中国科学技术大学
University of Science and Technology of China

Ultra-Light Dark Matter with Non-Canonical Kinetics

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September 11, 2023

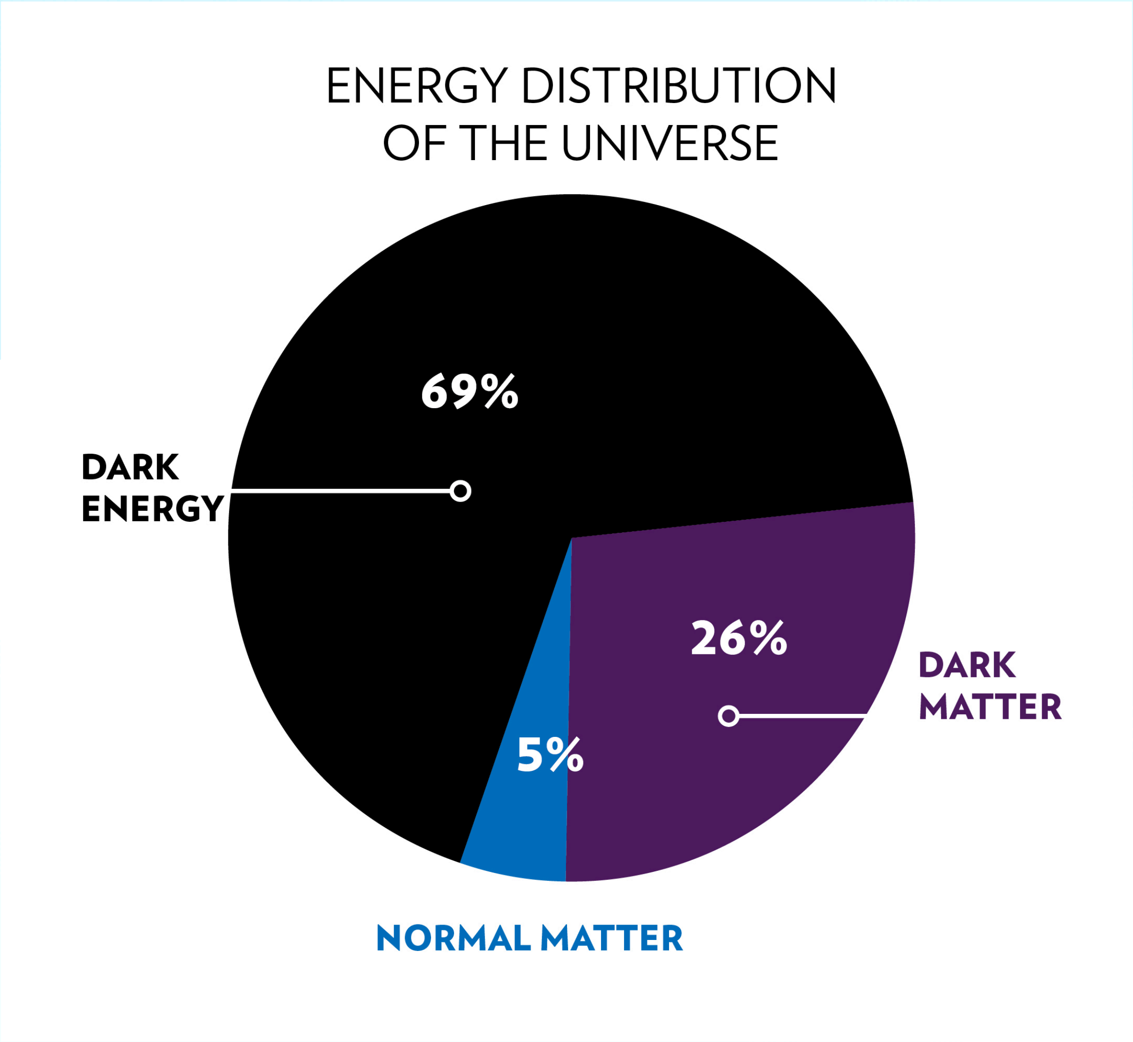
Based on the work with Yi-Fu Cai, Elisa G. M. Ferreira et al.
arXiv: Coming very soon

Introduction and Motivation

Introduction and Motivation

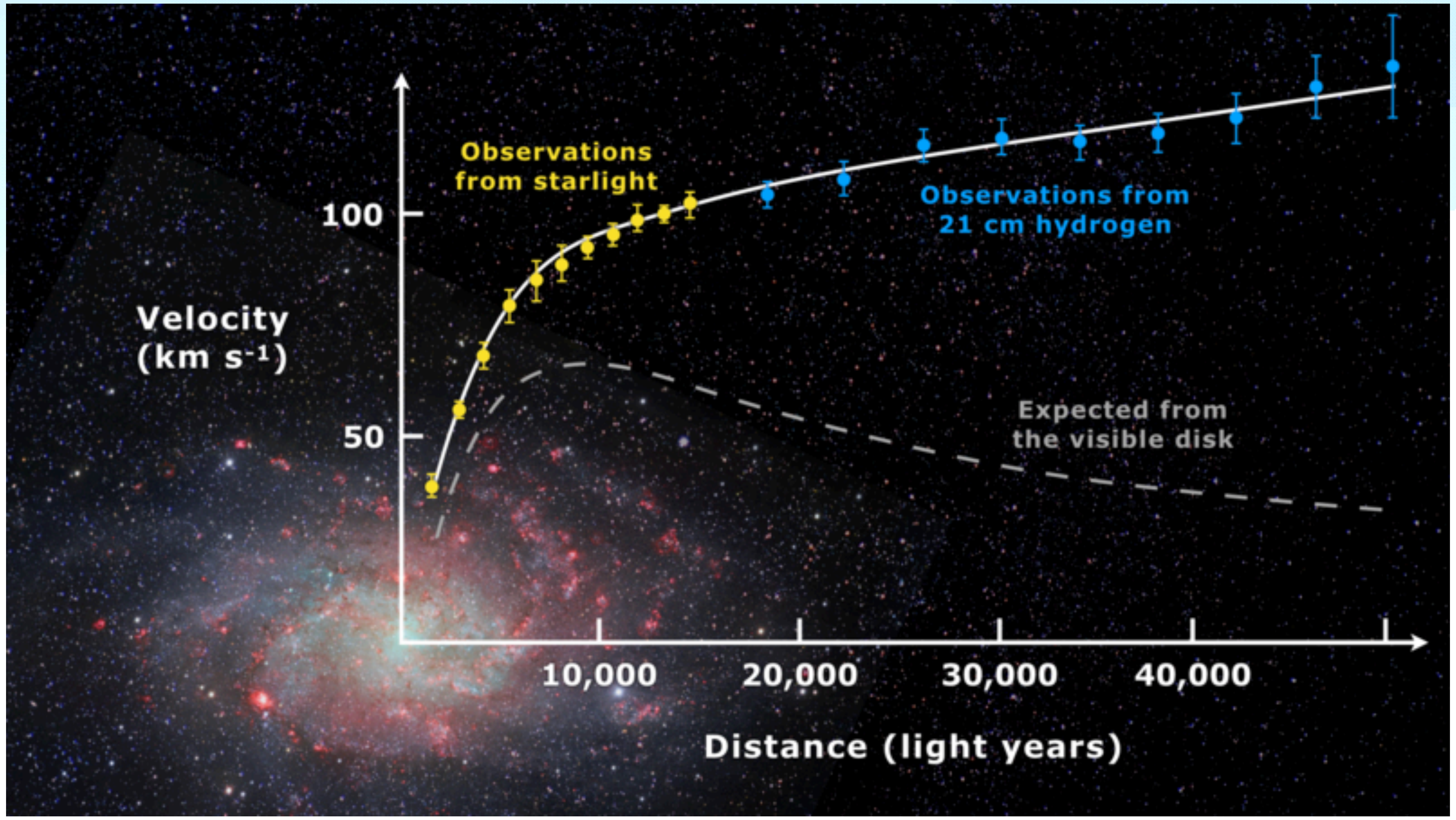
Dark matter

- CMB and Type Ia determine the $\Omega_{\rm DM}$



(Credit: NASA/CXC/K.Divona)

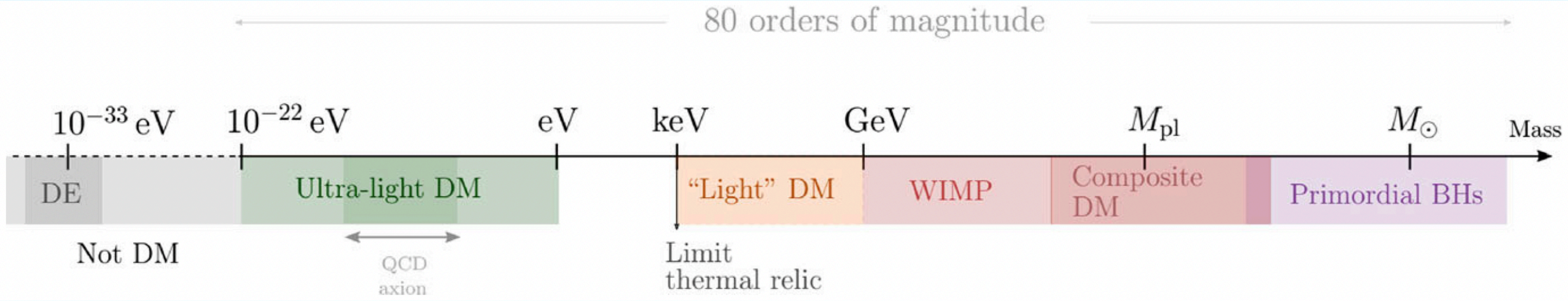
- Galaxy rotation curve



Rotation curve of spiral galaxy Messier 33

Introduction and Motivation

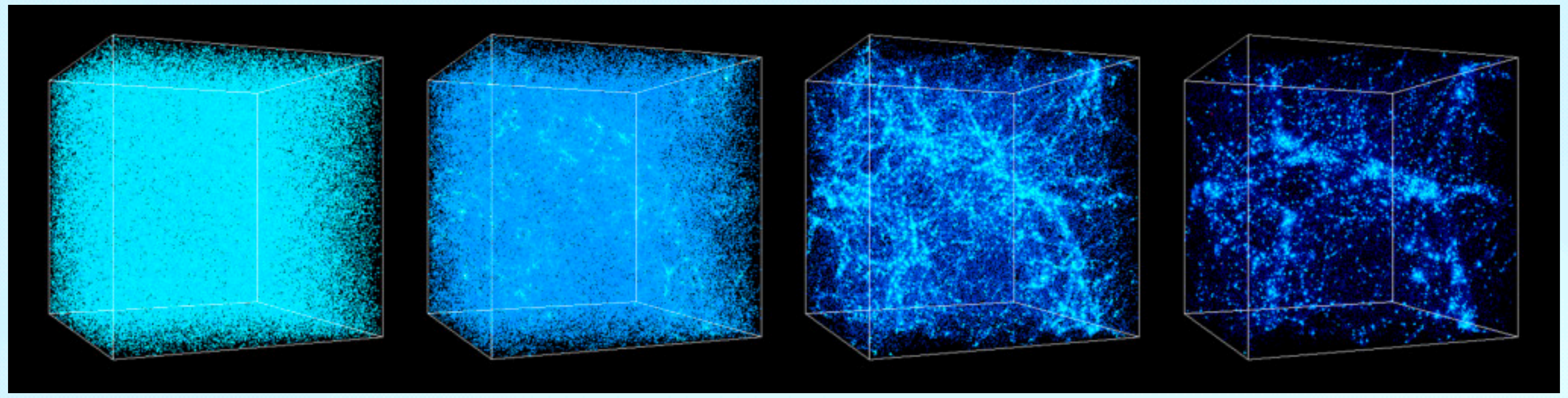
Dark matter models



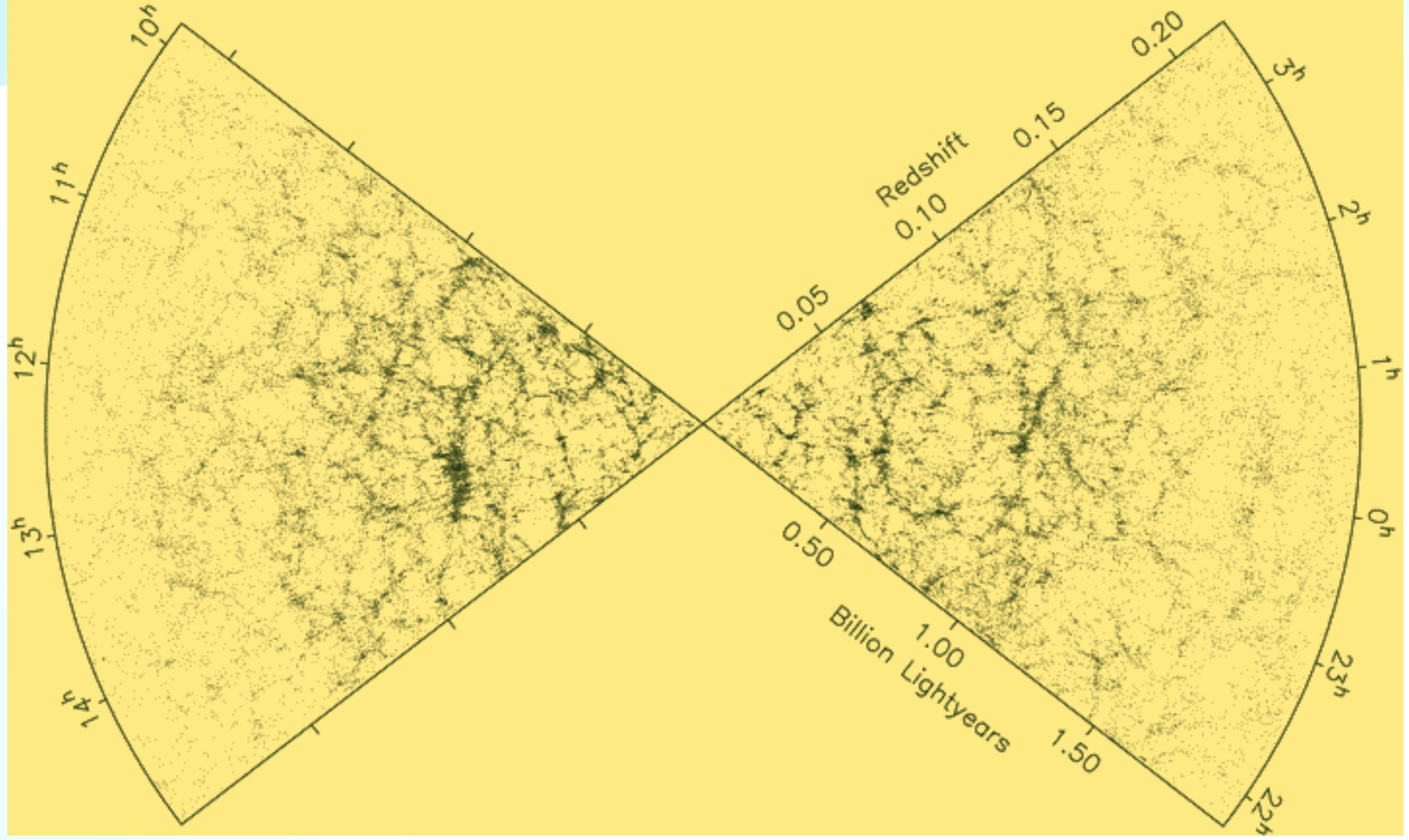
- WIMP; ULDM; …
- Most evidence are from gravitational effects, leaving the nature of DM unclear.
- Small-scale behavior?

