PRIMORDIAL MAGNETIC FIELDS IN BOUNCING COSMOLOGY

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MAGNETIC FIELDS IN THE UNIVERSE

- Magnetic fields (MF) $\simeq 1\mu G$ and coherence length $\lambda \simeq 1 kpc$ observed in galaxies and clusters.
- MF between $0 < z \leq 2$ have same strength: saturation mechanism $\implies$ "seed" field enhanced and maintained (*e.g.* through dynamo).
- Origin: astrophysical processes (after recombination)?
  primordial mechanisms (before recombination)?
- Primordial MF unaltered in voids $\implies$ still "seed" MF. Indirect evidence only (but see Broderick et al. Astrophys. J. 752, 22).

INFLATIONARY MAGNETOGENESIS

PMF in the very early Universe?

- Maxwell electromagnetism (EM)

$$\mathcal{L}_{Maxwell} = \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \tag{1}$$

  Conformally invariant: no EM fields during expansion.
- Way-out: break invariance through coupling (inflation, geometry, etc...).

$$\frac{1}{4} f(\phi) F_{\mu\nu} F^{\mu\nu} \,, \quad \frac{1}{4} R F_{\mu\nu} F^{\mu\nu} \qquad \text{B. Ratra(1988 – 1989)}$$

INFLATIONARY MAGNETOGENESIS

Inflationary magnetogenesis provide...

- ...seed MF compatible with observations
- ...naturally large coherence scale

BUT!

- ...simple models produce weak MF (*e.g.* $f(\phi)$ or R)
- ...the EM energy density usually big $\implies$ strong backreaction!
- ...alternative mechanisms exist :)

GR HAMILTONIAN (ADM FOLIATION)

$$H_T = \int d^3x \left(N\mathcal{H}_0 + N_i\mathcal{H}^i + \lambda P + \lambda^i P_i \right) \ , \tag{2}$$

<u>Constraints</u>

$$\mathcal{H}_0 = G_{ijkl}\Pi^{ij}\Pi^{kl} - h^{\frac{1}{2}}\, {}^3R \ , \tag{3a}$$

$$\mathcal{H}^i = -2\Pi^{ij}{}_{;j} \ . \tag{3b}$$

<u>DeWitt metric</u>

$$G_{ijkl} = \frac{1}{2}h^{-\frac{1}{2}}\left(h_{ik}h_{jl} + h_{il}h_{jk} - h_{ij}h_{kl}\right) \ . \tag{4}$$

CANONICAL QUANTISATION

Only non-zero commutator: $\left[\hat{h}_{ij}(x), \hat{\pi}^{kl}(x')\right] = i\hbar\delta^{kl}_{ij}\delta^3(x-x')$

Possible representation: $\hat{h}_{ij}(x) = h_{ij}(x)\ , \hat{\pi}^{ij}(x) = -i\hbar\frac{\delta}{\delta h_{ij}(x)}$

<u>Constraints</u>

$$\hat{\mathcal{H}}_0\psi = 0 = \left(\hat{G}_{ijkl}\hat{\pi}^{ij}\hat{\pi}^{kl} - \hat{h}^{\frac{1}{2}}\,{}^3\hat{R}\right)\psi \implies G_{ijkl}\frac{\delta^2\psi}{\delta h_{ij}\delta h_{kl}} + h^{\frac{1}{2}}\,{}^3\hat{R}\psi = 0\ , \qquad (5)$$

$$\hat{\mathcal{H}}^i\psi = 0 = \left(\hat{\pi}^{ij}{}_{,j} + {}^3\hat{\Gamma}^i{}_{ab}\hat{\pi}^{ab}\right)\psi \implies \left(\frac{\delta\psi}{\delta h_{ij}}\right)_{;j} = 0\ . \qquad (6)$$

MINISUPERSPACE APPROXIMATION

No solution (yet?) of $G_{ijkl}\frac{\delta^2\psi}{\delta h_{ij}\delta h_{kl}} + h^{\frac{1}{2}}\,{}^3\hat{R}\psi = 0$.

Approximation: keep only wavelength of the size of the Universe.

High degree of homogeneity in the observable Universe: a hint supporting this approximation?

