# Advances in the theoretical description of transfer reactions

Pierre Capel

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

JG|U

SFB 1245

1st December 2023

# Transfer reactions

Transfer reactions $(\mathrm{d,p}), (\mathrm{p,d}), (\mathrm{t,p}) \ldots$
used to study nuclear structure far from stability
e.g. halo nuclei, shell inversion...

Recent review : [Wimmer JPG 45, 033002 (2018)]

The ISS @ ISOLDE is the ideal setup
to measure these reactions on exotic nuclei
in inverse kinematics

Warning : What is the reaction probing ?

- Spectroscopic factors or Asymptotic normalisation ?
- What are the uncertainties in reaction model ?
- Should we develop new, more accurate models of transfer ?

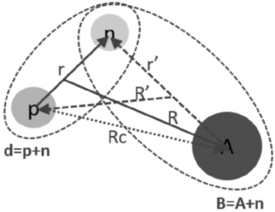
# Few-Body Model of Transfer Reactions

Transfer reactions are usually described with DWBA or ADWA

[Johnson & Tandy NPA 235, 56 (1974), review : Johnson JPG 41, 094005 (2014)]

Few-body model for $\mathrm{A(d,p)B}$, where $\mathrm{B \equiv A + n}$

$$
\begin{aligned}
H_{3b} &= T_R + T_r + U_{\mathrm{An}} + U_{\mathrm{Ap}} + V_{\mathrm{pn}} \\
T_{3b} &= \langle \phi_{\mathrm{B}} \chi^{(-)}_{\mathrm{Bp}} | V_{\mathrm{pn}} | \phi_{\mathrm{d}} \chi^{(+)}_{\mathrm{Ad}} \rangle,
\end{aligned}
$$




where

- $\chi^{(+)}_{\mathrm{Ad}}$ is incoming $\mathrm{A\text{-}d}$ distorted wave
  @ ADWA generated by $U_{\mathrm{Ad}} = \frac{\langle \phi_{\mathrm{d}} | V_{\mathrm{np}} \left( U_{\mathrm{Ap}} + U_{\mathrm{An}} \right) | \phi_{\mathrm{d}} \rangle}{\langle \phi_{\mathrm{d}} | V_{\mathrm{pn}} | \phi_{\mathrm{d}} \rangle}$
- $\phi_{\mathrm{d}}$ is the deuteron bound state generated by $V_{\mathrm{pn}}$
- $\chi^{(-)}_{\mathrm{Bp}}$ is $\mathrm{B\text{-}p}$ outgoing distorted wave
- $\phi_{\mathrm{B}}$ is the final single-particle bound state of $\mathrm{B}$ by $V_{\mathrm{An}}$

# Single-particle approximation

$\mathrm{A}$-$\mathrm{n}$ overlap wave function of the bound state of $\mathrm{B}$
is approximated by a single-particle wave function $\phi_{nljm}$

$$[T_r + V_{\mathrm{An}}(r)]\,\phi_{nljm}(\boldsymbol{r}) = E_{nl}\,\phi_{nljm}(\boldsymbol{r})$$

with $\|\phi_{nljm}\| = 1$

In reality, there is admixture of configurations :

$$\Psi_{\mathrm{B}}(J^\pi) = \Phi_{\mathrm{A}}(J^\pi_{\mathrm{A}}) \otimes \psi_{ljm}(\boldsymbol{r}) + \dots$$

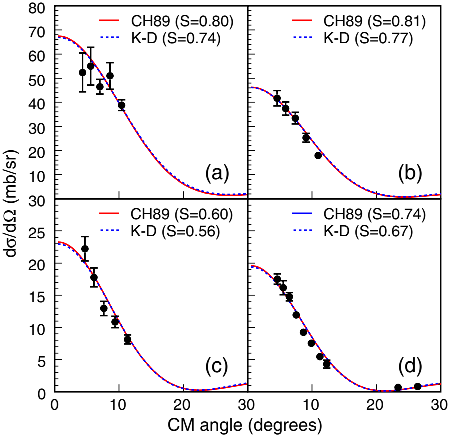
where $\psi_{ljm}$ is the overlap wave function
Spectroscopic Factor : $\mathcal{S}_{lj} = \|\psi_{ljm}\|^2$