Introduction and Motivation

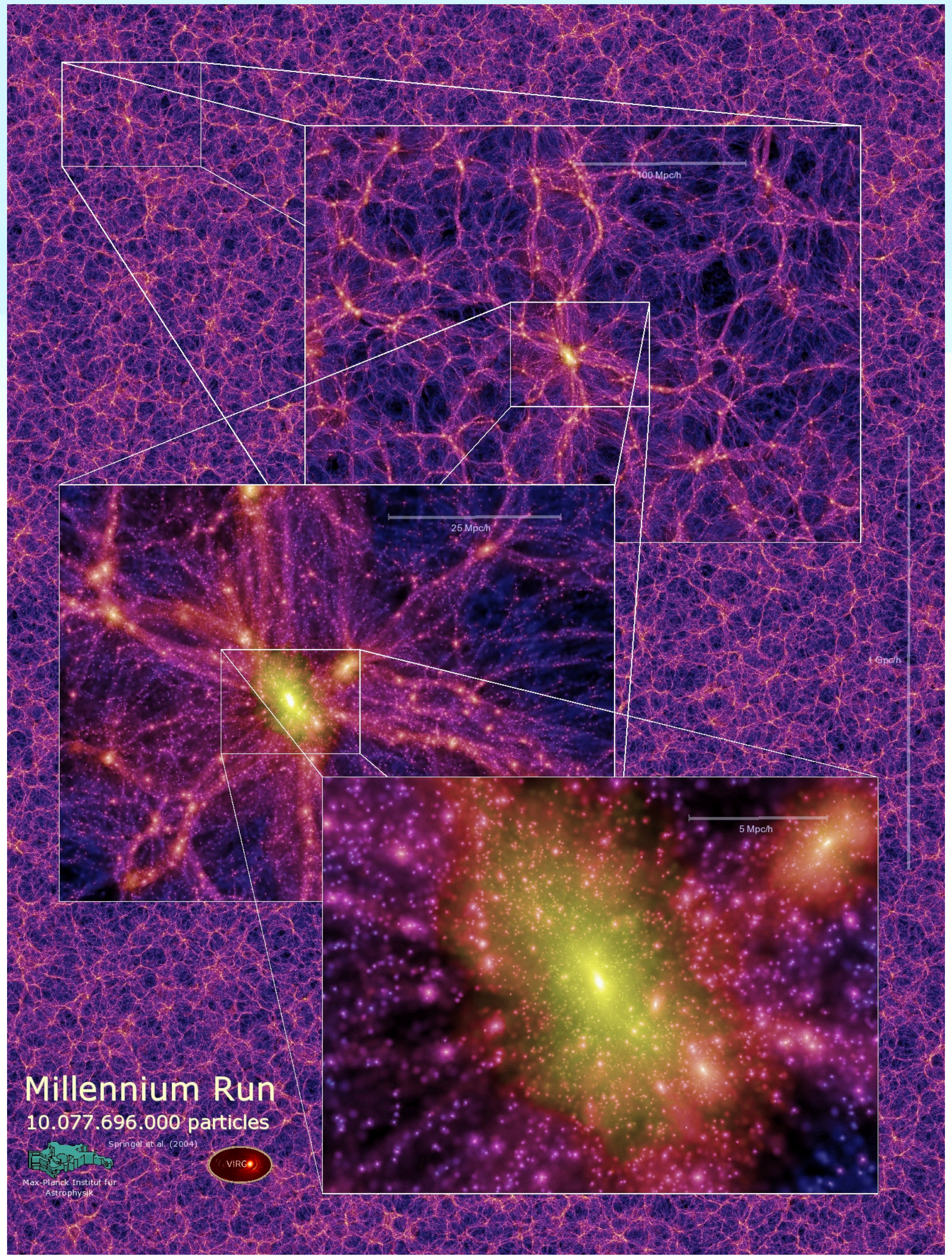
Structure formation



Andrey Kravtsov, Anatoly Klypin, National Center for Supercomputer Applications (NCSA)



The 2dF Galaxy Redshift Survey



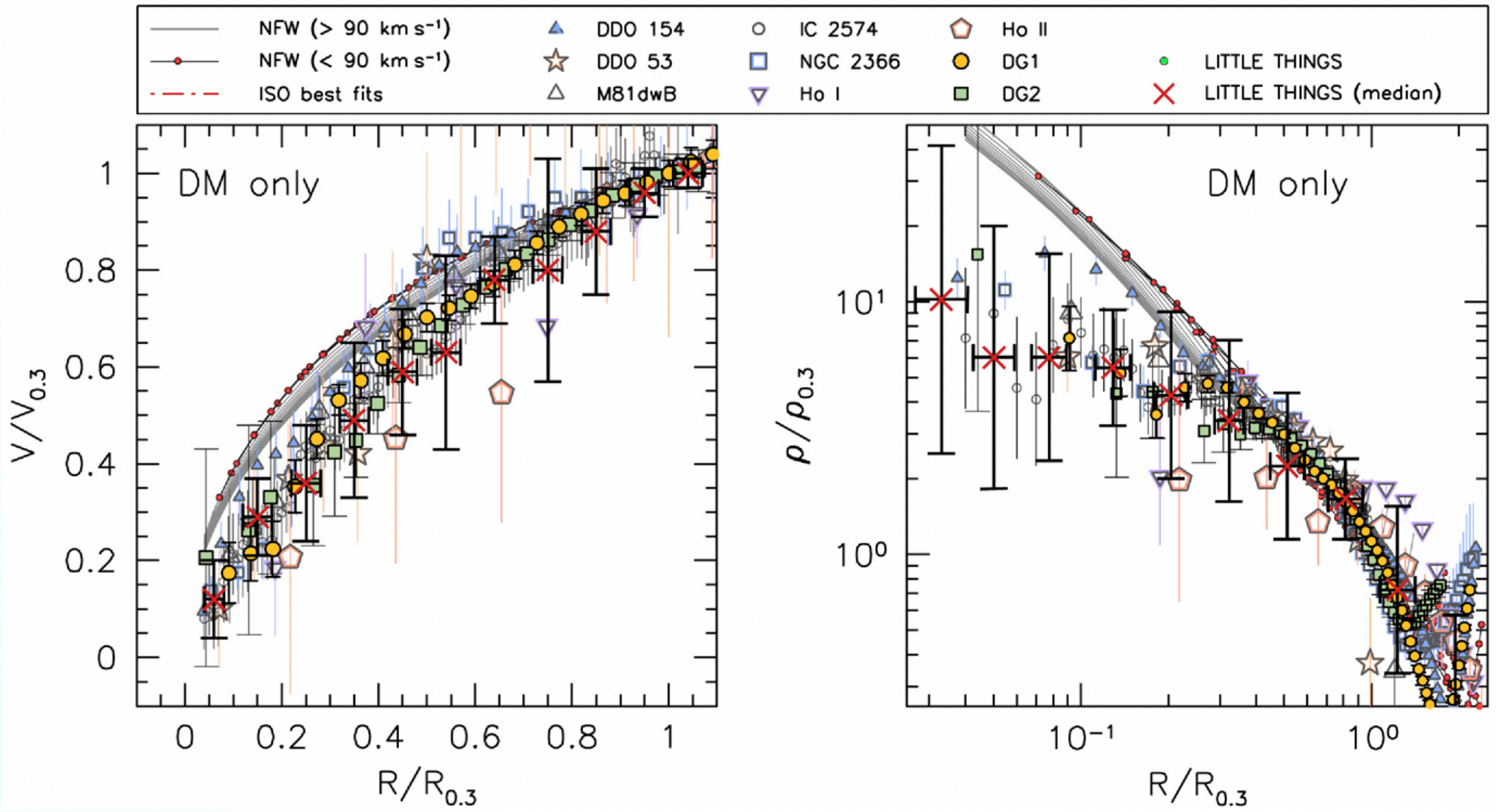
Springel et al., Virgo Consortium (simulation)

Introduction and Motivation

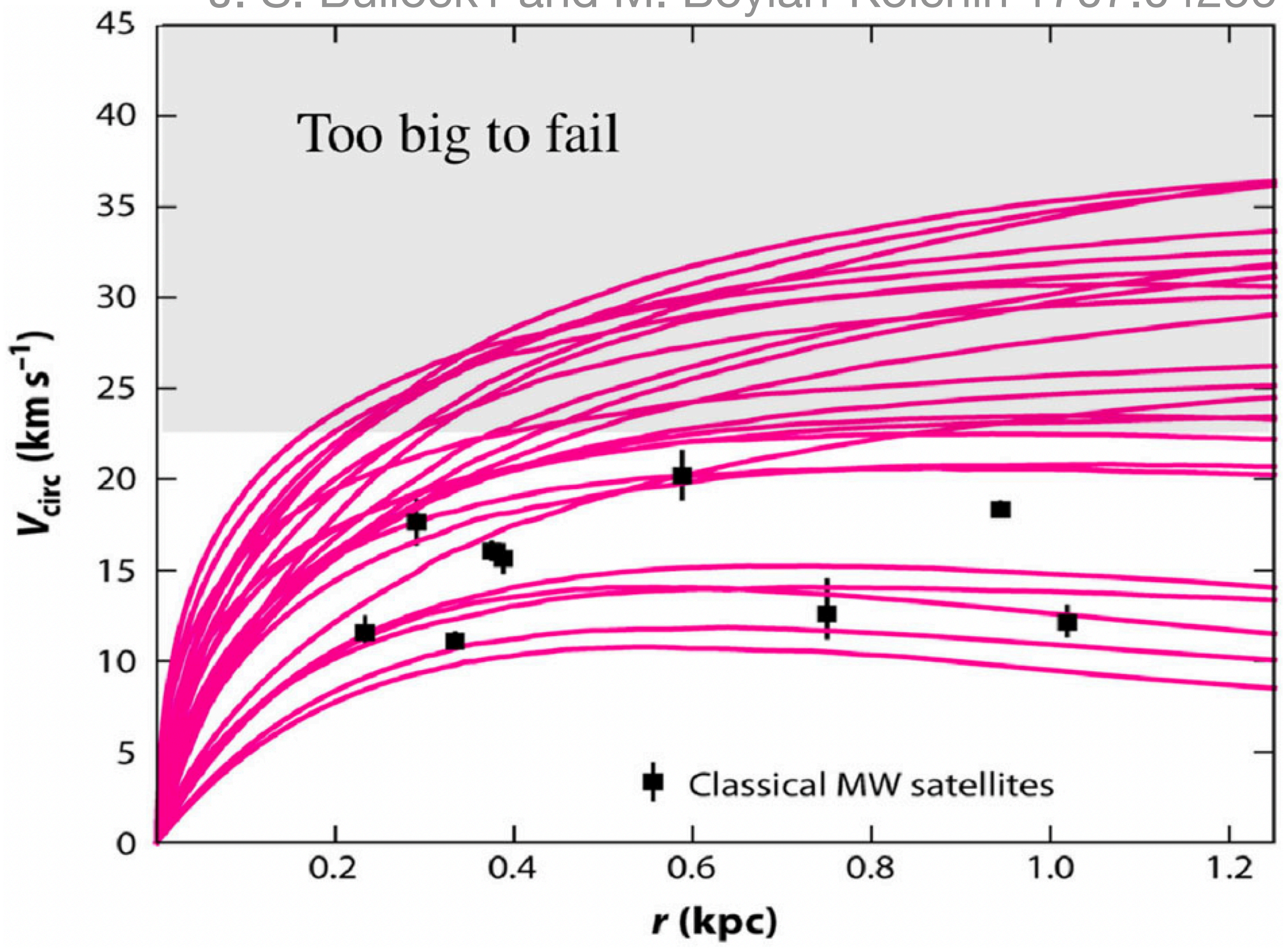
CDM crisis

- ΛCDM: cosmological standard model
- ΛCDM meets challenges in recent observations. e.g. small-scale curiosities.
  - Cusp-core problem (NFW v.s. core)
  - Missing satellites (or "Too big to fail" problem)

S.-H. Oh et al. 1011.2777

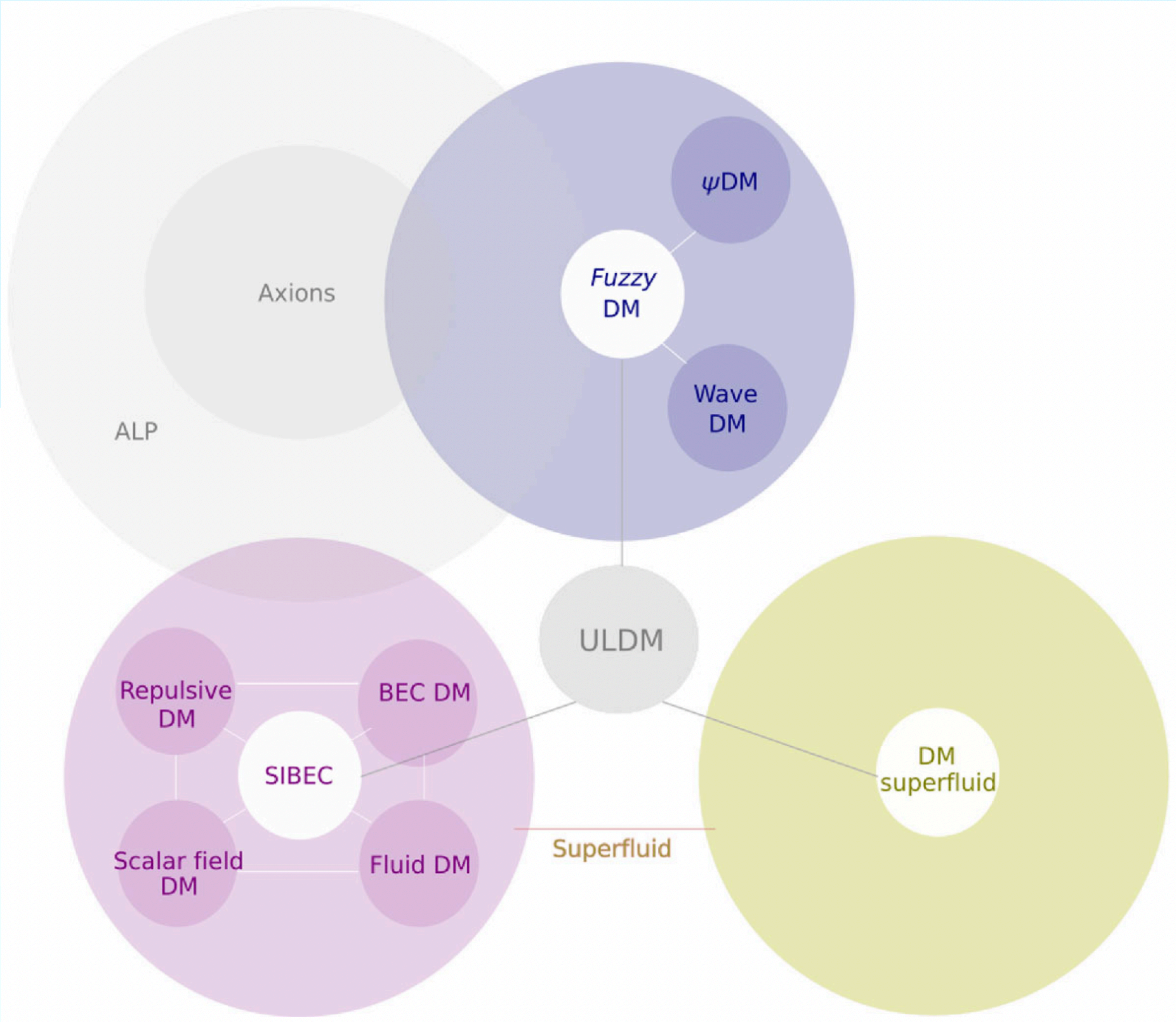


J. S. Bullock1 and M. Boylan-Kolchin 1707.04256



Introduction and Motivation

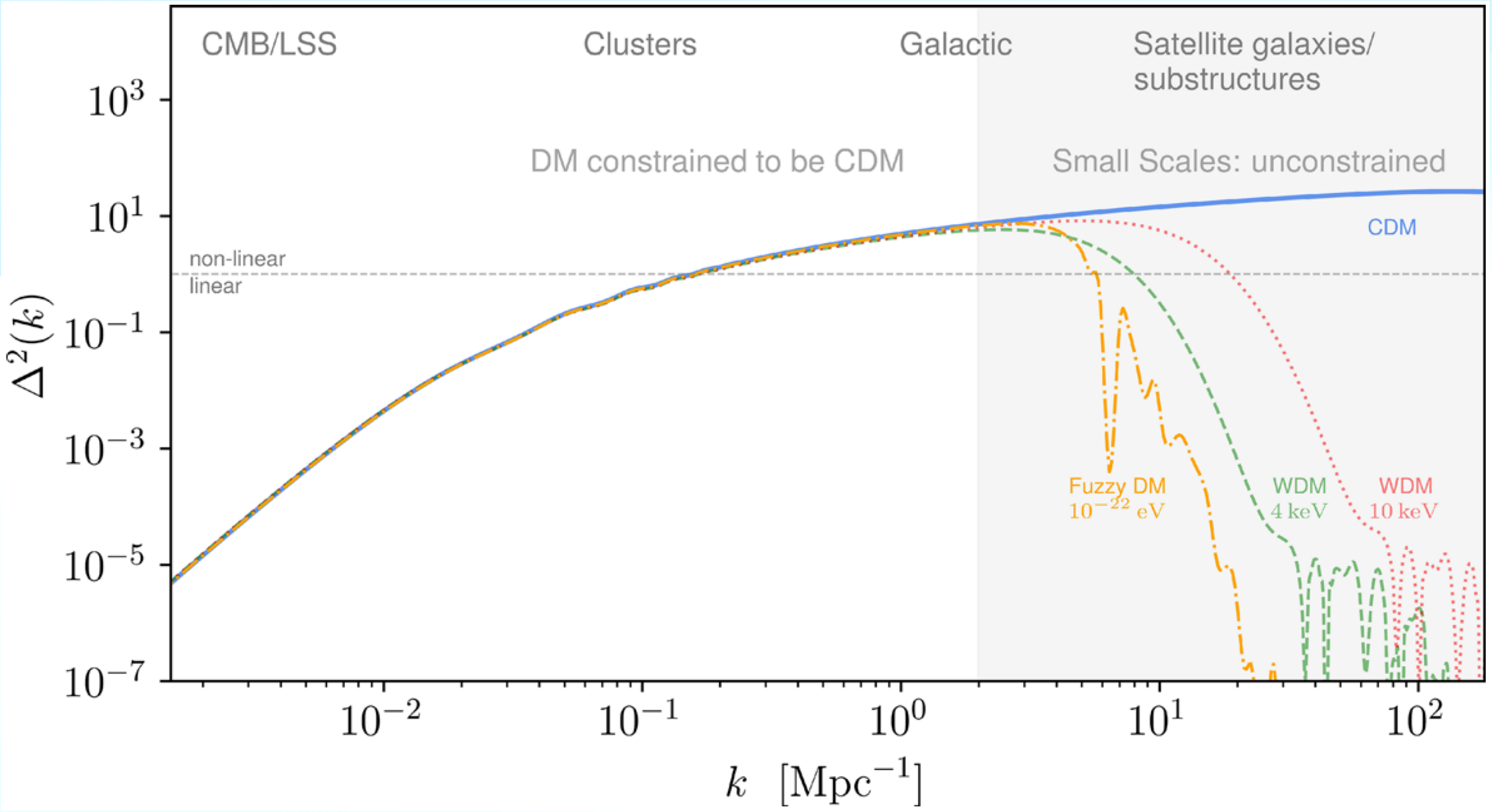
ULDM



- Ultra-light DM: small mass, wave nature, condensate structure
- Axion models (and ALPs) can produce FDM.
- Fuzzy DM (FDM): quantum pressure v.s. gravitational attraction.
- Differ from CDM at small scales.

