kappa G_0(1+z)^{-\delta}} - h\left[\alpha\left(\frac{h^2\Omega_m(1+z)^{\delta}}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right]}\left[-hh'(1+z) + \frac{2}{3}h^2\Omega_\chi - \frac{\gamma}{3}(1+z)^2\right]\Bigg]\Bigg\}\mathcal{Z}$$

$$+\frac{3}{(1+z)}\Bigg[\frac{\Omega_m h}{2}(1+3w) - \frac{3}{2}\frac{\alpha n\kappa G_0}{(1+z)^{\delta}}\left(\frac{h^2\Omega_m(1+z)^{\delta}}{\Omega_{m0}}\right)^n$$

$$+\frac{\alpha n\left(\frac{h^2\Omega_m(1+z)^{\delta}}{\Omega_{m0}}\right)^n - \frac{wh\Omega_m}{\kappa G_0(1+z)^{-\delta}}}{\frac{(1+w)h^2\Omega_m}{\kappa G_0(1+z)^{-\delta}} - h\left\{\alpha\left(\frac{h^2\Omega_m(1+z)^{\delta}}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right\}}\left[-hh'(1+z) + \frac{2}{3}h^2\Omega_\chi - \frac{\gamma}{3}(1+z)^2\right]\Bigg]\Delta$$

$$-\frac{4\sqrt{\Omega_\sigma}}{(1+z)}\mathcal{S} = 0 \qquad (7)$$

Evolution of the shear perturbations [11]:

$$\mathcal{S}' - \frac{3}{(1+z)}\mathcal{S} - \frac{\sqrt{3\Omega_\sigma}}{(1+z)}\left[1 + \frac{\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{4\beta}{3}h}{\frac{(1+w)h\Omega_m}{\kappa G_0(1+z)^{-\delta}} - \left[\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right]}\right]\mathcal{Z}$$
$$- \frac{3h\sqrt{3\Omega_\sigma}}{(1+z)}\left[\frac{\alpha n\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{wh\Omega_m}{\kappa G_0(1+z)^{-\delta}}}{\frac{(1+w)h\Omega_m}{\kappa G_0(1+z)^{-\delta}} - \left[\alpha\left(\frac{h^2\Omega_m(1+z)^\delta}{\Omega_{m0}}\right)^n - \frac{2\beta}{3}h\right]}\right]\Delta = 0 \tag{8}$$

[11] From here onwards, we will set $\kappa G_0 = 1$ for simplicity

Solutions of the matter perturbations

- Fix the background expansion history
- Set initial conditions at some redshift z_{in}, solve the system of perturbation equations for $\Delta(z)$ and compare it with that of standard GR/ΛCDM

$$\Delta(z_{in}) = 10^{-5}\ , \quad \mathcal{Z}(z_{in}) = 10^{-5}\ , \quad \Sigma(z_{in}) = 10^{-5}$$

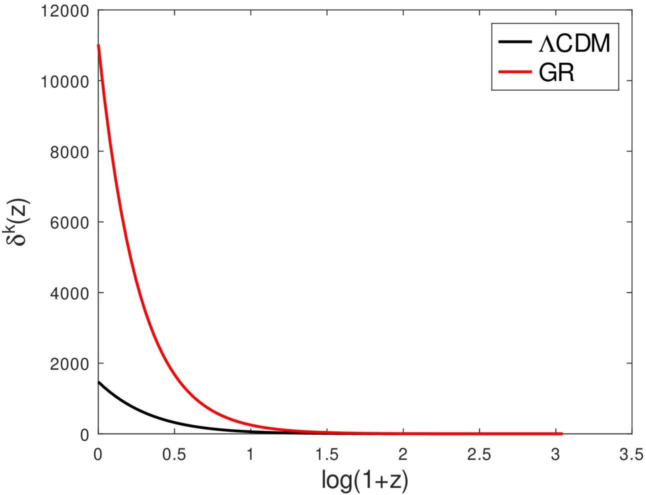
- Let's first define the normalized matter density contrast

