

# Gravitational Laboratories for Nuclear Physics

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2024 CAP Congress  
Computational Advances in Astrophysics and Cosmology

# Outline

Types of observations → set up the inverse problem

Difficulties with inverse problem

[PRD 105, 043016 \(2022\)](#)

Parametric model → reduce dimensionality, traditional sampling methods

Nonparametric representation → high dimensionality, novel sampling methods

Incorporating theoretical predictions within the prior

[PRC 102, 055803 \(2020\)](#)

Exotic high-density behavior with efficient TOV sequences

[arXiv:2405.05395 \(2024\)](#)

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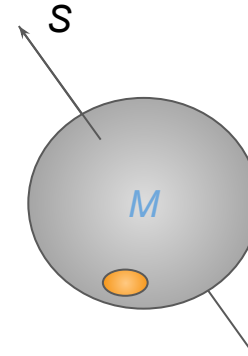
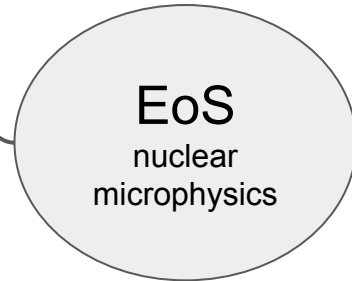
Exotic high-density behavior with efficient TOV sequences

arXiv:2405.05395 (2024)

# NS Observables: mass

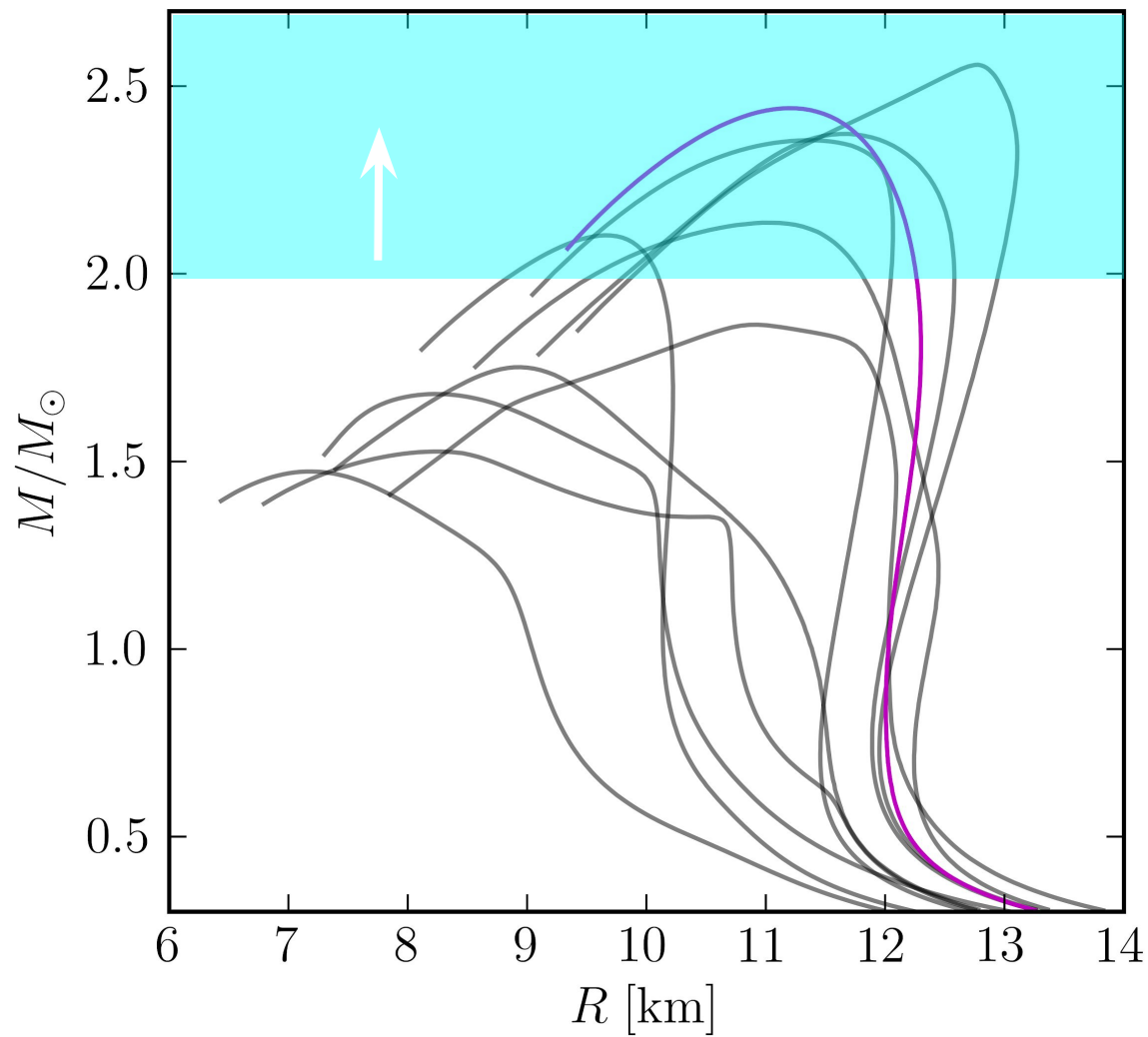
$M_{max}$

Massive pulsars (PSRs)  
J0740+6620 Cromartie+(2019)  
Fonseca+(2021)

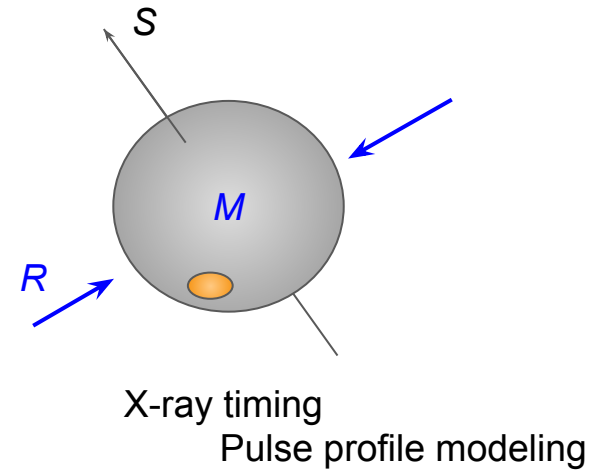
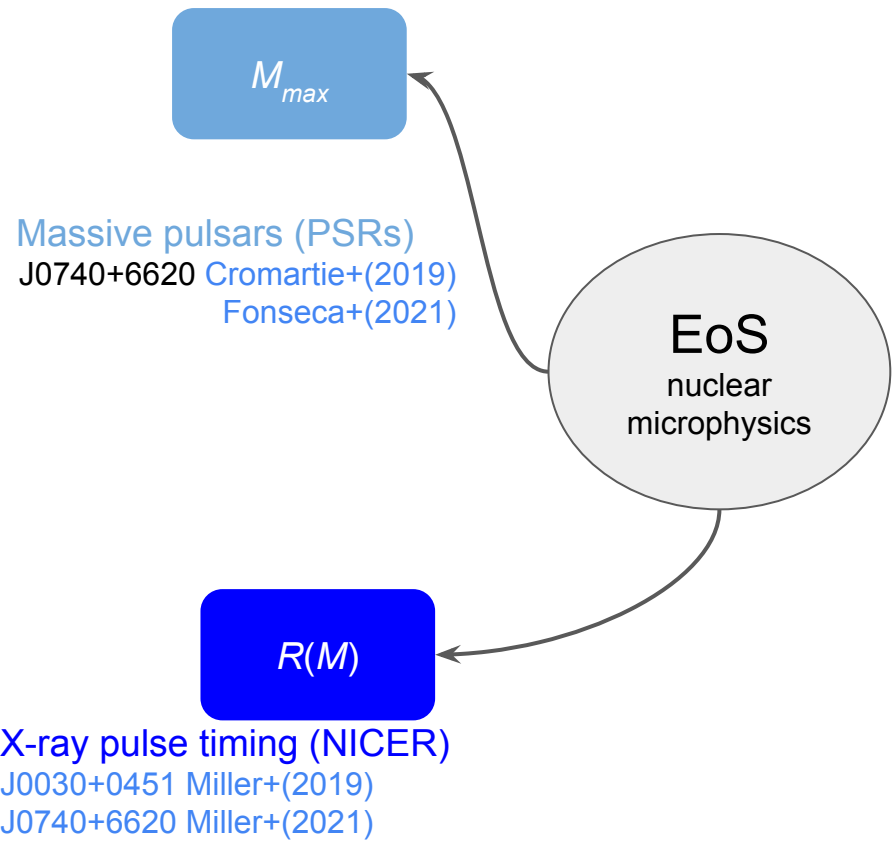