Introduction and Motivation

Small-scale suppression



- CDM: a perfect fluid with $w \approx 0$ and sound speed $c_s \approx 0$.

- FDM: $\langle w \rangle \approx 0$ and the effective sound speed $\langle c_s^2 \rangle_{\text{eff}} \simeq \dfrac{\frac{k^2}{4a^2m^2}}{\frac{k^2}{4a^2m^2}+1}$.

- Small-scale suppression

Introduction and Motivation

Addressing small-scale challenges

- Small-scale suppression (wave nature) $\Rightarrow$
  - Cusp-core:

    BE condensate (soliton solution). DM halo density profile is changed to

    $$\rho_{\text{halo}} \simeq \begin{cases} \rho_c & r < r_c \\ \rho_{\text{NFW}} & r > r_c \end{cases},$$

    with a central core instead of a cusp.
  - Missing satellite:

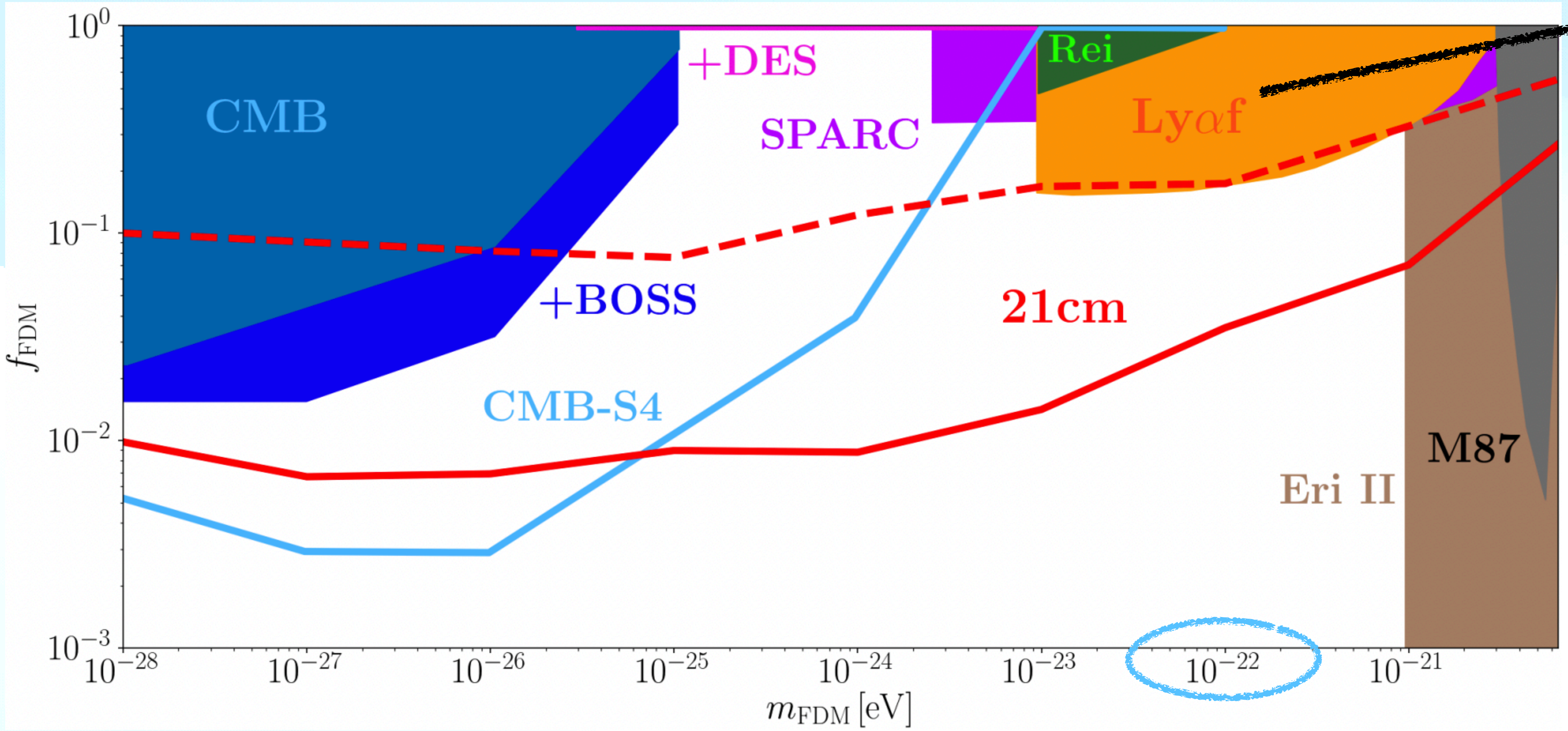
    A suppression of FDM halo formation, giving a bound

    $$M_{\text{lin}} = 4 \times 10^{10} M_\odot \left(\frac{m}{10^{-22}\text{eV}}\right)^3 \left(\frac{\Omega_m h^2}{0.14}\right)$$

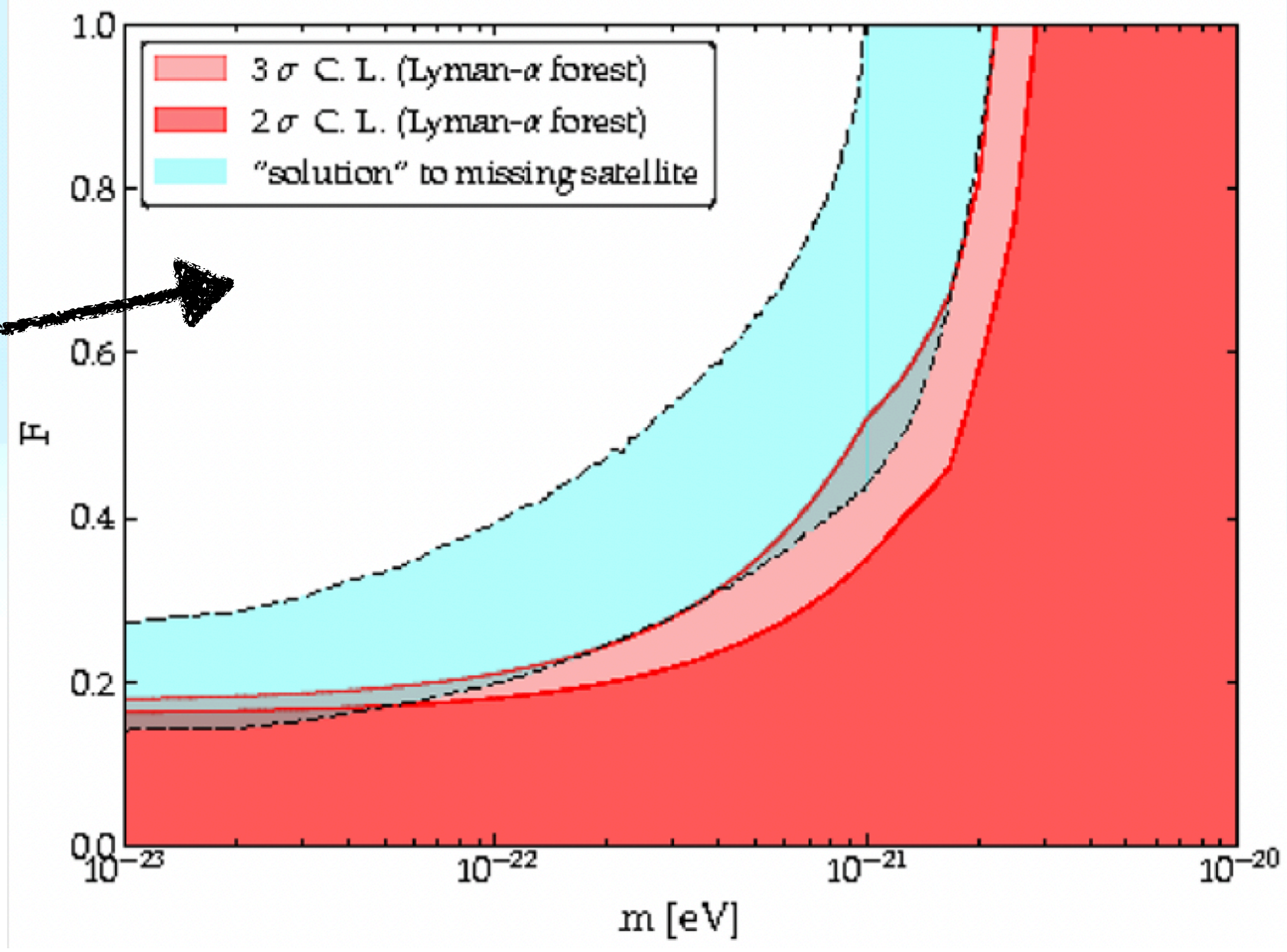
    a large suppression of small halos with $M < M_{\text{lin}}$.
- FDM with $m \sim 10^{-22}$ eV

Introduction and Motivation

Closing window on FDM



J. Flitter, E. D. Kovetz 2207.05083



T. Kobayashi et al. 1708.00015

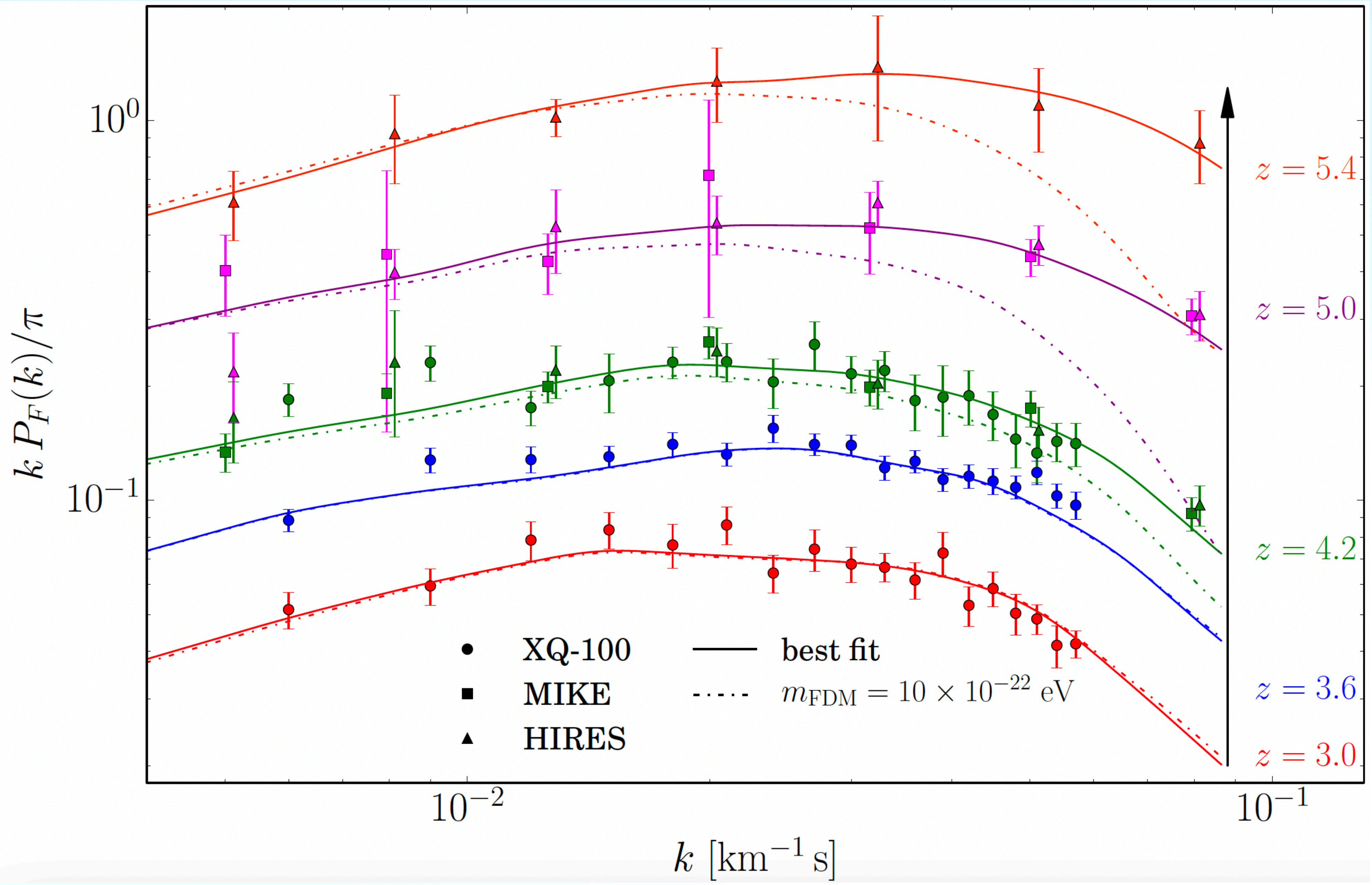
- Mass “tension” of FDM between missing satellite solution and Lyα

Introduction and Motivation

Closing window on FDM

- CDM preferred than FDM $(10^{-22}$ eV)?

