Reconstructing the Dark Energy

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Introduction

$$H^2(a) = H_0^2\left[\Omega_{r,0}\left(\frac{a_0}{a}\right)^4 + \Omega_{m,0}\left(\frac{a_0}{a}\right)^3 + \Omega_\Lambda + \Omega_{k,0}\left(\frac{a_0}{a}\right)^2\right]$$

At this point, it is common knowledge that the LCDM model is considered the standard model in Cosmology.

LCDM is very good at explaining most observations and phenomena.

But as data becomes more abundant and precise it starts showing some problems, such as:

- LCDM has problems at small scales.
- Hubble and S8 tensions.
- Coincidence problem.
- Fine-tuning problem.
- Just to mention a few…

And also not to mention the unexplained nature of the Dark Sector (Matter and Energy).

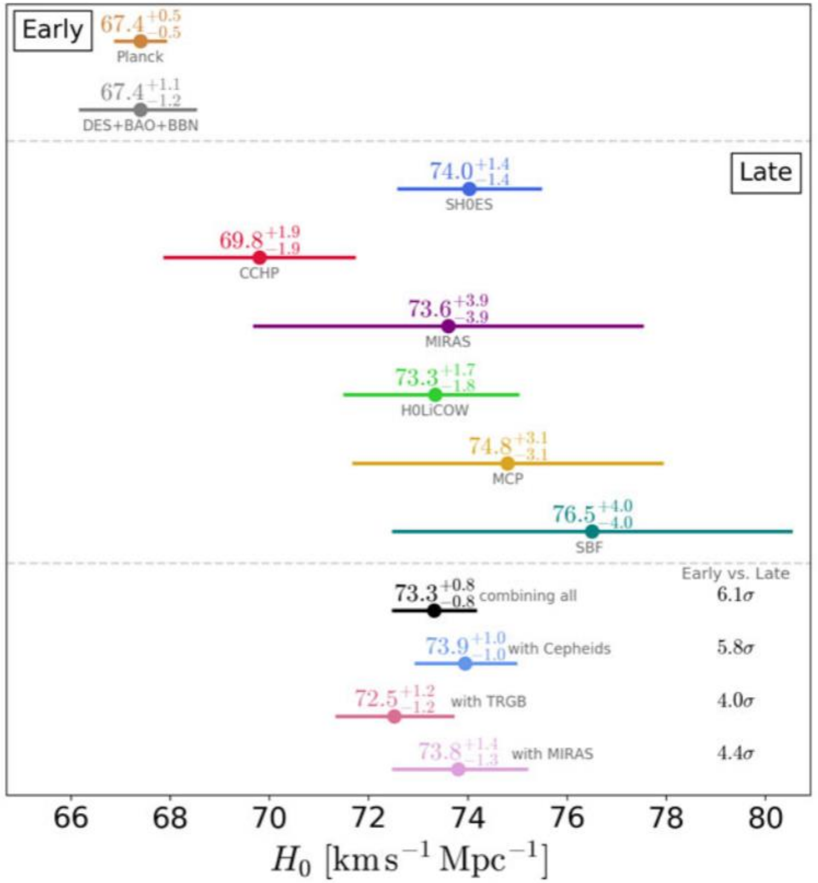


Image from:
"Tensions between the Early and the Late Universe", Nature Astronomy 2009. L. Verde, et. al.

What can we do?

Two inferences we can make are:

- There is something wrong with the data, or…
- There is something wrong with the model.

If the data is the problem, then one can only expect future better and bigger surveys to solve the problem.

If, on the other hand, the model or theory are the problem then we have a whole different issue.

Changing the theory

Depending on how deep one assumes the theory to be wrong we have several approaches.

One way to find a solution relies in a method referred to as **Reconstructions** and is the focus of this work. They can be broadly classified in 3 types:

-Parametric
-Non-parametric
-Model-independent

Reconstructions

Parametric reconstructions

In this kind of reconstruction, a parametric or functional form is proposed. Among the widely employed parameterizations in the literature, two prominent ones are the wCDM and the CPL parameterizations.

To determine the values of these functional forms' parameters, cosmological observations and data are utilized for inference.

They can be used to study a particular type of behavior such as:

- Oscillations
- Exponential growth.
- Just to mention a few…

$$w(z) = w_0 + w_1 z$$
$$w(a) = w_0 + w_a(1-a) = w_0 + w_a \frac{z}{1+z}$$
$$w_a = \sum_{i=0}^{N}(1-a)^i w_i$$
$$\Omega_{DE} = A_1 + A_2 x + A_3 x^2 \text{ con } x = z+1$$
$$w(z) = \sum_{i=0}^{N} w_i z^i$$
$$w(z) = w_i + \frac{w_f - w_i}{1+e^{\frac{z-z_t}{\Delta}}}$$
$$w(\log a) = w_0 + w_1 \cos\left(A \log \frac{a}{a_c}\right)$$
$$w(a) = w_0 e^{a-1}$$
$$w(a) = w_0 a(1-\log(a))$$
$$w(a) = w_0 a e^{a-1}$$
$$w(a) = w_0 a(1+\sin(1-a))$$
$$w(a) = w_0 a(1+\arcsin(1-a))$$
…

Non-parametric reconstructions

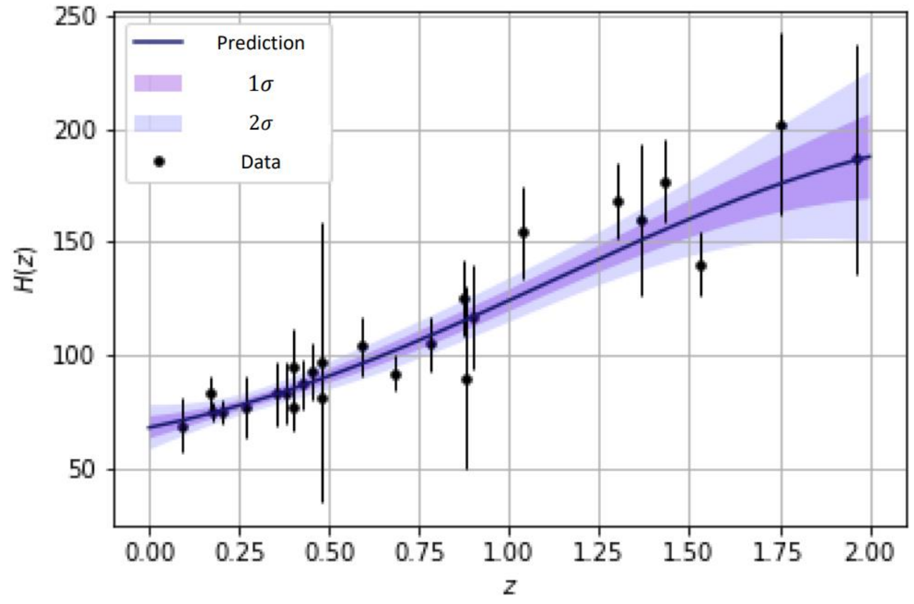
This approach to reconstructions does not assume a specific functional form or parametric model.

They typically rely on statistical and computational methods to analyze observational data.

Some examples are:

- Gaussian Process
- LOESS+Simex
- Neural Networks

Example of a non-parametric reconstruction of the Hubble parameter with a Gaussian Process.



Model-independent reconstructions

Unlike their non-parametric counterparts, these reconstructions do possess parameters and a functional form, although they are given much more freedom of shape and can be used to perform a model-selection procedure.

Some examples are:

- Bins reconstruction
- Spline reconstruction
- Fourier series
- Padé approximation
- Nodal reconstruction

These methods are the focus of this work, so lets delve deeper.

Reconstructing the EoS and the Energy Density

EUR. PHYS. J. C 83, 251 (2023)

ARXIV:2111.10457

Nodal Reconstruction

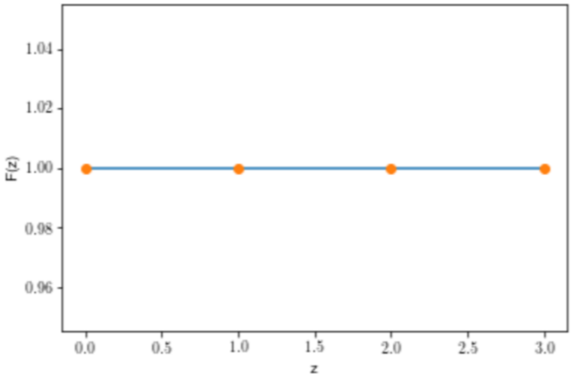
We use “nodes” which are then interpolated

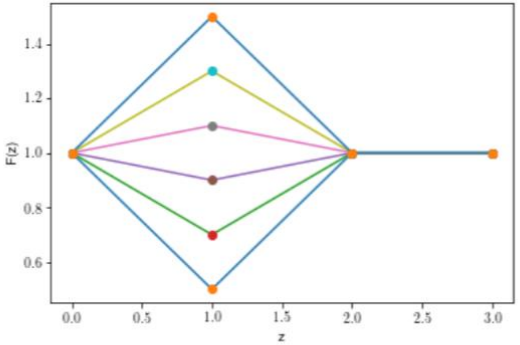
When using a straight line to interpolate:

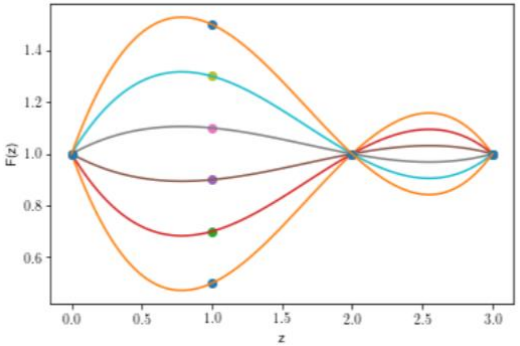
$$L_i(z) = \frac{w_{i+1} - w_i}{z_{i+1} - z_i}(z - z_i) + w_i, \quad z \in [z_i, z_{i+1}].$$

We can also use higher order polynomials to to the interpolations, such as the Cubic Spline interpolation. But the Cubic Spline could introduce unwanted noise to the reconstructed quantity.

