Double-copy constructions for Yang-Mills-Einstein supergravities with non-compact gauge groups

QCD meets gravity, CERN, 11/12/23

Based on work with M. Günaydin, H. Johansson and R. Roiban

Plan of the talk

1. Introduction
2. $\mathcal{N} = 2$ YME theories with non-compact gauge groups in 5D
3. Double-copy construction – generalities
4. Spinor-helicity formalism in 5D
5. The dictionary
6. Explicit example: the complex magical supergravity
7. Conclusions
Amplitudes and YME theories

YME amplitudes have been a major focus of research for the past decade

[MC, Günaydin, Johansson and Roiban, 2014, 2017] [Cachazo, He and Yuan, 2014]
[Casali, Geyer, Mason, Monteiro and Roehrig, 2015] [Stieberger and Taylor, 2016]
[Nandan, Plefka, Schlotterer and Wen, 2016] [Fu, Du, Huang and Feng, 2017]
[Teng and Feng, 2017] [Nandan, Plefka, Travaglini, 2018] [Faller, Plefka, 2018]
[Feng, Li, Zhou 2019] [Du and Hou, 2019] [Feng, Li, Zhou, 2019] [Feng, Li, Huang, 2020]
[Cheung and Mangan, 2021] [Ma, Dong, Du, 2022] [Porkert and Schlotterer, 2022]
[Mazloumi and Stieberger, 2022]

They are an important node in the web of double-copy theories
[See Donal's talk]

Usually, there are implicit assumptions on the choice of theory and gauge group
Compact vs non-compact gauge groups

YM theories with non-compact gauge groups are affected by problems due to "wrong" signs in the quadratic Lagrangian (non-positive-definite metric)

In supergravity, there are examples in which non-compact gauge groups are possible

Many supergravities possess some non-compact global isometry group (e.g. $E_{7(7)}$ in 4D maximal sugra) ⇒ Non-compact gauge groups are not only admissible, but also natural

We focus on 5D, $\mathcal{N} = 2$ supergravities, which provide a more tractable arena [Günaydin, Sierra, Townsend, 1984]
\( \mathcal{N} = 2 \) YME theories with non-compact gauge groups (5D)

General 5D Maxwell-Einstein \( \mathcal{N} = 2 \) sugra (bosonic terms)

[Günaydin, Sierra, Townsend, 1984]

\[
e^{-1} \mathcal{L} = -\frac{R}{2} - \frac{1}{4} \hat{a}_{ij}(\varphi) F^i_{\mu\nu} F^j_{\mu\nu} - \frac{1}{2} g_{xy}(\varphi) \partial_\mu \varphi^x \partial^\mu \varphi^y + \frac{e^{-1}}{6\sqrt{6}} C_{ijk} \epsilon^{\mu\nu\rho\sigma\lambda} F^i_{\mu\nu} F^j_{\rho\sigma} A^K_{\lambda}
\]

- \( n_V \) vector multiplets \( (A^x_\mu, \lambda^x_i, \varphi^x) \) \( x, y = 1, \ldots, n_V \)
- graviton multiplet \( (h_{\mu\nu}, \psi_\mu, A^0_\mu) \) \( l, j = 0, \ldots, n_V \)

- All quantities in the Lagrangian expressed in terms of sym tensors \( C_{ijk} \)

- if the \( C_{ijk} \) tensor admits a group \( G_0 \) of isometries, we can promote a subgroup \( G \subset G_0 \) to a local symmetry by covariantizing derivatives and adding some extra interaction terms.

- \( G \) can be non-compact! The choice of base-point spontaneously breaks the non-compact gauge symmetry
\( \mathcal{N} = 2 \) YME theories with non-compact gauge groups (5D)

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- \( G \) can be non-compact! The choice of base-point spontaneously breaks the non-compact gauge symmetry
Caution: massive tensors may appear

In more complex examples, vectors transforming in non-adjoint representations need to be dualized to massive tensors

\[ \mathcal{H}^I_{\mu\nu} = (\mathcal{F}^s_{\mu\nu}, B^N_{\mu\nu}) \quad I = (s, N) \]

Transformation matrices for the symmetries to be gauged:

\[ (T_s)^I_J = \begin{pmatrix} f^t_{su} & 0 \\ 0 & (T_s)^N_P \end{pmatrix} \]