$$\delta(z) \equiv \frac{\Delta(z)}{\Delta(z_{in})} \tag{9}$$

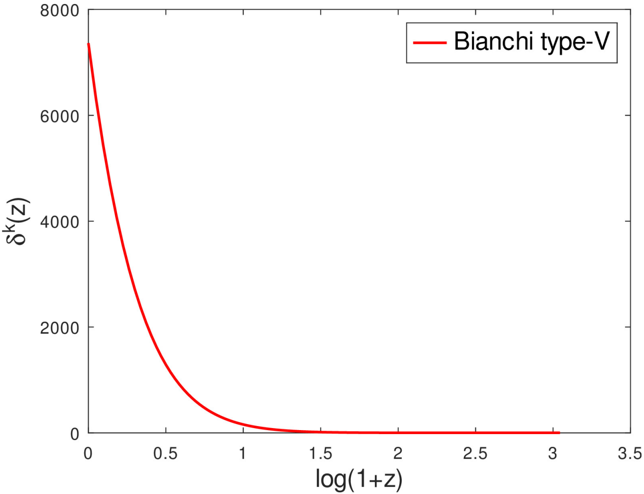
  with $z_{in} = 20$ in both GR/ΛCDM and our current models
- We have also used the following dimensionless viscosity parameters: $\alpha = 0.312, \beta = 1, n = 0.2$, as well as the current values from PLANCK2018:

$$\Omega_{m0} = 0.3111\ , \quad \Omega_{\Lambda 0} = 0.6889\ , \quad \Omega_{\chi 0} = -0.0007\ , \quad \Omega_{\sigma 0} = 1 - \Omega_{m0} - \Omega_{\Lambda 0} - \Omega_{\chi 0}$$

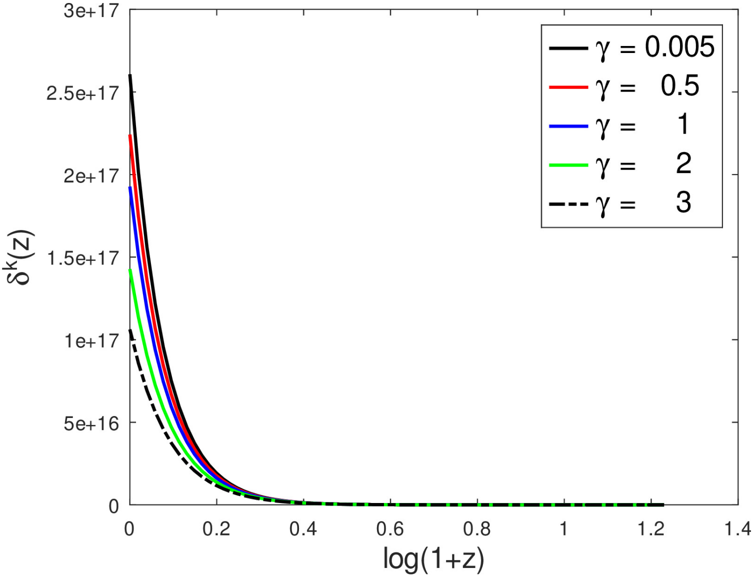
- The following are some of the highlights of our observations:
  - ✓ Increasing α decreases the late-time perturbation amplitude in the short-wavelength regime, but this effect is reversed for $z \gtrsim 0.65$
  - ✓ Increasing α increases the perturbation amplitude in the long-wavelength regime
  - ✓ Increasing β increases the perturbation amplitudes in both the short- and long-wavelength regimes
  - ✓ Increasing n increases the perturbation amplitudes in both the short- and long-wavelength regimes



