Thin Wall False Monopoles

by

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https://arxiv.org/abs/2312.17154

continuation/correction of

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arXiv:1006.0693 [hep-th]

Phys.Rev.D82:025022,2010

Various cosmological scenarios have the universe trapped in a false vacuum, at least for certain epochs of its evolution. The inflationary universe is explicitly dependent on the universe being in a false vacuum and then exponentially expanding for many e-foldings (which solves several major cosmological problems). String-based cosmology yields a persistent de Sitter expanding universe that is generically meta-stable, presumably to the present day. The model's phenomenological viability depends on the decay rate being sufficiently slow.

Generally speaking the false vacuum can decay via tunnelling to the true vacuum, and this tunnelling is mediated by instantons which correspond to Euclidean trajectories where a bubble of true vacuum forms inside the false vacuum, grows to a maximum size and then bounces back to the false vacuum.

False vacuum decay induced by topological solitons

- A topological soliton requires the vacuum manifold to be non-trivial. We imagine this occurs for the false vacuum, ie. at a local minimum of the potential. A topologically non-trivial texture in the false vacuum manifold then generally requires that the field passes through a region where it must vanish.
- If the true vacuum occurs at this point, one would imagine the region would grow without impediment.
- This can be avoided in certain scenarios with thin-walled topological solitons.

- Consider the Georgi-Glashow model, with 't Hooft Polyakov monopoles

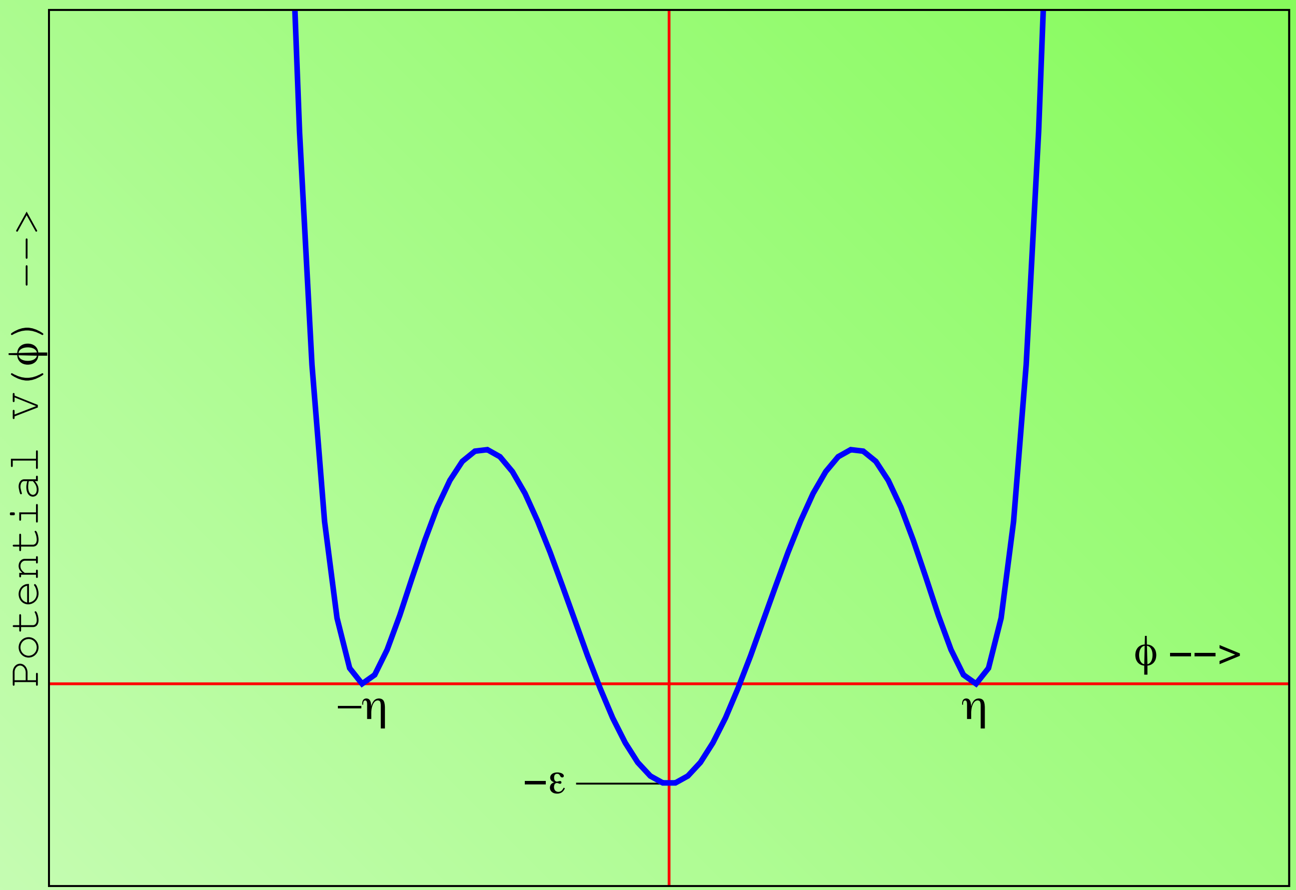
$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} + \frac{1}{2}(D_\mu\phi^a)(D^\mu\phi^a) - V(\phi^a\phi^a)$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + e\epsilon^{abc}A^b_\mu A^c_\nu,$$