pulsar observations  
radial velocity  
**Shapiro delay**

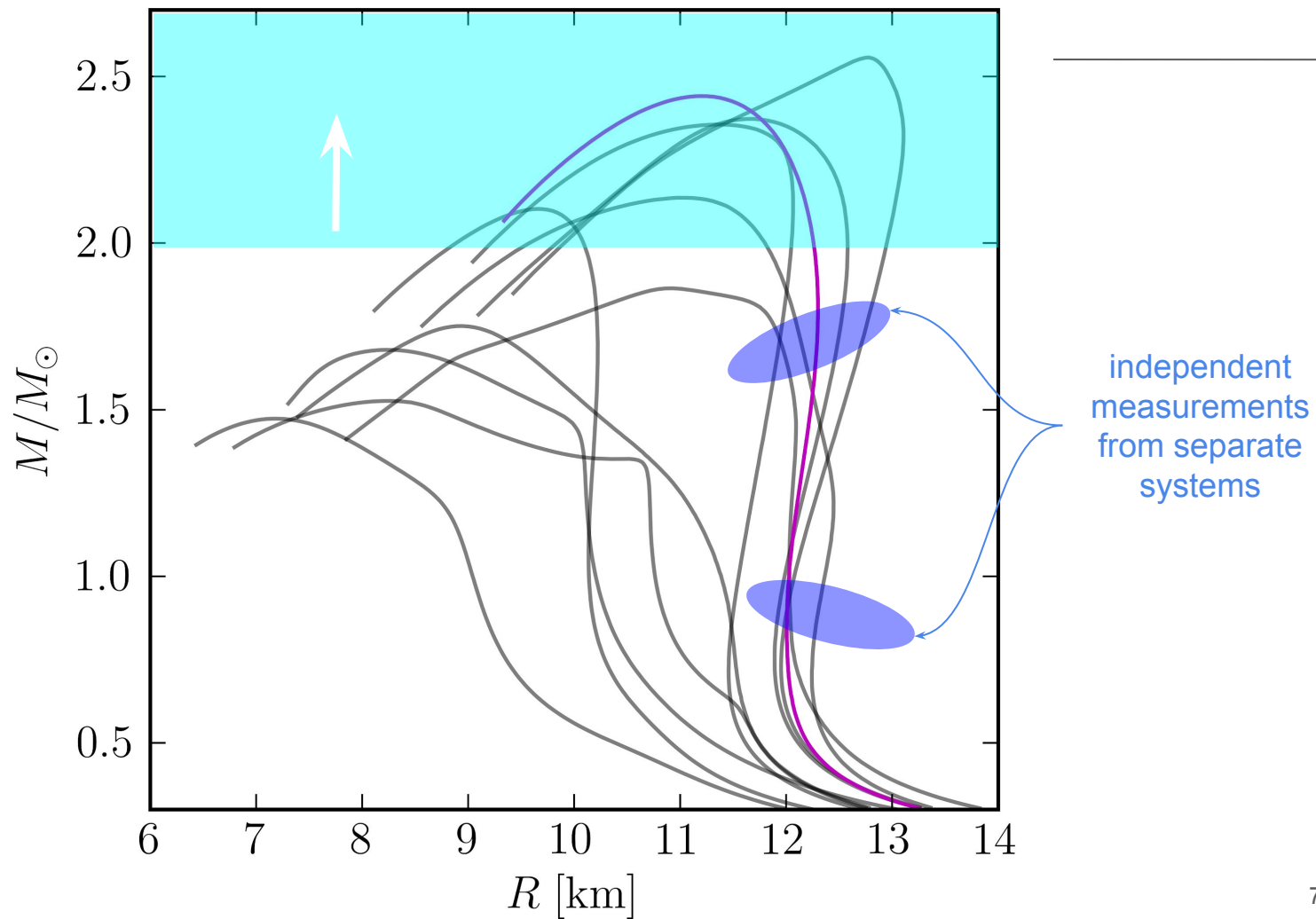
# NS Observables



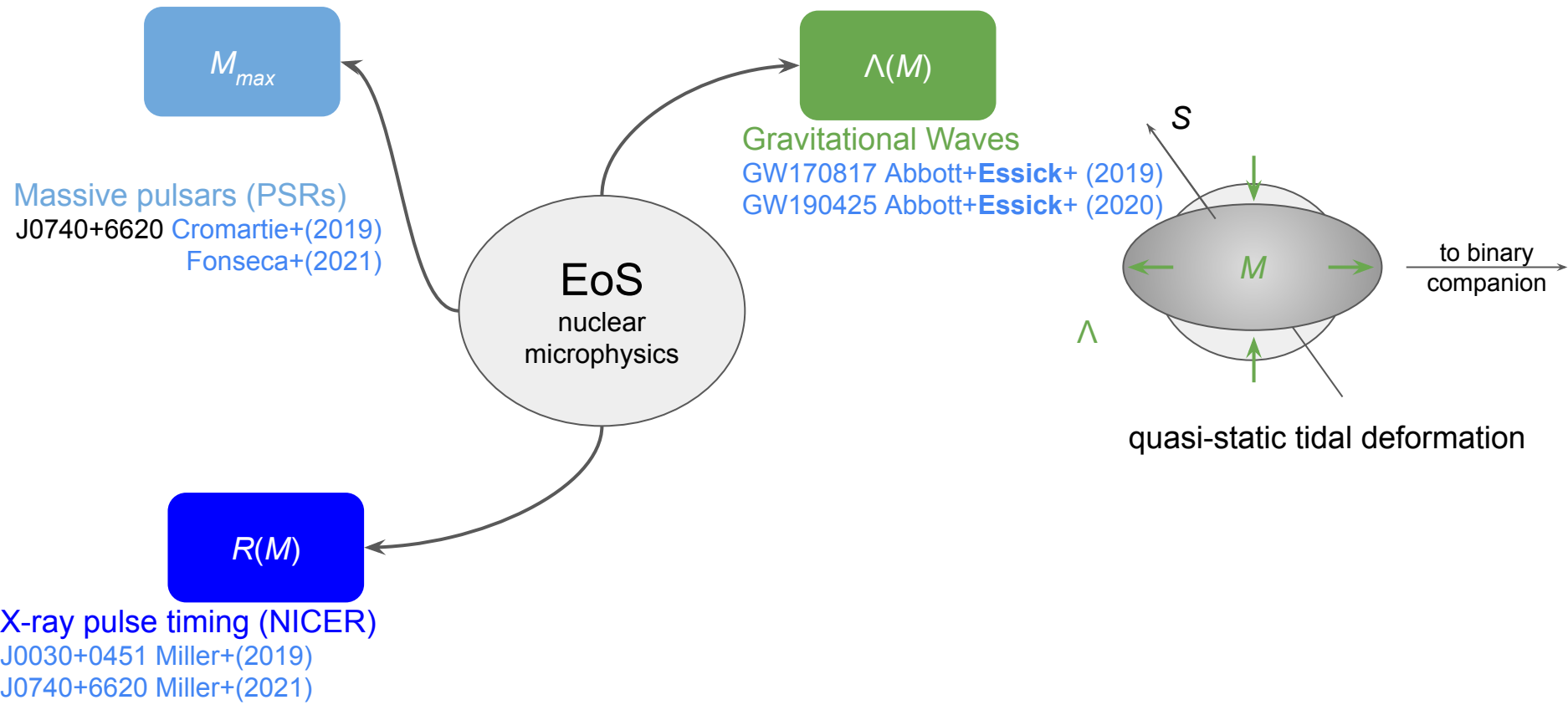
# NS Observables: mass and radius



# NS Observables

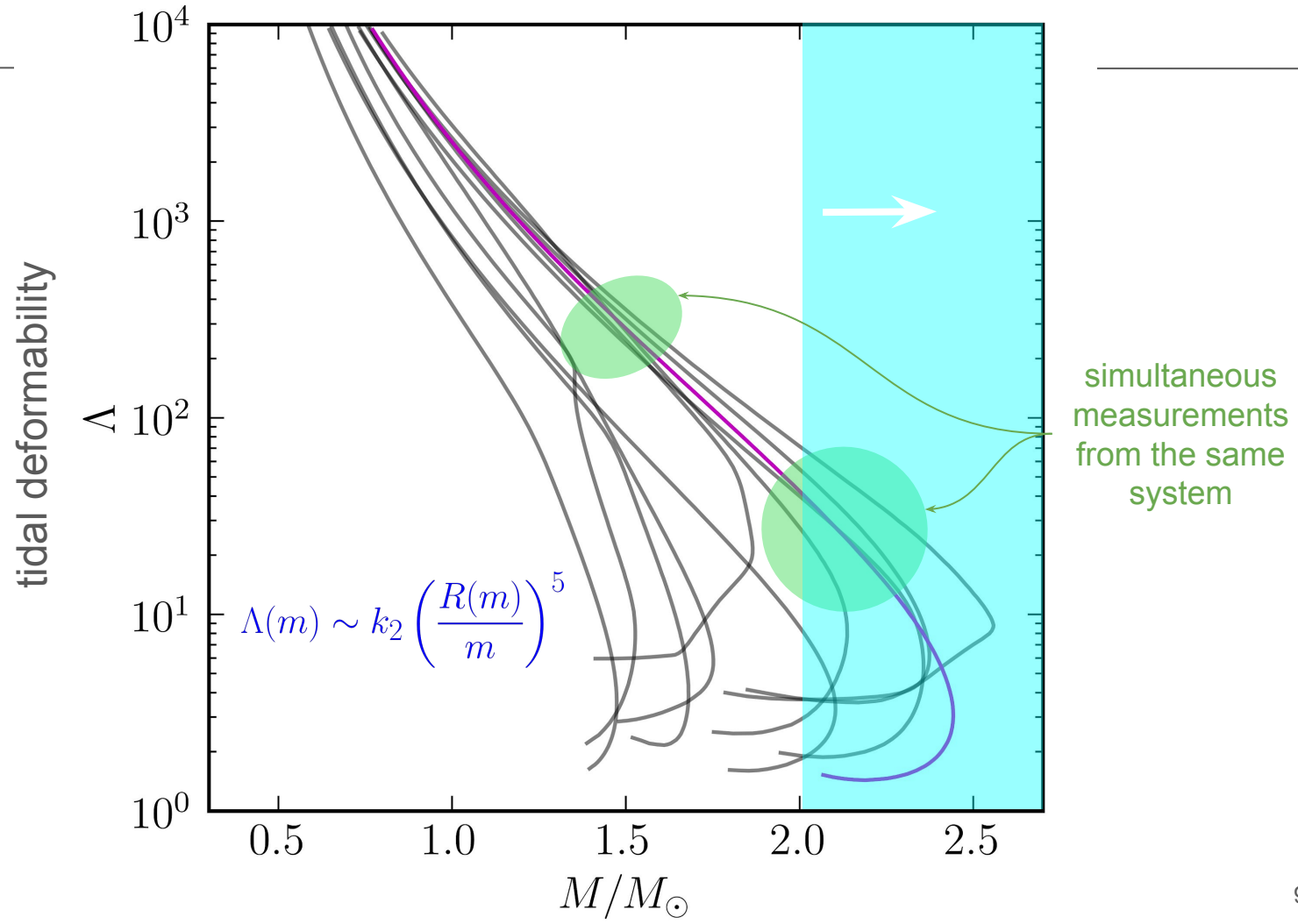


# NS Observables: mass and tidal deformability





# NS Observables



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consider a toy model:

→ fitting a 1D function (pressure vs. energy density)  
without constraints

linear parameterizations

**point+slope**

$$p(\varepsilon) = p_a + c_s^2(\varepsilon - \varepsilon_a)$$

**two-point**

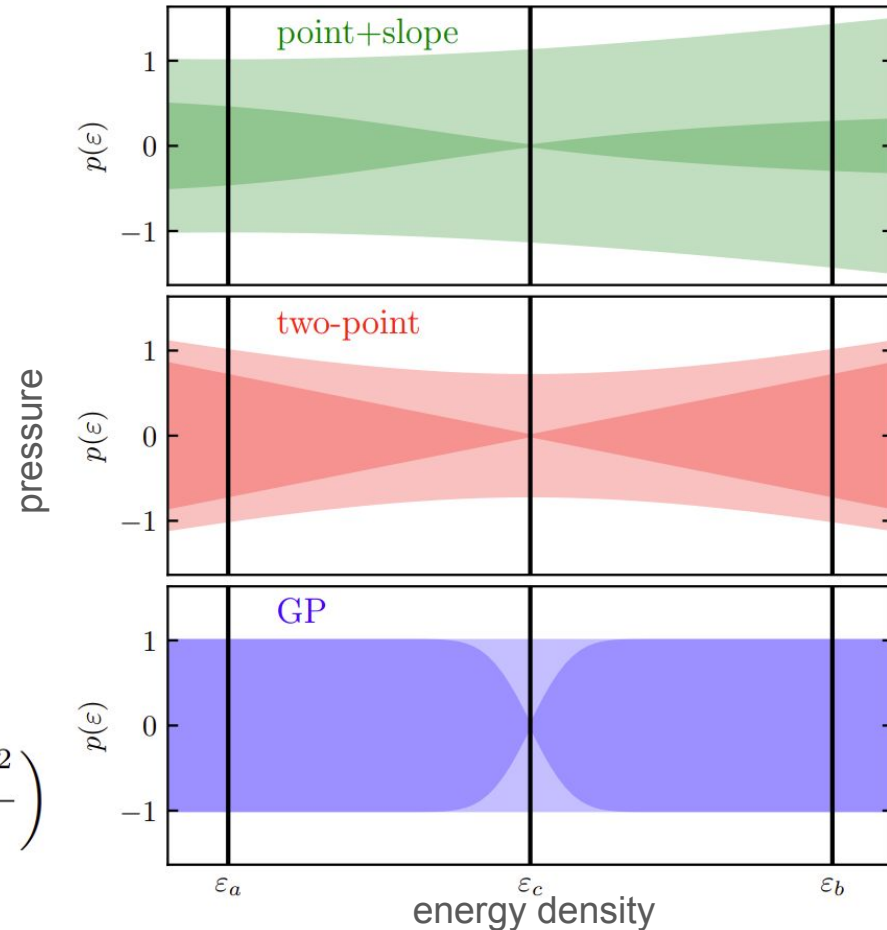
$$p(\varepsilon) = p_a + \frac{p_b - p_a}{\varepsilon_b - \varepsilon_a}(\varepsilon - \varepsilon_a)$$

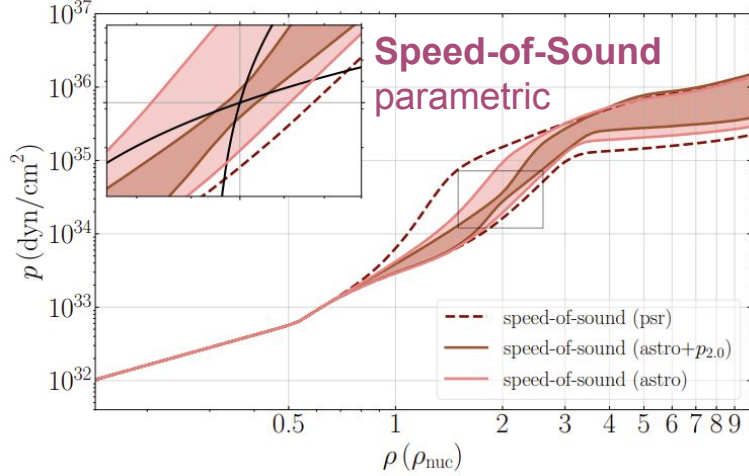
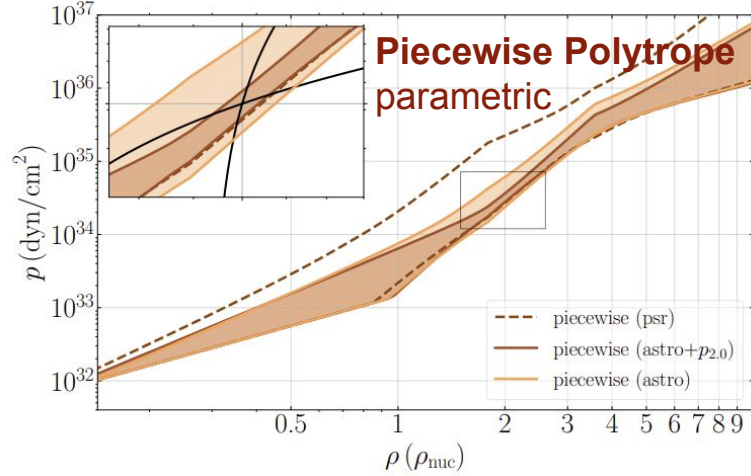
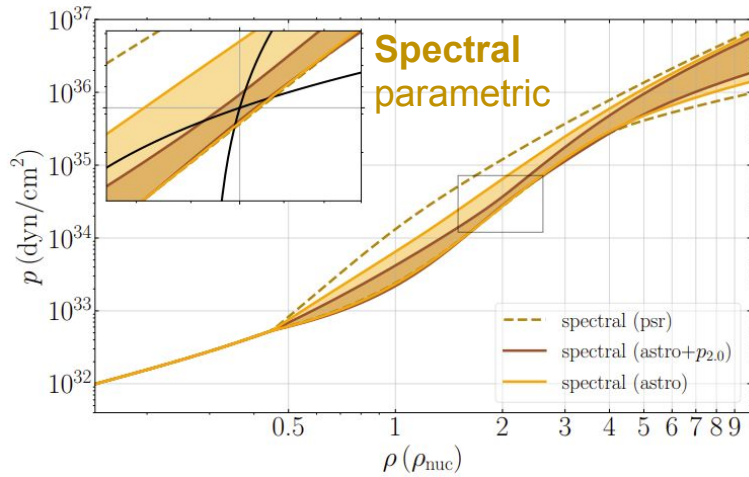
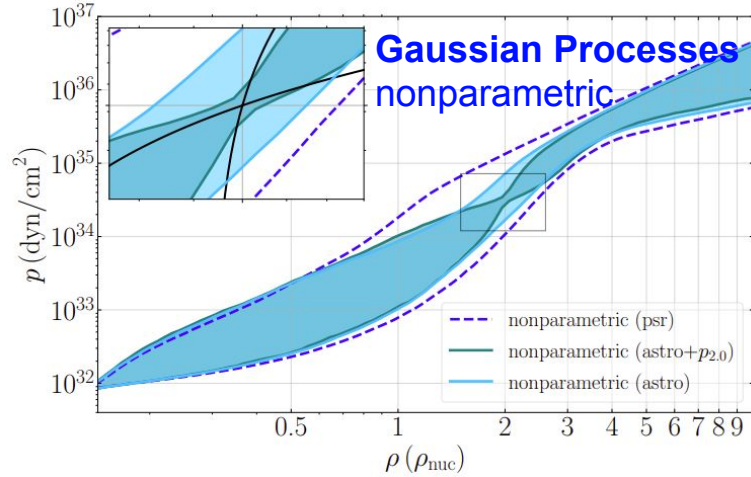
**Gaussian Process**

$$\vec{p} \sim \mathcal{N}(\vec{\mu}, \Sigma)$$

$$\Sigma_{ij} = \text{Cov}(p_i, p_j)$$

$$= K_{\text{se}}(\varepsilon_i, \varepsilon_j) = \sigma^2 \exp\left(-\frac{(\varepsilon_i - \varepsilon_j)^2}{l^2}\right)$$