Example with 4 nodes.







Bins reconstruction

Similar to the nodal one, but with the advantage of being easily differentiable.

It uses step functions (bins) joined via hyperbolic tangents.

Its functional form is:

$$w(z) = w_1 + \sum_{i=1}^{N-1} \frac{w_{i+1} - w_i}{2} \left(1 + \tanh\left(\frac{z - z_i}{\xi}\right)\right)$$

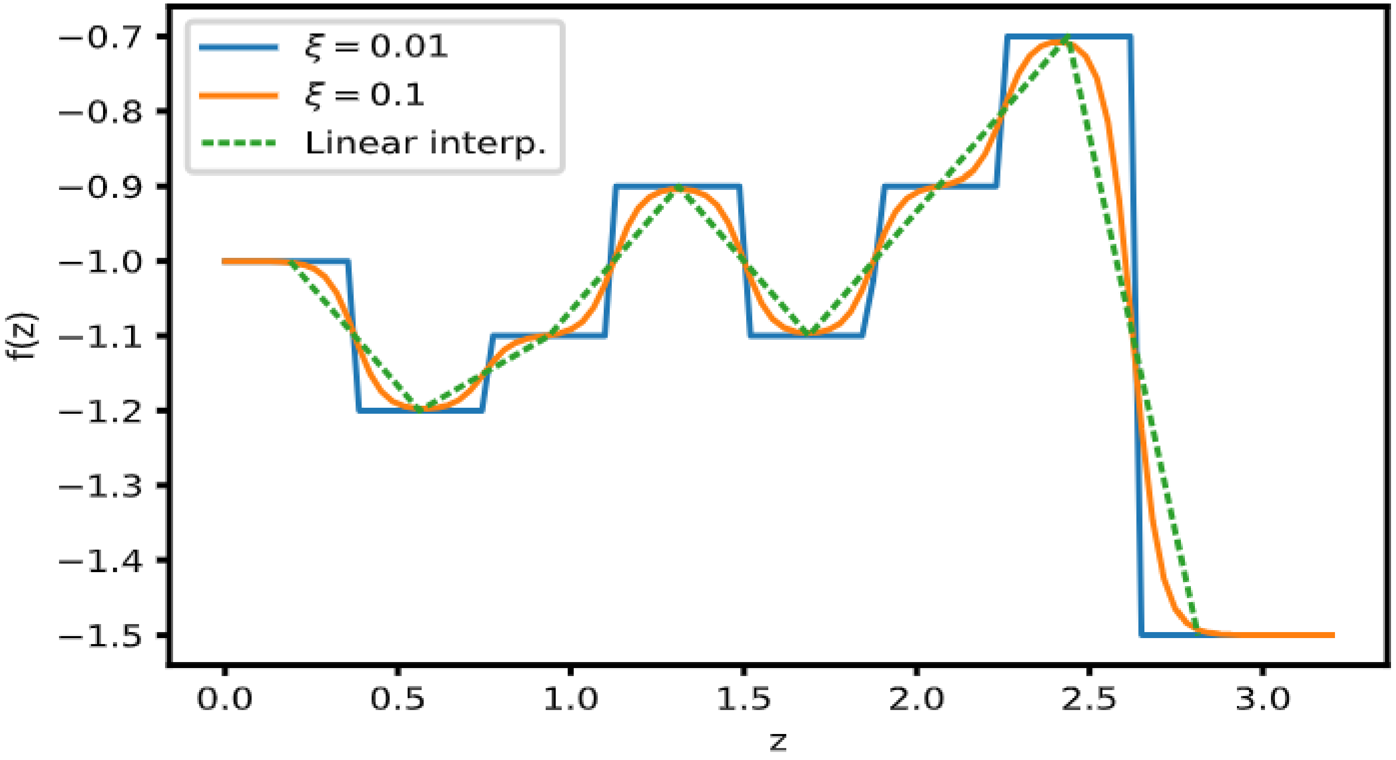
The parameter ξ allows us to control the “smoothness” of the transition from one bin to another.

Using bins and nodes

Example of the “shape” to be used as a model-independent reconstruction.

Here we have 8 nodes/bins.

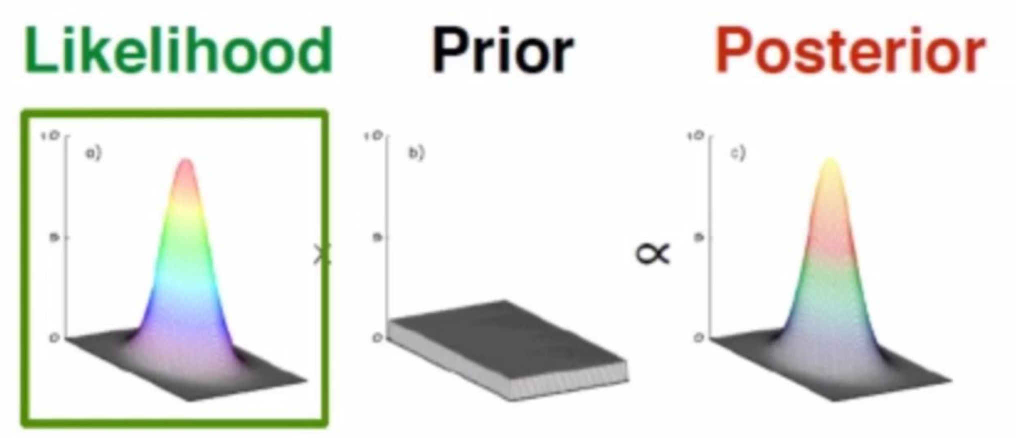
Each node’s/bin’s height is a free parameter, and to infer its value through data we need Bayesian Statistics.



Bayesian Statistics

To perform the parameter inference procedure we used Bayesian Statistics, which is a well-known statistical and numerical tool in Cosmology.

$$P(u, M|D) = \frac{\mathcal{L}(D|u, M)P(u, M)}{E(D|M)}$$



Cosmological parameter inference with Bayesian statistics

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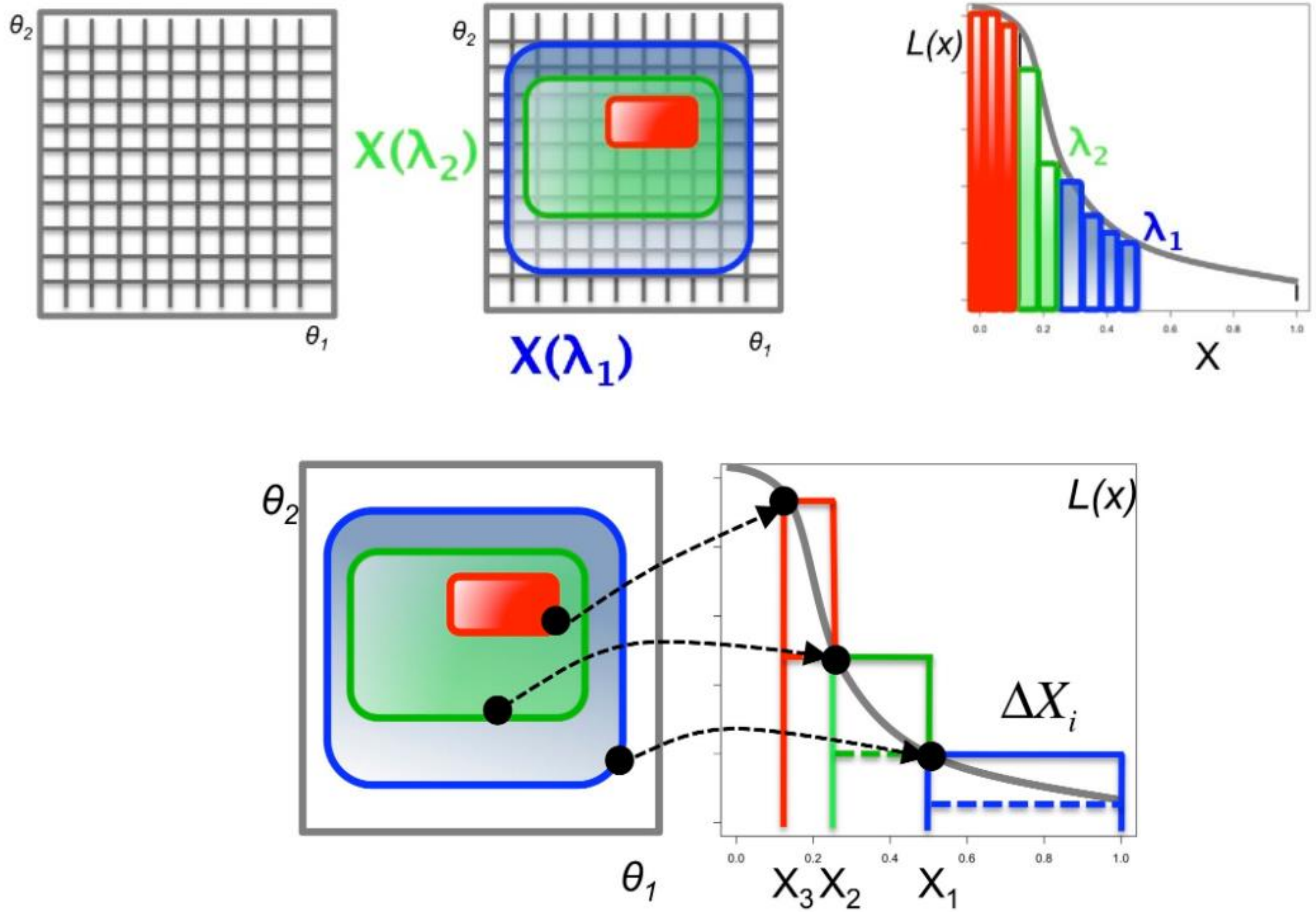
(Dated: March 28, 2019)