$$D_\mu\phi^a = \partial_\mu\phi^a + e\epsilon^{abc}A^b_\mu\phi^c.$$

$$V(\phi) = \lambda\phi^2(\phi^2 - a^2)^2 + \gamma^2\phi^2 - \epsilon$$

- The potential is taken to have the form

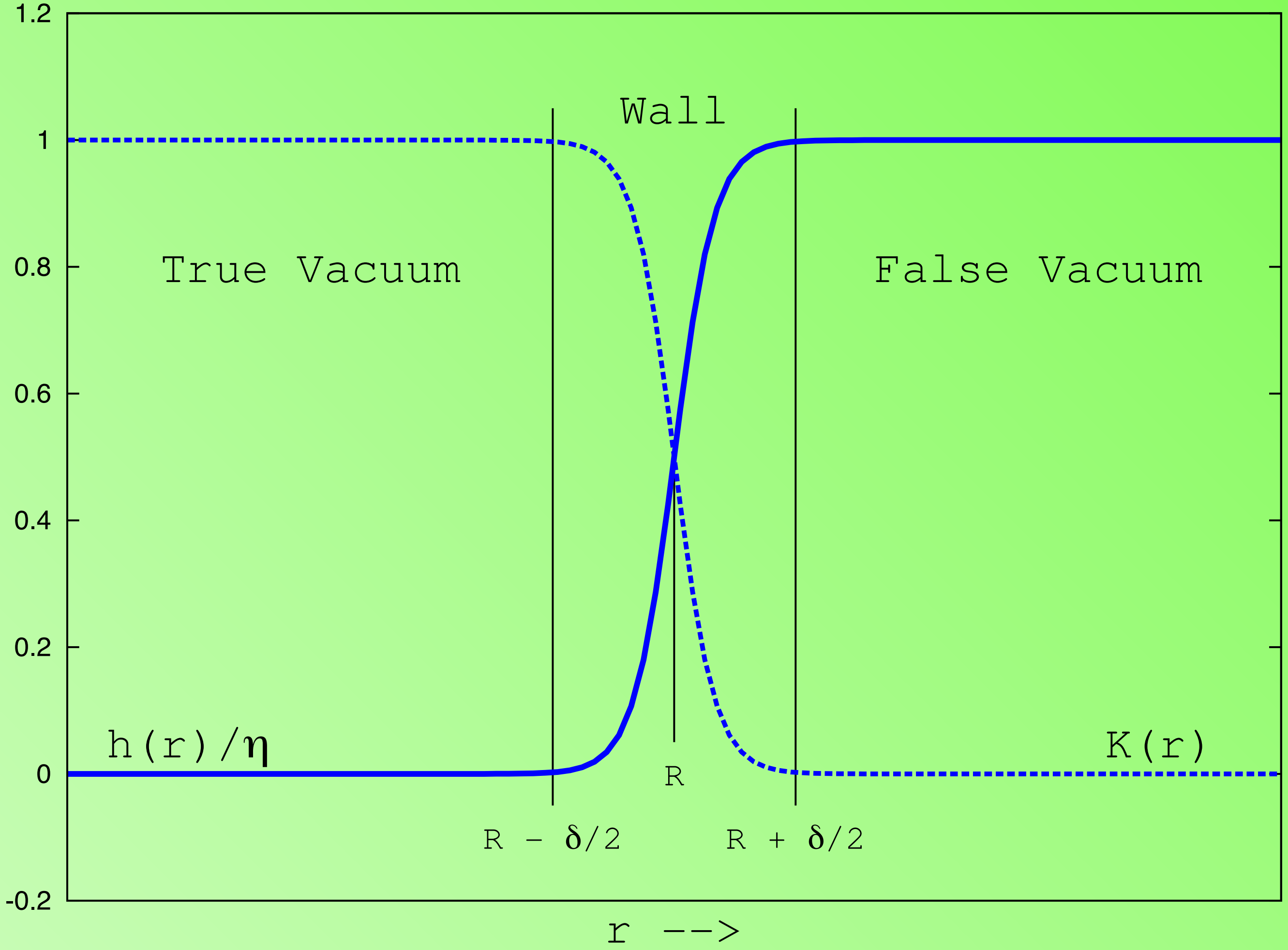


False Monopoles

- We can imagine soliton solutions to this theory which exist in the false vacuum:

$$
\begin{aligned}
\phi_a &= \hat{r}_a \, h(r) \\
A_\mu^a &= \epsilon_{\mu a b} \, \hat{r}_b \, \frac{1 - K(r)}{er} \\
A_0 &= 0
\end{aligned}
$$