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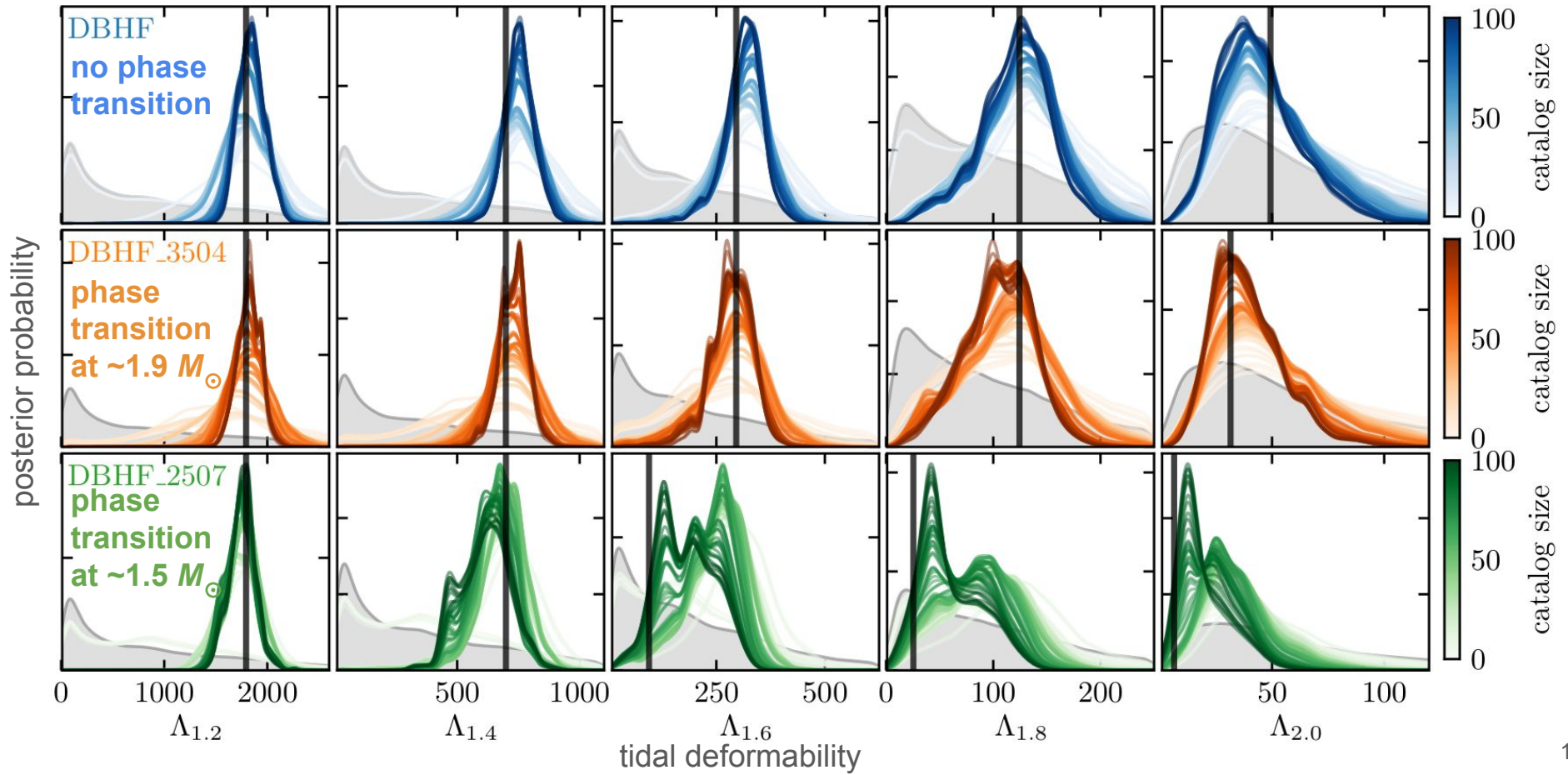
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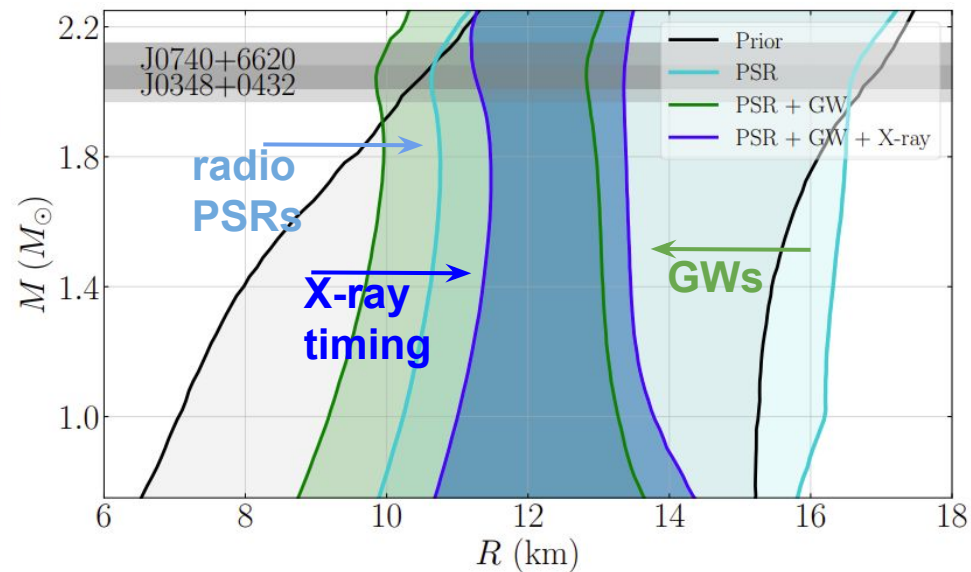
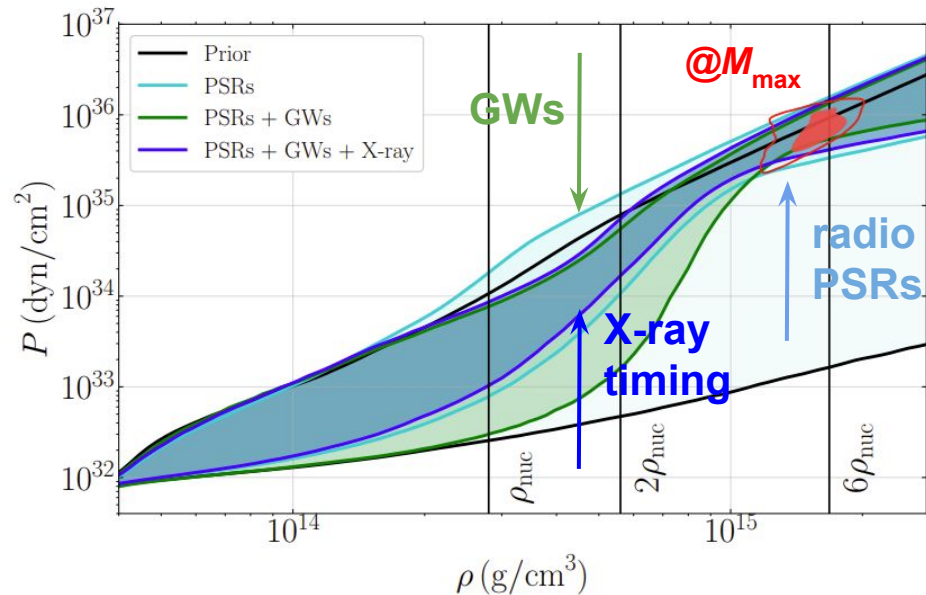
Exotic high-density behavior with efficient TOV sequences

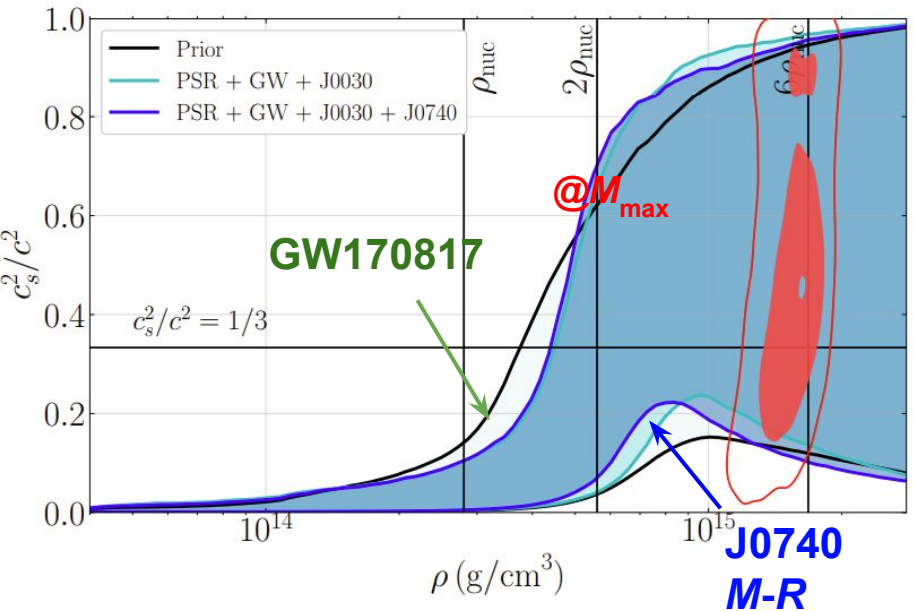
[arXiv:2405.05395 \(2024\)](#)

# Inference of the NS EoS: no systematics with nonparametrics

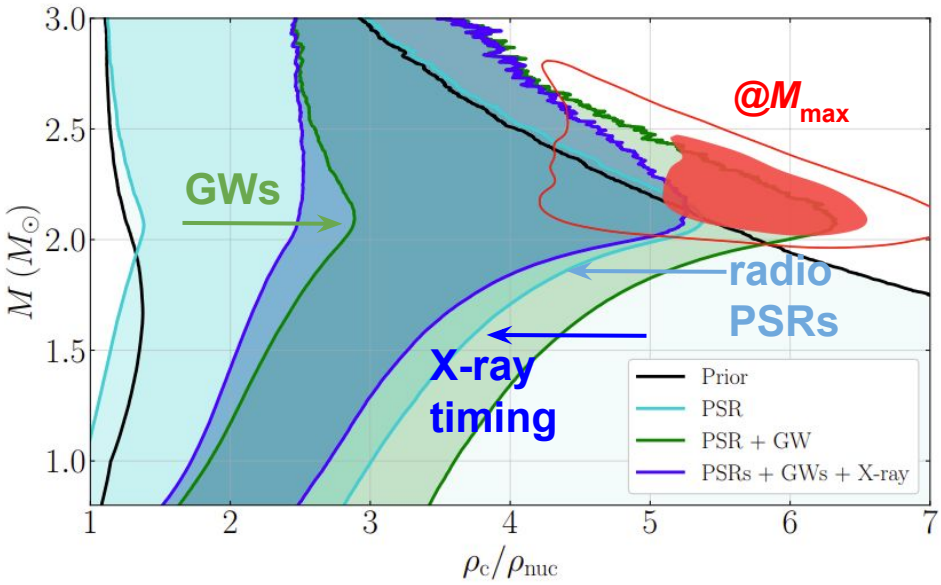


## Current *Theory Agnostic* Constraints





supranuclear sound speed almost certainly exceeds the conformal limit  
→strongly-coupled interactions



maximum central density is likely  $\sim 6\rho_{\text{nuc}}$



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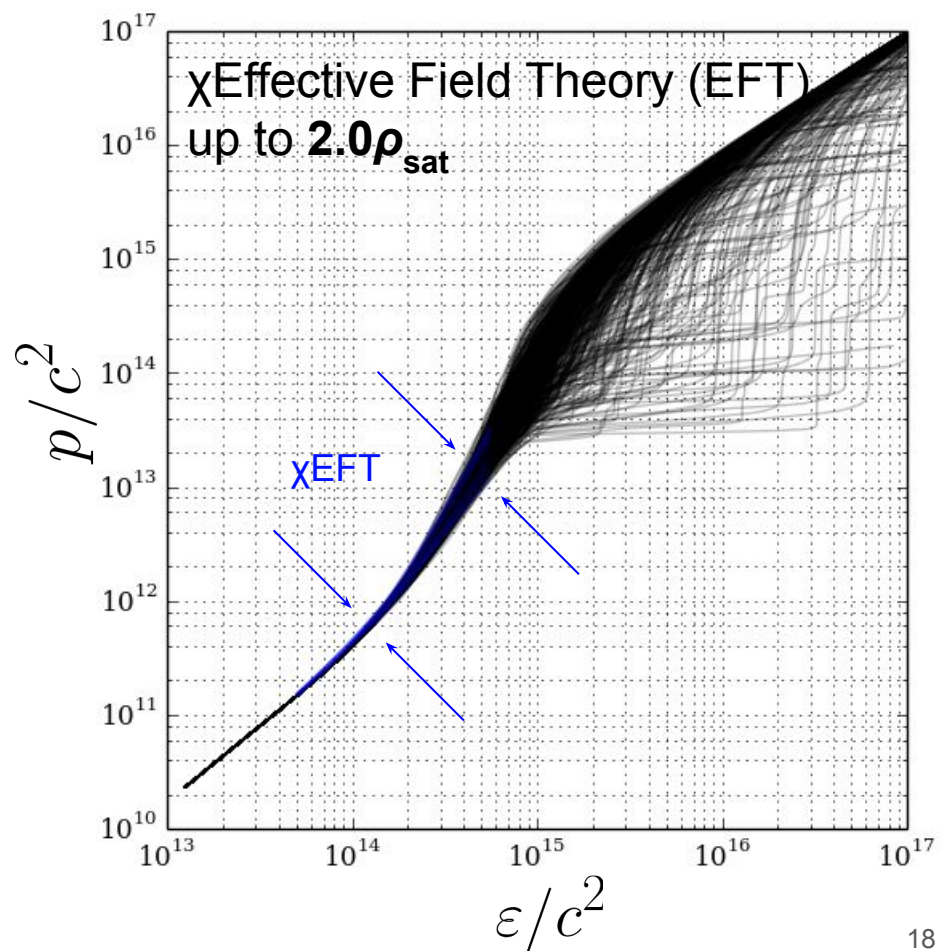
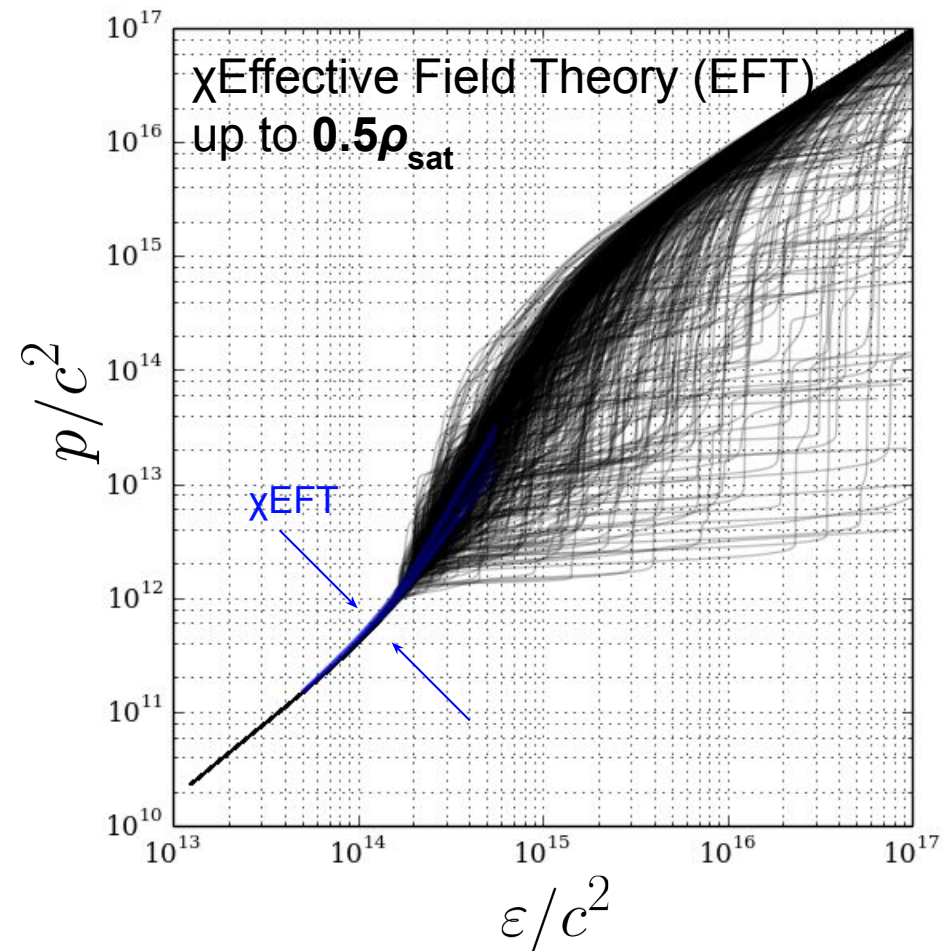
Difficulties with inverse problem PRD 105, 043016 (2022)

