Double-copy constructions for Yang-Mills-Einstein supergravities with non-compact gauge groups

UPPSALA
UNIVERSITET

Marco Chiodaroli

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Based on work with M. Günaydin, H. Johansson and R. Roiban
(arXiv:2202.08257, arXiv:2312.xxxx)

Plan of the talk

Amplitudes and YME theories

YME amplitudes have been a major focus of research for the past decade

[MC, Günaydin, Johansson and Roiban, 2014, 2017] [Cachazo, He and Yuan, 2014] [Casali, Geyer, Mason, Monteiro and Roehrig, 2015] [Stieberger and Taylor, 2016] [Nandan, Plefka, Schlotterer and Wen, 2016] [Fu, Du, Huang and Feng, 2017] [Teng and Feng, 2017] [Nandan, Plefka, Travaglini, 2018] [Faller, Plefka, 2018] [Feng, Li, Zhou 2019] [Du and Hou, 2019] [Feng, Li, Zhou, 2019] [Feng, Li, Huang, 2020] [Cheung and Mangan, 2021] [Ma, Dong, Du, 2022] [Porkert and Schlotterer, 2022] [Mazloumi and Stieberger, 2022]

They are an important node in the web of double-copy theories
[See Donal's talk]

Usually, there are implicit assumptions on the choice of theory and gauge group

Compact vs non-compact gauge groups

YM theories with non-compact gauge groups are affected by problems due to "wrong" signs in the quadratic Lagrangian (non-positive-definite metric)

In supergravity, there are examples in which non-compact gauge groups are possible

Many supergravities possess some non-compact global isometry group (e.g. $E_{7(7)}$ in 4D maximal sugra) $\Rightarrow$ Non-compact gauge groups are not only admissible, but also natural

We focus on 5D, $\mathcal{N} = 2$ supergravities, which provide a more tractable arena [Günaydin, Sierra, Townsend, 1984]

$\mathcal{N} = 2$ YME theories with non-compact gauge groups ($5D$)

General $5D$ Maxwell-Einstein $\mathcal{N} = 2$ sugra (bosonic terms)
[Günaydin, Sierra, Townsend, 1984]

$$e^{-1}\mathcal{L} = -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}(\varphi)F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{2}g_{xy}(\varphi)\partial_\mu\varphi^x\partial^\mu\varphi^y + \frac{e^{-1}}{6\sqrt{6}}C_{IJK}\epsilon^{\mu\nu\rho\sigma\lambda}F^I_{\mu\nu}F^J_{\rho\sigma}A^K_\lambda$$

- n_V vector multiplets $\left(A^x_\mu, \lambda^x_i, \varphi^x\right)$ $\quad x, y = 1, \ldots, n_V$
  graviton multiplet $\left(h_{\mu\nu}, \psi_\mu, A^0_\mu\right)$ $\quad I, J = 0, \ldots, n_V$
- All quantities in the Lagrangian expressed in terms of sym tensors C_{IJK}
- if the C_{IJK} tensor admits a group G_0 of isometries, we can promote a subgroup $G \subset G_0$ to a local symmetry by covariantizing derivatives and adding some extra interaction terms.
- G can be non-compact! The choice of base-point spontaneously breaks the non-compact gauge symmetry

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Caution: massive tensors may appear

In more complex examples, vectors transforming in non-adjoint representations need to be dualized to massive tensors

$$\mathcal{H}^I_{\mu\nu} = (\mathcal{F}^s_{\mu\nu}, B^N_{\mu\nu}) \qquad I = (s, N)$$

Transformation matrices for the symmetries to be gauged:

$$(T_s)^I{}_J = \begin{pmatrix} f^t_{su} & 0 \\ 0 & (T_s)^N_P \end{pmatrix}$$

The bosonic part of the SUGRA Lagrangian is:

$$\begin{aligned} e^{-1}\mathcal{L} \;=\; & -\frac{R}{2} - \frac{1}{4}\mathring{a}_{IJ}\mathcal{H}^I_{\mu\nu}\mathcal{H}^{J\mu\nu} - \frac{1}{2}g_{xy}\mathcal{D}_\mu\varphi^x\mathcal{D}^\mu\varphi^y + + \frac{e^{-1}}{4g}\epsilon^{\mu\nu\rho\sigma\lambda}\,\Omega_{NP}B^N_{\mu\nu}\mathcal{D}_\rho B^P_{\sigma\lambda} \\ & + \frac{e^{-1}}{6\sqrt{6}}C_{stu}\epsilon^{\mu\nu\rho\sigma\lambda}\left\{F^s_{\mu\nu}F^t_{\rho\sigma}A^u_\lambda + \frac{3}{2}gF^s_{\mu\nu}f^t{}_{t'u'}A^u_\rho A^{t'}_\sigma A^{u'}_\lambda\right\} + \mathcal{O}(g^2) \end{aligned}$$

This is a specific feature of five dimensions!

