Workshop on Tensions in Cosmology

COSMOLOGICAL TENSIONS AND STRONG COUPLING ISSUE IN THE F(T) GRAVITY

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OUTLINE

- **Modified Gravity: motivation and common questions**
- **Teleparallel gravity and beyond**
  - TEGR, f(T) and f(T,B) gravity
  - An alternative EFT formulation: cosmology tension.
- **Extra DoFs in f(T) gravity**
  - Hamiltonian analysis: field configuration dependence
  - Perturbation behavior: strong coupling problem. ← my work

I.BACKGROUND

At the beginning, I’d like to give some background about modified gravity

Motivation?

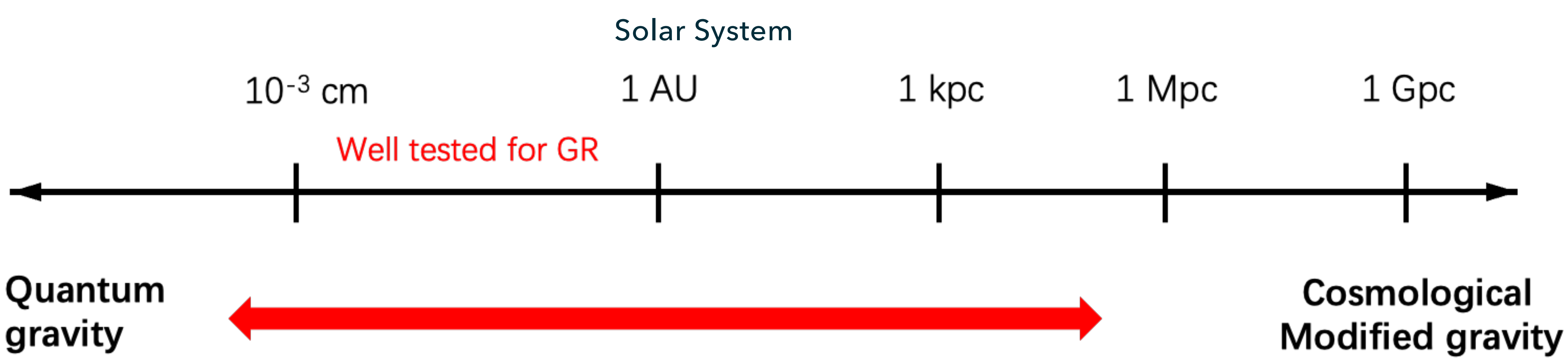
- **Discrepancies between GR predictions and observations:**
  - Dark Sector: Dark matter and dark energy…

  Plus a cosmological constant (ΛCDM):
  - Cosmological constant problem…
  - Cosmology tensions…
- **Self-consistency:**
  - Singularity…
  - Quantum gravity…

Different scales?

- **L. Heisenberg 1807.01725,**

Assuming that General Relativity is still the right effective theory for the intermediate scales, one can tackle these problems by modifying the gravitational interactions in the IR and UV.



Yi-Fu Cai 2021

How to modify?

Lovelock 1971, 1972: Lovelock's theorem

- 4d diffeomorphism space-time
- Local action
- Only metric field
- Up to second order derivatives of metric

The only possible second order equations of motion are the Einstein's equations and/or a cosmological constant.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} - g_{\mu\nu}\Lambda$$

How to modify?

- T. Clifton, et al. 1106.2476

- Different dimension/ violation of Lorentz-invariance
- Non-locality
- Fields other than the metric
- Higher-order derivatives

Modified gravity = Modified EoMs + additional degrees of freedom

relax one or more…

Modified gravity?

- T. Clifton, et al. 1106.2476

Modified gravity
= Modified EoMs
+ additional
degrees of freedom

Q1: How many physical DoFs?

Comparing with GR

Q2: How do these DoFs behave?

ghosts,…

Different Geometry

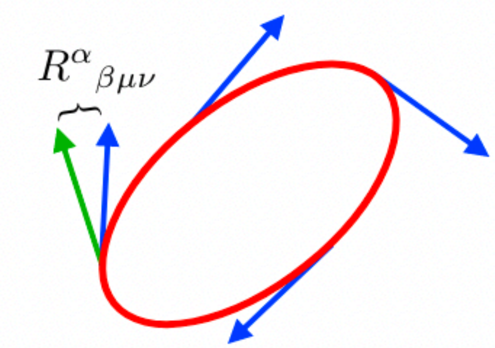
- J. Beltran Jiménez, et. al. 1903.06830

Q1: How many physical DoFs?