Universe 2021, 7(7), 213, arXiv:1903.11127

Nested Sampling

Particularly we use the Nested-Sampling algorithm.

This approach to parameter inference provides us with the Bayesian Evidence, which tells us how good our model performs against LCDM taking into account the added “complexity” in our reconstructions.



Data and code

The datasets used for the reconstructions are:

-Cosmic Chronometers.

-Pantheon's full catalogue of type 1a supernovae.

-SDSS BAO "low-redshift" data ($z<2$).

-Lyman-alpha "high-redshift" data ($z>2$).

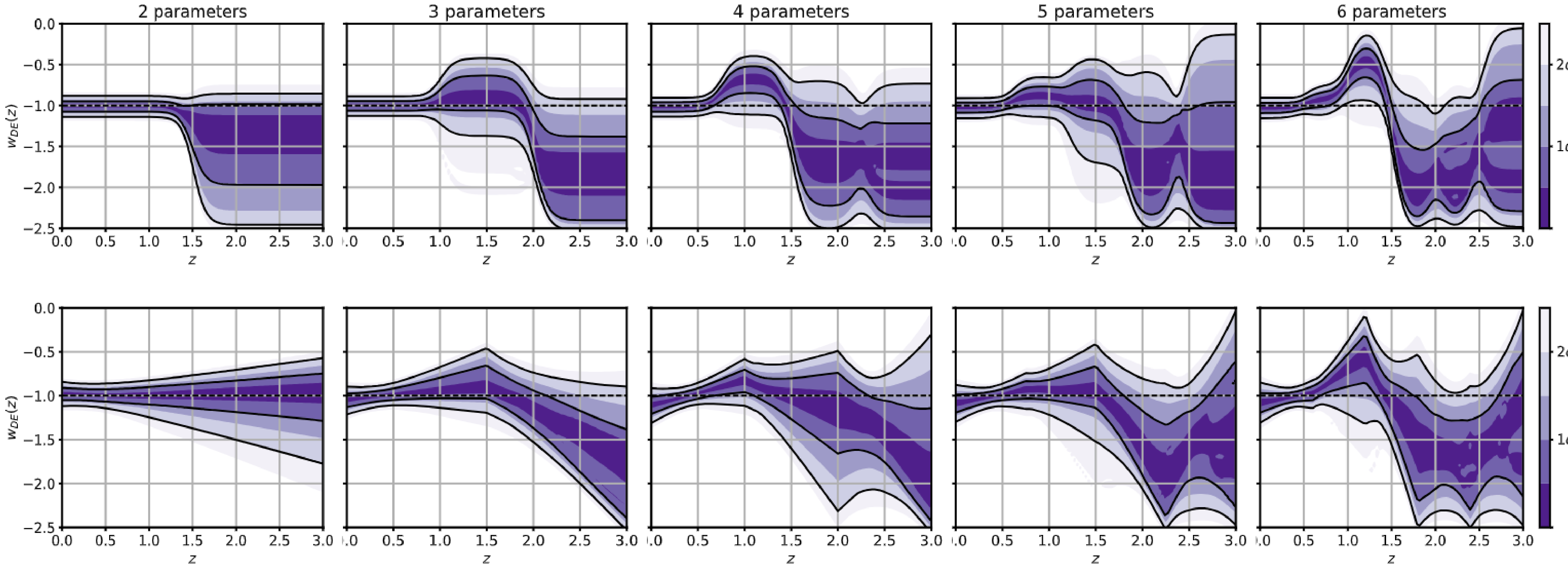
(for a full explanation of the data used please refer to arXiv:2111.10457)

Every reconstruction was made using our own Bayesian Inference code

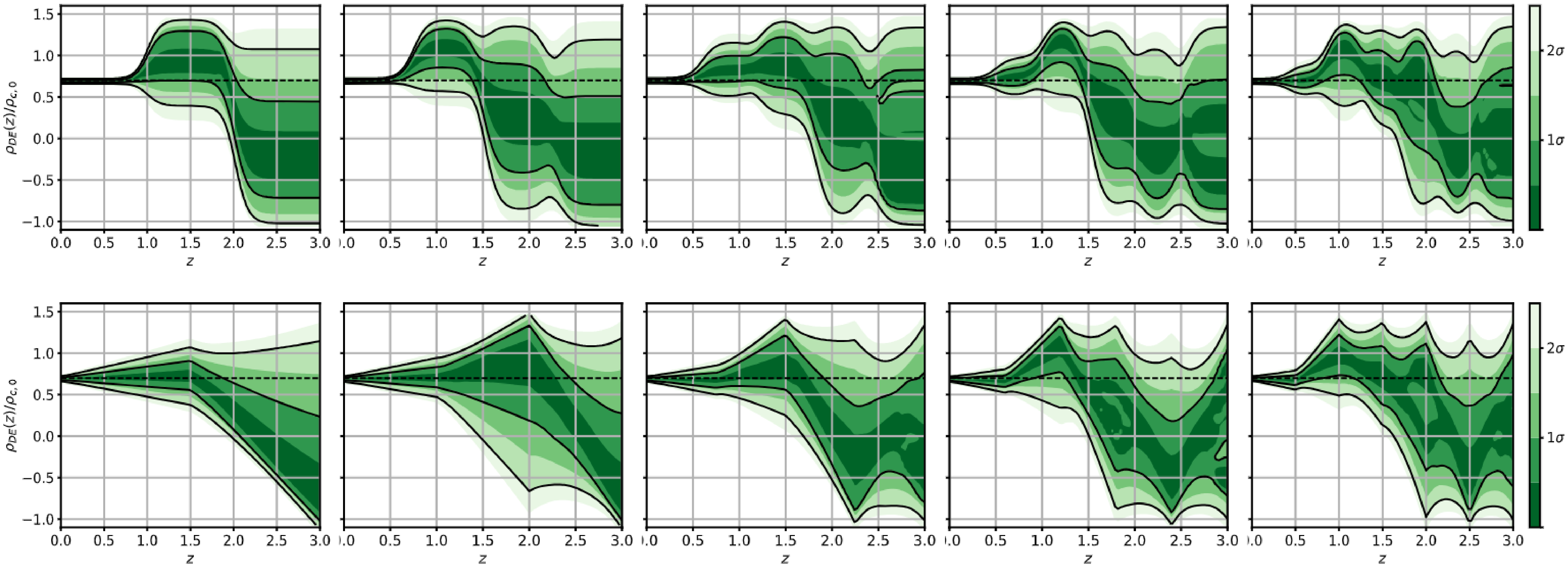
https://github.com/ja-vazquez/SimpleMC

RESULTS UP TO THIS POINT…

Results for the bins and nodes (EoS)

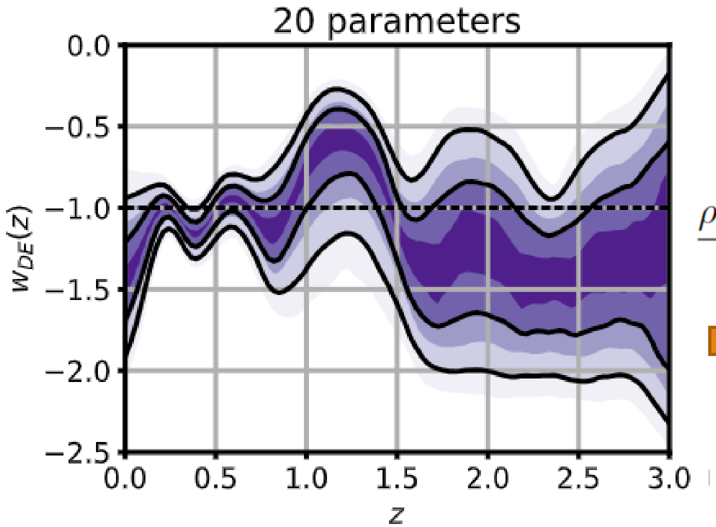


Results for the bins and nodes (density)