Single-particle approximation $\equiv \psi_{ljm} = \sqrt{\mathcal{S}_{lj}}\,\phi_{nljm}$
$\Rightarrow$ usual idea : $\mathcal{S}_{lj} = \sigma^{\mathrm{exp}}_{\mathrm{bu}}/\sigma^{\mathrm{th}}_{\mathrm{bu}}$

# Example on $^{11}\mathrm{Be}$

$^{10}\mathrm{Be(d,p)}^{11}\mathrm{Be}$ in inverse kinematics @ Oak Ridge

Schmidt *et al.* PRL 108, 192701 (2012)]




$E_{\mathrm{d}}$ = (a) 12 MeV (b) 15 MeV (c) 18 MeV (d) 21.4 MeV

SF varies with beam energy and optical potential

# ANC vs SF

Is $\mathcal{S}_{lj} = \sigma_{\rm bu}^{\rm exp}/\sigma_{\rm bu}^{\rm th}$ ?

Are transfer reactions really sensitive to SF ?
i.e. do we probe the whole overlap wave function ?

Isn't transfer rather peripheral ?
i.e. sensitive only to asymptotics ?

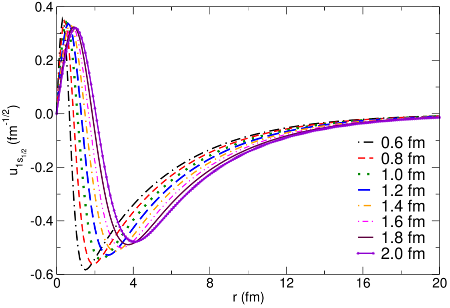
$$\psi_{lj}(r) \underset{r\to\infty}{\longrightarrow} C_{lj}\, e^{-\kappa r}$$

Asymptotic Normalisation Coefficient : $C_{lj}$

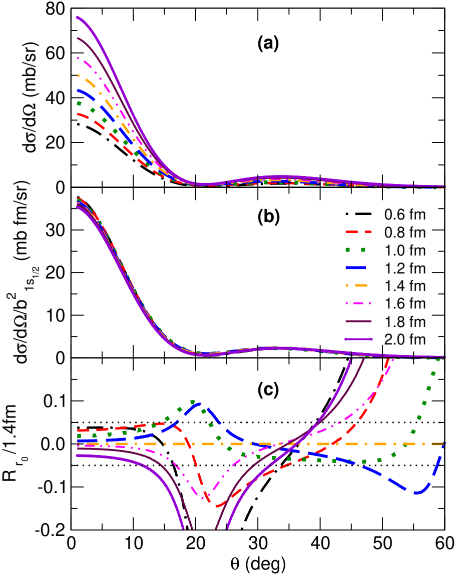
Study this on $^{14}\mathrm{C(d,p)}^{15}\mathrm{C}$
forming the one-neutron halo nucleus $^{15}\mathrm{C}$ (see Maria Borge's talk)
[Yang, PhD ; Moschini,Yang, PC PRC 100, 044615 (2019)]

# Test on $^{14}\mathrm{C}(\mathrm{d},\mathrm{p})^{15}\mathrm{C}$

Consider many $V_{^{14}\mathrm{Cn}}$ to describe one-neutron halo nucleus $^{15}\mathrm{C}$




[Moschini,Yang, PC PRC 100, 044615 ('19)]




⇒ different $\phi_{^{14}\mathrm{Cn}}$ ($\mathcal{S}_{s1/2} = 1$)

