

Downloading many-body continuous variable entanglement to qubits

Zhihua Han, Kero Lau
Simon Fraser University

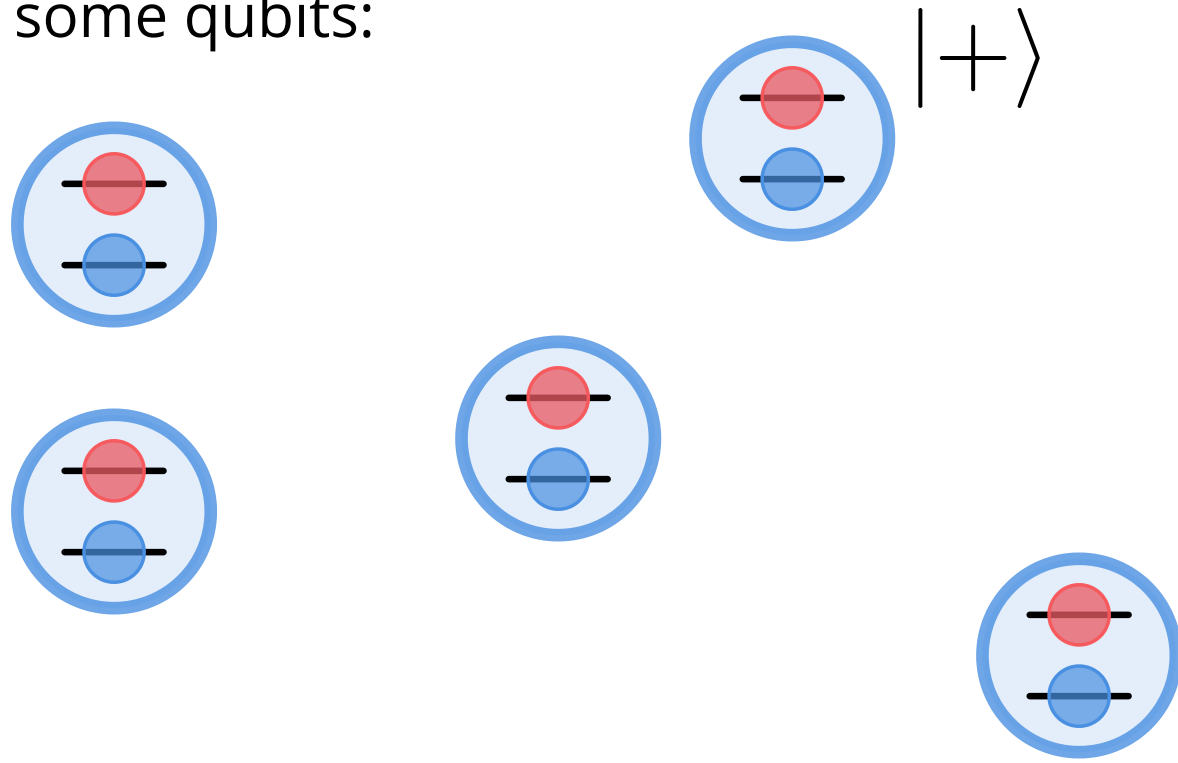


Canadian Association
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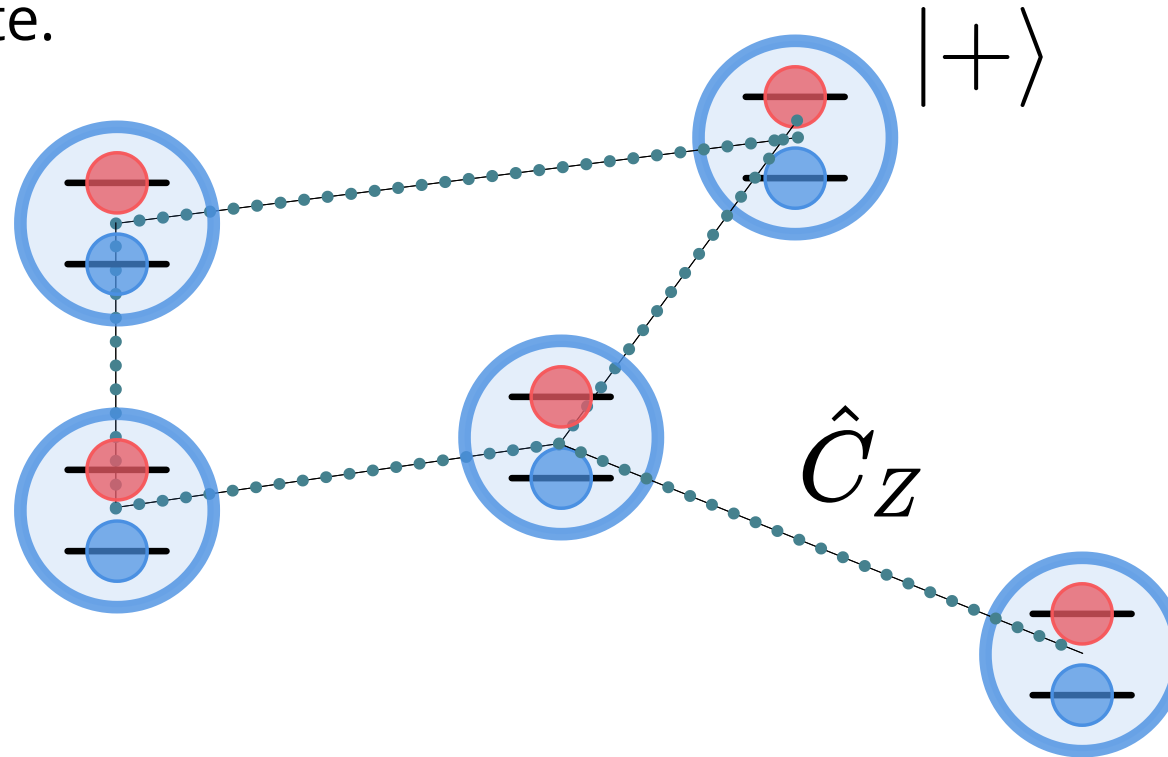
Qubit cluster state

Imagine I have some qubits:



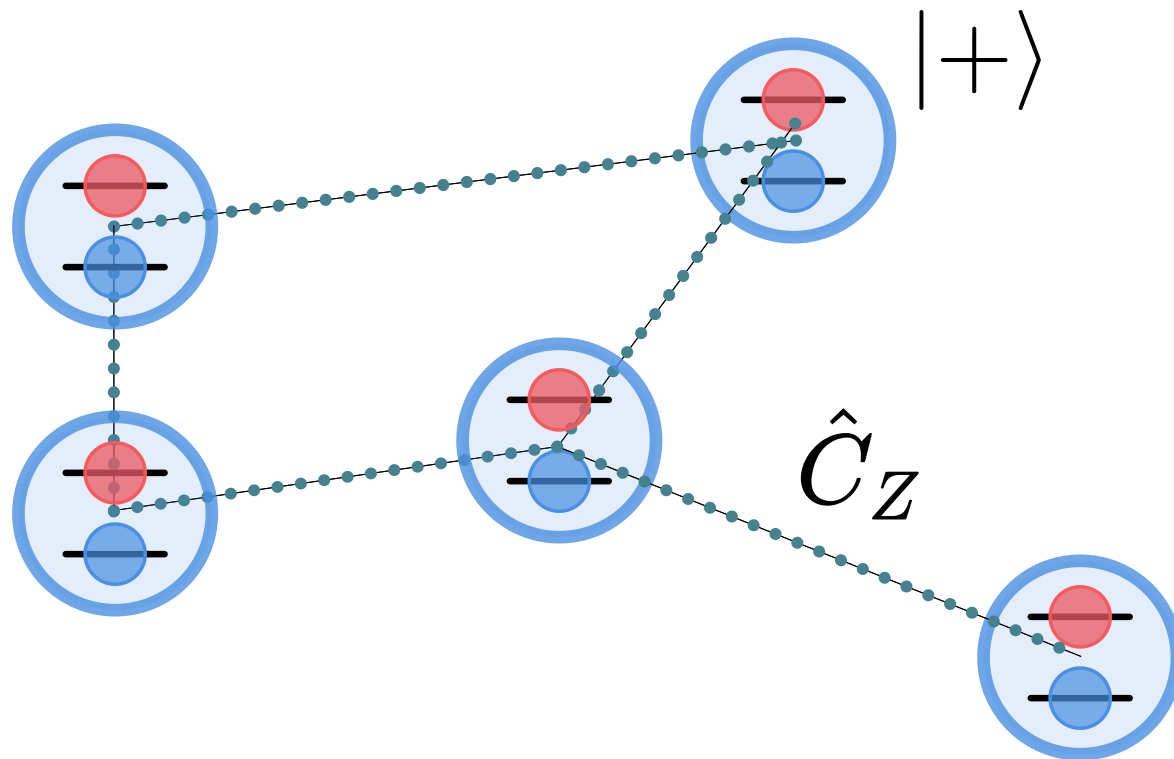
Qubit cluster state

and now I entangle the edges with the CZ gate.



Qubit cluster state

The quantum state specified by G is called a **qubit cluster state**.

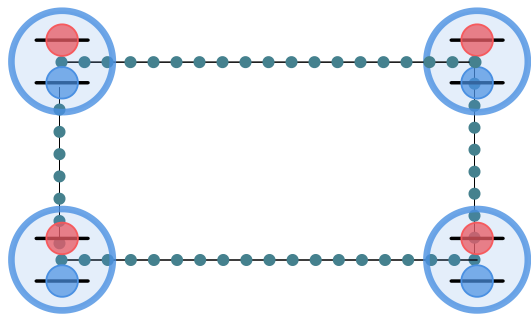


$$= |G\rangle$$
$$= \prod_{i,j \in E} \hat{C}_{Z_{ij}} |+\rangle^{\otimes N}$$

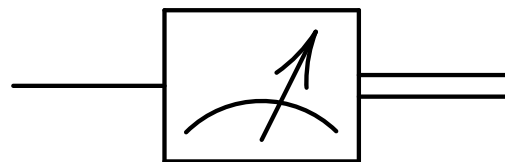
Why we need qubit cluster state

Qubit cluster state

Single qubit measurements



+



Fault tolerant
universal
quantum
computation

A One-Way Quantum Computer

Robert Raussendorf and Hans J. Briegel

Theoretische Physik, Ludwig-Maximilians-Universität München, Germany

(Received 25 October 2000)

We present a scheme of quantum computation that consists entirely of one-qubit measurements on a particular class of entangled states, the cluster states. The measurements are used to imprint a quantum logic circuit on the state, thereby destroying its entanglement at the same time. Cluster states are thus one-way quantum computers and the measurements form the program.

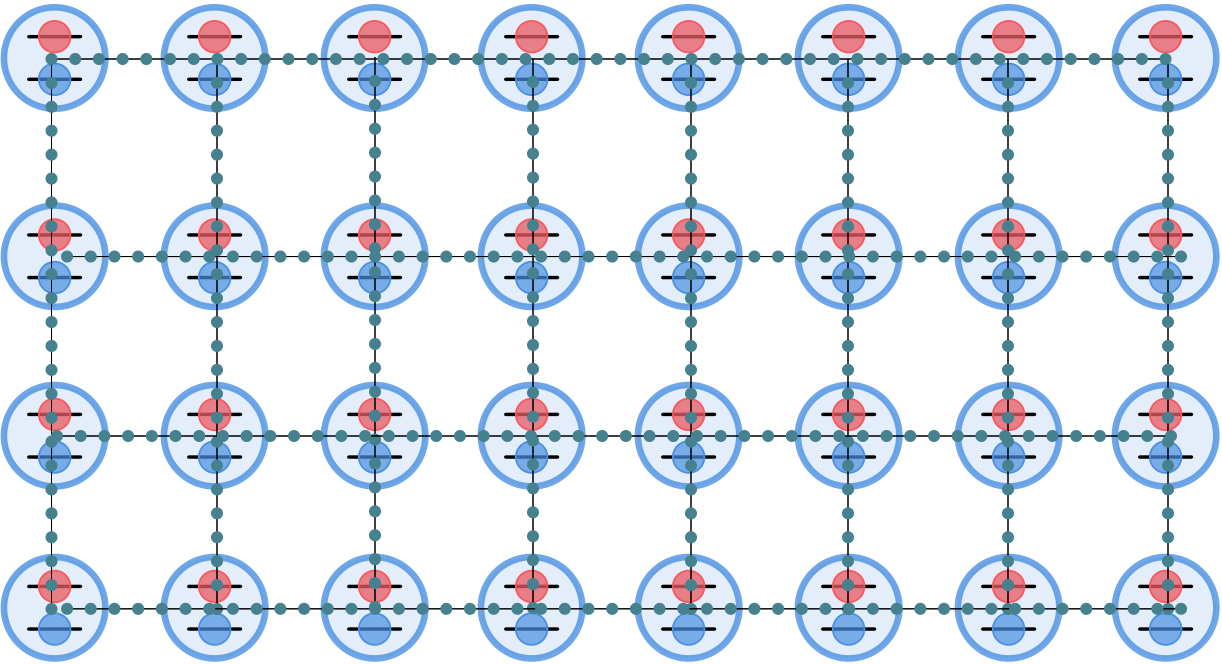
DOI: 10.1103/PhysRevLett.86.5188

PACS numbers: 03.67.Lx, 03.65.Ud

(Raussendorf 2001)

Why we need qubit cluster state

Qubit cluster state

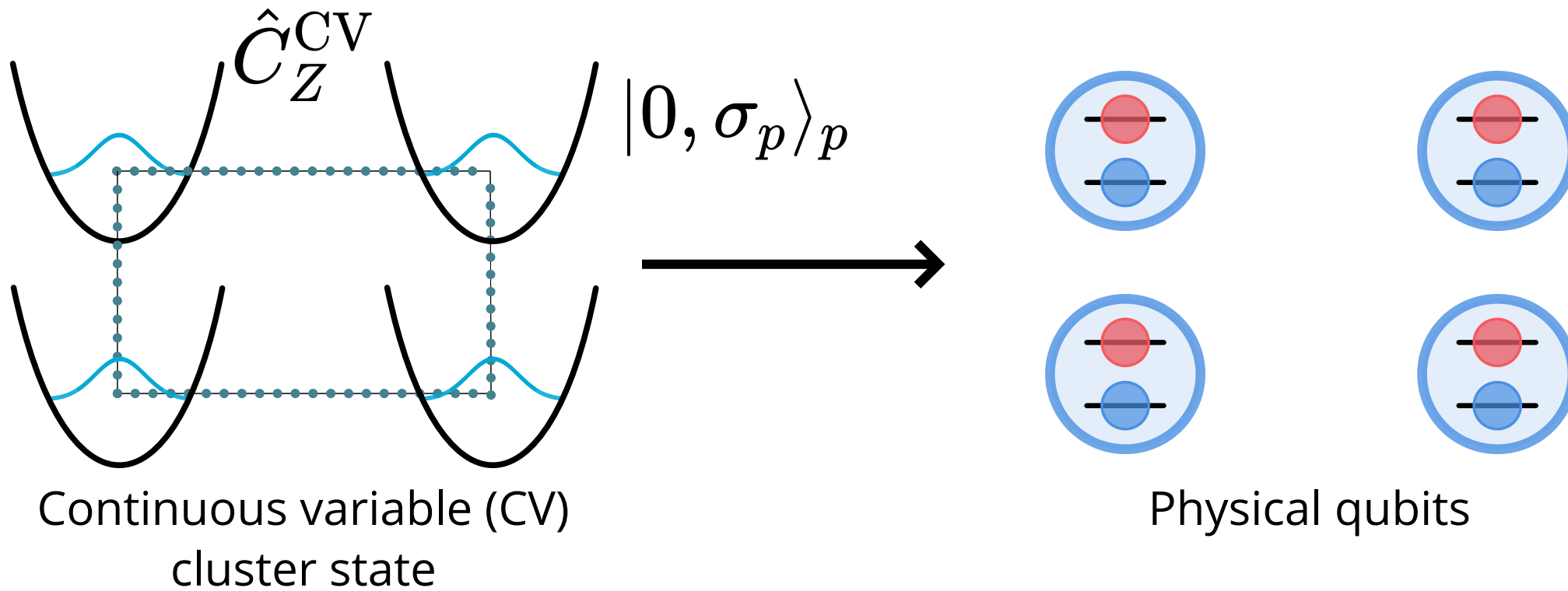


X10000...

Goal: Make many body entanglement in physical qubits

Entanglement Transfer Protocol

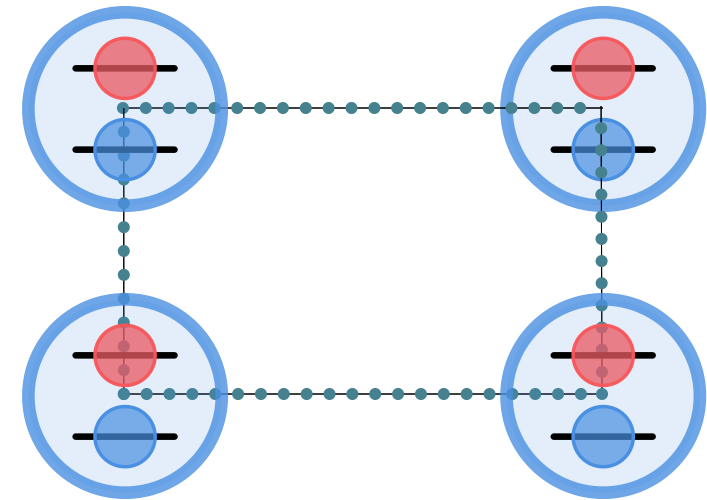
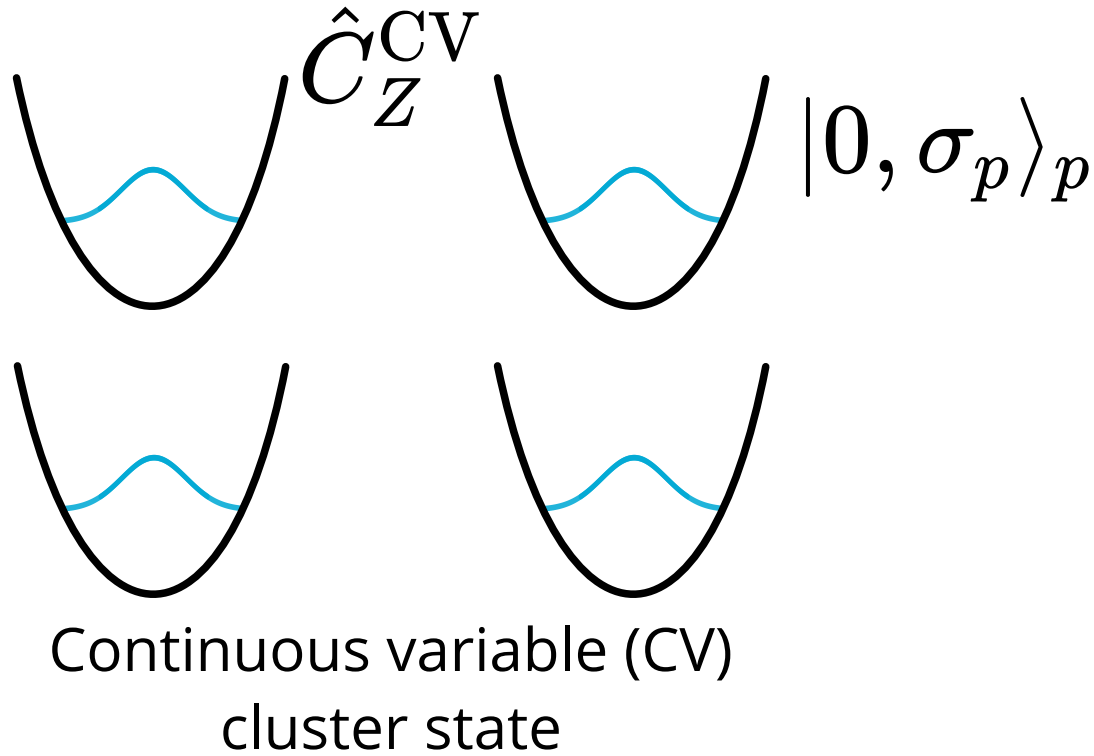
How do we make scalable qubit cluster states?



"Downloading entanglement from a CV cluster state"

Entanglement Transfer Protocol

How do we make scalable qubit cluster states?

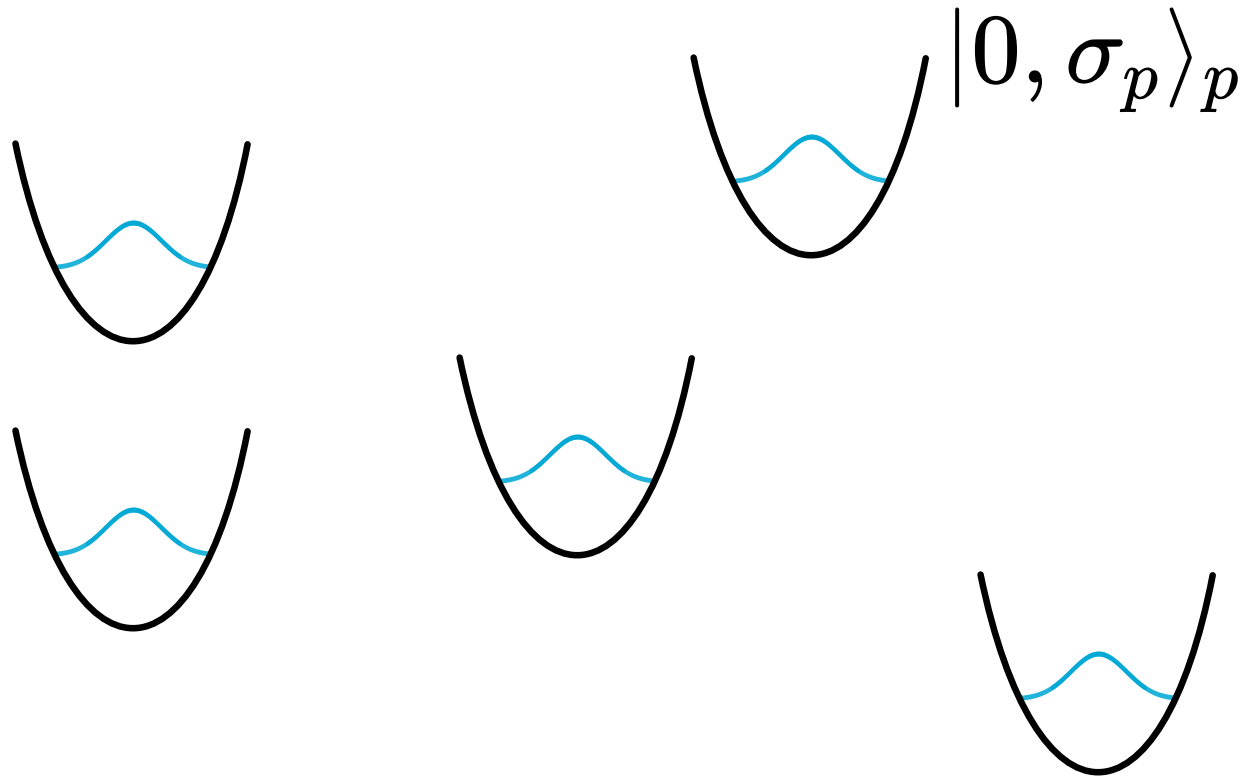


Qubit cluster state

Entanglement Transfer Protocol

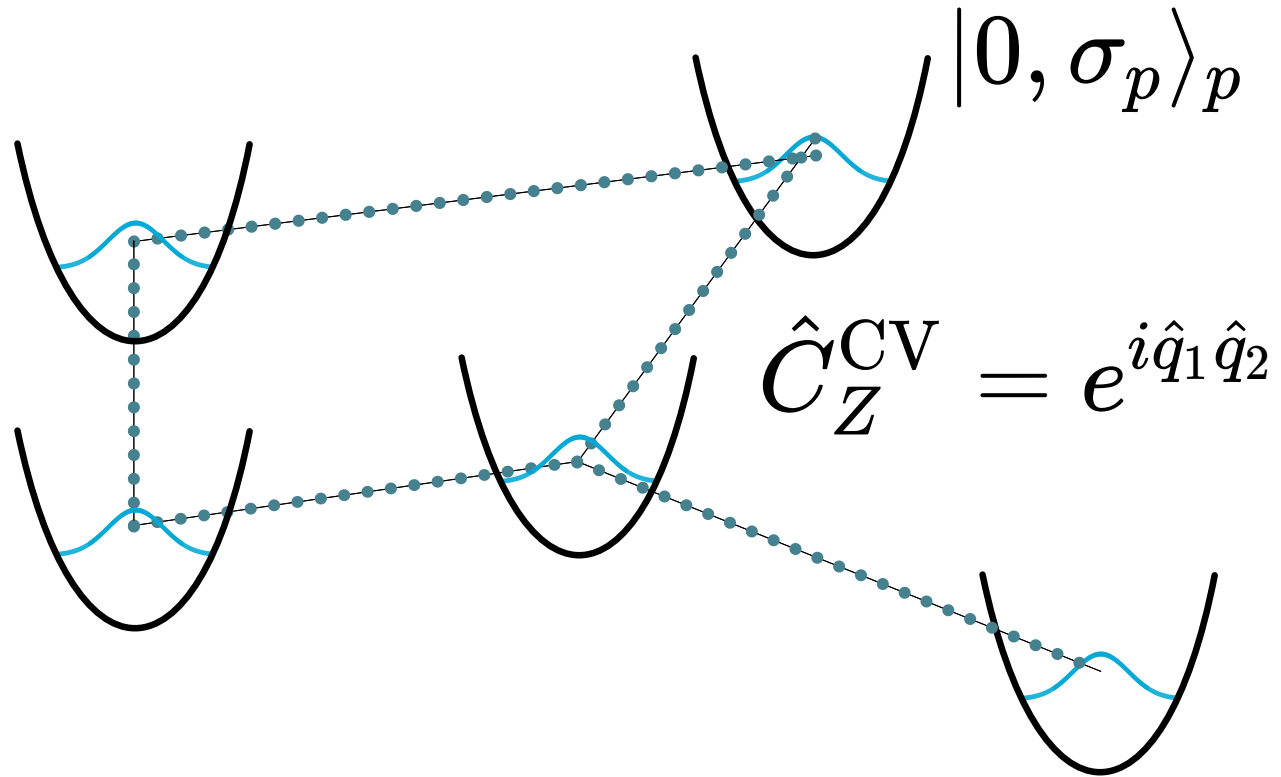
CV cluster state

Now if I have some bosons:



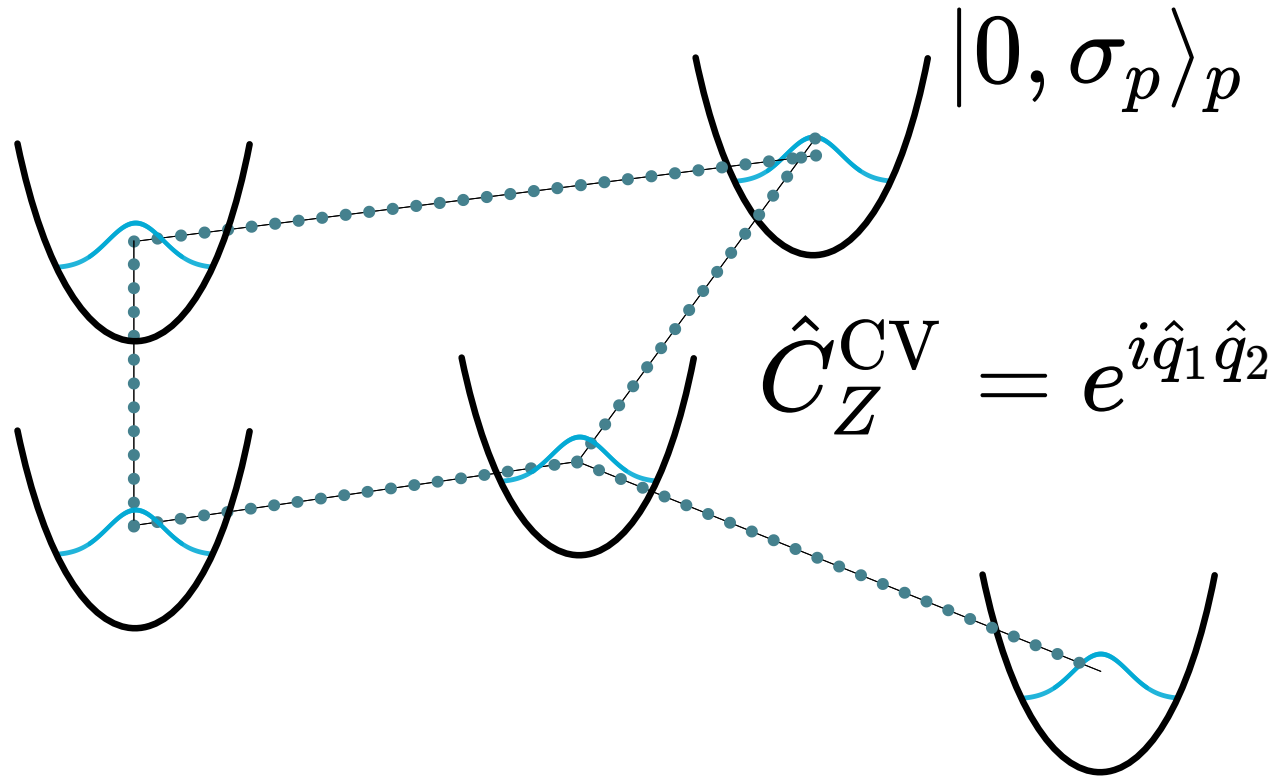
CV cluster state

and entangle them with CV CZ gate:



CV cluster state

We say it is a **CV cluster state**.

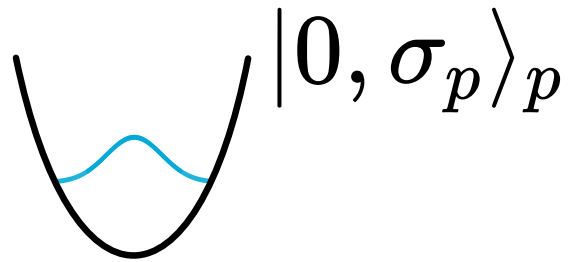


$$= |G\rangle^{\text{CV}}$$

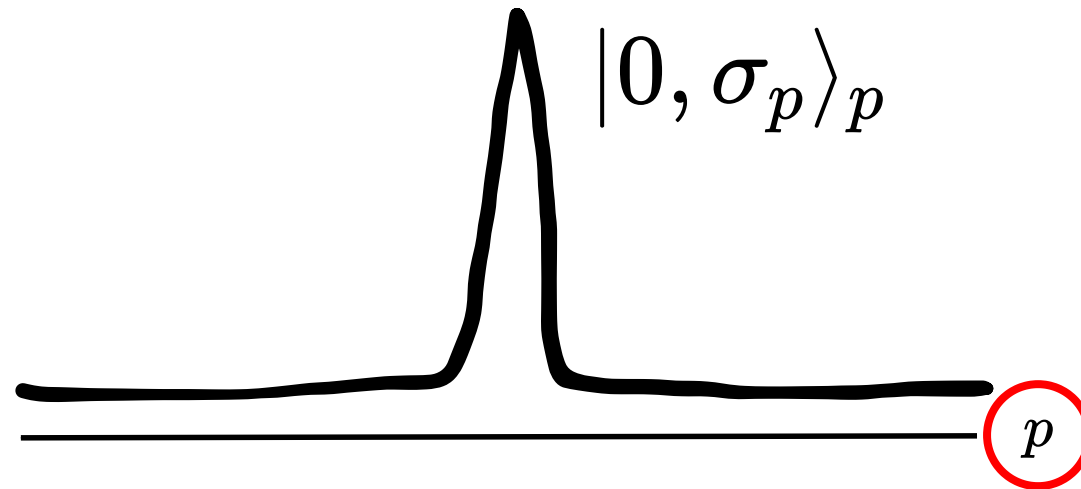
$$= \prod_{i,j \in E} \hat{C}_{Z_{ij}}^{\text{CV}} |0, \sigma_p\rangle_p^{\otimes N}$$

Finite vs Ideal CV cluster state

σ_p represents the variance of the squeezed state.

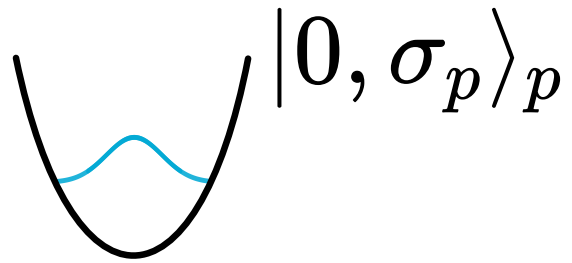


When $\sigma_p \rightarrow 0$, the CV cluster state is an **ideal CV cluster state**.

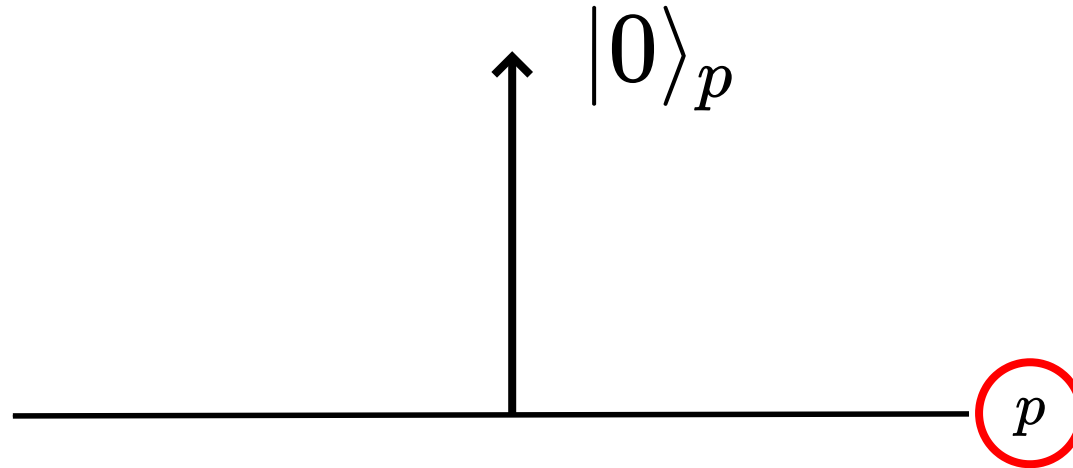


Finite vs Ideal CV cluster state

σ_p represents the variance of the squeezed state.



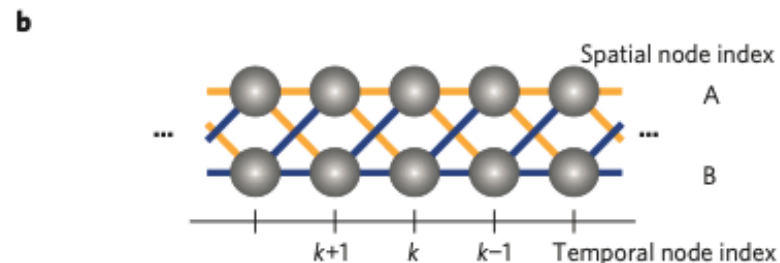
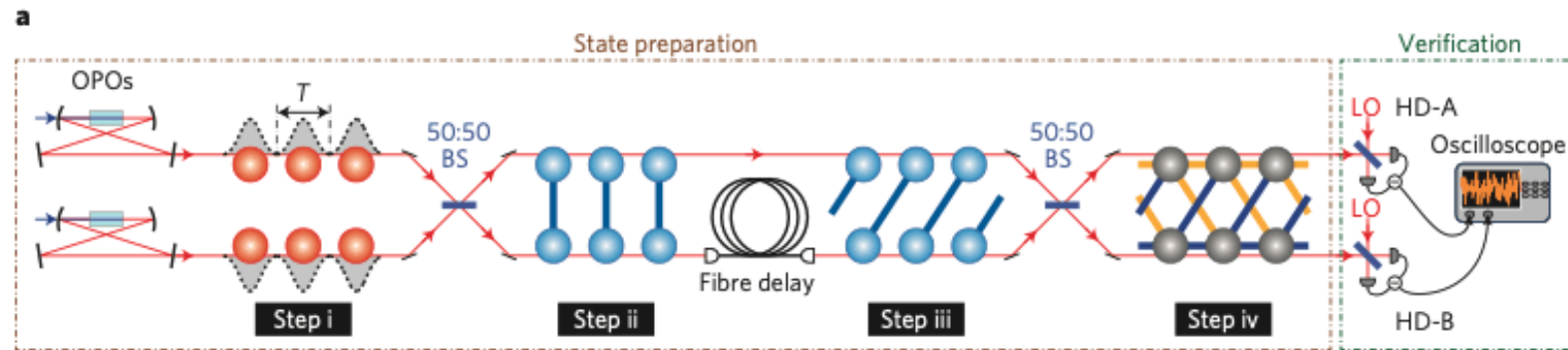
When $\sigma_p \rightarrow 0$, the CV cluster state is an **ideal CV cluster state**.