V. Iršič et al. 1703.04683



  - Flux power spectrum from Lyα measurement $(z \approx 3 \sim 5.4)$

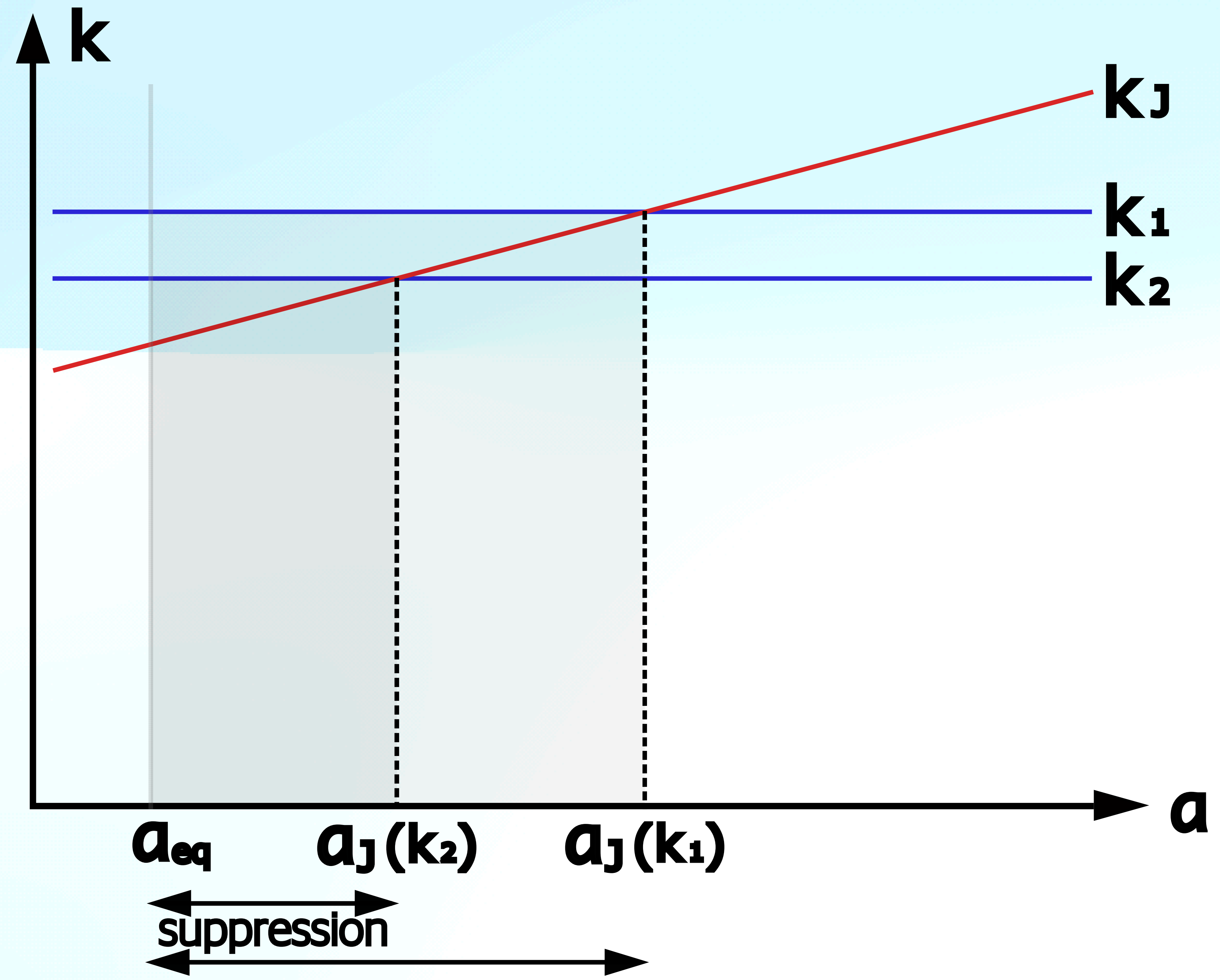
- FDM $(10^{-22}$ eV) preferred than CDM?
  - Missing satellite problem: THINGS (The HI Nearby Galaxy Survey).
  - Cusp-core problem: MW and the Local group.
  - $z \approx 0$.

Introduction and Motivation

Redshift dependence?

Introduction and Motivation

A Delayed suppression?



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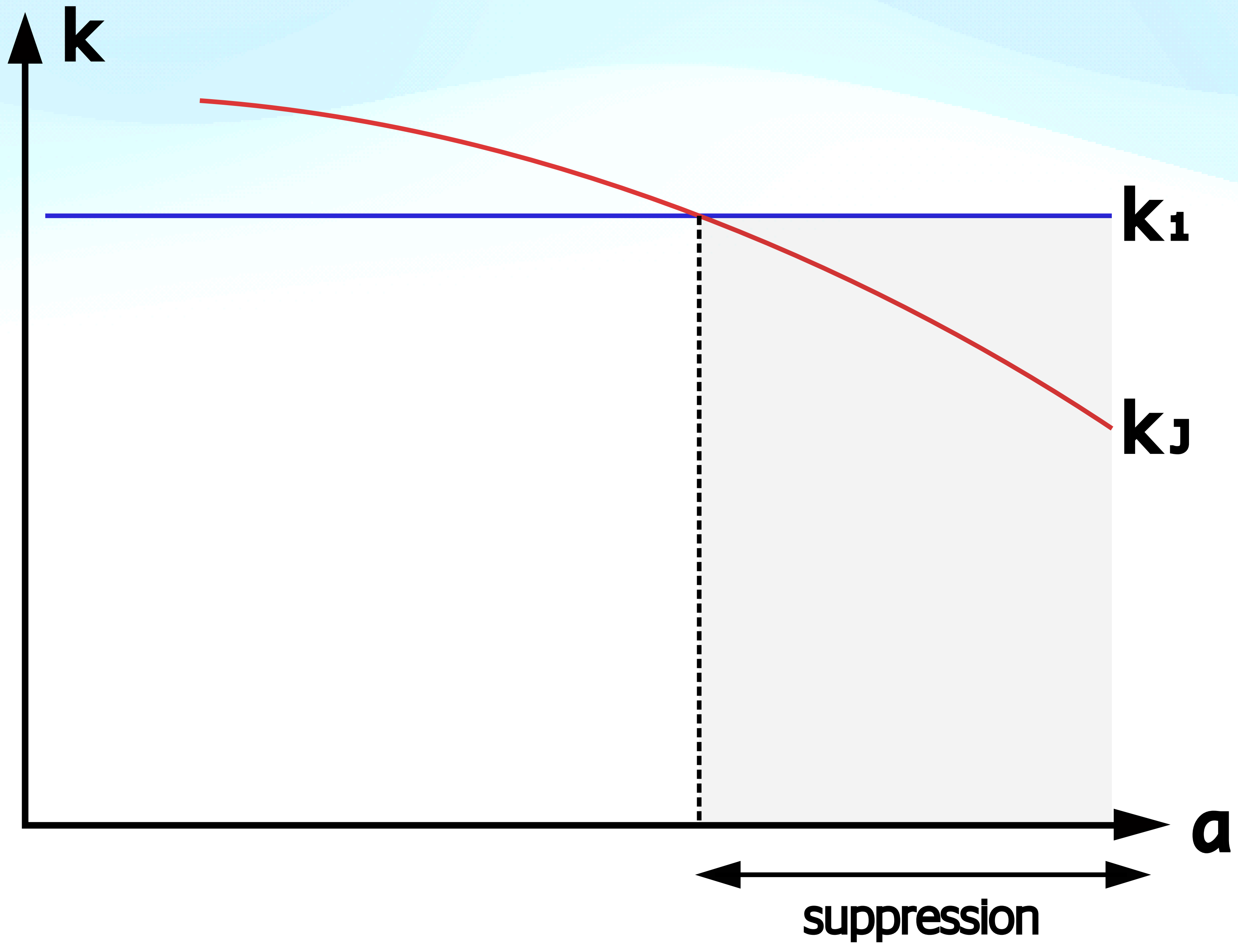


- Suppression period for FDM.
- An example to delay the suppression.

Ultra-Light Dark Matter

ULDM

Axions serving as DM

- The action for FDM

$$S = S_{EH} + S_\phi = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} m^2 \phi^2 \right] \ ,$$

- Axion during Inflation:

  PQ symmetry broken during inflation (if $f_a \gtrsim H_I/2\pi$), giving an initial misalignment angle for our patch, so

$$\langle \phi_i^2 \rangle = f_a^2 \theta_i^2 + \langle \delta\phi^2 \rangle \ ,$$

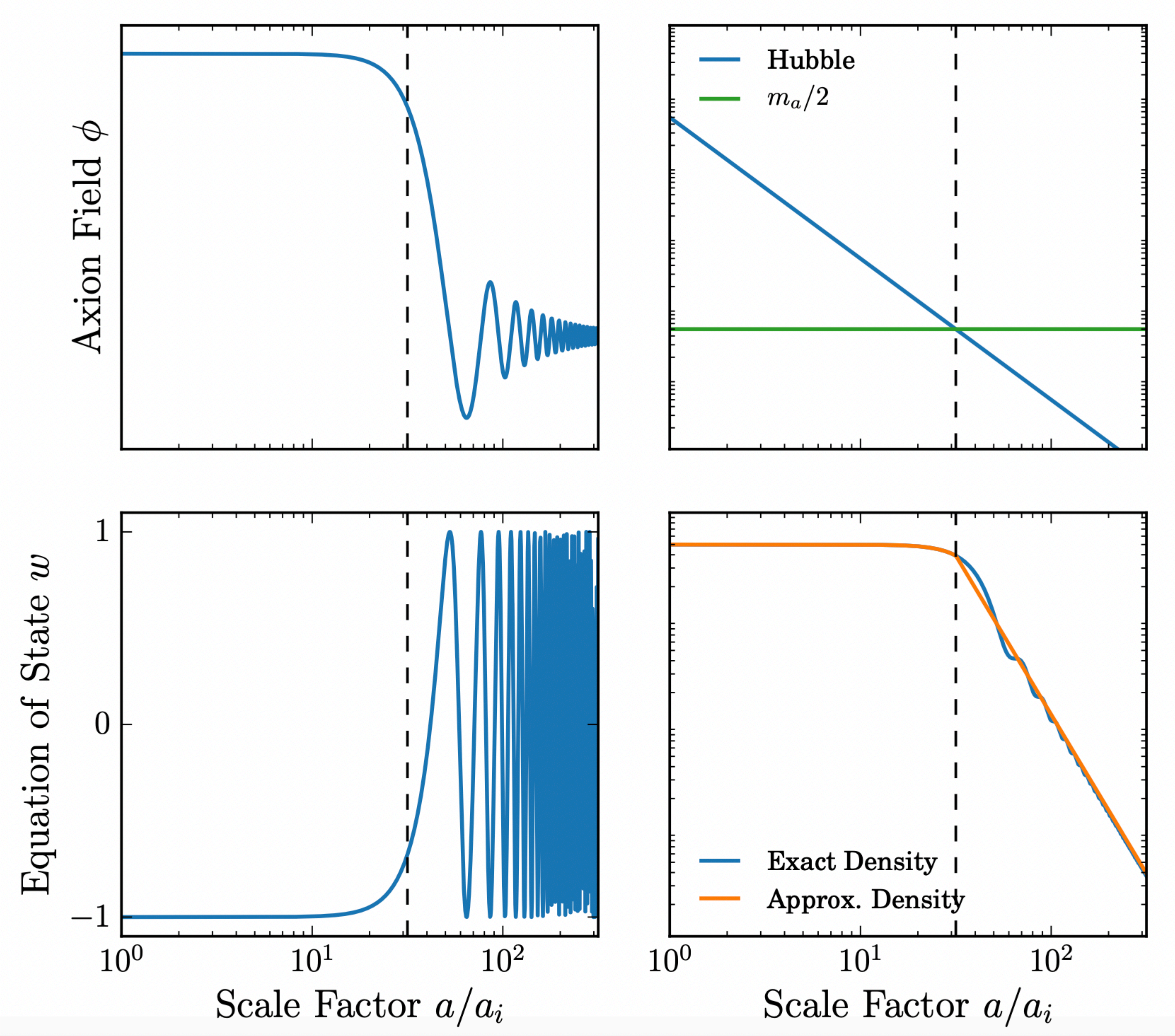
- Background evolution after inflation

$$\ddot{\phi}_0 + (3H + \Gamma)\dot{\phi}_0 + m_a^2 \phi_0 = 0 \ .$$

ULDM

Axions serving as DM

David J. E. Marsh 1510.07633



- When $H \gg m_a$,
  - ϕ_0 background rolls down very slowly $\dot{\phi}_0 \simeq -\frac{m_a^2}{3H}\phi_0$.
  - $w_a \simeq -1$, like DE.
- When $H \approx m_a$,
  - axion background begins oscillating.
- Some time after oscillation begins, $H \ll m_a$,
  - $\rho_a \sim a^{-3}$, as DM (fuzzy).

ULDM

Structure growth

- From 1st PT of $\nabla_\mu T^{\mu\nu} = 0$, e.o.m for fluids

$$\begin{cases} \dot{\delta} + 3H(c_{s,g}^2 - w)\delta = -(1+w)(\theta + \dot{h}/2) \ , \\ \dot{\theta} + \left[\frac{\dot{w}}{1+w} + (2-3w)H\right]\theta = \frac{k^2}{a^2}\left(\frac{c_{s,g}^2}{1+w}\delta + \Phi\right) \ , \end{cases}$$

$\delta \equiv \delta\rho/\rho, \theta \equiv -\frac{k^2}{a^2}\frac{\delta q}{p+\rho}, w \equiv p/\rho$, gauge-dependent $c_{s,g}^2 \equiv \frac{\delta p}{\delta \rho}$, and $\dot{h}/2 = -3\dot{\Psi} + k^2(B/a - \dot{E})$.

- For axions δ_a
  - $\delta\phi$ originated from inflation $\Rightarrow$ all to isocurvature perturbations;
  - The adiabatic mode $\delta_a = 0$ initially; can grow (from δ_r during RD) only when $w_a \neq -1$.

ULDM

Structure growth

- For FDM (axions after $t_{\rm osc}$), antasz by WKB

$$\phi(t) = a(t)^{-3/2}(\phi_0 \cos(mt + \varphi)) ,$$

- $H \ll m_a$, so we can average over the oscillations $\langle \rho_a \rangle \sim a^{-3}$, and

$$c_{s,\rm eff}^2 = \left\langle \frac{\delta p}{\delta \rho} \right\rangle = \frac{\frac{k^2}{4m^2a^2}}{1 + \frac{k^2}{4m^2a^2}} \simeq \begin{cases} \frac{k^2}{4m^2a^2} & \text{when } k < 2am \\ 1 & \text{when } k > 2am \end{cases} ,$$

in axion comoving gauge that $\langle \delta q \rangle = 0$.

ULDM

Structure growth

- To linear order, e.o.m of axion overdensities during MD (after $t_{\rm osc}$)

$$\ddot{\delta}_a + 2H\dot{\delta}_a + \left(\frac{k^2}{a^2}c_{s,g}^2 - 4\pi G\rho_a\right)\delta_a = 0\ ,$$

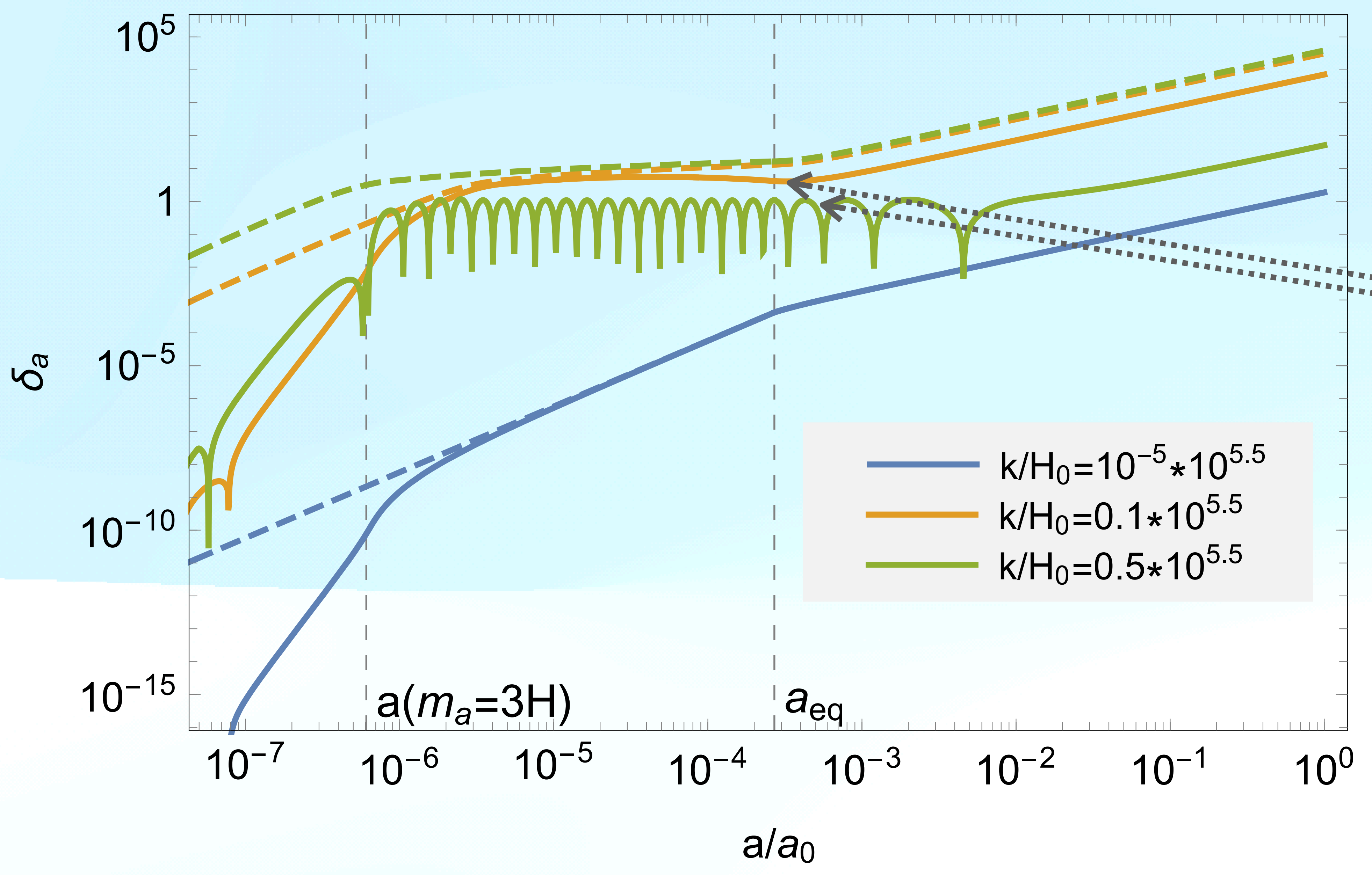
$$\frac{k^2}{a^2}c_{s,g}^2 \simeq \frac{k^2}{a^2}\frac{k^2}{4a^2m^2} \quad \text{v.s.} \quad 4\pi G\rho_a \propto a^{-3}\ ,$$

