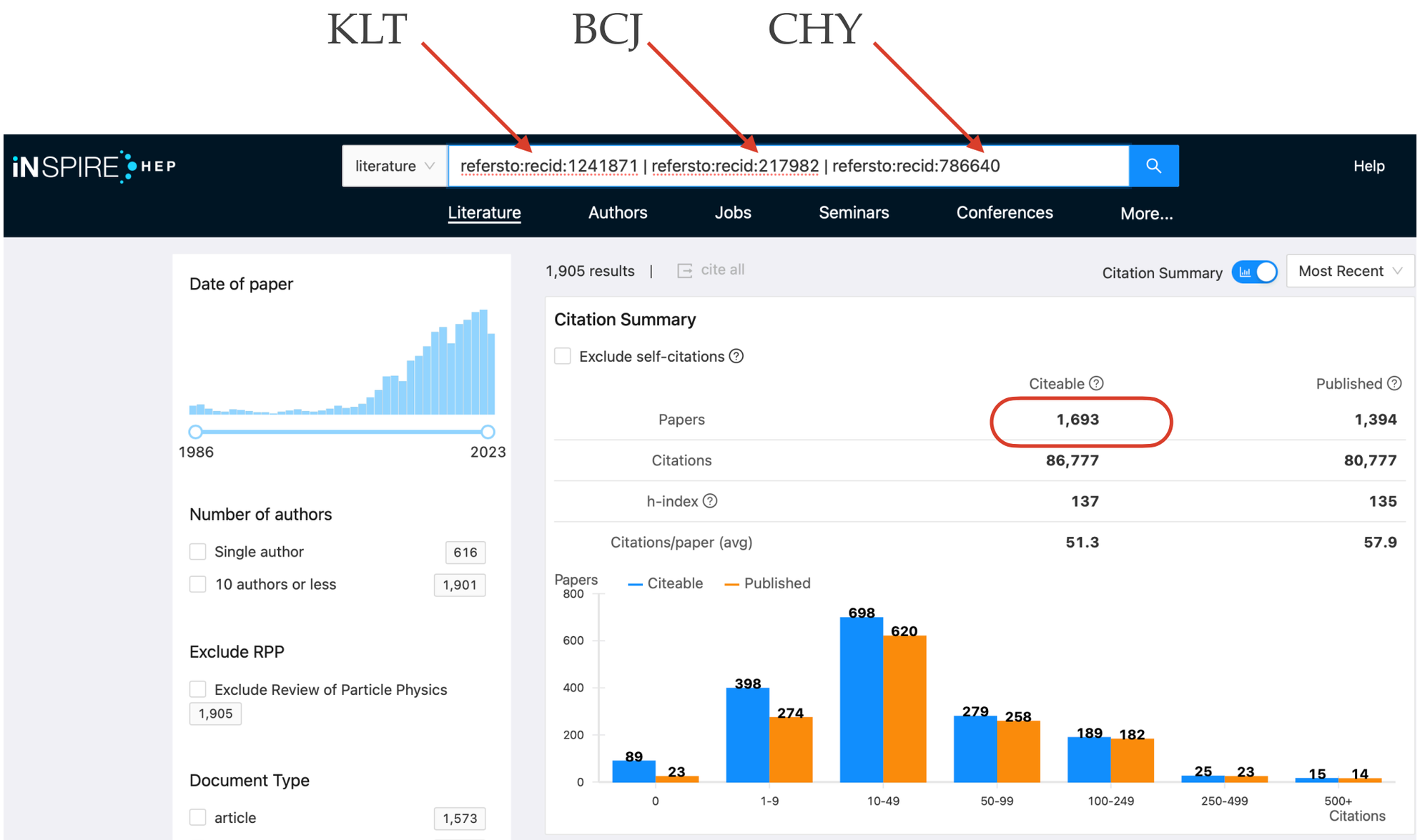
QCD meets Gravity, CERN, December 2023

The Double Copy: a Review

Donal O'Connell
Edinburgh

The Double Copy: a Review



The Double Copy: a Review

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Many important topics totally omitted! (Cosmology, string theory, generalised double copy, massive gravity, UV behaviour of supergravity, double copy on backgrounds, …)

What is the Double Copy?