The bosonic part of the SUGRA Lagrangian is:

\[ e^{-1} \mathcal{L} = -\frac{R}{2} - \frac{1}{4} \hat{a}_{ij} \mathcal{H}^I_{\mu\nu} \mathcal{H}^J_{\mu\nu} - \frac{1}{2} g_{xy} \mathcal{D}_{\mu} \varphi^x \mathcal{D}^\mu \varphi^y + \frac{e^{-1}}{4g} \epsilon_{\mu\nu\rho\sigma\lambda} \Omega_{NP} B^N_{\mu\nu} \mathcal{D}_\rho B^P_{\sigma\lambda} + \frac{e^{-1}}{6\sqrt{6}} C_{stu} \epsilon^{\mu\nu\rho\sigma\lambda} \left\{ F^s_{\mu\nu} F^t_{\rho\sigma} A^u_\lambda + \frac{3}{2} g F^s_{\mu\nu} f^t_{t'u'} A^u_\rho A^t'_{\sigma} A^u'_{\lambda} \right\} + \mathcal{O}(g^2) \]

This is a specific feature of five dimensions!

[Günaydin and Zagermann, 1999]
Double-copy construction for homogeneous Maxwell-Einstein supergravities:

\[ \mathcal{N} = 2 \text{ homogeneous supergravity} = (\mathcal{N} = 2 \text{ SYM} + \text{hyper}_R) \otimes (\text{YM} + n_s \text{ scalars} + n_f \text{ massive fermions}_{\bar{R}}) \]

Important variant – if \( R \) is pseudo-real, we can take half-hyper.

We will take \( R \) to be a complex representation and consider a massive deformations of both gauge-theory factors

\[ \left( \text{Higgsed } \mathcal{N} = 2 \text{ SYM} + \text{hyper}_R \right) \otimes \left( \text{YM} + n_s \text{ scalars} + n_f \text{ massive fermions}_{\bar{R}} \right) \]

[MC, Günaydin, Johansson, Roiban, to appear]
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[MC, Günaydin, Johansson, Roiban, to appear]
Supersymmetric theory (GT1)

$\mathcal{N} = 2$ SYM theory with hypermultiplet in fundamental representation with mass $m$

Consider spontaneous symmetry breaking with VEV

$$\langle \phi^\hat{a} \rangle^{\hat{a}} = \begin{pmatrix} u_1 l_{N_1} & 0 \\ 0 & u_2 l_{N_2} \end{pmatrix}$$

$$U(N_1 + N_2) \rightarrow SU(N_1) \times SU(N_2) \times U(1)^2, \quad R \rightarrow (N_1, 1) \oplus (1, N_2)$$

Physical masses $m_1 = m + u_1$ and $m_2 = m + u_2$ obey the relation

$$m_1 - m_2 = m_W$$
Non-supersymmetric theory (GT2)

We consider a massive deformation of a YM-fermion theory in higher dimension:

\[\mathcal{L} = -\frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} (D_\mu \phi_{\hat{a}l})^2 - \frac{1}{2} m_{ij} \phi_{\hat{a}i} \phi_{\hat{a}j} - \frac{g^2}{4} f_{\hat{a}\hat{b}\hat{c}} f_{\hat{a}\hat{d}\hat{e}} \phi_{\hat{a}i} \phi_{\hat{b}j} \phi_{\hat{c}l} \phi_{\hat{d}m} \phi_{\hat{e}n} - \frac{g \lambda}{3!} f_{\hat{a}\hat{b}\hat{c}} F^{IJK} \phi_{\hat{a}i} \phi_{\hat{b}j} \phi_{\hat{c}k} + \frac{i}{2} \bar{\chi} \partial \chi - \frac{1}{2} \bar{\chi} M \chi + \frac{g}{2} \phi_{\hat{a}i} \bar{\chi}^a \Gamma^l t_{\hat{a}l} \chi \]  

(\hat{a}, \hat{b} adjoint ind)

Fermions are in rep $\bar{R}$. Imposing color/kinematics duality gives the constraints:

[MC, Günaydin, Johansson, Roiban, 2018]

- Two fermions—two scalars:

\[\{ \Gamma^i, \Gamma^j \} = -2 \delta^{ij} \]
\[\{ \{ \Gamma^i, M \}, \Gamma^j \} + i \lambda F^{IJK} \Gamma^K = 0 \]