Homogeneous and isotropic metric

$$ds^2 = -N^2(t)dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2\sin^2\theta d\phi^2\right] . \qquad (7)$$

TOTAL SUPER-HAMILTONIAN

Spatially flat spacetime ($k = 0$) + perfect fluid:

$$H_T = N\mathcal{H}_T = N\left(-\frac{\Pi_a^2}{24a} + \frac{\Pi_T}{a^{3w}}\right) = 0$$

with T a global time (unitary evolution)

WHEELER-DE WITT EQUATION

Choose ordering before quantising constraint:

$$\frac{N}{a^{3w}}\left(-\frac{1}{24}a^{\frac{3w-1}{2}}\Pi_a a^{\frac{3w-1}{2}}\Pi_a+\Pi_T\right)=0\,. \tag{8}$$

Physically interesting! After gauge choice $N=a^{3w}$:

$$\frac{1}{24}\left[a^{\frac{3w-1}{2}}\frac{\partial}{\partial a}\left(a^{\frac{3w-1}{2}}\frac{\partial}{\partial a}\right)\right]\Psi(a,T)=i\frac{\partial\Psi}{\partial T}(a,T)\,. \tag{9}$$

Covariant upon field redefinition.

WHEELER-DE WITT EQUATION

Time-reversed wavefunction evolution for a 1D free particle:

$$i\frac{\partial\Psi}{\partial T}(\chi, T) = \frac{1}{24}\frac{\partial^2\Psi}{\partial\chi^2}(\chi, T)\,, \quad \chi = -\frac{2}{3w-3}a^{\frac{3w-3}{2}}\,. \tag{10}$$

Choose normalised IC with arbitrary bounce timescale T_b:

$$\Psi_{init}(\chi) = \left(\frac{8}{T_b\pi}\right)^{\frac{1}{4}}\exp\left(-\frac{\chi^2}{T_b}\right)\,, \tag{11}$$

WHEELER-DE WITT EQUATION

Normalised wavefunction of the Universe at all times

$$\Psi(a,T) = \left(\frac{8\,T_b}{\pi\left(T^2+T_b^2\right)}\right)^{\frac{1}{4}} \exp\left(\frac{-4\,T_b a^{3(1-w)}}{9\left(T^2+T_b^2\right)(1-w)^2}\right) \times \exp\left\{-i\left[\frac{4\,T a^{3(1-w)}}{9\left(T^2+T_b^2\right)(1-w)^2} + \frac{1}{2}\mathrm{atan}\left(\frac{T_b}{T}\right) - \frac{\pi}{4}\right]\right\}. \tag{12}$$

How to interpret the quantum mechanics of the Universe?

DE BROGLIE-BOHM INTERPRETATION

Guidance equation with $\Psi := Re^{iS/\hbar}$:

$$\frac{\mathrm{d}a}{\mathrm{d}T} = -\frac{a^{3w-1}}{12}\frac{\partial S}{\partial a} \,. \tag{13}$$

Insert phase of wavefunction:

$$a(T) = a_0 \left[1 + \left(\frac{T}{T_0}\right)^2\right]^{\frac{1}{3(1-w)}} \,. \tag{14}$$

Non-singular, symmetrical bouncing cosmology

THE ELECTROMAGNETIC SECTOR

- Problem of time: choose dust ($w = 0$) as clock $\implies T = t$.
- Coupling EM / curvature (Frion et al. (2020))

$$\mathcal{L}_{EM} = -\left(\frac{1}{4} + \frac{R}{m_\star^2}\right) F_{\mu\nu} F^{\mu\nu} \; . \tag{15}$$

Mass scale $m_\star$ to be determined by observations.

What is the physically interesting space of parameters?

THE ELECTROMAGNETIC SECTOR

Quantising EM in Coulomb gauge ($A_0 = 0$ and $\partial_i A^i = 0$)

- Potential vector with *a priori* two helicities σ

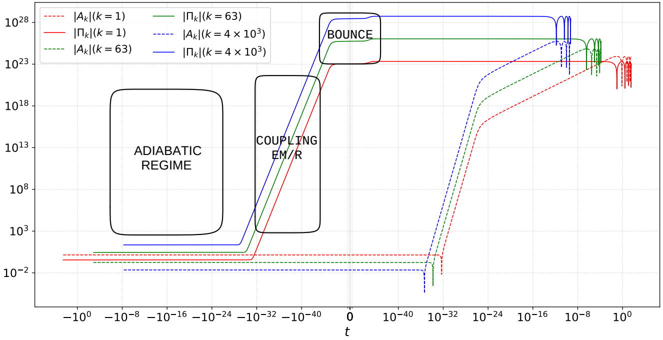
$$\hat{A}_i(t,\mathbf{x}) = \sum_{\sigma=1,2} \int \frac{d^3k}{(2\pi)^{3/2}} \left[\epsilon_{i,\sigma}(\mathbf{k}) \hat{a}_{\mathbf{k},\sigma} A_{k,\sigma}(t) e^{i\mathbf{k}\cdot\mathbf{x}} + H.C. \right] , \tag{16}$$