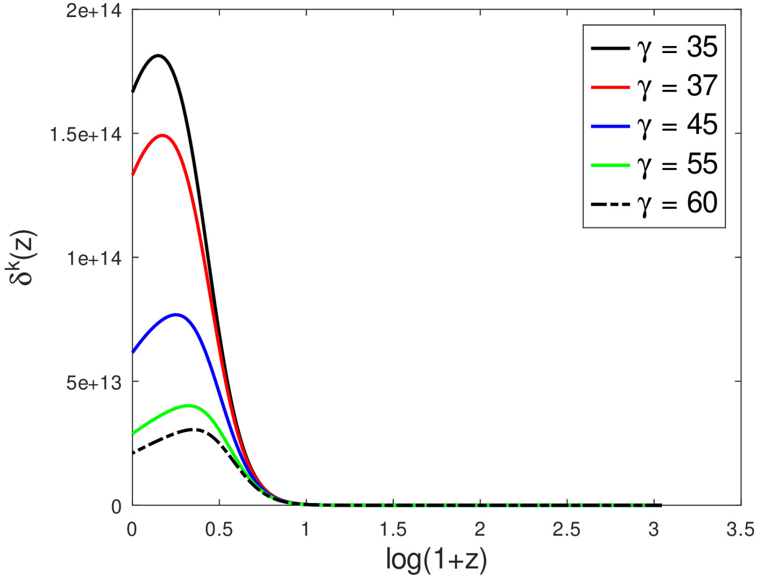
Growth of matter density perturbations $\delta(z)$ vs z for ΛCDM and GR without Λ ($\Omega_m = 1$, $\Omega_\Lambda = 0$)



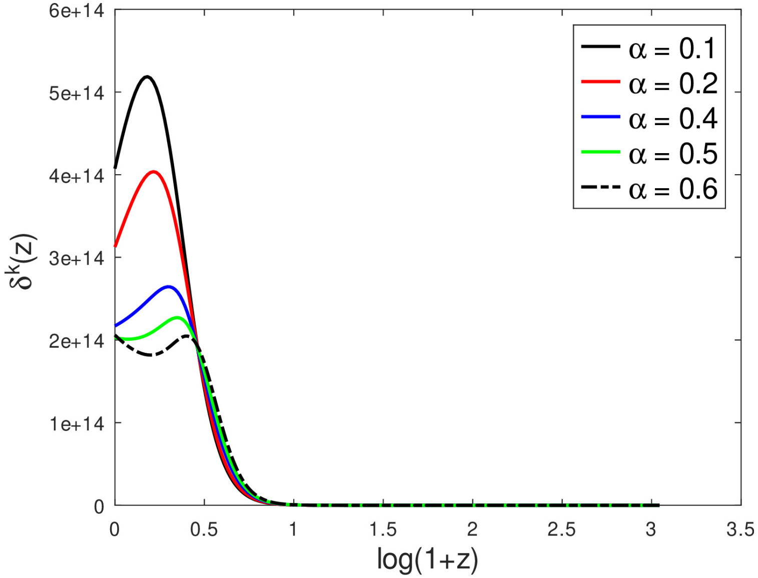
Growth of matter density perturbations $\delta(z)$ vs z for the Bianchi type-V model for non-viscous ($\alpha = 0 = \beta$) fluid, but with changing G and Λ



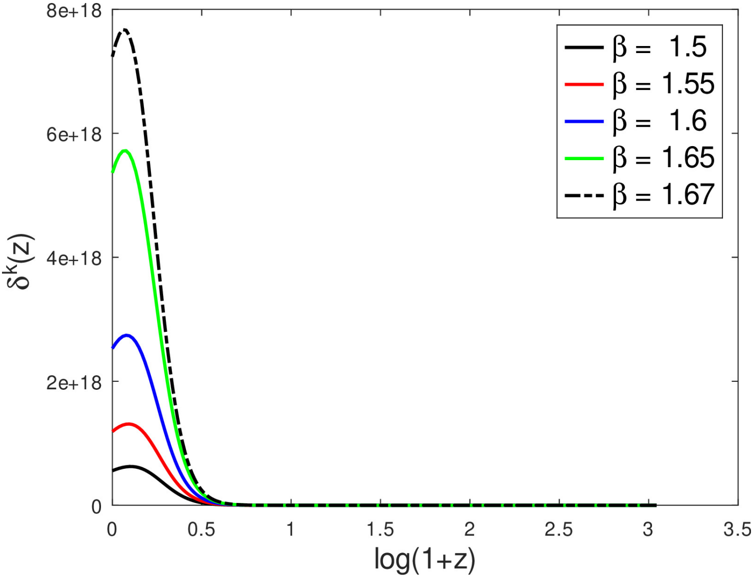
Growth of matter density perturbations $\delta(z)$ vs z for the viscous Bianchi type-V cosmological model in long-wavelength regimes



