EXORCIZING THE GHOSTS IN HIGHER DERIVATIVE GRAVITY

ANAMARIA HELL

LUDWIG-MAXIMILIANS-UNIVERSITÄT

IN COLABORATION WITH

DIETER LÜST AND GEORGE ZOUPANOS

The Basics

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$$S_{HDG} = \int d^4x\sqrt{-g}\left[M_{pl}^2(R-2\Lambda) + \alpha_{CG}W_{\mu\nu\rho\sigma}W^{\mu\nu\rho\sigma} + \beta R^2\right]$$

$$W_{\mu\nu\rho\sigma} = R_{\mu\nu\rho\sigma} + \frac{1}{6}R(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}) - \frac{1}{2}(g_{\mu\rho}R_{\nu\sigma} - g_{\mu\sigma}R_{\nu\rho} - g_{\nu\rho}R_{\mu\sigma} + g_{\nu\sigma}R_{\mu\rho}).$$

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$$h_k = A_T e^{ik\eta} + B_T e^{-ik\eta} + \eta \left[C_T e^{ik\eta} + D_T e^{-ik\eta} \right]$$

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- **Can we recover a general solution?**

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- **WHAT DETERMINES THE BACKGROUND?**

- AH, D. Lüst, G. Zoupanos, JHEP **08** (2023), 168

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$$S_{CG} = \alpha_{CG} \int d^4x \sqrt{-g} \left[2H_{\mu\nu}H^{\mu\nu} - \frac{4}{3}\Lambda\left(R - 2\Lambda\right) - 24\left(\frac{R}{12} - \frac{\Lambda}{3}\right)^2 \right] + \alpha_{CG} S_{GB}$$

$$R_{\mu\nu} = H_{\mu\nu} - \frac{g_{\mu\nu}}{4} R$$

$$\Lambda = 3H_\Lambda^2$$

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The Starobinski Expansion

- Expansion:

$$ds^2 = -dt^2 + \gamma_{ij}(\vec{x})dx^i dx^j, \qquad \text{at} \qquad t \to \infty$$

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$$ds^2 = a^2(\eta)\left[-d\eta^2 + \sum_{n=0}^{\infty}\frac{(-1)^n}{n!}(\eta H_\Lambda)^n g_{ij}^{(n)}(\vec{x})dx^i dx^j\right]$$

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- Relation:

$$g_{\mu\nu}^{(0)} = g_{\mu\nu}\big|_{\eta=0} \qquad g_{\mu\nu}^{(1)} = g'_{\mu\nu}\big|_{\eta=0}$$

$$H_\Lambda^2 g_{\mu\nu}^{(2)} = g''_{\mu\nu}\big|_{\eta=0}$$

$$R = \frac{1}{a^2}\left[6\frac{a''}{a} + R^{(0)} + H_\Lambda^2 g^{(2)}\left(3\eta\frac{a'}{a}\right) - H_\Lambda^3 g^{(3)}\left(\eta + \frac{3}{2}\eta^2\frac{a'}{a}\right) + \mathcal{O}(\eta^2)\right]$$

$$H_{00} = 3\frac{(a')^2}{a^2} - \frac{3}{2}\frac{a''}{a} + \frac{1}{4}R^{(0)} + \frac{H_\Lambda^2}{4}g^{(2)}\left(\eta\frac{a'}{a} - 1\right) + H_\Lambda^3\eta g^{(3)}\left(\frac{1}{4} - \frac{a'}{8a}\eta\right) + \mathcal{O}(\eta^2)$$

$$H_{0i} = \frac{H_\Lambda^2}{2}\eta\left(D_j g_i^{j(2)} - D_i g^{(2)}\right) + \mathcal{O}(\eta^2)$$

$$\begin{aligned}H_{ij} = {} & R_{ij}^{(0)} - \frac{1}{4}R^{(0)} + g_{ij}^{(0)}\left(\frac{(a')^2}{a^2} - \frac{a''}{2a}\right)\\ & + H_\Lambda^2\left[\frac{1}{2}g_{ij}^{(2)} + \frac{1}{2}\frac{(a')^2}{a^2}\eta^2 g_{ij}^{(2)} + \frac{a'}{a}\eta g_{ij}^{(2)} - \frac{1}{4}\frac{a''}{a}\eta^2 g_{ij}^{(2)} - \frac{g_{ij}^{(0)}}{4}g^{(2)}\left(1 + \frac{a'}{a}\eta\right)\right]\\ & - H_\Lambda^3\left[g_{ij}^{(3)}\left(\frac{\eta}{2} - \frac{a''\eta^3}{12a} + \frac{(a')^2\eta^3}{6a^2} + \frac{a'\eta^2}{2a}\right) - g_{ij}^{(0)}g^{(3)}\frac{\eta}{4}\left(1 + \frac{a\eta}{2a}\right)\right]\end{aligned}$$