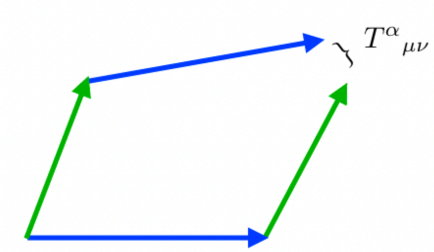
Comparing with GR

Q2: How these DoFs behave?

Teleparallel: vanishing curvature

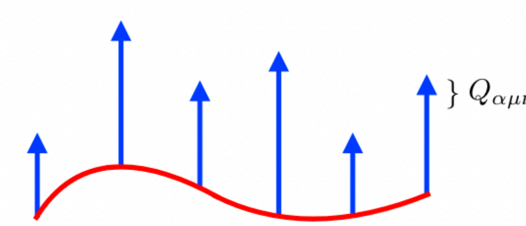


The rotation of a vector transported along a closed curve is given by the curvature: General Relativity.



The non-closure of parallelograms formed when two vectors are transported along each other is given by the torsion: Teleparallel Equivalent of General Relativity.

$$T^{\alpha}_{\mu\nu} \equiv \Gamma^{\alpha}_{\mu\nu} - \Gamma^{\alpha}_{\nu\mu}.$$

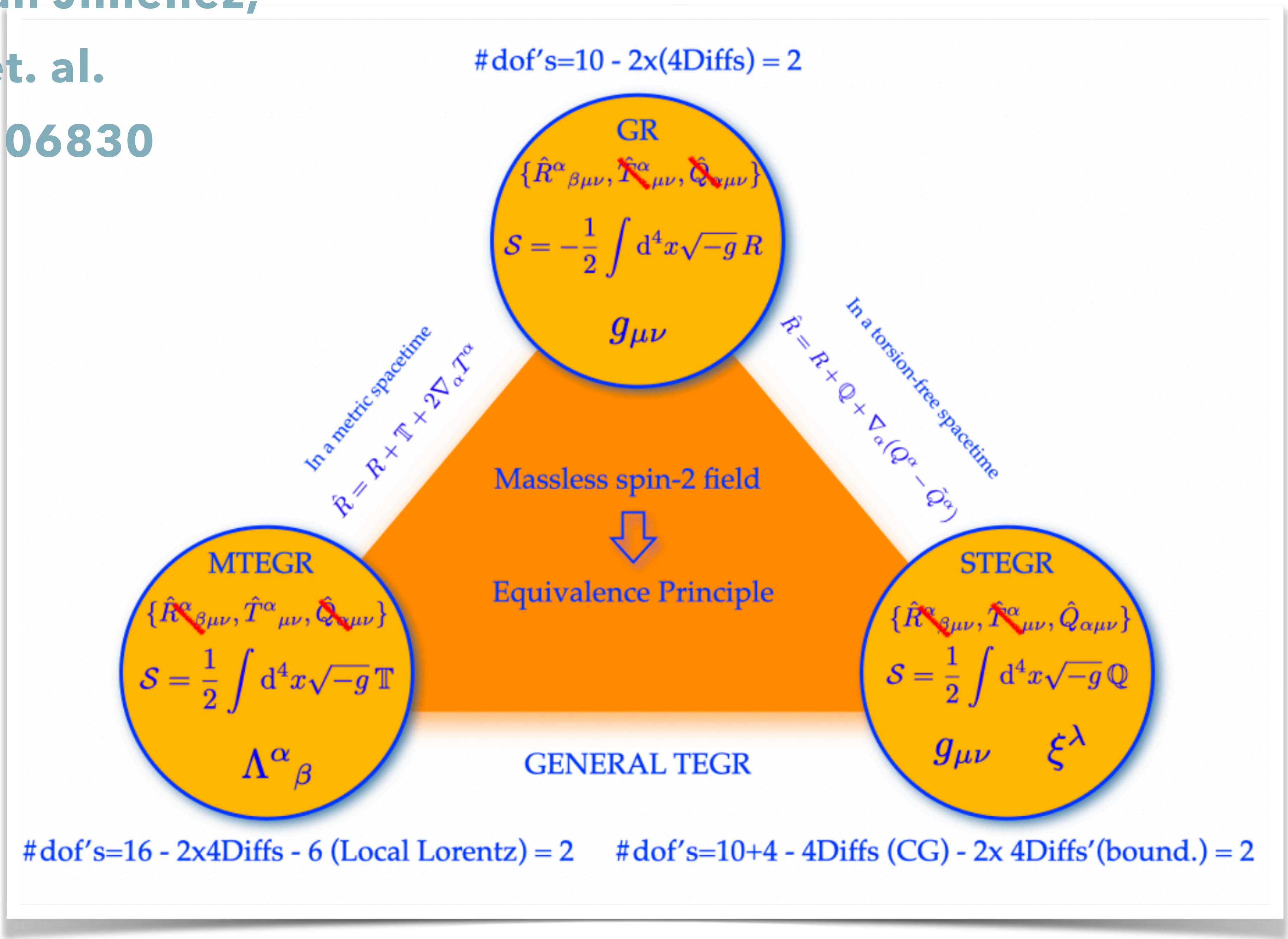


The variation of the length of a vector as it is transported is given by the non-metricity: Symmetric Teleparallel Equivalent of General Relativity.