⇒ different $d\sigma_{(\mathrm{d,p})}/d\Omega$

Difference due to the tail of $\phi_{^{14}\mathrm{Cn}}$

⇒ can infer the ANC from data

by selecting forward angles

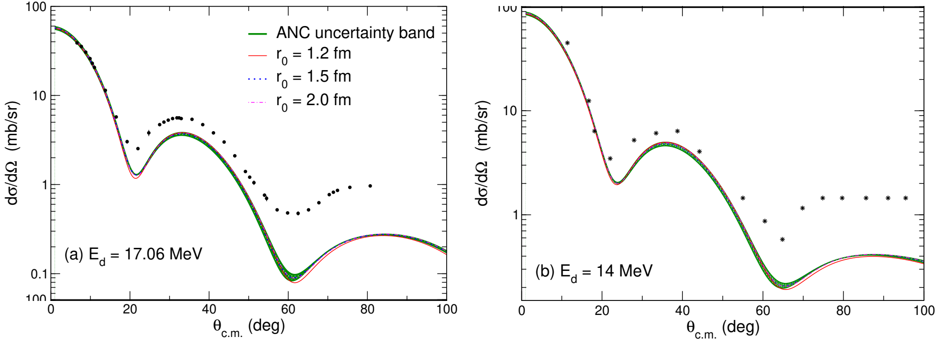
and low energy

# Re-analysis of $^{14}C(d,p)^{15}C$ @ $E_d = 17$ MeV

Scaling theory to experiment,

we infer $C^2_{1/2^+} = 1.59 \pm 0.06\ \text{fm}^{-1}$ Moschini, Yang, PC PRC 100, 044615 (2019)

agrees with *ab initio* $C^2_{1/2^+} = 1.644\ \text{fm}^{-1}$ Navrátil *et al.* (2019)



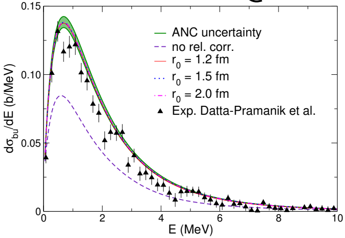

Ex : Mukhamedzhanov *et al.* PRC 84, 024616 (11) Ex : Goss *et al.* PRC 12, 1730 (1975)

Larger angles sensitive to short-range physics

Good agreement also with previous data @ $E_d = 14$ MeV

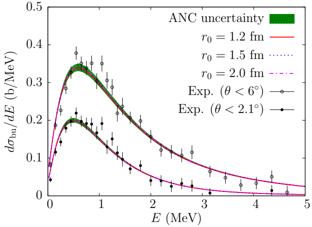
# And with other experiments on $^{15}$C

$^{15}\mathrm{C} + \mathrm{Pb} \rightarrow {}^{14}\mathrm{C} + \mathrm{n} + \mathrm{Pb}$ @ 605*A*MeV



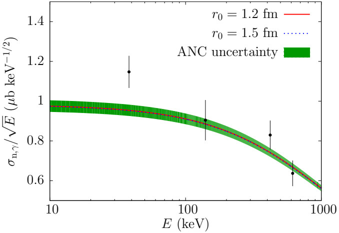

Ex : Datta Pramanik *et al.* PLB 551, 63 ('01)

$^{15}\mathrm{C} + \mathrm{Pb} \rightarrow {}^{14}\mathrm{C} + \mathrm{n} + \mathrm{Pb}$ @ 68*A*MeV




Ex : Nakamura *et al.* PRC 79, 035805 ('03)

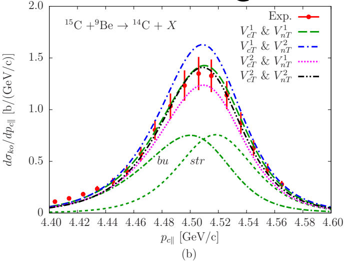
$^{14}\mathrm{C}(\mathrm{n},\gamma)^{15}\mathrm{C}$




Ex : Reifarth *et al.* C 77, 015804 (2008)

Th : Moschini *et al.* PRC 100, 044615 ('19)

$^{15}\mathrm{C} + {}^{9}\mathrm{Be} \rightarrow {}^{14}\mathrm{C} + \mathrm{X}$ @ 54*A*MeV




Ex : Tostevin *et al.* PRC 66, 024607 (2002)

Th : Hebborn, PC PRC 104, 024616 ('21)