Ultra-large-scale continuous-variable cluster states multiplexed in the time domain

10000 modes! 1D,
(Furusawa 2013)

Shota Yokoyama¹, Ryuji Ukai¹, Seiji C. Armstrong^{1,2}, Chanond Sornphiphatphong¹, Toshiyuki Kaji¹, Shigenari Suzuki¹, Jun-ichi Yoshikawa¹, Hidehiro Yonezawa¹, Nicolas C. Menicucci³ and Akira Furusawa^{1*}



How to make CV cluster state

RESEARCH ARTICLE | SEPTEMBER 27 2016

Invited Article: Generation of one-million-mode continuous-variable cluster state by unlimited time-domain multiplexing F



Jun-ichi Yoshikawa ; Shota Yokoyama; Toshiyuki Kaji; Chanond Sornphiphatphong; Yu Shiozawa; Kenzo Makino; Akira Furusawa



[+ Author & Article Information](#)

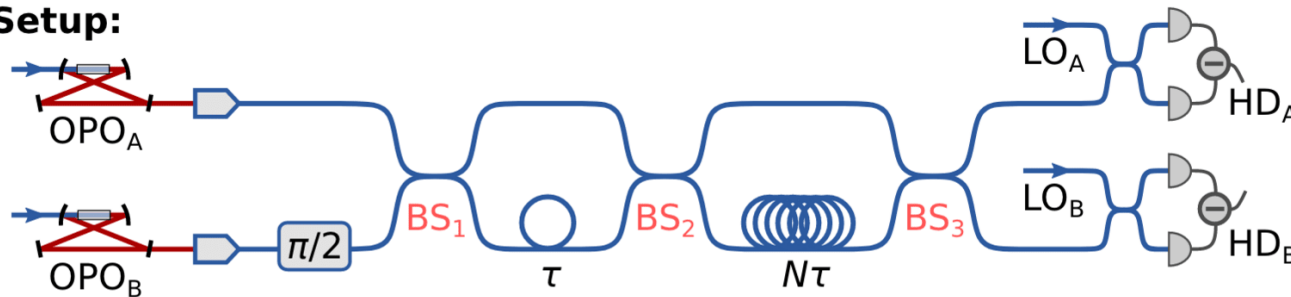
APL Photonics 1, 060801 (2016)

<https://doi.org/10.1063/1.4962732> [Article history](#)

CHORUS

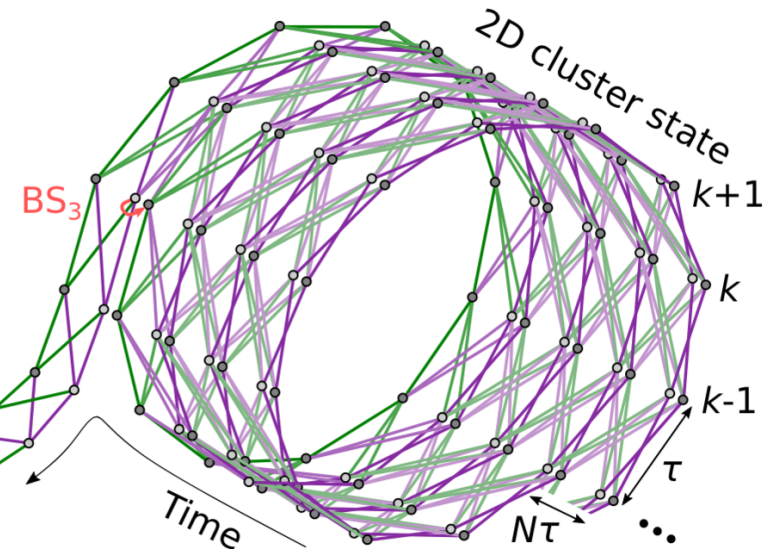
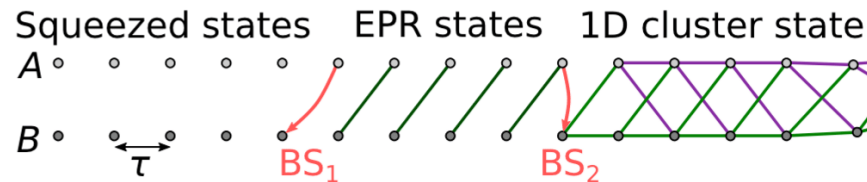
1 million modes, 1D, 2016

Setup:



Resulting graph:

- 1 —
- 1/2 — 1/2 —
- 1/4 — 1/4 —



How to make CV cluster state

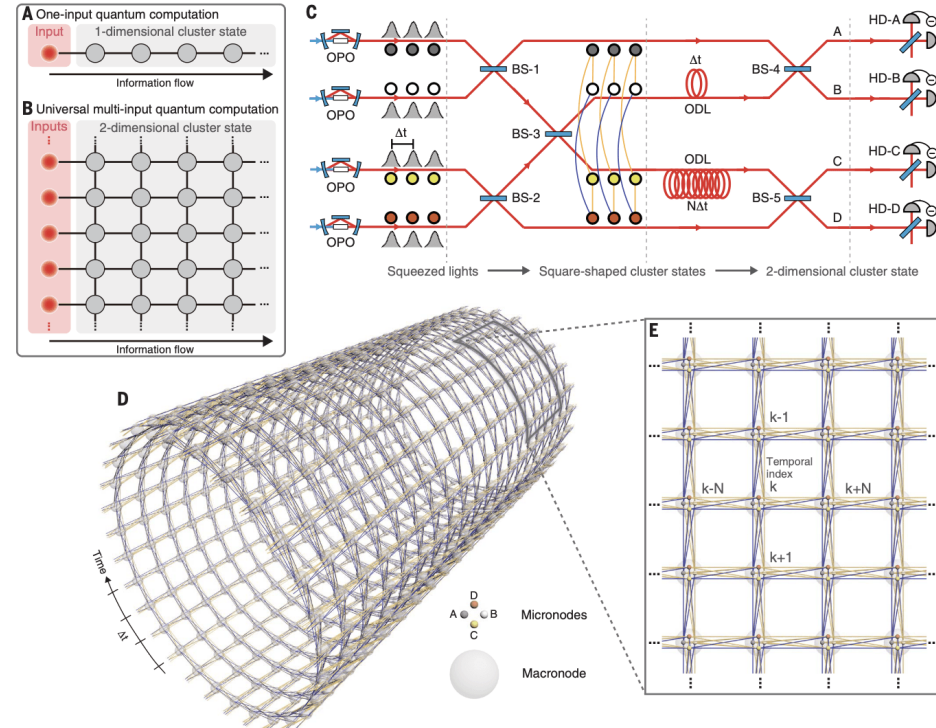
QUANTUM COMPUTING

Generation of time-domain-multiplexed two-dimensional cluster state

Warit Asavanant¹, Yu Shiozawa¹, Shota Yokoyama², Baramée Charoensombutamon¹, Hiroki Emura¹, Rafael N. Alexander³, Shuntaro Takeda^{1,4}, Jun-ichi Yoshikawa¹, Nicolas C. Menicucci⁵, Hidehiro Yonezawa², Akira Furusawa^{1*}

Entanglement is the key resource for measurement-based quantum computing. It is stored in quantum states known as cluster states, which are prepared offline and enable quantum computing by means of purely local measurements. Universal quantum computing requires cluster states that are both large and possess (at least) a two-dimensional topology. Continuous-variable cluster states—based on bosonic modes rather than qubits—have previously been generated on a scale exceeding one million modes, but only in one dimension. Here, we report generation of a large-scale two-dimensional continuous-variable cluster state. Its structure consists of a 5- by 1240-site square lattice that was tailored to our highly scalable time-multiplexed experimental platform. It is compatible with Bosonic error-correcting codes that, with higher squeezing, enable fault-tolerant quantum computation.

5x1240 modes, 2D, (Furusawa 2019)



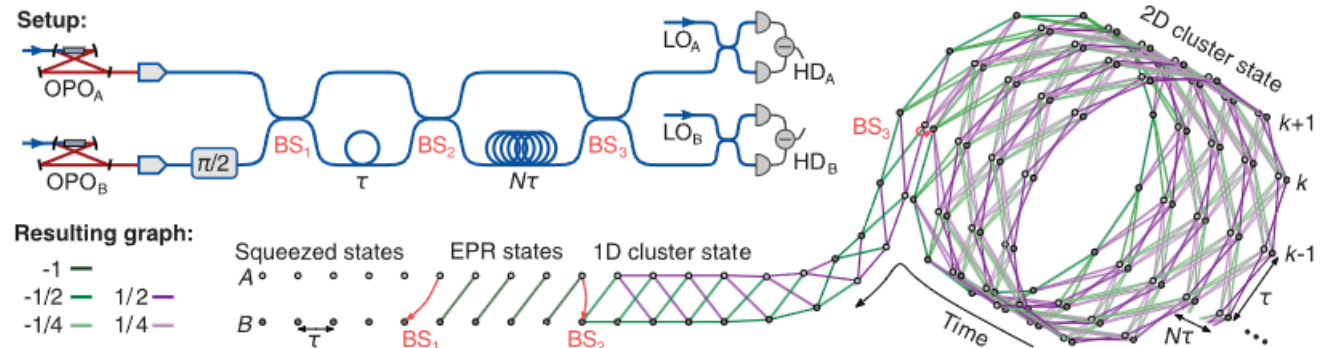
24x1250, 2D (Andersen 2019)

QUANTUM COMPUTING

Deterministic generation of a two-dimensional cluster state

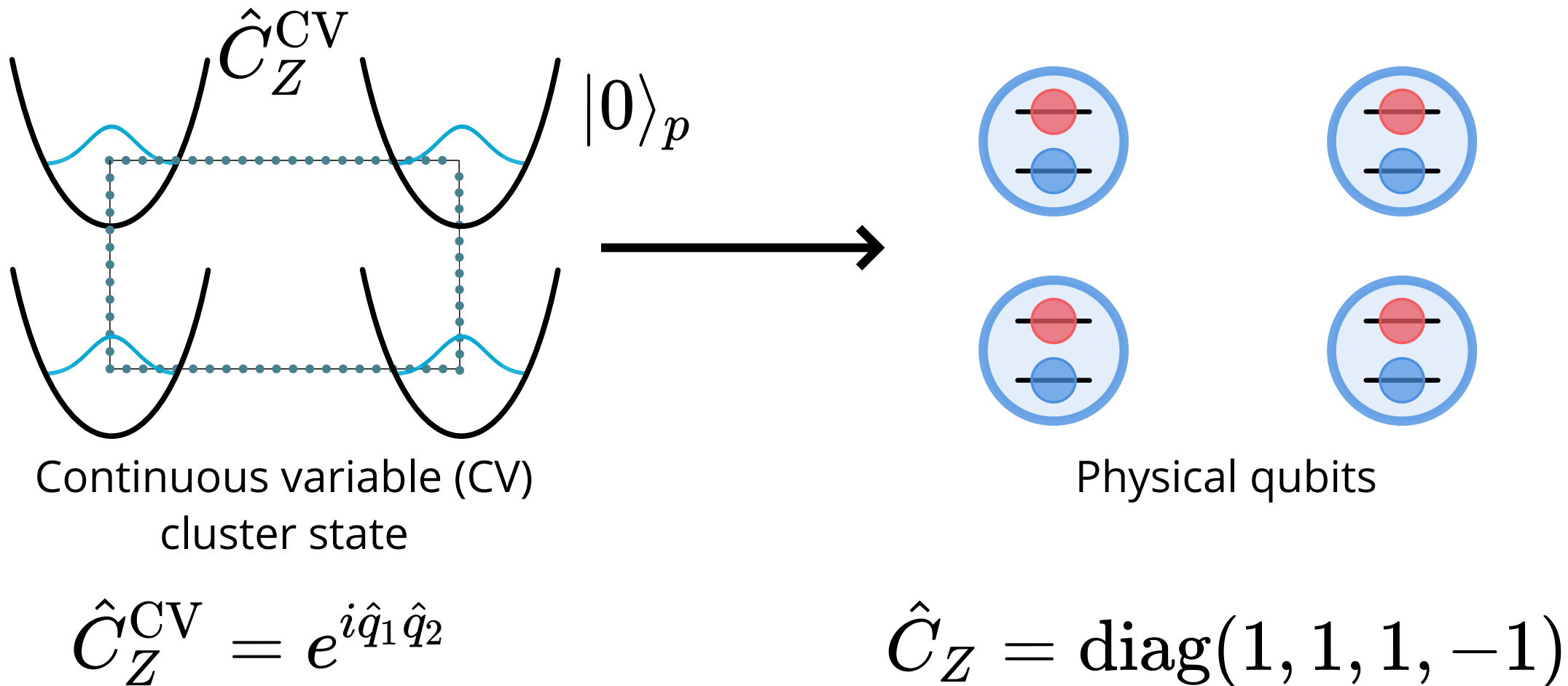
Mikkel V. Larsen*, Xueshi Guo, Casper R. Breum, Jonas S. Neergaard-Nielsen, Ulrik L. Andersen*

Measurement-based quantum computation offers exponential computational speed-up through simple measurements on a large entangled cluster state. We propose and demonstrate a scalable scheme for the generation of photonic cluster states suitable for universal measurement-based quantum computation. We exploit temporal multiplexing of squeezed light modes, delay loops, and beam-splitter transformations to deterministically generate a cylindrical cluster state with a two-dimensional (2D) topological structure as required for universal quantum information processing. The generated state consists of more than 30,000 entangled modes arranged in a cylindrical lattice with 24 modes on the circumference, defining the input register, and a length of 1250 modes, defining the computation depth. Our demonstrated source of two-dimensional cluster states can be combined with quantum error correction to enable fault-tolerant quantum computation.



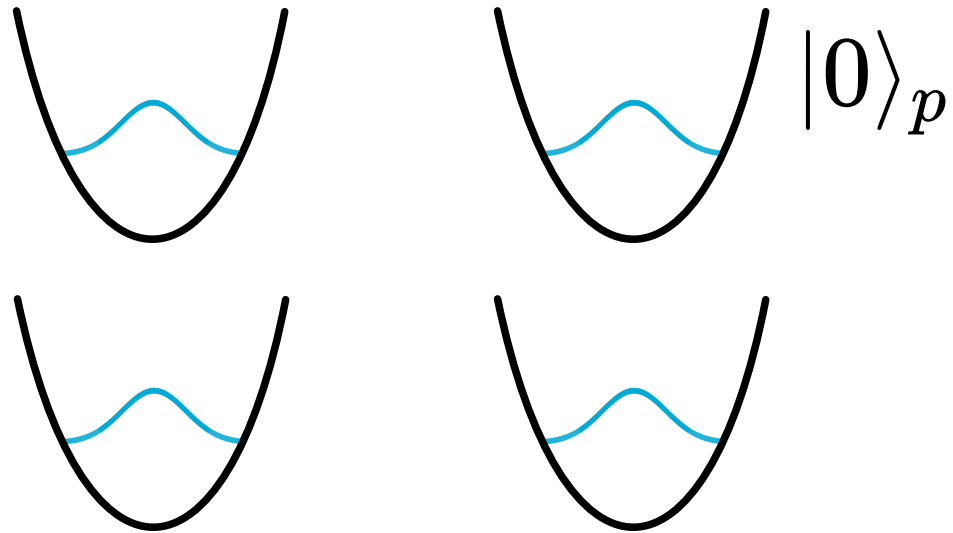
Entanglement transfer protocol

How to perform entanglement transfer?



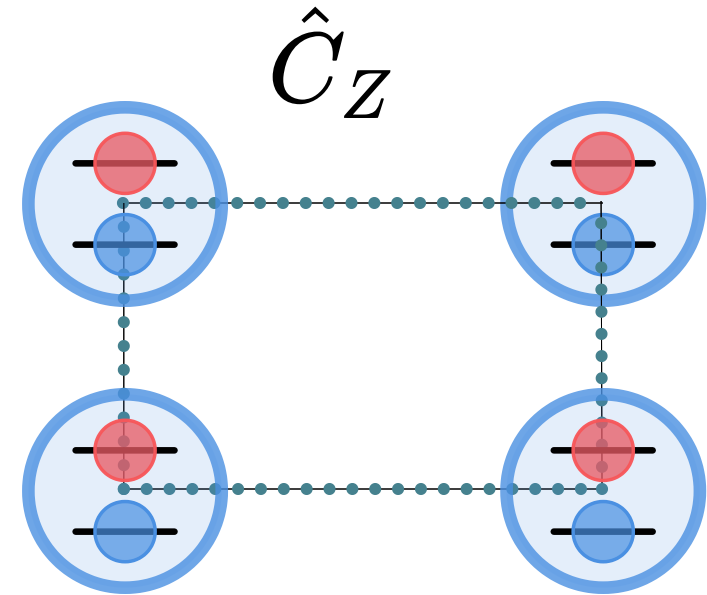
Entanglement transfer protocol

How to perform entanglement transfer?



Continuous variable (CV)
cluster state

$$\hat{C}_Z^{\text{CV}} = e^{i\hat{q}_1\hat{q}_2}$$



Qubit cluster state

$$\hat{C}_Z = \text{diag}(1, 1, 1, -1)$$

Entanglement transfer protocol

How to perform entanglement transfer?

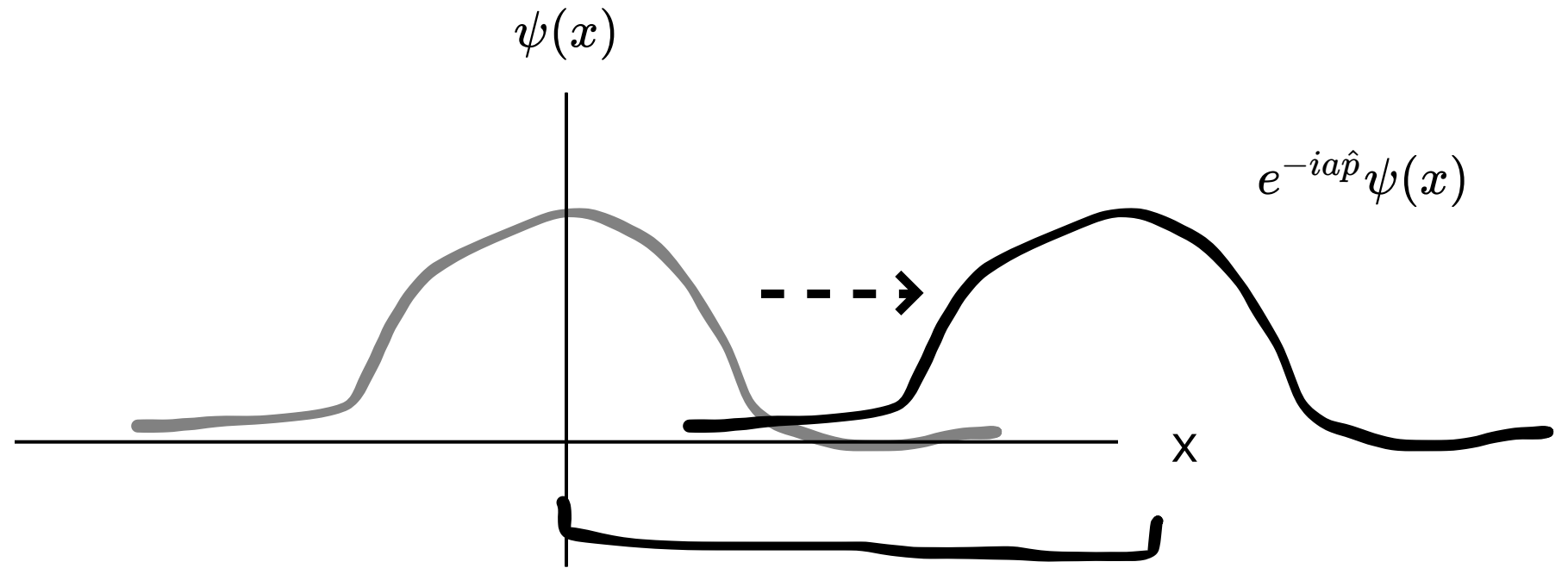
We need:

- A CV cluster state*
- \hat{q} quadrature homodyne detection
- Conditional displacement gate \hat{C}_D

$$\hat{C}_D = |\mathbf{0}\rangle\langle\mathbf{0}| \hat{I} + |\mathbf{1}\rangle\langle\mathbf{1}| \hat{D}_q(\sqrt{\pi})$$

ETP: Displacement Gate

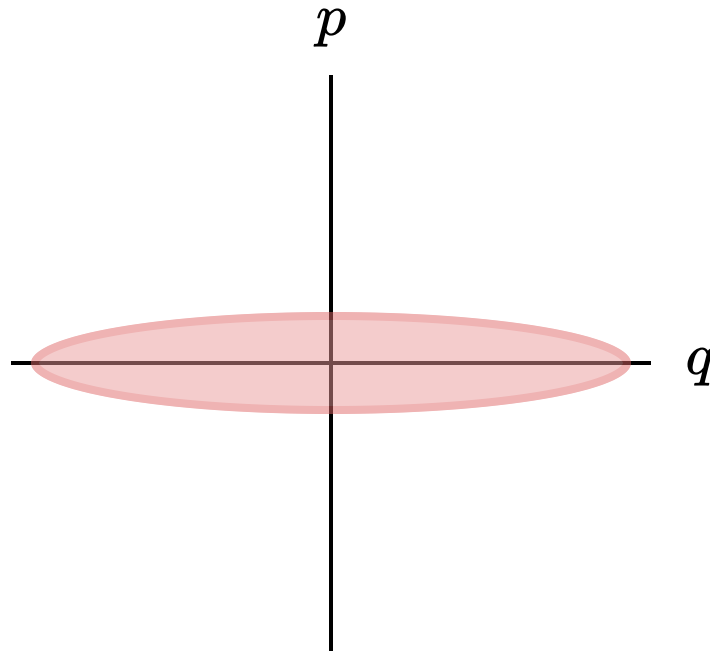
Displacement gate of strength a shifts the state.



$$\hat{D}_q(a)|x\rangle := |x + a\rangle$$

ETP: Displacement Gate

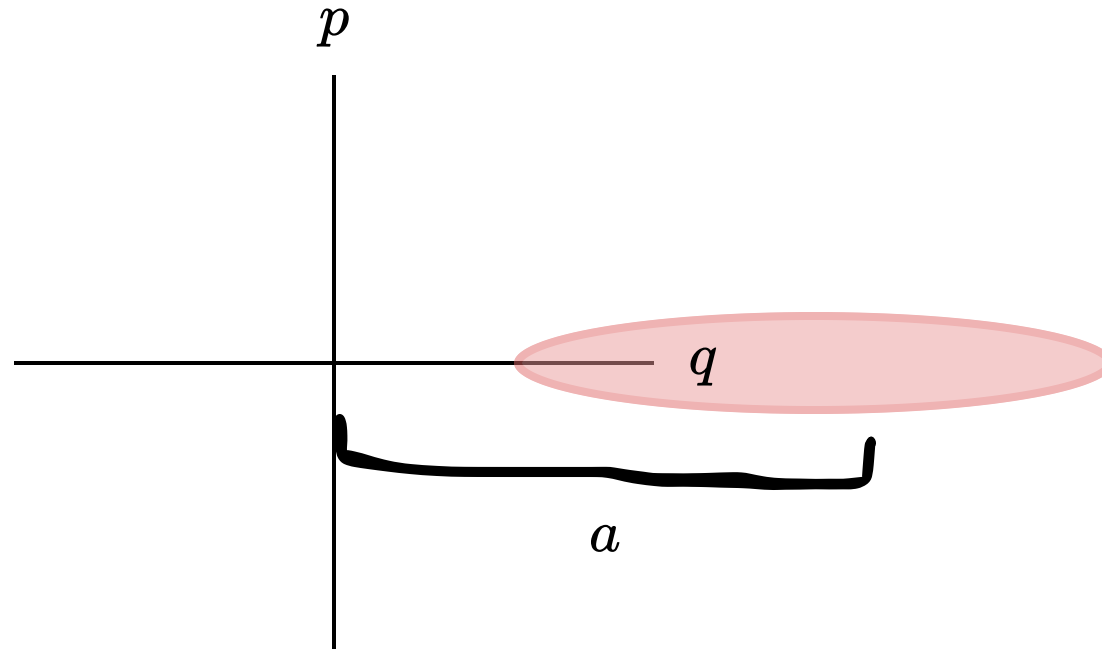
Displacement gate of strength a shifts the state.



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ETP: Displacement Gate

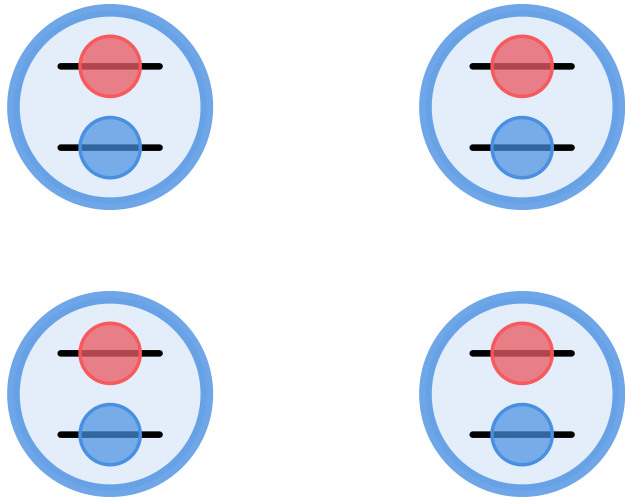
Displacement gate of strength a shifts the state.



$$\hat{D}_q(a)|x\rangle := |x + a\rangle$$

ETP: Overview

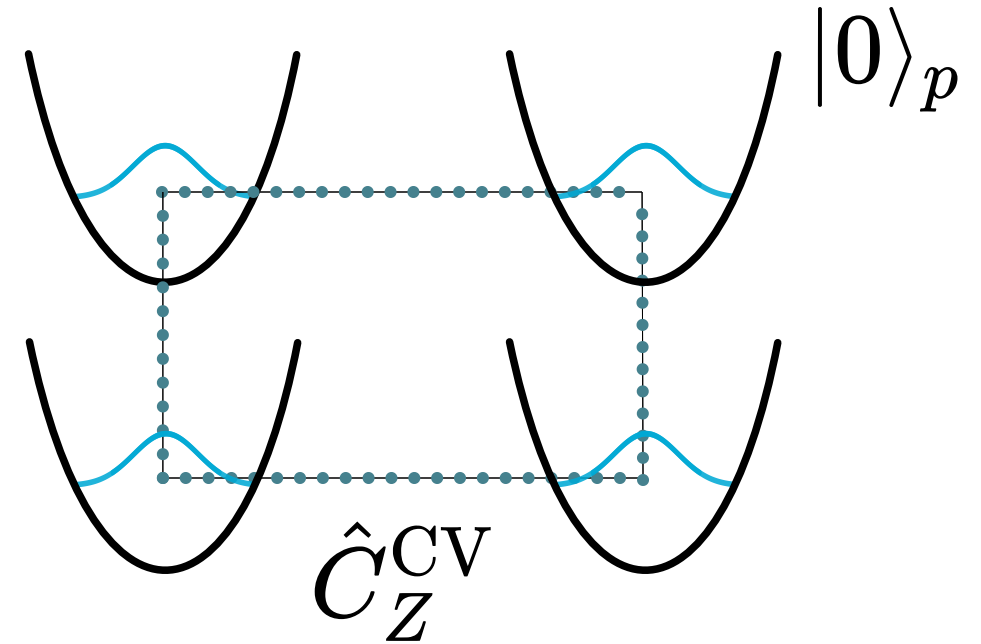
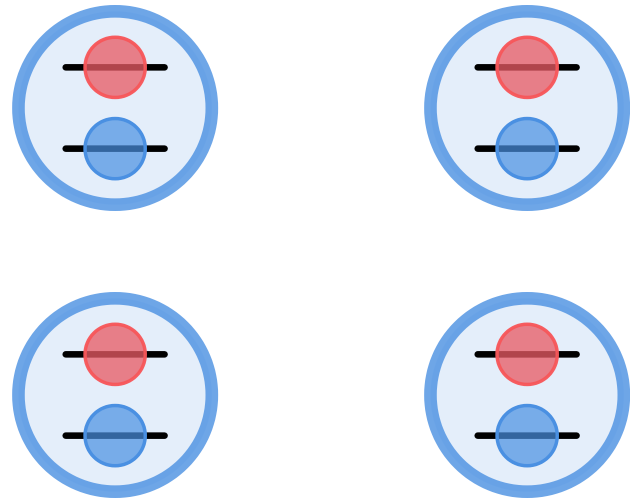
$|+\rangle$



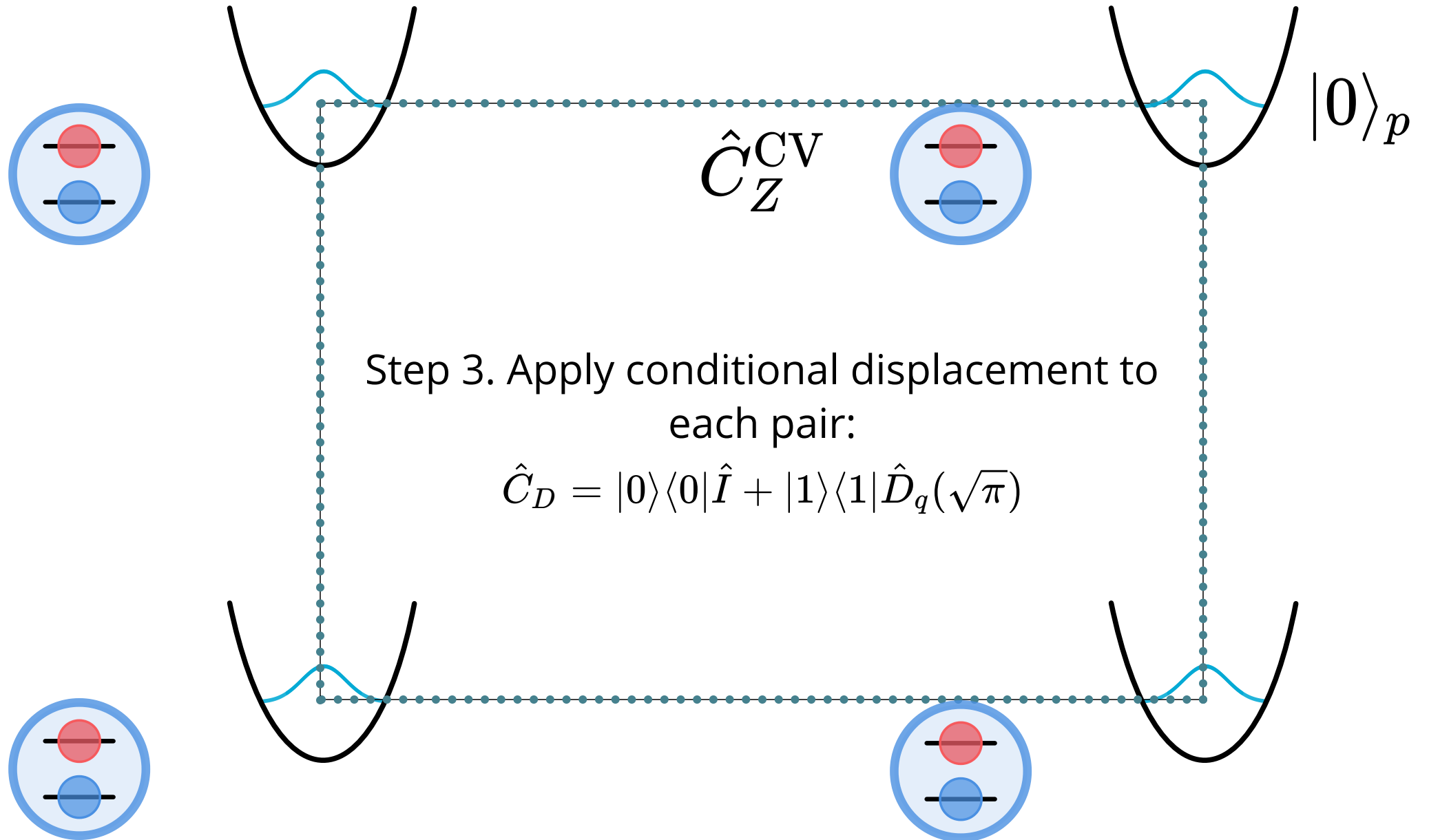
1. Initialize all qubits to $|+\rangle$.

ETP: Overview

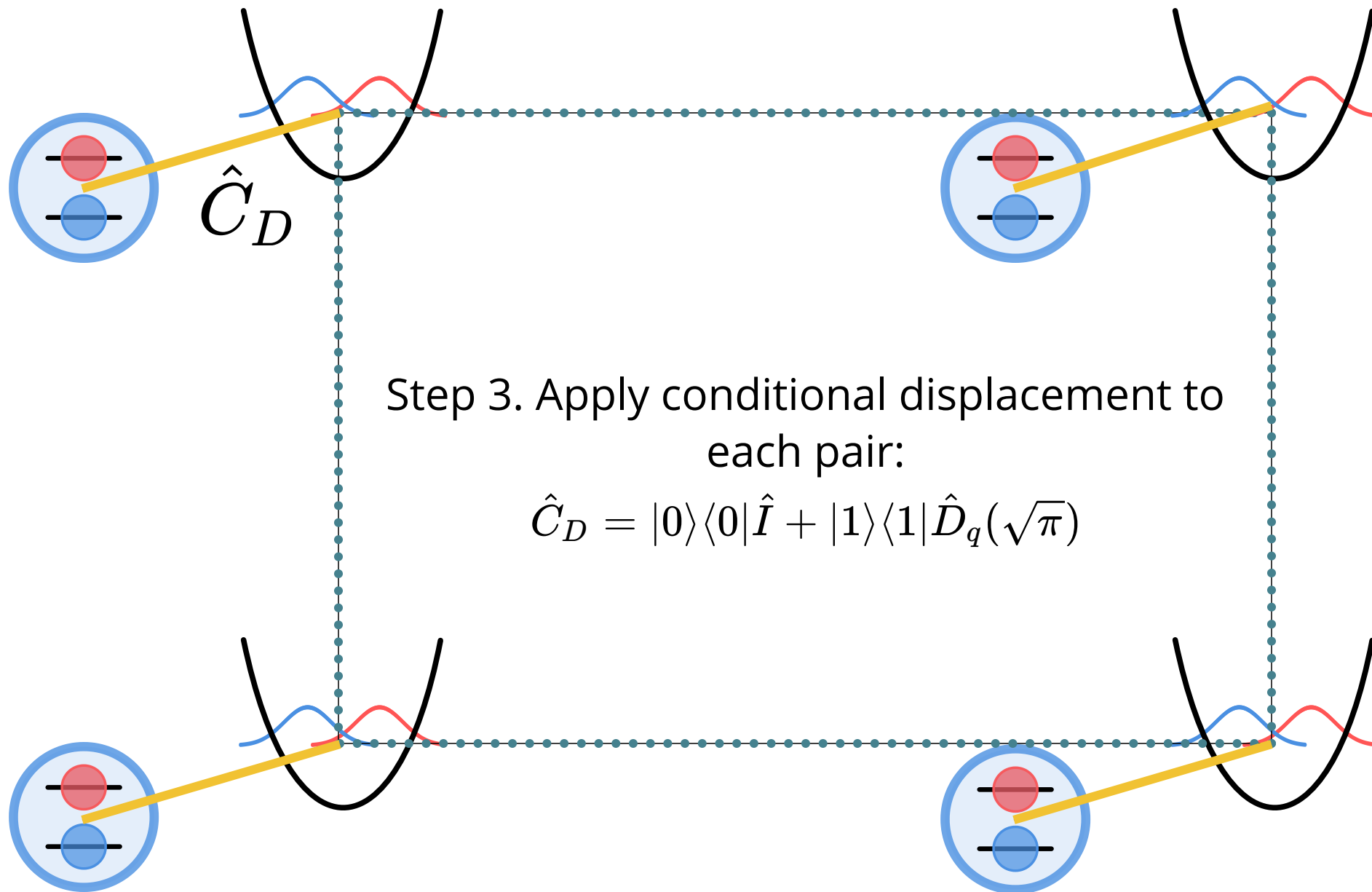
Step 2: Get a CV cluster state.