- The functions $h(r)$ and $K(r)$ have the following behaviour:



Thin Walled Monopoles

- We first just assume the existence of thin walled monopoles. This means the functions *h(r)* and *K(r)* have the following profiles:

$$
\begin{aligned}
h \approx 0\,,\; K \approx 1 \qquad & r < R - \frac{\delta}{2} \\
h \approx \eta\,,\; K \approx 0 \qquad & r > R + \frac{\delta}{2} \\
0 < h < \eta\,,\; 0 < K < 1 \qquad & R - \frac{\delta}{2} \leq r \leq R + \frac{\delta}{2}
\end{aligned}
$$

Energy functional

- The energy of the thin-wall monopole will be given by:

$$E(K,h) = 4\pi \int_0^\infty dr \Big(\frac{(K')^2}{e^2} + \frac{(1-K^2)^2}{2e^2r^2} + \frac{1}{2}r^2(h')^2 + K^2h^2 + r^2V(h) \Big)$$

$$E = 4\pi \Big[\int_0^{R-\frac{\delta}{2}} dr\, r^2V(h) + \int_{R+\frac{\delta}{2}}^\infty dr\, \frac{1}{2e^2r^2} + \int_{R-\frac{\delta}{2}}^{R+\frac{\delta}{2}} dr \Big(\frac{(K')^2}{e^2} + \frac{(1-K^2)^2}{2e^2r^2} + \frac{1}{2}r^2(h')^2 + K^2h^2 + r^2V(h) \Big) \Big].$$

Energetics

- Then the total energy of the configuration must have the following behaviour in *R*

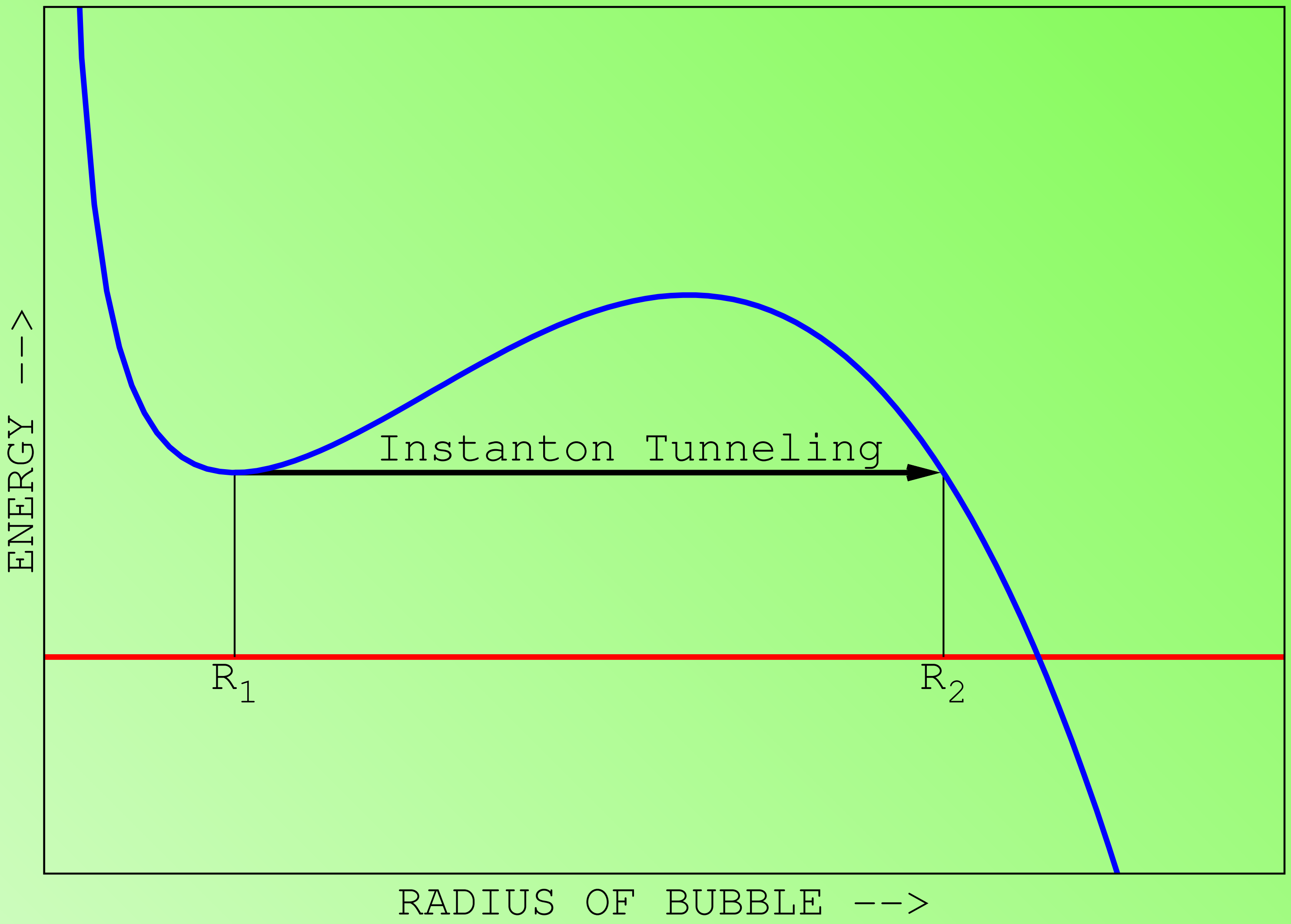
$$E(R) = -\alpha R^3 + 4\pi\sigma R^2 + \frac{C}{R}$$

