PRIMORDIAL MAGNETIC FIELDS IN BOUNCING COSMOLOGY

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MAGNETIC FIELDS IN THE UNIVERSE

- Magnetic fields (MF) \( \sim 1 \mu G \) and coherence length \( \lambda \sim 1 \text{kpc} \) observed in galaxies and clusters.
- MF between \( 0 < z \leq 2 \) have same strength: saturation mechanism \( \Rightarrow \) “seed” field enhanced and maintained (e.g. through dynamo).
- Origin: astrophysical processes (after recombination)?
  - primordial mechanisms (before recombination)?
- Primordial MF unaltered in voids \( \Rightarrow \) still “seed” MF. Indirect evidence only (but see Broderick et al. Astrophys. J. 752, 22).
INFLATIONARY MAGNETOGENESIS

PMF in the very early Universe?

• Maxwell electromagnetism (EM)

\[ \mathcal{L}_{\text{Maxwell}} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \]  

Conformally invariant: no EM fields during expansion.

• Way-out: break invariance through coupling (inflation, geometry, etc...).

\[ \frac{1}{4} f(\phi) F_{\mu\nu} F^{\mu\nu}, \quad \frac{1}{4} R F_{\mu\nu} F^{\mu\nu} \]  

Inflationary magnetogenesis provide...

- ...seed MF compatible with observations
- ...naturally large coherence scale

BUT!

- ...simple models produce weak MF (e.g. $f(\phi)$ or $R$)
- ...the EM energy density usually big $\Rightarrow$ strong backreaction!
- ...alternative mechanisms exist :)

PMF in bouncing cosmology / Emmanuel FRION efrion@uwo.ca May 29, 2024
GR HAMILTONIAN (ADM FOLIATION)

\[ H_T = \int d^3x \left( N\mathcal{H}_0 + N_i \mathcal{H}^i + \lambda P + \lambda^i P_i \right), \quad (2) \]

Constraints

\[
\mathcal{H}_0 = G_{ijkl} \Pi^i \Pi^{kl} - h^{1/3} R, \quad (3a)
\]

\[
\mathcal{H}^i = -2\Pi^{ij,j}. \quad (3b)
\]

DeWitt metric

\[
G_{ijkl} = \frac{1}{2} h^{-1/2} \left( h_{ik} h_{jl} + h_{ij} h_{lk} - h_{ij} h_{kl} \right). \quad (4)
\]
Only non-zero commutator: \[
\left[ \hat{h}_{ij}(x), \hat{\pi}^{kl}(x') \right] = i\hbar \delta_{ij}^{kl} \delta^3(x - x')
\]

Possible representation:
\[
\hat{h}_{ij}(x) = h_{ij}(x) , \hat{\pi}^{ij}(x) = -ih \frac{\delta}{\delta h_{ij}(x)}
\]

Constraints

\[
\hat{\mathcal{H}}_0 \psi = 0 = \left( \hat{G}_{ijkl} \hat{\pi}^{ij} \hat{\pi}^{kl} - \hat{h}_{ij}^1 3 \hat{R} \right) \psi \implies G_{ijkl} \frac{\delta^2 \psi}{\delta h_{ij} \delta h_{kl}} + h_{ij}^1 3 \hat{R} \psi = 0 , (5)
\]

\[
\hat{\mathcal{H}}^i \psi = 0 = \left( \hat{\pi}^{ij} + 3 \hat{\Gamma}^i_{ab} \hat{\pi}^{ab} \right) \psi \implies \left( \frac{\delta \psi}{\delta h_{ij}} \right)_{;j} = 0 . (6)
\]
No solution (yet?) of $G_{ijkl} \frac{\delta^2 \psi}{\delta h^i_j \delta h^k_l} + h^{1/2} 3 \hat{R} \psi = 0$.

Approximation: keep only wavelength of the size of the Universe.

High degree of homogeneity in the observable Universe: a hint supporting this approximation?

Homogeneous and isotropic metric

$$ds^2 = -N^2(t)dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]. \quad (7)$$
Spatially flat spacetime \((k = 0) + \) perfect fluid:

\[
H_T = \mathcal{N} \mathcal{H}_T = \mathcal{N} \left( - \frac{\Pi^2_a}{24a} + \frac{\Pi_T}{a^{3w}} \right) = 0
\]

with \(T\) a global time (unitary evolution)
Choose ordering before quantising constraint:

\[
\frac{N}{a^{3w}} \left( -\frac{1}{24} a^{\frac{3w-1}{2}} \Pi a^{\frac{3w-1}{2}} \Pi + \Pi_T \right) = 0 .
\] (8)

Physically interesting! After gauge choice \( N = a^{3w} \):

\[
\frac{1}{24} \left[ a^{\frac{3w-1}{2}} \frac{\partial}{\partial a} \left( a^{\frac{3w-1}{2}} \frac{\partial}{\partial a} \right) \right] \psi(a, T) = i \frac{\partial \psi}{\partial T}(a, T) .
\] (9)