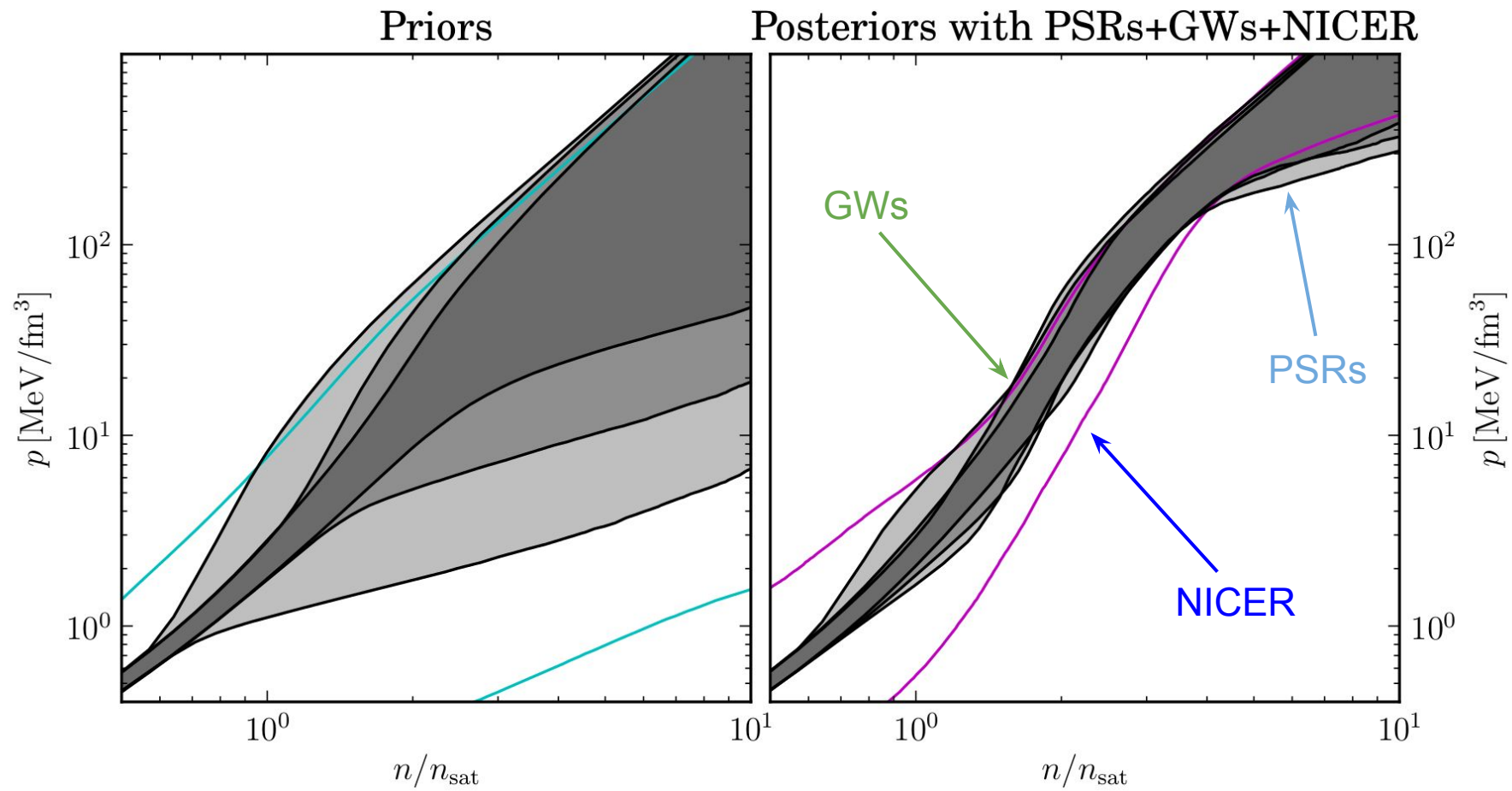
Parametric model → reduce dimensionality, traditional sampling methods

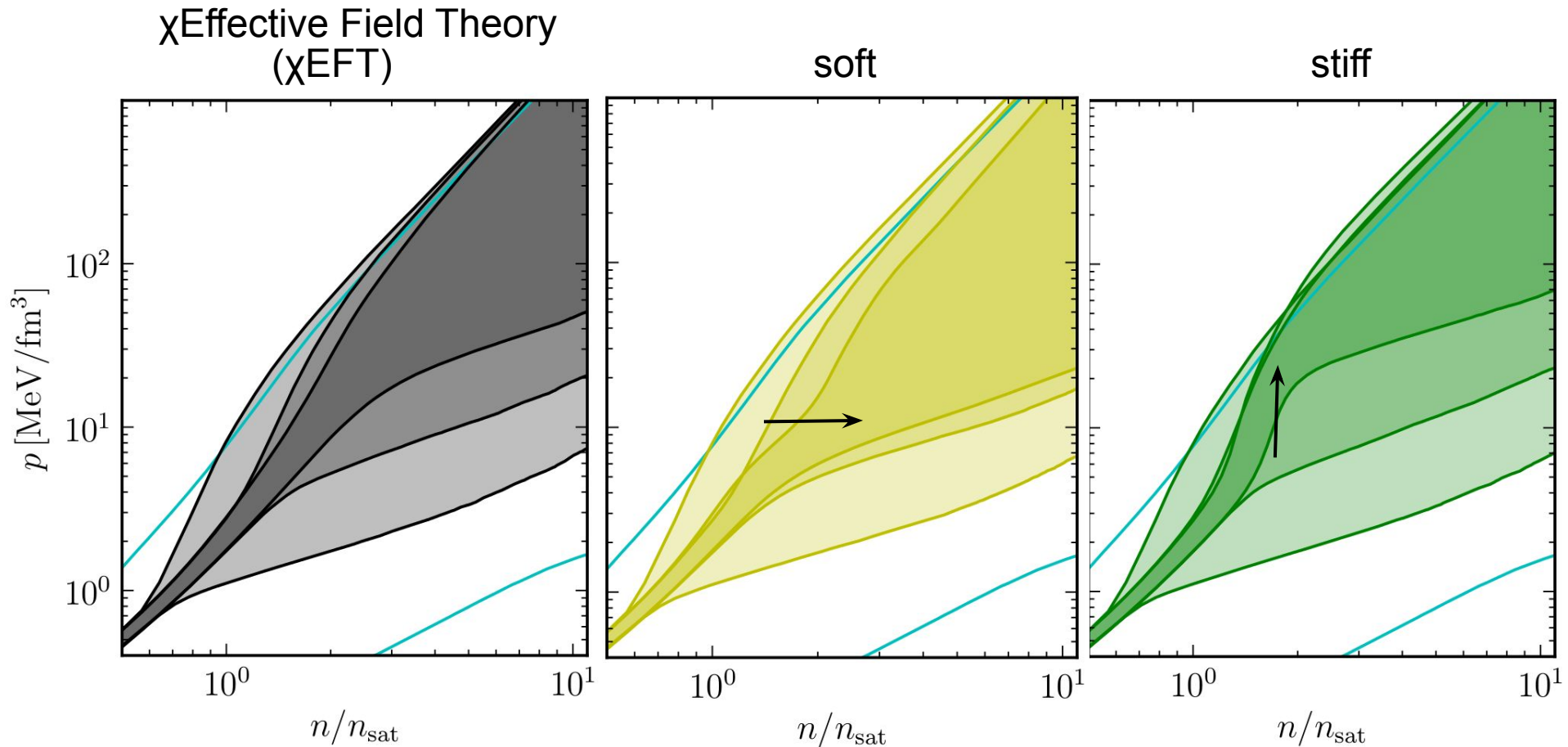
Nonparametric representation → high dimensionality, novel sampling methods

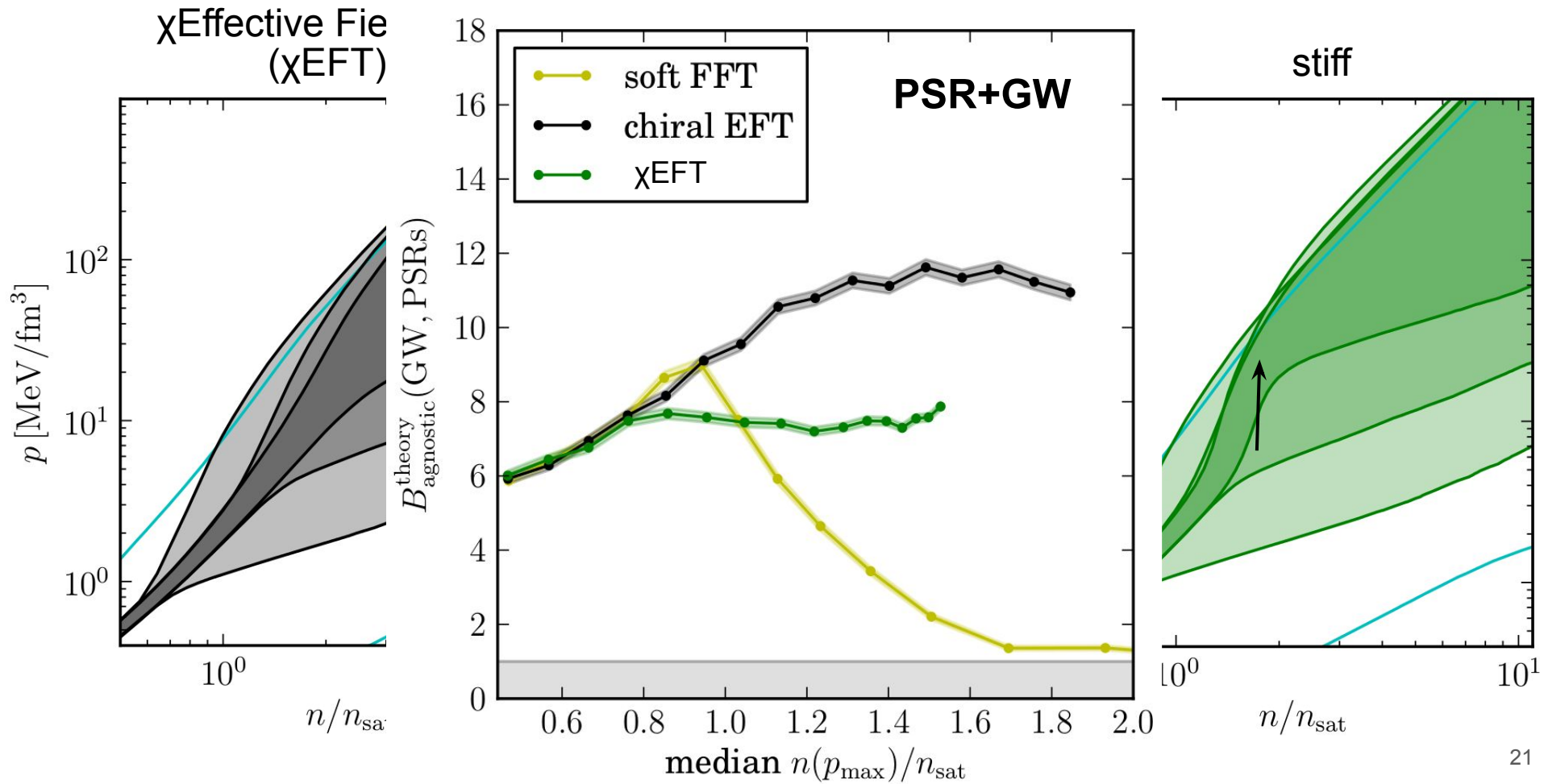
Incorporating theoretical predictions within the prior [PRC 102, 055803 \(2020\)](#)

Exotic high-density behavior with efficient TOV sequences arXiv:2405.05395 (2024)







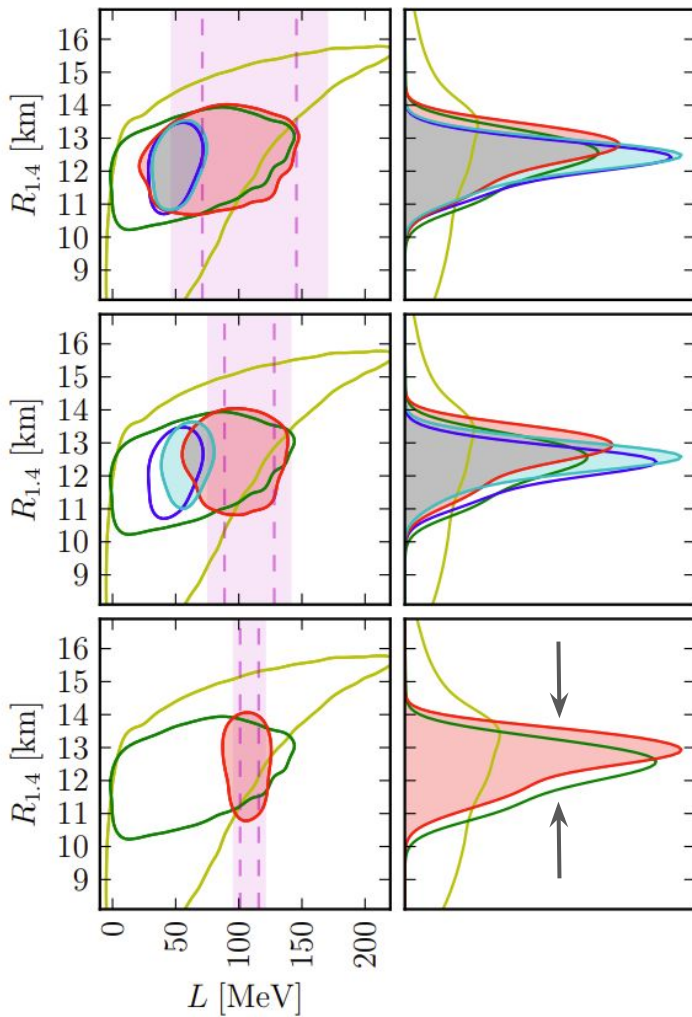


# Inference of the NS EoS

current  $R_{\text{skin}}$  uncertainty

$R_{\text{skin}}$  uncertainty improved  
by a factor of 2

hypothetical perfect  
 $R_{\text{skin}}$  measurement



nonparametric prior  
nonparametric astro-only posterior  
 $\chi$ EFT+astro posterior  
nonparametric astro+ $R_{\text{skin}}$  posterior  
 $\chi$ EFT+astro+ $R_{\text{skin}}$  posterior

improved precision in nuclear experiments is unlikely to affect our knowledge of NS radii without improved theoretical calculations