$$Q_{\alpha\mu\nu} \equiv \nabla_{\alpha} g_{\mu\nu}.$$

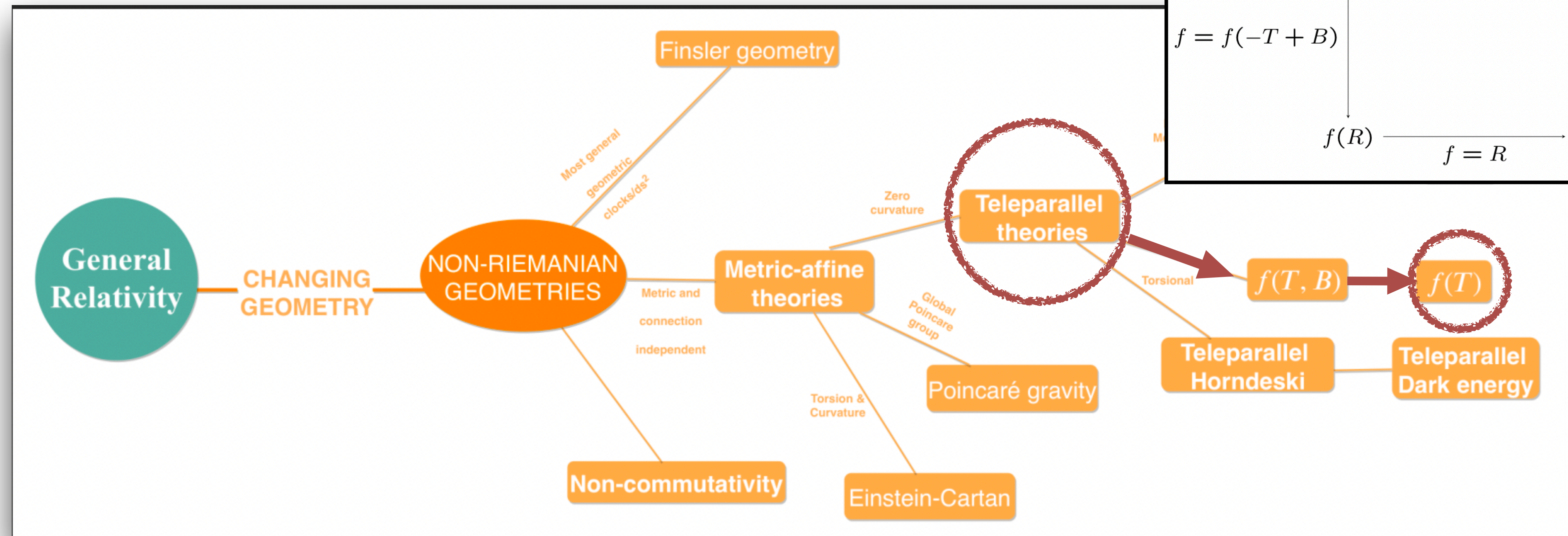
Geometrical trinity

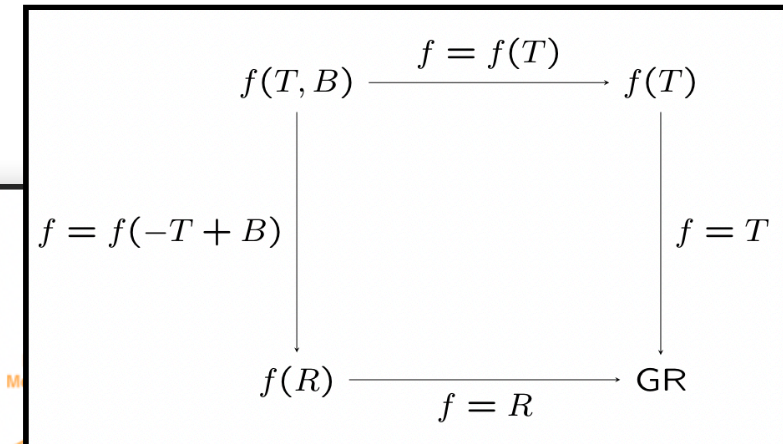
- J. Beltran Jiménez, et. al. 1903.06830



Teleparallel gravity

- CANTATA. 2105.12582; S. Bahamonde, et.al. 1508.05120





Teleparallel : general connection with vanishing curvature

The EFT approach?

- Chun-Long Li et al. 1803.09818

A general EFT action of torsional gravity:

$$S=\int d^4x\sqrt{-g}\Big[\frac{M_P^2}{2}\Psi(t)R-\Lambda(t)-b(t)g^{00}+\frac{M_P^2}{2}d(t)T^0\Big]+S^{(2)},$$

→ all terms that start quadratic in perturbations

$$R=-T-2\nabla_\mu T^\mu,\qquad \int d^4x\sqrt{-g}h(t)\nabla_\mu T^\mu=-\int d^4x\sqrt{-g}\dot{h}(t)T^0$$

$$f(T)=f(T^{(0)})+f_T(T^{(0)})\left(T-T^{(0)}\right)+\frac{1}{2}f_{TT}(T^{(0)})\left(T-T^{(0)}\right)^2+\frac{1}{6}f_{TTT}(T^{(0)})\left(T-T^{(0)}\right)^3+\cdots,$$

f(T) gravity

Sheng-Feng Yan, et. al. 1909.06388; Xin Ren, et. al. 2203.01926

Interpreting cosmological tensions from the effective field theory of torsional gravity

Sheng-Feng Yan,[1,2,3] Pierre Zhang,[1,2,3] Jie-Wen Chen,[1,2,3] Xin-Zhe Zhang,[1,2,3] Yi-Fu Cai,[1,2,3,*] and Emmanuel N. Saridakis[4,5,1,†]