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Method of constructing amplitudes in theory T from amplitudes in two other theories t^L, t^R

$$\mathcal{M}_n = \mathcal{A}_n^L \otimes \mathcal{A}_n^R$$

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Bilinear in amplitudes

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Could be the same

$$t^L = t^R$$

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Focus here: “QCD meets Gravity”!

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Various formulations…

Formulations of the Double Copy

1. KLT (Kawai, Lewellen, Tye) 1986

$$\mathcal{M}_n = \sum_{a,b} A_n^L[a] S_n[a|b] A_n^R[b]$$

KLT kernel: function of Mandelstams
Inverse of biadjoint scalar amplitudes

Stringy derivation. Here field theory limit

Bjerrum-Bohr, Bern, Cachazo, Damgaard, Dixon, He, Perelstein, Rozowsky, Sondergaard, Yuan, Vanhove

Formulations of the Double Copy

2. BCJ (Bern, Carrasco, Johansson) 2008-2010

$$\mathcal{A}^L = \sum_i \frac{n_i^L c_i^L}{D_i} \qquad \mathcal{A}^R = \sum_i \frac{n_i^R c_i^R}{D_i}$$

- n_i are kinematic numerators (momenta, polarisations)
- c_i are color factors (structure constants)
- D_i are Feynman propagators
- Sum over cubic/trivalent diagrams

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provided color-kinematics duality holds:

if $c_\alpha + c_\beta + c_\gamma = 0$

then $n_\alpha + n_\beta + n_\gamma = 0$

Formulations of the Double Copy

2. BCJ (Bern, Carrasco, Johansson) 2008-2010

- Much work on existence of color-dual numerators
- Evident loop-level generalisation, many explicit examples

Bautista, Bern, Boels, Boucher-Veronneau, Bourjaily, Carrasco, Casali, Chen, Davies, Dennen, Dixon, Douglas, Edison, Guevara, He, von Hippel, Huang, Isermann, Kalyanapuram, Kiermaier, Johansson, Kosower, Mangan, Mizera, Mogull, Monteiro, Nandan, Nohle, Ochirov, DOC, Patatoukos, Parra-Martinez, Pavao, Plefka, Plesser, Roiban, Schlotterer, Sminov, Teng, Tourkine, Travaglini, Vazquez-Holm, Zeng, Zhang,..

Formulations of the Double Copy

3. CHY (Cachazo, He, Yuan) 2013-2014

$$A_n^L = \int \mathrm{d}\mu_n(k,\sigma)\frac{I^L(k,\epsilon,\sigma)}{\sigma_{12}\sigma_{23}\cdots\sigma_{n1}} \qquad A_n^R = \int \mathrm{d}\mu_n(k,\sigma)\frac{I^R(k,\epsilon,\sigma)}{\sigma_{12}\sigma_{23}\cdots\sigma_{n1}}$$

Integrate over sphere

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Integrate over sphere

- $I^{L,R}$ function of momenta, polarisation objects
- Measure localises σ_i integral to solutions of *scattering equations*

$$\sum_{j\neq i} \frac{k_i \cdot k_j}{\sigma_{ij}} = 0 \text{ for all } i = 1, 2, \ldots, n$$

Fairlie, Roberts (1972)

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Clarified that double copy involves a web of theories…

Web of theories

Double copy touches many theories; examples: → Marco's talk

	YM	Max sYM	ChiPT	Biadjoint scalar
YM	Gravity +			
Max sYM	Half max sugra	Max sugra		
ChiPT	Born-Infeld	super DBI	sGalileon	
Biadjoint scalar	YM	Max sYM	ChiPT	Biadjoint scalar

Anastasiou, Borsten, Cachazo, Carrasco, Chen, Cheung, Chiodaroli, Diaz-Jaramillo, Du, Duff, Elvang, Engelbrecht, Fu, Günaydin, Hadjiantonis, He, Hohm, Hughes, Jin, Johansson, Jones, Mafra, Menezes, Nagy, Ochirov, Paranjape, Plefka, Roiban, Schlotterer, Shen…

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Connection to AdS/CFT?

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Computes (asymptotically Mink.) observables — where is geometry?