- Four scalars:

\[F^{IJM} F^{KLM} + F^{KLM} F^{JLM} + F^{JKM} F^{ILM} = 0 \Rightarrow F \text{ are structure constants for the supergravity gauge symmetry} \]

Truncate the theory to have fields in the same representations as GT1

[MC, Günaydin, Roiban, 2013]
Non-supersymmetric theory (GT2)

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\[ \mathcal{L} = -\frac{1}{4} (F_{\mu\nu})^2 + \frac{1}{2} (D_{\mu} \hat{\phi}^{\hat{a} l})^2 - \frac{1}{2} m_{ij} \hat{\phi}^{\hat{a} l} \hat{\phi}^{\hat{a} j} - \frac{g^2}{4} f^{\hat{a} \hat{b} \hat{c}} f^{\hat{a} \hat{d} \hat{e}} \hat{\phi}^{\hat{a} l} \hat{\phi}^{\hat{b} j} \hat{\phi}^{\hat{c} l} \hat{\phi}^{\hat{d} j} \\
- \frac{g \lambda}{3!} f^{\hat{a} \hat{b} \hat{c}} F^{IJK} \hat{\phi}^{\hat{a} l} \hat{\phi}^{\hat{b} j} \hat{\phi}^{\hat{c} k} + \frac{i}{2} \bar{\chi} \slashed{D} \chi - \frac{1}{2} \bar{\chi} M \chi + \frac{g}{2} \phi^{\hat{a} l} \bar{\chi}^a \Gamma^I t^{\hat{a}} \chi \]  

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Fermions are in rep \( \bar{R} \). Imposing color/kinematics duality gives the constraints:

[MC, Günaydin, Johansson, Roiban, 2018]

- Two fermions—two scalars:

  \begin{align*}
  (1) \quad \{ \Gamma^I, \Gamma^J \} &= -2 \delta^{I J} \\
  (2) \quad [\{ \Gamma^I, M \}, \Gamma^J] + i \lambda F^{IJK} \Gamma^K &= 0
  \end{align*}

- Four scalars:

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Truncate the theory to have fields in the same representations as GT1

[MC, Günaydin, Roiban, 2013]
Massive spinors (solve Dirac equation with opposite sign for the mass):

\[ |p^a\rangle_A \quad a \text{ is a } SU(2)_L \text{ little group index} \]
\[ |p^{\dot{a}}\rangle_A \quad \dot{a} \text{ is a } SU(2)_R \text{ little group index} \]
\[ A, B \text{ are } USp(4) \text{ indices} \]

Massive momenta:

\[ \Gamma^B_A \cdot p = \frac{1}{2} |p^a\rangle_A \langle p^a|^B + \frac{1}{2} |p^{\dot{a}}\rangle_A \langle p^{\dot{a}}|^B \]

Massive vector/tensor polarizations:

\[ \varepsilon_{a\dot{a}}^{\mu} = \frac{[p^a|\Gamma^\mu|p^{\dot{a}}]}{\sqrt{2m}} \]
\[ \varepsilon_{ab}^{\mu\nu}(p) = \frac{\langle p^a|\Gamma^{\mu\nu}|p^b\rangle}{4\sqrt{2m}} \]
\[ \varepsilon_{\dot{a}\dot{b}}^{\mu\nu}(p) = \frac{[p^{\dot{a}}|\Gamma^{\mu\nu}|p^{\dot{b}}]}{4\sqrt{2m}} \]

Spinors:

\[ u(p) = |p^a\rangle \quad \bar{v}(p) = \langle p^a| \]

See also similar formalisms: [Dennen, Huang, Siegel, 2009] [Cheung and O'Connell, 2009] [Czech, Huang, Rozali 2012] [Boels and O'Connell, 2012]
The dictionary – fields

We focus on vector fields. Some vectors come from bosonic double copies:

\[ A^{ab} = \phi \otimes A^{ab} \]
\[ A^{ab} = A^{ab} \otimes \phi \]
\[ A^{ab} = \epsilon_{cd} A^{ac} \otimes A^{bd} \]

Fermionic double copies provide additional options:

1. \[ W^{\dot{a} \dot{a}} = |p^{\dot{a}}\rangle \otimes |p^{\dot{a}}\rangle \]
2. \[ W^{a \dot{a}} = |p^{\dot{a}}\rangle \otimes |p^{a}\rangle \]

spinors from the two sides have opposite sign for the mass

1. \[ B^{ab} = |p^{a}\rangle \otimes |p^{b}\rangle \]
2. \[ B^{\dot{a} \dot{b}} = |p^{\dot{b}}\rangle \otimes |p^{\dot{a}}\rangle \]

spinors from the two sides have same sign for the mass
Supergravity vectors with different double-copy origins couple differently!

\[
\mathcal{M}_3(1\overline{W}, 2W, 3A) = -iA_3(1\overline{W}, 2W, 3A)A_3(1\overline{\varphi}, 2\varphi, 3\phi)
\]

\[
= i \frac{\lambda}{4m^2} \left\{ \langle 12 \rangle [21](p_1 \cdot \varepsilon_3) + \frac{1}{2} \langle 1|p_2|1 \rangle \langle 2|\varepsilon_3|2 \rangle - (1 \leftrightarrow 2) \right\}
\]

\[
= \frac{\lambda}{2} \langle WWA \rangle_{|DW|^2} + \frac{\lambda}{2} \langle WWA \rangle_{\overline{W} \cdot F \cdot W}
\]

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\mathcal{M}_3(1\overline{W}, 2W, 3A) = -iA_3(1\overline{W}, 2W, 3\phi)A_3(1\varphi, 2\varphi, 3A)
\]

\[
= i \frac{1}{2m} \left\{ \langle 12 \rangle [21](p_1 \cdot \varepsilon_3) - (1 \leftrightarrow 2) \right\}
\]

\[
= m \langle WWA \rangle_{|DW|^2} - m \langle WWA \rangle_{\overline{W} \cdot F \cdot W}
\]

\[
\mathcal{M}_3(1\overline{W}, 2W, 3A) = -i \epsilon^{ab}_{\mu} \frac{\partial}{\partial z^{a}_3}A_3(1\overline{W}, 2W, 3A) \frac{\partial}{\partial z^{b}_3}A_3(1\overline{\varphi}, 2\varphi, 3A)
\]

\[
= -\langle WWA \rangle_{\overline{DW} \wedge A \wedge DW}
\]

Auxiliary variables to dress little group indices, e.g. \( \epsilon^{\mu}(z) = \epsilon^{\mu}_{ab}z^{a}z^{b} \)
Supergravity vectors with different double-copy origins couple differently!

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\[ = \frac{\lambda}{2} \langle \overline{W}WA \rangle_{DW}^2 + \frac{\lambda}{2} \langle \overline{W}WA \rangle_{\overline{W}F\overline{W}} \]

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\[ = m \langle \overline{W}WA \rangle_{DW}^2 - m \langle \overline{W}WA \rangle_{\overline{W}F\overline{W}} \]

\[ \mathcal{M}_3(1\overline{W}, 2W, 3A) = -i \frac{\epsilon_{\mu}}{4} \frac{\partial}{\partial z_3^a} A_3(1\overline{W}, 2W, 3A) \frac{\partial}{\partial z_3^b} A_3(1\varphi, 2\varphi, 3A) \]

\[ = -\langle \overline{W}WA \rangle_{\overline{DW}^\wedge A^\wedge DW} \]

Auxiliary variables to dress little group indices, e.g. \( \varepsilon^\mu (z) = \varepsilon^\mu_{ab} z^a z^b \)
Fermionic double-copies are particularly interesting:

\[ \mathcal{M}_3(1\overline{W}, 2W, 3A) = -i \frac{\epsilon^{ab}}{4} \frac{\partial}{\partial z^a_3} A_3(1\overline{\chi}, 2\chi, 3A) \frac{\partial}{\partial z^b_3} A_3(1\chi, 2\overline{\chi}, 3A) \]

\[ = \frac{i}{4\sqrt{2}m} \left( \langle 1|p_2|1 \rangle \langle 2|\varepsilon_3|2 \rangle - (1 \leftrightarrow 2) \right) \]

\[ = \sqrt{2}im\langle W\overline{WA}\rangle_{W\cdot F\cdot W} \]

\[ \mathcal{M}_3(1\overline{W}, 2W, 3A) = -iA(1\overline{\chi}, 2\chi, 3A)A(1\chi, 2\overline{\chi}, 3\phi) \]

\[ = \frac{i}{2} [12] \langle 1|\varepsilon_3|2 \rangle \]

\[ = m\langle W\overline{WA}\rangle|_{DW}|^2 + \frac{1}{4} \langle W\overline{WA}\rangle_{D\overline{W}\wedge A\wedge DW} \]
We will focus on a simple example

\[ \left( \mathcal{N} = 2 \text{ SYM} + \text{hyper}_R \right) \otimes \left( \text{YM} + 3 \text{ scalars} + 2 \text{ matter fermions}_R \right) \]