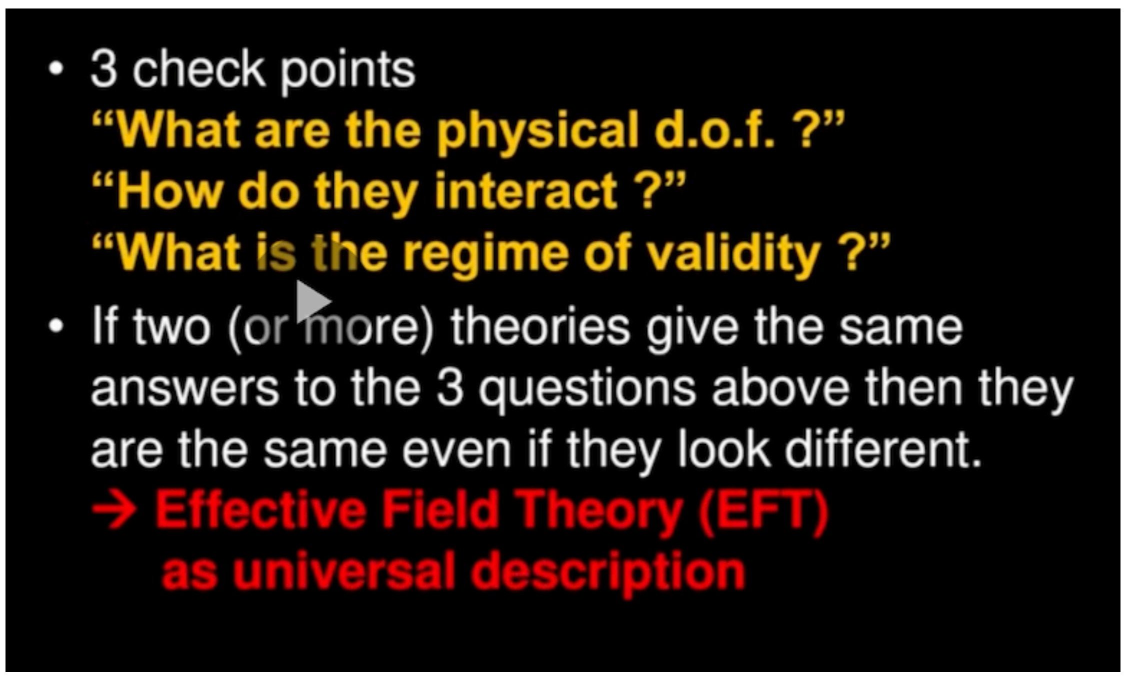
Cosmological tensions can arise within ΛCDM scenario amongst different observational windows, which may indicate new physics beyond the standard paradigm if confirmed by measurements. In this article, we report how to alleviate both the H_0 and σ_8 tensions simultaneously within torsional gravity from the perspective of effective field theory (EFT). Following these observations, we construct concrete models of Lagrangians of torsional gravity. Specifically, we consider the parametrization $f(T) = -T - 2\Lambda/M_P^2 + \alpha T^\beta$, where two out of the three parameters are independent. This model can efficiently fit observations solving the two tensions. To our knowledge, this is the first time where a modified gravity theory can alleviate both H_0 and σ_8 tensions simultaneously, hence, offering an additional argument in favor of gravitational modification.

Gaussian processes and effective field theory of $f(T)$ gravity under the H_0 tension

Xin Ren,[1,2] Sheng-Feng Yan,[3,4,1] Yaqi Zhao,[1,2] Yi-Fu Cai,[1,2] and Emmanuel N. Saridakis[5,1,2]

The EFT approach?

- **Shinji Mukohyama. 2022:**



Q1: How many physical DoFs?

Q3:what is the regime of validity?

Q2: How do these DoFs behave?

EFT method should be a good unified way to answer this question

II. EXTRA DOFS IN F(T) GRAVITY

Other hints

- **Inversible transformaion: dynamically equivalent**

Matthew Wright. 1602.05764

$$S = \frac{1}{16\pi G}\int \left[-\hat{T} - \frac{\psi}{\sqrt{3}}\hat{B} + \frac{1}{2}g^{\mu\nu}\psi_\mu\psi_\nu - U(\psi)\right]\hat{e}\, d^4x$$
$$= \frac{1}{16\pi G}\int \left[-\hat{T} - \frac{2}{\sqrt{3}}\hat{T}^\mu\psi_\mu + \frac{1}{2}g^{\mu\nu}\psi_\mu\psi_\nu - U(\psi)\right]\hat{e}\, d^4x,$$

the presence of a scalar field with a coupling term

Hamiltonian analysis

- **Miao Li, et al.1105.5934:** 5
- **Pisin Chen, et al. 1412.8383:** 5, 4, 2
- **Rafael Ferraro, et al. 1802.02130:** 3
- **Milutin Blagojević, et al. 2006.15303:** 5, 4, 2, 0

Controversial in some cases

Inconsistent results?

- **Pisin Chen, et al. 1412.8383**

the number of physical DoFs and the classes of Dirac constraints,

can and do change depending on the values of the fields.

That is, they are expected to be different on different background geometries.

Generally there are 5 DoFs in f(T) gravity, however,

there would exist solutions where the Poisson bracket matrix has less rank.

this could be a generic feature of teleparallel theories,

Hard to solve?