Color-Kinematics Duality

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BCJ formulation: numerators have algebraic structure

$$\mathcal{A}^L = \sum_i \frac{n_i^L c_i^L}{D_i} = \sum_i \frac{(n_i^L + \Delta_i) c_i^L}{D_i}$$

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Gauge invariance: algebra of c_i

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$$\mathcal{M} = \sum_i \frac{n_i^L n_i^R}{D_i} = \sum_i \frac{(n_i^L + \Delta_i) n_i^R}{D_i}$$

Gauge invariance ⇒ numerators have same algebraic properties

$$c_\alpha + c_\beta + c_\gamma = 0 \Rightarrow n_\alpha^R + n_\beta^R + n_\gamma^R = 0$$

Color-Kinematics Duality

Consequence: *Yang-Mills is simpler than seems*. Only $(n-3)!$ n-point ordered tree amplitudes

Bern, Carrasco, Johansson

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$$A_1 = \frac{n_s}{s} - \frac{n_t}{t} \qquad A_2 = \frac{n_t}{t} - \frac{n_u}{u}$$

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Simplification in YM because of numerator algebra

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Simplification in YM because of numerator algebra

Gravitational origin

Bern, Carrasco, Johansson

Color-Kinematics Duality

"Kinematic algebra": elusive algebra which computes numerators

Self-dual:

$$\mathcal{A}_3 = f^{abc}\frac{[12]^4}{[12][23][31]}\delta^4(p_1+p_2+p_3) \qquad \lambda_1 \propto \lambda_2 \propto \lambda_3$$

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Kinematic stucture constant

Momentum conservation is $\lambda_1(\tilde{\lambda}_1 + \tilde{\lambda}_2 + \tilde{\lambda}_3) = 0$

Monteiro, DOC

Color-Kinematics Duality

$$F_{p_1 p_2 p_3} = [12]\delta^4(p_1 + p_2 + p_3)$$

$$F_{p_1 p_2}{}^{k} F_{k p_3 p_4} = \int d^4k \, [12]\delta^4(p_1 + p_2 - k) \, [34]\delta^4(k + p_3 + p_4)$$

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Interesting that a diffeomorphism algebra leads to YM simplifications

Color-Kinematics Duality

Full kinematic algebra: still unclear. Directions:

- Full YM: hard, more complicated structures? NMHV works
- Scalar models (NLSM); color-kinematics of currents, field strengths
- Chern-Simons
- Heavy mass EFTs (HQET)

Ben Shahar, Bjerrum-Bohr, Borsten, Bonezzi, Brandhuber, Brown, Chen, Cheung, Cho, Damgaard, Diaz-Jaramillo, Feng, Fu, Garozzo, Gowdy, Helset, Johansson, Jurco, Kim, Krasnov, Lee, Lin, Macrelli, Mafra, Mangan, Mizera, Monteiro, Nagy, Parra-Martinez, Queimada, Remmen, Rodina, Saemann, Schlotterer, Shah, Shen, Shi, Skrzypek, Teng, Travaglini, Wang, Wen, DOC, …