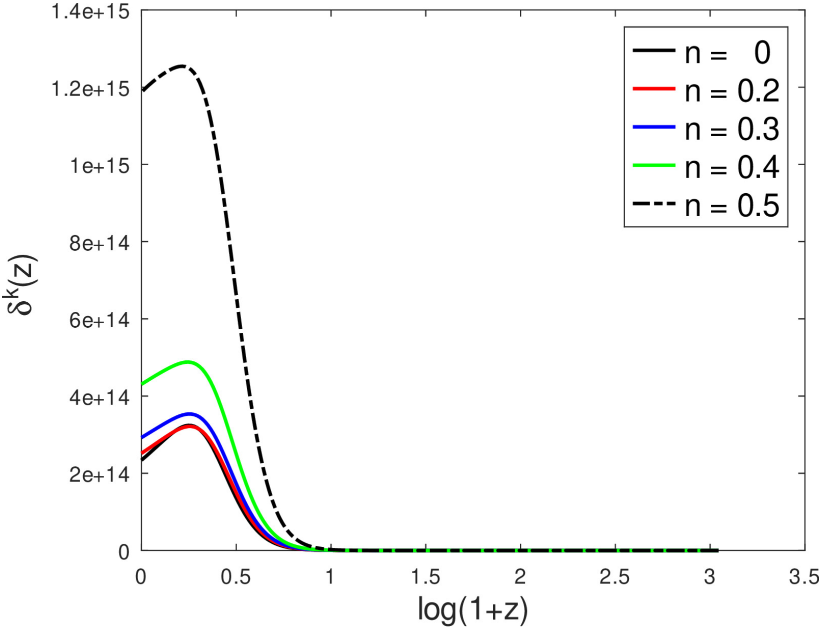
Growth of matter density perturbations $\delta(z)$ vs z for the viscous Bianchi type-V cosmological mode in short-wavelength regimes



Growth of matter density perturbations $\delta(z)$ vs z for the viscous Bianchi type-V cosmological model for varying values of α



Growth of matter density perturbations $\delta(z)$ vs z for the viscous Bianchi type-V cosmological model for varying values of β



The variation of the matter density perturbations $\delta(z)$ for a viscous Bianchi type-V cosmological model vs. redshift for $\gamma = 50$, $\alpha = 0.3$, $\beta = 1$ and different values of n.

Summary

- The Bianchi type$-V$ cosmological model in the presence of shear η and bulk ξ viscosities in the cosmic fluids for time-varying G and Λ
  - ✓ Describes a universe that starts off with some anisotropic universe in the past, and isotropic at late times, possibly indistinguishable from the FLRW universe.
  - ✓ Also describes a universe that starts off with a negative cosmological term, dominated by non-relativistic matter and decelerated, that eventually becomes dark energy-dominated and hence expanding with acceleration, in concordance with current observations
- Introducing viscosity to the cosmic fluid not only affects the background expansion history, but also the rate at which structures grow
  - ✓ Demonstrated this by first looking at the rate of structure growth in pure GR with and without the cosmological constant (and assuming a gravitational constant G)
  - ✓ The amplitudes' comparison shows structure growth in ΛCDM is slower compared to pure GR, as structures have less time to coalesce and grow in an accelerated background
  - ✓ More structures can be expected in a Bianchi-V universe with evolving Λ and G compared to both ΛCDM and pure GR cases
  - ✓ Introduced the viscosity, and showed that structures grow even faster in this case, perhaps even suggesting nonlinear effects in the perturbations
- Our results suggest that the longer the wavelength, the larger the perturbation amplitudes, ceteris paribus
- In the short-wavelength regime, the perturbation amplitudes peak at about the same redshift that the fractional background matter density peaked
  - ✓ Not observed in the long-wavelength regime, and this may be because the wavelength-dependent contributions to the perturbations are negligible compared to the other terms in the equation
- Future directions: putting more stringent constraints on the values of the defining parameters of the model, with more rigorous data and statistical analysis – using existing and upcoming cosmological data– including large-scale structure data