- where the parameters are calculable:

$$\alpha = 4\pi\epsilon/3 \qquad\qquad C = 2\pi/e^2$$

$$\begin{aligned}\sigma \;&=\; \frac{1}{R^2}\int_{R-\frac{\delta}{2}}^{R+\frac{\delta}{2}} dr\,\Big(\frac{(K')^2}{e^2} + \frac{(1-K^2)^2}{2e^2r^2} \\ &+\; \frac{1}{2}r^2(h')^2 + K^2h^2 + r^2V(h)\Big).\end{aligned}$$

Graph of the energy



Technical Details

- We tried to prove the existence of the thin wall monopoles solutions using the classic analysis of Coleman for thin wall bubbles.
- Indeed, our monopoles are simply magnetically charged, thin wall bubbles.
- The equations of motion are:

$$
\begin{aligned}
h'' + \frac{2}{r}h' - \frac{2h}{r^2}K^2 - \frac{\partial V}{\partial h} &= 0 \\
K'' - \frac{K}{r^2}(K^2 - 1) - e^2h^2K &= 0.
\end{aligned}
$$

- We assumed that we can take $K(r)=1$ in the equation for $h(r)$. For large r obviously the terms involving $K(r)$ have r in the denominator and are negligible, while for small r the approximation $K(r)=1$ is simply valid. Not true!
- The assumption was that $K(r)$ does not greatly affect the behaviour of h, however $h(r)$ has a great effect on $K(r)$. Not true!
- Indeed, as $h(r)$ increases to its (false) vacuum value at large r, while $K(r)$ is forced to vanish by its equation. Sort of true!

Numerical solution

- We find numerically that the potential chosen does not give thin wall monopoles.
- We had to modify the potential to the following:

$$V(h) = \lambda\Big((h^2 - a^2)^2(h^2 - \frac{\epsilon^{\frac{1}{2n+1}}}{a^4})\Big)^{2n+1}$$

- which has the graph:

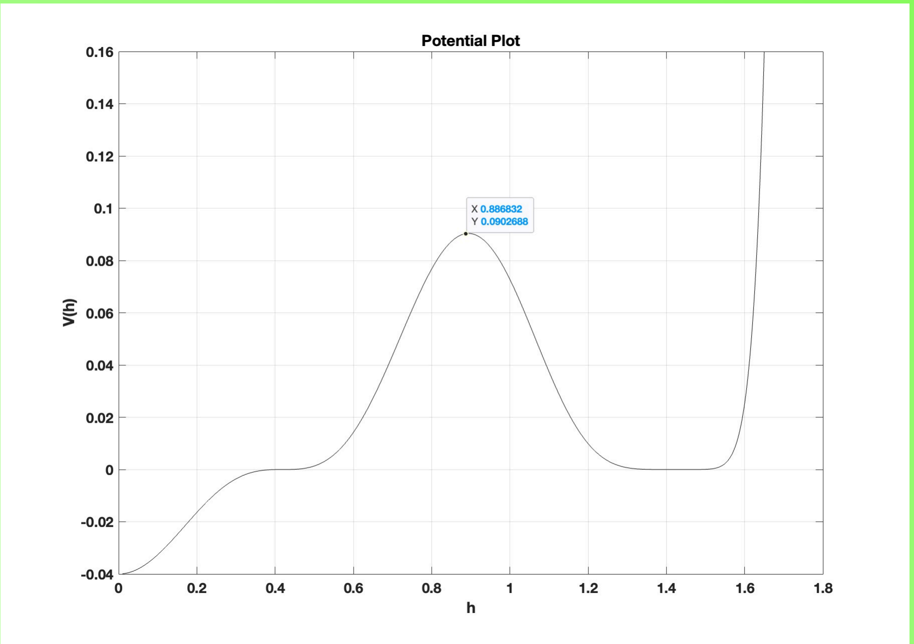


FIG. 2: Potential for the scalar field with $n = 1$, $a = 1.43$, $\lambda = 0.1$ and $\epsilon = .4$