# Quantification of parametric uncertainties in reactions

Bayesian approaches are very useful
to quantify theoretical uncertainties

[Furnstahl *et al.* JPG 42, 034028 (2015) & PRC 92, 024005 (2015)]

Used often with EFT to quantify uncertainty in

- NN $\chi$EFT interactions
- EoS

Thanks to ADWA low computational cost, extended to study propagation of optical model uncertainties to $(\mathrm{d},\mathrm{p})$

[Lowell, Nunes PRC 97, 064612 (2018)]

More recently to KO [Hebborn *et al.* PRC 108, 014601 (2023)]

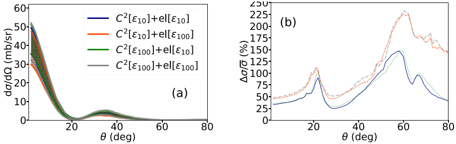
and breakup reactions [Sürer *et al.* PRC 106, 024607 (2023)]

The original study of Lowell and Nunes on $(\mathrm{d},\mathrm{p})$ expanded to single-particle structure model [Catacora-Rios *et al.* PRC 108, 024601 (2023)]

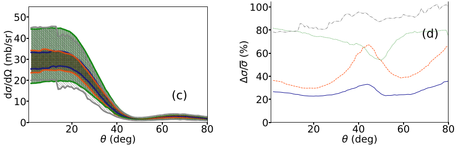
# Complete study including A-n and optical potentials

[Catacora-Rios, Lowell, Nunes PRC 108, 024601 (2023)]

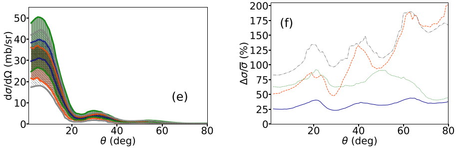
$^{14}\mathrm{C(d,p)}^{15}\mathrm{C}$ @ $E_\mathrm{d}$ = 17 MeV




$^{16}\mathrm{O(d,p)}^{17}\mathrm{O}$ @ $E_\mathrm{d}$ = 15 MeV




$^{48}\mathrm{Ca(d,p)}^{49}\mathrm{Ca}$ @ $E_\mathrm{d}$ = 24 MeV




Authors vary parameters of A-n and optical potentials constrained by ANC and $d\sigma_{\mathrm{el}}/d\Omega$ to get posterior distributions

- constraining optical potential is not enough
- ANC plays a significant role
- should constrain both

Important for nuclear-structure studies using (d, p)

⇒ make sure to know what calculations are sensitive to

# Summary an take-home message

- Transfer reaction used to study nuclei far from stability (ISS)
- Usually described within DWBA or ADWA
  with a single-particle description of nucleus
  Spectroscopic factors inferred from data : $\mathcal{S}_{lj} = \sigma_{\rm bu}^{\rm exp} / \sigma_{\rm bu}^{\rm th}$
- In the case of halo nucleus $^{15}\mathrm{C}$ [Moschini *et al.* PRC 100, 044615 (2019)]
  $^{14}\mathrm{C(d,p)}$ purely peripheral at low energy and forward angles
  ⇒ can be used to infer ANC NOT SF !!!
  ⇒ agreement with other reactions : breakup, $(\mathrm{n},\gamma)$, KO...
- Sensitivity to inputs studied with Bayesian approach
  [Catacora-Rios *et al.* PRC 108, 024601 (2023)]
  ⇒ ANC is important even for deeply-bound nuclei
  uncertainty in optical potentials are significant
  ⇒ affects structure information inferred from experiment
- Need transfer model that goes beyond single-particle model
  [Gomez-Ramos & Moro PRC 95, 044612 (2017) ; Punta *et al.* PRC 108, 024613 (2023)]

Be sure to know to what the reaction is sensitive...

# Thanks to my collaborators

Jiecheng Yang — 湖州师范学院 Huzhou University

Laura Moschini — University of Surrey

Chloë Hebborn — NSCL

Hans-Werner Hammer — Technische Universität Darmstadt

Daniel Phillips — Ohio University