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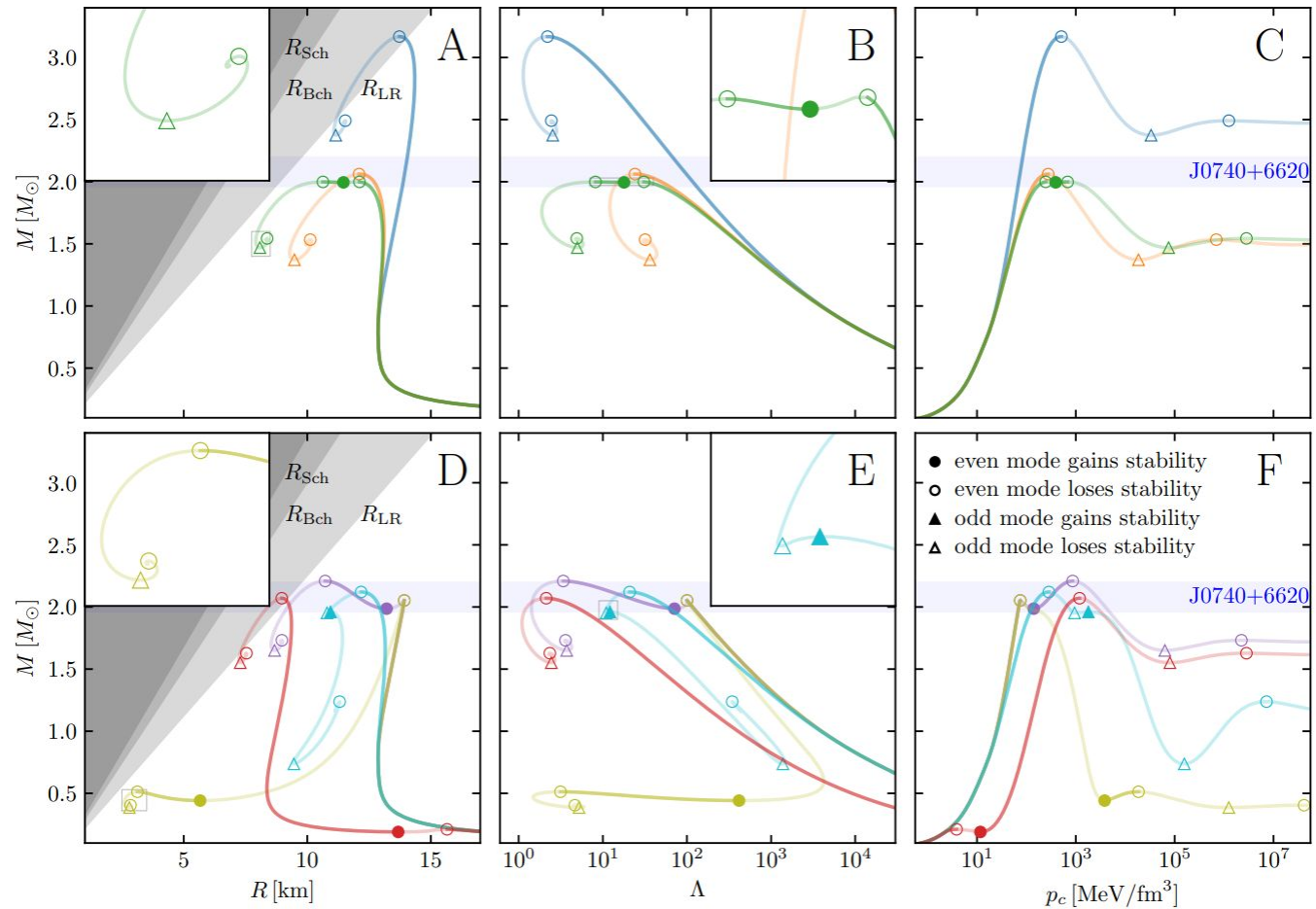
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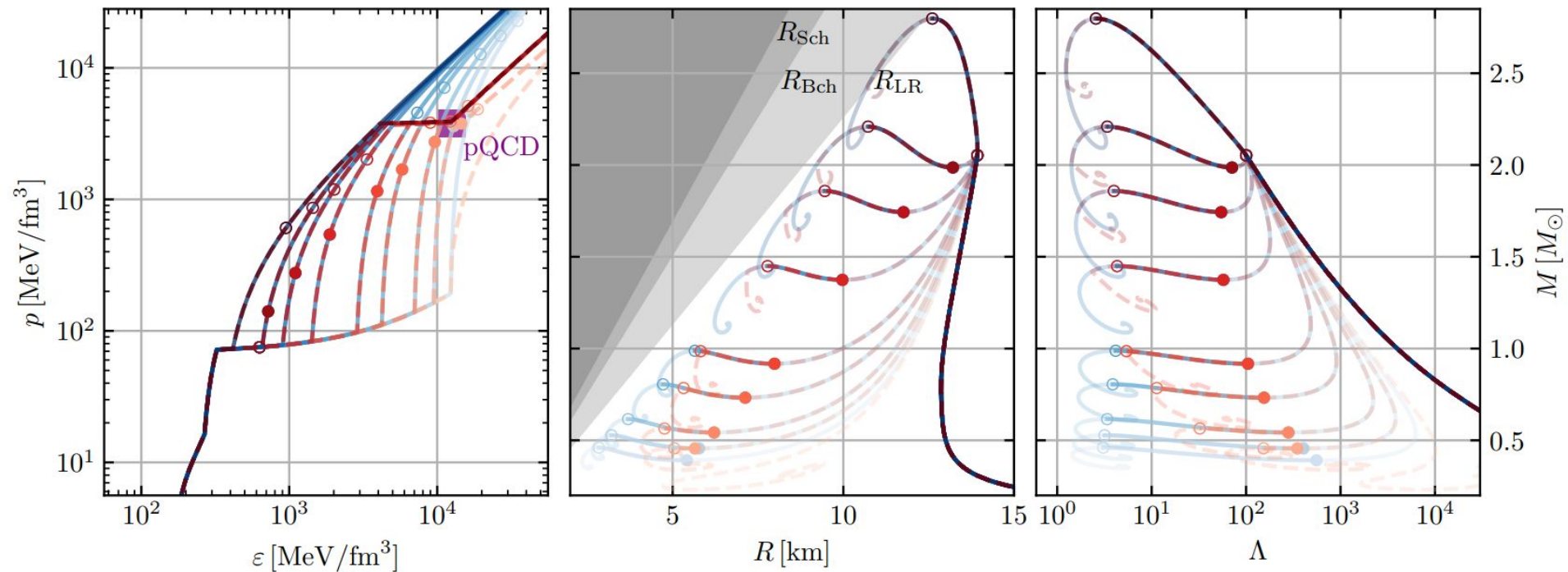
Exotic high-density behavior with efficient TOV sequences

[arXiv:2405.05395](https://arxiv.org/abs/2405.05395) (2024)

# Exotic Behavior with Efficient TOV Sequences







# References

- I. Legred, K. Chatziioannou, R. Essick, P. Landry, *Implicit Correlations within Phenomenological Parametric Models of the neutron Star Equation of State*, PRD 105, 043016 (2022)
- R. Essick, I. Tews, P. Landry, S. Reddy, D. E. Holz, *Direct Astrophysical Tests of Chiral Effective Field Theory at Supranuclear Densities*, PRC 102, 055803 (2020)
- R. Essick, I. Tews, P. Landry, A. Schwenk, *Astrophysical Constraints on the Symmetry Energy and the Neutron Skin of  $^{208}\text{Pb}$  with Minimal Modeling Assumptions*, PRL 127, 192701 (2021)
- R. Essick, P. Landry, A. Schwenk, I. Tews, *Detailed Examination of Astrophysical Constraints on the Symmetry Energy and Neutron Skin of  $^{208}\text{Pb}$  with Minimal Modeling Assumptions*, PRC 104, 065804 (2021)
- R. Essick, *Exotic Stable Branches and Efficient TOV Sequences*, arXiv:2405.05395 (2024)



consider a toy model:

- fitting a 1D function (pressure vs. energy density)
- without constraints

point+slope

$$p(\varepsilon) = p_a + c_s^2(\varepsilon - \varepsilon_a)$$

two-point

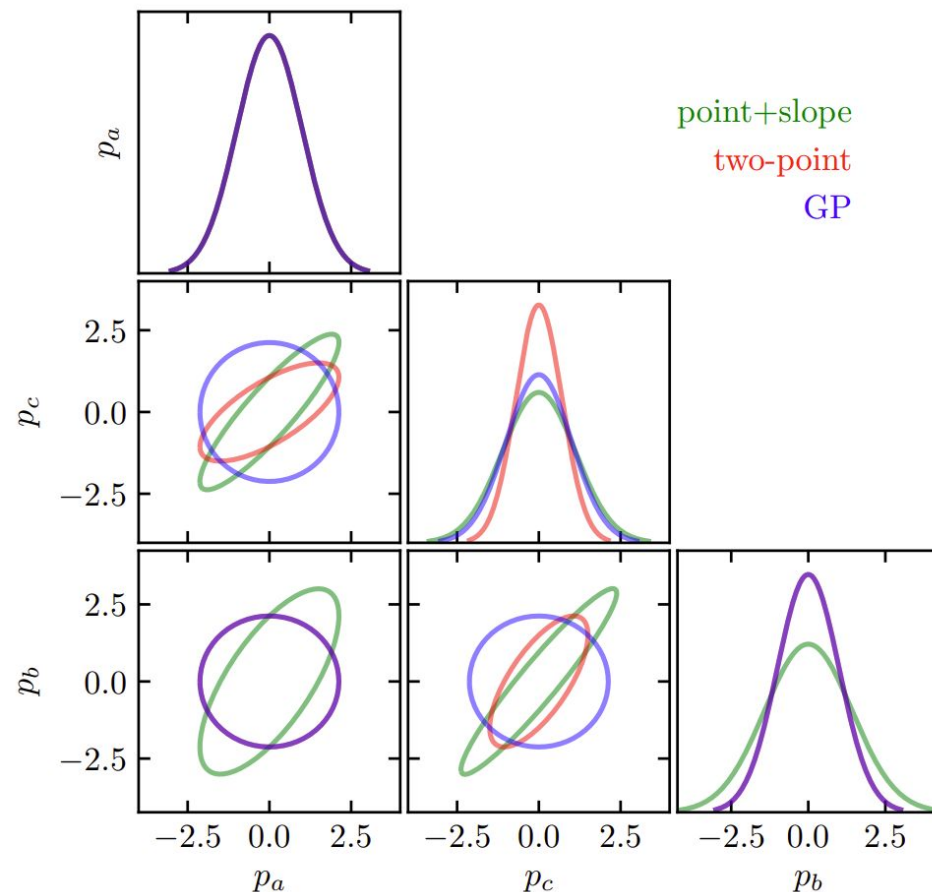
$$p(\varepsilon) = p_a + \frac{p_b - p_a}{\varepsilon_b - \varepsilon_a}(\varepsilon - \varepsilon_a)$$

GP

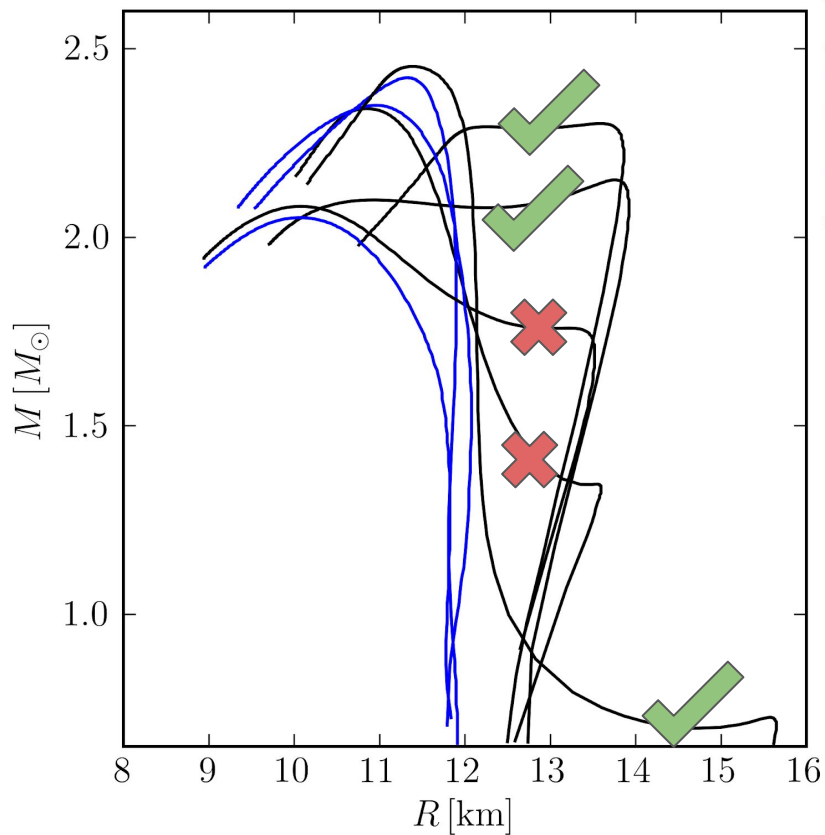
$$\vec{p} \sim \mathcal{N}(\vec{\mu}, \Sigma)$$

$$\Sigma_{ij} = \text{Cov}(p_i, p_j)$$

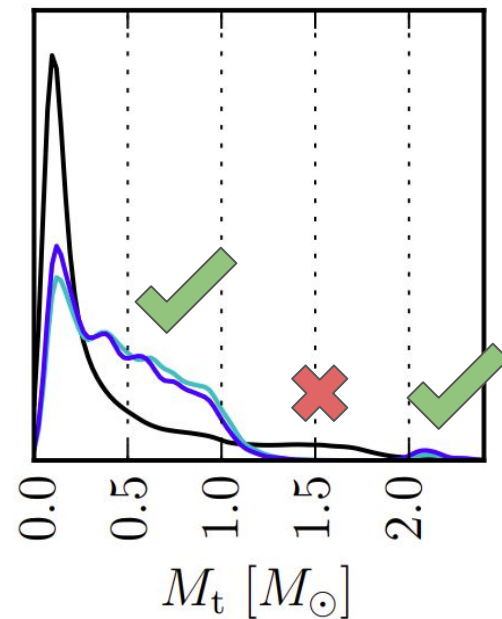
$$= K_{\text{se}}(\varepsilon_i, \varepsilon_j) = \sigma^2 \exp\left(-\frac{(\varepsilon_i - \varepsilon_j)^2}{l^2}\right)$$



**only the GP has independent marginal distributions** for all pressures

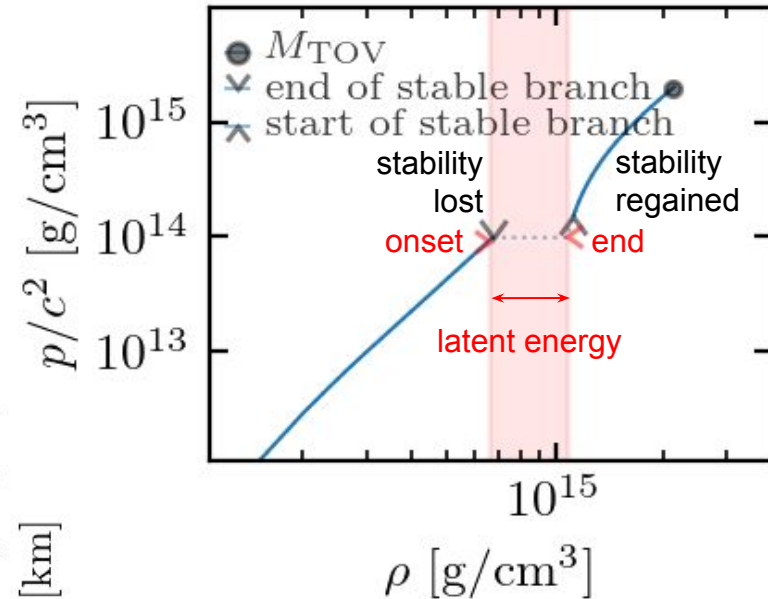
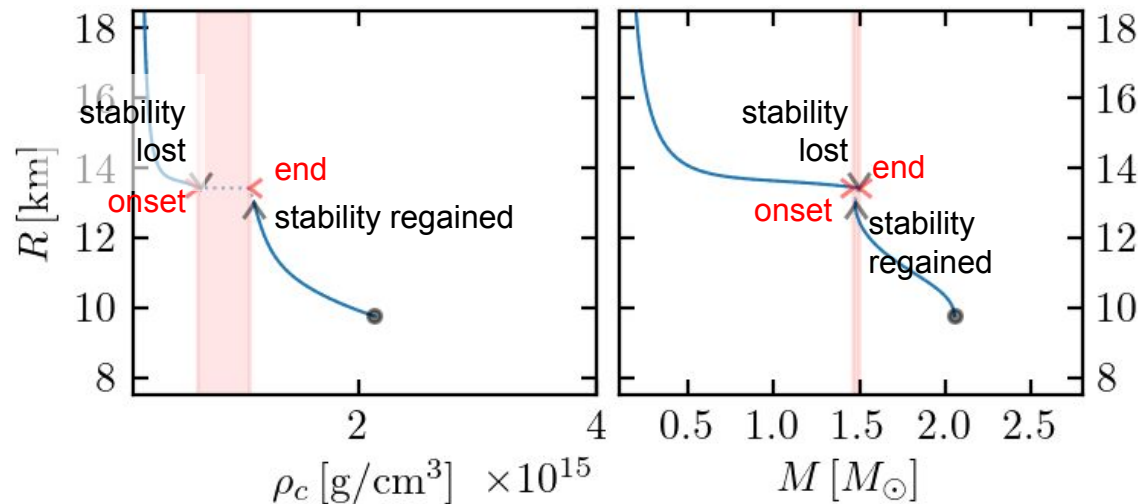