Double Copy & Gravitational Waves

Double Copy & GWs

Double copy classically unexpected: explore classical implications

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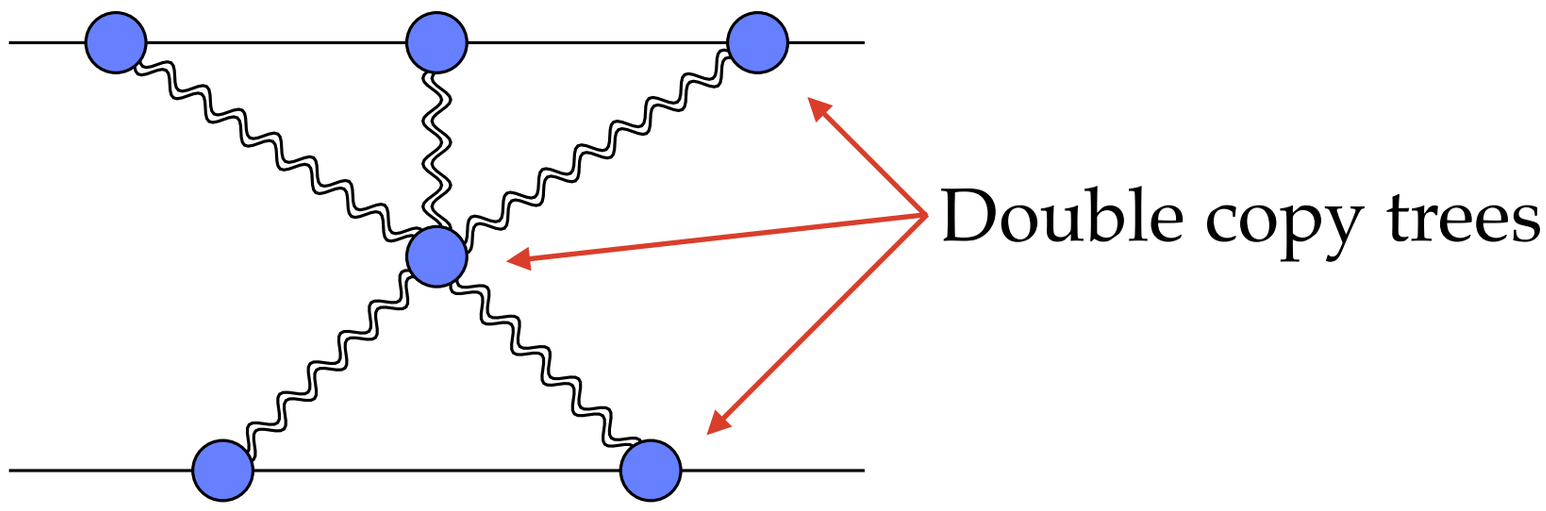
Double copy classically unexpected: explore classical implications

1. Interaction potential



Aoude, Bern, Bjerrum-Bohr, Chen, Cheung, Chung, Cristofoli, Damgaard, de la Cruz, Di Vecchia, Febres Cordero, Gatica, Haddad, Helset, Herrmann, Huang, Ita, Kim, Kosmopoulos, Lee, Luna, Neill, Parra-Martinez, Planté, Roiban, Rothstein, Ruf, Scheopner, Shen, Smirnov, Solon, Teng, Vanhove, Vines, Zeng

Double Copy & GWs

Double copy classically unexpected: explore classical implications

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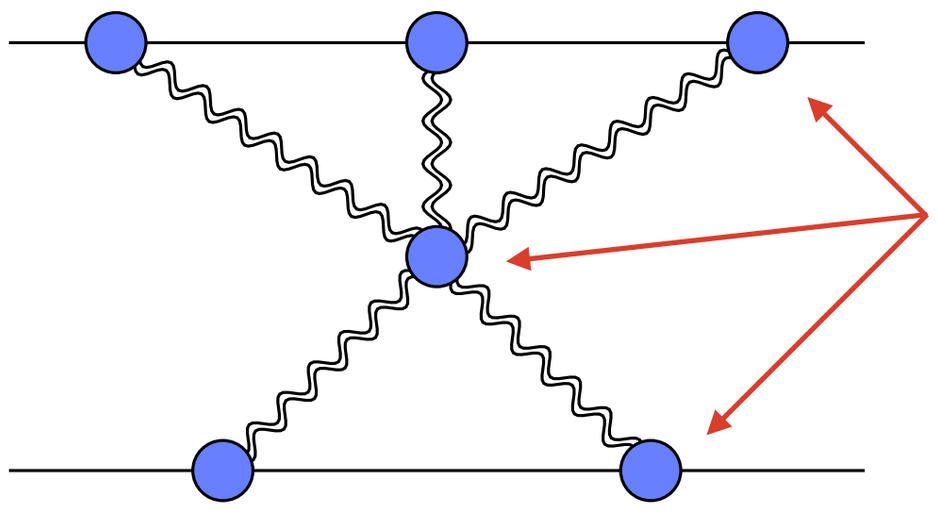
Spin!