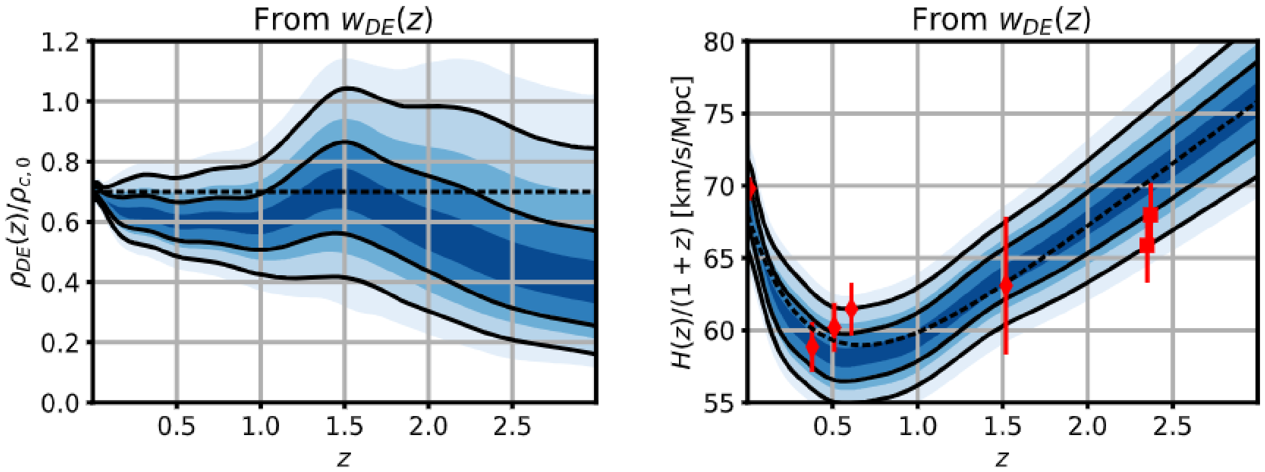


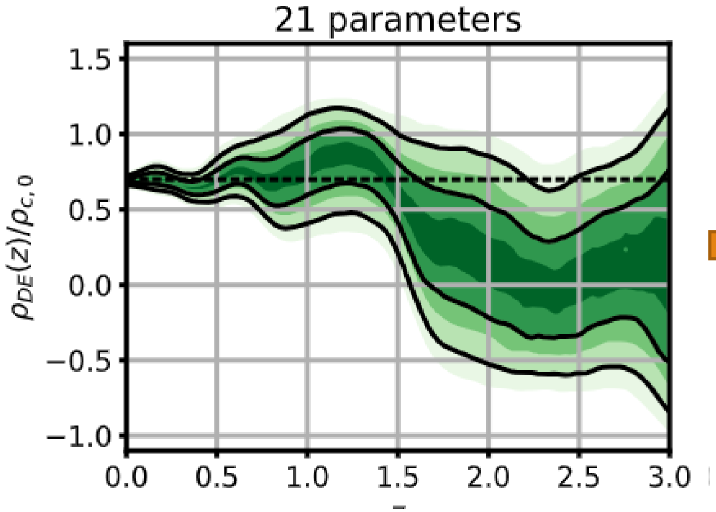
Results (20 bins)

arXiv:2111.10457

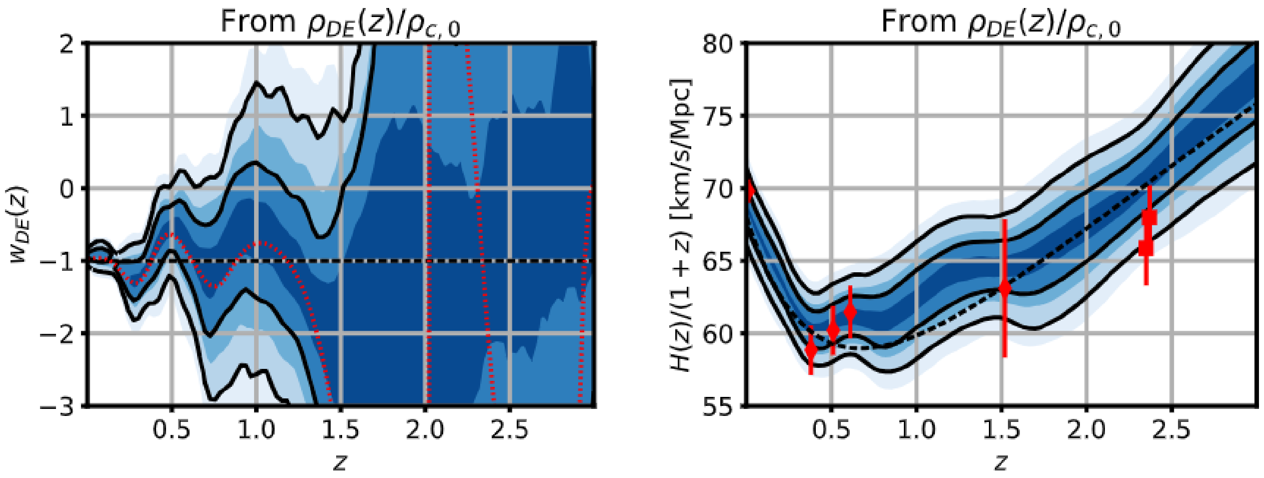


$$\frac{\rho_{\mathrm{DE}}(z)}{\rho_{\mathrm{c},0}} = \Omega_{\mathrm{DE},0}\, e^{3\int_0^z \frac{dz'}{1+z'}(1+w_{\mathrm{DE}}(z'))}$$





$$w_{\mathrm{DE}}(z) = -1 + \frac{1}{3}\frac{d\ln\rho_{\mathrm{DE}}(z)}{d\ln(1+z)}$$



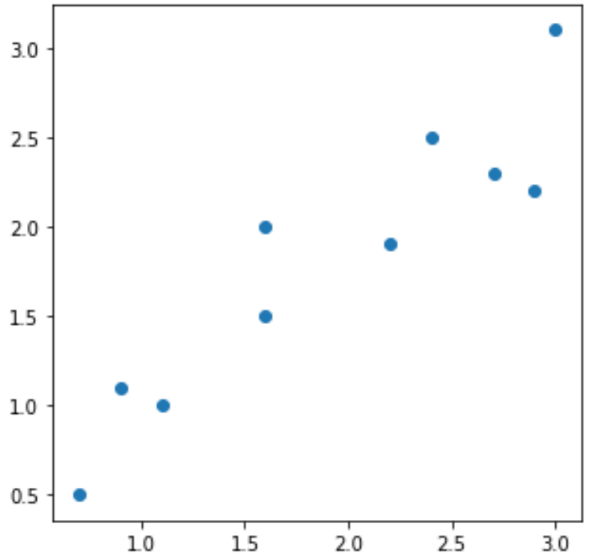
PCA

After conducting our reconstructions, a procedure called Principal Component Analysis (PCA) can be performed.

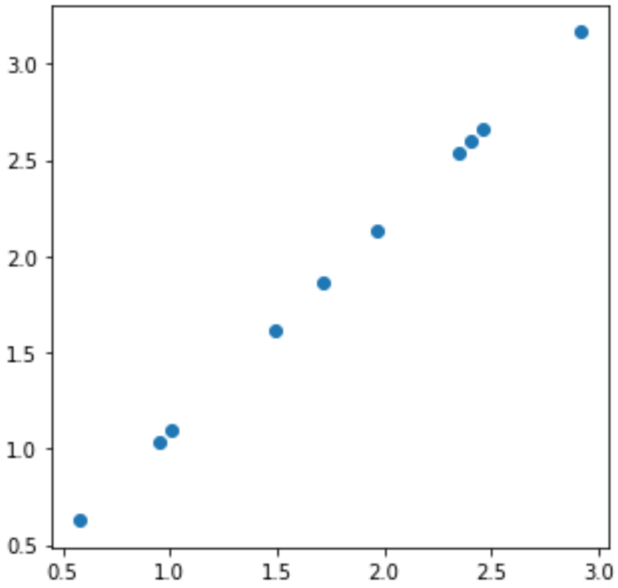
When it comes to data sets, PCA is used to eliminate 'noise,' extract 'hidden dynamics,' or even compress information by removing the least-contributing part to the total variance (e.g., in images).

We can obtain information about the constraint on our parameters, and therefore draw conclusions regarding the data used.

PCA



Eliminating 1 PC

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(876L, 1136L)
51
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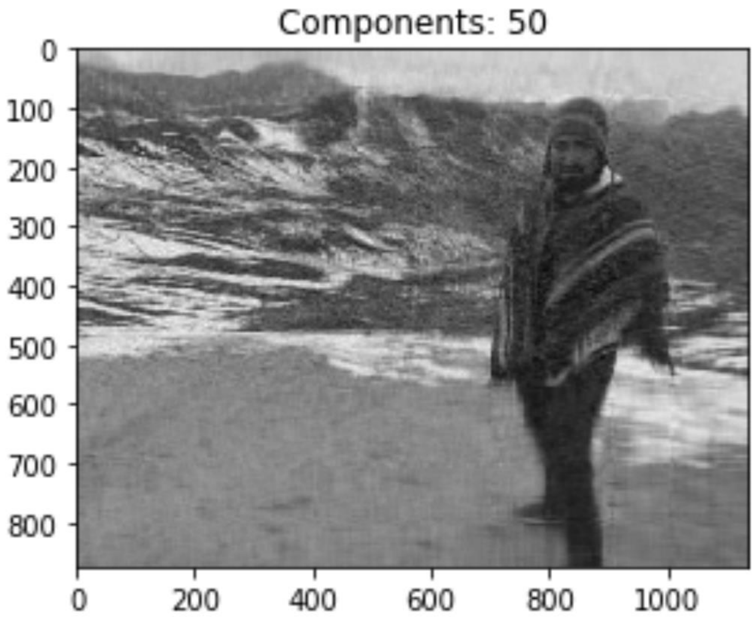
Eliminating 826 PCs



Results (PCA)

The transitions occur after z=2.2

In this redshift the only available data is the Lyman-alpha BAO.