- Xian Gao et al. 2019: degenerate conditions in 3-DoF SCG gravity

$$\mathcal{S}(\vec{x},\vec{y}) := \frac{\delta^2 S}{\delta N(\vec{x})\,\delta N(\vec{y})} - \int \mathrm{d}^3x' \int \mathrm{d}^3y'\, N(\vec{x}')\, \frac{\delta}{\delta N(\vec{x})}\left(\frac{1}{N(\vec{x}')}\frac{\delta S}{\delta K_{i'j'}(\vec{x}')}\right) \times \mathcal{G}_{i'j',k'l'}(\vec{x}',\vec{y}')\, N(\vec{y}')\, \frac{\delta}{\delta N(\vec{y})}\left(\frac{1}{N(\vec{y}')}\frac{\delta S}{\delta K_{i'j'}(\vec{y}')}\right),$$

$$\mathcal{J}(\vec{x},\vec{y}) := \int \mathrm{d}^3x' \int \mathrm{d}^3y' \int \mathrm{d}^3x'' \int \mathrm{d}^3y''\, \frac{\delta C(\vec{x})}{\delta K_{ij}(\vec{x}')}\mathcal{G}_{i'j',k'l'}(\vec{x}',\vec{x}'') \times N(\vec{x}'')\, \frac{\delta^2 S}{\delta h_{i'j'}(\vec{x}'')\,\delta K_{k'l'}(\vec{y}'')}\mathcal{G}_{k'l',kl}(\vec{y}'',\vec{y}')\, \frac{\delta C(\vec{y})}{\delta K_{ij}(\vec{y}')} - \int \mathrm{d}^3x' \int \mathrm{d}^3y'\, \frac{\delta C'(\vec{x})}{\delta K_{ij}(\vec{x}')}\mathcal{G}_{ij,kl}(\vec{x}',\vec{y}')\, N(\vec{y}')\, \frac{\delta C(\vec{y})}{\delta h_{kl}(\vec{y}')} - (\vec{x}\leftrightarrow\vec{y}),$$

Difficult to solve when applying for specific model

Alternative method is needed → Perterbation method

Perturbation

- **Linear order:** **no extra DoF or strong coupling problem ?**

K. Izumi and Y.C. Ong, 1212.5774

- **In flat FLRW background:**

  All possible modes of perturbations up to second order action, including pseudoscalar and pseudovector modes in addition to the usual scalar, vector, and tensor modes.

- **In Minkowski spacetim:**

  By taking the limit $H \to 0$, the extra degrees of freedom do not appear in this level.

Perturbation

Higher order: **strong coupling problem ?**

J. Beltran Jimene, et.al. 2004.07536

- **In Minkowski spacetime: 4-th order perturbation action**

consider the general Minkowski solution as perturbation around the trivial tetrad

$$\Lambda = \exp(\lambda) = I + \lambda + \frac{1}{2}\lambda^2 + \frac{1}{3!}\lambda^3 + \dots ,$$

$\lambda \in so(1, 3)$

Q1: How many extra DoFs? → **At least one scalar mode, (not only in M)**

What will happen in higher order around cosmology background ?

New DoF would appear in 3rd order or 4-th order ?

STRONG COUPLING ISSUE IN F(T) GRAVITY BY EFT METHOD

Based on the work arXiv: 2302.03545

in collaboration with Yaqi Zhao, Xin Ren, Bo Wang, E. N. Saridakis, Yi-Fu Cai

Perturbed tetrads

- **K. Izumi, Y.Ong. 1212.5774;**

only focus on the scalar perturbations

$$e^0_\mu = \delta^0_\mu + \delta^0_\mu \phi + a\delta^i_\mu \partial_i \chi\,,$$
$$e^a_\mu = a\delta^i_\mu \delta^a_i + \delta^0_\mu \delta^a_i \partial^i \mathcal{E} + a\delta^i_\mu \delta^a_j \left[\epsilon_{ijk}\partial_k \sigma - \psi\delta_{ij} + \frac{1}{2}\partial_i\partial_j F\right]$$

$$e^0{}_\mu = \delta^0_\mu\,,$$
$$e^a{}_\mu = a\delta^i_\mu \delta^a_i\,.$$

The gauge transformation

$$\delta\tilde{e}^a_\mu = \delta e^a_\mu - \xi^\alpha \bar{e}^a_{\mu,\alpha} - \xi^\rho_{,\mu}\bar{e}^a_\rho\,,$$