dS Selection

- Conditions

$$H_{\mu\nu} = 0 \qquad R = 12H_\Lambda^2$$

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$$g_{ij}^{(1)} = 0 \qquad g_{ij}^{(2)} = \frac{2}{H_\Lambda^2}\left(R_{ij}^{(0)} - \frac{1}{4} g_{ij}^{(0)} R^{(0)}\right) \qquad g^{(3)} = 0$$

$$a = -\frac{1}{H_\Lambda \eta}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Vector modes:

$$V_i|_{\eta=0} = 0 \qquad V_i'|_{\eta=0} = 0$$

$$\rightarrow \quad V_i = 0$$

- Tensor modes:

$$h_{ij}'\big|_{\eta=0} = 0 \quad \& \quad h_k''|_{\eta=0} = k^2 \; h_k|_{\eta=0}$$

DS Selection

- Conditions

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$$h_k(\eta) = A(1 - ik\eta)e^{ik\eta} + B(1 + ik\eta)e^{-ik\eta}$$

Minkowski Selection

$$S_{CG} = \int d^4x\sqrt{-g}\left[2\alpha_{CG}H_{\mu\nu}H^{\mu\nu} - \frac{\alpha_{CG}}{2}R\left(\frac{R}{3} + \frac{M_{pl}^2}{\alpha_{CG}}\right) + \frac{M_{pl}^2}{2}R\right] + \alpha_{CG}S_{GB}$$

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- Conditions

$$H_{\mu\nu} = 0 \qquad\qquad R = 0$$

- Expansion

$$ds^2 = \left(g_{\mu\nu}^{(0)} + \sum_{n=1}^{\infty}\frac{1}{n!}h_{\mu\nu}^{(n)}\right)dx^\mu dx^\nu$$

$$h_{\mu\nu}^{(n)} = (-1)^n(\eta)^n g_{\mu\nu}^{(n)}$$

RECOVERING THE MINKOWSKI SPACETIME

- **CAN WE IMPOSE THE NBC?**

Recovering the Minkowski Spacetime

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$$h_k = A_T \left(e^{ik\eta} + e^{-ik\eta}\right)$$

Recovering the Minkowski Spacetime

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$$\boxed{g_{ij}^{(1)} \neq 0}$$

$$R = R^{(0)} + g^{(2)} + \frac{1}{4} g^{(1)} g^{(1)} - \frac{3}{4} g^{(1)}_{ij} g^{(1)ij} +$$

$$+ \eta \left(g^{(1)}_{ij} R^{(0)ij} + D_i D^i g^{(1)} - D_i D_j g^{(1)ij} \right) +$$

$$+ \eta \left(-g^{(3)} - \frac{1}{2} g^{(2)} g^{(1)} + \frac{5}{2} g^{(1)ij} g^{(2)}_{ij} + \frac{1}{2} g^{(1)ij} g^{(1)}_{ij} g^{(1)} - \frac{3}{2} g^{(1)ij} g^{(1)k}_{i} g^{(1)}{}_{jk} \right)$$

$$H_{0i} = \frac{1}{2} \left(D_i g^{(1)} - D_j g^{(1)j}_{i} \right) +$$

$$+ \frac{\eta}{2} \left(\frac{3}{2} g^{(1)kl} D_i g^{(1)}_{kl} + \frac{1}{2} g^{(1)}_{ij} D^j g^{(1)} - g^{(1)}_{ij} D_k g^{(1)jk} - g^{(1)jk} D_j g^{(1)}_{ik} + D^j g^{(2)}_{ij} - D_i g^{(2)} \right)$$

$$H_{00} = \frac{1}{4} R^{(0)} - \frac{1}{4} g^{(2)} + \frac{1}{16} \left(g^{(1)ij} g^{(1)}_{ij} + g^{(1)} g^{(1)} \right) +$$

$$+ \frac{\eta}{4} \left(g^{(1)ij} R^{(0)}_{ij} - D_i D_j g^{(1)ij} + D_i D^i g^{(1)} \right) +$$

$$+ \frac{\eta}{8} \left(2 g^{(3)} - g^{(2)} g^{(1)} - 3 g^{(1)ij} g^{(2)}_{ij} + g^{(1)ij} g^{(1)k}_{i} g^{(1)}_{jk} + g^{(1)ij} g^{(1)}_{ij} g^{(1)} \right)$$