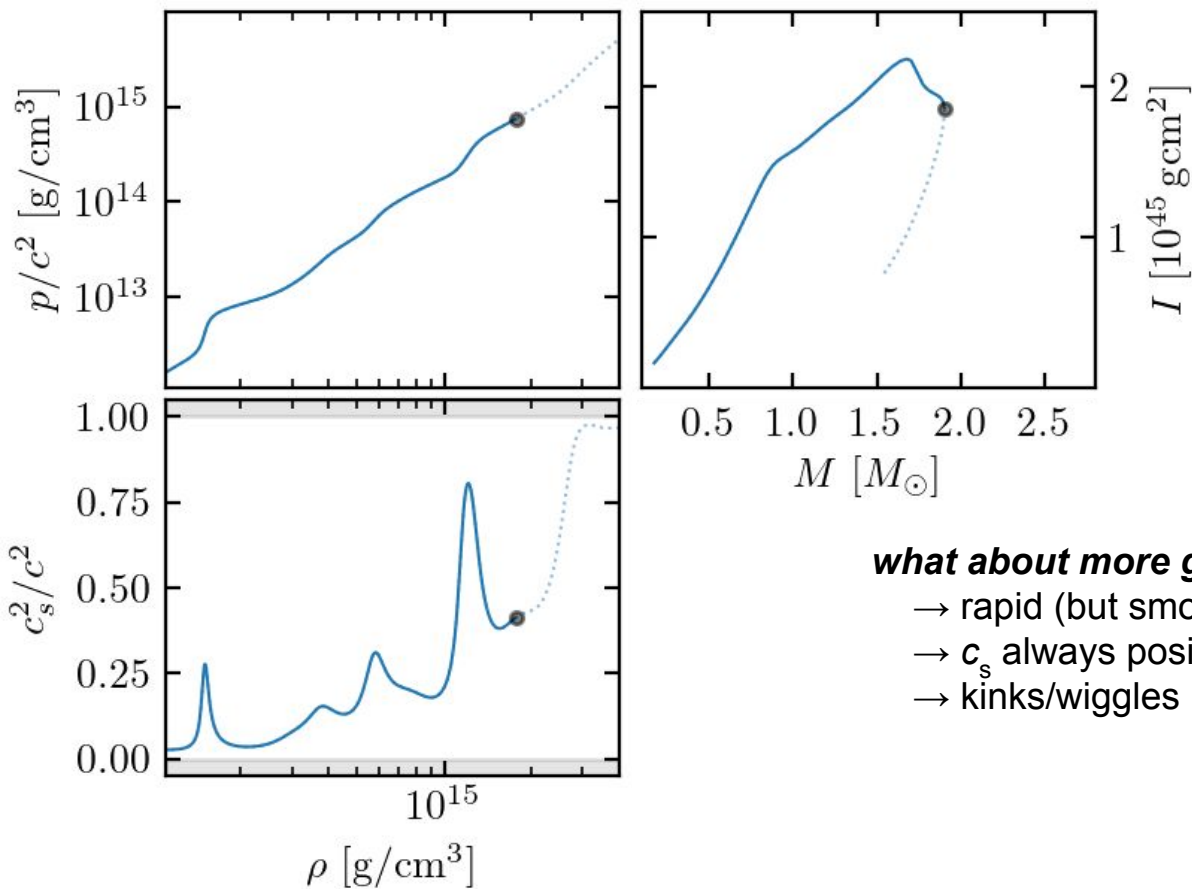
Data	$\max \mathcal{L}_{n=1}^{n>1}$	$\mathcal{B}_{n=1}^{n>1}$	$\max \mathcal{L}_{\substack{c_s^2 > c^2/3 \\ c_s^2 \leq c^2/3}}$	$\mathcal{B}_{\substack{c_s^2 > c^2/3 \\ c_s^2 \leq c^2/3}}$
w/PSRs	1.00	$0.120 \pm 0.002$	1.0	$10.2 \pm 0.5$
w/o J0740+6620	0.97	$0.220 \pm 0.007$	50.8	$2220 \pm 790$
w/J0740+6620	Miller+	$0.60$	26.7	$1000 \pm 340$
	Riley+	0.94	$0.185 \pm 0.006$	72.7



# Inference of the NS EoS: phase transitions

the sudden appearance of new (degenerate) degrees of freedom produces a sharp drop in the sound speed ( $c_s$ )  
 → “loss of pressure support”

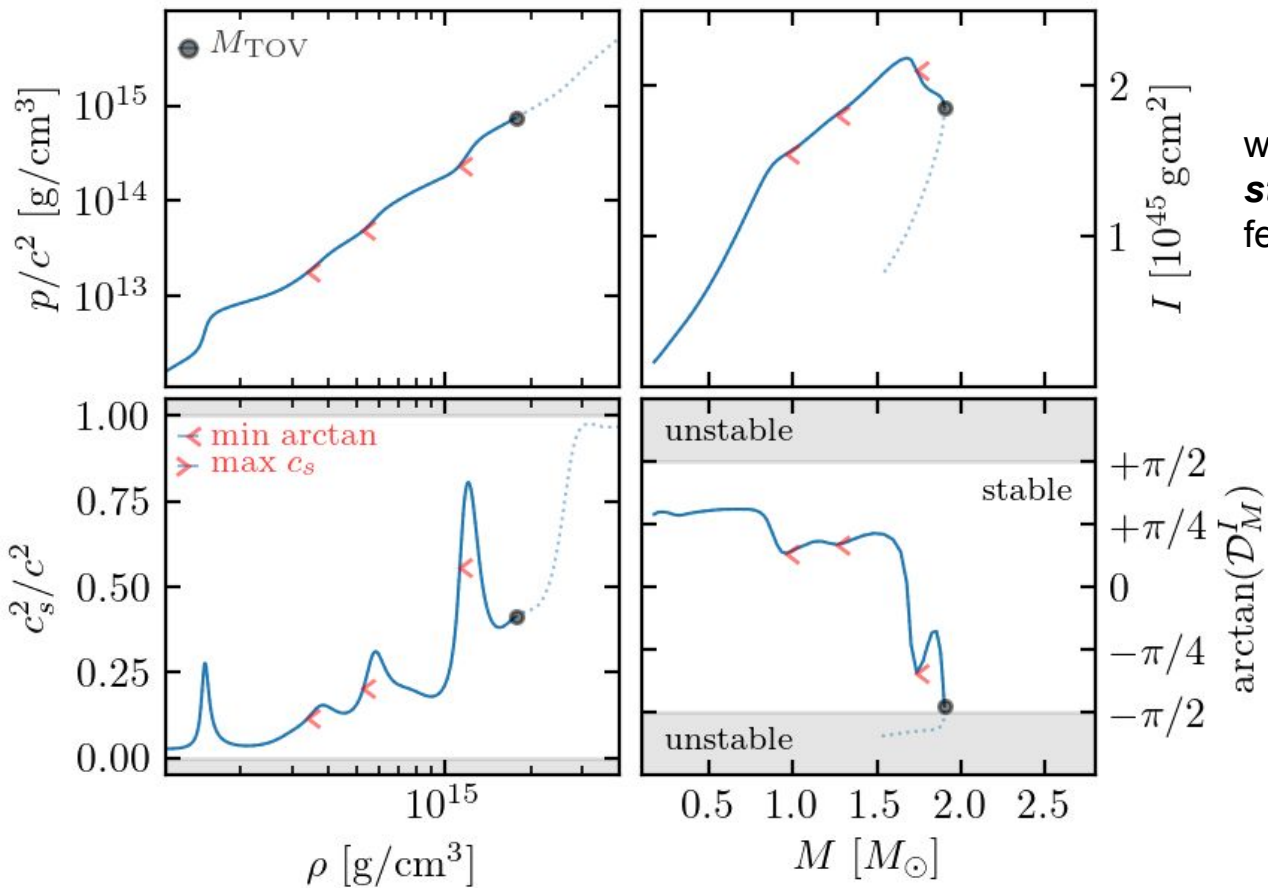




***what about more general phenomenology?***

- rapid (but smooth) changes in  $c_s$ ?
- $c_s$  always positive definite?
- kinks/wiggles in stellar sequence but no loss of stability?

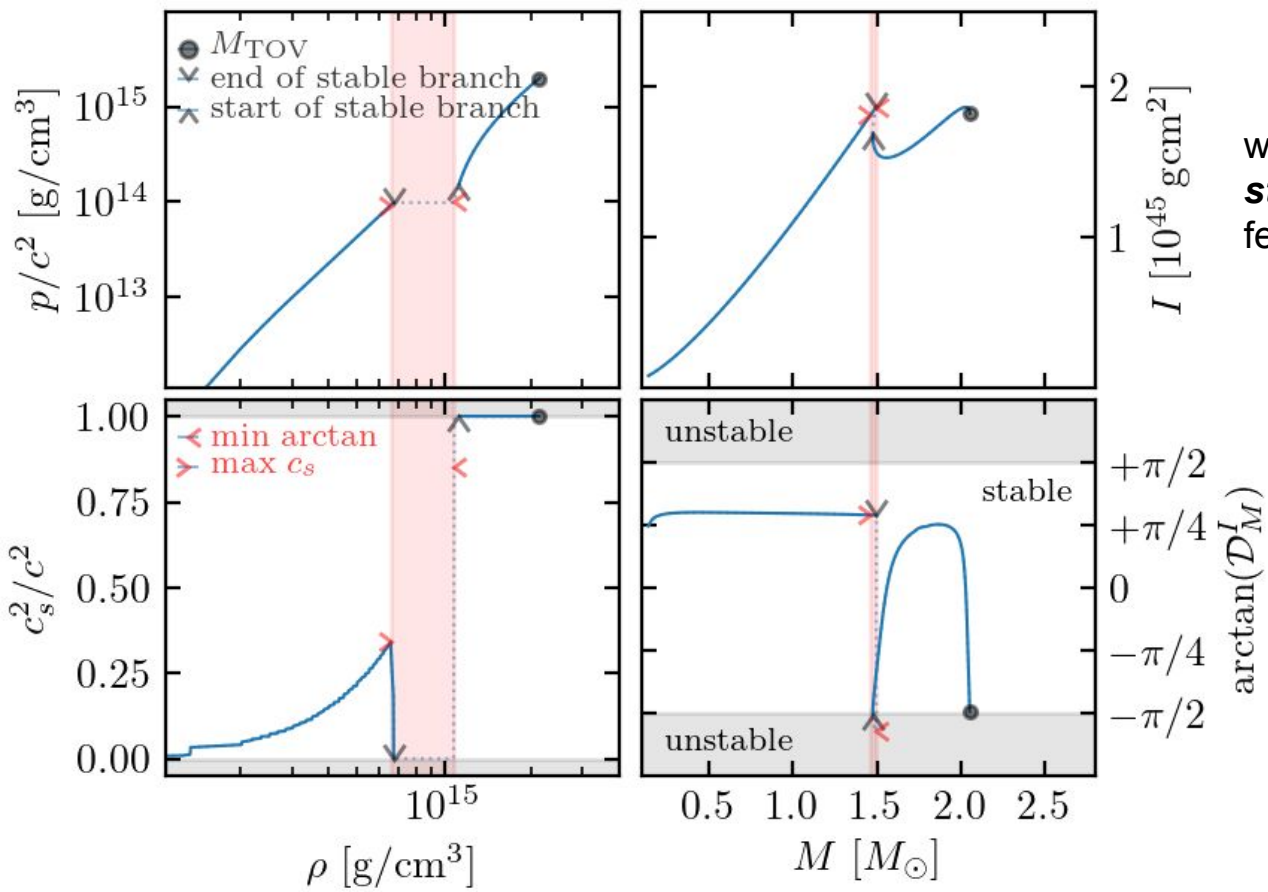
## Inference of the NS EoS: phase transitions



we search for **features based on stellar properties** and connect these to features in  $c_s$