When performing PCA and eliminating the noisy part of our reconstruction we observe an “overestimation” of the information in this region.

We can conclude that Lyman-alpha data does not constrain our parameters.

This implies heavy systematic errors in the data used.

It is possible that most of the behavior found in this region is only due to noisy data.

We need more data in this region to discern real responses from noise.

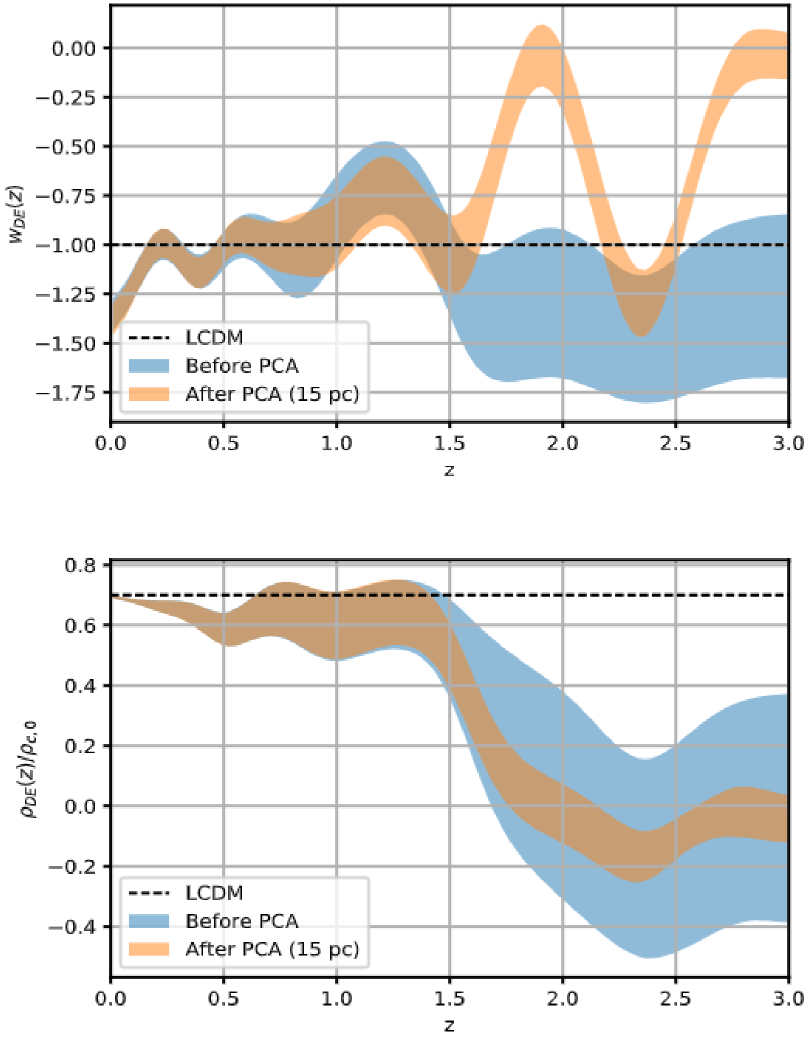


FIG. 7: Applying the principal component technique to our reconstructions of the EoS and the energy density with 20 bins. By eliminating 5 PCs (which add up to about 5% of the total variance for each reconstruction) we obtain the orange figures with slightly overestimated errors localized in $z > 1.5$.

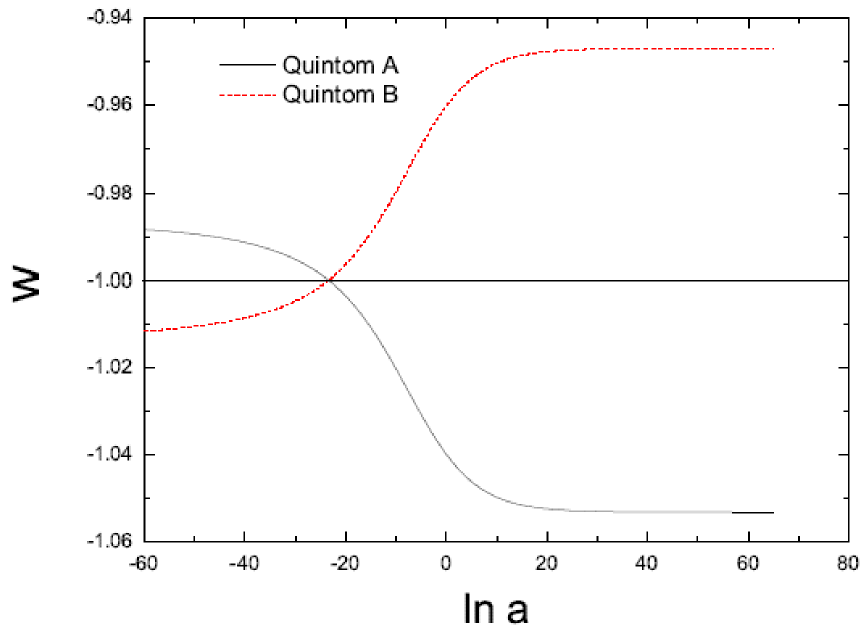
Results

Every reconstruction presents dynamic behavior, sometimes being preferred over LCDM by 2-sigma.

Also some type of oscillations are observed.

The EoS reconstructions present a possible crossing of the Phantom Divide Line in a manner similar to a Quintom Scalar Field. This transition is impossible for “simple” scalar fields.

The effective densities present possible transitions from positive to null or negative values, providing further motivation to study models (like gDE) that allow such a feature.

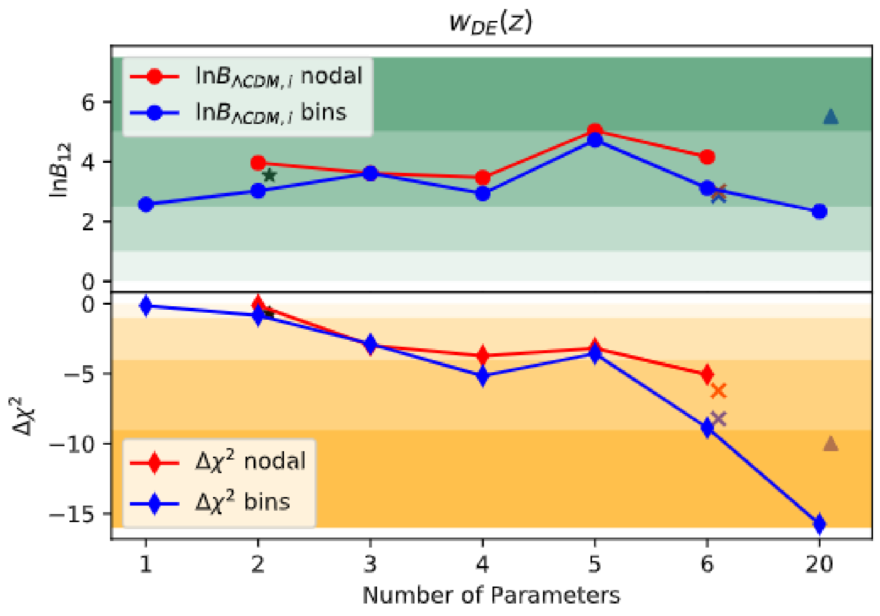


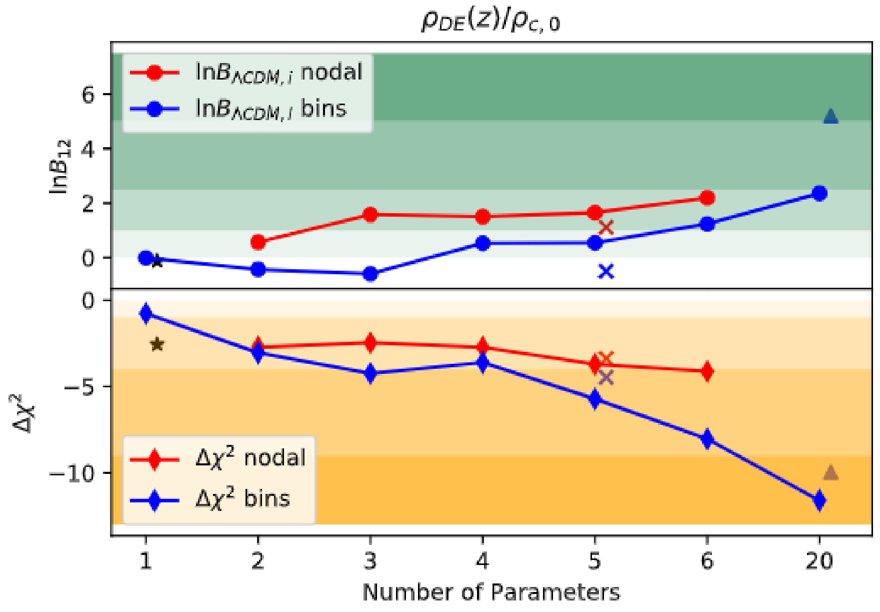
Results

Every reconstruction is worse than the LCDM model, although not by a long margin.