Graph of the potential for different values of the parameters

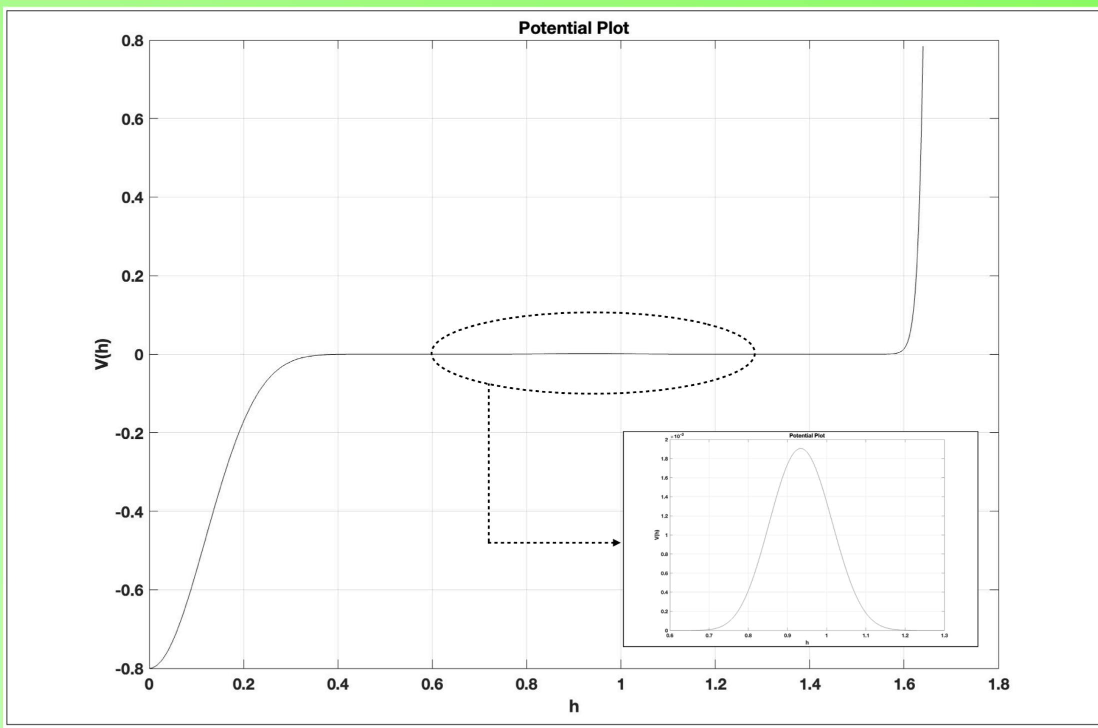


FIG. 12: Graph of the potential (including a zoom around $r = 1$), for $\epsilon = 8$, $\lambda = 0.1$, $a = 1.4$ and $n = 4$.