$$H_{ij} = R^{(0)}_{ij} - \frac{1}{4} g^{(0)}_{ij} R^{(0)} + \frac{1}{2} \left[g^{(2)}_{ij} - g^{(1)}_{jk} g^{(1)k}_{i} - \frac{1}{2} g^{(0)}_{ij} g^{(2)} + \frac{1}{2} g^{(1)} g^{(1)}_{ij} + \frac{g^{(0)}_{ij}}{8} \left(3 g^{(1)}_{kl} g^{(1)kl} - g^{(1)} g^{(1)} \right) \right]$$

$$- \frac{\eta}{4} \left[g^{(0)}_{ij} g^{(1)kl} R^{(0)}_{kl} - g^{(1)}_{ij} R^{(0)} + 2 D_k D_j g^{(1)k}_{i} + 2 D_k D_i g^{(1)k}_{j} - 2 D_k D^k g^{(1)}_{ij} - 2 D_i D_j g^{(1)} \right.$$

$$\left. + g^{(0)}_{ij} \left(D_k D^k g^{(1)} - D_k D_l g^{(1)kl} \right) \right] + \frac{\eta}{16} \left[-8 g^{(3)}_{ij} + 4 g^{(0)}_{ij} g^{(3)} - 4 g^{(1)} g^{(2)}_{ij} + 8 g^{(1)}_{ik} g^{(2)k}_{j} \right.$$

$$+ 8 g^{(1)}_{jk} g^{(2)k}_{i} + 2 g^{(0)}_{ij} \left(g^{(1)} g^{(2)} - 5 g^{(1)kl} g^{(2)}_{kl} \right) + g^{(1)}_{ij} \left(g^{(1)} g^{(1)} + g^{(1)kl} g^{(1)}_{kl} \right) - 8 g^{(1)kl} g^{(1)}_{jk} g^{(1)}_{il}$$

$$\left. + 2 g^{(0)}_{ij} \left(3 g^{(1)kl} g^{(1)}_{ks} g^{(1)s}_{l} - g^{(1)kl} g^{(1)}_{kl} g^{(1)} \right) \right]$$

$$D_i g^{(1)} = D_j g_i^{(1)j}$$

$$g_{ij}^{(2)} = -2R_{ij}^{(0)} + g_{ik}^{(1)} g_j^{(1)k} - \tfrac{1}{2} g^{(1)} g_{ij}^{(1)}$$

$$g^{(2)} = \frac{g_{ij}^{(1)} g^{(1)ij}}{2} \qquad R^{(0)} = \tfrac{1}{4}\left(g^{(1)ij} g_{ij}^{(1)} - g^{(1)} g^{(1)}\right)$$

$$g_{ij}^{(3)} = -D_k D_j g_i^{(1)k} - D_k D_i g_j^{(1)k} + D_k D^k g_{ij}^{(1)} + D_i D_j g^{(1)}$$

$$+ g_{ik}^{(1)} g_j^{(2)k} + g_{jk}^{(1)} g_i^{(2)k} + \frac{1}{2}\left(g_{ij}^{(1)} g^{(2)} - g^{(1)} g_{ij}^{(2)}\right) - g^{(1)kl} g_{jk}^{(1)} g_{il}^{(1)}$$

Recovering the Minkowski Spacetime

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$$h_k = A_T \left(e^{ik\eta} + e^{-ik\eta}\right)$$

$$g^{(1)}_{ij} \neq 0$$

$$g^{(0)}_{\mu\nu} = g_{\mu\nu}\big|_{\eta=0} \qquad g^{(1)}_{\mu\nu} = -\, g'_{\mu\nu}\big|_{\eta=0}$$

$$g^{(2)}_{\mu\nu} = g''_{\mu\nu}\big|_{\eta=0} \qquad g^{(3)}_{\mu\nu} = -\, g'''_{\mu\nu}\big|_{\eta=0}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

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Recovering the Minkowski Spacetime

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$$V_i|_{\eta=0} = 0 \qquad V_i'|_{\eta=0} = 0 \qquad V_i''|_{\eta=0} = 0$$

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Einstein-Weyl Theory

$$S = \int \sqrt{-g}\left(M_{pl}^2 R + \frac{1}{g_W^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