$$\mathcal{D}_M^I \equiv \frac{d \log I / d \log p_c}{d \log M / d \log p_c}$$

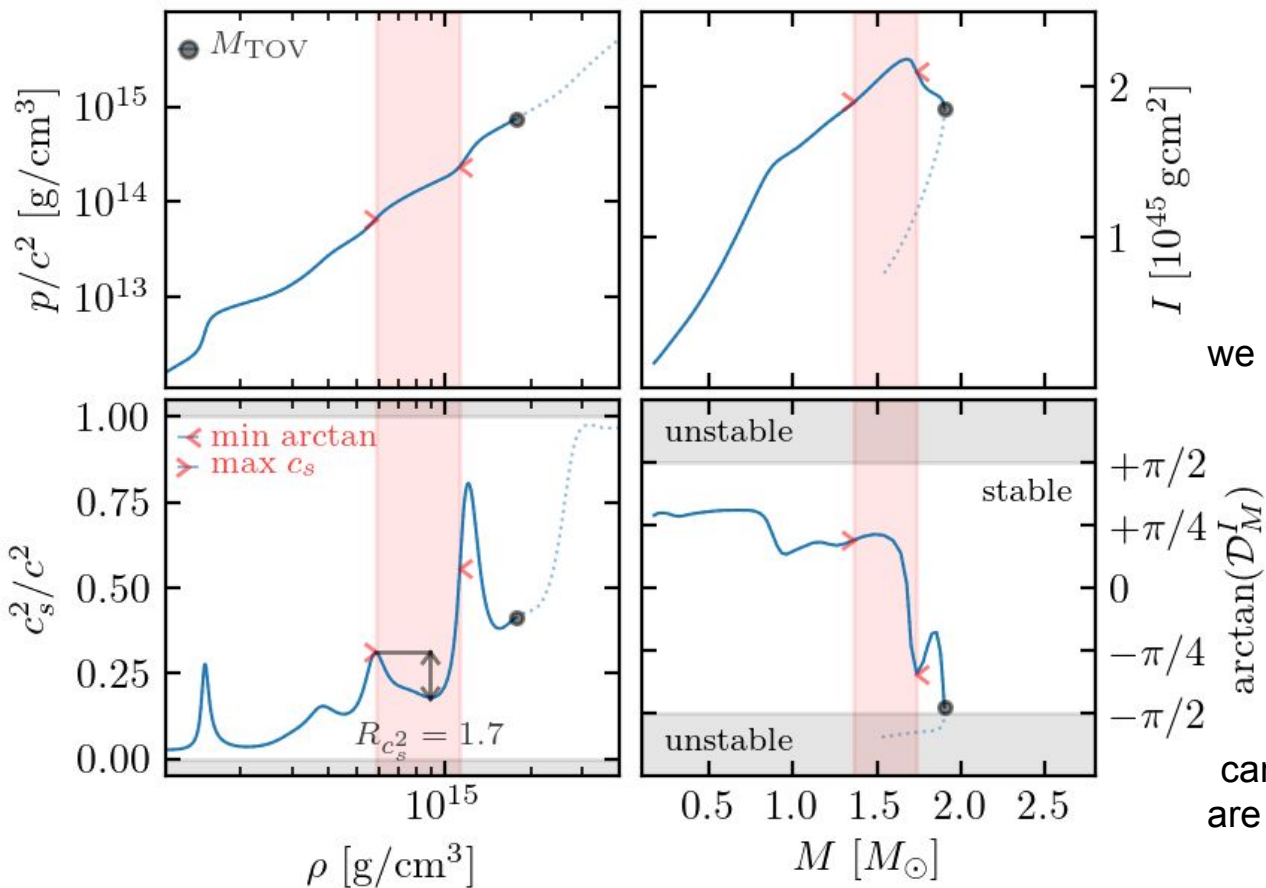




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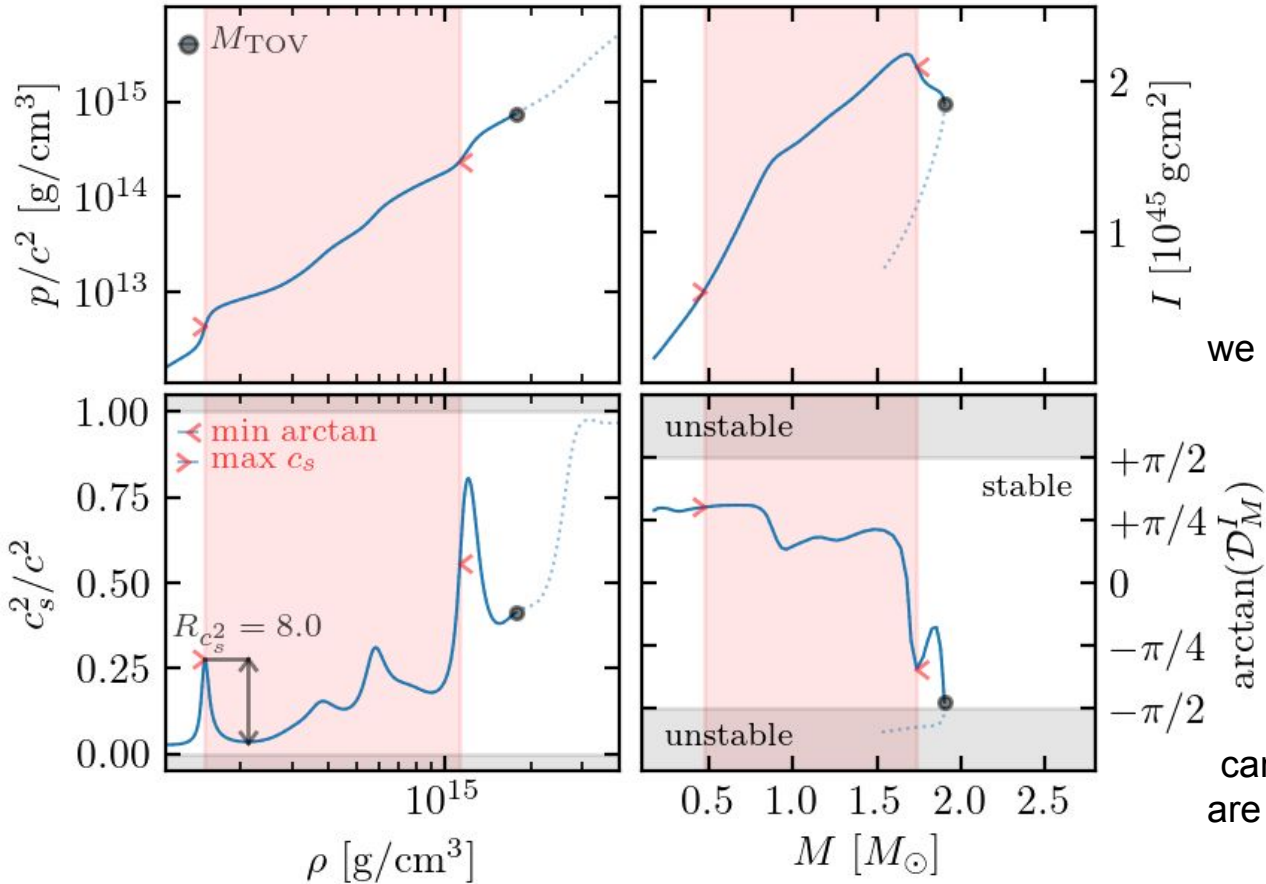
**local minima** → **end** of phase trans

and do so more precisely than where stability is regained



we associate a **local minimum (end)** with the most recent preceding **“running maximum” in  $c_s$  (onset)**

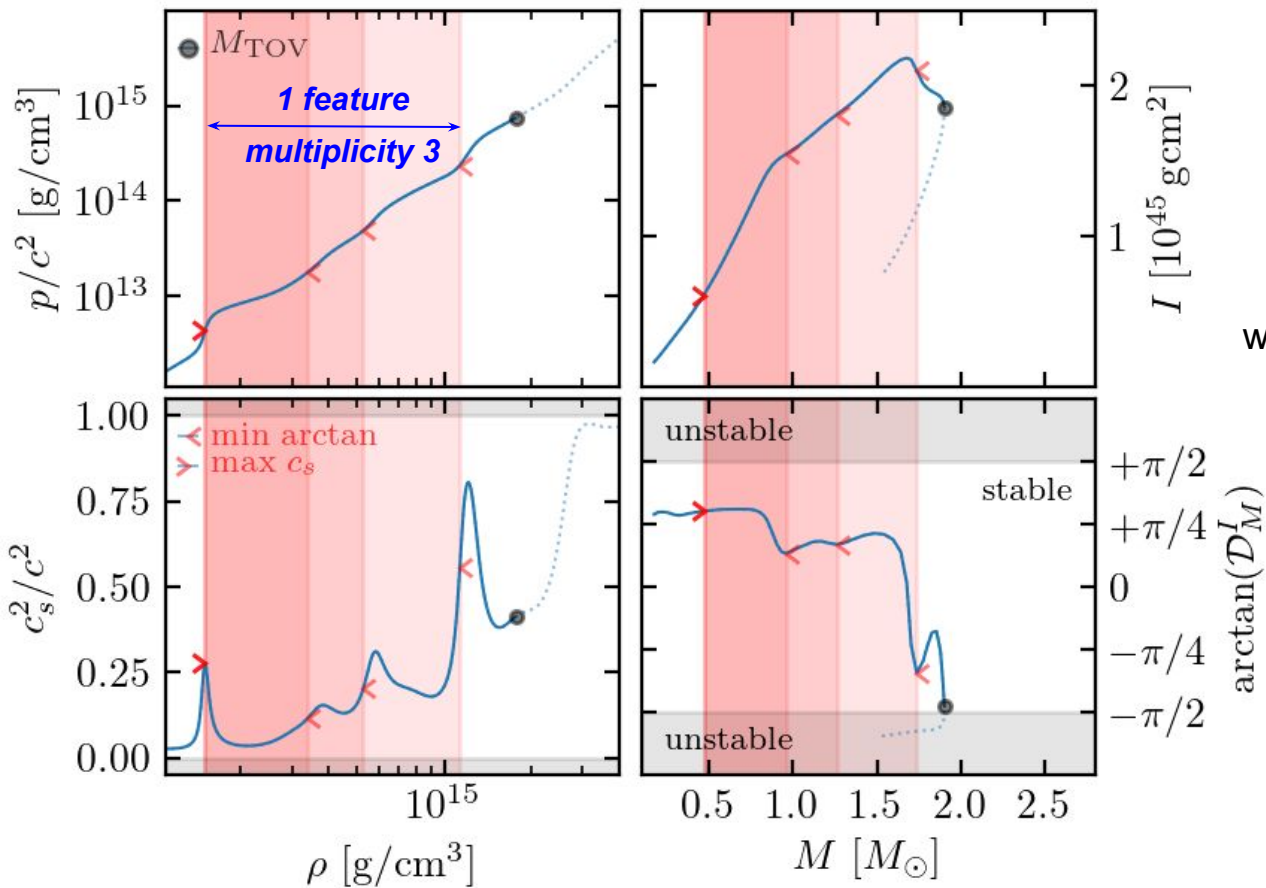
candidate **running maxima in  $c_s$  (onset)** are accepted only if they are followed by a **large drop in  $c_s$**



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candidate **running maxima in  $c_s$  (onset)** are accepted only if they are followed by a **large drop in  $c_s$**

## Inference of the NS EoS: phase transitions



we iterate until all **local minimum (end)**  
 have an associated  
**"running maximum" in  $c_s$  (onset)**

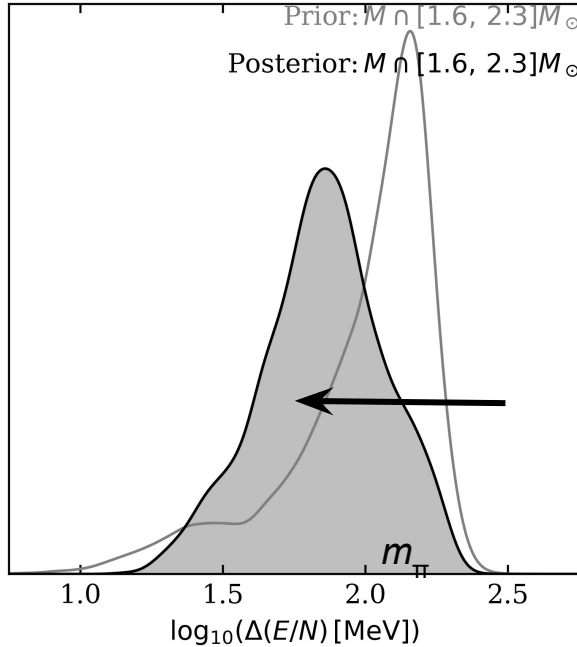
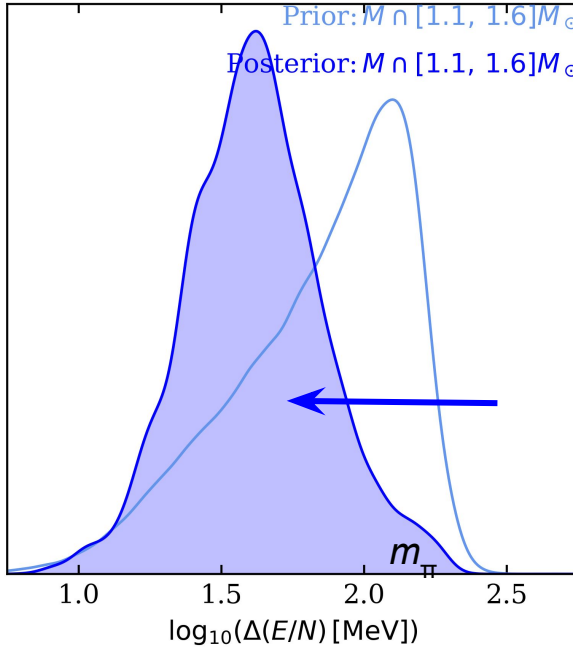
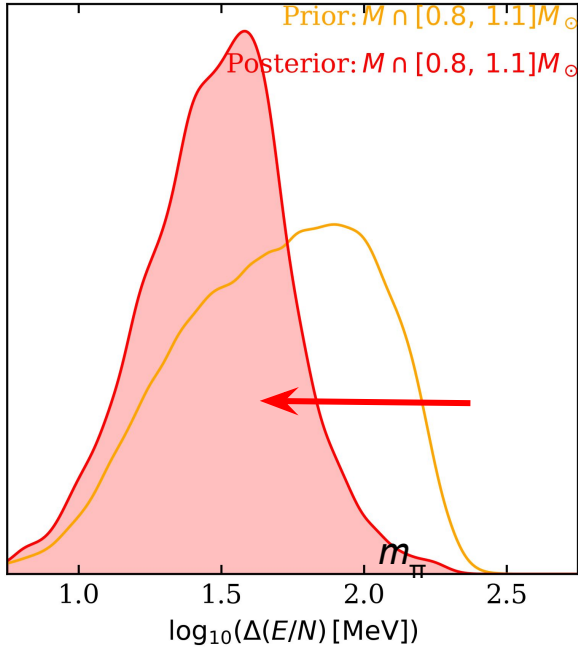
this EoS has  
**1 feature**  
 with  
**multiplicity 3**

# Inference of the NS EoS: phase transitions

masses from J0348, J0740

LIGO-Virgo-KAGRA GWs from GW170817, GW190425

NICER X-ray Timing from J0030, J0740



## Inference of the NS EoS: phase transitions

masses from J0348, J0740

LIGO-Virgo-KAGRA GWs from GW170817, GW190425

NICER X-ray Timing from J0030, J0740

$M$ [ $M_{\odot}$ ]	Stable Branches			$\min \Delta(E/N)$ [MeV]	$\mathcal{D}_M^I$ Features		
	$\max \mathcal{L}_{n=1}^{n \geq 2}(\text{PGX})$	$\mathcal{B}_{n=1}^{n \geq 2}(\text{PGX})$	$\mathcal{B}_{n=1}^{n \geq 2}(\text{GX P})$		$\max \mathcal{L}_{n=0}^{n \geq 1}(\text{PGX})$	$\mathcal{B}_{n=0}^{n \geq 1}(\text{PGX})$	$\mathcal{B}_{n=0}^{n \geq 1}(\text{GX P})$
0.8–1.1	0.47	$0.362 \pm 0.036$	$2.219 \pm 0.162$	10	0.57	$1.222 \pm 0.020$	$0.684 \pm 0.011$
				50	0.49	$0.366 \pm 0.011$	$0.588 \pm 0.016$
				100	0.26	$0.117 \pm 0.008$	$0.292 \pm 0.021$
1.1–1.6	0.14	$0.030 \pm 0.006$	$0.291 \pm 0.055$	10	0.57	$1.043 \pm 0.020$	$0.552 \pm 0.010$
				50	0.49	$0.463 \pm 0.013$	$0.552 \pm 0.010$
				100	0.26	$0.152 \pm 0.009$	$0.267 \pm 0.017$
1.6–2.3	0.20	$0.147 \pm 0.028$	$0.120 \pm 0.026$	10	0.52	$1.012 \pm 0.035$	$0.385 \pm 0.013$
				50	0.49	$0.898 \pm 0.034$	$0.385 \pm 0.013$
				100	0.29	$0.383 \pm 0.023$	$0.256 \pm 0.016$