Covariant upon field redefinition.
WHEELER-DE WITT EQUATION

Time-reversed wavefunction evolution for a 1D free particle:

\[ i \frac{\partial \Psi}{\partial T}(\chi, T) = \frac{1}{24} \frac{\partial^2 \Psi}{\partial \chi^2}(\chi, T), \quad \chi = -\frac{2}{3w-3} a^{\frac{3w-3}{2}} . \]  

(10)

Choose normalised IC with arbitrary bounce timescale \( T_b \):

\[ \Psi_{\text{init}}(\chi) = \left( \frac{8}{T_b \pi} \right)^{\frac{1}{4}} \exp\left( -\frac{\chi^2}{T_b} \right) , \]  

(11)
WHEELER-DE WITT EQUATION

Normalised wavefunction of the Universe at all times

$$\psi(a, T) = \left( \frac{8 T_b}{\pi (T^2 + T_b^2)} \right)^{\frac{1}{4}} \exp \left( \frac{-4 T_b a^{3(1-w)}}{9 (T^2 + T_b^2) (1 - w)^2} \right) \times \exp \left\{ -i \left[ \frac{4 T a^{3(1-w)}}{9 (T^2 + T_b^2) (1 - w)^2} + \frac{1}{2} \tan(\frac{T_b}{T}) - \frac{\pi}{4} \right] \right\} . \quad (12)$$

How to interpret the quantum mechanics of the Universe?
Guidance equation with $\Psi := \text{Re} e^{iS/\hbar}$:

$$\frac{da}{dT} = -\frac{a^{3w-1}}{12} \frac{\partial S}{\partial a}.$$  \hspace{1cm} (13)

Insert phase of wavefunction:

$$a(T) = a_0 \left[ 1 + \left( \frac{T}{T_0} \right)^2 \right]^{\frac{1}{3(1-w)}}.$$  \hspace{1cm} (14)

Non-singular, symmetrical bouncing cosmology
THE ELECTROMAGNETIC SECTOR

• Problem of time: choose dust \((w = 0)\) as clock \(\Rightarrow T = t\).

• Coupling EM / curvature (Frion et al. (2020))

\[
\mathcal{L}_{EM} = - \left(\frac{1}{4} + \frac{R}{m_*^2}\right) F_{\mu\nu} F^{\mu\nu}.
\]  

(15)

Mass scale \(m_*\) to be determined by observations.

What is the physically interesting space of parameters?
Quantising EM in Coulomb gauge \((A_0 = 0\) and \(\partial_i A^i = 0\))

- Potential vector with \textit{a priori} two helicities \(\sigma\)

\[
\hat{A}_i(t, x) = \sum_{\sigma=1,2} \int \frac{d^3k}{(2\pi)^{3/2}} \left[ \epsilon_{i,\sigma}(k) \hat{A}_{k,\sigma} A_{k,\sigma}(t) e^{ik \cdot x} + H.C. \right],
\]

No external charge + isotropy

\[
\ddot{A}_k + \left( \frac{\dot{a}}{a} + \frac{\dot{f}}{f} \right) A_k + \frac{k^2}{a^2} A_k = 0.
\]
EVOLUTION OF THE MODES

\( A_k \): magnetic field, \( \Pi_k \): electric field
Magnetic power spectrum $\mathcal{P}_B \propto k^6$. Very blue!
PARAMETER SPACE AT 1 MPC

Orange region: lower limit set by cosmic rays
Blue region: lower limit set by dynamo effect
Scalar field + exponential potential + Bohmian cosmology = Bouncing cosmology with dark energy (Bacalhau et al (2017))!

\[ \hat{H} \Psi(\alpha, \phi) = 0 \Rightarrow \left[ -\frac{\partial^2}{\partial \alpha^2} + \frac{\partial^2}{\partial \phi^2} \right] \Psi(\alpha, \phi) = 0. \]  

(18)

Current work \( \Rightarrow \) adding PMF to this model
Wave function (Gaussian superposition):

\[ \Psi = \sigma \sqrt{\pi} \left\{ \exp \left[ -\left( \frac{\alpha + \phi}{2} \right)^2 \right] \exp \left[ id(\alpha + \phi) \right] + \exp \left[ -\left( \frac{\alpha - \phi}{2} \right)^2 \right] \exp \left[ -id(\alpha - \phi) \right] \right\}. \]  

Guidance equations (\( \alpha = \log a \)):

\[ \dot{\alpha} = -\frac{N}{l_p} e^{-3\alpha} \frac{\partial S}{\partial \alpha}, \quad \dot{\phi} = \frac{N}{l_p} e^{-3\alpha} \frac{\partial S}{\partial \phi} \]  

(19)
Gaussian and Cauchy couplings:

\[
\begin{align*}
    f & \equiv \frac{1}{4} + B e^{-\frac{\phi^2}{\beta^2}}, \\
    f & \equiv \frac{1}{4} + \frac{B}{1 + (\phi/\beta)^2}
\end{align*}
\]

\[ (21) \quad (22) \]

\[ B, \beta: \text{free parameters} \]
PRELIMINARY RESULTS

\[ \beta = 1.0e^{100}, \ \beta = 3.6e^{-4} \]
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