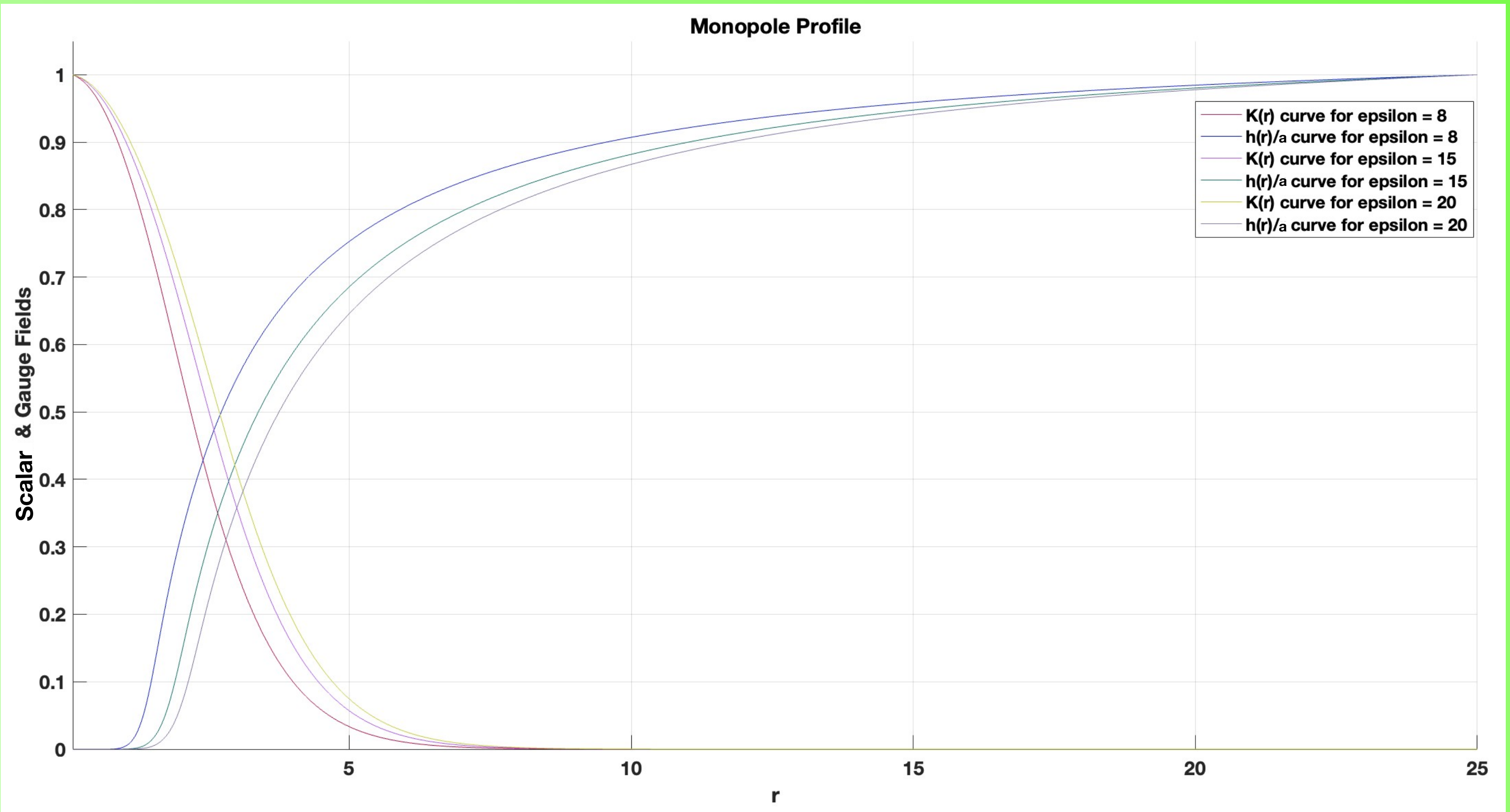


FIG. 4: The profiles for h/a and K for various values of ϵ and for $\lambda = 0.1$, $a = 1.4$ and $n = 4$.