This is the complex magical Maxwell-Einstein supergravity. Target space in 5D

\[ \mathcal{M}_{5D} = \frac{\text{SL}(3, \mathbb{C})}{\text{SU}(3)} \]

Matter content: 8 vector multiplets and 9 total vectors (i=1,2,3):

\[
\begin{align*}
A^0 &= \frac{\epsilon^{ab}}{4} \frac{\partial}{\partial z^a} A \otimes \frac{\partial}{\partial z^b} A \\
A^1 &= \phi \otimes A \\
A^{i+1} &= A \otimes \phi^i \\
W_1 &= \bar{\chi} \otimes \chi_1 \\
\bar{W}_1 &= \chi \otimes \bar{\chi}_1 \\
W_2 &= \bar{\chi} \otimes \chi_2 \\
\bar{W}_2 &= \chi \otimes \bar{\chi}_2
\end{align*}
\]
Parameter counting

\[ \text{GT}_1 \text{ masses for hypermultiplets: } m_1, m_2 \]

mass for W bosons: \( m_W \)

\[ m_1 - m_2 = m_W \quad \text{(Higgs mechanism)} \]
Parameter counting

**GT1** masses for hypermultiplets: $m_1, m_2$
mass for $W$ bosons: $m_W$

\[
m_1 - m_2 = m_W \quad \text{(Higgs mechanism)}
\]

**GT2** masses for fermions: $\tilde{m}_1, \tilde{m}_2$
mass for scalars $\tilde{m}_\varphi$
trilinear couplings: $\lambda$

\[
\tilde{m}_1 + \tilde{m}_2 = \frac{\lambda}{2} \quad \text{(C/K duality constraint)}
\]
Parameter counting

**GT1**  masses for hypermultiplets: $m_1, m_2$
mass for W bosons: $m_W$

$$m_1 - m_2 = m_W \quad \text{(Higgs mechanism)}$$

**GT2**  masses for fermions: $\tilde{m}_1, \tilde{m}_2$
mass for scalars $\tilde{m}_\phi$
trilinear couplings: $\lambda$

$$\tilde{m}_1 + \tilde{m}_2 = \frac{\lambda}{2} \quad \text{(C/K duality constraint)}$$

- Supergravity fields correspond to double copies with conjugate gauge-theory representations
- Masses between the two gauge theories can be matched up to a sign
Explicit example: the complex magical supergravity

**Option one: only massive vectors**

<table>
<thead>
<tr>
<th>Rep.</th>
<th>GT 1</th>
<th>mass 1</th>
<th>GT 2</th>
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<tr>
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Fermionic masses are matched with opposite sign
Fun with Heisenberg algebras (take $u_1 = u_2$)

★ Inspection of three-point amplitudes reveals unbroken $U(2)$ gauge group with gluons $A^1, A^2, A^3, A^4$

★ $W$ and $\bar{W}$ transform as $SU(2)$ doublets and with opposite $U(1)$ charge

★ There is an extra nonzero amplitude

$$\mathcal{M}_3(1\bar{W}, 2W, 3A^0) = im \langle W\bar{W}A \rangle_{\bar{W}·F·W}$$

★ Reverse-engineering of structure constants reveals the appearance of the central charge in the Heisenberg group $\mathcal{H}_5$

This is a $U(2) \rtimes \mathcal{H}_5$ gauging!