No external charge + isotropy

$$\ddot{A}_k + \left(\frac{\dot{a}}{a} + \frac{\dot{f}}{f} \right) \dot{A}_k + \frac{k^2}{a^2} A_k = 0 . \tag{17}$$

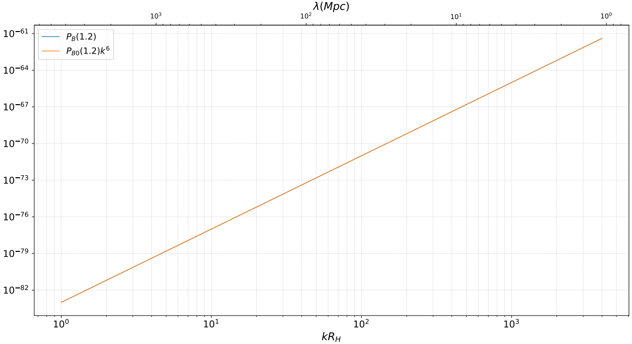
EVOLUTION OF THE MODES

A_k: magnetic field, Π_k: electric field



MAGNETIC POWER SPECTRUM

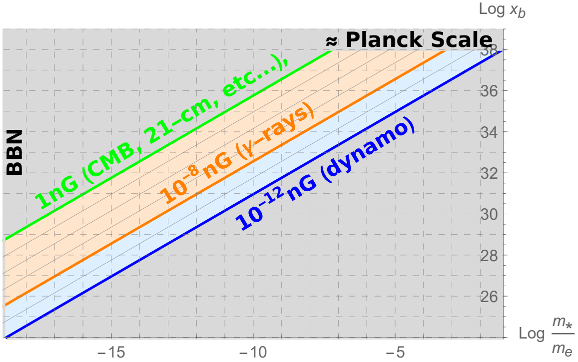
Magnetic power spectrum $\mathcal{P}_B \propto k^6$. Very blue!



PARAMETER SPACE AT 1 MPC

Orange region: lower limit set by cosmic rays

Blue region: lower limit set by dynamo effect



BOUNCING COSMOLOGY, DARK ENERGY AND PMF

Scalar field + exponential potential + Bohmian cosmology = Bouncing cosmology with dark energy (Bacalhau et al (2017))!

$$\hat{\mathcal{H}}\Psi(\alpha,\phi) = 0 \Rightarrow \left[-\frac{\partial^2}{\partial\alpha^2} + \frac{\partial^2}{\partial\phi^2}\right]\Psi(\alpha,\phi) = 0. \tag{18}$$

Current work $\Longrightarrow$ adding PMF to this model

BOUNCING COSMOLOGY, DARK ENERGY AND PMF

Wave function (Gaussian superposition):

$$\Psi = \sigma\sqrt{\pi}\left\{\exp\left[-\frac{(\alpha+\phi)^2\sigma^2}{4}\right]\exp[id(\alpha+\phi)] + \exp\left[-\frac{(\alpha-\phi)^2\sigma^2}{4}\right]\exp[-id(\alpha-\phi)]\right\}. \tag{19}$$

Guidance equations ($\alpha = \log a$):

$$\dot{\alpha} = -\frac{N}{l_p}\mathrm{e}^{-3\alpha}\frac{\partial S}{\partial \alpha}, \quad \dot{\phi} = \frac{N}{l_p}\mathrm{e}^{-3\alpha}\frac{\partial S}{\partial \phi} \tag{20}$$

BOUNCING COSMOLOGY, DARK ENERGY AND PMF

Gaussian and Cauchy couplings:

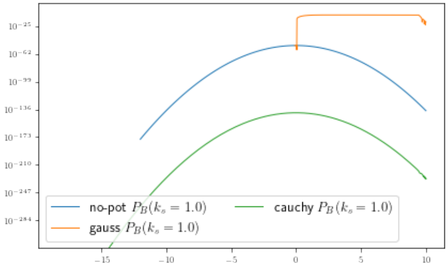
$$f \equiv \frac{1}{4} + Be^{-\frac{\phi^2}{\beta^2}} , \tag{21}$$

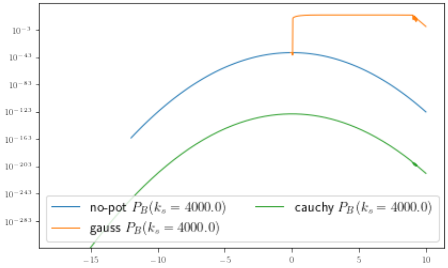
$$f \equiv \frac{1}{4} + \frac{B}{1 + (\phi/\beta)^2} \tag{22}$$

B, β: free parameters

PRELIMINARY RESULTS

$\beta = 1.0e^{100}$, $\beta = 3.6e^{-4}$





PRELIMINARY RESULTS

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