- Degrees of Freedom

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

$$\mathcal{L}_S = -6M_{pl}^2(\dot{\psi}\dot{\psi} - \partial_j\psi\partial_j\psi + m_g^2\psi^2)$$

$$\mathcal{L}_V = \frac{1}{2g_W^2}(\dot{V}_{in}\dot{V}_{in} - \partial_j V_{in}\partial_j V_{in} + m_g^2 V_{in}V_{in})$$

$$\mathcal{L}_T = \frac{1}{g_W^2} h_{ij}^T(\partial^2 + m_g^2)\partial^2 h_{ij}^T$$

$$m_g^2 = g_W^2 M_{pl}^2$$

- General Solutions

$$\psi_k = A_S e^{i\omega_k t} + B_S e^{-i\omega_k t}$$

$$v_k = A_V e^{i\omega_k t} + B_V e^{-i\omega_k t}$$

$$h_k = A_T e^{it\omega_k} + B_T e^{-it\omega_k} + C_T e^{ikt} + D_T e^{-ikt}$$

$$\omega_k = \sqrt{|\vec{k}|^2 - m_g^2}$$

Einstein-Weyl Theory

Conditions – Einstein gravity

$$\psi'_k|_{t=0} = 0 \qquad 3\psi''_k + (k^2 + 3m_g^2)\psi_k|_{t=0} = 0$$

$$\psi''_k + 3(k^2 + 3m_g^2)\psi_k|_{t=0} = 0 \qquad \psi'''_k + (k^2 + 3m_g^2)\psi'_k|_{t=0} = 0$$

$$\rightarrow \quad \psi_k = 0$$

$$v_k|_{t=0} = 0 \qquad v'_k|_{t=0} = 0 \qquad v''_k|_{t=0} = 0$$

$$\rightarrow \quad v_k = 0$$

$$h''_k|_{t=0} = -k^2\, h_k|_{t=0} \qquad -k^2\, h'_k|_{t=0} = h'''_k|_{t=0}$$

$$\rightarrow \quad h_k(t) = C_T e^{ikt} + D_T e^{-ikt}$$

Massive gravity ?

$$S = \int \sqrt{-g}\left(-M_{pl}^2 R + \frac{1}{g_W^2} W_{\mu\nu\rho\sigma} W^{\mu\nu\rho\sigma}\right)$$

$$\psi''_k|_{t=0} = -\omega_k^2\, \psi_k|_{t=0} \qquad -\omega_k^2\, \psi'_k|_{t=0} = \psi'''_k|_{t=0}$$

$$v''_k|_{t=0} = -\omega_k^2\, v_k|_{t=0} \qquad -\omega_k^2\, v'_k|_{t=0} = v'''_k|_{t=0}$$

$$h''_k|_{t=0} = -\omega_k^2\, h_k|_{t=0} \qquad -\omega_k^2\, h'_k|_{t=0} = h'''_k|_{t=0}$$

$$\rightarrow \quad \psi_k = A_S e^{i\omega_k t} + B_S e^{-i\omega_k t}$$

$$v_k = A_V e^{i\omega_k t} + B_V e^{-i\omega_k t} \qquad \omega_k = \sqrt{|\vec{k}|^2 + m_g^2}$$

$$h_k = A_T e^{it\omega_k} + B_T e^{-it\omega_k}$$

Conclusion

- **Ghosts?**
- Well, yes…

CONCLUSION

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- Well, yes…

- **BUT CAN WE REMOVE THEM?**
- Well, yes…

CONCLUSION

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HOW?

- **BOUNDARY CONDITIONS**
- In both dS and Minkowski space!

  But with different BC!

Conclusion

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  But with different BC!

- **DS** $h'_{ij}\big|_{\eta=0} = 0$ $\quad h''_k|_{\eta=0} = k^2\ h_k|_{\eta=0}$
- **MS** $-k^2 h'_k\big|_{\eta=0} = \frac{\partial^3 h_k}{\partial\eta^3}\Big|_{\eta=0}$ $\quad h''_k|_{\eta=0} = -k^2 h_k\big|_{\eta=0}$

CONCLUSION

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CURRENTLY IN MOTION

- **EXTENSION TO OTHER HIGHER-DERIVATIVE GRAVITATIONAL THEORIES IN MINKOWSKI AND DS SPACE-TIME**

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CURRENTLY IN MOTION

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THANK YOU!

Backup

$$S_{CG} = \alpha_{CG} \int d^4x\sqrt{-g}\left[2H_{\mu\nu}H^{\mu\nu} - \frac{4}{3}\Lambda\left(R - 2\Lambda\right) - 24\left(\frac{R}{12} - \frac{\Lambda}{3}\right)^2\right] + \alpha_{CG}S_{GB}$$

$$\alpha_{CG}S_{GB} = \frac{8}{3}\alpha_{CG}\Lambda^2 \int d^4x\sqrt{-g}$$

$$-\alpha_{CG}\frac{4}{3}\Lambda\left(R - 2\Lambda\right) = -\alpha_{CG}\frac{8}{3}\Lambda^2$$

$$M_{pl}^2 = -\frac{8}{3}\alpha_{CG}\Lambda$$

$$S_{EH} = \frac{M_{pl}^2}{2} \int d^4x\sqrt{-g}(R - 2\Lambda) = M_{pl}^2\Lambda \int d^4x\sqrt{-g}$$