[Günaydin and Zagermann, 1999]

Double-copy construction – generalities

Double-copy construction for homogeneous Maxwell-Einstein supergravities:
[MC, Günaydin, Johansson, Roiban, 2015]

$$\left(\begin{array}{c}\mathcal{N}=2\text{ homogeneous}\\ \text{supergravity}\end{array}\right) \quad = \quad \left(\begin{array}{c}\mathcal{N}=2\,\text{SYM}\\ +\,\text{hyper}_R\end{array}\right)\otimes\left(\begin{array}{c}\text{YM}+n_s\text{ scalars}\\ +n_f\text{ fermions}_{\bar{R}}\end{array}\right)$$

Important variant – if R is pseudo-real, we can take half-hyper.

We will take R to be a complex representation and consider a massive deformations of both gauge-theory factors

$$\left(\text{Higgsed}\,\mathcal{N}=2\,\text{SYM}+\text{hyper}_R\right)\otimes\left(\text{YM}+n_s\text{ scalars}+n_f\text{massive fermions}_{\bar{R}}\right)$$

[MC, Günaydin, Johansson, Roiban, to appear]

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$$\begin{pmatrix} \mathcal{N} = 2 \text{ homogeneous} \\ \text{supergravity} \end{pmatrix} = \begin{pmatrix} \mathcal{N} = 2 \text{ SYM} \\ + \text{hyper}_R \end{pmatrix} \otimes \begin{pmatrix} \text{YM} + n_s \text{ scalars} \\ + n_f \text{ fermions}_{\bar{R}} \end{pmatrix}$$

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[MC, Günaydin, Johansson, Roiban, to appear]

Supersymmetric theory (GT1)

$\mathcal{N} = 2$ SYM theory with hypermultiplet in fundamental representation with mass m

Consider spontaneous symmetry breaking with VEV

$$\langle\phi^{\hat{a}}\rangle t^{\hat{a}} = \begin{pmatrix} u_1 I_{N_1} & 0 \\ 0 & u_2 I_{N_2} \end{pmatrix}$$

$$U(N_1 + N_2) \to SU(N_1) \times SU(N_2) \times U(1)^2 \,, \qquad R \to (N_1, 1) \oplus (1, N_2)$$

Physical masses $m_1 = m + u_1$ and $m_2 = m + u_2$ obey the relation

$$m_1 - m_2 = m_W$$

Non-supersymmetric theory (GT2)

We consider a massive deformation of a *YM*-fermion theory in higher dimension:

$$\mathcal{L} = -\frac{1}{4}(F^{\hat{a}}_{\mu\nu})^2 + \frac{1}{2}(D_\mu \phi^{\hat{a}I})^2 - \frac{1}{2} m^2_{IJ}\phi^{\hat{a}I}\phi^{\hat{a}J} - \frac{g^2}{4} f^{\hat{a}\hat{b}\hat{e}} f^{\hat{c}\hat{d}\hat{e}} \phi^{\hat{a}I}\phi^{\hat{b}J}\phi^{\hat{c}I}\phi^{\hat{d}J}$$
$$-\frac{g\lambda}{3!} f^{\hat{a}\hat{b}\hat{c}} F^{IJK}\phi^{\hat{a}I}\phi^{\hat{b}J}\phi^{\hat{c}K} + \frac{i}{2}\bar{\chi}\not{D}\chi - \frac{1}{2}\bar{\chi}M\chi + \frac{g}{2}\phi^{\hat{a}I}\bar{\chi}^{a}\Gamma^{I} t^{\hat{a}}_{\bar{R}}\chi \qquad (\hat{a}, \hat{b} \text{ adjoint ind})$$

Fermions are in rep $\bar{R}$. Imposing color/kinematics duality gives the constraints:
[MC, Günaydin, Johansson, Roiban, 2018]

- Two fermions—two scalars:

$$(1) \qquad \{\Gamma^I, \Gamma^J\} = -2\delta^{IJ}$$
$$(2) \qquad \left[\{\Gamma^I, M\}, \Gamma^J\right] + i\lambda F^{IJK}\Gamma^K = 0$$