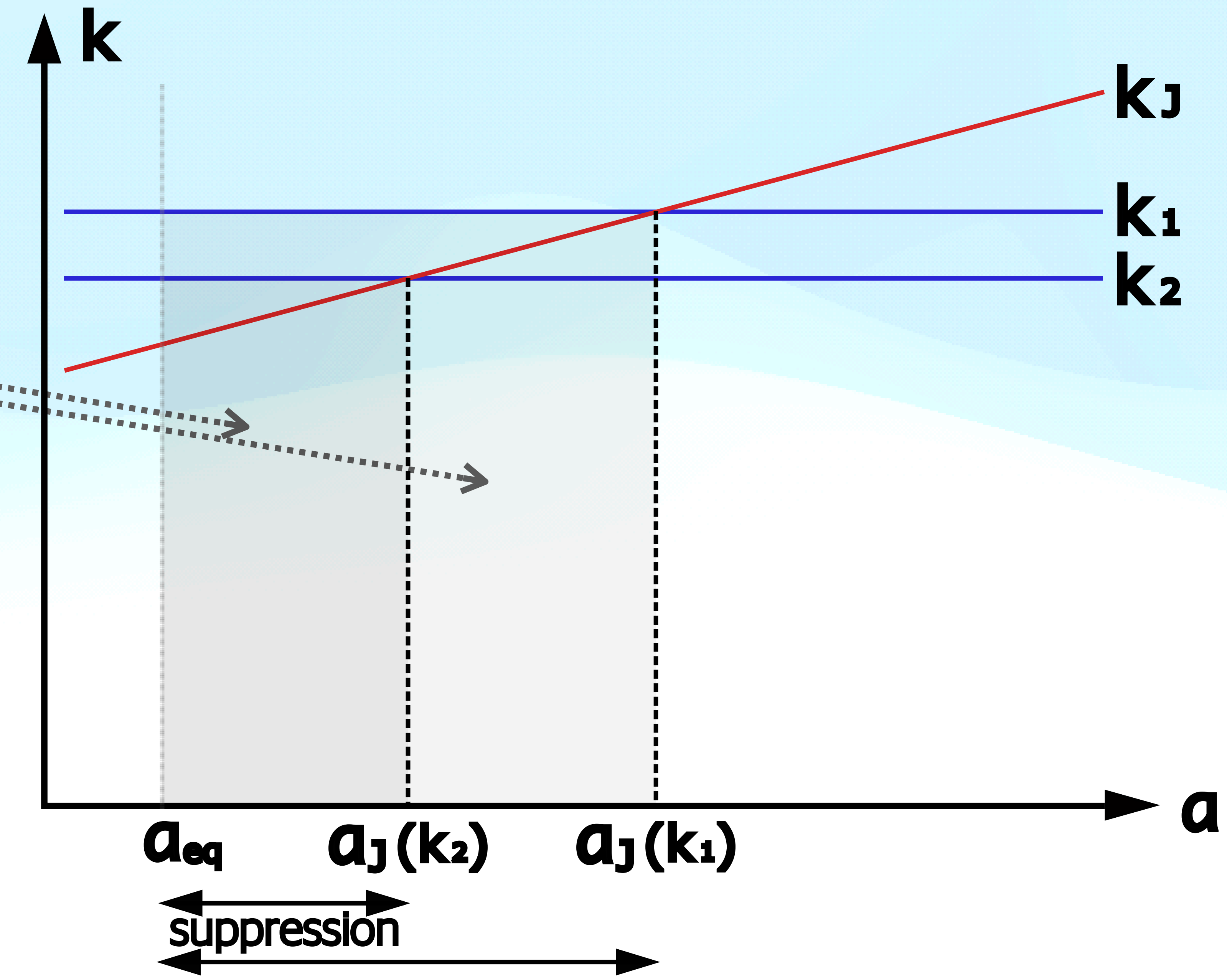
defining a Jeans scale $k_J = 66.5\left(\frac{\Omega_a h^2}{0.12}\right)^{1/4} a^{1/4}\left(\frac{m_a}{10^{-22}\text{eV}}\right)^{1/2}\text{Mpc}^{-1}$.

  - For $k < k_J$, gravitational term $4\pi G\rho_a$ dominates, same as CDM. $\rightarrow \delta_a \sim a$ (growing mode);
  - For $k > k_J$, sound speed term $k^2c_{s,g}^2/a^2$ dominates, showing the wave nature. $\rightarrow |\delta_a| \sim a^0$.

ULDM

Structure growth





- Overdensity evolution of FDM with $m_a = 10^{-22}\mathrm{eV}$ ($\Omega_a/\Omega_d = 1$) compared to standard CDM.

- Structure is suppressed for $k > k_J(a) = 66.5a^{1/4}\mathrm{Mpc}^{-1}$. (or $k_J(a = a_0) \simeq \sqrt{m_a H_0} = 10^{5.5} H_0$.)

- Scale comparison for k-modes. ($k_J \propto a^{1/4}$ during MD.)

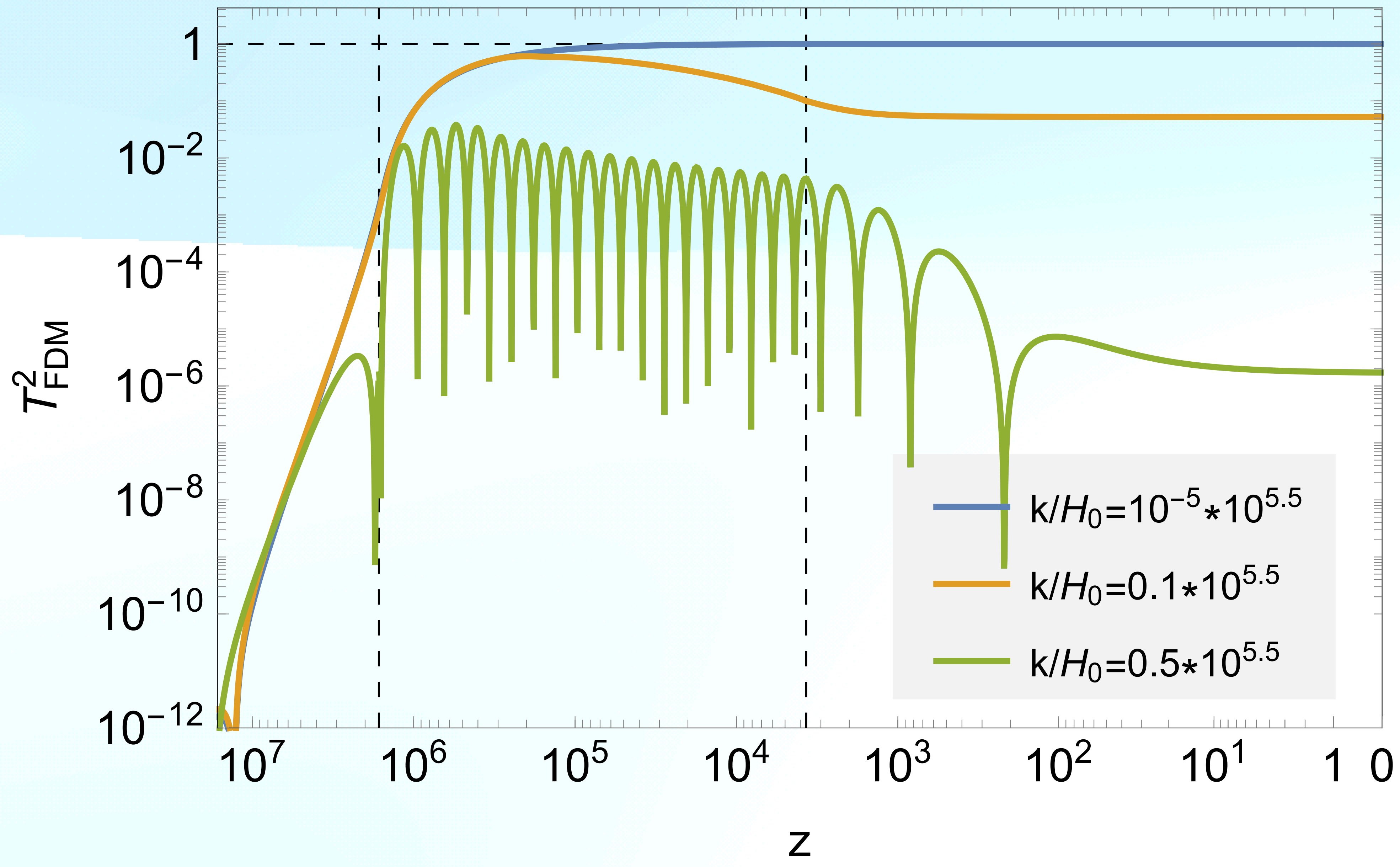
- Suppression is integrated when $k > k_J(a)$ (earlier time).

ULDM

Structure growth

- Transfer function defined from the suppression of linear matter power spectrum

$$P_{\rm FDM}(k,z) = T^2_{\rm FDM}(k,z) P_{\Lambda{\rm CDM}}(k,z)$$



$T^2_{\rm FDM}(k,z)$ for FDM with $m_a = 10^{-22}$ eV.

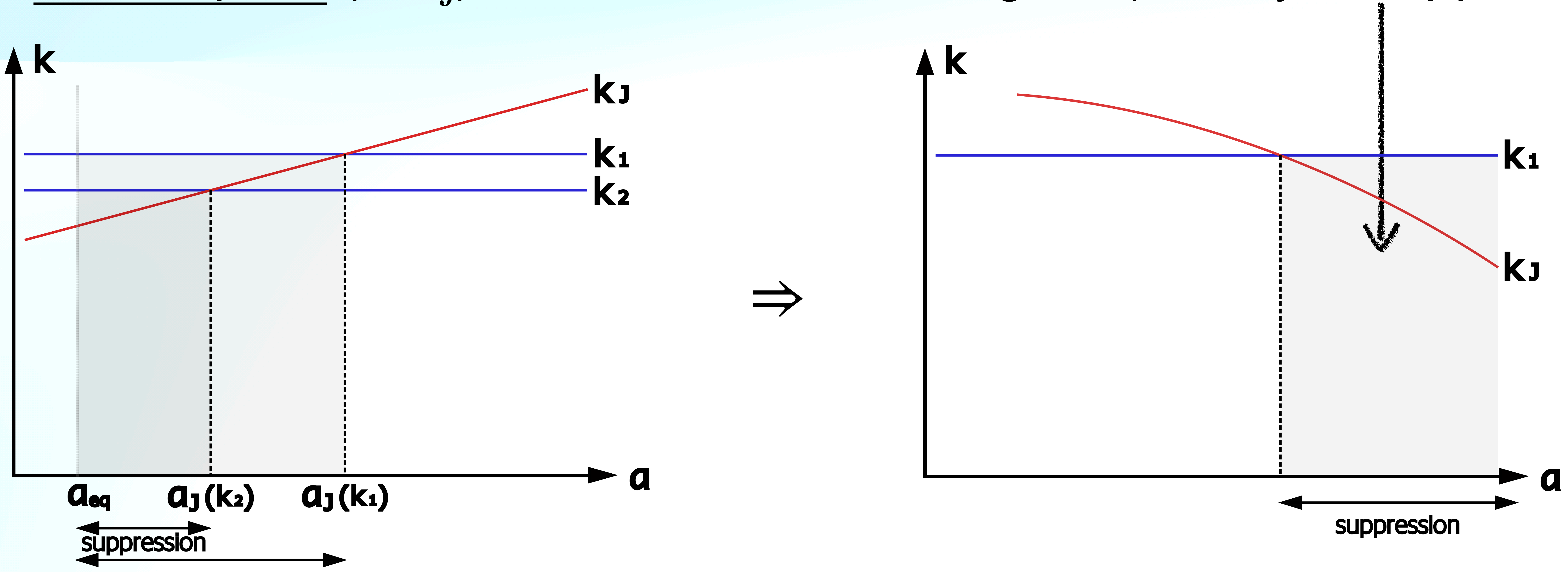
- Late time z-dependence is often ignored in FDM study.
- By numerical method, there is an approximation of $T^2_{\rm FDM} = \cos(x_J^3(k))/(1+x_J^8(k))$, ($x_J = 1.61k/(9{\rm Mpc}^{-1})$).
- The half-mode is $k_{1/2} \approx 5.1\left(\frac{m_a}{10^{-22}{\rm eV}}\right)^{4/9}$ Mpc^{-1}.

ULDM

Structure growth

- The linear suppression at small scale is accumulated when $k > k_J$ (or when the "quantum pressure" term dominates: $\underline{k^2 c_{s,g}^2}/a^2 \gg 4\pi G\rho_a$).

- The observation prefers CDM at high z but FDM at low z, expecting new ULDM model: modified sound speed (or k_J) → an earlier structure grow (a delayed suppression)



ULDM with Non-Canonical Kinetics

ULDM with Non-Canonical Kinetics

Modified c_s from Theory

- For canonical scalar field, c_s^2 defined from Mukhanov-Sasaki variables $\nu = z\zeta$ should be 1 (gauge-invariant);
- $c_{s,g}^2 \equiv \delta p/\delta \rho$ is gauge-dependent (discussed in comoving gauge or synchronous gauge).