Even though our reconstructions present evidence against them when compared to LCDM, we see a clear improvement in the fitness to the data.

Finally, two of our reconstructions present a positive Bayes Factor, but the evidence in their favor is inconclusive at best.





Interacting Dark Energy with reconstructions

ARXIV:2305.16290

Phenomenological interaction

When working with DE and DM one usually assumes that they are conserved separately

$$\dot{\rho}_{\rm DM} + 3H\rho_{\rm DM} = 0.$$

$$\dot{\rho}_{\rm DE} + 3H\rho_{\rm DE}(1 + w_{\rm DE}) = 0.$$

But there is really nothing stopping us from proposing a phenomenological interaction between them:

$$\dot{\rho}_{\rm DM} + 3H\rho_{\rm DM} = Q,$$

$$\dot{\rho}_{\rm DE} + 3H\rho_{\rm DE}(1 + w_{\rm DE}) = -Q$$

Under this sign convention:

$$Q \begin{cases} > 0 \\ < 0 \end{cases} \rightarrow energy\ transfer\ is \quad \begin{cases} dark\ energy \rightarrow dark\ matter \\ dark\ matter \rightarrow dark\ energy \end{cases}$$

IDE model

$$\dot{\rho}_{\rm DM} + 3H(1 + w_{\rm eff,DM})\rho_{\rm DM} = 0$$

$$\dot{\rho}_{\rm DE} + 3H(1 + w_{\rm eff,DE})\rho_{\rm DE} = 0$$

$$w_{\rm eff,DM} = \frac{-Q}{3H\rho_{\rm DM}}, \quad w_{\rm eff,DE} = w_{\rm DE} + \frac{Q}{3H\rho_{\rm DE}}$$

$$\Pi_{\rm DM} = \frac{-Q}{3H\rho_{\rm c,0}} = -\Pi_{\rm DE}$$

$$\frac{{\rm d}(\rho_{\rm DM}/\rho_{\rm c,0})}{{\rm d}z} = \frac{3}{1+z}\left(\frac{\rho_{\rm DM}}{\rho_{\rm c,0}} + \Pi_{\rm DM}\right),$$

$$\frac{{\rm d}(\rho_{\rm DE}/\rho_{\rm c,0})}{{\rm d}z} = \frac{3}{1+z}\left[(1 + w_{\rm DE})\frac{\rho_{\rm DE}}{\rho_{\rm c,0}} + \Pi_{\rm DE}\right]$$

$$\frac{H^2(z)}{H_0^2} = \Omega_{b,0}(1+z)^3 + \frac{\rho_{\rm DM}(z)}{\rho_{\rm c,0}} + \frac{\rho_{\rm DE}(z)}{\rho_{\rm c,0}}$$

We will reconstruct the interaction kernel of an Interacting Dark Energy (IDE) model.

The interaction

If working from a “parametric reconstruction” point of view it is frequently assumed that the interaction should be a function of the energy densities and time (through the Hubble Parameter). Some examples:

$$Q = 3H\sigma(\rho_{\rm DE} - \alpha\rho_{\rm DM})$$

$$Q = 3H\gamma\frac{\rho_{dm}^2}{\rho}$$

$$Q = q(\alpha\dot{\rho}_m + 3\beta H\rho_m)$$

$$Q = q(\alpha\dot{\rho}_{tot} + 3\beta H\rho_{tot})$$

$$Q = q(\alpha\dot{\rho}_{_{DE}} + 3\beta H\rho_{_{DE}})$$

But again we opted for a model-independent approach.

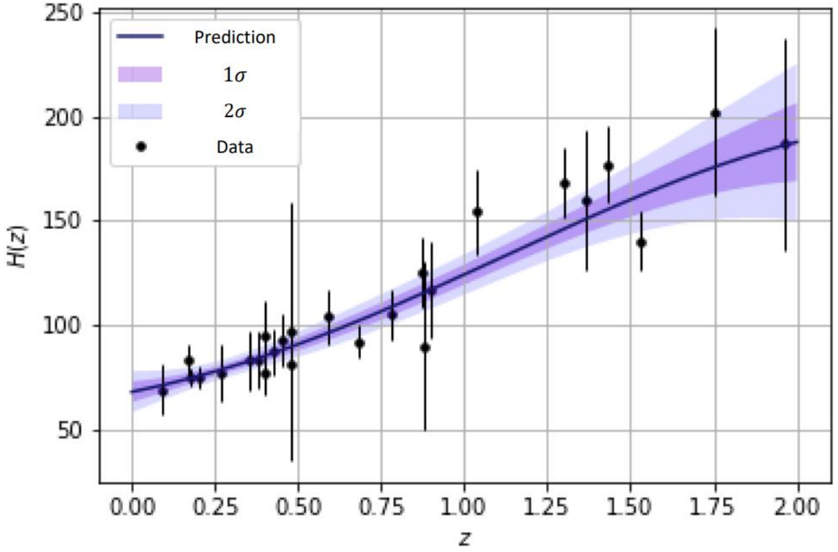
We will use again the binning scheme

$$w(z) = w_1 + \sum_{i=1}^{N-1} \frac{w_{i+1} - w_i}{2} \left(1 + \tanh\left(\frac{z - z_i}{\xi}\right)\right)$$

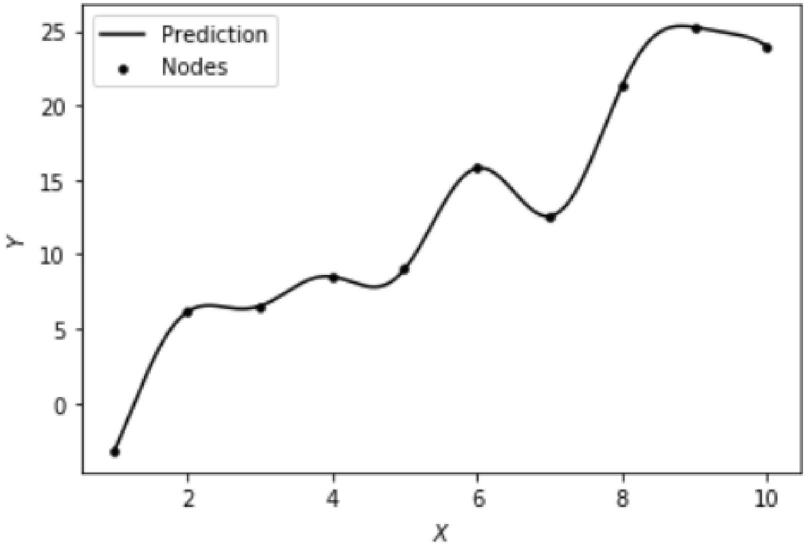
And also we will use a Gaussian Process, but in an unusual way…

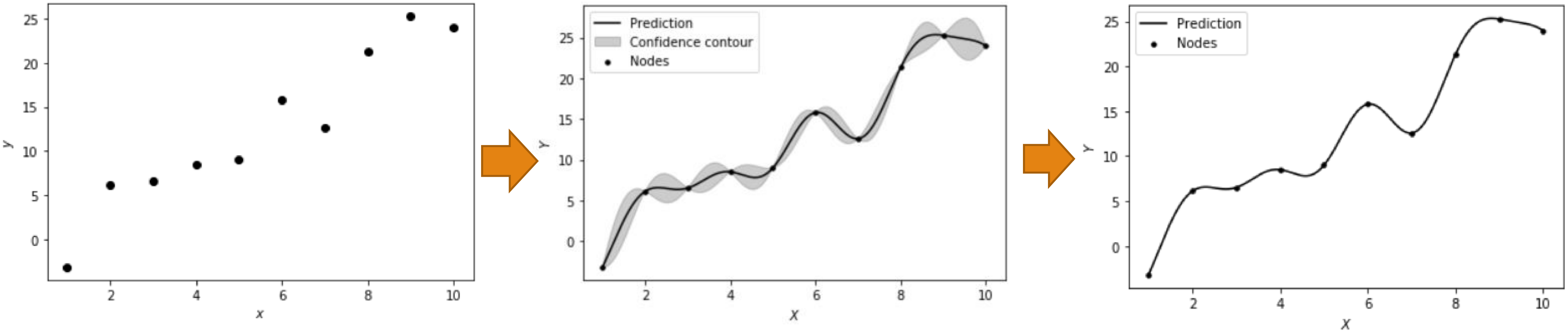
Gaussian Process as an interpolation

For the IDE reconstruction we also used a Gaussian Process (GP) but in a different approach than the usual one.