- Four scalars:

$$F^{IJM}F^{KLM} + F^{KIM}F^{JLM} + F^{JKM}F^{ILM} = 0 \qquad \Rightarrow$$

F are structure constants for the supergravity gauge symmetry

Truncate the theory to have fields in the same representations as GT1
[MC, Günaydin, Roiban, 2013]

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$$-\frac{g\lambda}{3!}f^{\hat{a}\hat{b}\hat{c}}F^{IJK}\phi^{\hat{a}I}\phi^{\hat{b}J}\phi^{\hat{c}K} + \frac{i}{2}\bar{\chi}\not{D}\chi - \frac{1}{2}\bar{\chi}M\chi + \frac{g}{2}\phi^{\hat{a}I}\bar{\chi}^a\Gamma^I t^{\hat{a}}_{\bar{R}}\chi \qquad (\hat{a}, \hat{b} \text{ adjoint ind})$$

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Spinor-Helicity formalism in 5D [MC, Günaydin, Johansson, Roiban, 2022]

Massive spinors (solve Dirac equation with opposite sign for the mass):

$|\boldsymbol{p}^a\rangle_A$ — a is a $SU(2)_L$ little group index

$|\boldsymbol{p}^{\dot{a}}]_A$ — $\dot{a}$ is a $SU(2)_R$ little group index — A, B are $USp(4)$ indices

Massive momenta: $$\Gamma_A^{\ B} \cdot p = \frac{1}{2}|\boldsymbol{p}_a\rangle_A \langle \boldsymbol{p}^a|^B + \frac{1}{2}|\boldsymbol{p}_{\dot{a}}]_A [\boldsymbol{p}^{\dot{a}}|^B$$

Massive vector/tensor polarizations:

$$\varepsilon^{\mu}_{a\dot{a}} = \frac{[\boldsymbol{p}_{\dot{a}}|\Gamma^{\mu}|\boldsymbol{p}_{a}\rangle}{\sqrt{2}m} \qquad \varepsilon^{\mu\nu}_{ab}(p) = \frac{\langle \boldsymbol{p}_a|\Gamma^{\mu\nu}|\boldsymbol{p}_b\rangle}{4\sqrt{2}m} \qquad \varepsilon^{\mu\nu}_{\dot{a}\dot{b}}(p) = \frac{[\boldsymbol{p}_{\dot{a}}|\Gamma^{\mu\nu}|\boldsymbol{p}_{\dot{b}}]}{4\sqrt{2}m}$$

Spinors: $u(p) = |\boldsymbol{p}_{\dot{b}}]$ $\qquad \bar{v}(p) = \langle \boldsymbol{p}_a|$

See also similar formalisms: [Dennen, Huang, Siegel, 2009] [Cheung and O'Connell, 2009] [Czech, Huang, Rozali 2012] [Boels and O'Connell, 2012]

The dictionary – fields

We focus on vector fields. Some vectors come from bosonic double copies:

$$\begin{aligned} A^{ab} &= \phi \otimes A^{ab} \\ A^{ab} &= A^{ab} \otimes \phi \\ A^{ab} &= \epsilon_{cd} A^{ac} \otimes A^{bd} \end{aligned} \qquad W^{a\dot{a}} = W^{a\dot{a}} \otimes \varphi$$

Fermionic double copies provide additional options:

1. $W^{a\dot{a}} = |\boldsymbol{p}^a\rangle \otimes |\boldsymbol{p}^{\dot{a}}] \qquad W^{a\dot{a}} = |\boldsymbol{p}^{\dot{a}}] \otimes |\boldsymbol{p}^a\rangle$

   spinors from the two sides have opposite sign for the mass

2. $B^{ab} = |\boldsymbol{p}^a\rangle \otimes |\boldsymbol{p}^b\rangle \qquad B^{\dot{a}\dot{b}} = |\boldsymbol{p}^{\dot{a}}] \otimes |\boldsymbol{p}^{\dot{a}}]$

   spinors from the two sides have same sign for the mass

The dictionary – interactions

Supergravity vectors with different double-copy origins couple differently!