$$\xi^i = a^{-1}(\partial_i \xi + \xi^{tr}_i)$$

$$\tilde{\phi} = \phi - \xi^0{}_{,0}\,, \quad \tilde{\chi} = \chi - \frac{1}{a}\xi^0\,, \quad \tilde{\mathcal{E}} = \mathcal{E} - \xi_{,0} a\,,$$
$$\tilde{\psi} = \psi - \xi^0 \frac{\dot{a}}{a}\,, \quad \tilde{F} = F - 2\xi\,, \quad \tilde{\sigma} = \sigma\,,$$

$$e^0{}_\mu = \delta^0_\mu + \delta^0_\mu \phi + a\delta^i_\mu \partial_i \chi\,,$$
$$e^a{}_\mu = a\delta^i_\mu \delta^a_i (1-\psi) + \delta^0_\mu \delta^a_i \partial^i \chi\,,$$

written in the Newtonian gauge

setting $B = \mathcal{E} - \chi = 0$ and $F = 0$

Perturbed tetrads

- Yu-Min Hu et al. 2302. 3545

higher order expension: up to cubic order

$$
\begin{aligned}
e^0{}_\mu &= \delta^0_\mu + \delta^0_\mu \phi + a\delta^i_\mu \partial_i \chi \;, \\
e^a{}_\mu &= a\delta^i_\mu \delta^a_i (1-\psi) + \delta^0_\mu \delta^a_i \partial^i \chi \;,
\end{aligned}
$$

$\rightarrow$

$$
e^\nu{}_\mu = \delta^\nu{}_\mu + \delta e^\nu{}_\mu + \frac{1}{2}\delta e^\nu{}_\rho \delta e^\rho{}_\mu + \cdots \;,
$$

$$
\begin{aligned}
e^0_\mu =& \delta^0_\mu\big(1+\phi+\frac{1}{2}\phi^2+\frac{1}{2}\partial_i\chi\partial_i\chi\big) + a\delta^i_\mu\Big[\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big] \;, \\
e^a_\mu =& a\delta^i_\mu\delta^a_i\big(1-\psi+\frac{1}{2}\psi^2\big) + \frac{a}{2}\delta^i_\mu\delta^a_j\partial_i\chi\partial_j\chi + \delta^0_\mu\delta^a_i\Big[\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big] \;, \\
e^\mu_0 =& \delta^\mu_0\big(1-\phi+\frac{1}{2}\phi^2+\frac{1}{2}\partial_i\chi\partial_i\chi\big) + \frac{1}{a}\delta^\mu_i\Big[-\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big] \;, \\
e^\mu_a =& \frac{1}{a}\delta^\mu_i\delta^i_a\big(1+\psi+\frac{1}{2}\psi^2\big) + \frac{1}{2a}\delta^\mu_i\delta^j_a\partial_i\chi\partial_j\chi + \delta^\mu_0\delta^i_a\Big[-\partial_i\chi+\frac{1}{2}(\phi\partial_i\chi-\psi\partial_i\chi)\Big]
\end{aligned}
$$

with cubic contribution skipped in this slide

Strong coupling issue

- Yu-Min Hu et al. 2302. 3545

$$S = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}\Psi(t)R - \Lambda(t) - b(t)g^{00} + \frac{M_P^2}{2}d(t)T^0\right] + S^{(2)} .$$

$$f(T) = f(T^{(0)}) + f_T(T^{(0)})\left(T - T^{(0)}\right) + \frac{1}{2}f_{TT}(T^{(0)})\left(T - T^{(0)}\right)^2 + \frac{1}{6}f_{TTT}(T^{(0)})\left(T - T^{(0)}\right)^3 + \cdots ,$$

linear order perturbation actions:

$$\mathcal{L}_2^{kin} = M_p{}^2 a\left(f_T\left(3a^2\dot{\psi}^2 + \partial_i\psi\left(2\partial^i\phi - 4aH\partial^i\chi - \partial^i\psi\right)\right) + 2aH\left(6f_{TT}H\left(3a\dot{\psi}^2 - 2H\partial_i\chi\partial^i\phi\right) + \dot{f}_T\partial_i\chi\partial^i\pi\right)\right)$$

$$\chi = -\frac{\psi}{aH} ,$$

$$\phi = \frac{\partial^2\psi}{3a^2H^2} - \frac{\dot{\psi}}{H} + \frac{\dot{f}_T}{f_T}\pi \qquad \text{with} \qquad \pi = -\frac{\psi}{H}$$