- Example of non-canonical scalar: k-essence

$$\mathscr{L} = V(\phi)F(X) \; , \; X = (\partial\phi)^2/2 \; ,$$

has equation of state $w = F/(2XF_X - F)$, and non-trivial sound speed

$$c_s^2 = \frac{\partial_X p}{\partial_X \rho} = \frac{F_X}{F_X + 2XF_{XX}} \; .$$

ULDM with Non-Canonical Kinetics

Modified c_s from DBI

- As an example to change k_J, we use DBI theory to construct DM with modified c_s (by $f(\phi)$)

$$S = \int d^4x\sqrt{-\mathrm{g}}\left[f(\phi)^{-1}(1-\sqrt{1-2f(\phi)X})-\frac{1}{2}m^2\phi^2\right] ,$$

where $X = -\,g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi/2$. So

$$\rho = \frac{1}{c_s}\frac{1}{c_s+1}\dot{\phi}^2+\frac{1}{2}m^2\phi^2 ,$$

$$p = \frac{1}{c_s+1}\dot{\phi}^2-\frac{1}{2}m^2\phi^2 ,$$

with sound speed

$$c_s^2 = \frac{\partial_X p}{\partial_X \rho} = 1 - f(\phi)\dot{\phi}^2 .$$

ULDM with Non-Canonical Kinetics

DM-like DBI

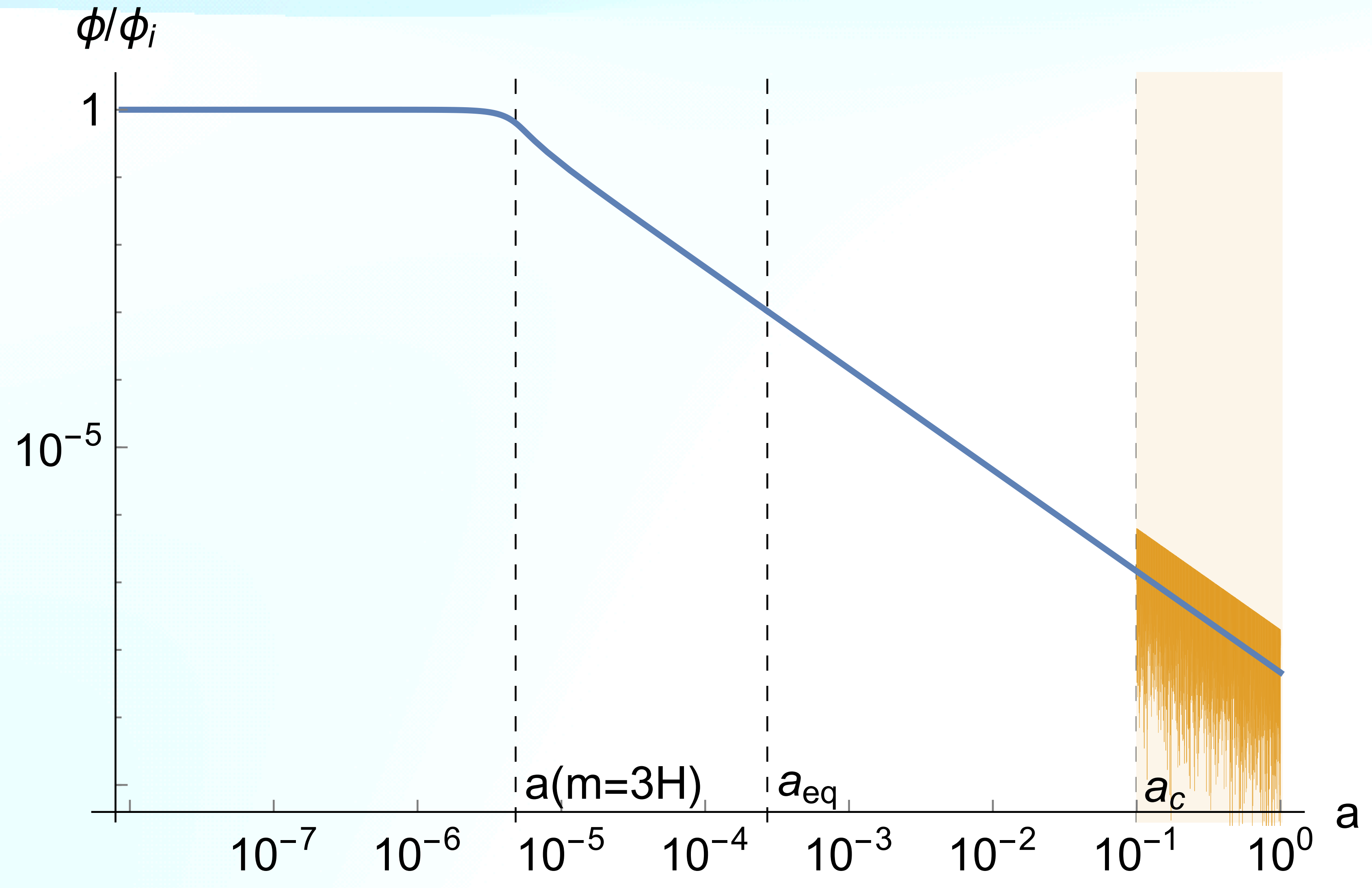
- Trivial case (canonical scalar): $f(\phi) = 0$, so $c_s = 1$.
  - Background evolution determined by H v.s. m, like in axion model.
  - After $t_{\rm osc}$, $c_{s,g}^2 = \frac{k^2}{4m^2a^2} \Big/ \left(1 + \frac{k^2}{4m^2a^2}\right)$, the structure formation is like FDM.
- Small c_s case ("relativistic limit"): $c_s^{-1} = (1 - f\dot{\phi}^2)^{-1/2} \gg 1$.
  - $w = p/\rho \simeq c_s \to 0$, and the kinetic term $\frac{1}{c_s}\frac{1}{c_s+1}\dot{\phi}^2$ dominates ρ.
  - $c_{s,g}^2 = c_s^2$, so the structure formation can be similar to CDM.

ULDM with Non-Canonical Kinetics

Our k-ULDM: example 1 (“Phase transition” case)

$$f(\phi) = \begin{cases} \frac{1}{(m/H_0)^2}\phi^{-2}(\phi/\phi_i)^{-8/3} & \text{for } \mathrm{t_i < t < t_{eq}}\ , \\ \left(\frac{4m}{3H_0}\right)^{-3/2}(t_{eq}/t_0)^{1/2}\phi^{-2}(\phi/\phi_i)^{-2} & \text{for } \mathrm{t_{eq} < t < t_c}\ , \\ 0 & \text{for } \mathrm{t > t_c}\ , \end{cases}$$

$\phi_i = \phi(t_i)$; t_0 is the time today.



- Solving e.o.m of DBI scalar

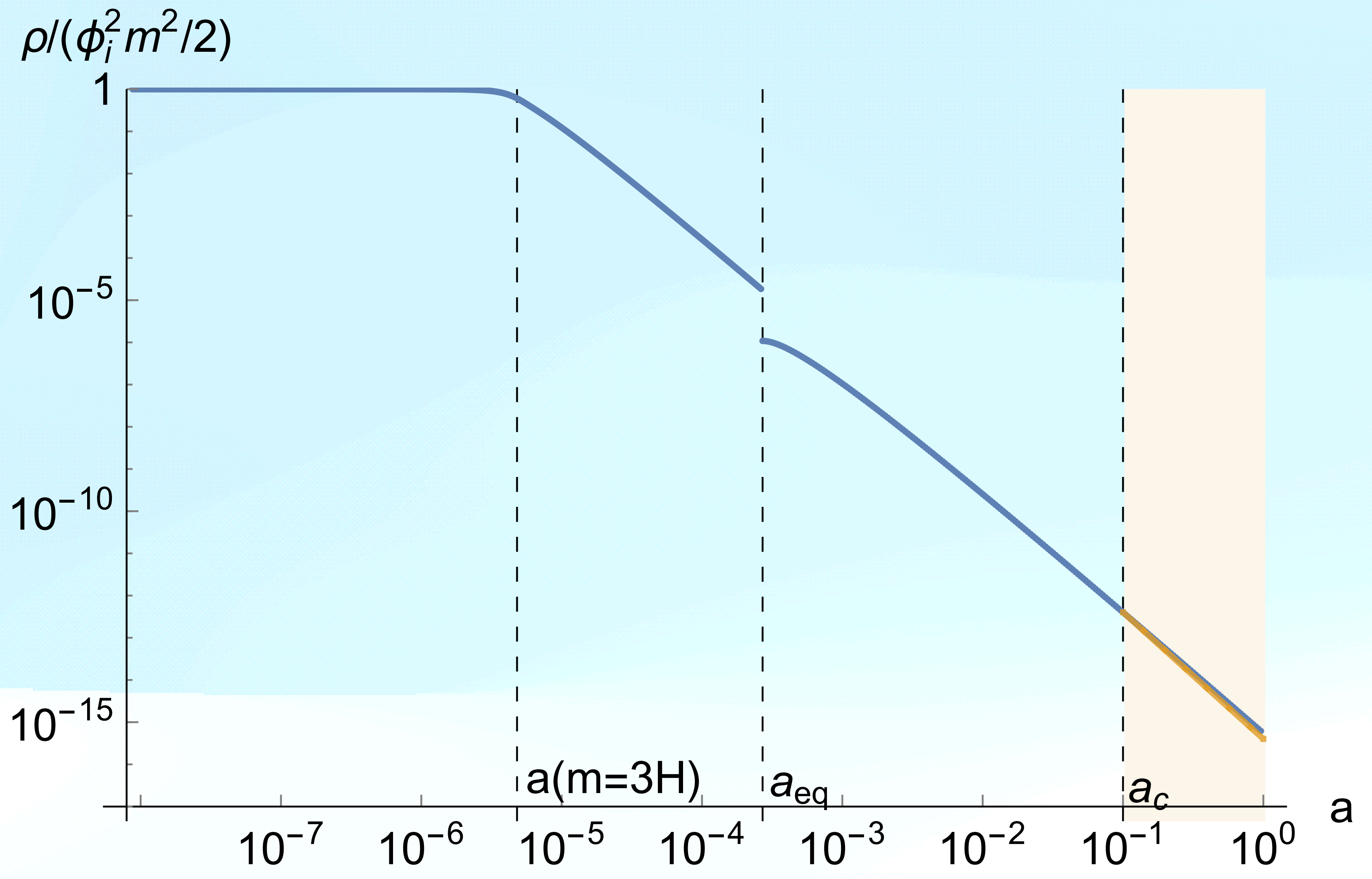
$$\ddot{\phi} + 3Hc_s^2\dot{\phi} + c_s^3 V'(\phi) + \frac{f'(\phi)}{2f(\phi)}\left(1 - \frac{2c_s^2}{1+c_s}\right)\dot{\phi}^2 = 0\ ,$$

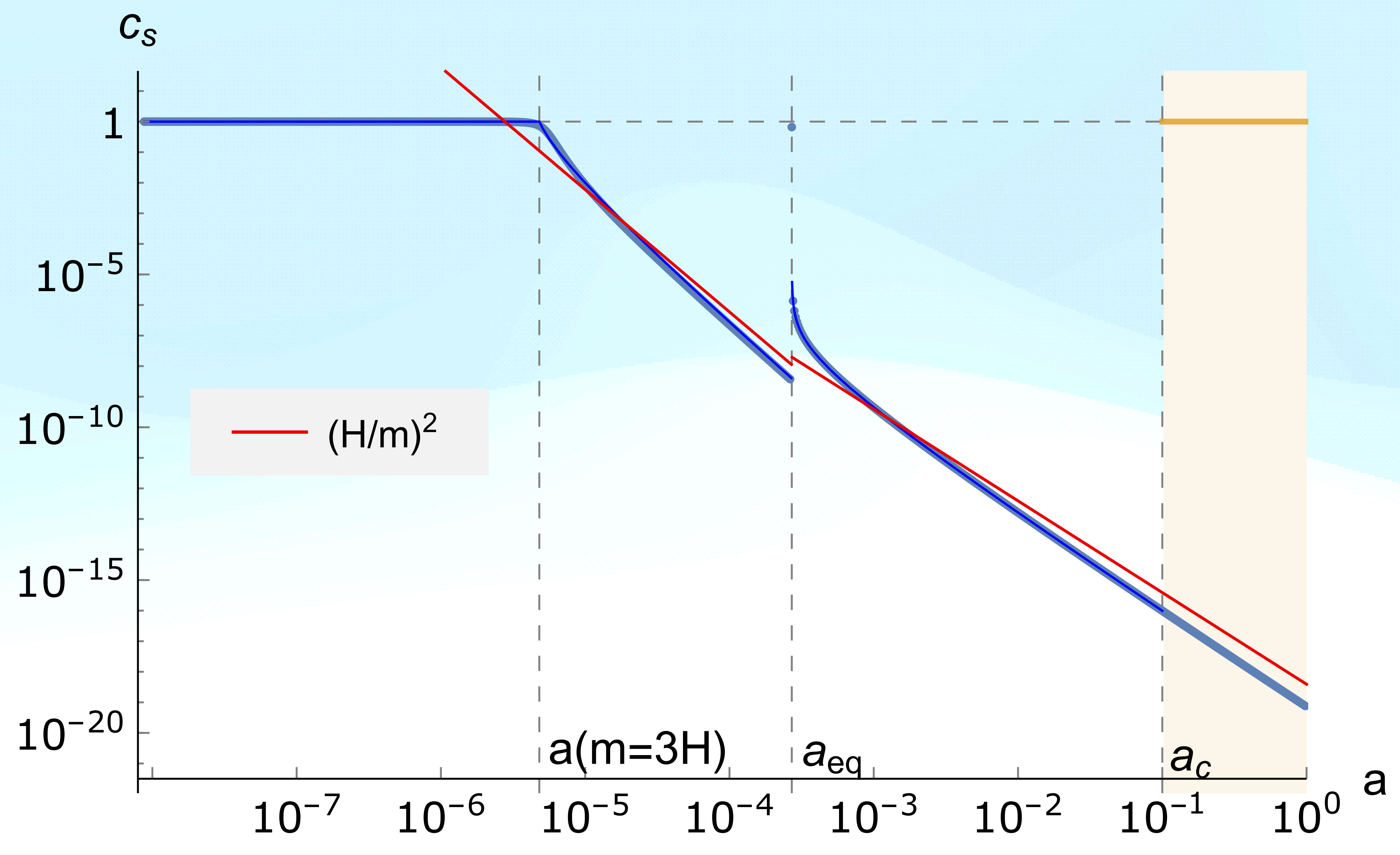
  - Before t_c, we have $c_s \ll 1$. The oscillation is switched off by $f(\phi)$.
  - After t_c, as long as the small enough $f(\phi)$ keep $f(\phi)\dot{\phi}^2 \ll 1$ when the oscillation resumes,

    the CDM-like field $\phi \Rightarrow$ FDM-like afterwards.

ULDM with Non-Canonical Kinetics

Our k-ULDM: example 1 (“Phase transition” case)





- $\rho \propto a^{-3}$. CDM-like (blue lines) transit to FDM-like (yellow lines) at t_c.
- $c_s \ll 1 \Rightarrow c_s = 1$. Late time suppression by switching to FDM-like phase.
- (Discontinuity from the inconsistency by our treatment of $H(t)$ at $t_{\rm eq}$ (sudden transition from RD to MD).)

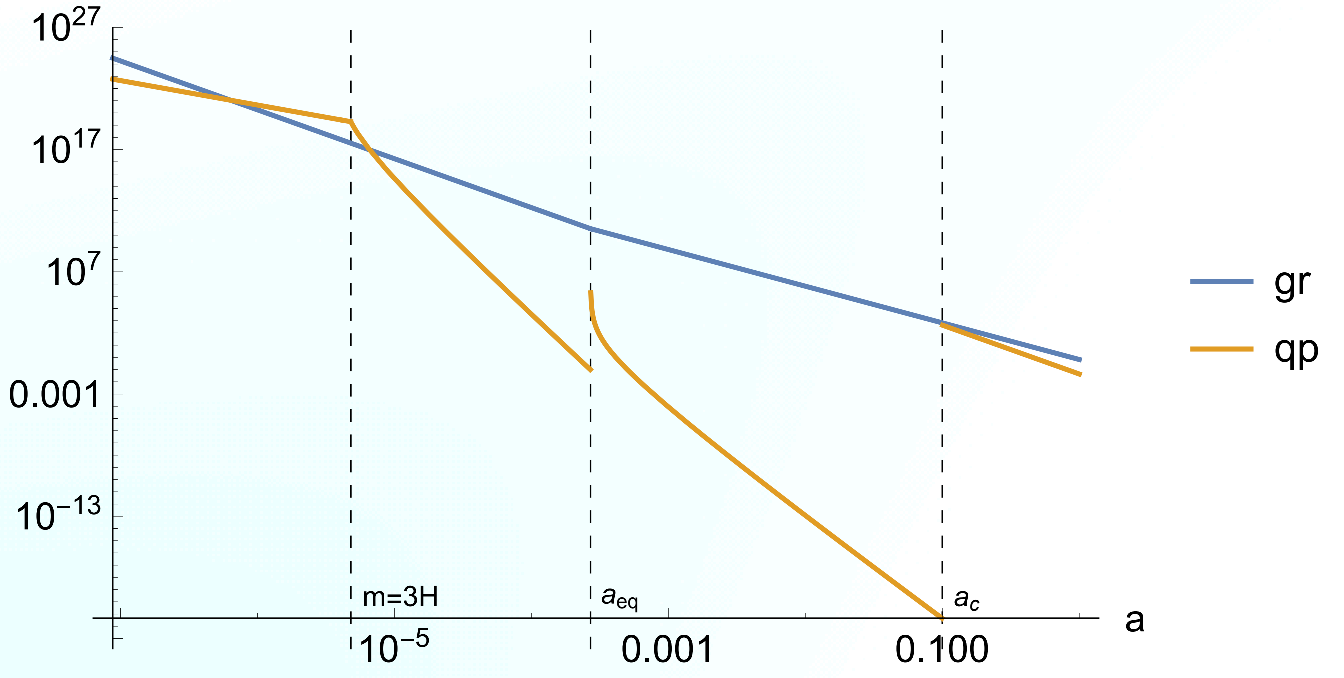
ULDM with Non-Canonical Kinetics

Our k-ULDM: example 1 (“Phase transition” case)