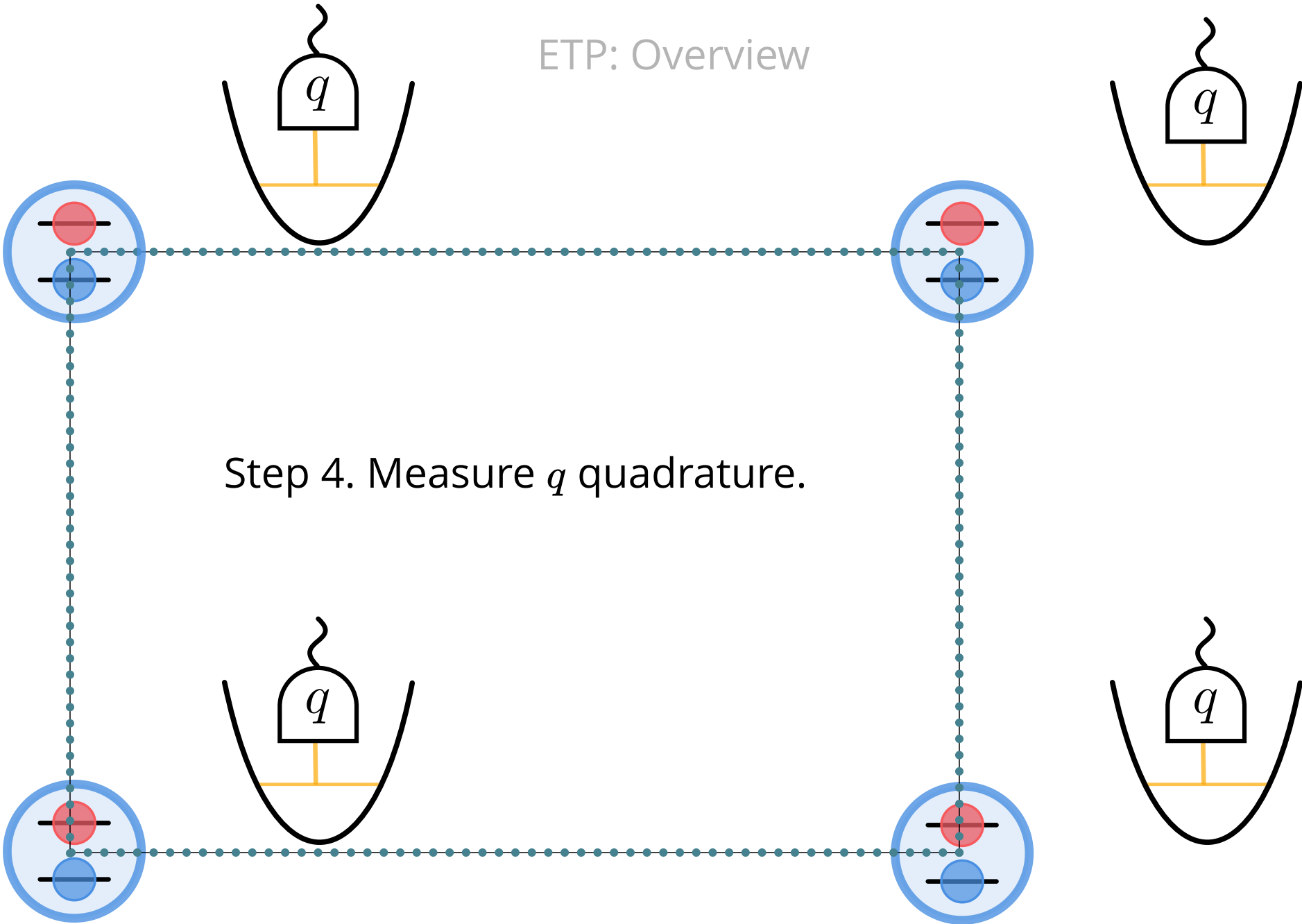
ETP: Overview



ETP: Overview

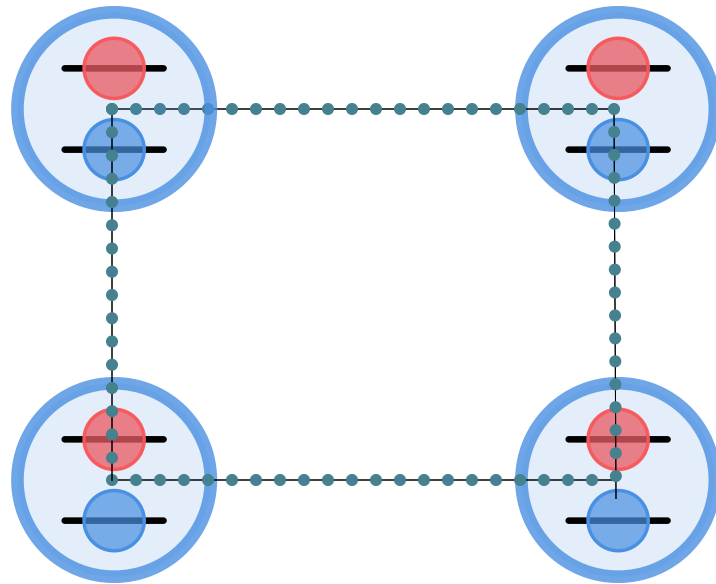


ETP: Overview



ETP: Overview

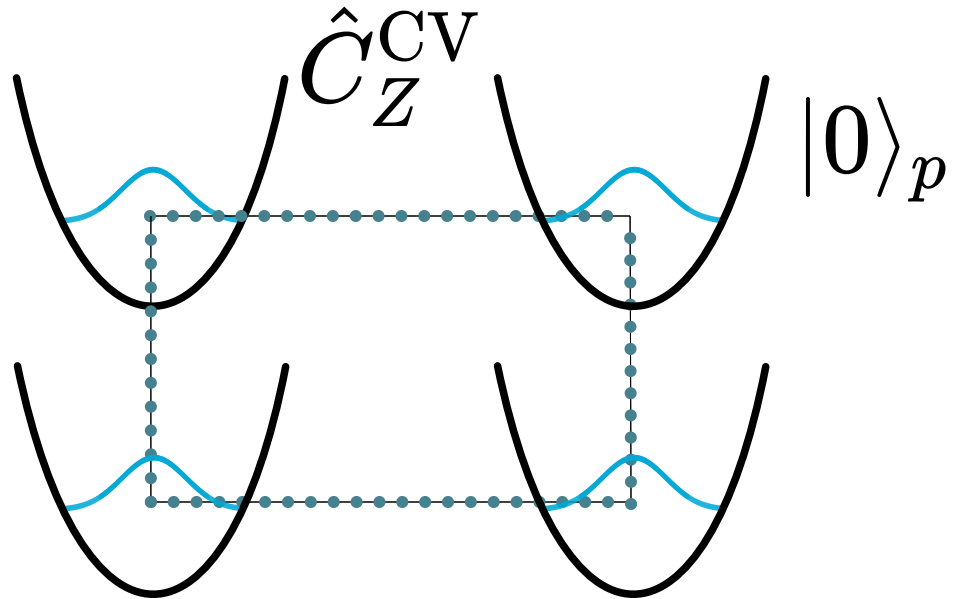
You now have a qubit cluster state!



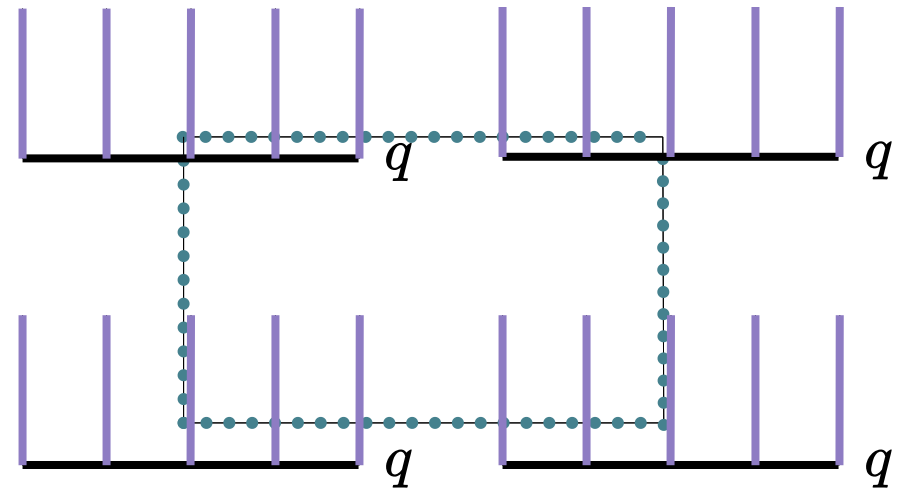
But why does it work?

Qubit cluster inside CV cluster

We show there is a hidden qubit cluster state inside a CV cluster state!

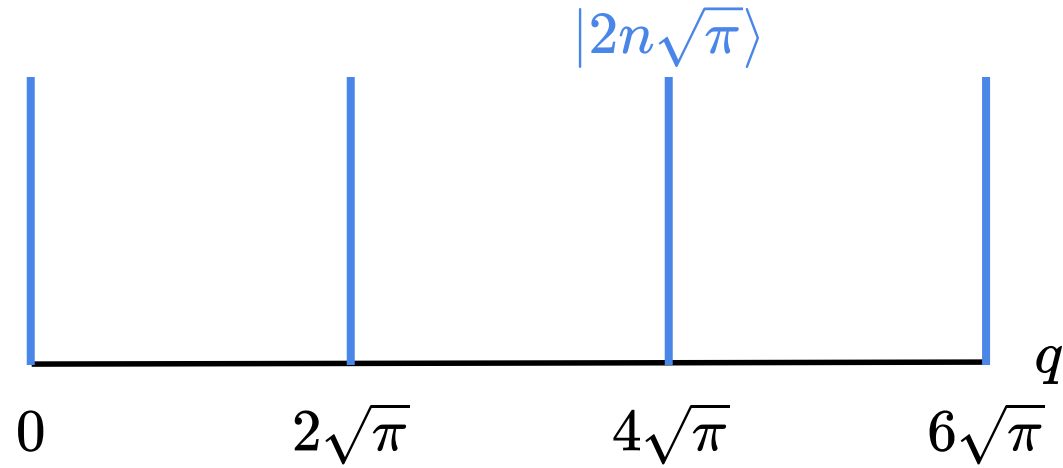


CV cluster state



GKP Background

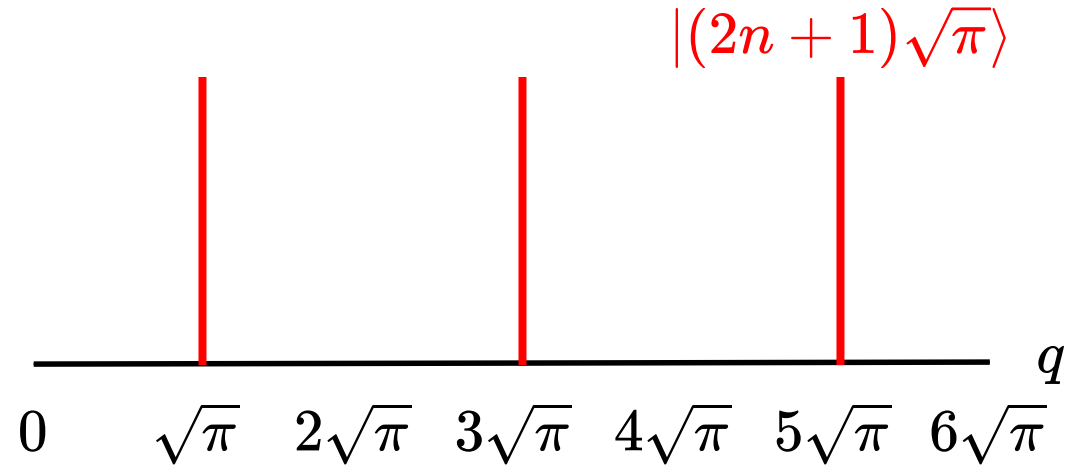
$$|0\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |2n\sqrt{\pi}\rangle_q$$



Gottesman-Kitaev-Preskill (GKP state)

GKP Background

$$|1\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |(2n+1)\sqrt{\pi}\rangle_q$$

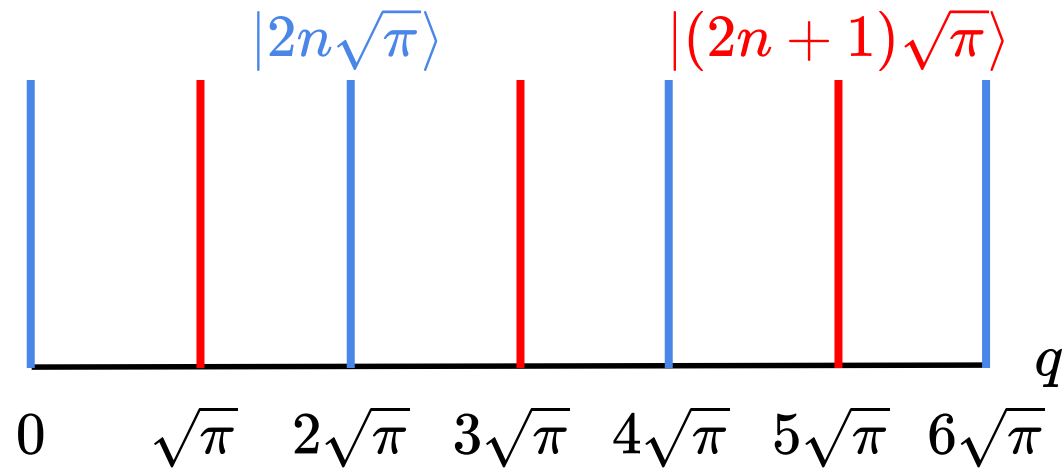


$$\hat{X}^{\text{GKP}} = e^{-i\sqrt{\pi}\hat{p}} = \hat{D}_q(\sqrt{\pi})$$

GKP Background

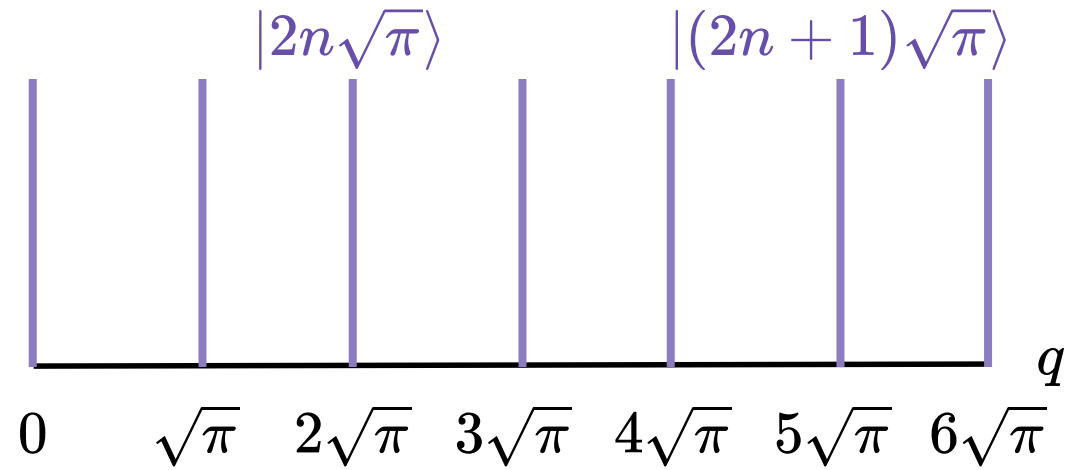
$$|0\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |2n\sqrt{\pi}\rangle_q$$

$$|1\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |(2n+1)\sqrt{\pi}\rangle_q$$

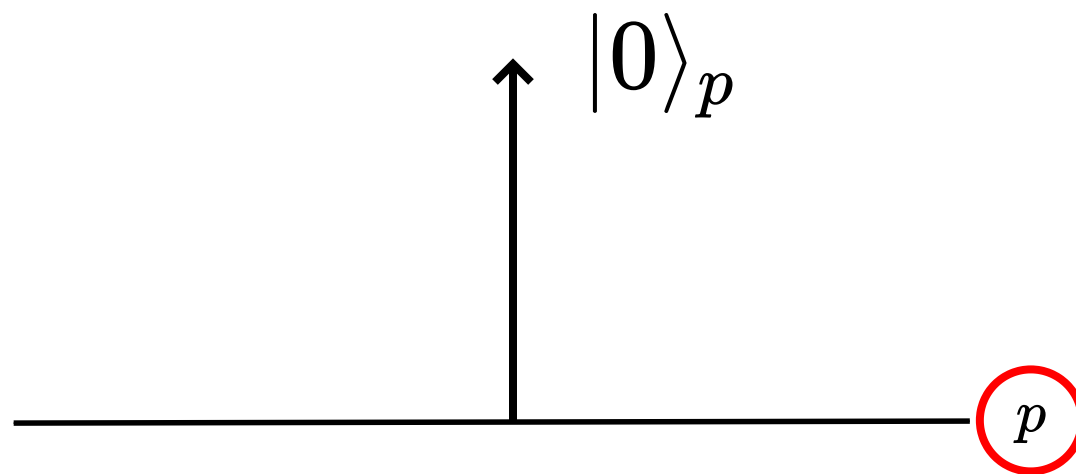


GKP Background

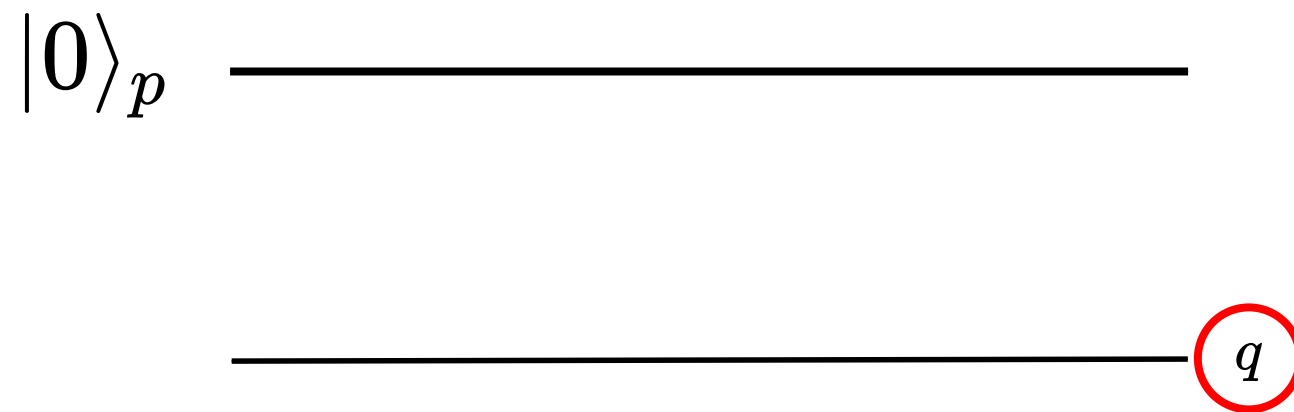
$$|+\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |n\sqrt{\pi}\rangle_q = \frac{1}{\sqrt{2}} (|0\rangle_{\text{GKP}} + |1\rangle_{\text{GKP}})$$



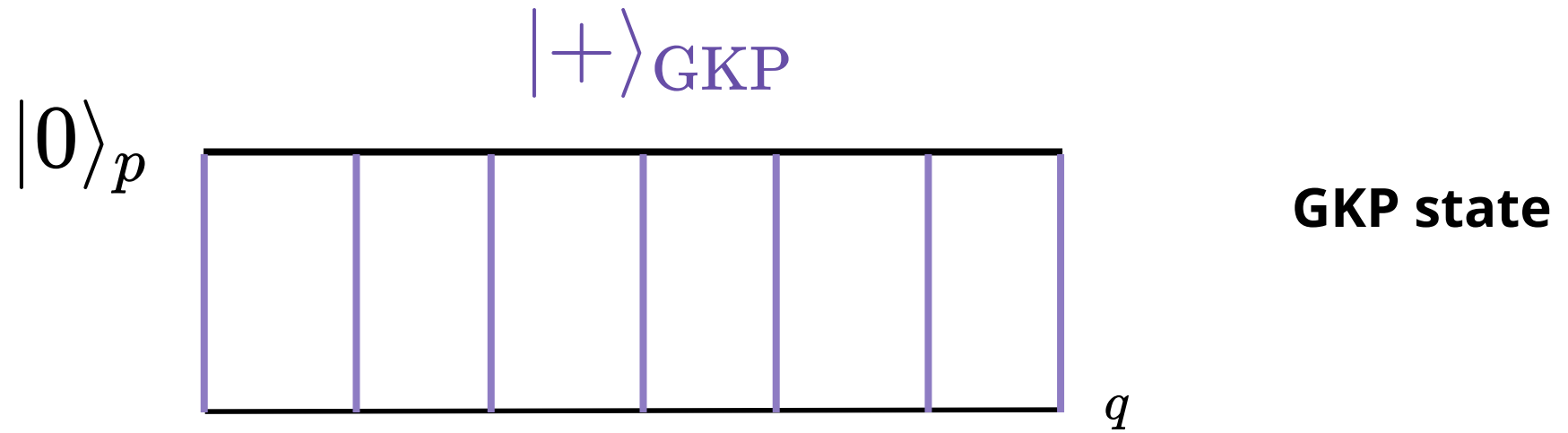
Node of ideal CV cluster



Node of ideal CV cluster



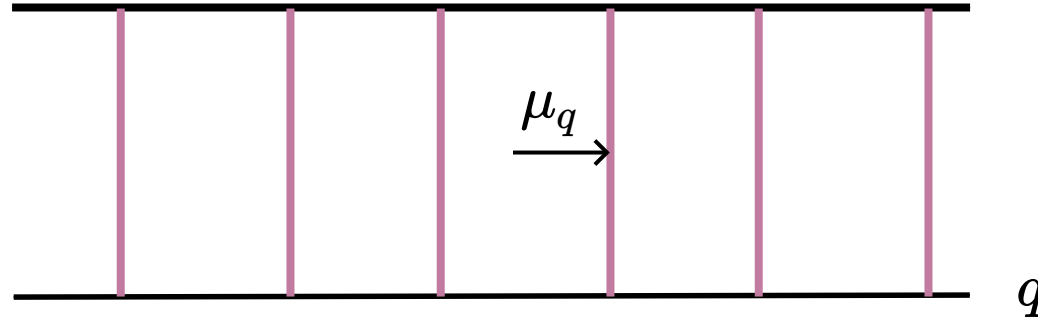
Node of ideal CV cluster



Displaced GKP

$$|+\mu_q, 0\rangle_{\text{GKP}} \equiv \hat{D}_q(\mu_q)|+\rangle_{\text{GKP}}$$

$|0\rangle_p$



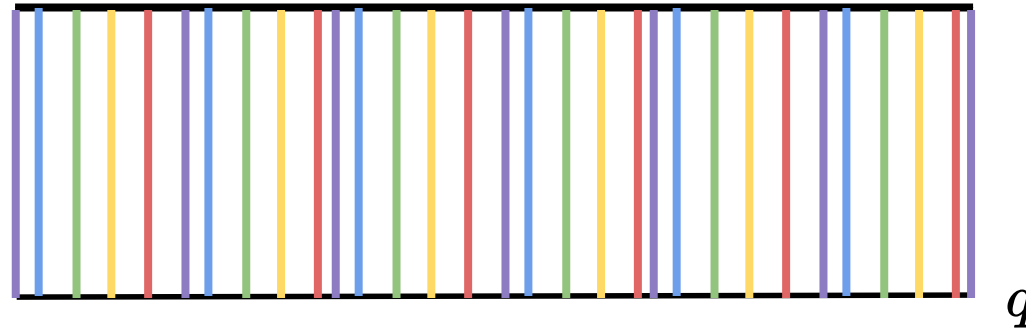
Displaced GKP state
(Glancy 2006)

$$\mu_q \in \left[-\frac{\sqrt{\pi}}{2}, -\frac{\sqrt{\pi}}{2}\right)$$

Node of ideal CV cluster is superposition of displaced GKP

$$|+\mu_q, 0\rangle_{\text{GKP}} \equiv \hat{D}_q(\mu_q)|+\rangle_{\text{GKP}}$$

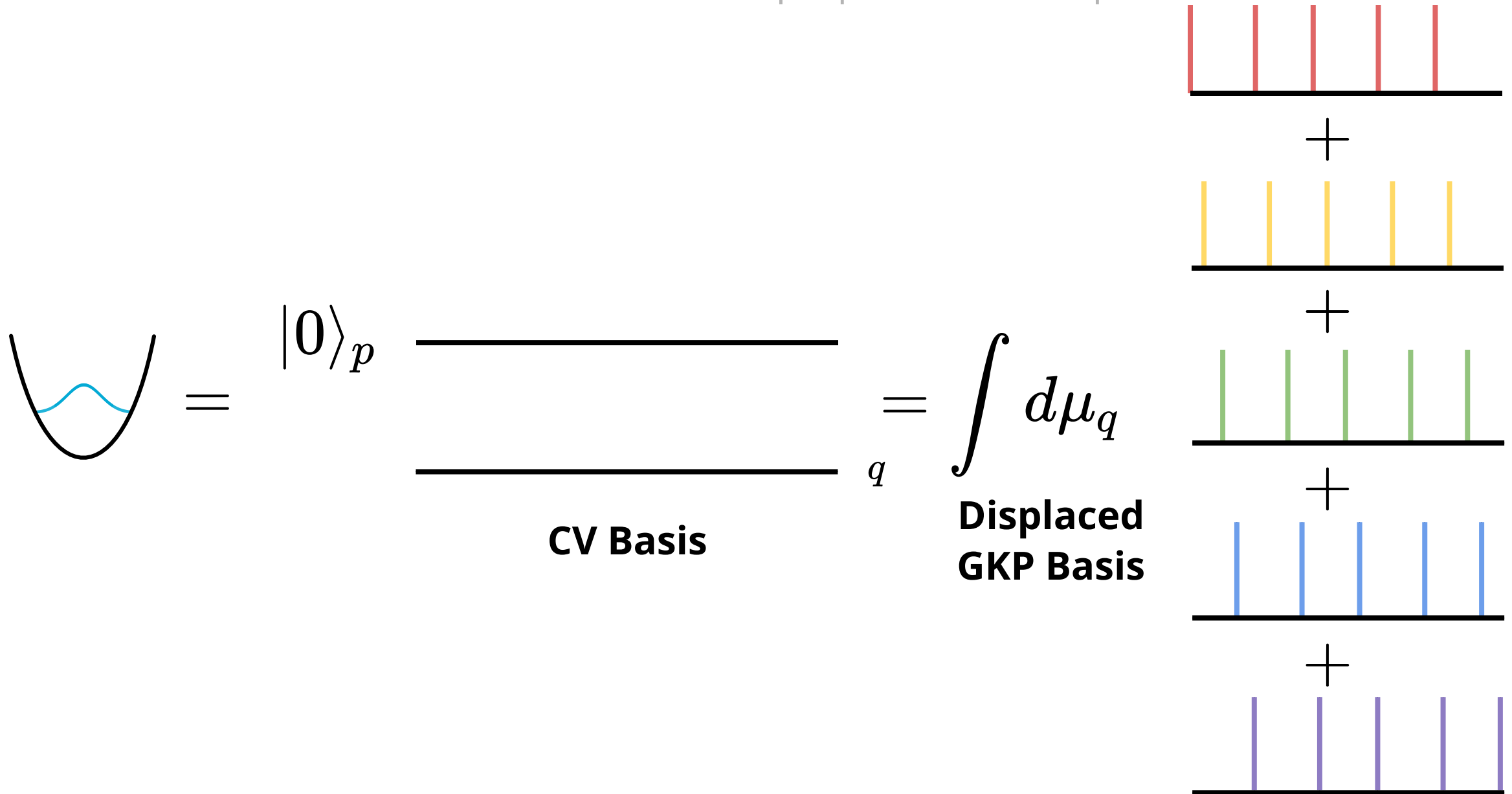
$|0\rangle_p$



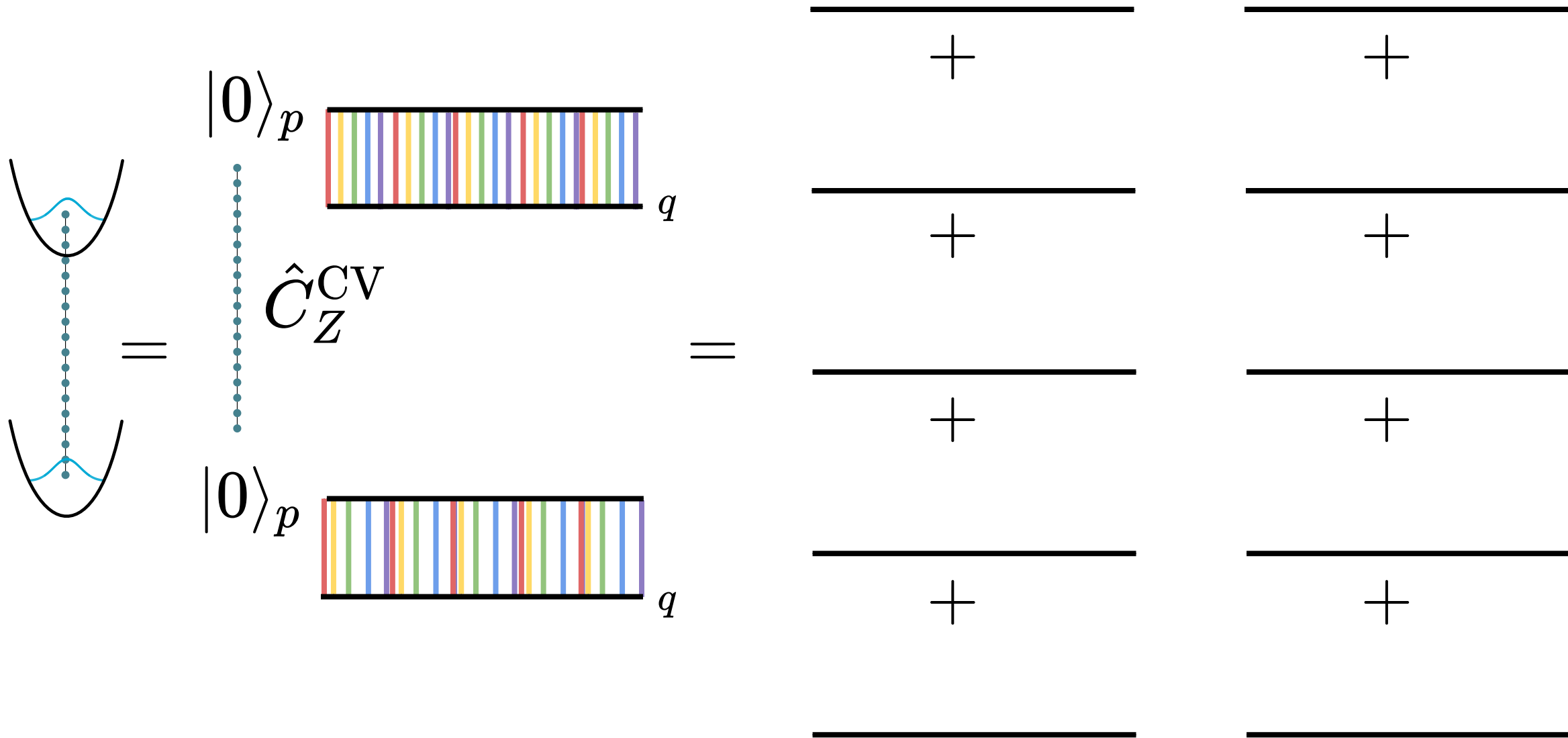
Displaced GKP Basis
(Glancy 2006)

So if we integrate over μ_q , we should form
an ideal $|0\rangle_p$ state.

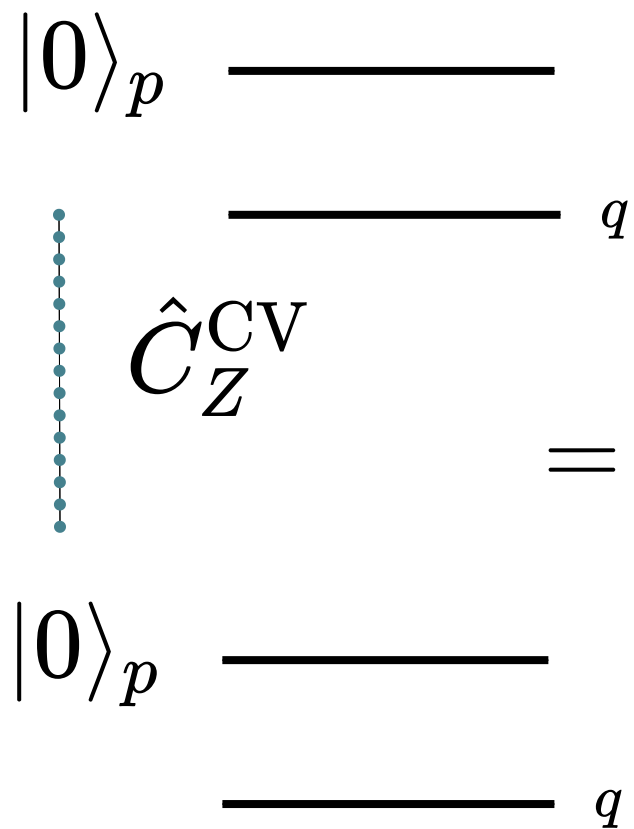
Node of ideal CV cluster is superposition of displaced GKP