Solve all the constraint equations of three non-dynamical variables

Strong coupling issue

- Yu-Min Hu et al. 2302. 3545

both linear and second order perturbation actions:

$$\mathcal{L}_2 = -M_P^2 \frac{1}{3aH^2} f_T \left(\partial^2\psi\right)^2$$

hint for 4-th order

$$\mathcal{L}_3 = M_P^2 a^3 \Big[-\frac{f_T}{3H^3a^4}(\partial^2\psi)^2\dot{\psi} - \frac{2f_{TT}}{3H^2a^6}\partial^2\psi\left(5\partial^2\psi\partial^2\psi + 2\partial_i\partial_j\psi\partial^i\partial^j\psi\right) + \frac{f_T}{9H^2a^4}\psi(-2\partial^2\psi\partial^2\psi + 5\partial_i\partial_j\psi\partial^i\partial^j\psi) + \frac{2\dot{f}_T}{3H^3a^4}\psi(\partial^2\psi\partial^2\psi - 2\partial_i\partial_j\psi\partial^i\partial^j\psi) - \frac{8\dot{f}_{TT}}{3Ha^4}\psi(\partial^2\psi\partial^2\psi - \partial_i\partial_j\psi\partial^i\partial^j\psi)\Big]. \quad (4.47)$$

Q2: How do the DoF behave?

New DoF would not appear in cubic action around FLRW background

The EFT approach

- Yu-Min Hu et al. 2302. 3545

Strong coupling problem: 4-th order is needed

But equations become much involved and are hard to solve.

Q3: what is the regime of validity ?

When is perturbation method out of the valid regime?

If the ratio of cubic to quadratic Lagrangian becomes larger than one, then the theory is strongly coupled.

$$X = \frac{\mathcal{L}_3}{\mathcal{L}_2} \sim 1 \quad \rightarrow \quad E_{cubic}$$

Strong coupling scale

perform a energy scale estimation

Strong coupling issue

- **Yu-Min Hu et al. 2302. 3545**

$$S_2 \sim -\frac{M^4}{2}\int dt\, dx^3 a^3 \frac{(\partial^2\psi)^2}{H^4 a^4}$$

← In this notation $f_T = \frac{3M^4}{2H^2 M_P^2}$

the cut-off scale

In the quadratic action, the leading term is assumed to be of order one → $\psi \sim \frac{H^2 E^{\frac{1}{2}}}{M^2 p^{\frac{1}{2}}}$

Challenge: no time derivatives appear in linear order

Generally, the physical momentum p is related to E through

the dispersion relation obtained in quadratic order.

→ **introduce a dimensionless parameter** $\beta \sim \frac{EH}{p^2}$

Strong coupling issue

- **Yu-Min Hu et al. 2302. 3545**

introduce a dimensionless parameter $\beta \sim \frac{EH}{p^2}$

$X = \frac{\mathcal{L}_3}{\mathcal{L}_2} \sim 1$ → $E_{cubic} \sim (\frac{M^3}{\beta H^3})^{\frac{1}{5}} M$ $\gg (\frac{M}{H})^{\frac{1}{3}} M$ $\frac{M}{H} \gg 1$

focusing on the modes deep inside horizon

$\frac{p^2}{H^2} \sim \frac{E}{\beta H} \gg 1$ → $\beta \ll \frac{E}{H}$

much higher than the cutoff M

valid up to the scale M

SUMMARY

- **Q1: How many extra DoFs of f(T) gravity in flat FLRW?**
  - Strongly support one scalar mode at least, fail to confirm.
- **Q2: How do this scalar type DoF behave in flat FLRW?**
  - Scalar DoF do not appear up to cubic action
  - Strongly indicate it should appear in 4th action, then strong coupling
- **Q3: what is the validity regime when stong coupling appear?**
  - valid up to the EFT cut-off scale M (at least for deep inside modes)

THANK YOU FOR YOUR ATTENTION