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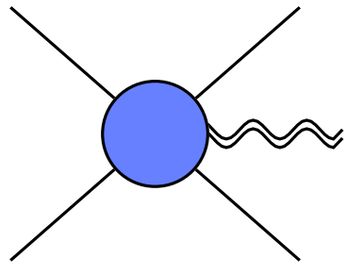
2. Observables eg waveform in scattering event

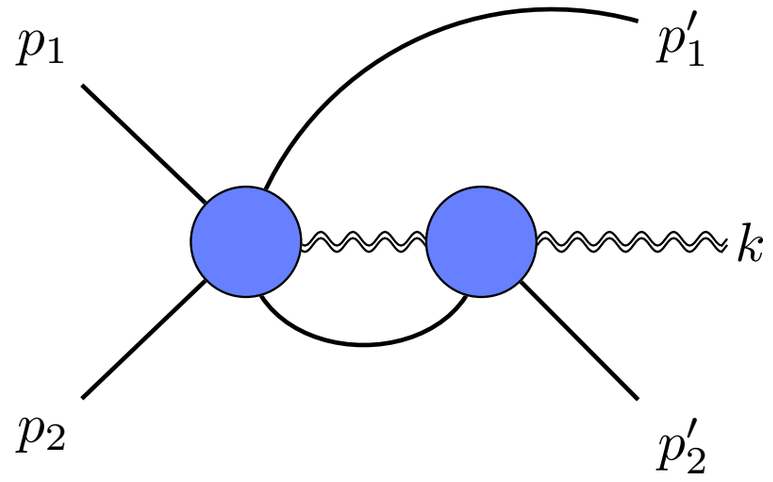
$$\text{waveform} = \frac{1}{\text{distance}} \int$$ 

Adamo, Aoude, Brandhuber, Brown, Caron-Huot, Chen, Cristofoli, De Angelis, Elkhidir, Giroux, Georgoudis, Gonzo, Gowdy, Haddad, Hannesdottir, Heissenberg, Helset, Herrmann, Ilderton, Klisch, Kosower, Maybee, Mizera, Novichkov, Parra-Martinez, Travaglini, Vazquez-Holm, Zeng, DOC

Double Copy & GWs

Some recent activity → Fei, Mathieu talks

1. NLO waveform



Subtleties with analytic continuation ($i\epsilon$) currently being sorted out

Bini, Brandhuber, Brown, Caron-Huot, Chen, Damour, De Angelis, Di Vecchia, Elkhidir, Geralico, Giroux, Gowdy, Hannesdottir, Heissenberg, Mizera, Russo, Travaglini, Vazquez-Holm, Veneziano, DOC

Double Copy & GWs

Some recent activity → Pavel's talk

2. LO waveform with spin

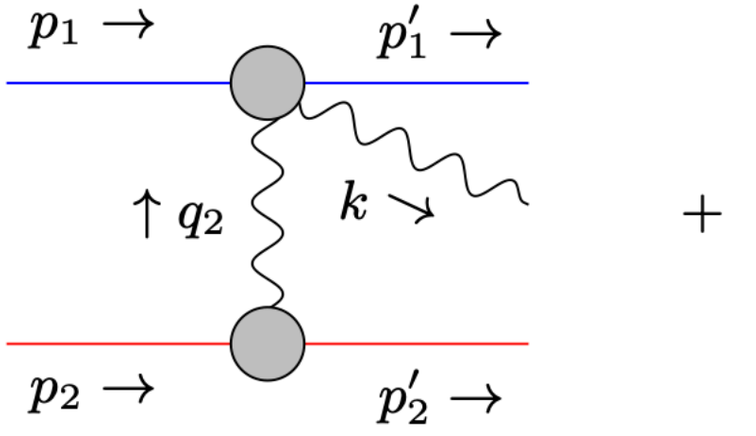


Figure from Aoude, Haddad, Heissenberg, Helset

Aoude, Brandhuber, Brown, Chen, De Angelis, Gowdy, Haddad, Heissenberg, Helset, Novichkov, Travaglini