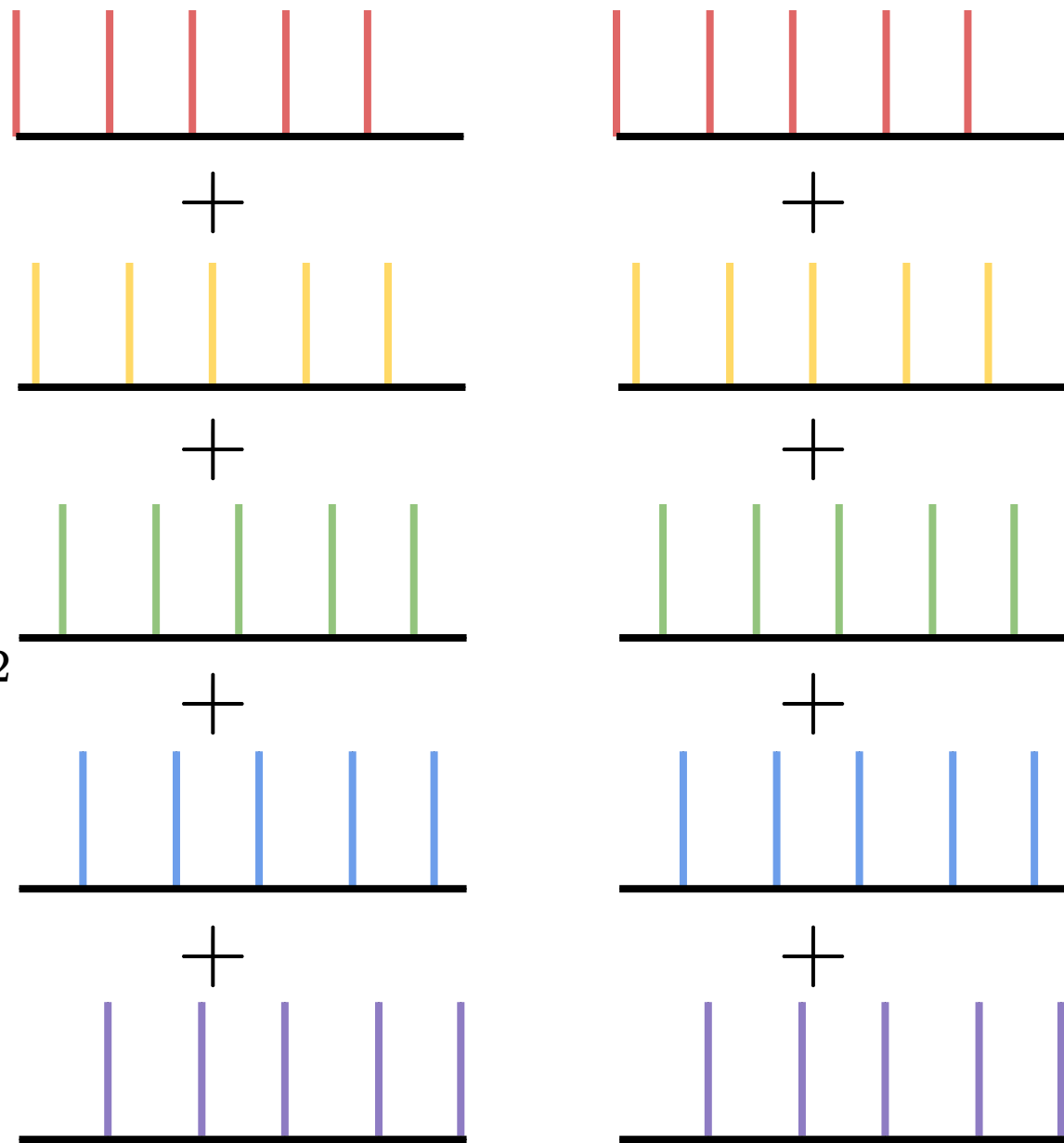
Edges of ideal CV cluster



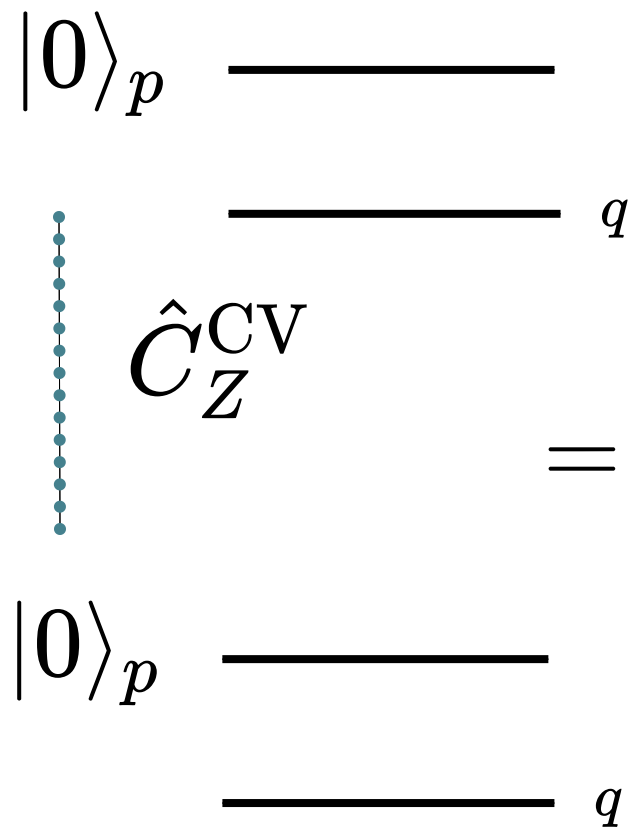
Edges of ideal CV cluster



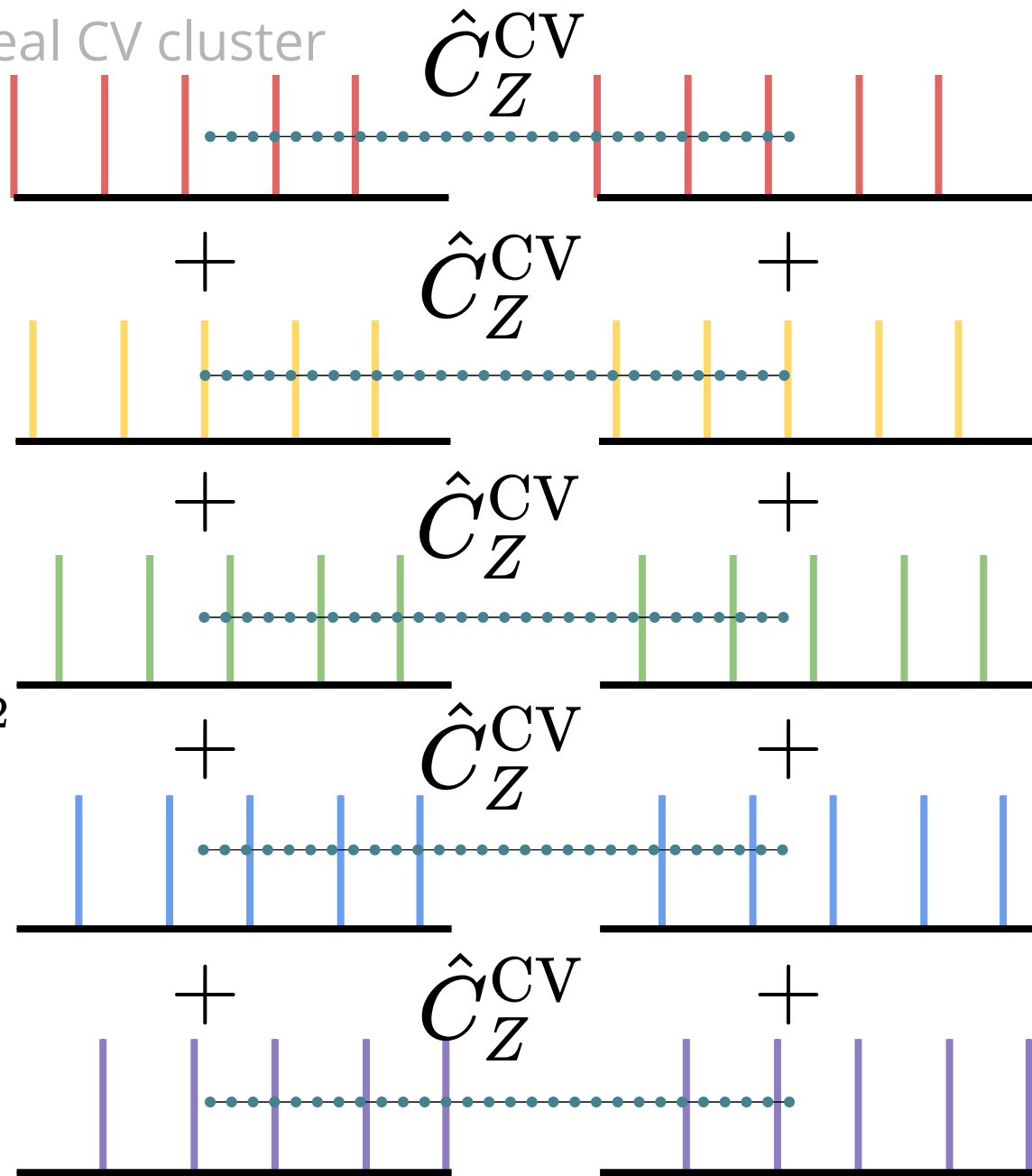
$$= \iint d\mu_{q_1} d\mu_{q_2}$$



Edges of ideal CV cluster

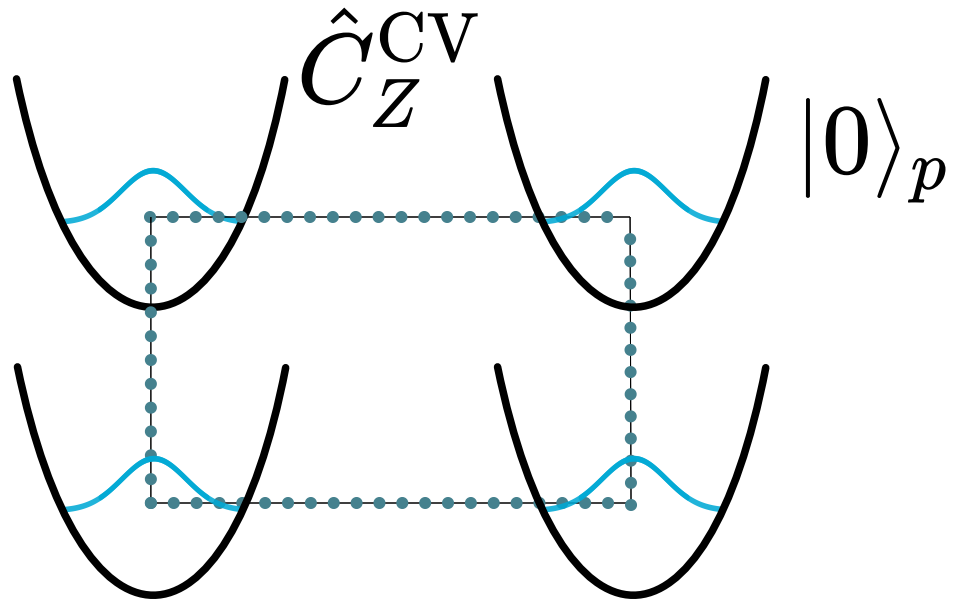


$$= \iint d\mu_{q_1} d\mu_{q_2}$$



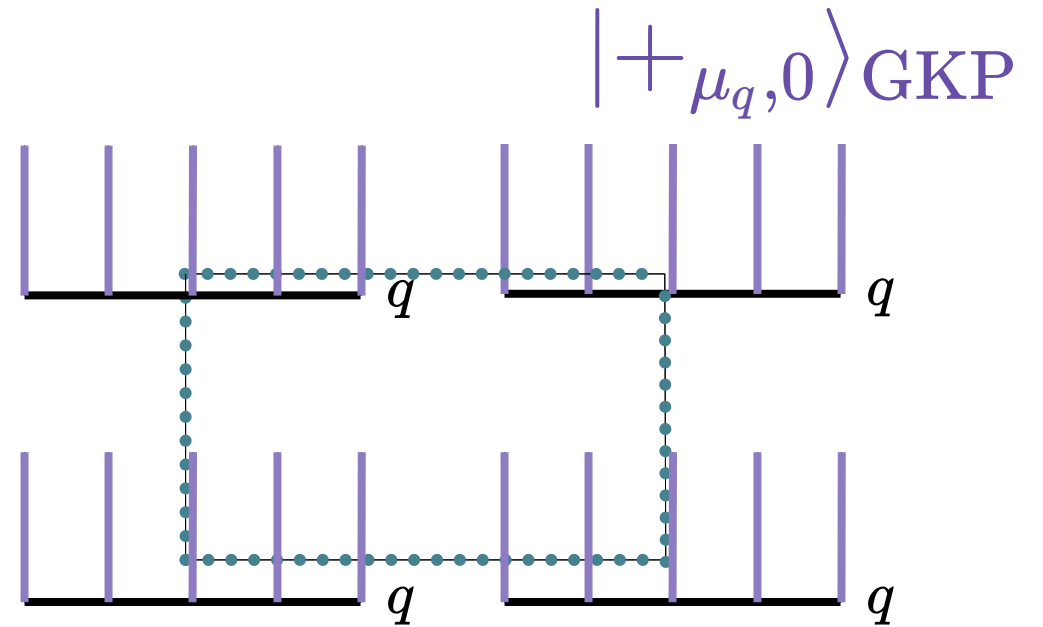
Node of ideal CV cluster is displaced GKP

Nodes of a ideal CV cluster state



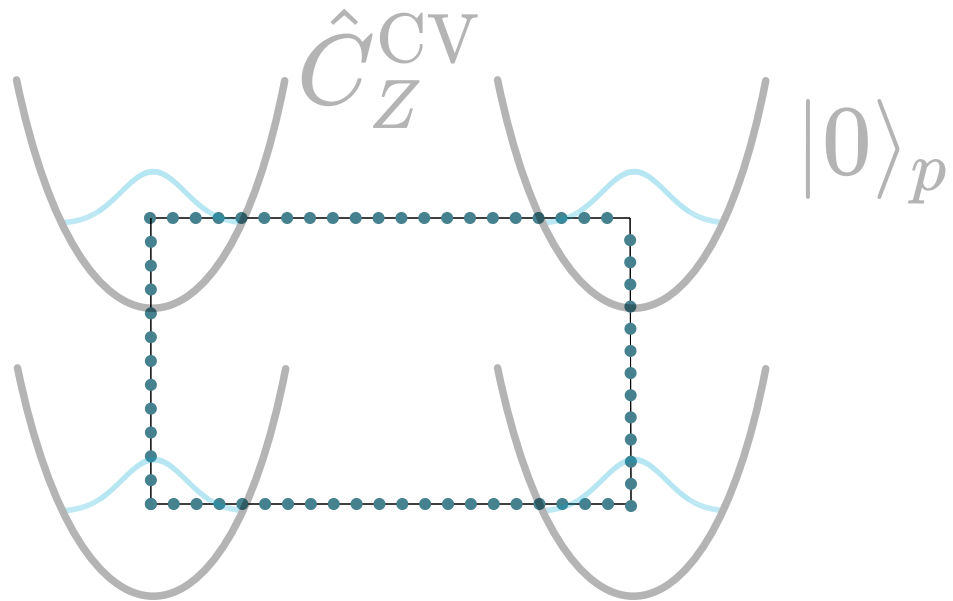
is a superposition of

Displaced GKP states

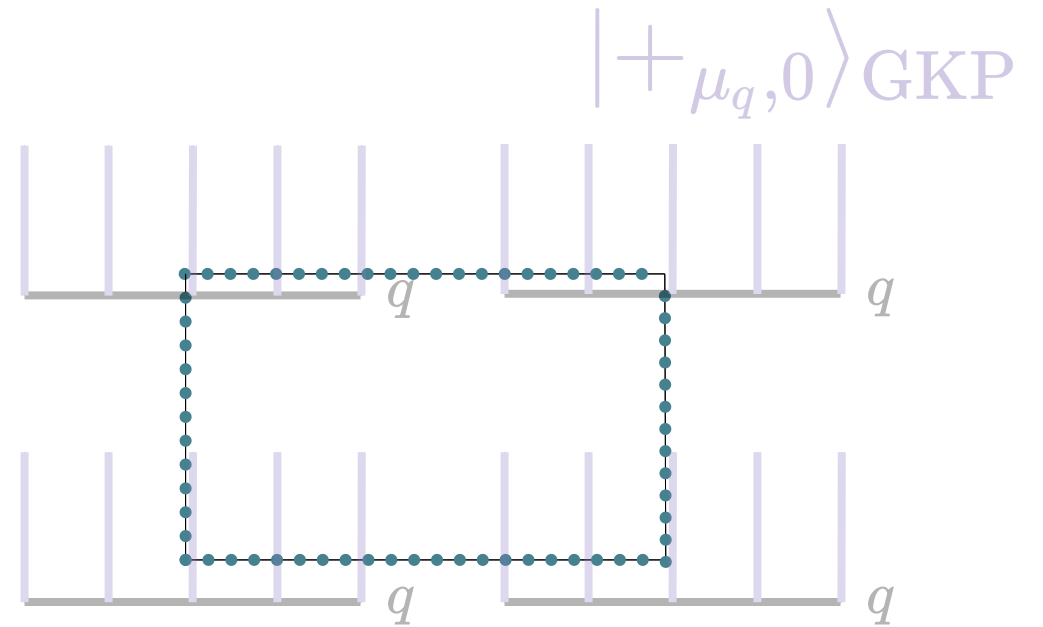


Edge of ideal CV cluster is GKP CZ

The edges of the CV cluster state?



GKP CZ gate



$$\hat{C}_Z^{\text{CV}} = e^{i\hat{q}_1\hat{q}_2} \quad |+\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |n\sqrt{\pi}\rangle_q$$

$$\hat{C}_Z^{\text{CV}} |++\rangle_{\text{GKP}} = ?$$

Substitute definition

$$\hat{C}_Z^{\text{CV}} = e^{i\hat{q}_1\hat{q}_2} \quad |+\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |n\sqrt{\pi}\rangle_q$$

$$e^{i\hat{q}_1\hat{q}_2} \sum_{n_1, n_2} |n_1\sqrt{\pi}\rangle_q |n_2\sqrt{\pi}\rangle_q = ?$$

Apply \hat{q}

$$\hat{C}_Z^{\text{CV}} = e^{i\hat{q}_1\hat{q}_2} \quad |+\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |n\sqrt{\pi}\rangle_q$$

$$e^{i\pi n_1 n_2} \sum_{n_1, n_2} |n_1\sqrt{\pi}\rangle_q |n_2\sqrt{\pi}\rangle_q = ?$$

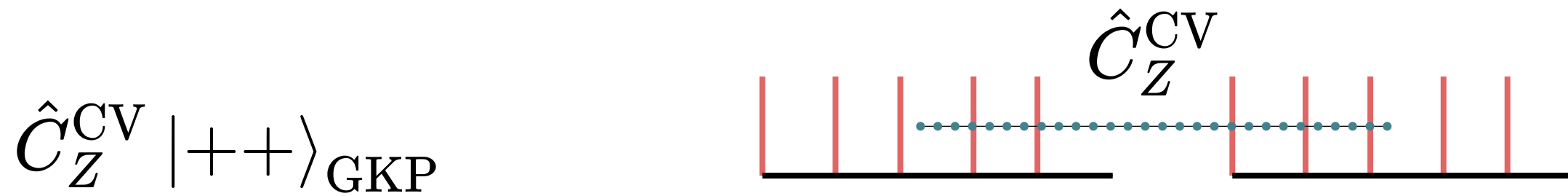
Expand into even and odd sums

$$e^{i\pi n_1 n_2} \sum_{n_1, n_2} |n_1 \sqrt{\pi}\rangle_q |n_2 \sqrt{\pi}\rangle_q = ?$$

n_1 or n_2 even $\implies n_1 n_2$ is even

$$\sum_{n_1 \text{ or } n_2 \text{ even}} |n_1 \sqrt{\pi}\rangle_q |n_2 \sqrt{\pi}\rangle_q = |00\rangle_{\text{GKP}} + |01\rangle_{\text{GKP}} + |10\rangle_{\text{GKP}}$$

Edge of ideal CV cluster is logical CZ



$$\hat{C}_Z^{CV} |++\rangle_{\text{GKP}}$$

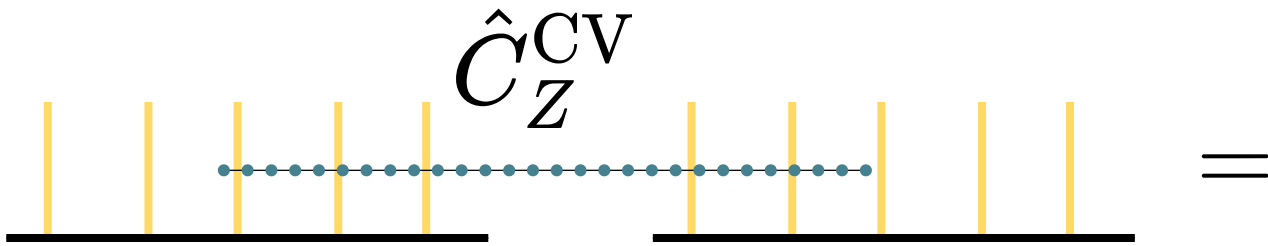
$$= \sum_{n_1, n_2} e^{i\pi n_1 n_2} |n_1 \sqrt{\pi}\rangle_q |n_2 \sqrt{\pi}\rangle_q$$

$$= \frac{1}{2} (|00\rangle_{\text{GKP}} + |01\rangle_{\text{GKP}} + |10\rangle_{\text{GKP}} - |11\rangle_{\text{GKP}})$$

$$\equiv \hat{C}_Z^{\text{GKP}} |++\rangle_{\text{GKP}}$$

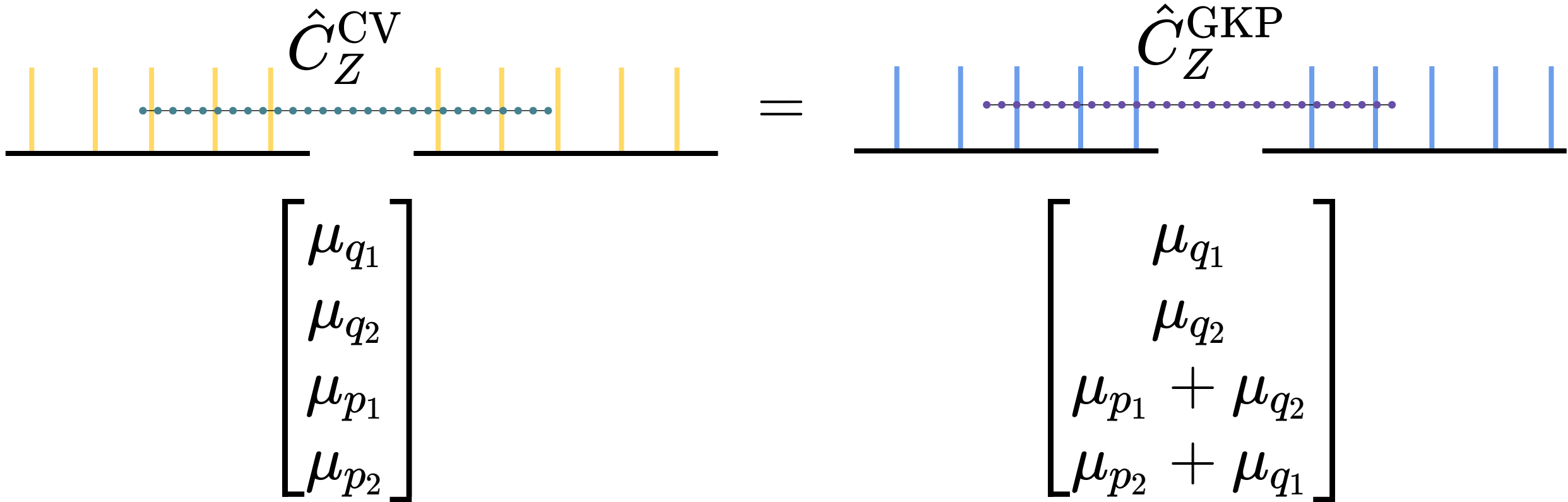
Logical qubit CZ gate on GKP states!

Edge of ideal CV cluster is logical CZ



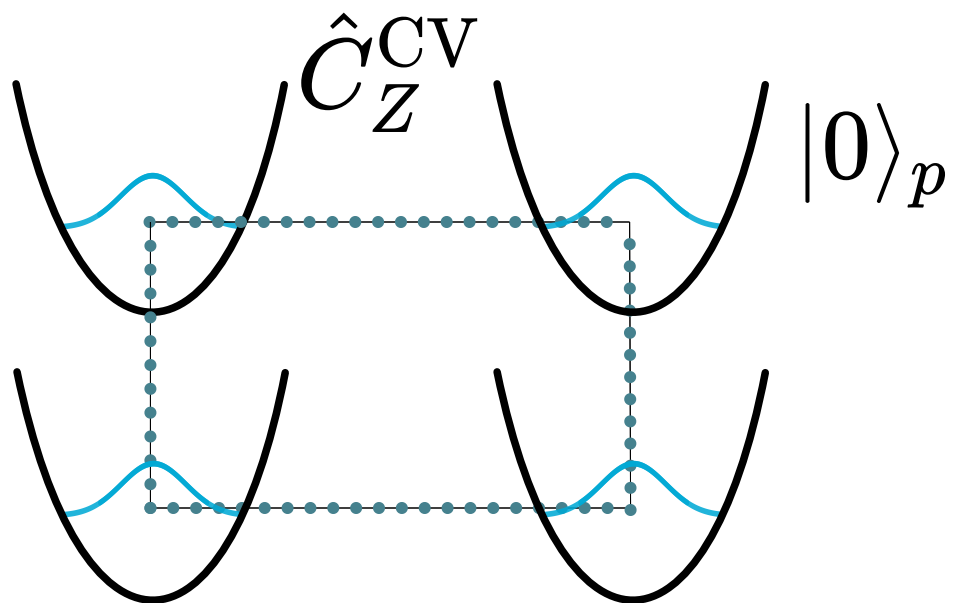
What about CV CZ on a displaced GKP state?

Edge of ideal CV cluster is logical CZ

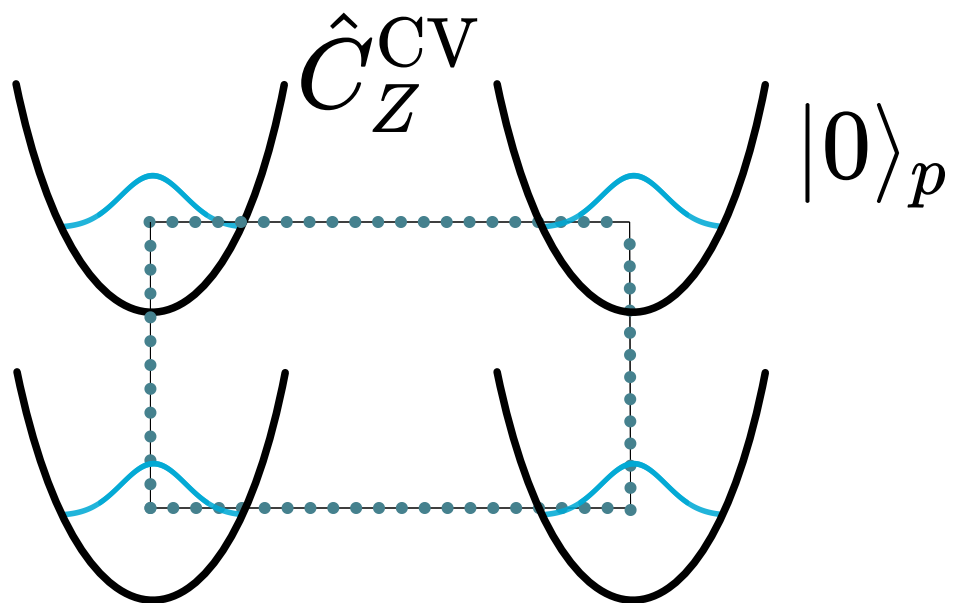


CV CZ gate on displaced GKP state = GKP CZ on displaced GKP state.

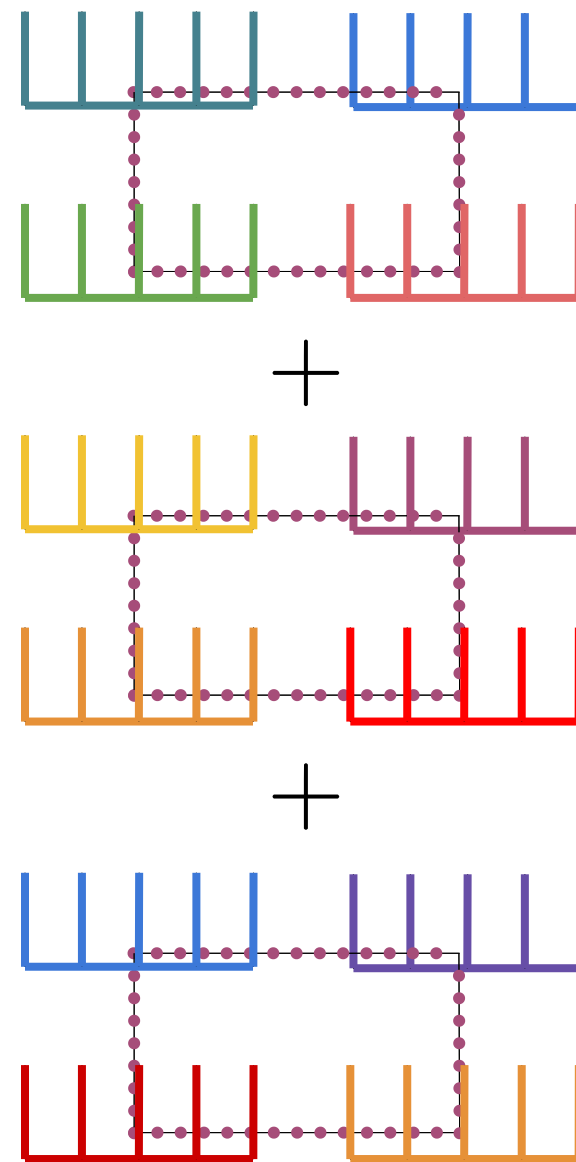
Displaced GKP cluster inside a CV cluster



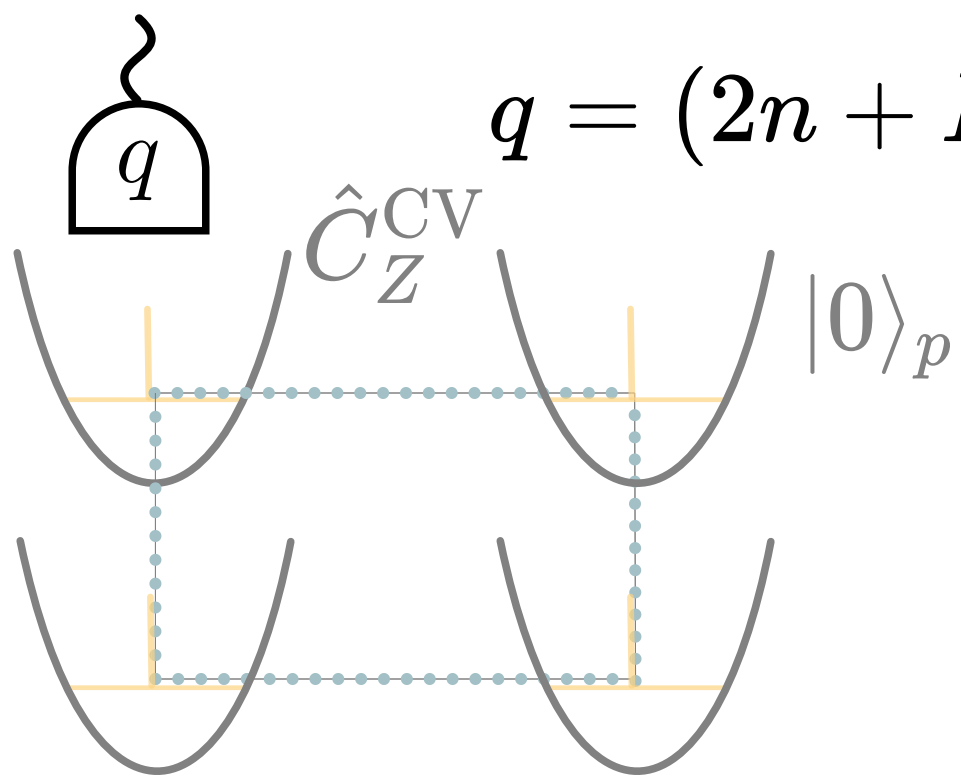
Displaced GKP cluster inside a CV cluster



$$= \int \dots$$

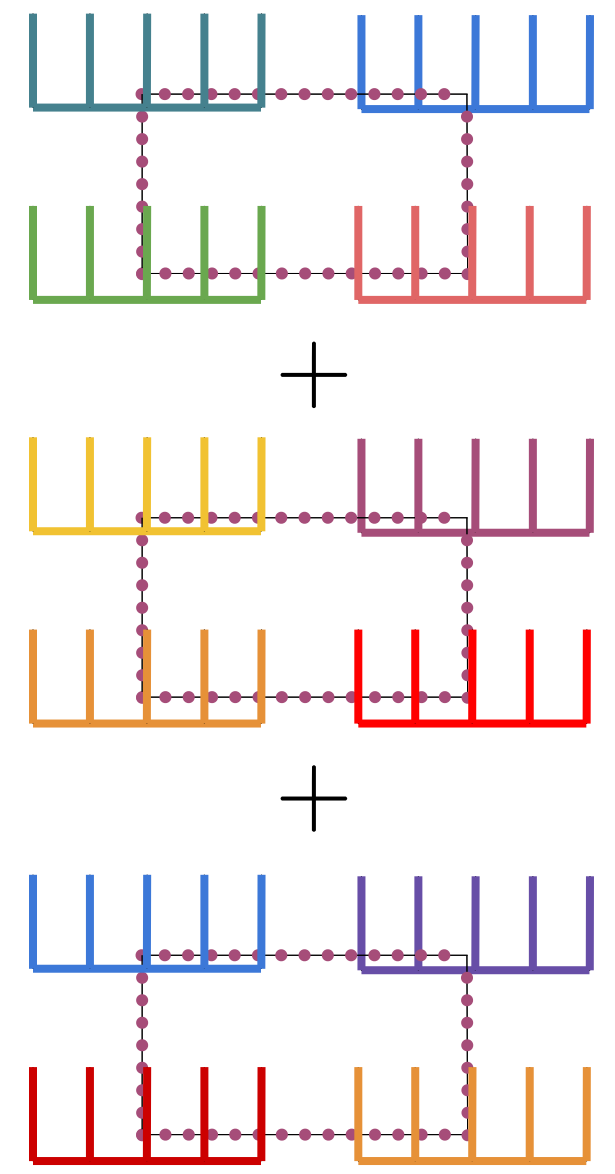


Homodyne detection collapses the GKP cluster

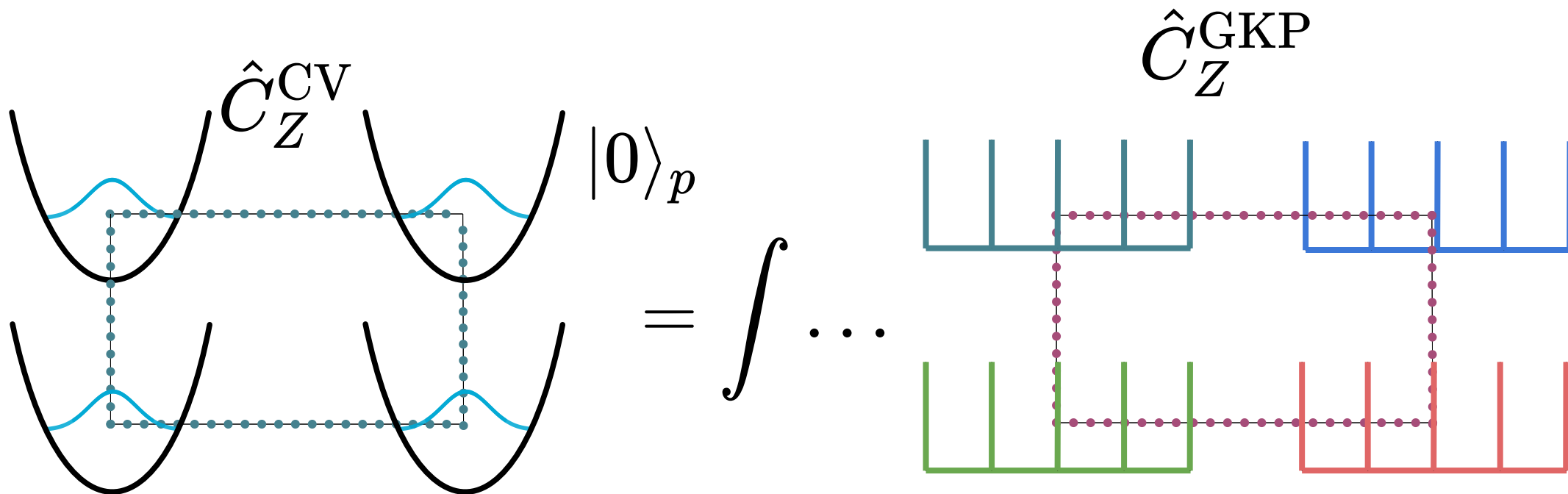


$$q = (2n + L)\sqrt{\pi} + \mu_q$$

$$= \int \dots$$



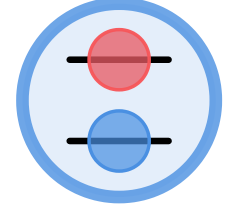
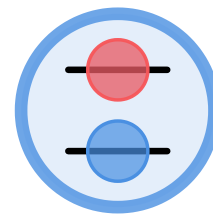
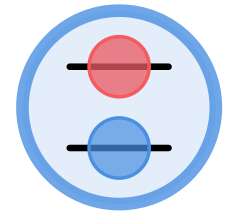
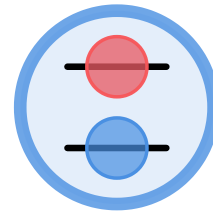
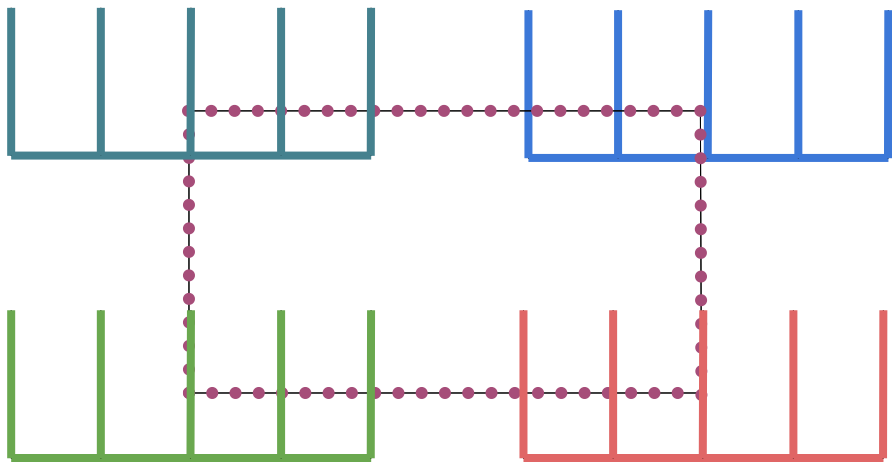
Displaced GKP cluster inside a CV cluster



Displaced GKP cluster state inside a CV cluster... How to get the entanglement out?

Displaced GKP cluster to qubit cluster

$$\hat{C}_Z^{\text{GKP}} \left| +_{\mu_q, \mu_p} \right\rangle_{\text{GKP}}$$

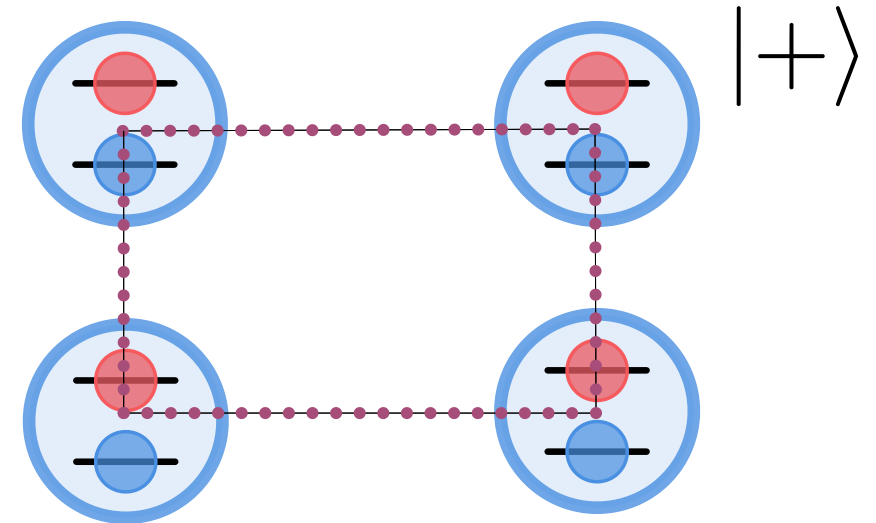
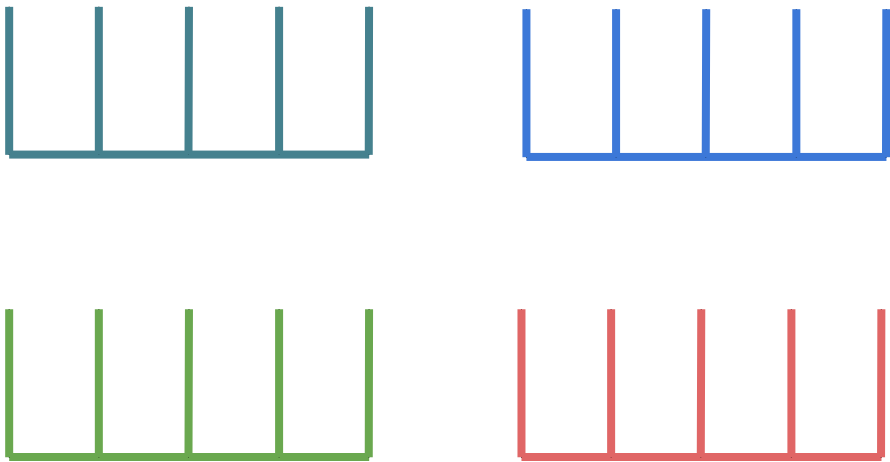


Interpret the GKP cluster as a qubit cluster.
We perform qubit-qubit quantum teleportation.