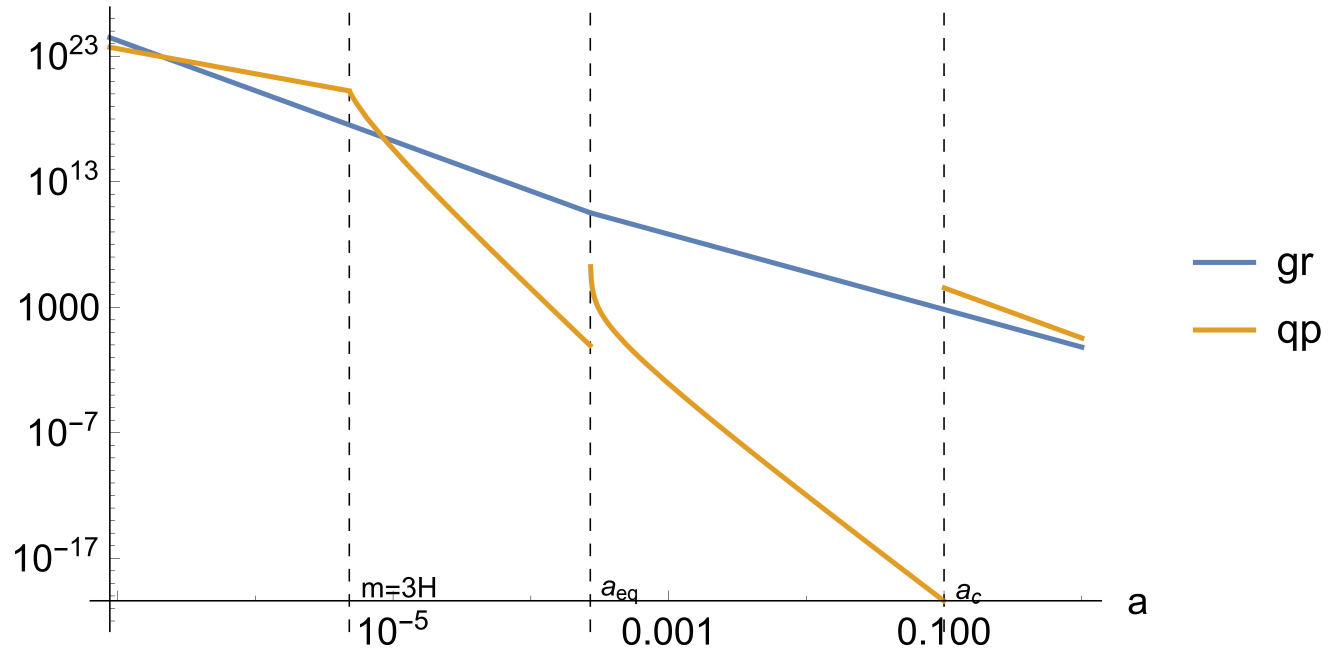
- The modified sound speed then writes

$$c_{s,g}^2 = \begin{cases} 1 & t < t_{\rm osc} \\ \sim (t/t_{\rm o})^{-4} & t_{\rm osc} < t < t_c \\ \dfrac{\frac{k^2}{4a^2m^2}}{1+\frac{k^2}{4a^2m^2}} & t > t_c \end{cases} .$$

- Structure formation is determined by the competition. For example



$k/H_0 = 1 \times 10^{4.5}$



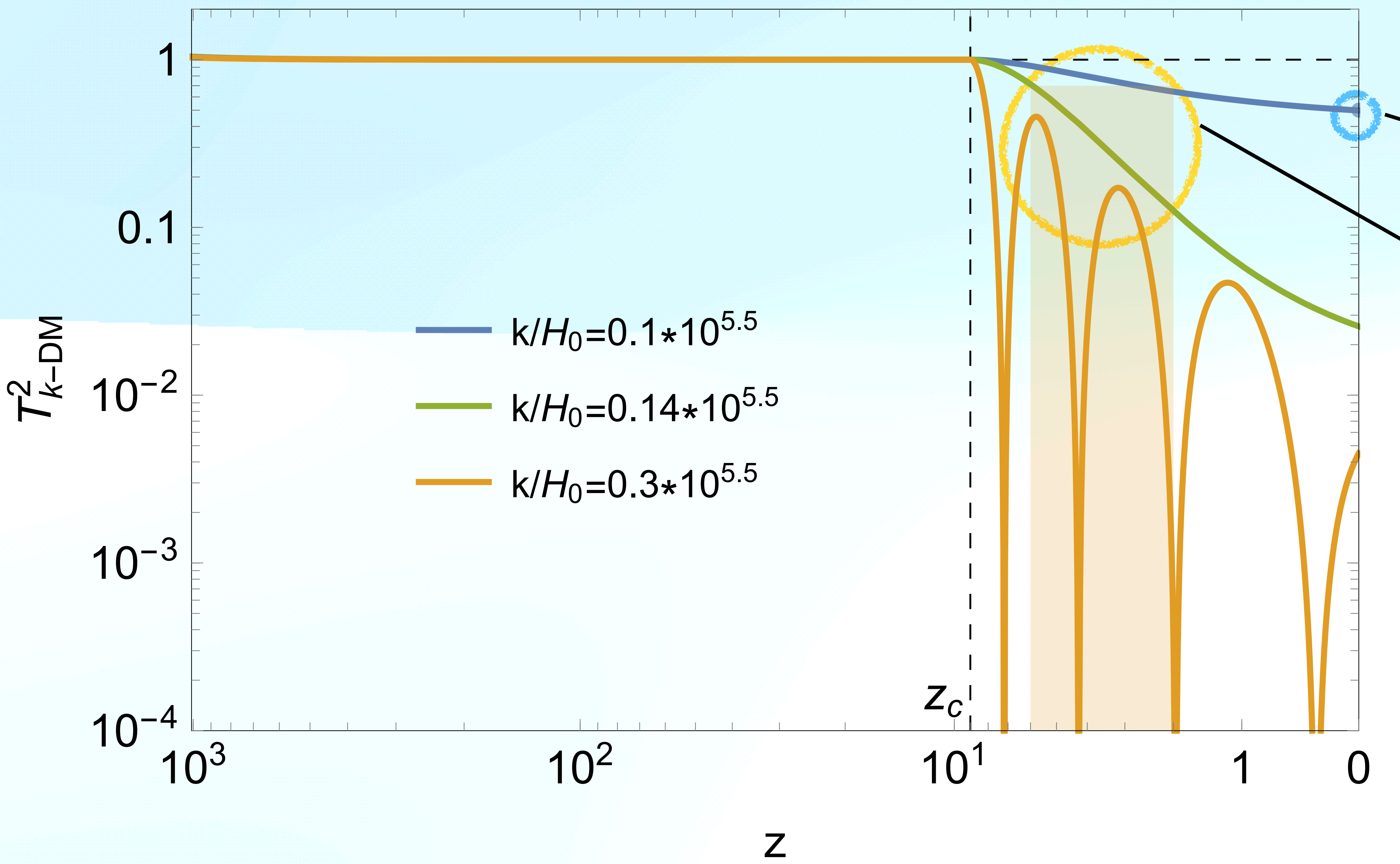
$k/H_0 = 3 \times 10^{4.5}$

- A comparison of gravitational term and sound speed term when taking $m = 10^{-24}$eV and $a(t_c) = 0.1$.

- $k = 0.1 \times 10^{5.5} H_0 \sim 6$ Mpc$^{-1} \sim k_{1/2}(10^{-22}$ eV FDM)

ULDM with Non-Canonical Kinetics

Our k-ULDM: example 1 ("Phase transition" case)



$m = 10^{-24}$eV and $a(t_c) = 0.1$.

- To interpret cusp-core (missing satellite) & Lyα at the same time:
  - $T^2(k = 4.5\ \mathrm{Mpc}^{-1}) \sim 0.5$ at $z = 0$;
  - $T^2(k = 20h$ Mpc$^{-1}) > 0.7$ at $z \approx 2 \sim 6$ (Lyα).
- The k-ULDM in "Phase transition" case can alleviate the Lyα problem,

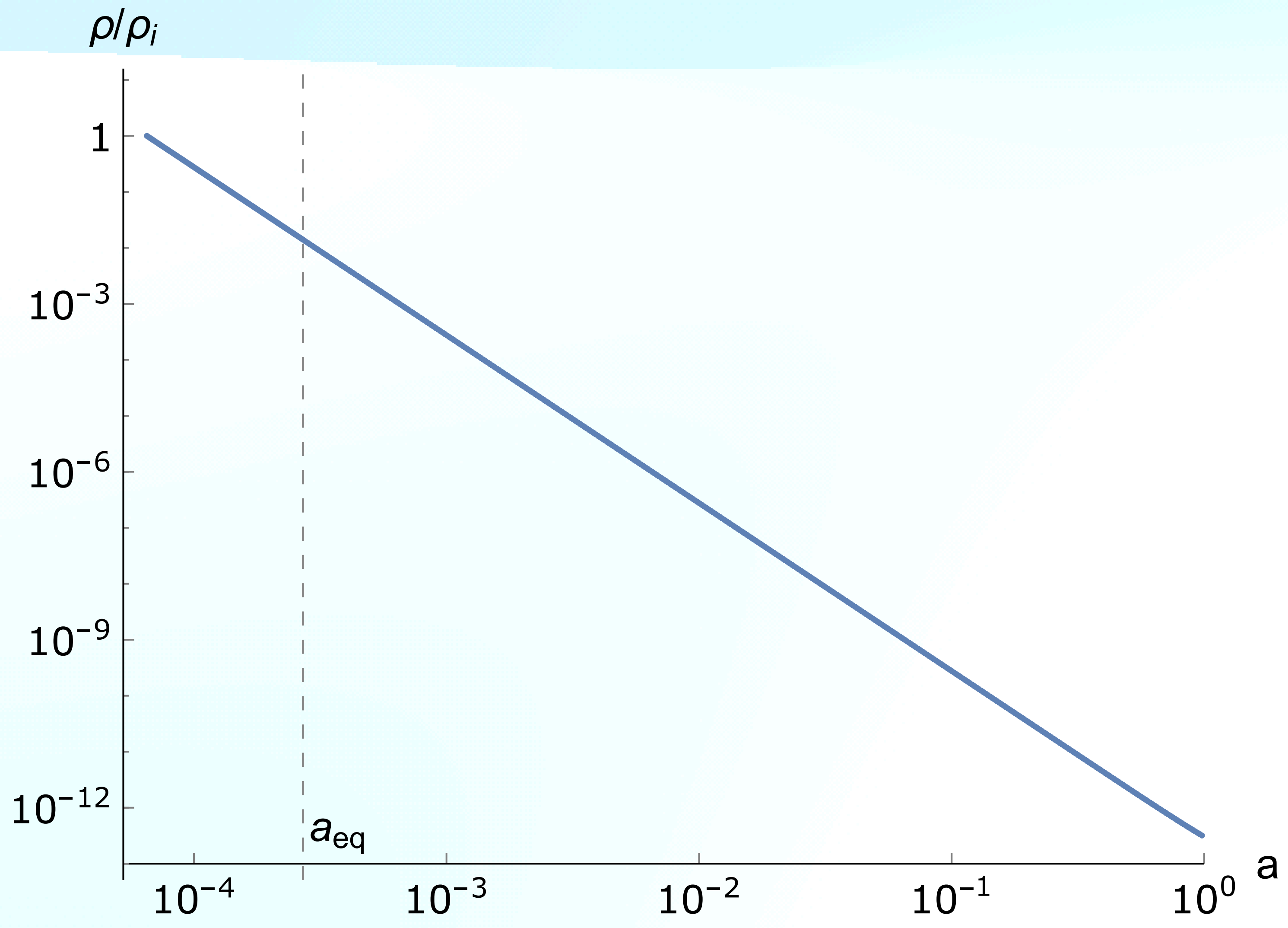
  compared to a z-indpendent T^2 at late time (in FDM).

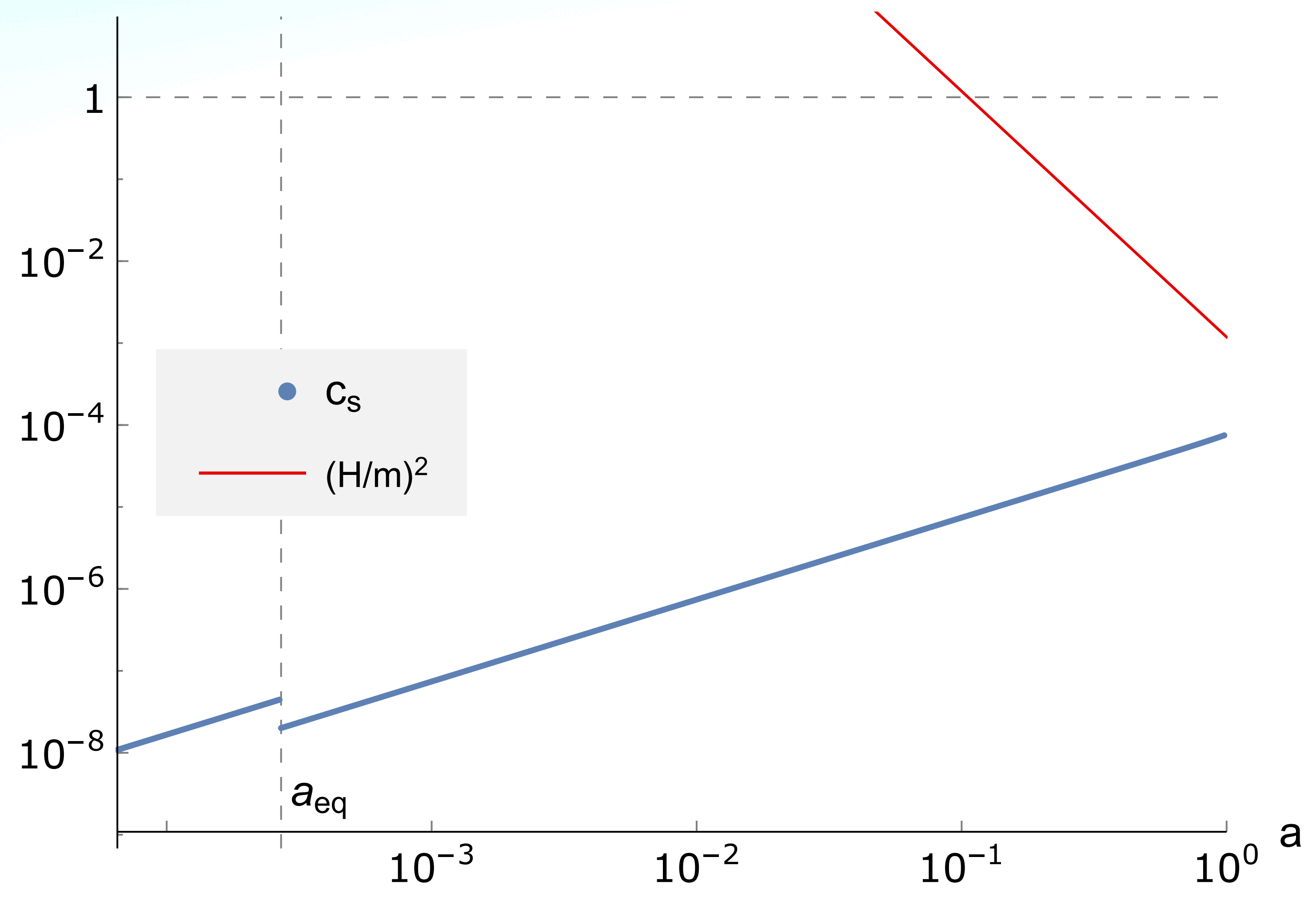
ULDM with Non-Canonical Kinetics

Our k-ULDM: example 2 ("Chaplygin-like" case)

$$f(\phi) = \begin{cases} \left(1 - c_{s,i}^2(\phi/\phi_i)^2\right)\left(2t_i\phi_i^{-1}(\phi/\phi_i)\right)^2 & \text{for } t_i < t < t_{\rm eq} \text{ during RD} \\ \left(1 - c_{s,\rm eq}^2(\phi/\phi_{\rm eq})^4\right)\left(3t_{\rm eq}\phi_{\rm eq}^{-1}(\phi/\phi_{\rm eq})^2\right)^2 & \text{for } t_{\rm eq} < t < t_c \text{ during MD} \end{cases},\quad \phi_i = \phi(t_i);\ t_c > t_0 \text{ here.}$$

- Then (taking $c_{s,i} = c_s(a_i = 6.5 \times 10^{-5}) = 1.09 \times 10^{-8}$ and $m/H_0 = 29.0$)





- $\rho \propto a^{-3}$. $|p| \ll \rho$. DM-like.

  $p \propto \rho^{2/3}$ (during MD).