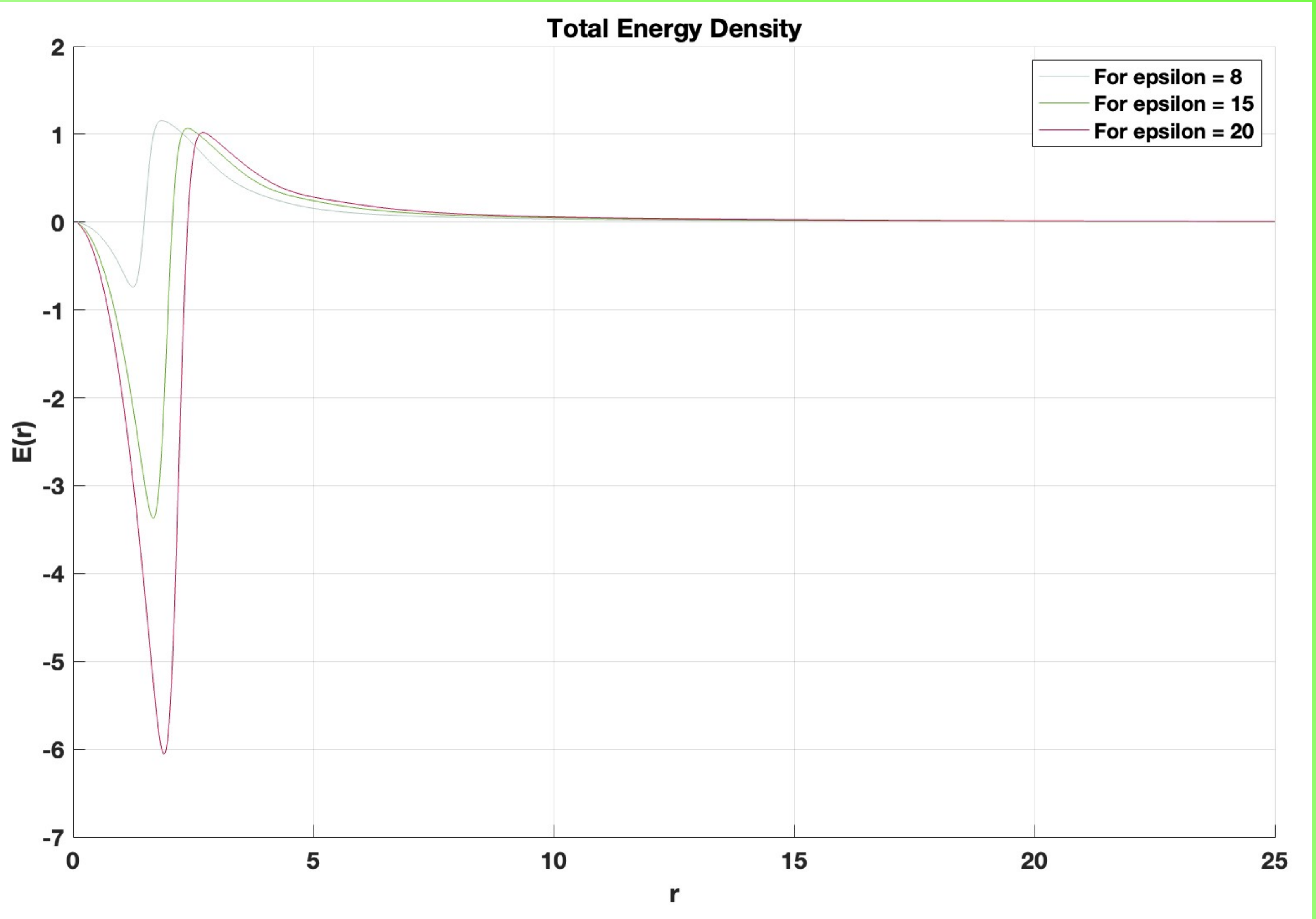


FIG. 5: The total energy density for the profiles in Fig.(4)for various values of ϵ and for $\lambda = 0.1$, $a = 1.4$ and and for $n = 4$.

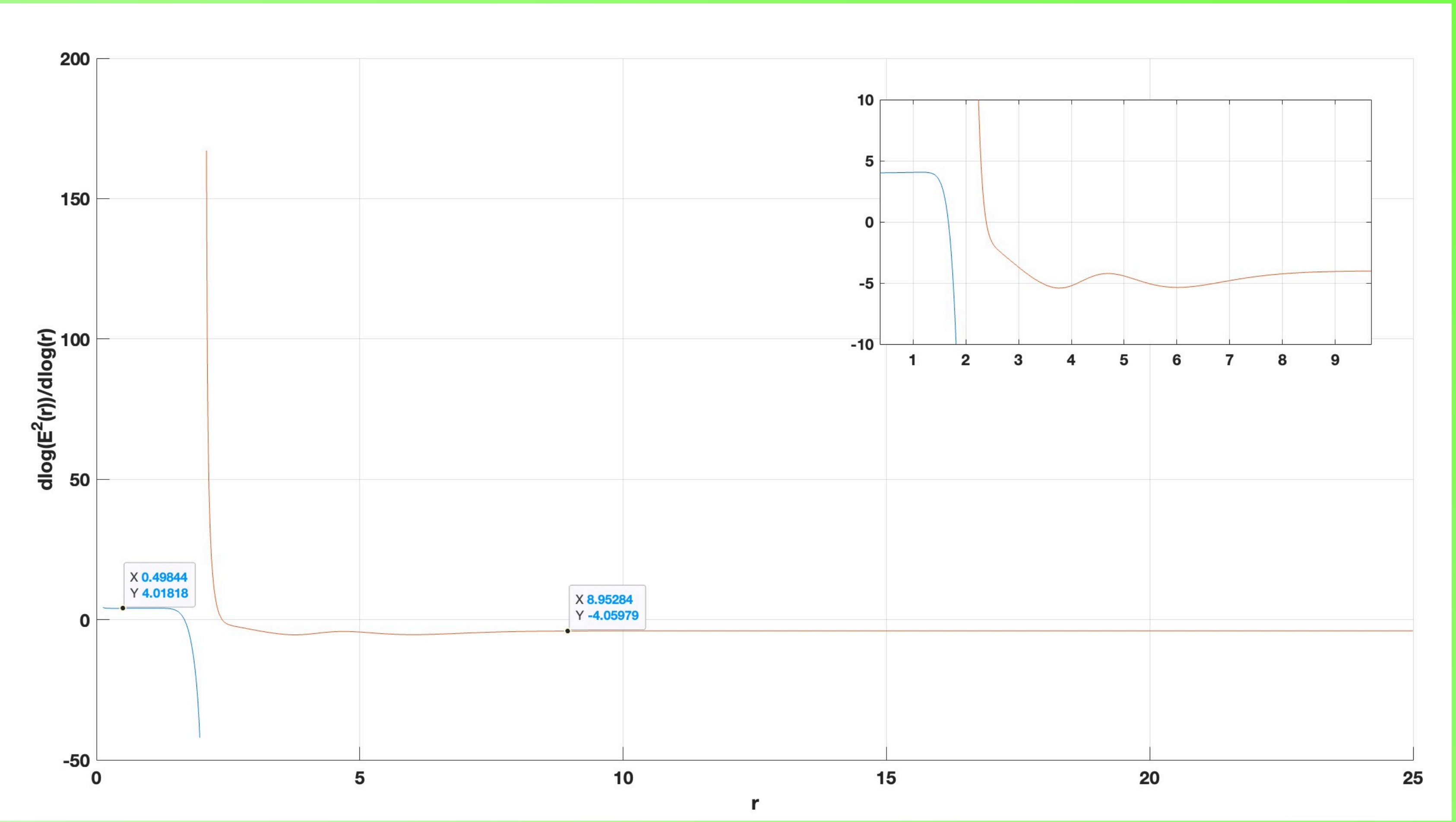


FIG. 6: The derivative of the log of the total energy density squared with respect to the log of r, for $\epsilon = 15$ and for $\lambda = 0.1$, $a = 1.4$ and and $n = 4$.