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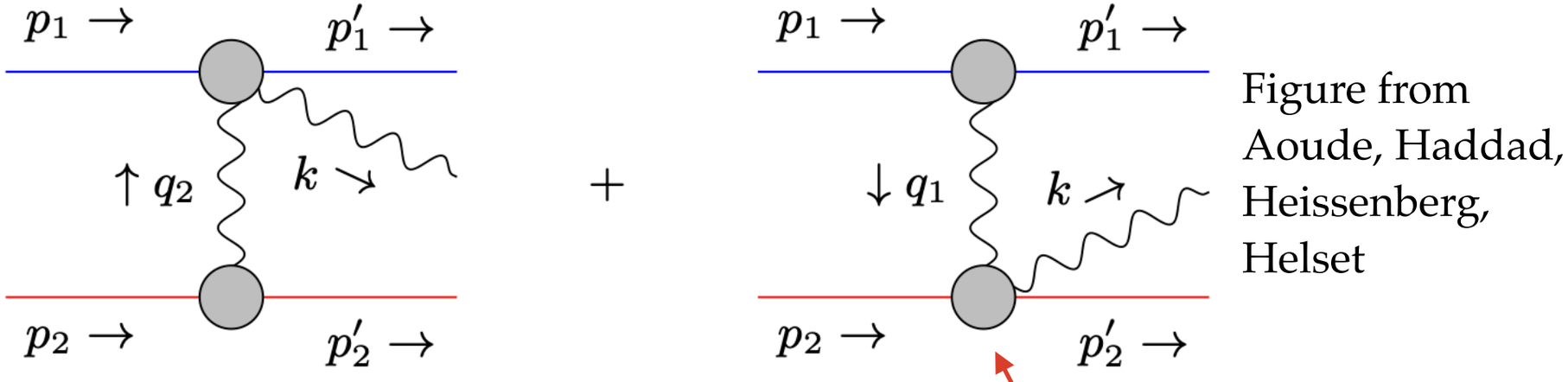

Figure from Aoude, Haddad, Heissenberg, Helset

Spinning Compton amplitude!

Aoude, Brandhuber, Brown, Chen, De Angelis, Gowdy, Haddad, Heissenberg, Helset, Novichkov, Travaglini

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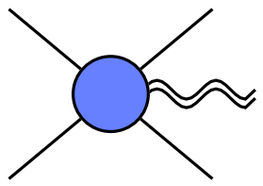
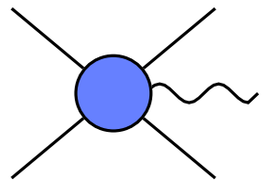
$$\text{curvature} = \frac{1}{\text{distance}} \int e^{iq\cdot x} \quad + \cdots$$



Classical Double Copy

$$\text{field strength} = \frac{1}{\text{distance}} \int e^{iq\cdot x} \quad + \cdots$$



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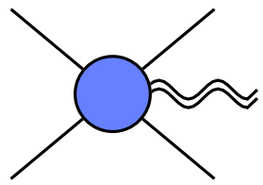

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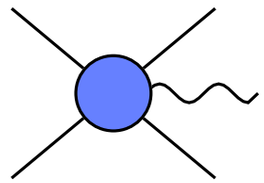

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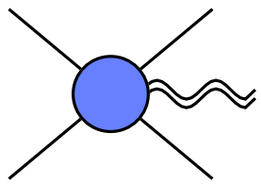
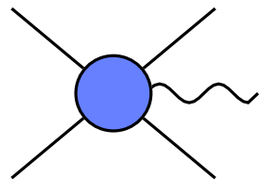


Classical Double Copy

$$\text{field strength} = \frac{1}{\text{distance}} \int e^{iq\cdot x} \quad + \cdots$$



Double copy

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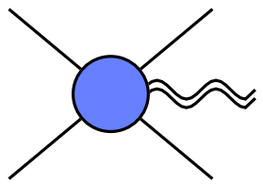


What about *static* field / curvature?