Displaced GKP cluster to qubit cluster

$$\hat{C}_Z^{\text{GKP}} = \text{diag}(1, 1, 1, -1)$$

$$\hat{C}_Z = \text{diag}(1, 1, 1, -1)$$



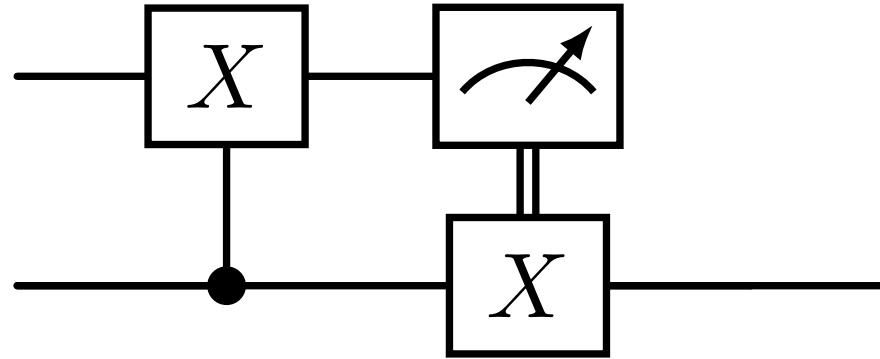
Interpret the GKP cluster as a qubit cluster.
We perform qubit-qubit quantum teleportation.

One bit teleportation



qubit

$|\psi\rangle$



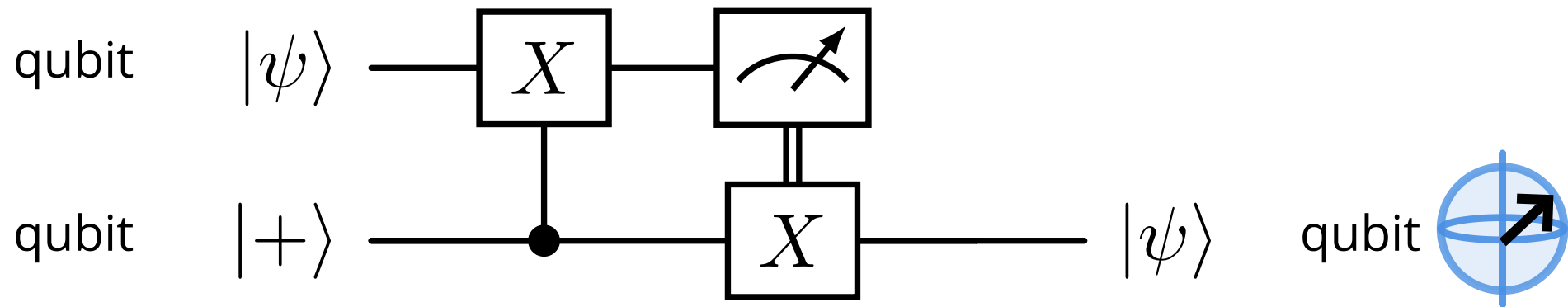
qubit

$|+\rangle$

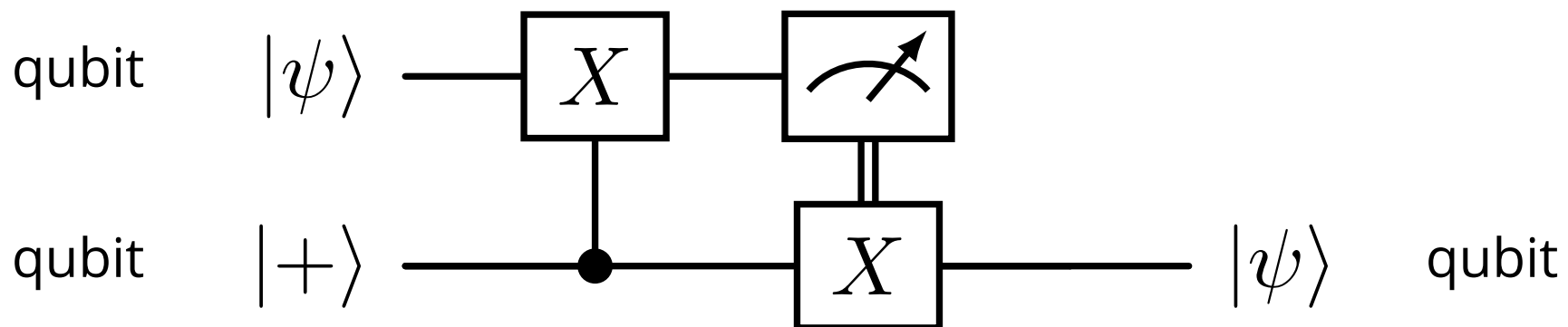
$|\psi\rangle$

qubit

One bit teleportation



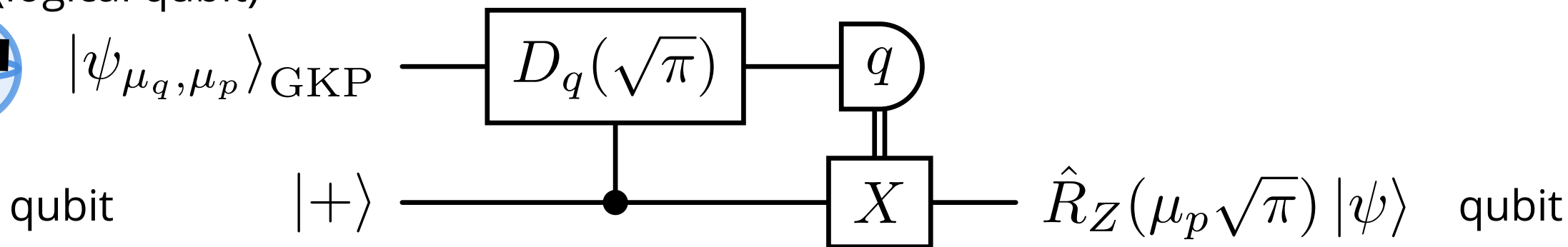
GKP-qubit one bit teleportation



GKP (logical qubit)



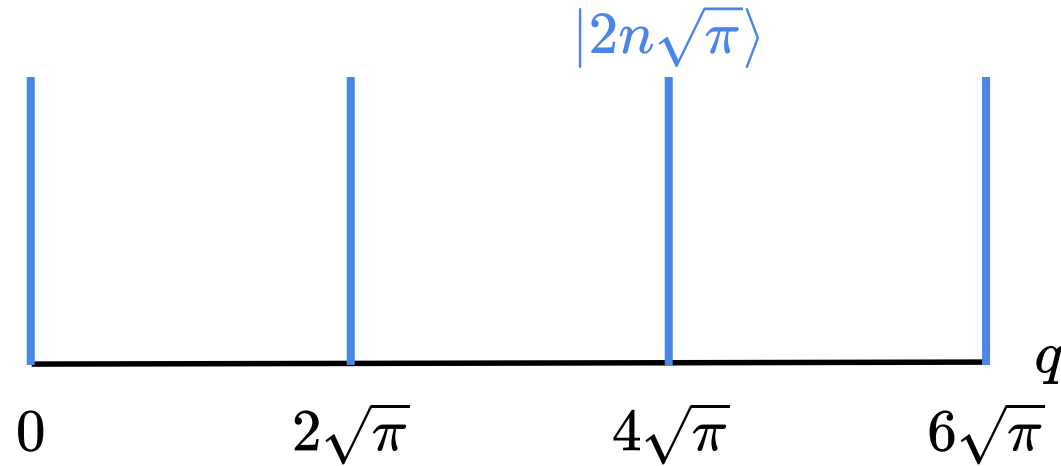
$|\psi_{\mu_q, \mu_p}\rangle_{\text{GKP}}$



teleportation by products

GKP-qubit one bit teleportation: X gate

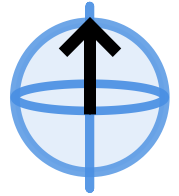
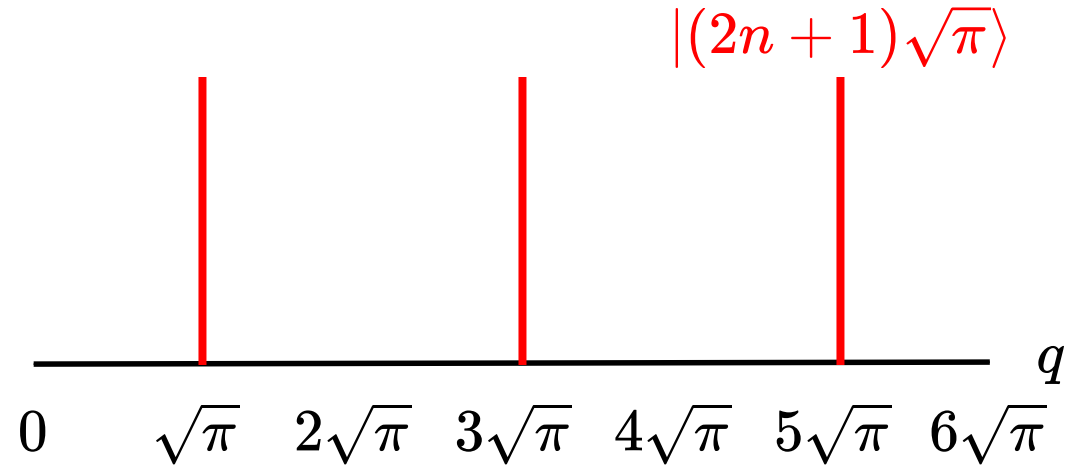
$$|0\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |2n\sqrt{\pi}\rangle_q$$



Gottesman-Kitaev-Preskill (GKP state)

GKP-qubit one bit teleportation: X gate

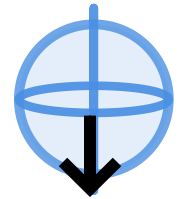
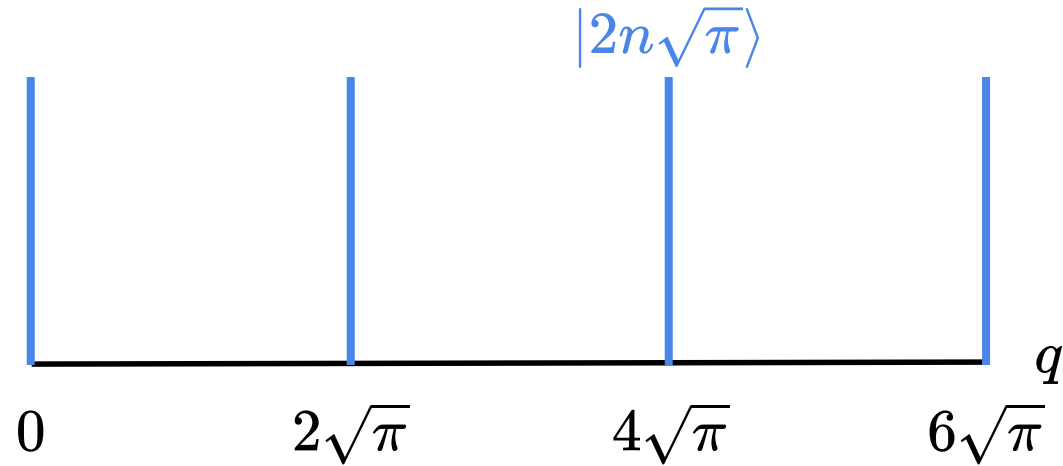
$$|1\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |(2n+1)\sqrt{\pi}\rangle_q$$



$$\hat{X}^{\text{GKP}} = e^{-i\sqrt{\pi}\hat{p}} = \hat{D}_q(\sqrt{\pi})$$

GKP-qubit one bit teleportation: μ_q, μ_p

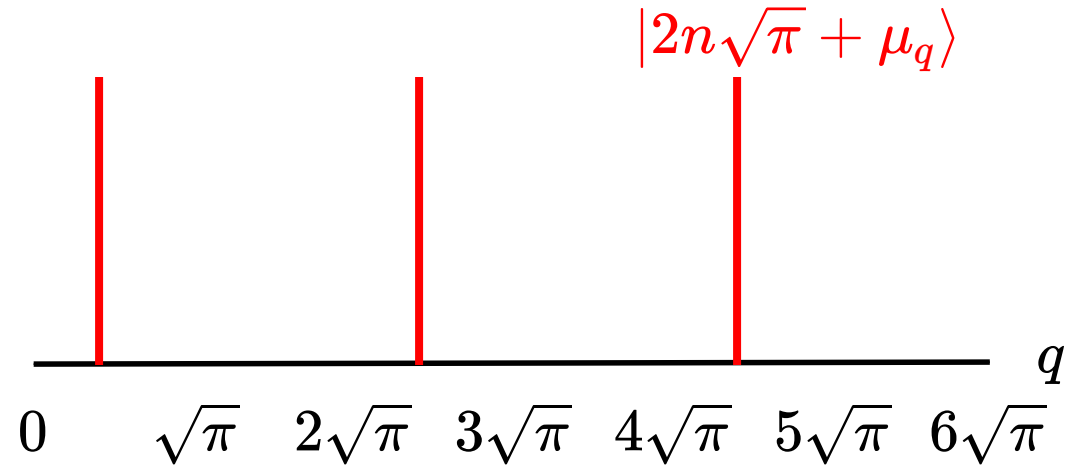
$$|0\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |2n\sqrt{\pi}\rangle_q$$



Gottesman-Kitaev-Preskill (GKP state)

μ_q, μ_p as rotational X, Z

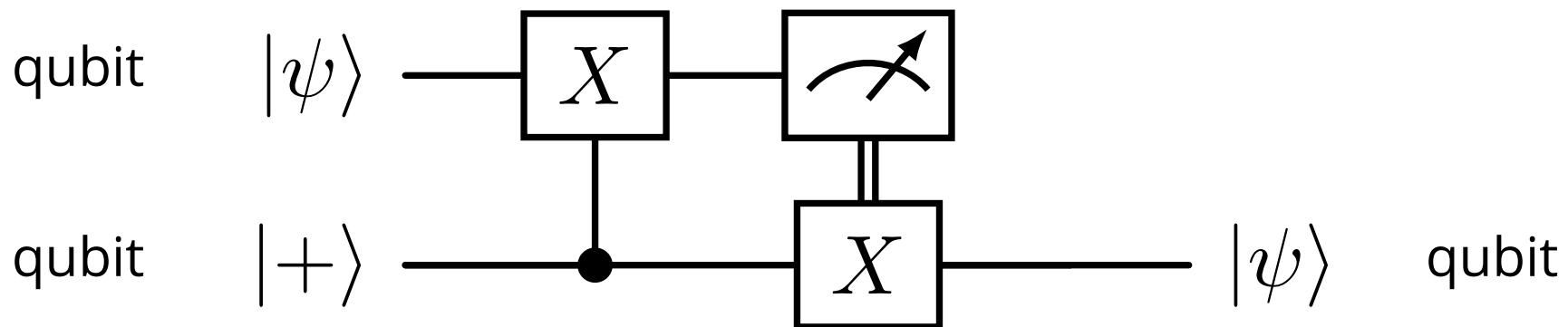
$$|1\rangle_{\text{GKP}} = \sum_{n=-\infty}^{\infty} |(2n+1)\sqrt{\pi}\rangle_q$$



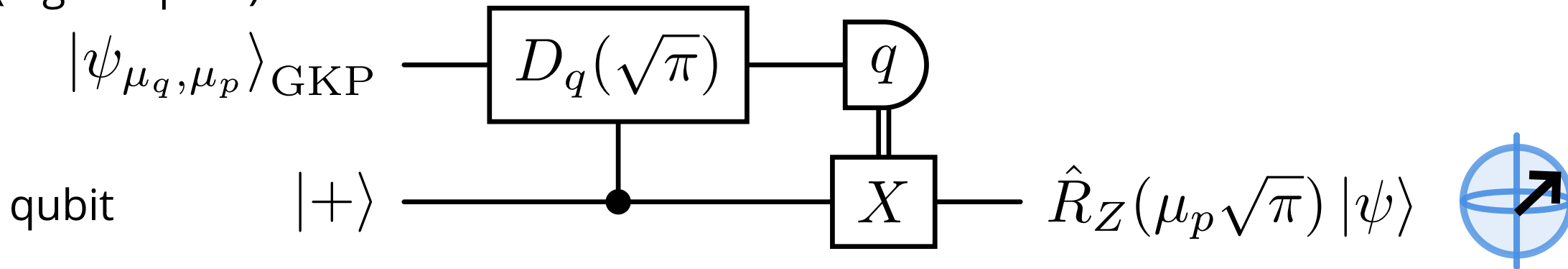
μ_q : Rotational X gate

μ_p : Rotational Z gate

GKP-qubit one bit teleportation



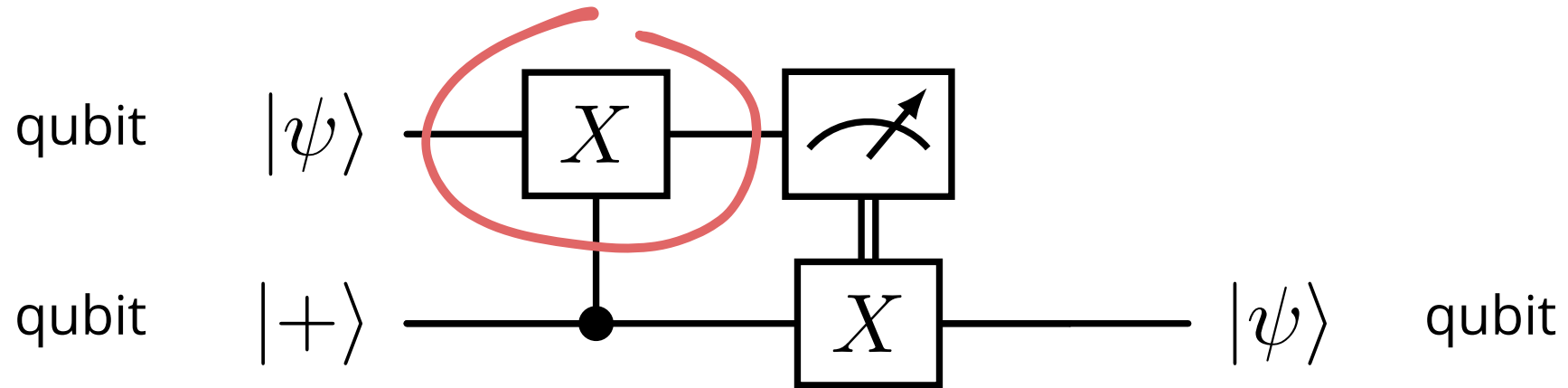
GKP (logical qubit)



teleportation by products

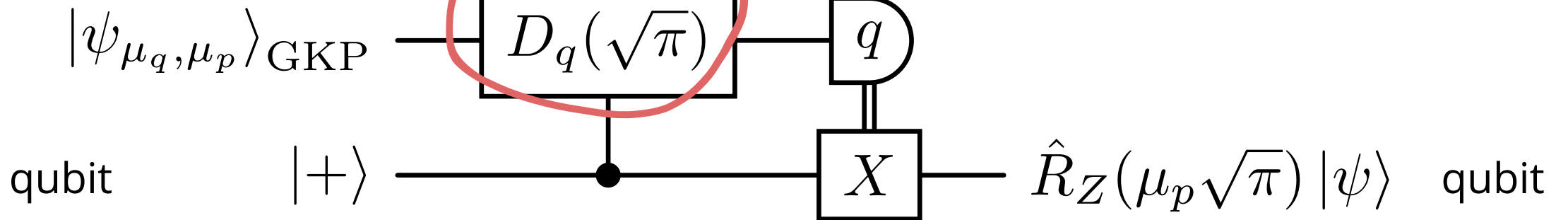


GKP-qubit one bit teleportation: X gate

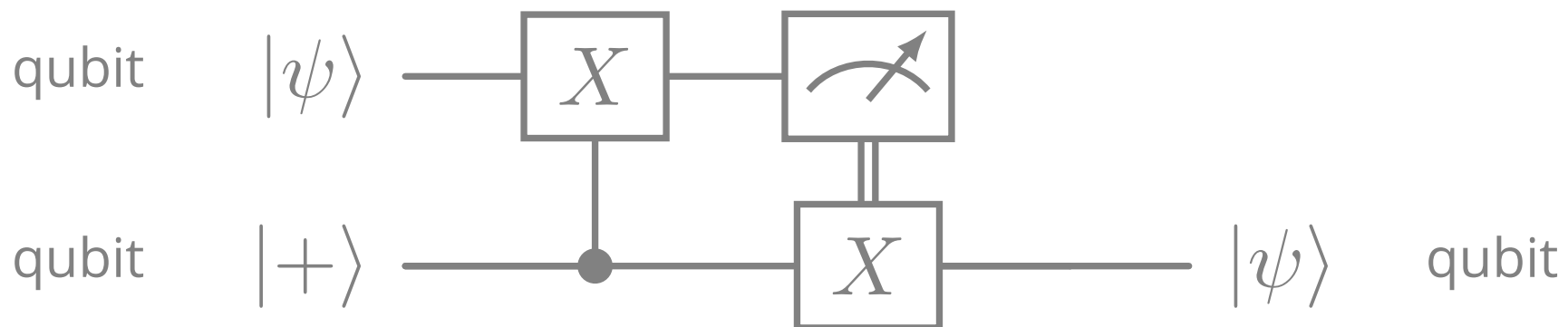


GKP X gate

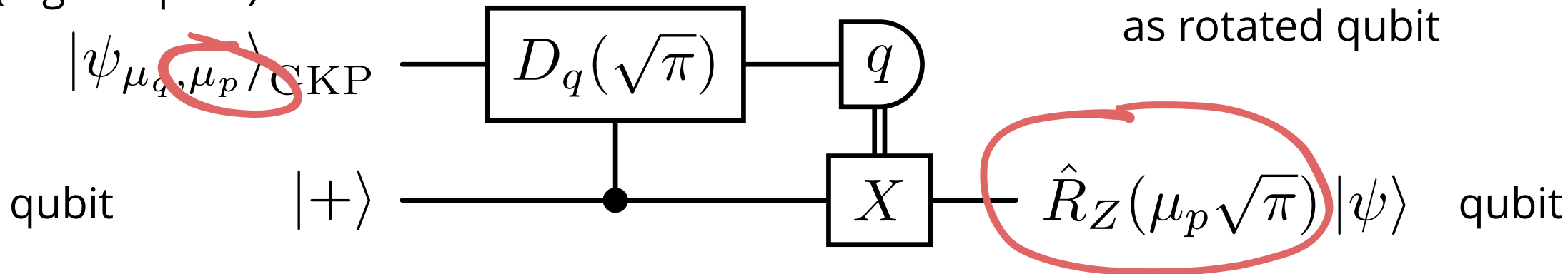
GKP (logical qubit)



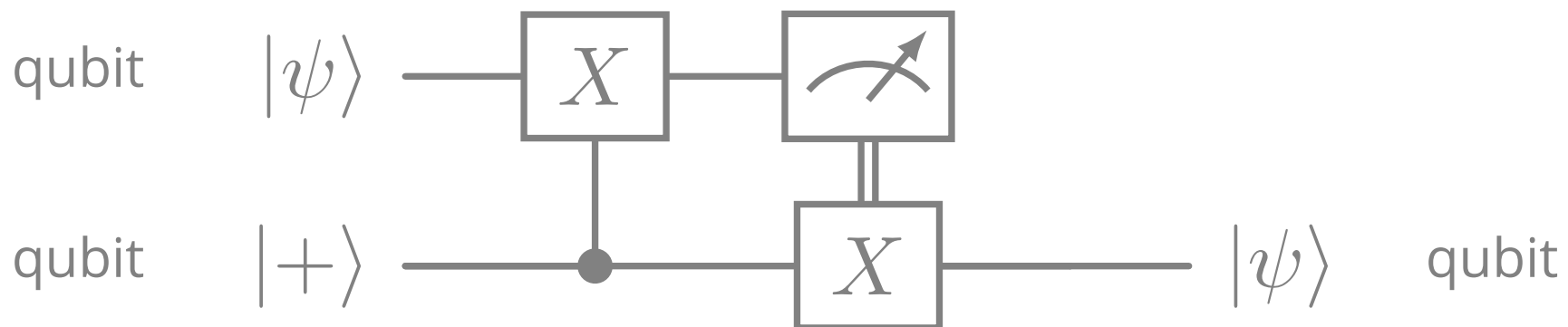
GKP-qubit one bit teleportation: μ_p



GKP (logical qubit)



GKP-qubit one bit teleportation: μ_q

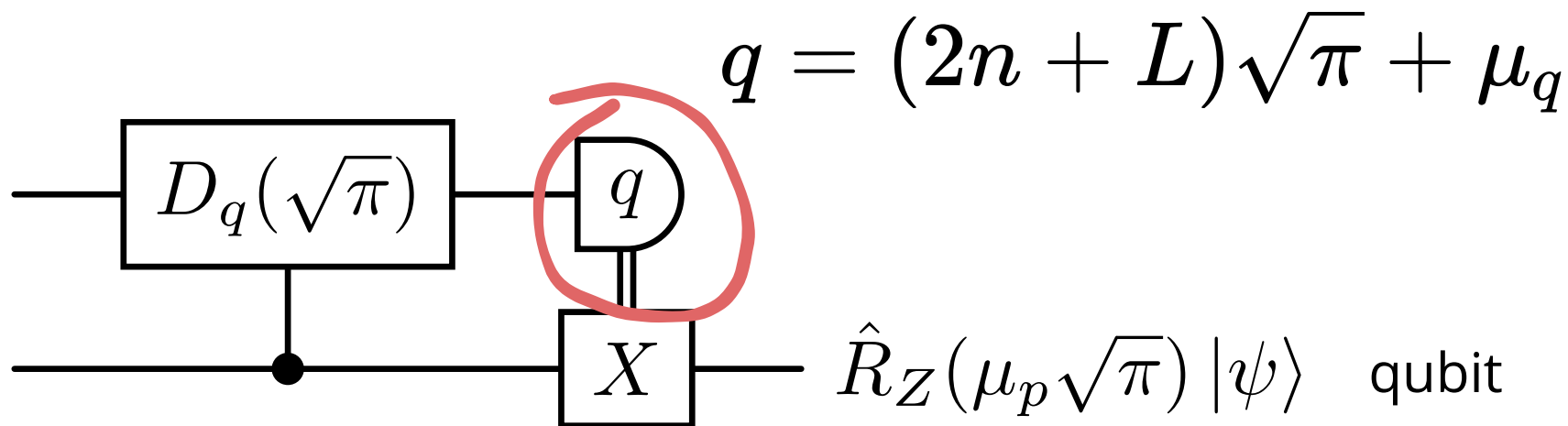


GKP (logical qubit)

$|\psi, \mu_q, \mu_p\rangle_{\text{GKP}}$

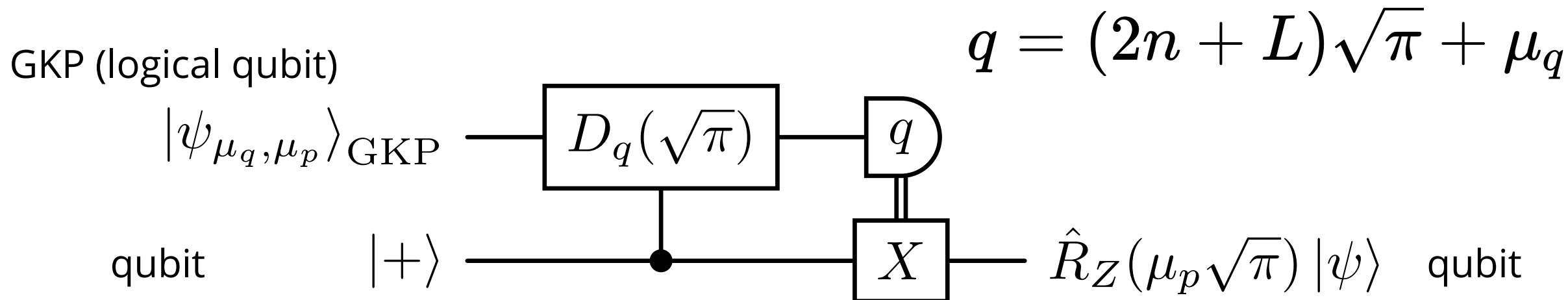
qubit

$|+\rangle$

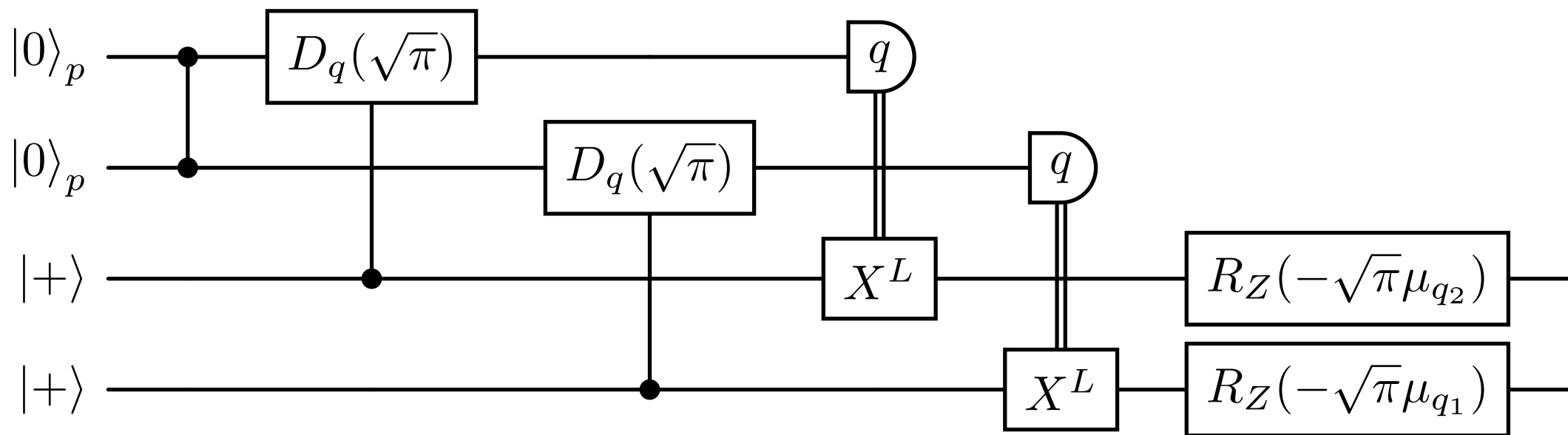


Homodyne detection roles:

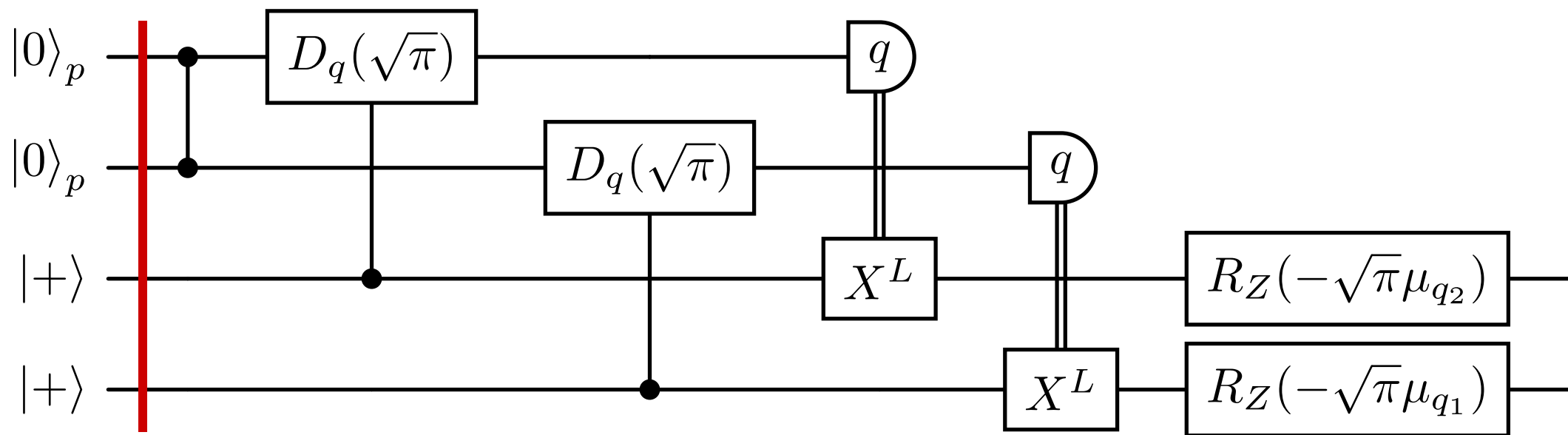
1. Collapsing the superposition into some GKP cluster
2. Quantum teleportation
3. Need μ_q to correct phase shifts due to CV CZ



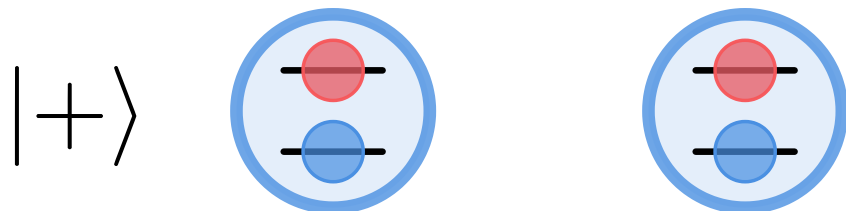
Entanglement transfer protocol: Recap



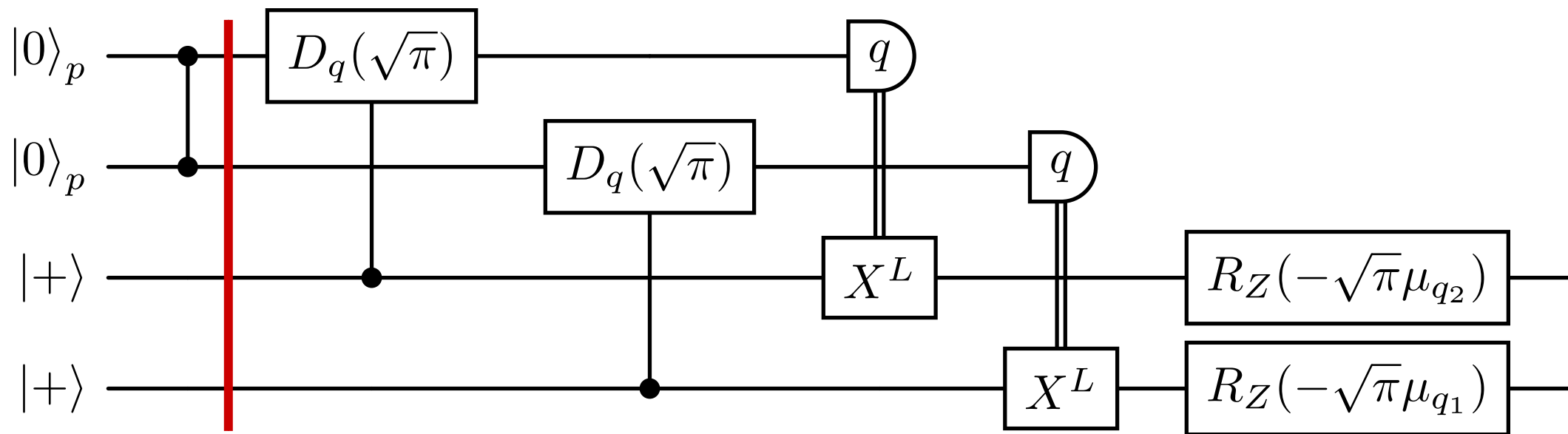
Entanglement transfer protocol: Recap



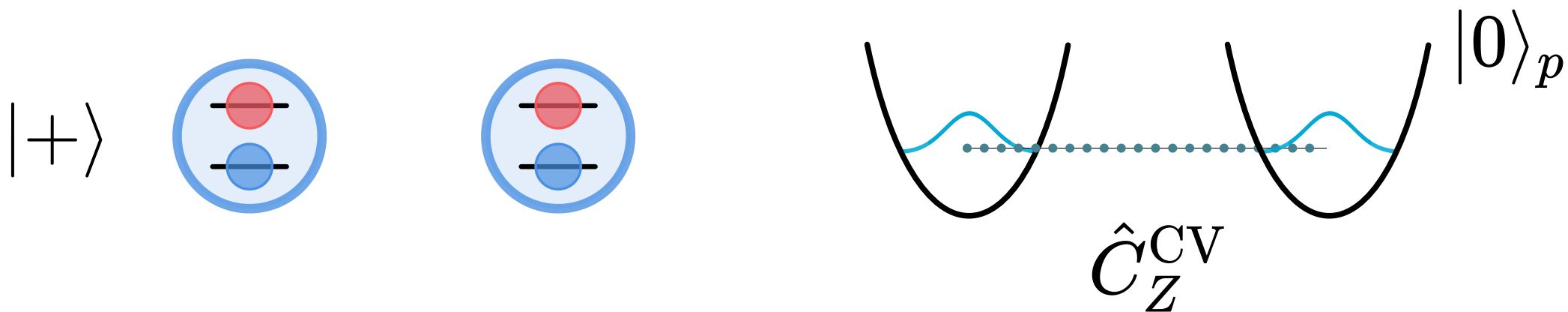
1. Initialize all qubits to $|+\rangle$.