[MC, Günaydin, Johansson, Roiban, to appear]
### Option two: only massive tensors

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<td>$-\tilde{m}_1$</td>
<td>$\mathcal{T}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
<tr>
<td>$(1, N_2)$</td>
<td>$\Phi_{\mathcal{N}=2}$</td>
<td>$m_2 = m + u_2$</td>
<td>$\chi_2$</td>
<td>$\tilde{m}_2 = m_2$</td>
<td>$\mathcal{T}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
<tr>
<td>$(1, \bar{N}_2)$</td>
<td>$\bar{\Phi}_{\mathcal{N}=2}$</td>
<td>$-m_2$</td>
<td>$\bar{\chi}_2$</td>
<td>$-\tilde{m}_2$</td>
<td>$\mathcal{T}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
</tbody>
</table>

Gauge group is $U(2) \rightarrow U(1)^2$, unbroken when $u_1 = u_2$. Massive tensors transform as doublets of $SU(2)$. This is a $U(2)$ gauging with tensors.
### Option three: two vectors and two tensors

<table>
<thead>
<tr>
<th>Rep.</th>
<th>GT 1</th>
<th>mass 1</th>
<th>GT 2</th>
<th>mass 2</th>
<th>SUGRA fields</th>
</tr>
</thead>
<tbody>
<tr>
<td>Adj.</td>
<td>$\mathcal{V}_{\mathcal{N}=2}$</td>
<td>0</td>
<td>$A_\mu \oplus \phi$</td>
<td>0</td>
<td>$\mathcal{H}<em>{\mathcal{N}=2} \oplus 2\mathcal{V}</em>{\mathcal{N}=2}$</td>
</tr>
<tr>
<td>$(N_1, \bar{N}_2)$</td>
<td>$\mathcal{V}_{\mathcal{N}=2}^{(m)}$</td>
<td>$m_W = u_1 - u_2$</td>
<td>$\varphi$</td>
<td>$\tilde{m}_\varphi = -m_W$</td>
<td>$\mathcal{V}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
<tr>
<td>$(\bar{N}_1, N_2)$</td>
<td>$\mathcal{V}_{\mathcal{N}=2}^{(m)}$</td>
<td>$-m_W$</td>
<td>$\bar{\varphi}$</td>
<td>$-\tilde{m}_\varphi$</td>
<td>$\mathcal{V}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
<tr>
<td>$(N_1, 1)$</td>
<td>$\Phi_{\mathcal{N}=2}$</td>
<td>$m_1 = m + u_1$</td>
<td>$\chi_1$</td>
<td>$\tilde{m}_1 = -m_1$</td>
<td>$\mathcal{V}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
<tr>
<td>$(\bar{N}_1, 1)$</td>
<td>$\bar{\Phi}_{\mathcal{N}=2}$</td>
<td>$-m_1$</td>
<td>$\bar{\chi}_1$</td>
<td>$-\tilde{m}_1$</td>
<td>$\mathcal{V}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
<tr>
<td>$(1, N_2)$</td>
<td>$\Phi_{\mathcal{N}=2}$</td>
<td>$m_2 = m + u_2$</td>
<td>$\chi_2$</td>
<td>$\tilde{m}_2 = m_2$</td>
<td>$\mathcal{T}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
<tr>
<td>$(1, \bar{N}_2)$</td>
<td>$\bar{\Phi}_{\mathcal{N}=2}$</td>
<td>$-m_2$</td>
<td>$\bar{\chi}_2$</td>
<td>$-\tilde{m}_2$</td>
<td>$\mathcal{T}_{\mathcal{N}=2}^{(m)}$</td>
</tr>
</tbody>
</table>

$$\frac{\lambda}{2} = \tilde{m}_1 + \tilde{m}_2 = u_1 - u_2$$

**Double-copy construction for** $U(1,1) \rtimes \mathcal{H}_3$ **gauging with tensors**

**Exotic matter couplings:** one tensor can turn into a vector (!)

[Bergshoeff, Cucu, de Wit, Gheerardyn, Vandoren, Van Proeyen, 2004]

$$\mathcal{M}_3(\bar{1}B, 2W, 3A) = \langle 12 \rangle \langle 1 | \varepsilon_3 | 2 \rangle$$
Conclusions

★ We have presented several examples of YME theories in 5D. Non-compact gauge groups and massive tensors can be obtained from the double copy. Signs matter!

★ These constructions rely on a simple massive deformation of the gauge theories involved – can this be studied with different approaches?

★ The appearance of the Heisenberg algebra and of the tensor-vector-vector couplings is a surprise. Known in SUGRA community, but regarded as exotic

★ On the other side, realizing more "conventional" gaugings such as $SU(3)$ or $SU(2, 1)$ for the complex magical theory as double copies remains an open problem

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