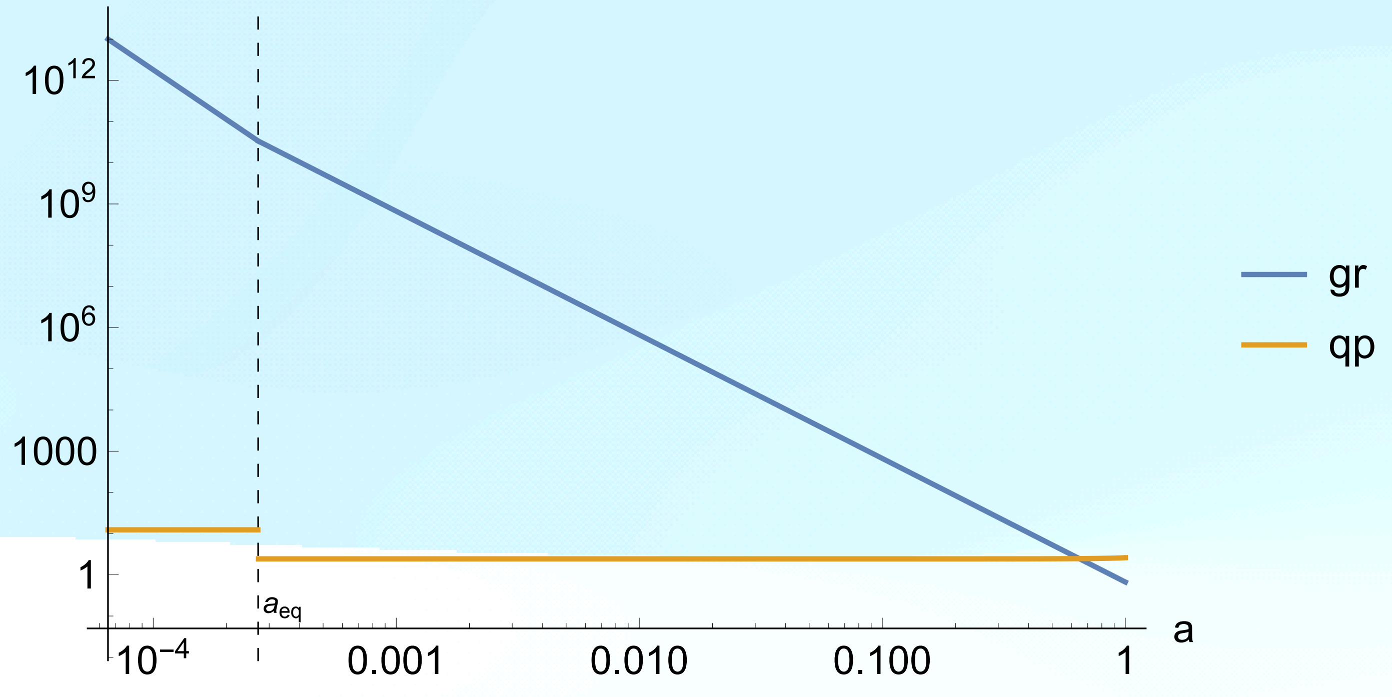
- We get an increasing c_s, without a hand-set t_c. $c_s = c_{s,i}(a/a_i)$ for $t > t_i$.

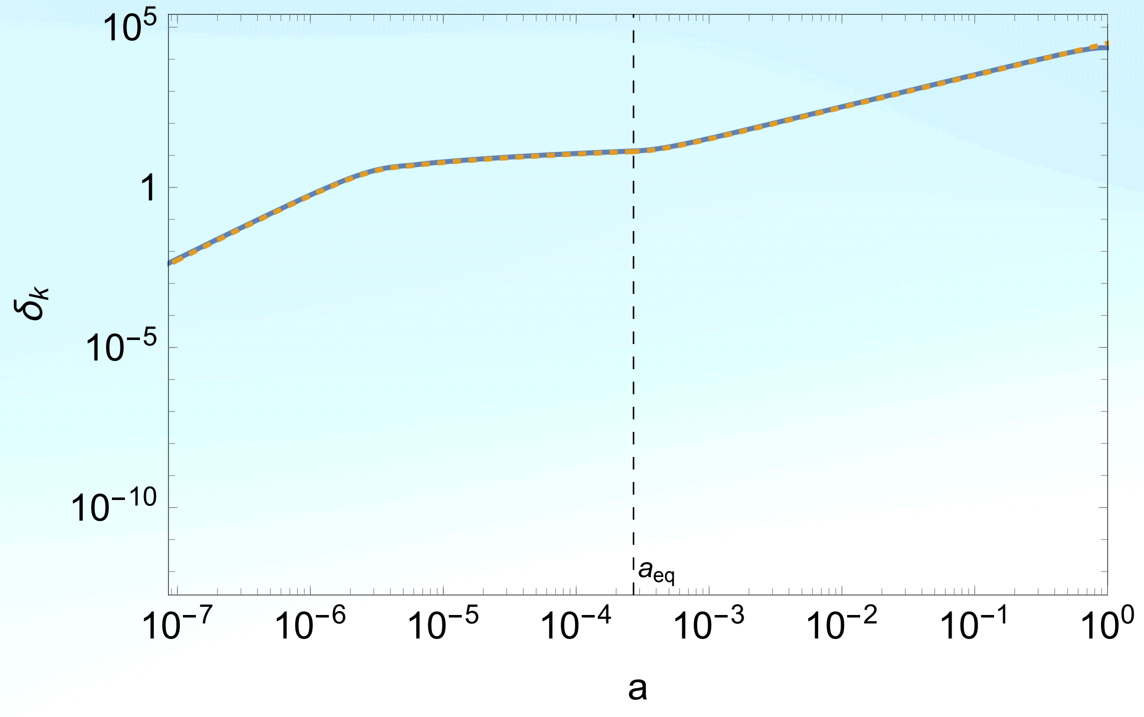
- Late time suppression by increasing c_s.

ULDM with Non-Canonical Kinetics

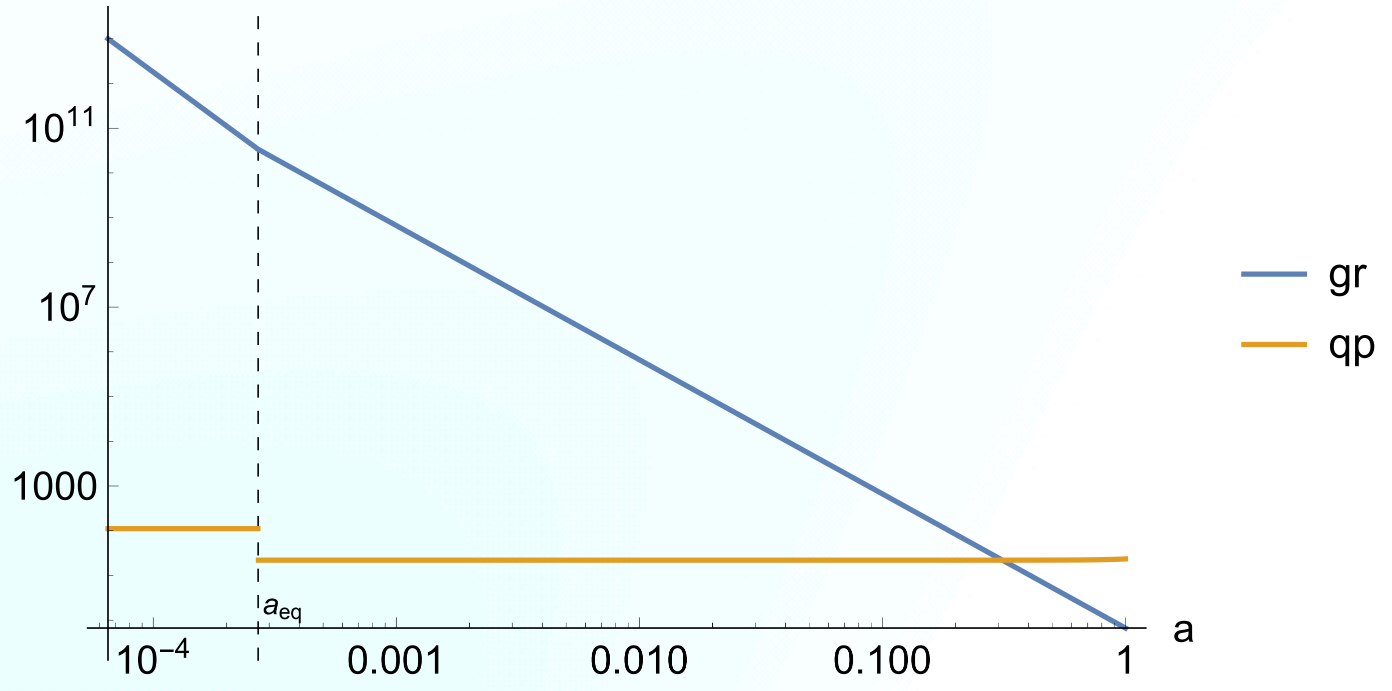
Our k-ULDM: example 2 ("Chaplygin-like" case)

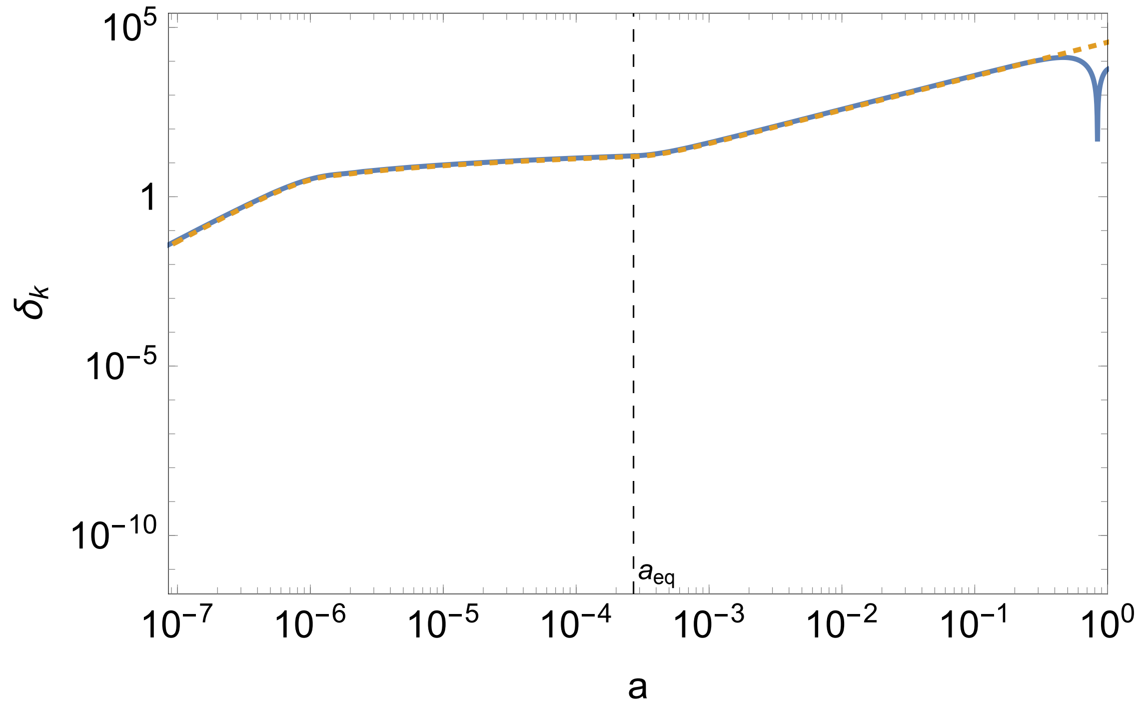
$k/H_0 = 1 \times 10^{4.5}$





$k/H_0 = 3 \times 10^{4.5}$

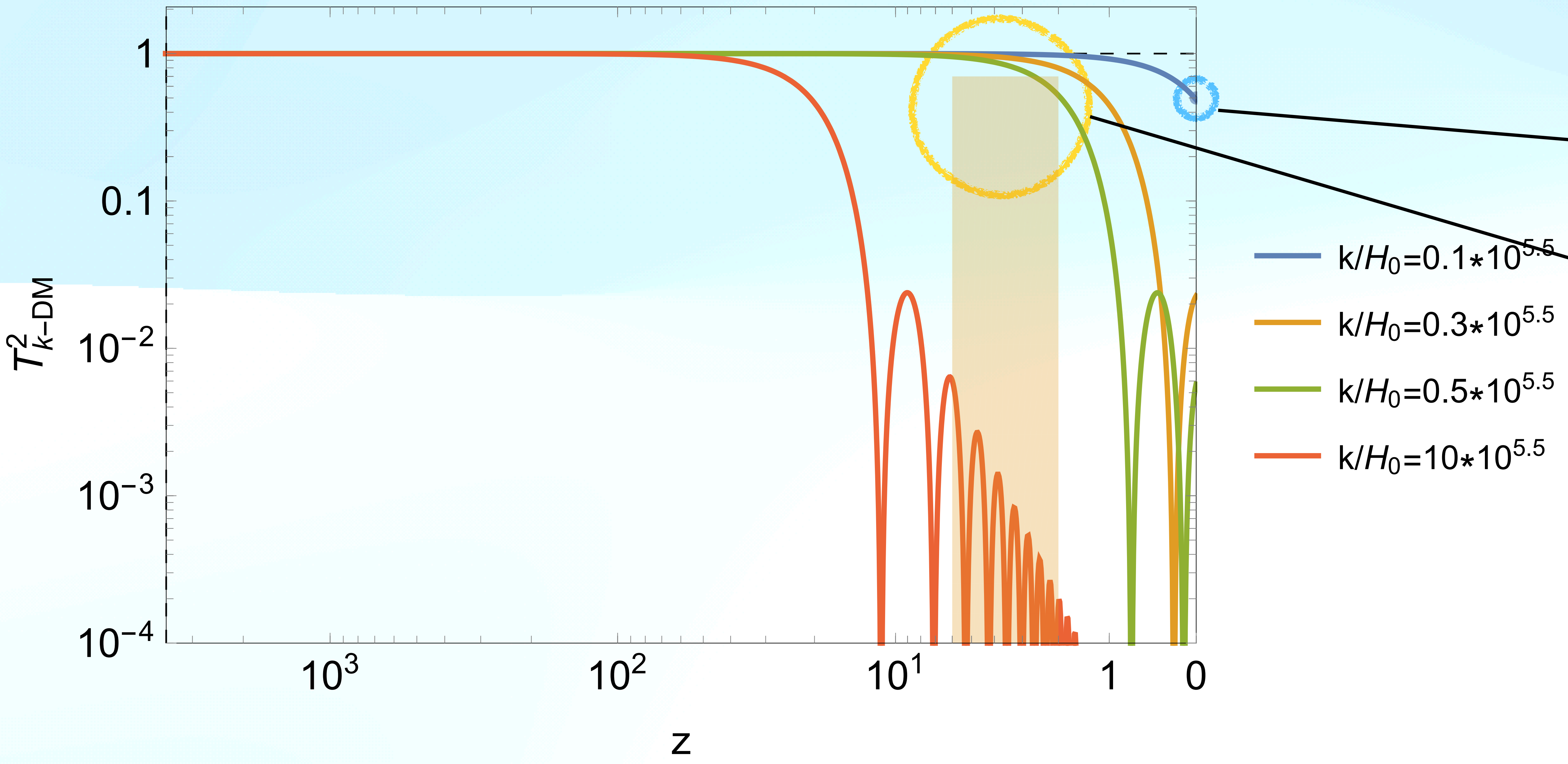




- A comparison of gravitational term and sound speed term when taking $c_{s,i} = c_s(a_i = 6.5 \times 10^{-5}) = 1.09 \times 10^{-8}$ and $m/H_0 = 29.0$.
- $c_{s,g} = c_s = c_{s,i}(a/a_i)$. Late time suppression.
- The mass of ϕ should be light enough to guarantee the large enough $\frac{k^2}{a^2}c_{s,g}^2$ at late time.

ULDM with Non-Canonical Kinetics

Our k-ULDM: example 2 ("Chaplygin-like" case)



$c_{s,i} = c_s(a_i = 6.5 \times 10^{-5}) = 1.09 \times 10^{-8}$, $m/H_0 = 29.0$

- To interpret cusp-core (missing satellite) & Lyα at the same time:
  - $T^2(k = 4.5\ \mathrm{Mpc}^{-1}) \sim 0.5$ at $z = 0$;
  - $T^2(k = 20h\ \mathrm{Mpc}^{-1}) > 0.7$ at $z \approx 2 \sim 6$ (Lyα).
- Such k-ULDM can truly reopen the window of ULDM constrained by Lyα.

Summary

- We have reviewed the small-scale challenges of ΛCDM model. Then we reviewed how the wave nature can hopefully solve the problems in CDM models.
- However, the preferred FDM mass solving CDM crisis are seemingly disfavored by recent observations. This is the motivation of our work.
- We noticed that the problem comes from observations at different redshift, and came to the thought that the small-scale suppression for FDM may be delayed.
- We found that, ULDM with non-canonical kinetic with modified sound speed can hopefully serve as the expected model.
- We found that examples of DM constructed from DBI theory can alleviate or even solve the constraint.

Thank you!