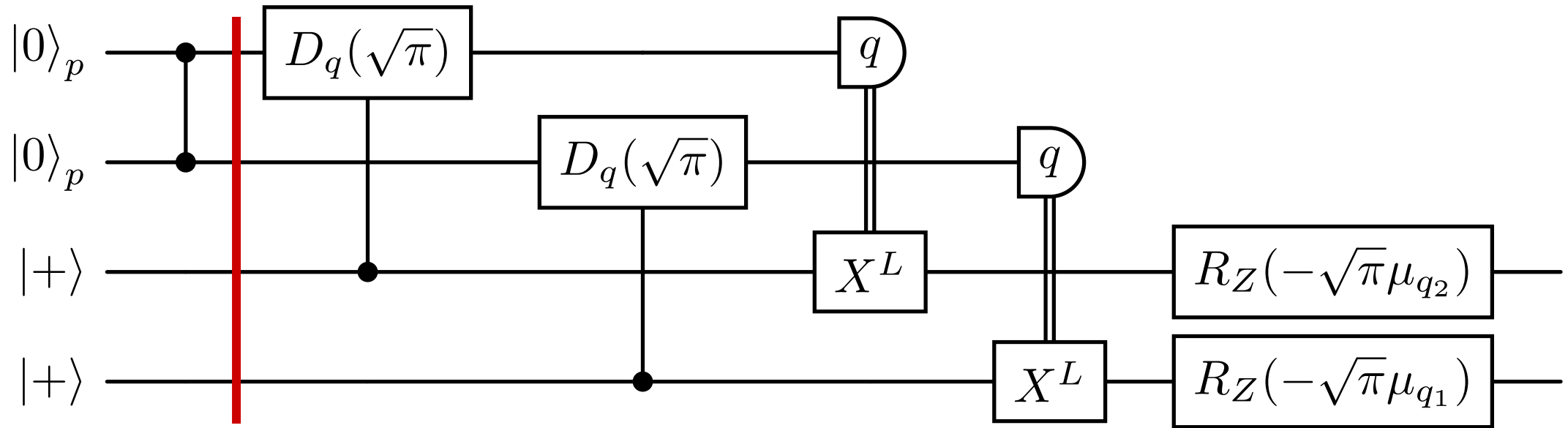
Entanglement transfer protocol: Recap



2. Create a CV cluster state.

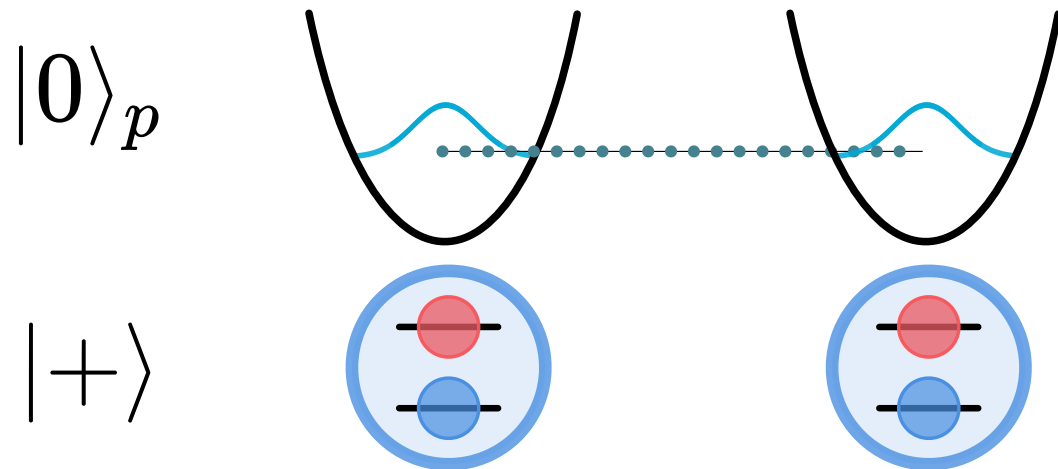


Entanglement transfer protocol: Recap

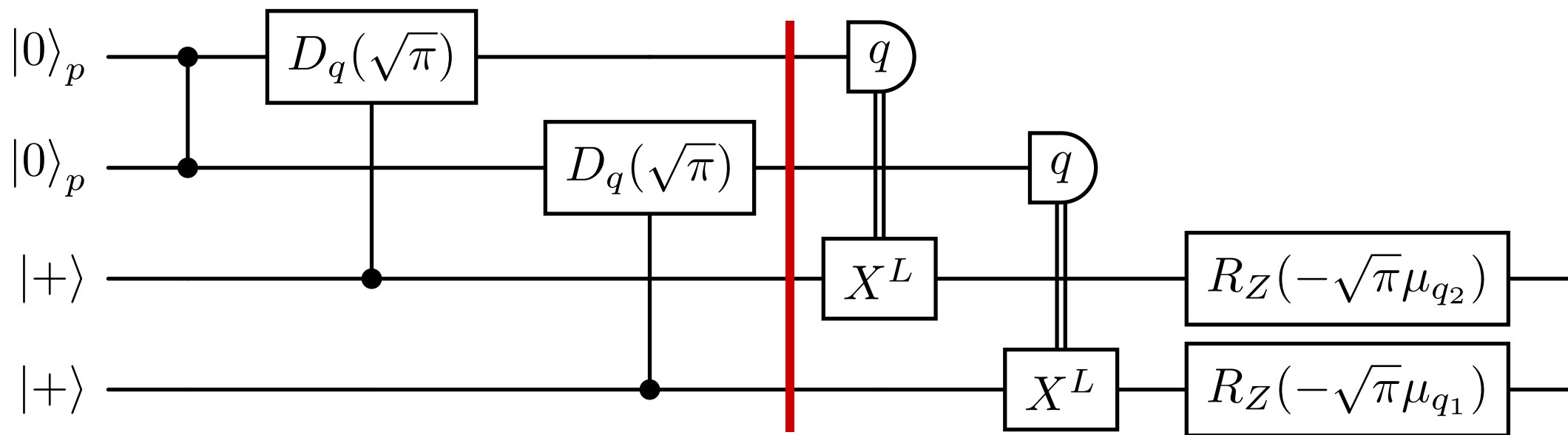


Step 3. Apply conditional displacement to each pair:

$$\hat{C}_D = |0\rangle\langle 0|\hat{I} + |1\rangle\langle 1|\hat{D}_q(\sqrt{\pi})$$

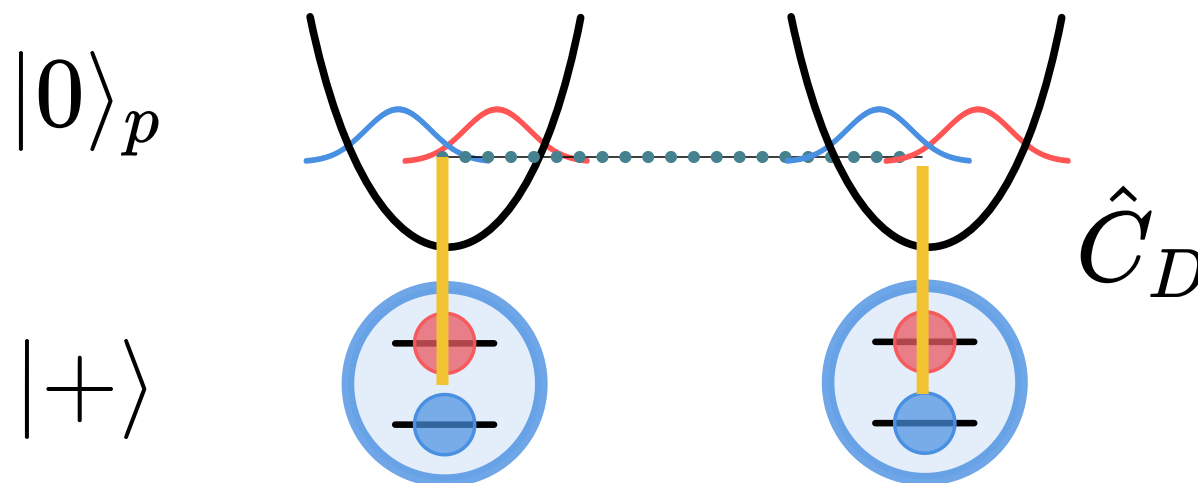


Entanglement transfer protocol: Recap

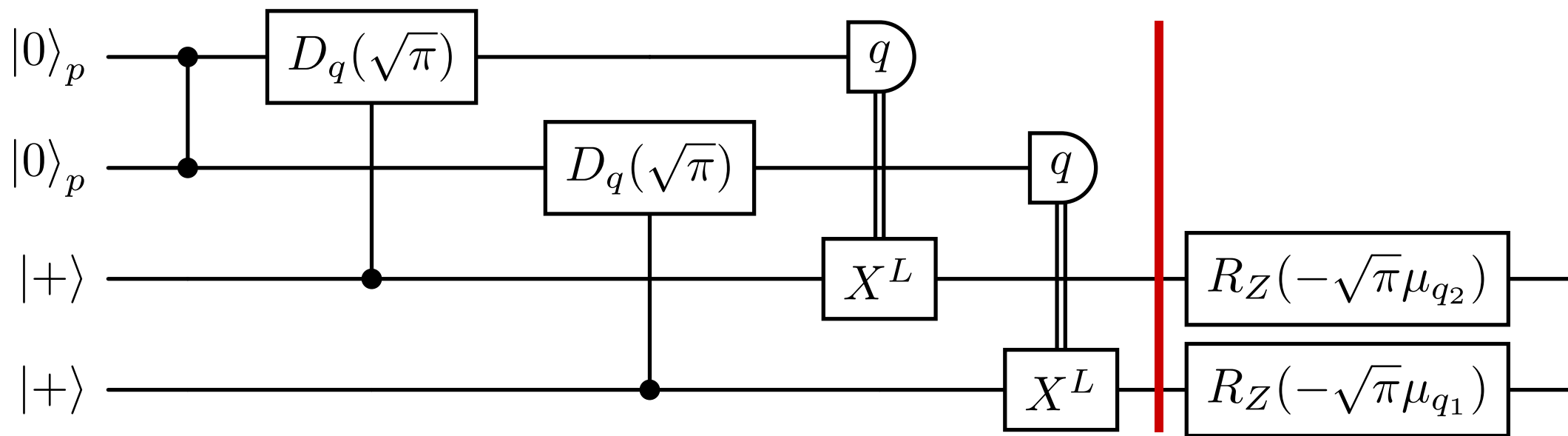


Step 3. Apply conditional displacement to each pair:

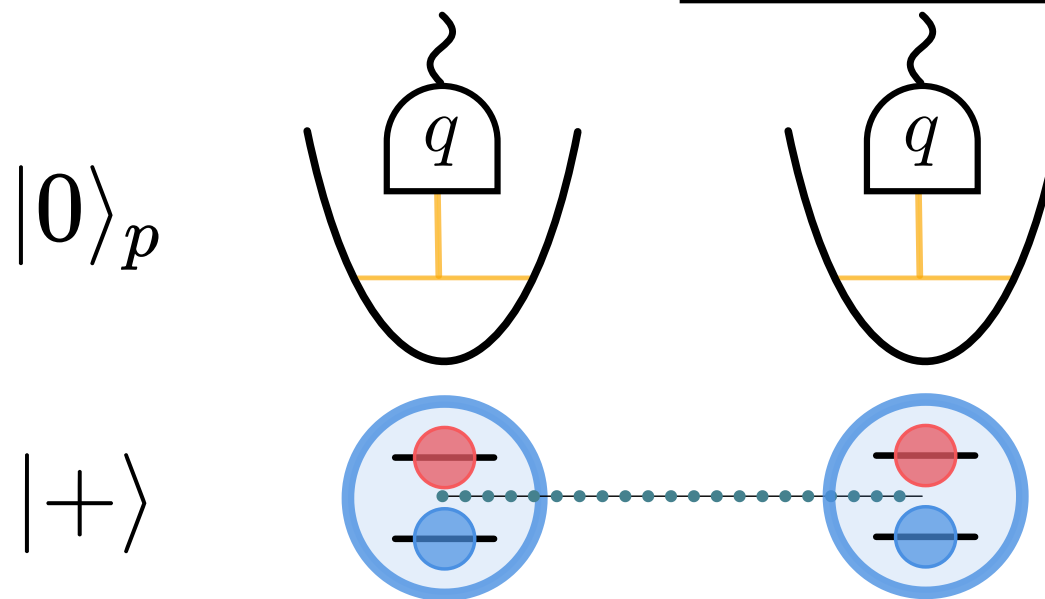
$$\hat{C}_D = |0\rangle\langle 0|\hat{I} + |1\rangle\langle 1|\hat{D}_q(\sqrt{\pi})$$



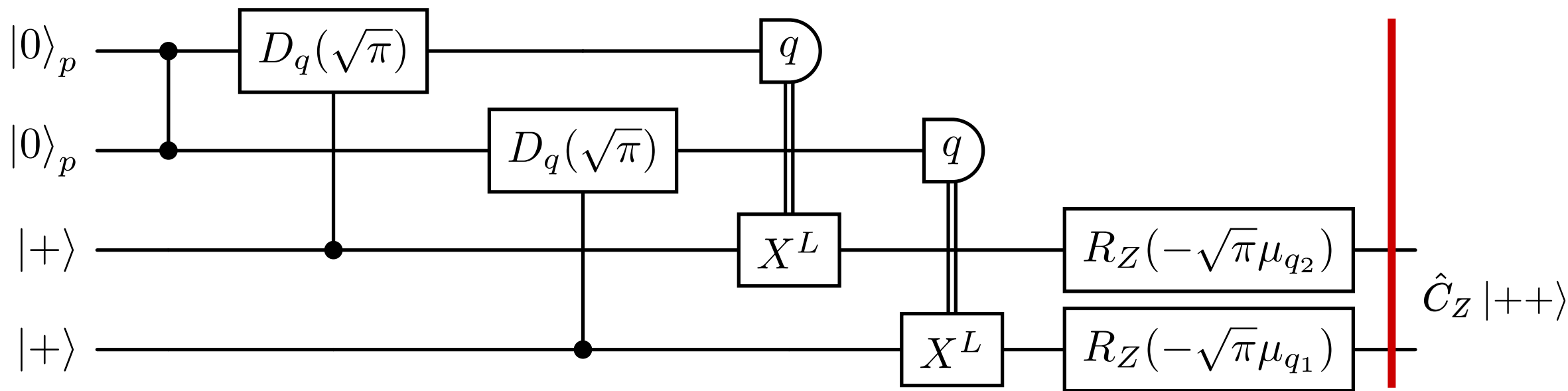
Entanglement transfer protocol: Recap



Step 4. Measure the q quadrature.



Entanglement transfer protocol: Recap



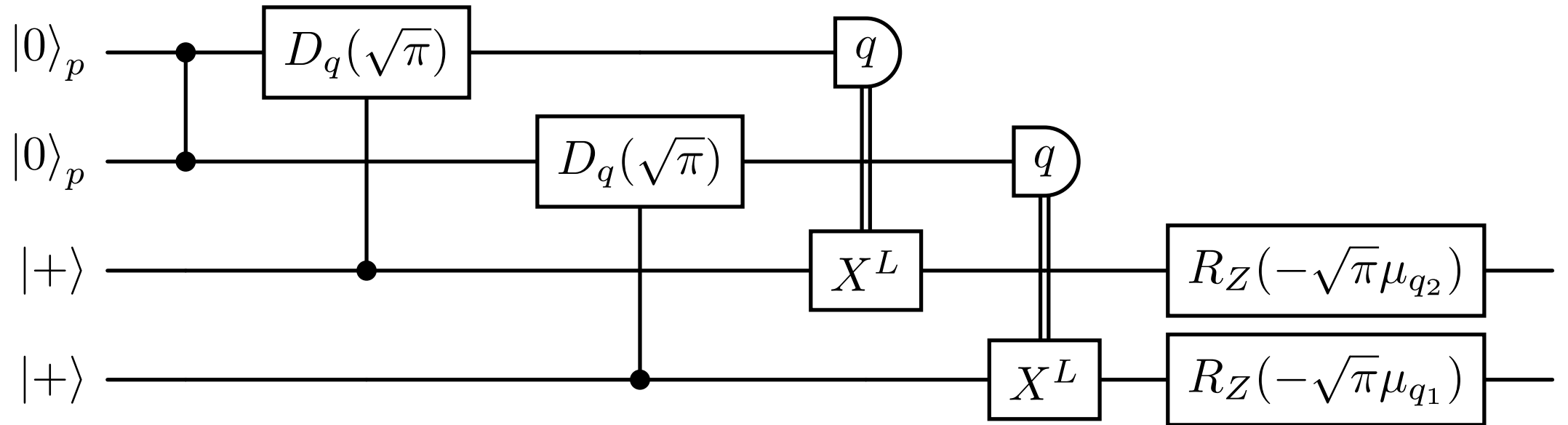
Step 5. Correct by products.

$|+\rangle$

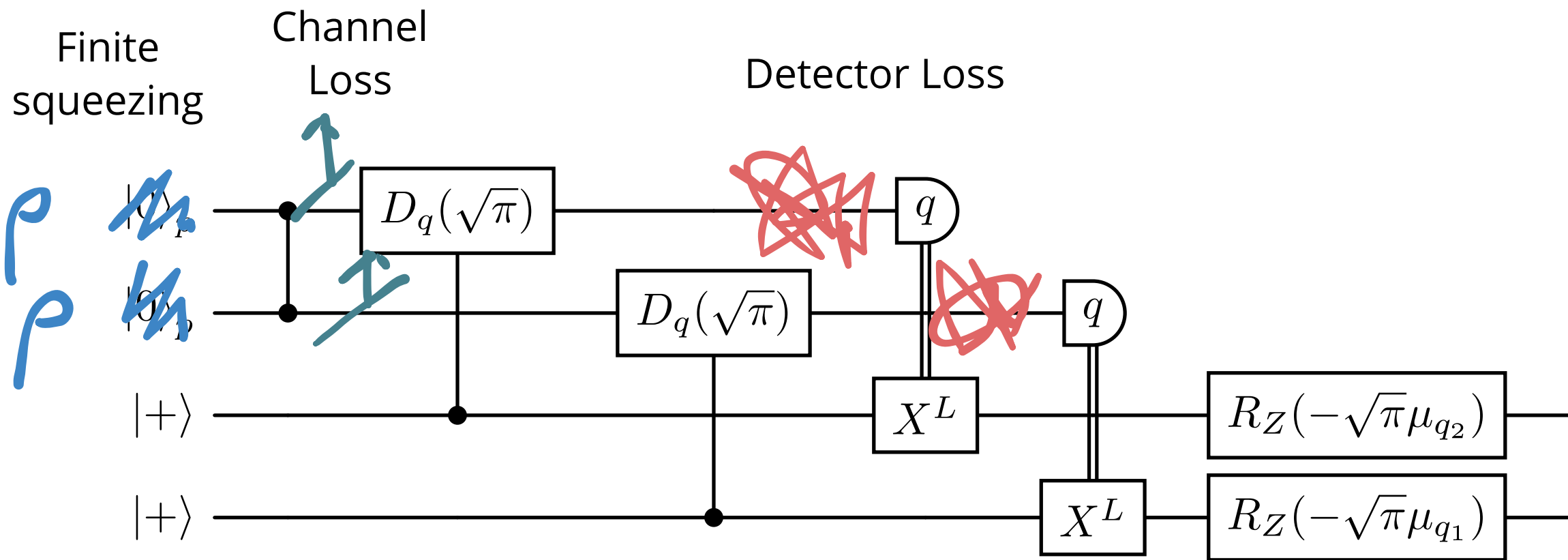


Entanglement transfer protocol: Loss

1. Ideal CV cluster \rightarrow perfect qubit cluster
2. No GKP states in the protocol

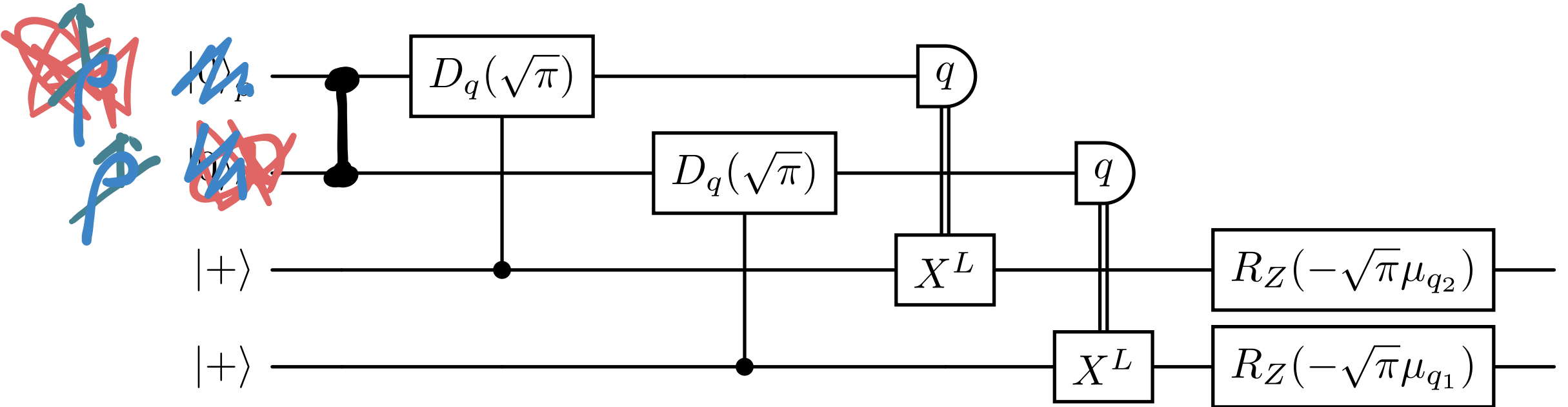


Entanglement transfer protocol: Loss



Entanglement transfer protocol: Loss

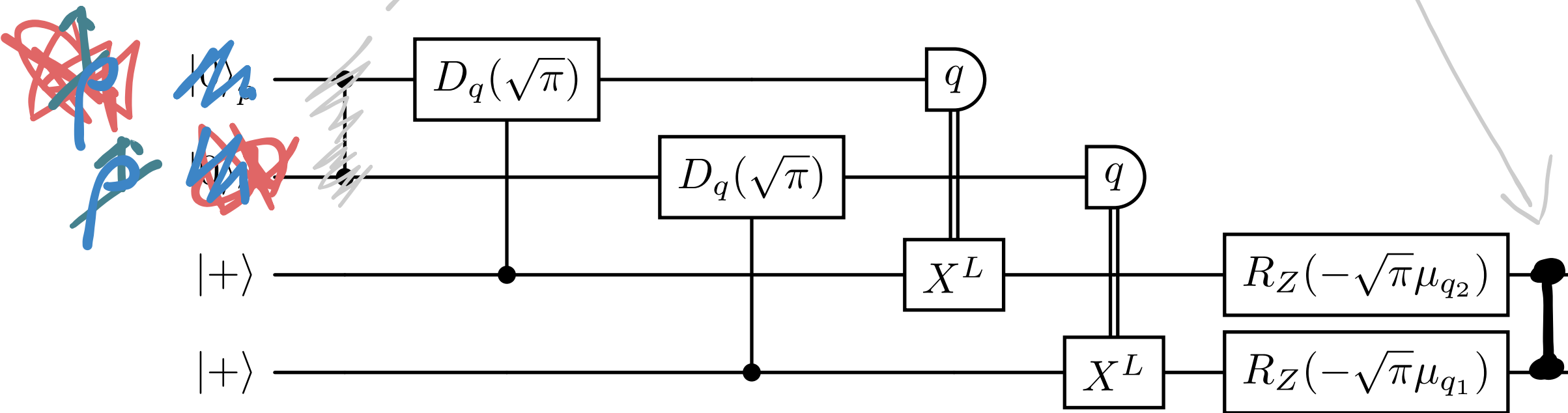
Client
Data Field
Losses
Compressing



Entanglement transfer protocol: Loss

Classical
Losses

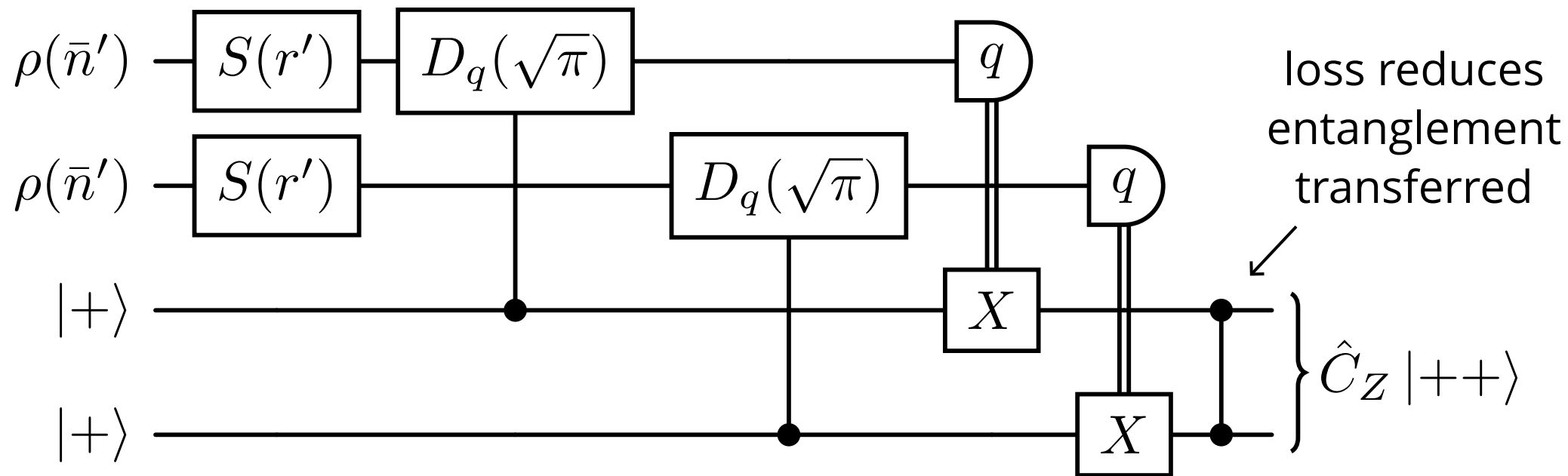
math ✨



Entanglement transfer protocol: Loss

Squeezed thermal state

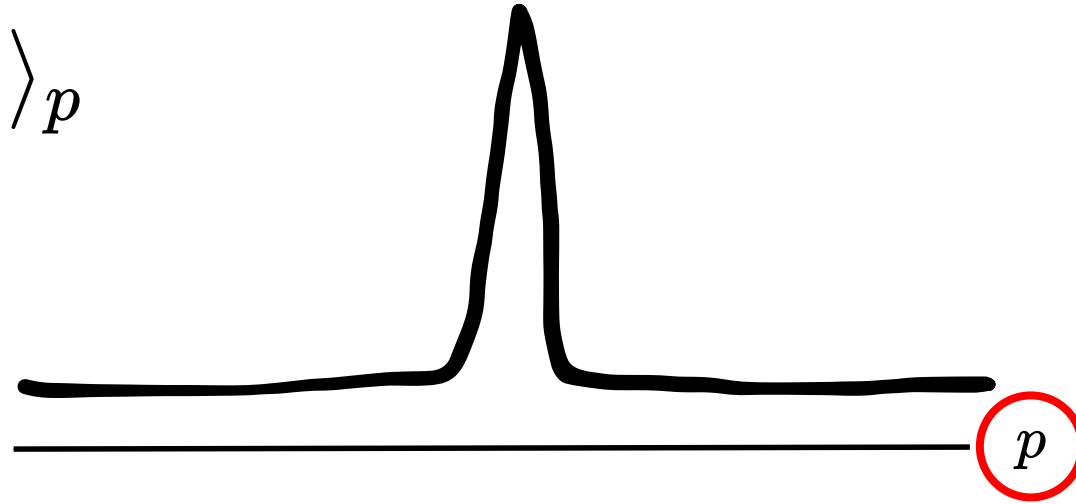
Equivalent circuit model



Loss: Finite Squeezing

Finite squeezing:

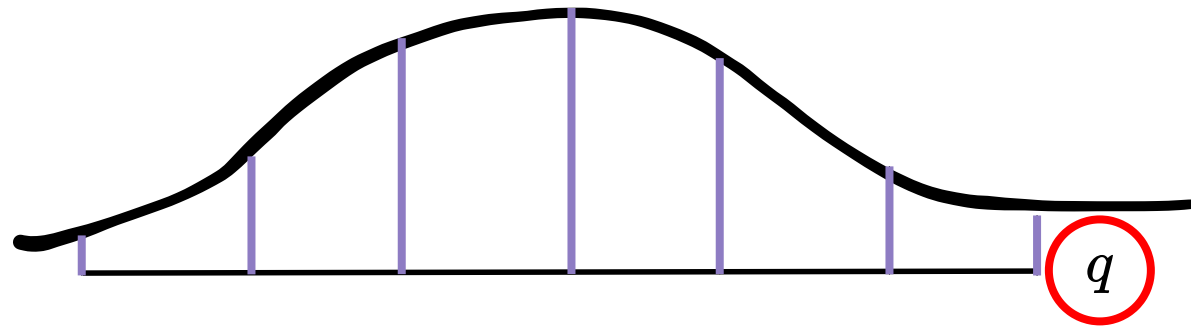
$$|0, \sigma_p\rangle_p$$



Loss: Finite Squeezing

Finite squeezing:

$$|0, \sigma_p\rangle_p$$

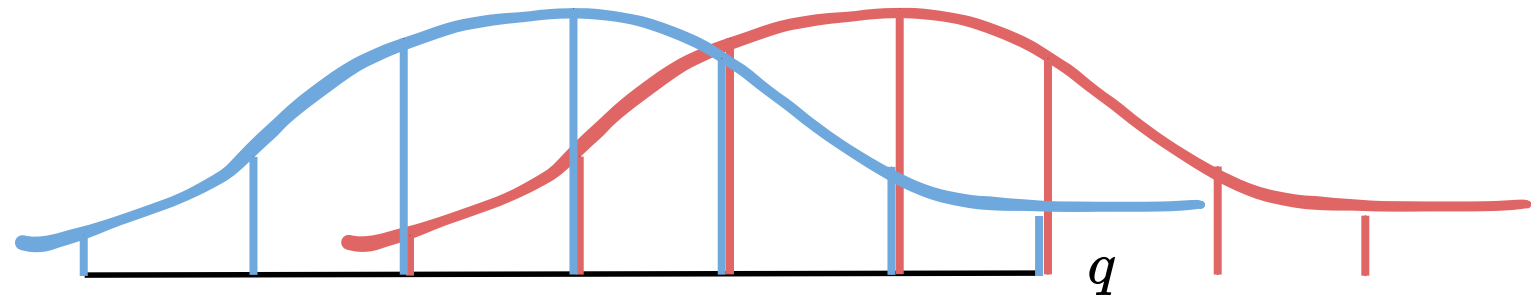


Loss: Finite Squeezing

After conditional displacement:

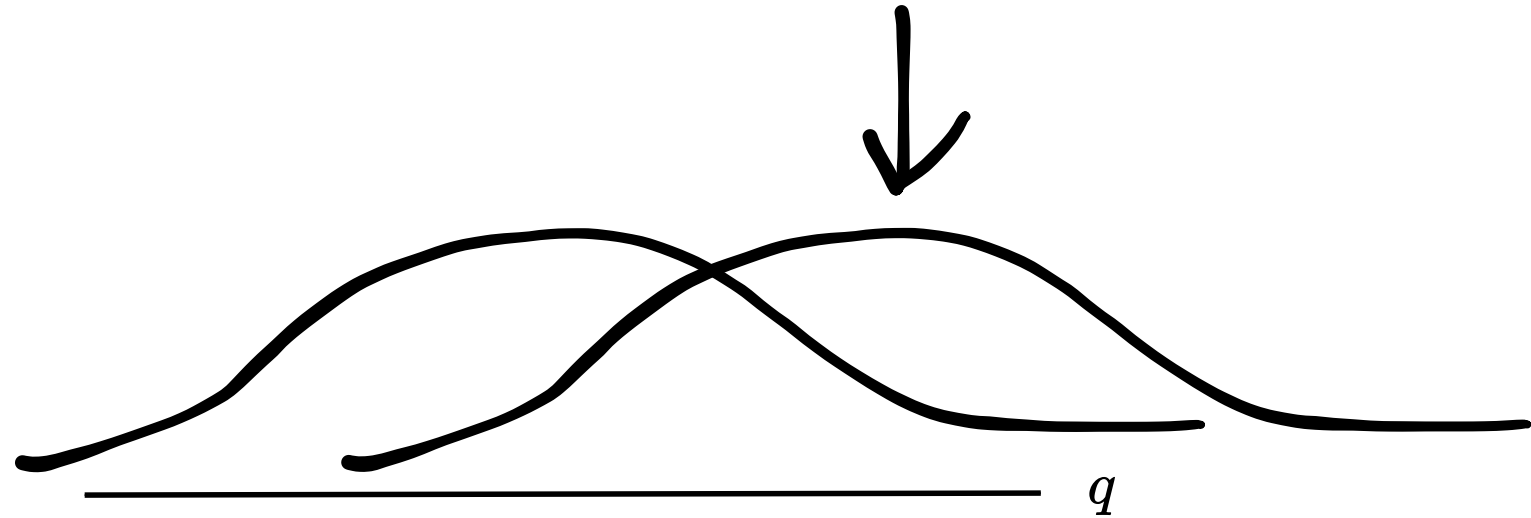
$$|0, \sigma_p\rangle_p |0\rangle^{\text{Qubit}}$$

$$\hat{D}_q(\sqrt{\pi}) |0, \sigma_p\rangle_p |1\rangle^{\text{Qubit}}$$



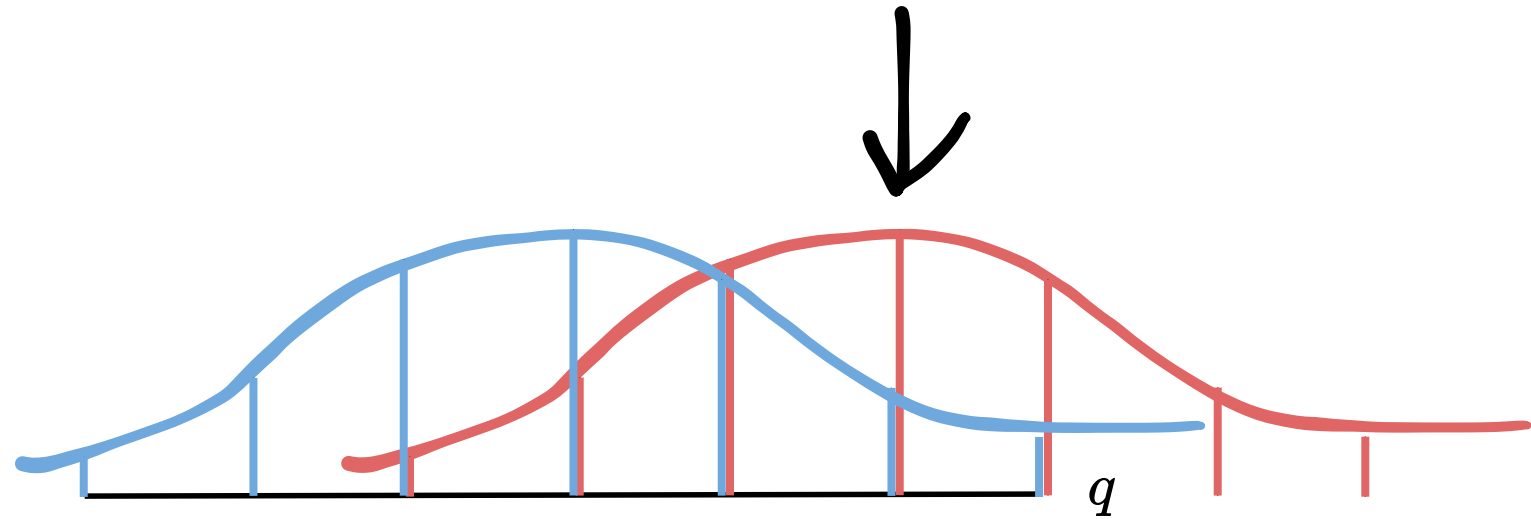
Loss: Finite Squeezing

Now, the probability of measuring q :