# Future Prospects: phase transitions

DBHF

DBHF\_3504

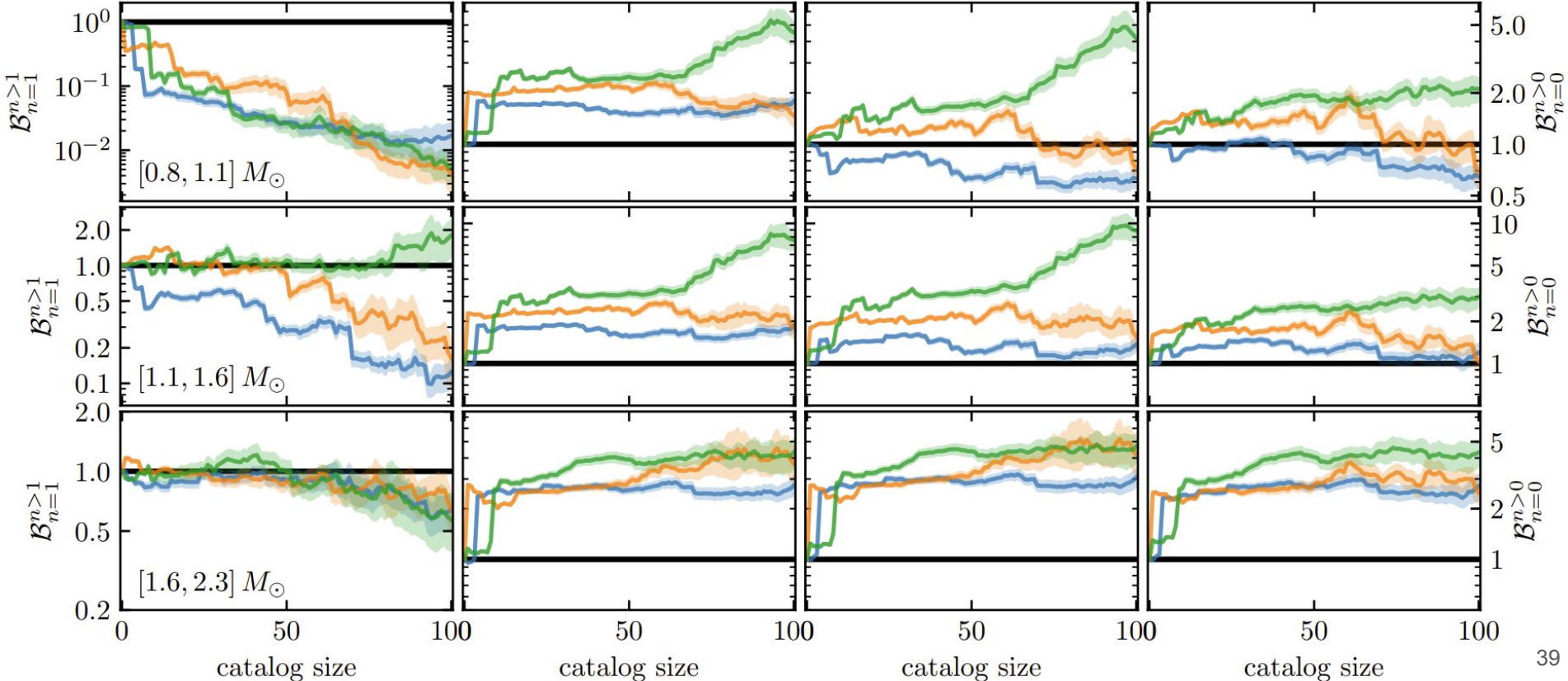
DBHF\_2507

Stable Branches

$\Delta(E/N) > 10$  MeV

$\Delta(E/N) > 50$  MeV

$\Delta(E/N) > 100$  MeV

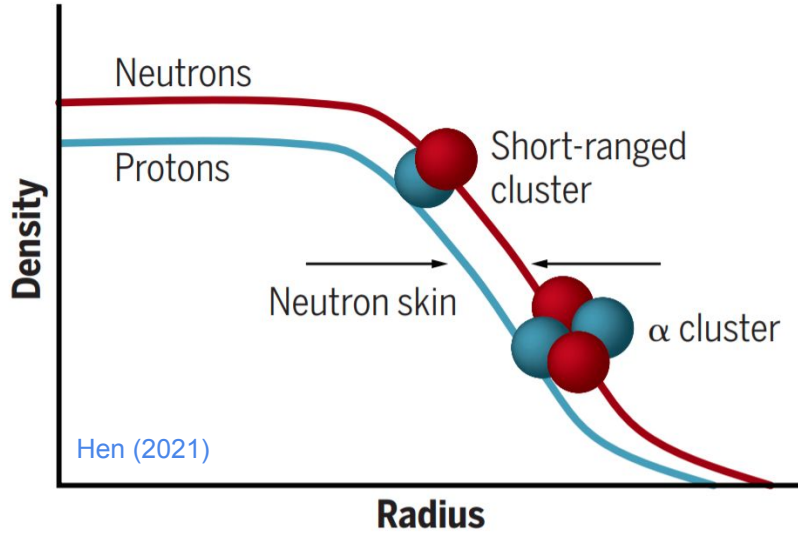


# Inference of the NS EoS: low-density nuclear experiment

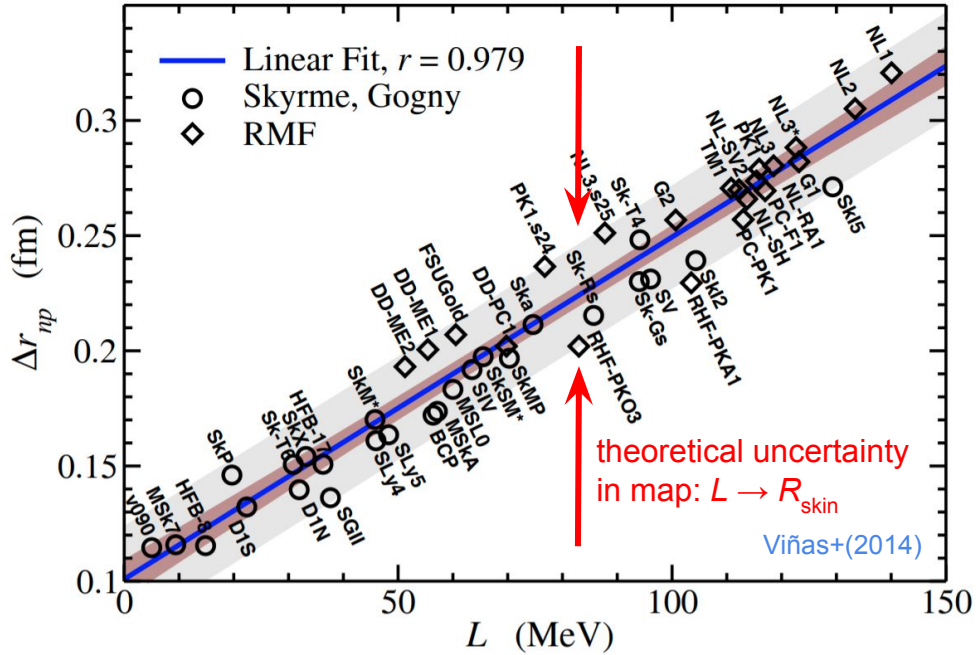
Connection to “new” experimental probes: Neutron Skin Thickness ( $R_{\text{skin}}$ )

Reed+(2021) infer  $L \geq 100$  MeV based on  $R_{\text{skin}} = 0.29 \pm 0.07$  fm. Suggest this implies  $R_{1.4} \geq 14$  km.

## Nucleon density in neutron-rich nuclei



Scattering experiments (PREX) measure this



we constrain this with  
astrophysical observations



# Inference of the NS EoS: low-density nuclear experiment

Map from nonparametric EoS in  $\beta$ -equilibrium to nuclear params describing the energy per particle near nuclear saturation ( $n_0$ : minimum of  $E_{\text{SNM}}$ )

$$x = n_p/n$$

$$E_{\text{nuc}}(n, x) = E_{\text{SNM}}(n) + (1 - 2x)^2 S_0(n) + \mathcal{O}(x^4)$$

$$= \frac{\varepsilon_\beta(n) - \varepsilon_e(n, x)}{n} - m_N$$

$$E_{\text{SNM}}(n) = E_0 + \frac{1}{2} K_0 \left( \frac{n - n_0}{3n_0} \right)^2 + \dots$$

$$\mu_n = \mu_p + \mu_e$$

proton fraction

nuclear energy per particle

symmetric-nuclear-matter energy per particle (local min at  $n_0$ )

condition for  $\beta$ -equilib

solve these self-consistently to obtain  $S_0(n)$  and then compute

$$L = 3n \left( \frac{dS_0}{dn} \right)$$

$$K_{\text{sym}} = 9n^2 \left( \frac{d^2S_0}{dn^2} \right)$$

constrained by astro observations (input from nonparametric analysis)

measured in the lab (input from terrestrial experiment)

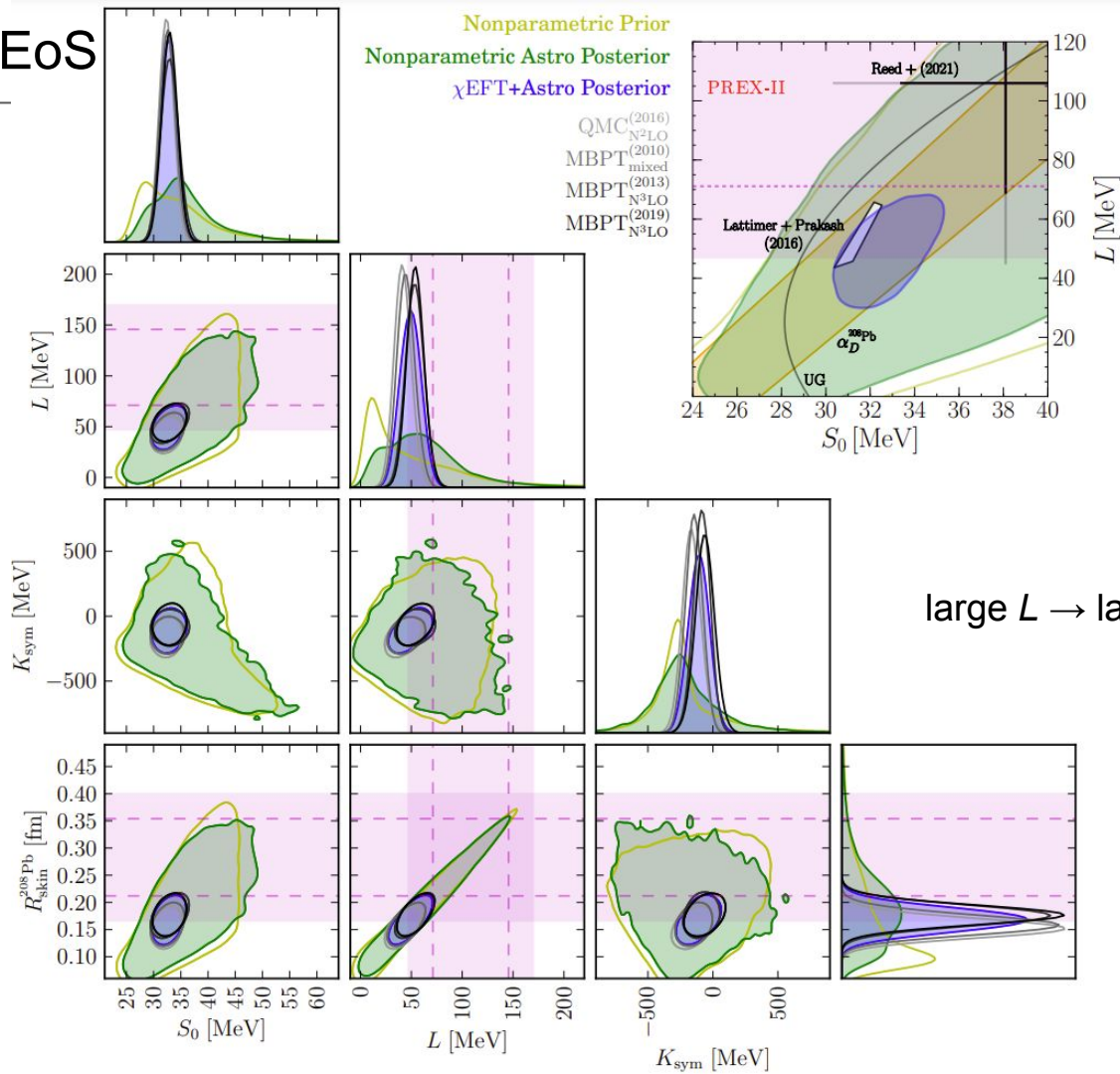
modeled as degenerate Fermi gas (input from theory)

expressed in terms of derivatives of  $E_{\text{nuc}}$

$$\mu_i = \frac{dE}{dN_i}$$

# Inference of the NS EoS

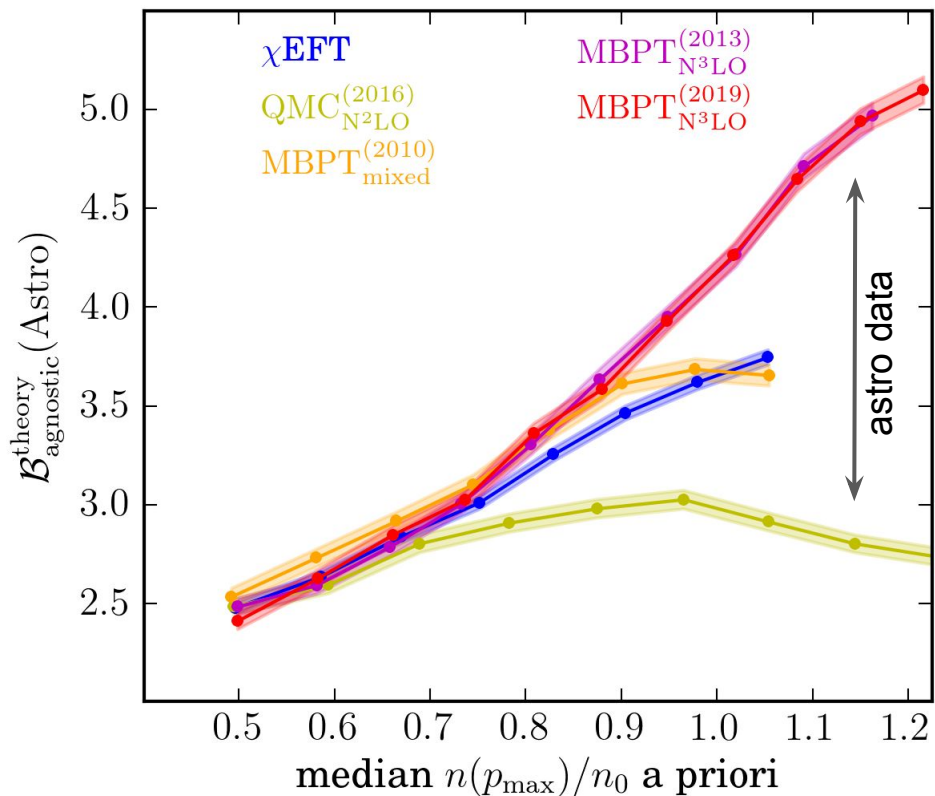
Essick+ (2021)  
Essick+ (2021)



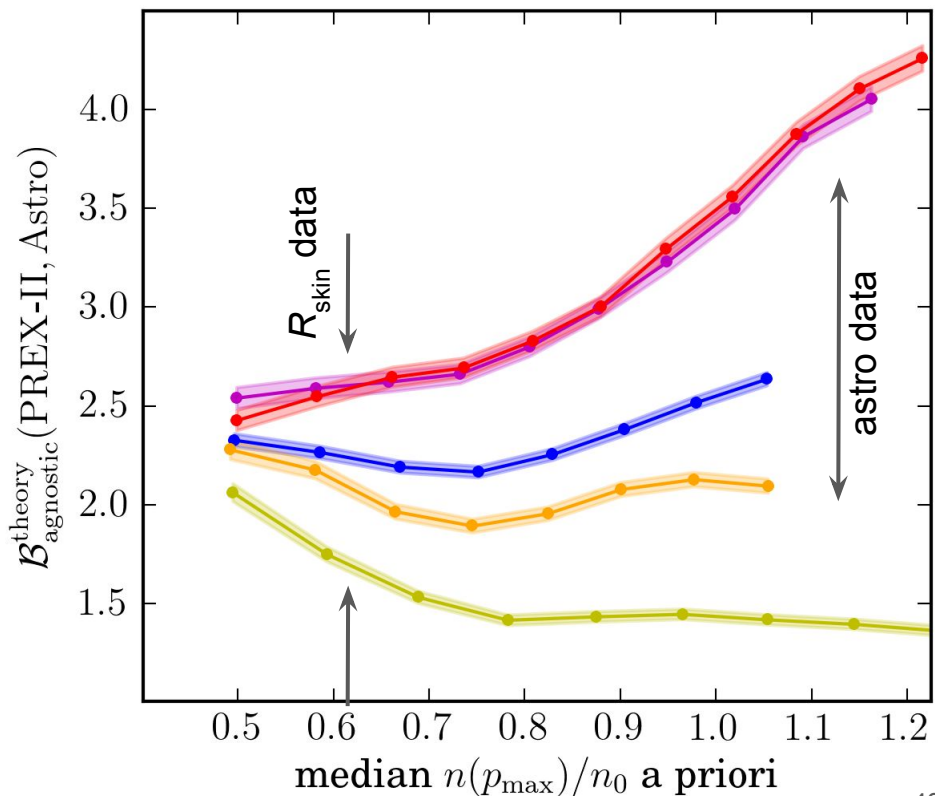
We can also extract  
“nuclear parameters”  
directly from  
nonparametric EoS  
without the need for  
parametrized EoS models

## Inference of the NS EoS: low-density nuclear experiment

astro data can distinguish between  
nuclear theories at high densities



nuclear experiments probe lower densities



# Inference of the NS EoS: low-density nuclear experiment

100 MeV < L  
30 MeV < L < 70 MeV  
All L