Classical Double Copy

Kerr-Schild double copy proposal

$$A_\mu = \phi k_\mu \quad \xleftrightarrow[\text{copy}]{\text{double}} \quad g_{\mu\nu} = \eta_{\mu\nu} + \phi k_\mu k_\nu$$

Simplest example: Coulomb & Schwarzschild

$$\phi \sim \frac{1}{r}, \quad k_\mu \, \mathrm{d}x^\mu = \mathrm{d}t - \mathrm{d}r$$

More gauge invariant formulation: Weyl double copy

Luna, Monteiro, Nicholson, White, DOC

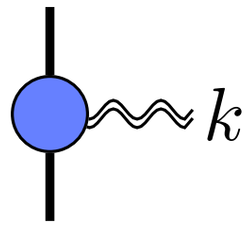
Classical Double Copy

Known relation to double copy of amplitudes

Classical Double Copy

Known relation to double copy of amplitudes

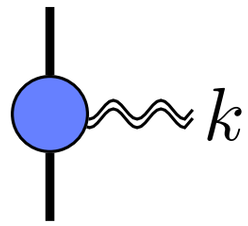
$$\int_{\text{on-shell}} \mathrm{d}k\; e^{-ik\cdot x}|k\rangle_\alpha|k\rangle_\beta|k\rangle_\gamma|k\rangle_\delta$$



Classical Double Copy

Known relation to double copy of amplitudes

$$\int_{\text{on-shell}} \mathrm{d}k\; e^{-ik\cdot x} |k\rangle_\alpha |k\rangle_\beta |k\rangle_\gamma |k\rangle_\delta$$

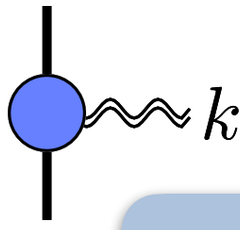


Analytic continuation
eg to metric (+,+,-,-)

Classical Double Copy

Known relation to double copy of amplitudes

$$\int_{\text{on-shell}} \mathrm{d}k\; e^{-ik\cdot x}|k\rangle_\alpha|k\rangle_\beta|k\rangle_\gamma|k\rangle_\delta$$



$$= \Psi^{\text{Schw.}}_{\alpha\beta\gamma\delta}(x)$$

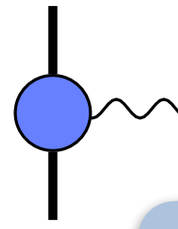
Analytic continuation
eg to metric (+,+,-,-)

Exact Schwarzschild
Weyl curvature spinor

Monteiro, Nagy, Sergola, Peinador Veiga, DOC

Classical Double Copy

Known relation to double copy of amplitudes

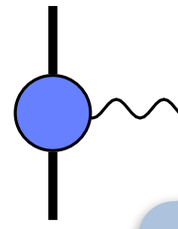
$$\int_{\text{on-shell}} \mathrm{d}k\; e^{-ik\cdot x}|k\rangle_\alpha|k\rangle_\beta$$  $$= \Phi_{\alpha\beta}(x)$$

Coulomb Maxwell spinor

Monteiro, Nagy, Sergola, Peinador Veiga, DOC

Classical Double Copy

Known relation to double copy of amplitudes

$$\int_{\text{on-shell}} \mathrm{d}k\; e^{-ik\cdot x}|k\rangle_\alpha|k\rangle_\beta$$  $$= \Phi_{\alpha\beta}(x)$$

Coulomb Maxwell spinor

Derive in same way as waveform: linearised

Unclear how to think of higher-order corrections in amplitudes terms

Monteiro, Nagy, Sergola, Peinador Veiga, DOC

Classical Double Copy

Area has been very active

- Twistor form; Cotton double copy 3d topologically massive theory
- Classical double copy & amplitudes on backgrounds
- Newman-Penrose map
- Locating horizons; isometries
- Deformed non-linear classical double copy in the EOB approach

Adamo, Armstrong-Williams, Bogna, Brown, Carillo González, Ceresole, Chacón, Chawla, Damour, Easson, Emond, Farnsworth, Goldberger, Gowdy, Graesser, Herczeg, Keeler, Lee, Luna, Manton, Mason, Momeni, Moynihan, Nagar, Nagy, Pezzelle, Plefka, Rettegno, Ridgway, Rumbutis, Sharma, Spence, Steinhoff, Wang, White, Wise, Wikeley, Wormsbecher,…

Final Remarks

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- More to learn about the double copy
  - Conservative or more radical?
- No doubt that double copy is a useful tool
  - Classical: GWs
  - Quantum: supergravity, pheno?
- Kinematic algebra has great potential but is still murky
- Progress needed on connection between classical double copy and amplitudes