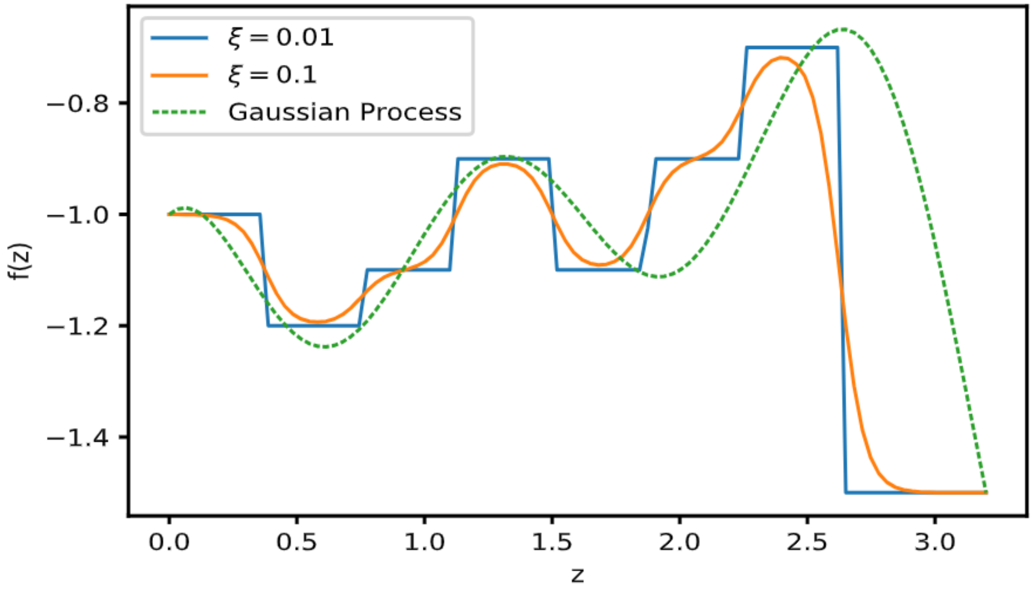


Our GP is used as an interpolation of nodes. This allow us to perform model comparison with LCDM (and others).





Visual comparison with the binning scheme



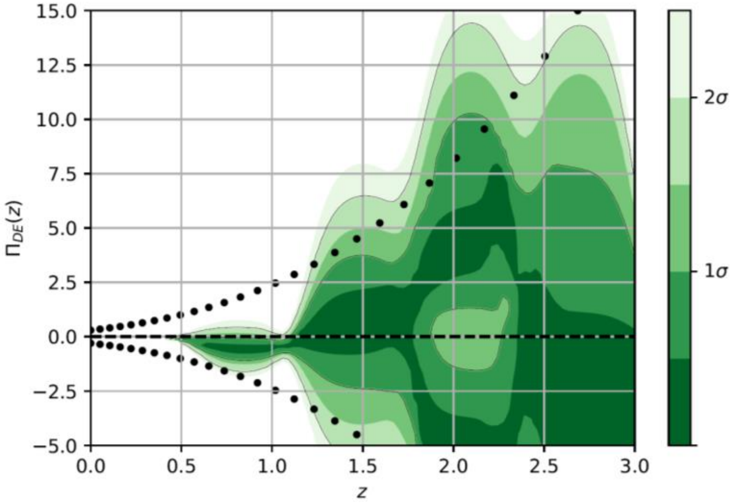
Results of the IDE reconstruction

We have 4 cases:

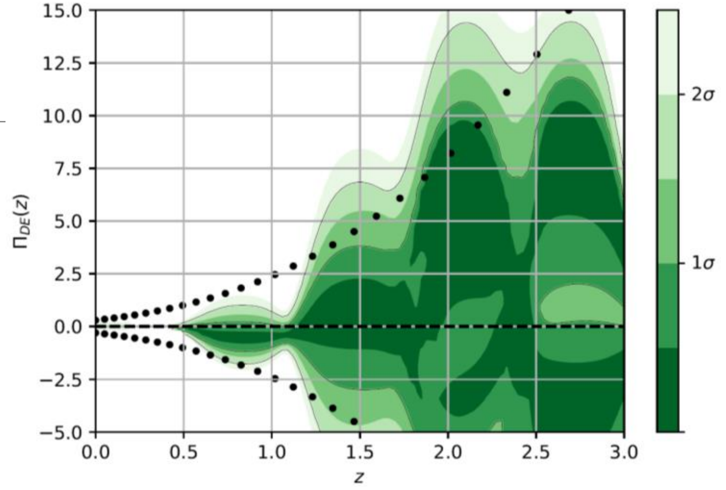
- Bins with $w_0 = -1$ for the EoS (case 1)
- Bins with a free parameter w_0 for the EoS (case 2)
- GP with $w_0 = -1$ for the EoS (case 3)
- GP with a free parameter w_0 for the EoS (case 4)