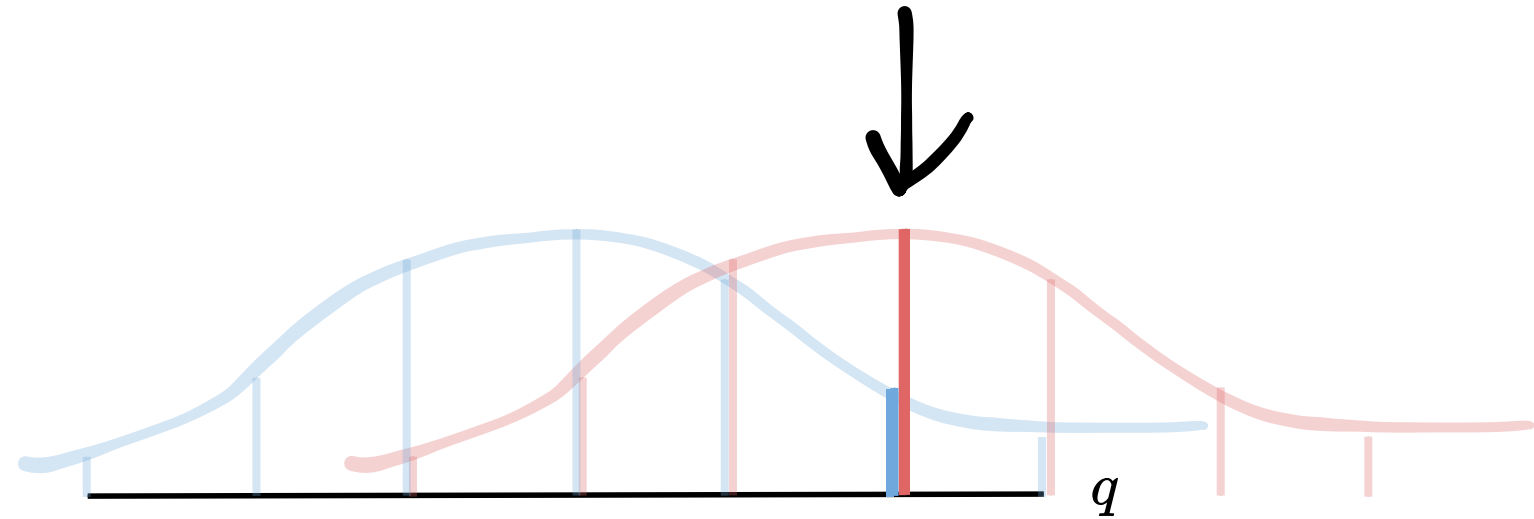
Loss: Finite Squeezing

The displaced GKP state after measuring q :



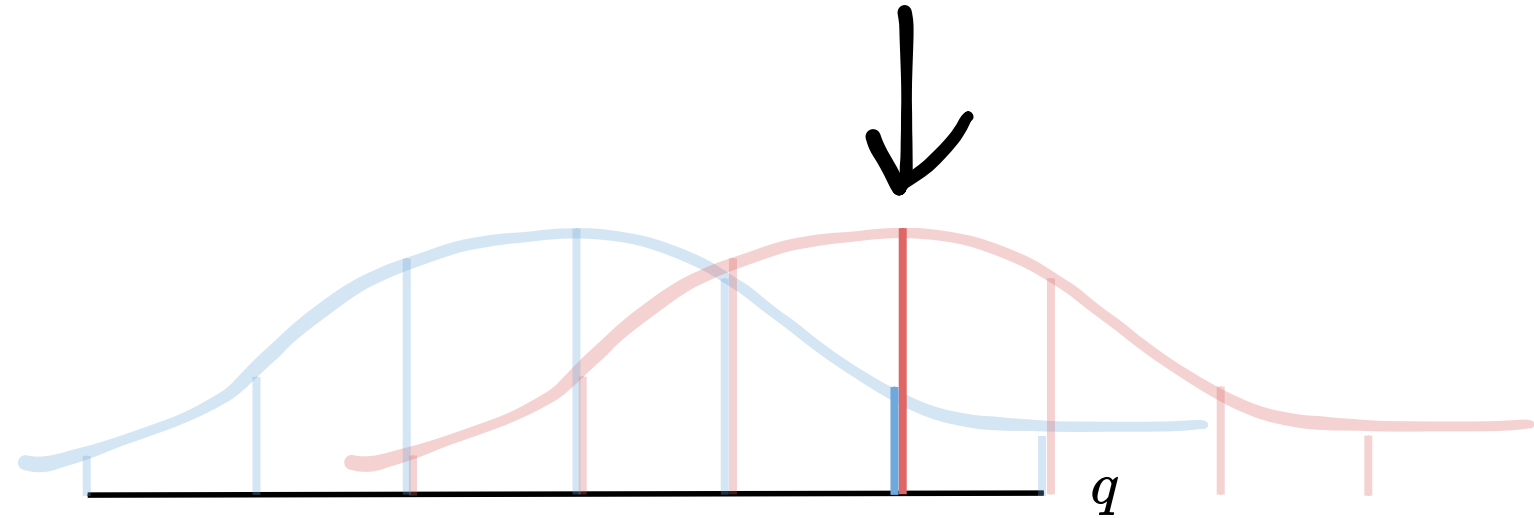
Loss: Finite Squeezing

The displaced GKP state after measuring q :



Loss: Finite Squeezing

The displaced GKP state after measuring q :



The qubit is:

$$|0\rangle + |1\rangle$$

Amplitude imbalance error

Loss: Finite Squeezing

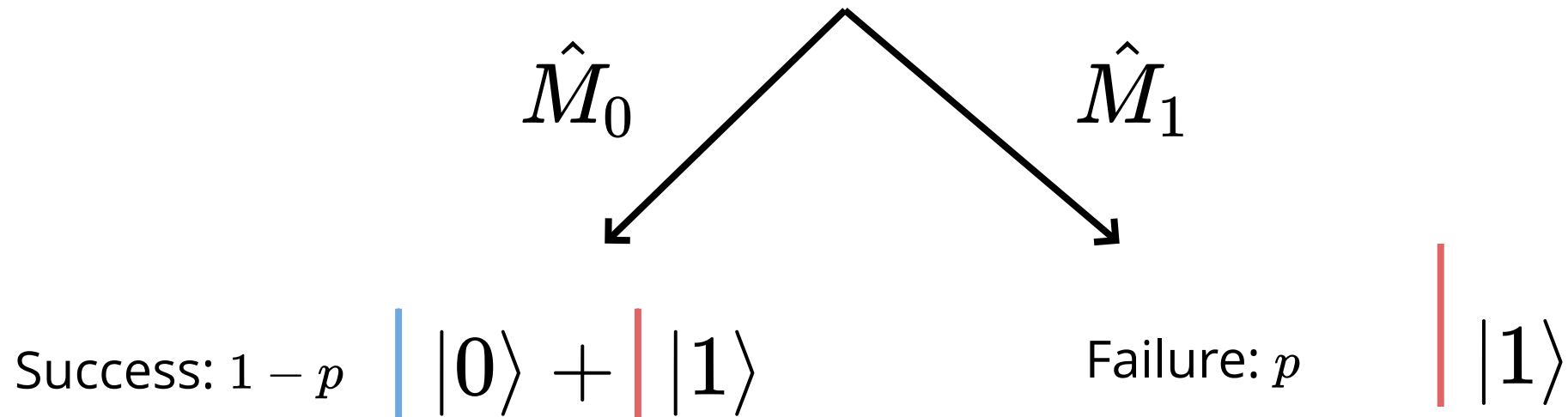
The qubit is: $|0\rangle + |1\rangle$ Amplitude imbalance error

We can correct the qubit by performing weak measurement POVMs M_0, M_1 .

Loss: Finite Squeezing

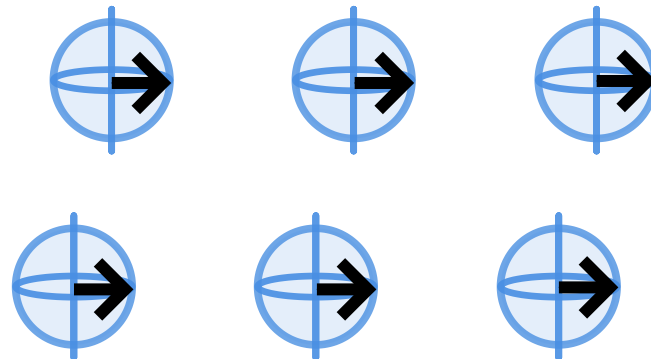
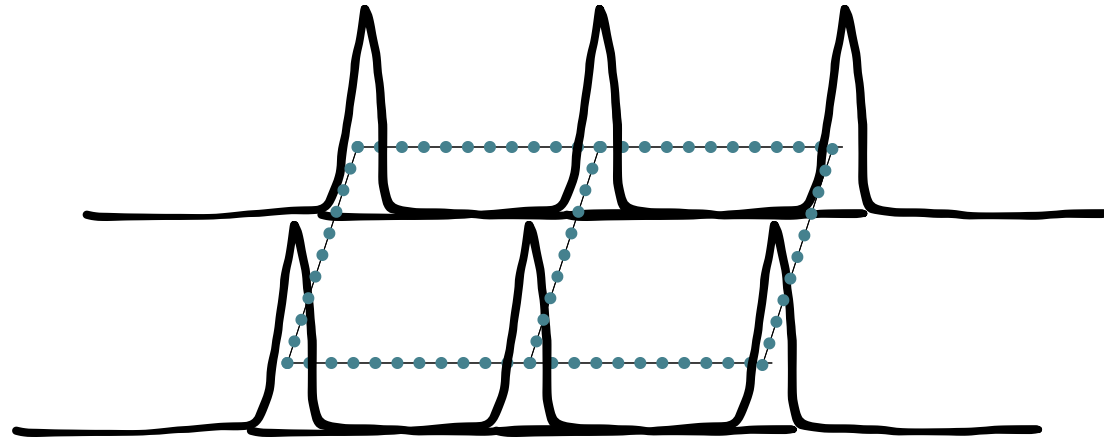
The qubit is: $|0\rangle + |1\rangle$ Amplitude imbalance error

We can correct the qubit by performing weak measurement POVMs M_0, M_1 .



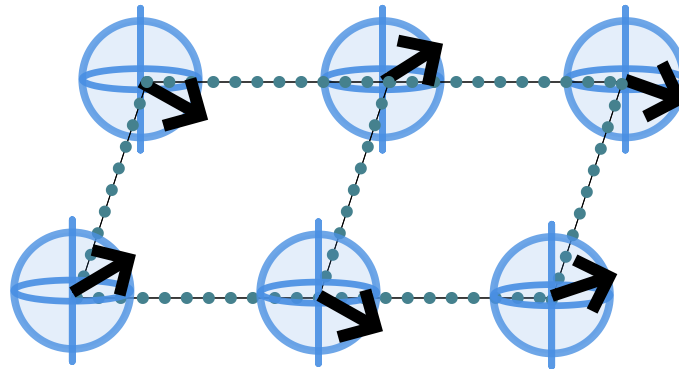
Loss: Finite Squeezing

Finitely squeezed CV cluster state



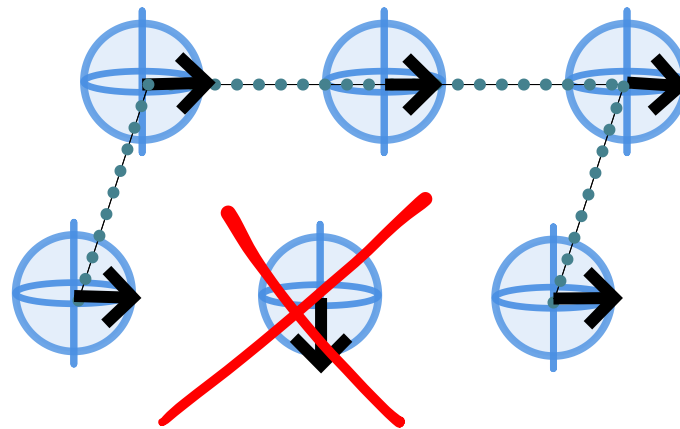
Loss: Finite Squeezing

Amplitude imbalance error!



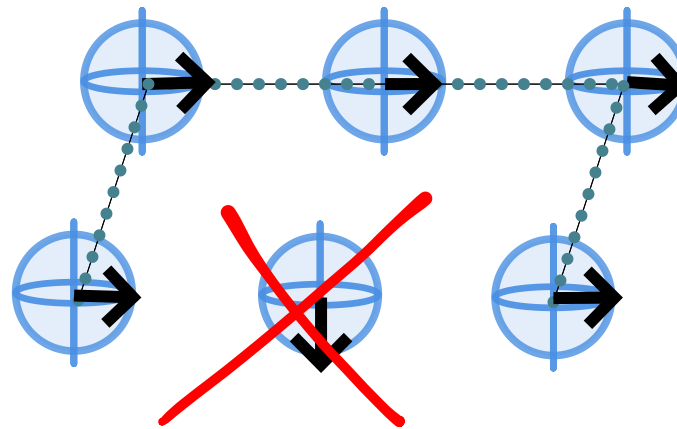
Loss: Finite Squeezing

After weak measurement:



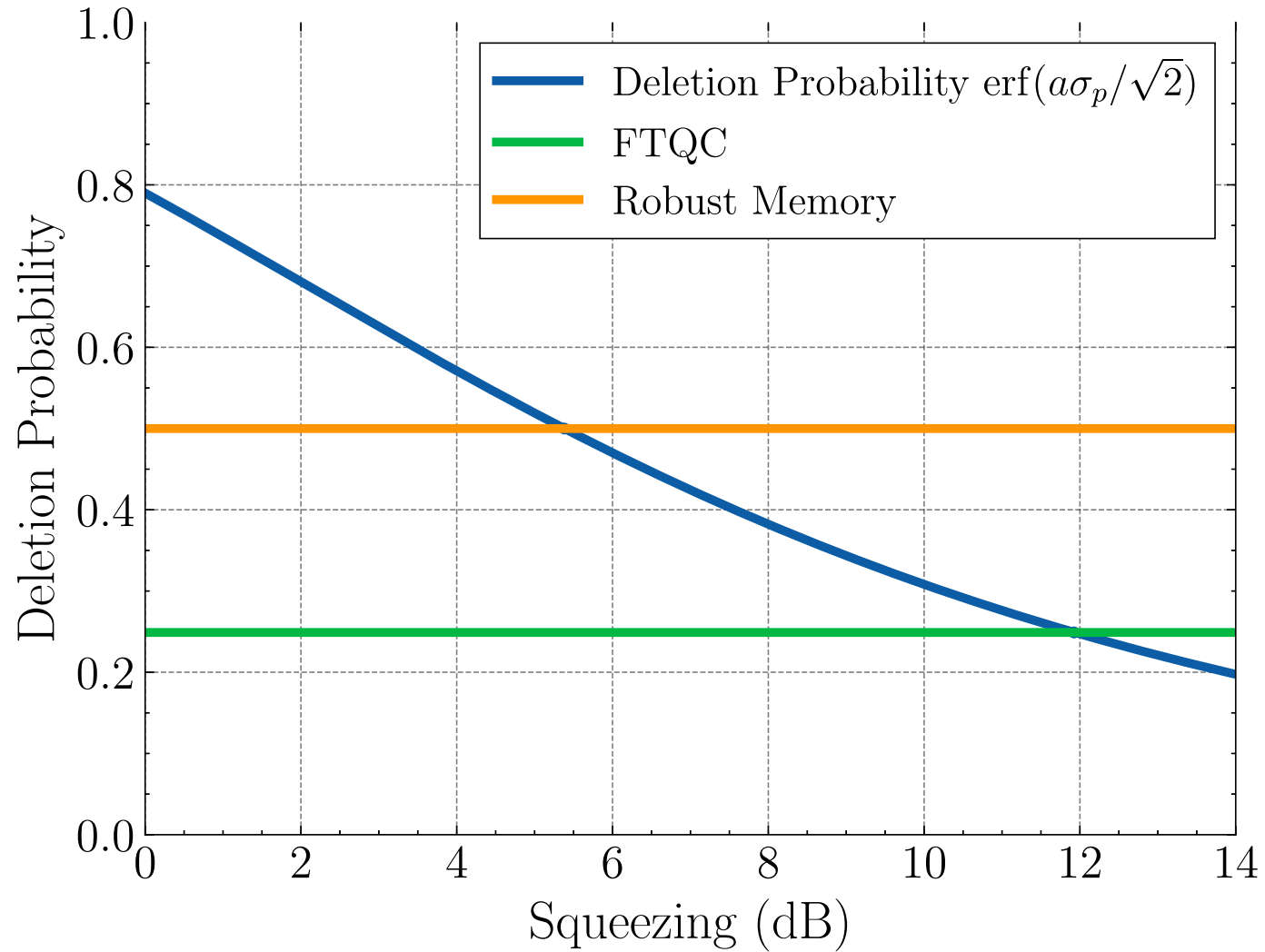
Loss: Finite Squeezing

Can convert initial squeezing error to deletion error!



Failure: p

Loss: Finite Squeezing

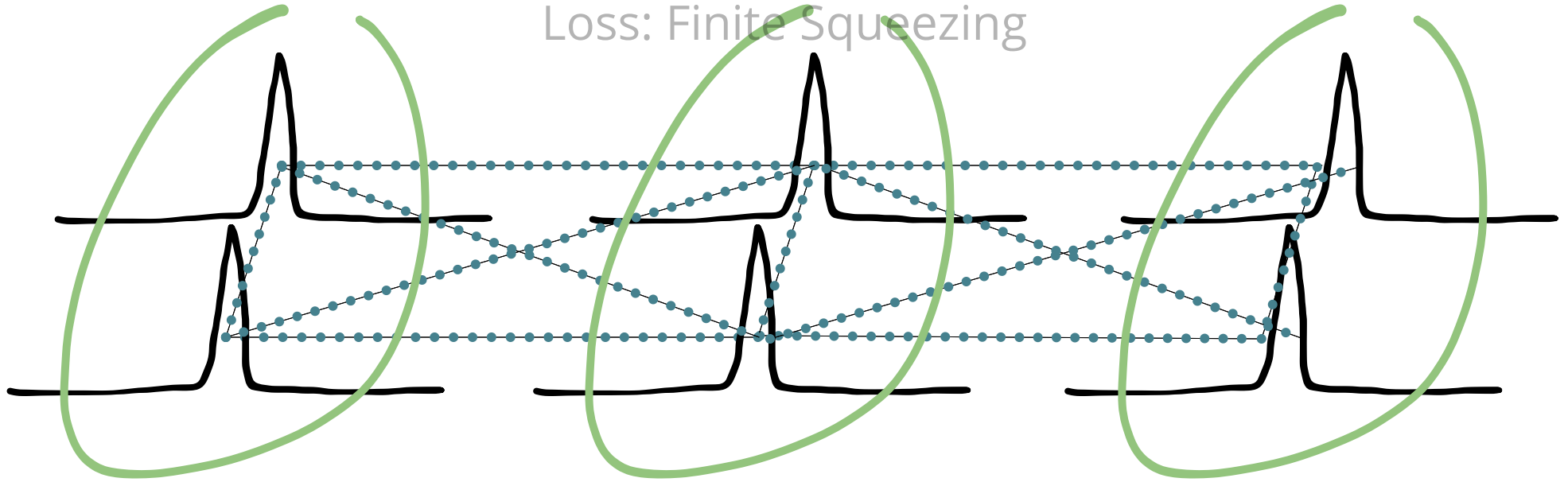


(Stace 2009)

(Barrett and
Stace 2010)

Failure: p

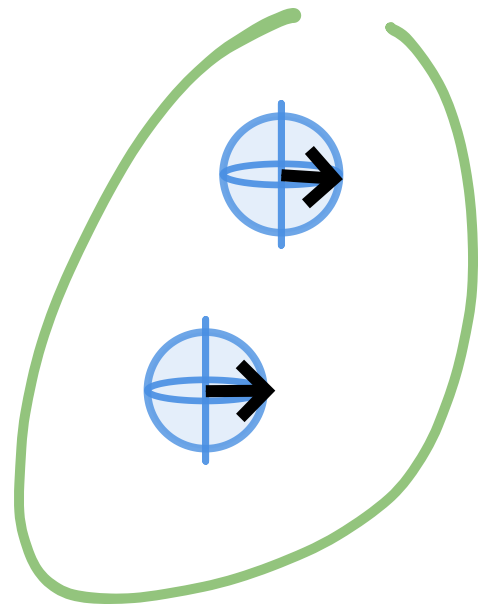
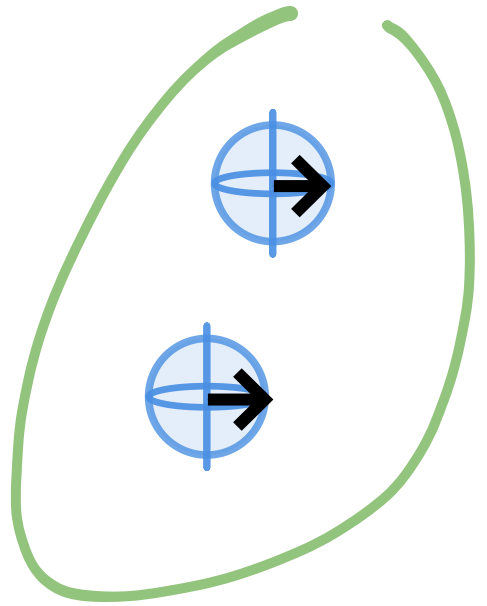
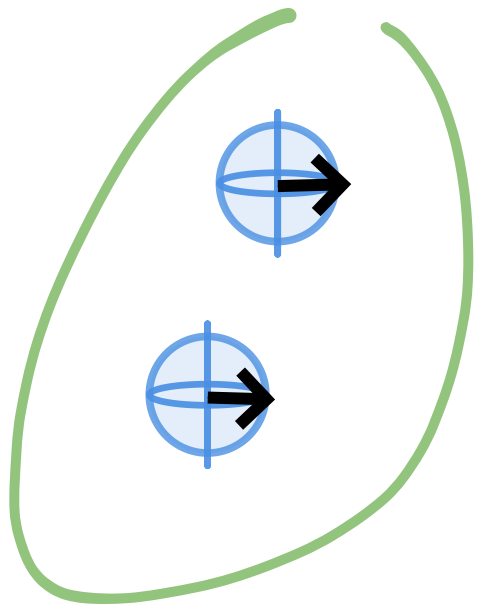
Loss: Finite Squeezing



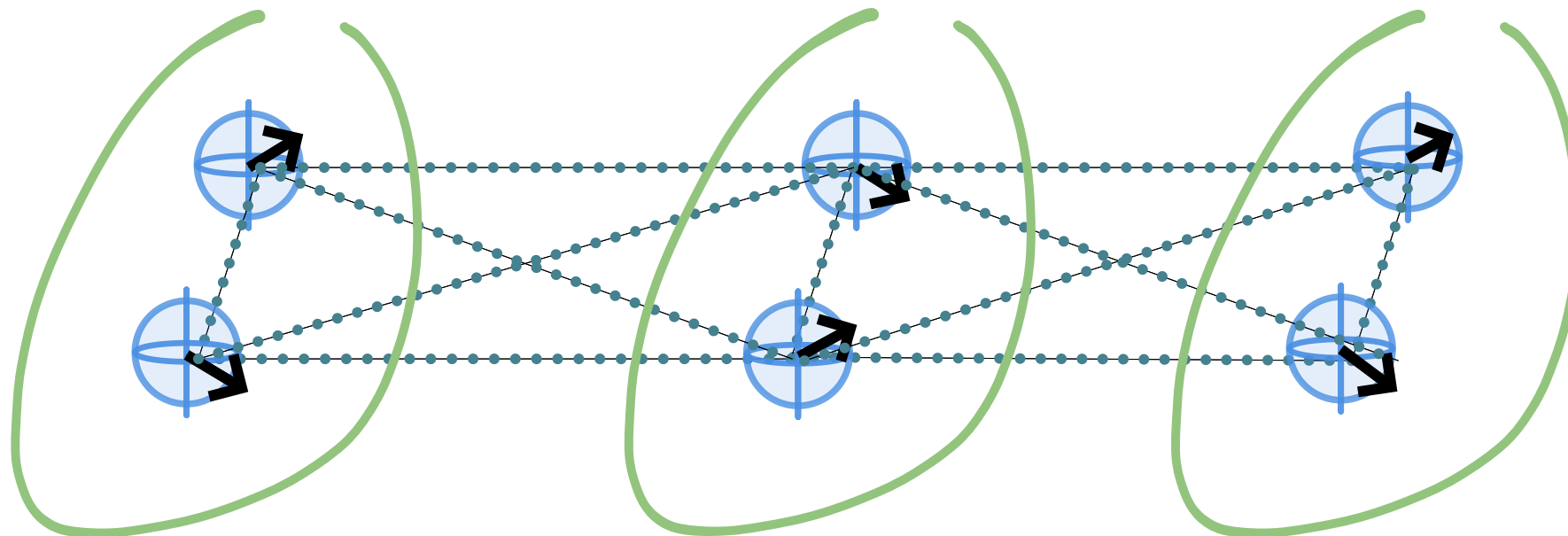
Site 1

Site 2

Site 3

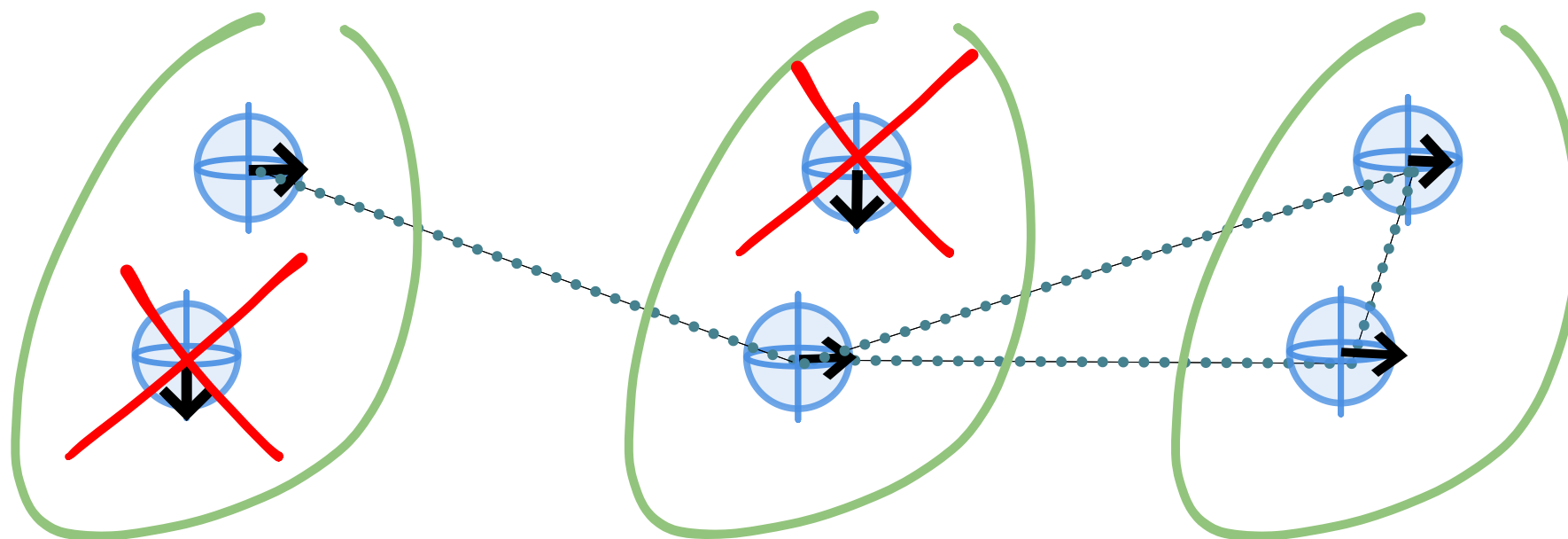


Loss: Finite Squeezing



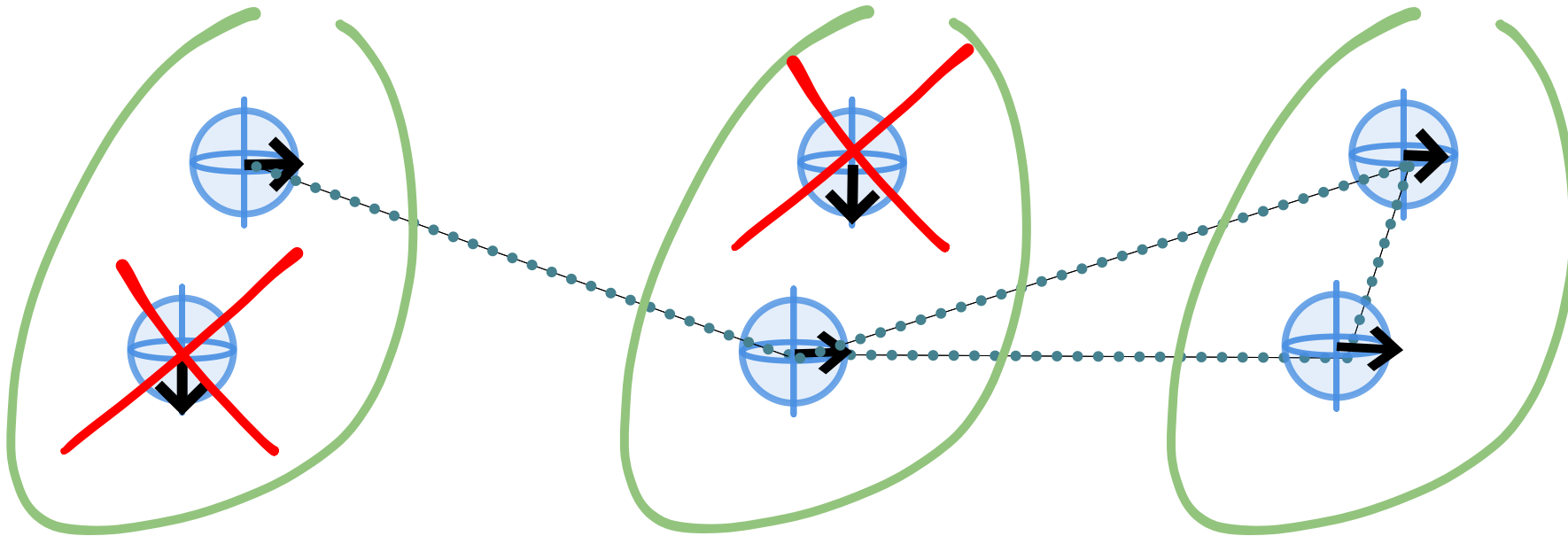
Loss: Finite Squeezing

After weak measurement:



Loss: Finite Squeezing

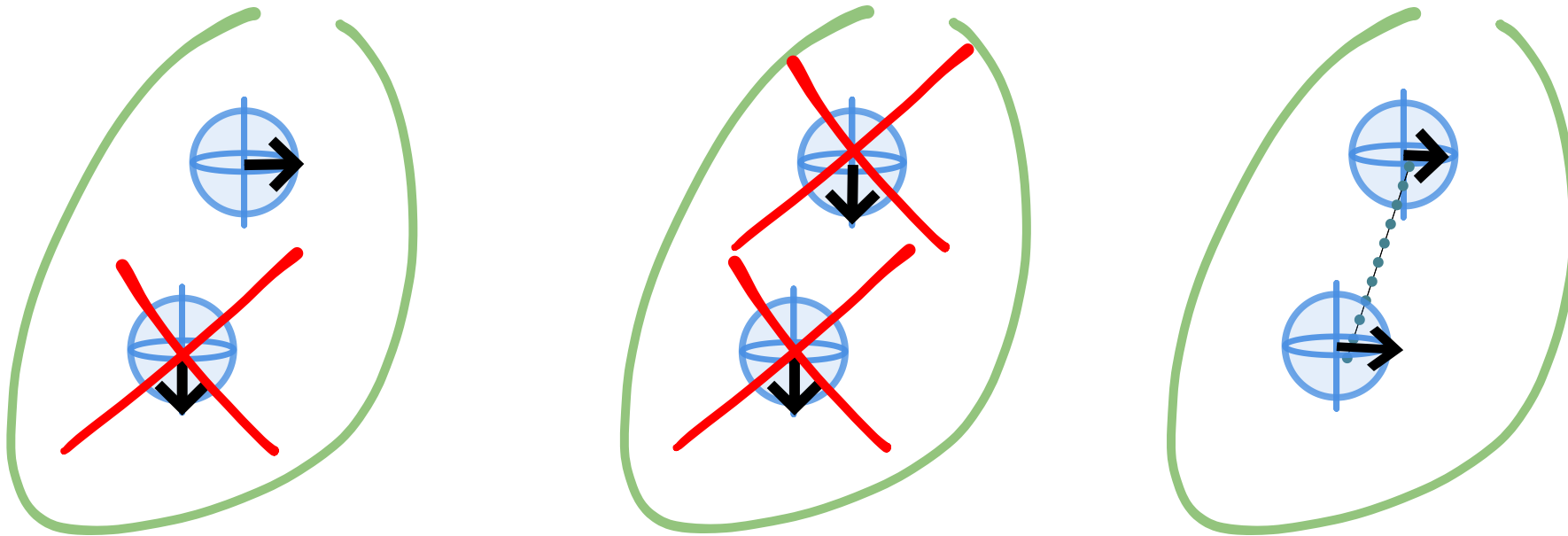
Dual rail encoding (n=2)



In order to break entanglement between site 1 and 3 both qubits has to be deleted.