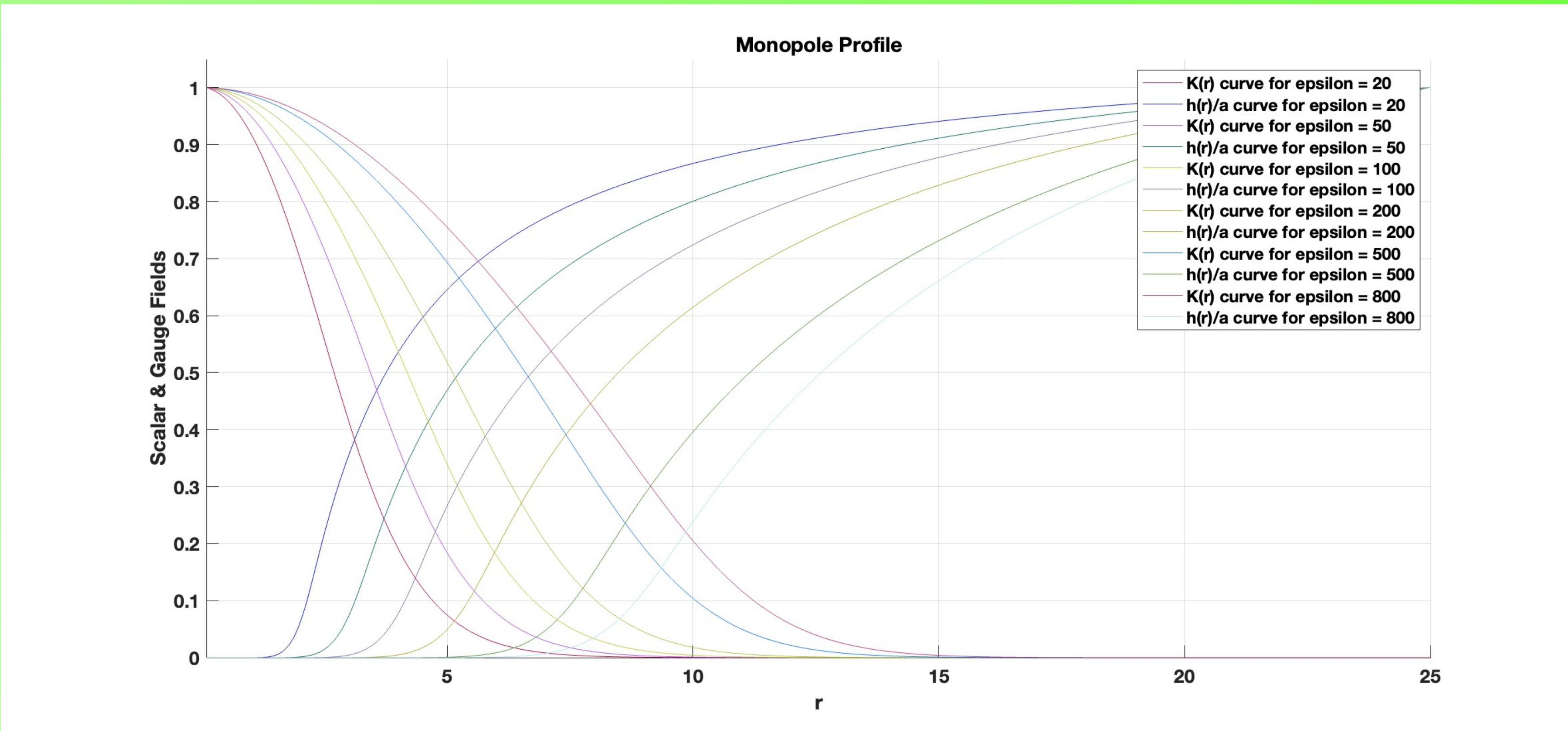


FIG. 11: Multiple solutions for h and$K$ for various values of ϵ and for $\lambda = 0.1$, $a = 1.4$ and and $n = 4$.

Conclusions

- We have computed the induced decay rate of the vacuum due to “false” monopoles of the false vacuum for thin wall monopoles.
- The rate can be unsuppressed for heavy monopoles.
- The monopole mass is controlled by the gauge coupling constant which has nothing to do with the scalar potential, which determines whether there is a false vacuum.
- We have numerically shown the existence of thin wall monopoles.