Results of the IDE reconstruction

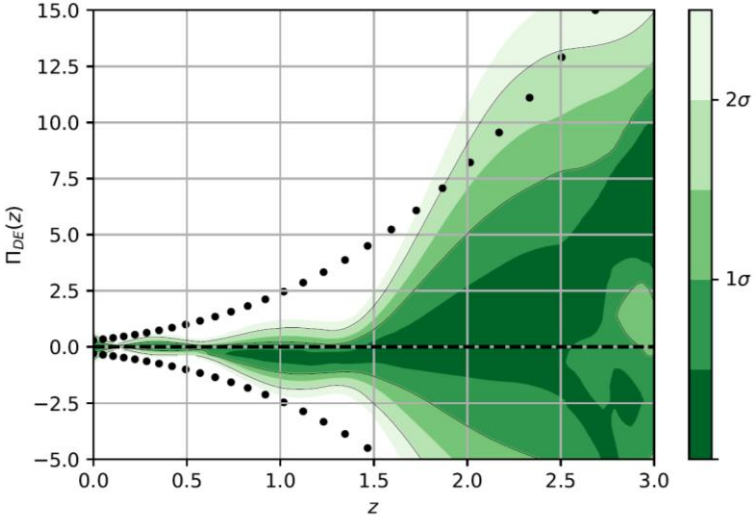
Case 1



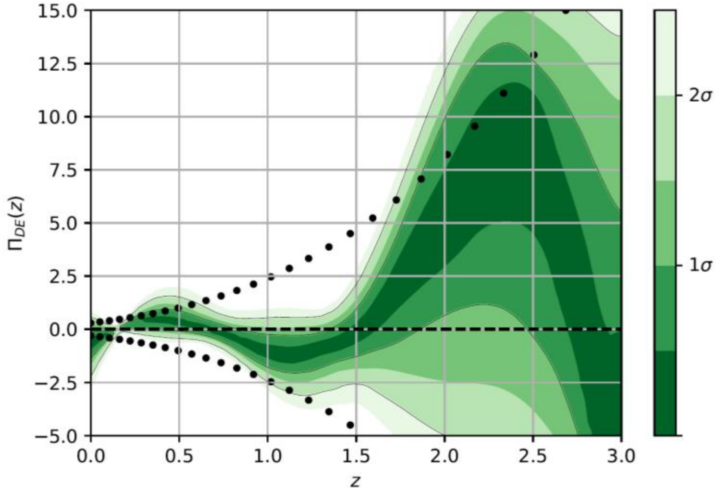
Case 2



Case 3

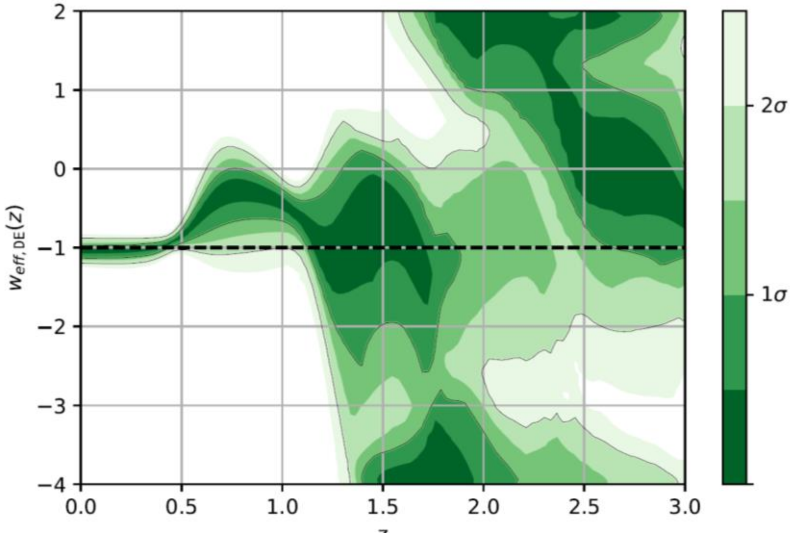


Case 4

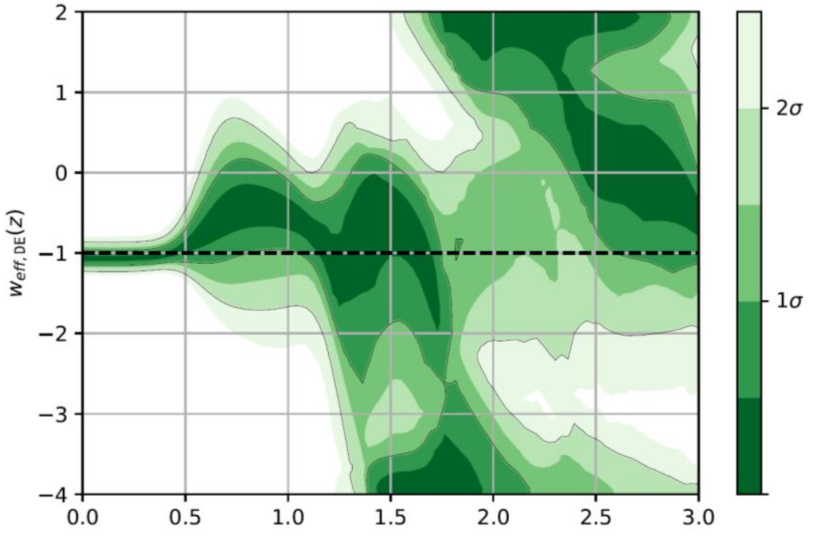


The effective EoS

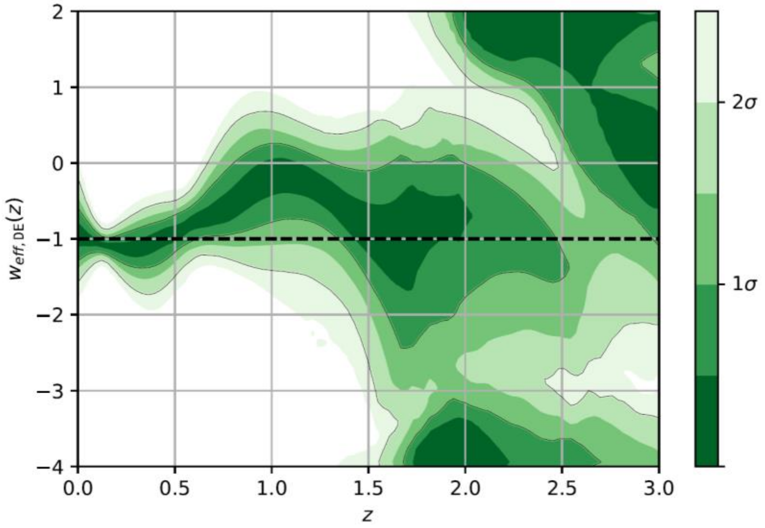
Case 1



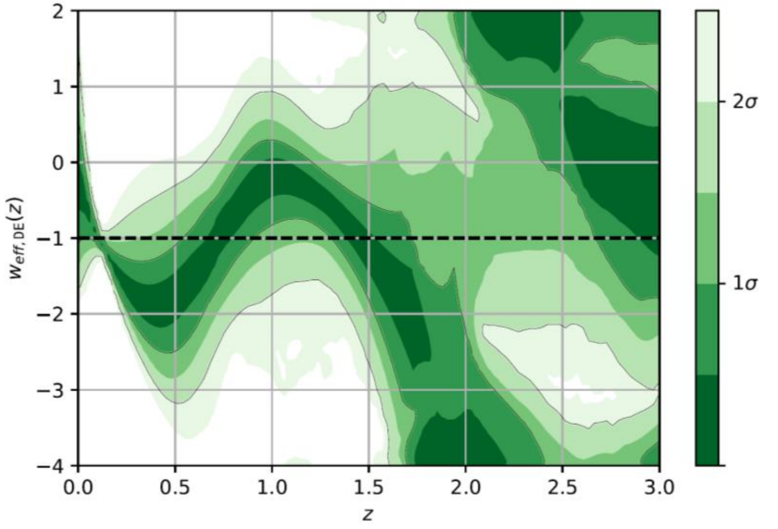
Case 2



Case 3

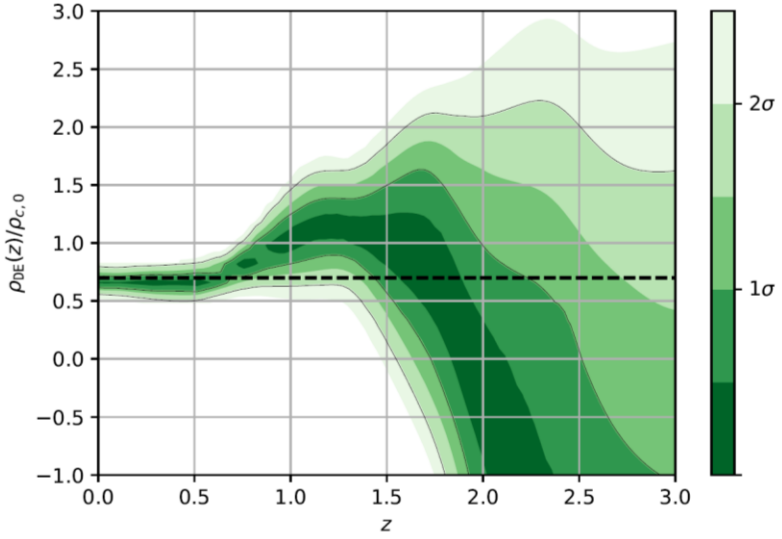


Case 4

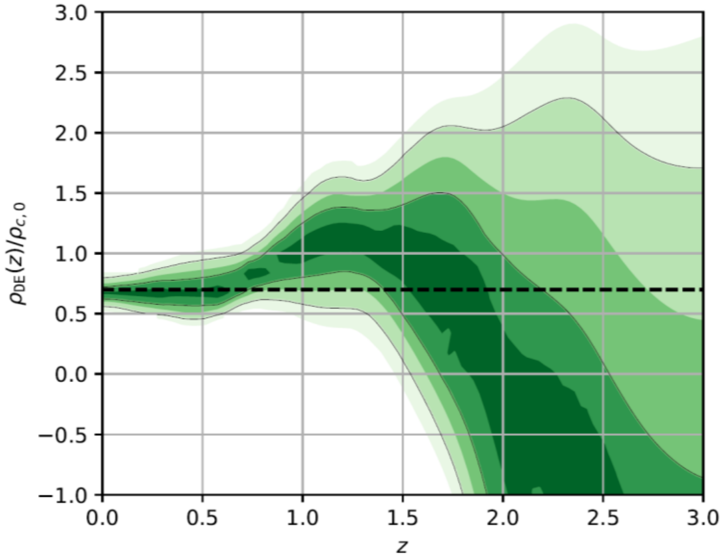


Their respective DE energy density

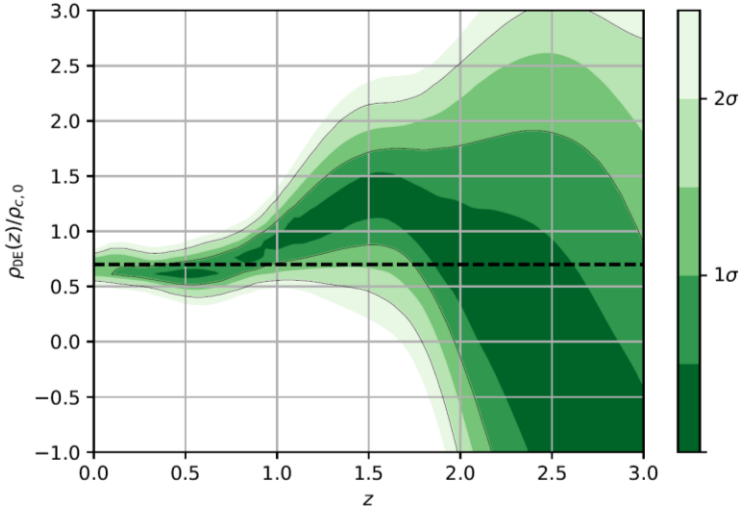
Case 1



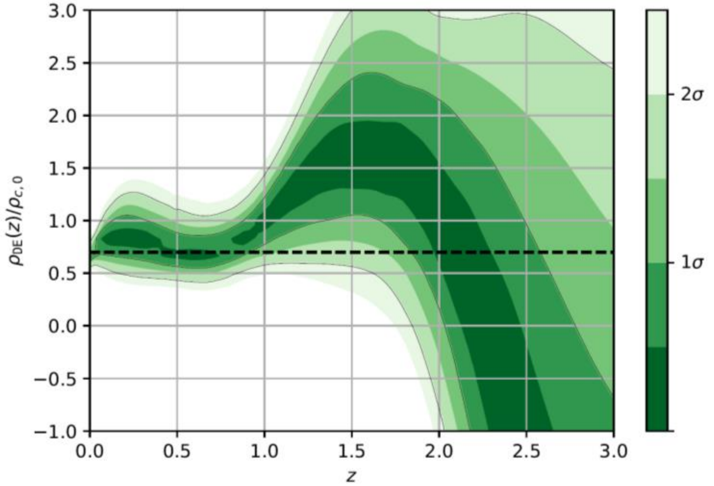
Case 2



Case 3



Case 4



Results of IDE reconstruction

Even though we are using an interactive DE we find a similar behavior: **negative density for DE**.

Also we find a possible oscillation in the interaction kernel, which implies several changes in the flux of energy from DE to DM.

Conclusions

-All reconstructions exhibit dependence on z (redshift).

-The reconstructions of the EoS show behavior similar to a Quintom-like dark energy.

-The reconstructions of the density parameter exhibit dynamic behavior. Its density may undergo a sign change at z>2 or a significant decrease.

-Bayesian evidence still favors LCDM (Lambda Cold Dark Matter).

-The analysis conducted with PCA indicates that the data at z>2 do not tightly constrain our parameters, suggesting that the observed dynamics could be due to systematic errors (with the Lyman-alpha data in this region).

-When using an interacting Dark Sector we recover similar features such as oscillations and a possible sing-switching density for DE.

Thank you for your attention!