Loss: Finite Squeezing

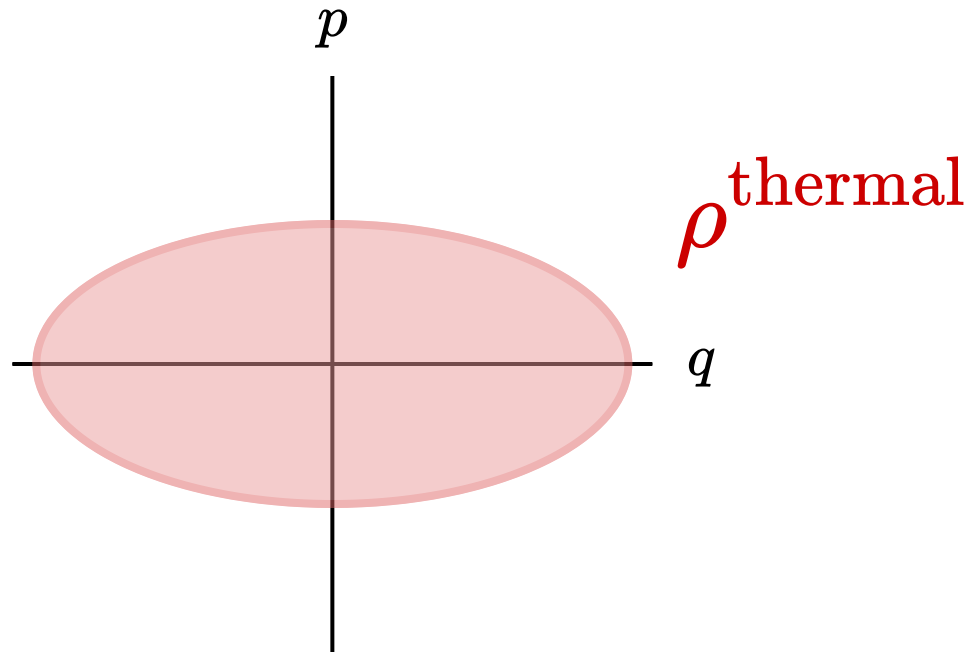
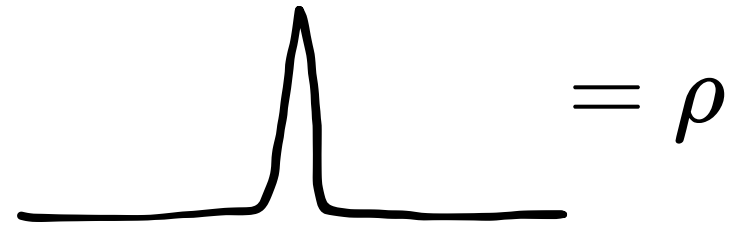
Dual rail encoding (n=2)



Deletion probability of a site: p^n

Loss: Channel and detector loss

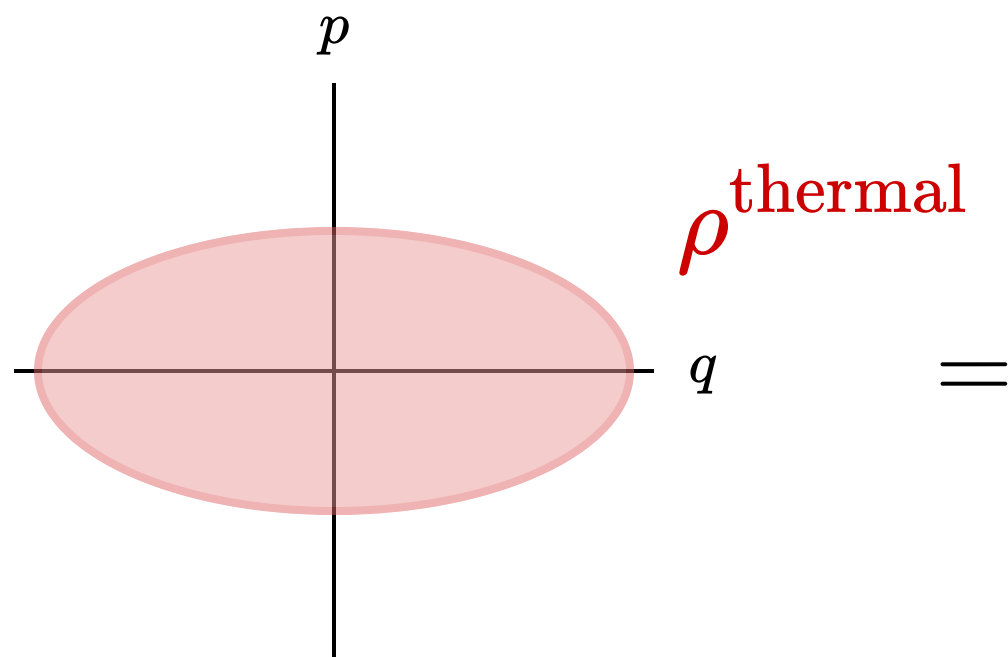
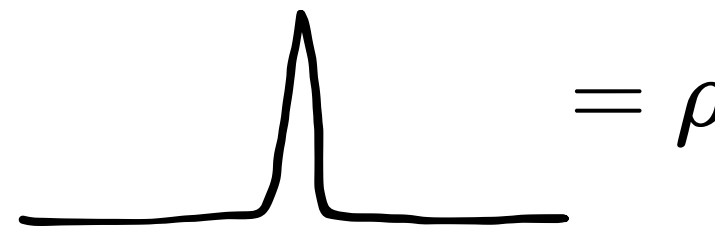
What happens to the qubit if you send in a squeezed thermal state?



Squeezed thermal state

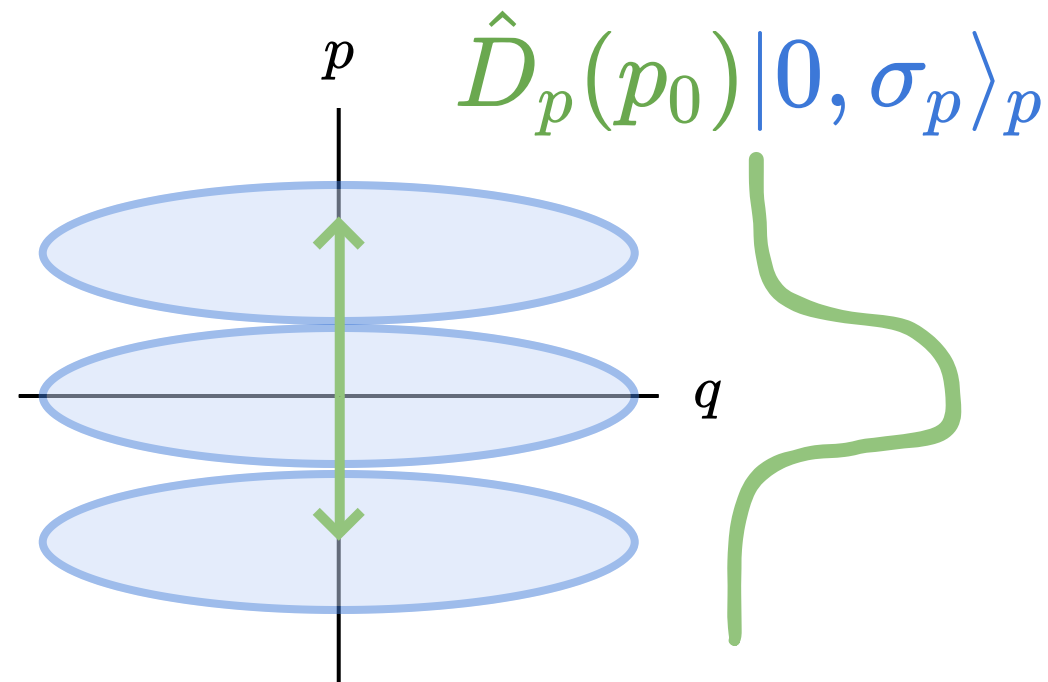
Loss: Channel and detector loss

What happens to the qubit if you send in a squeezed thermal state?



Squeezed thermal state

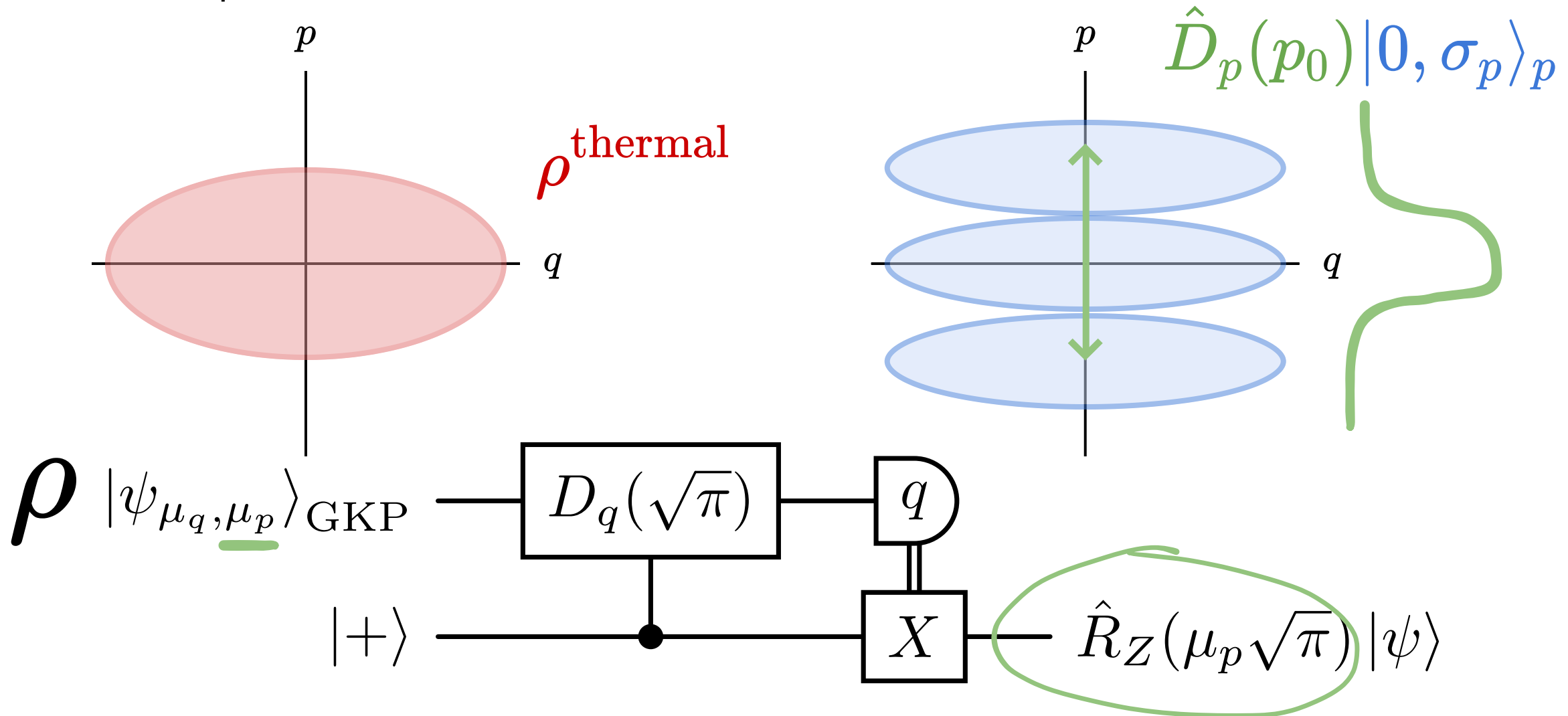
=



Mixture of squeezed states

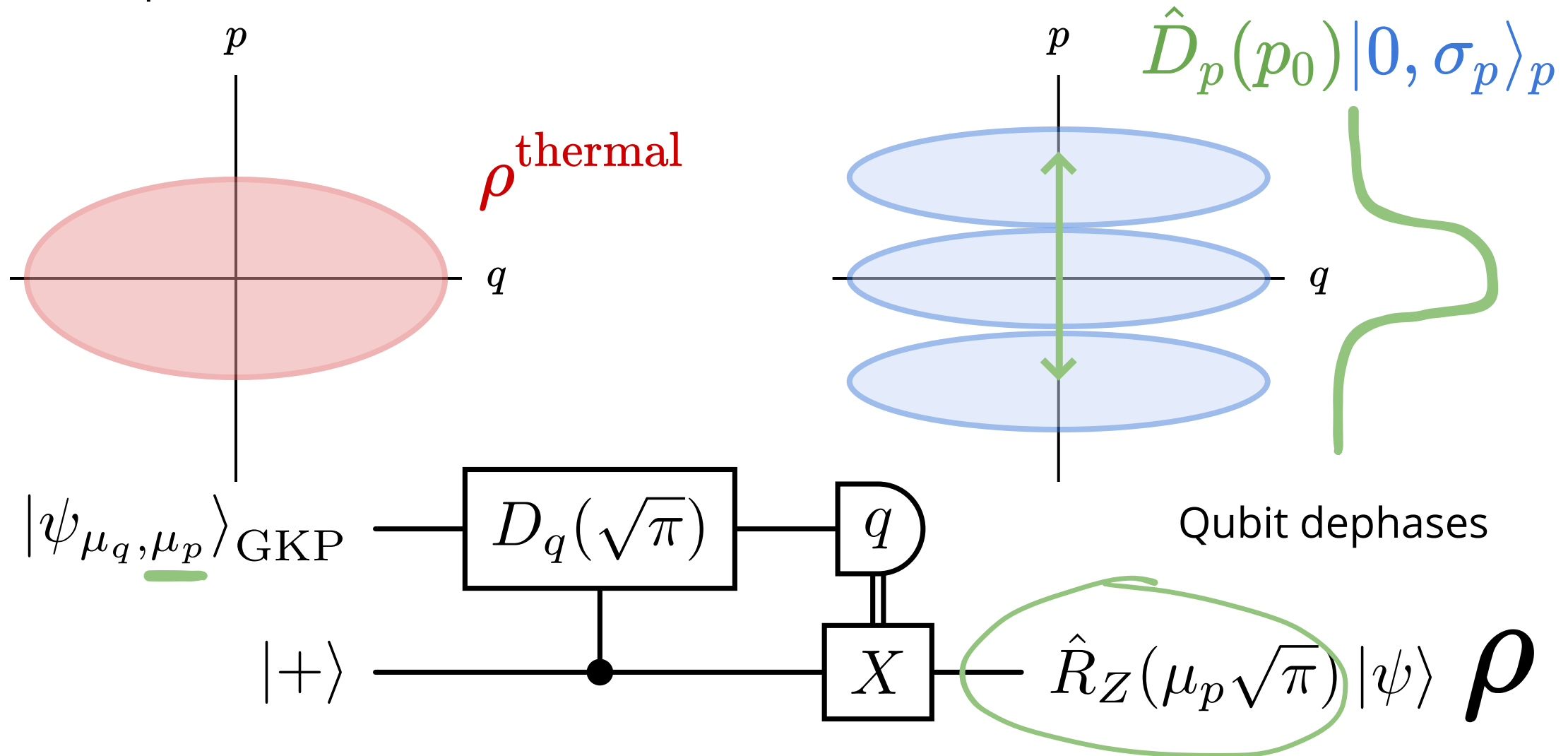
Loss: Channel and detector loss

What happens to the qubit if you send in a squeezed thermal state?



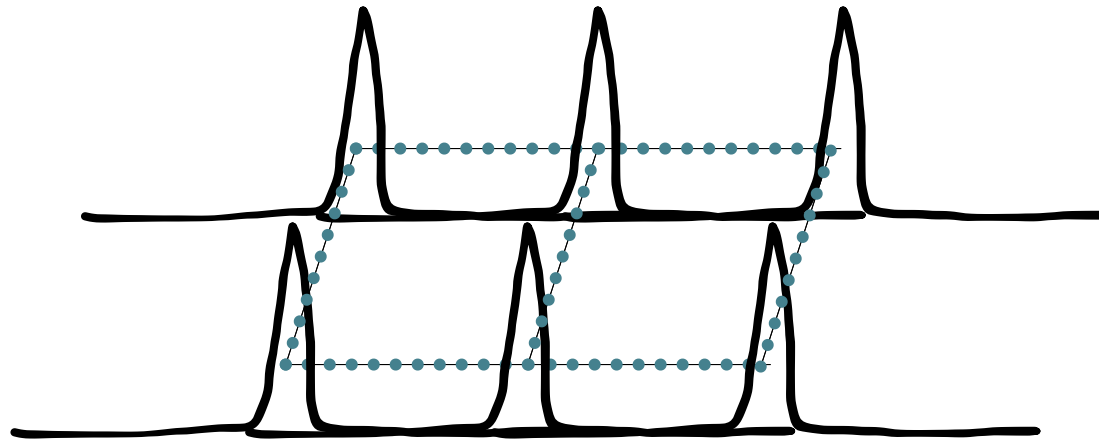
Loss: Channel and detector loss

What happens to the qubit if you send in a squeezed thermal state?



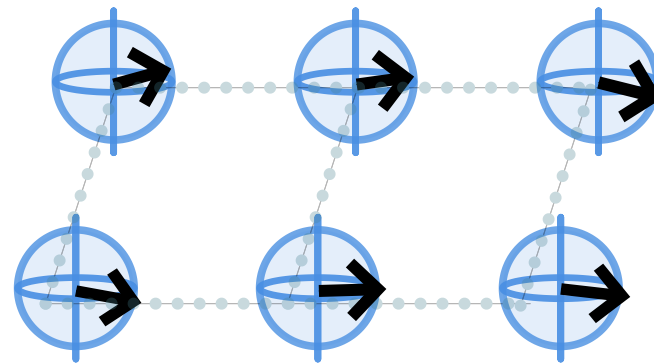
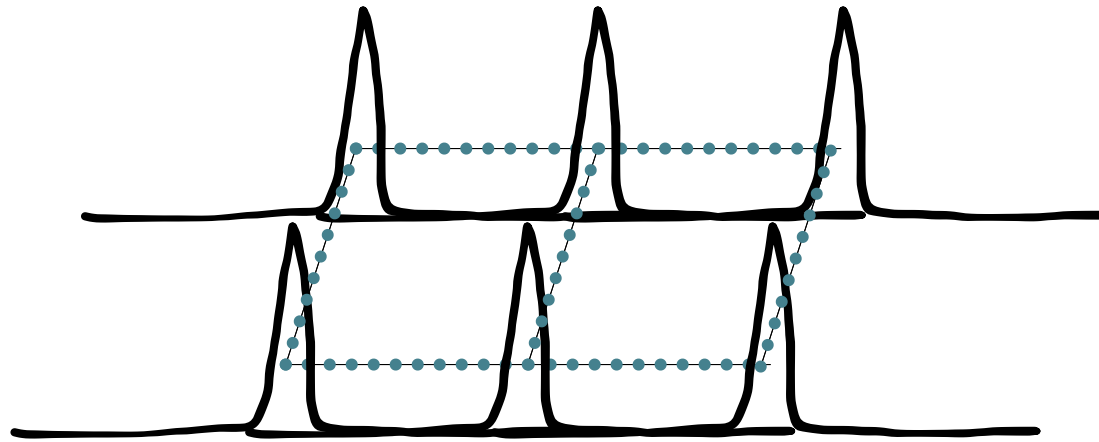
Weak conditional displacement

Suppose \hat{C}_D is 3 times weaker



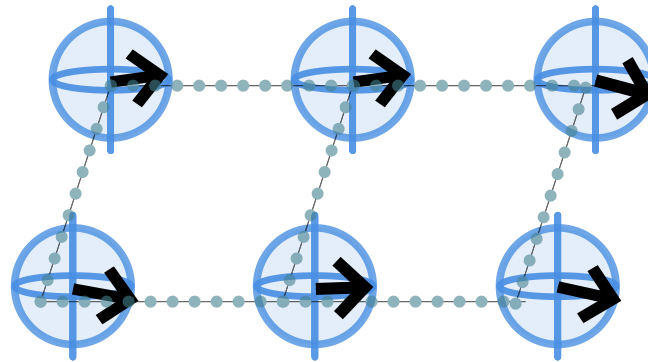
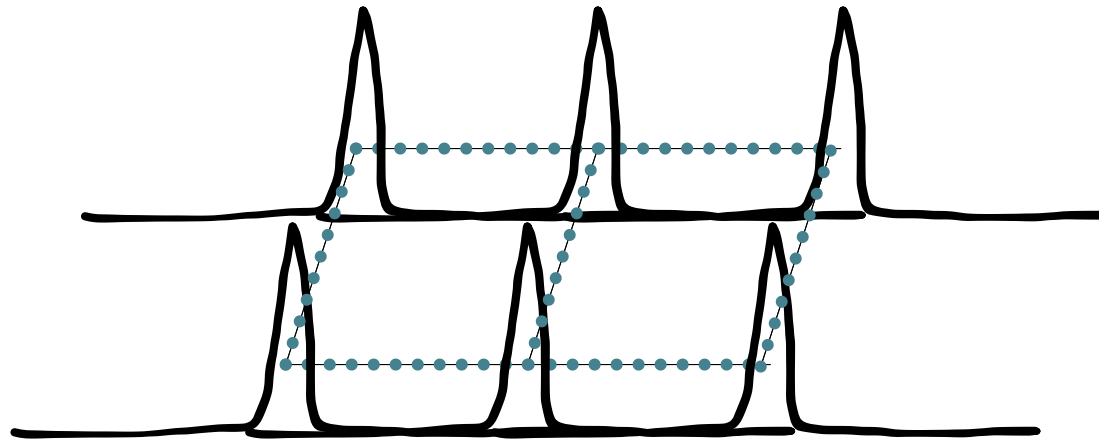
Weak conditional displacement

Suppose \hat{C}_D is 3 times weaker



Weak conditional displacement

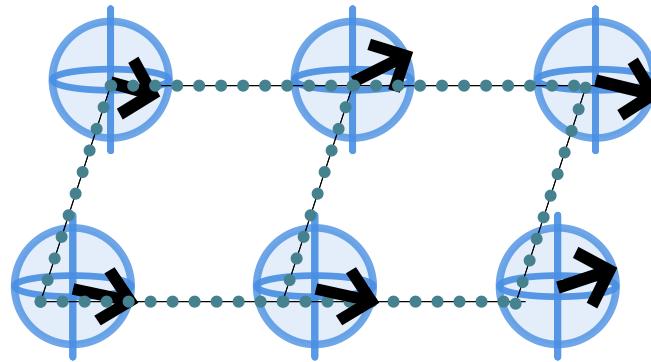
Suppose \hat{C}_D is 3 times weaker



Weak conditional displacement

Suppose \hat{C}_D is 3 times weaker

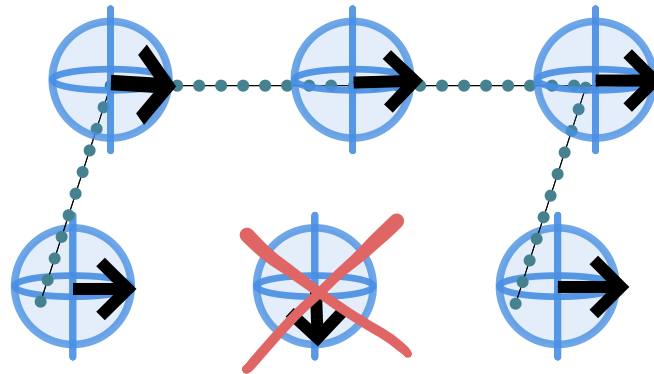
Weak conditional displacement can be cancelled out by performing entanglement transfer more times.



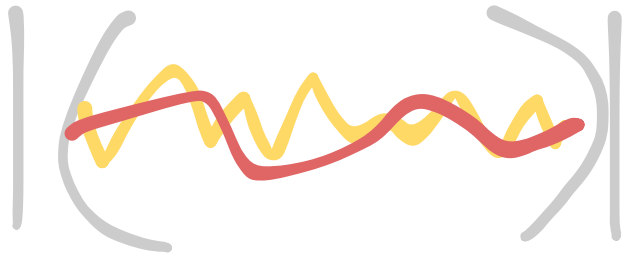
Weak conditional displacement

Suppose \hat{C}_D is 3 times weaker

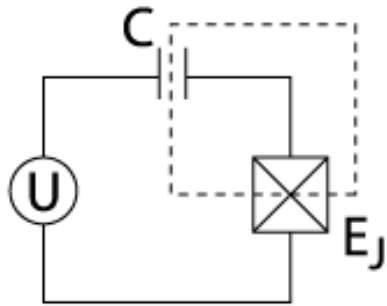
Only one round of weak measurement
correction.



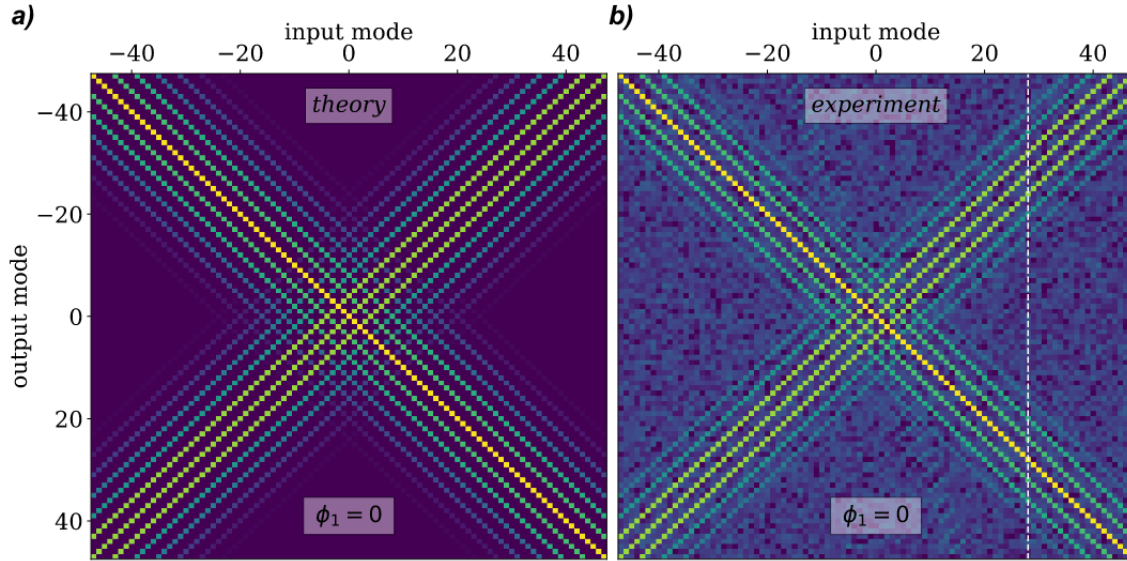
Possible implementations: Superconducting qubits



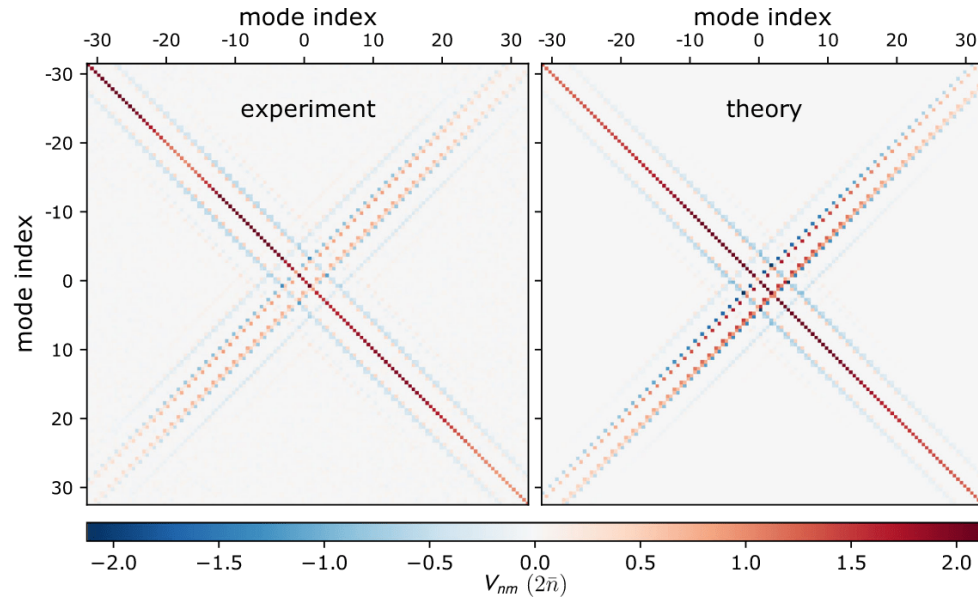
CV cluster: Frequency comb in microwave resonator



Transmon

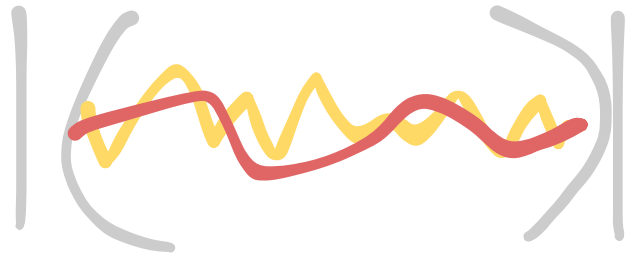


95 correlated modes
(Hernández 2024)

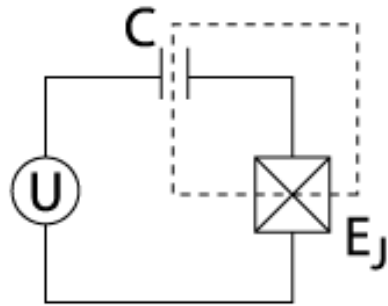


64 correlated modes
(Jolin 2023)

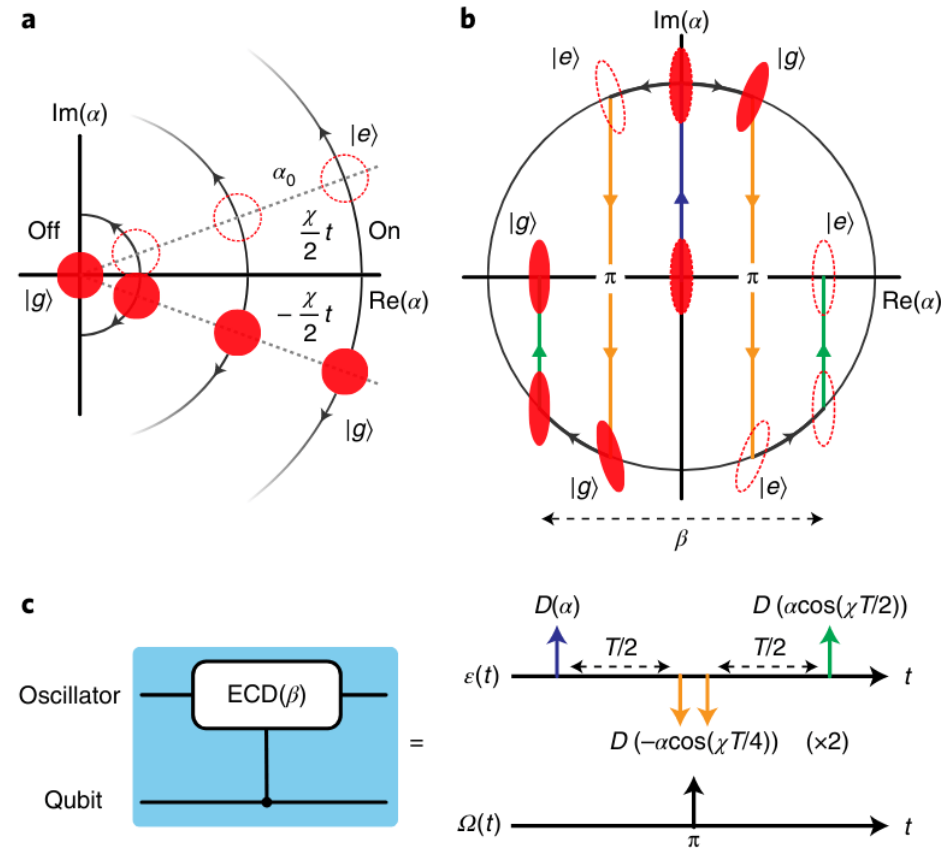
Possible implementations: Superconducting qubits



CV cluster: Frequency comb in microwave resonator

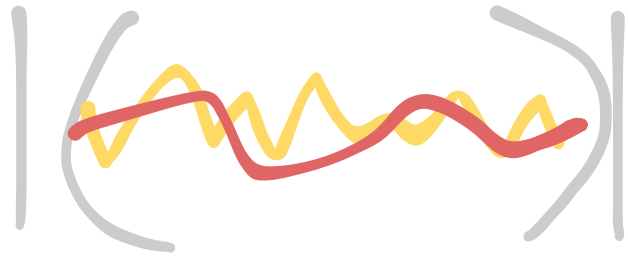


Transmon

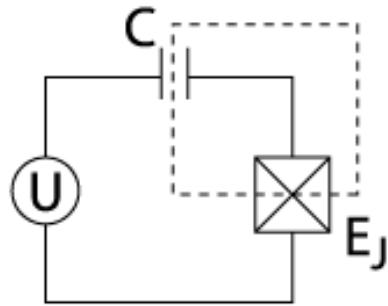


Conditional displacement:
ECD gate (A. Eickbusch 2018)

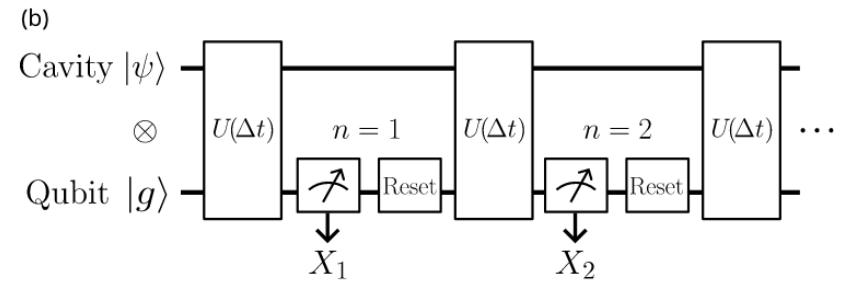
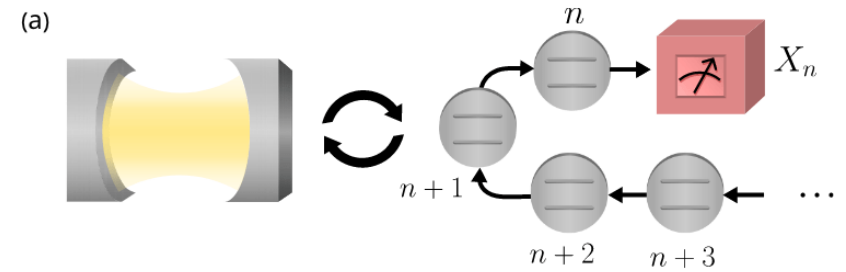
Possible implementations: Superconducting qubits



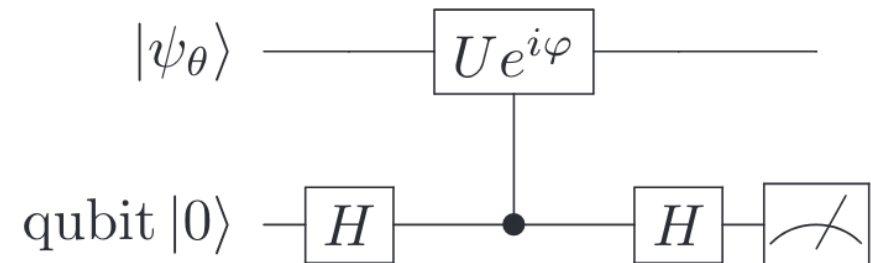
CV cluster: Frequency comb in microwave resonator



Transmon

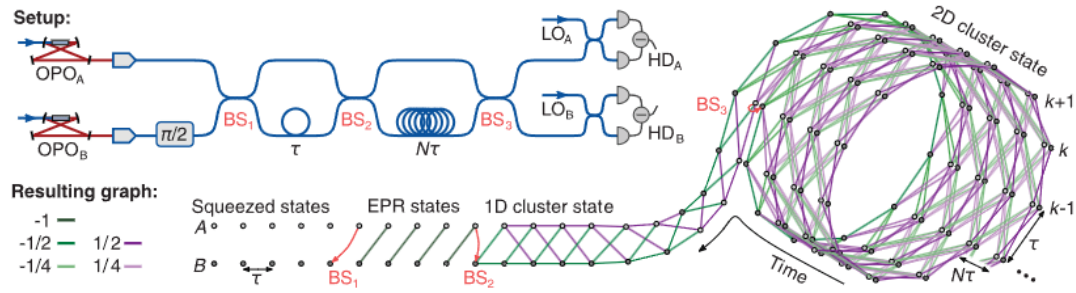


Qubitdyne detection (Strandberg 2023)



Quantum Phase Estimation (Terhal and Weigand 2016)

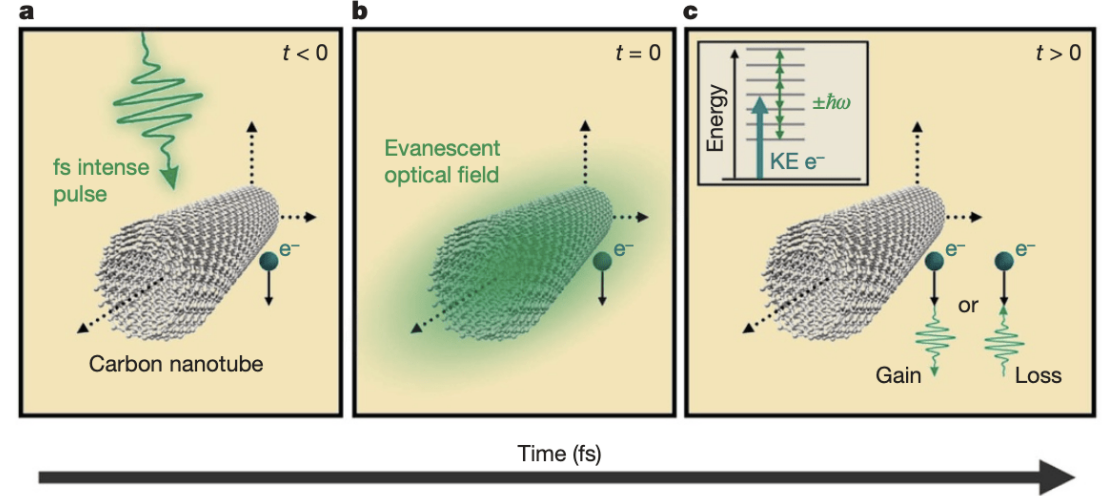
Possible implementations: Free electron qubits



CV cluster: Furusawa protocol



Free electron qubits
(Reinhardt 2021,
Baranes 2024)



CD gate: Photon-induced near-field electron microscopy (Barwick 2009)



Homodyne detection

Possible implementations: Summary

	Superconducting qubit + microwave cavity	Free electron qubits
CV cluster state	Frequency comb in cavity	Optics
Conditional displacement	Echoed conditional displacement gate (ECD gate)	PINEM (photon-induced near field electron microscopy)
Homodyne detection	Quantum phase estimation Qubitdyne detection	Homodyne detection
Qubit	Transmon	Free electrons



Downloading many-body continuous variable entanglement to qubits

- **We can make many body entanglement in qubits!!**
- Entanglement transfer from CV cluster state to qubit cluster state is possible
- Quality of the qubit cluster state depends on the initial state
- Weak measurement protocol and qubit deletion protocol can reduce requirements
- 6dB squeezing for robust quantum memory
- 12dB squeezing for fault tolerant quantum computing
- No GKP states needed in protocol
- arXiv in progress

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