$$\begin{aligned}\mathcal{M}_3(1\overline{W},2W,3A) &= -iA_3(1\overline{W},2W,3A)A_3(1\overline{\varphi},2\varphi,3\phi)\\ &= i\frac{\lambda}{4m^2}\left\{\langle\mathbf{12}\rangle\,[\mathbf{21}](p_1\cdot\varepsilon_3)+\frac{1}{2}\langle\mathbf{1}|p_2|\mathbf{1}]\,\langle\mathbf{2}|\varepsilon_3|\mathbf{2}]-(1\leftrightarrow 2)\right\}\\ &= \frac{\lambda}{2}\langle\overline{W}WA\rangle_{|DW|^2}+\frac{\lambda}{2}\langle\overline{W}WA\rangle_{\overline{W}\cdot F\cdot W}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_3(1\overline{W},2W,3A) &= -iA_3(1\overline{W},2W,3\phi)A_3(1\varphi,2\varphi,3A)\\ &= \frac{i}{2m}\left\{\langle\mathbf{12}\rangle\,[\mathbf{21}](p_1\cdot\varepsilon_3)-(1\leftrightarrow 2)\right\}\\ &= m\langle\overline{W}WA\rangle_{|DW|^2}-m\langle\overline{W}WA\rangle_{\overline{W}\cdot F\cdot W}\end{aligned}$$

$$\begin{aligned}\mathcal{M}_3(1\overline{W},2W,3A) &= -i\frac{\epsilon^{ab}}{4}\frac{\partial}{\partial z_3^a}A_3(1\overline{W},2W,3A)\frac{\partial}{\partial z_3^b}A_3(1\overline{\varphi},2\varphi,3A)\\ &= -\langle\overline{W}WA\rangle_{D\overline{W}\wedge A\wedge DW}\end{aligned}$$

Auxiliary variables to dress little group indices, e.g. $\varepsilon^\mu(z)=\varepsilon^\mu_{ab}z^a z^b$

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$$\mathcal{M}_3(1\overline{W},2W,3A) = -i\frac{\epsilon^{ab}}{4}\frac{\partial}{\partial z_3^a}A_3(1\overline{W},2W,3A)\frac{\partial}{\partial z_3^b}A_3(1\overline{\varphi},2\varphi,3A)$$
$$= -\langle\overline{W}WA\rangle_{D\overline{W}\wedge A\wedge DW}$$

Auxiliary variables to dress little group indices, e.g. $\varepsilon^\mu(z)=\varepsilon^\mu_{ab}z^az^b$

Fermionic double-copies are particularly interesting:

$$
\begin{aligned}
\mathcal{M}_3(1\overline{W}, 2W, 3A) &= -i\frac{\epsilon^{ab}}{4}\frac{\partial}{\partial z_3^a}A_3(1\bar{\chi}, 2\chi, 3A)\frac{\partial}{\partial z_3^b}A_3(1\chi, 2\bar{\chi}, 3A) \\
&= \frac{i}{4\sqrt{2}m}\Big(\langle \underline{\mathbf{1}}|p_2|\underline{\mathbf{1}}]\,\langle \underline{\mathbf{2}}|\varepsilon_3|\underline{\mathbf{2}}] - (1 \leftrightarrow 2)\Big) \\
&= \sqrt{2}im\langle W\overline{W}A\rangle_{\overline{W}\cdot F\cdot W}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{M}_3(1\overline{W}, 2W, 3A) &= -iA(1\bar{\chi}, 2\chi, 3A)A(1\chi, 2\bar{\chi}, 3\phi) \\
&= \frac{i}{2}[\underline{\mathbf{12}}]\,\langle \underline{\mathbf{1}}|\varepsilon_3|\underline{\mathbf{2}}\rangle \\
&= m\langle \overline{W}WA\rangle_{|DW|^2} + \frac{1}{4}\langle \overline{W}WA\rangle_{D\overline{W}\wedge A\wedge DW}
\end{aligned}
$$

Example: gauging the complex magical supergravity

[MC, Günaydin, Johansson, Roiban, to appear]

We will focus on a simple example

$$\left(\mathcal{N} = 2\,\text{SYM} + \text{hyper}_R\right) \otimes \left(\text{YM} + 3\,\text{scalars} + 2\,\text{matter fermions}_{\bar{R}}\right)$$

This is the complex magical Maxwell-Einstein supergravity. Target space in 5D

$$\mathcal{M}_{5D} = \frac{SL(3,\mathbb{C})}{SU(3)}$$

Matter content: 8 vector multiplets and 9 total vectors (i=1,2,3):

$$\begin{aligned} A^0 &= \frac{\epsilon^{ab}}{4}\frac{\partial}{\partial z^a}A \otimes \frac{\partial}{\partial z^b}A \\ A^1 &= \phi \otimes A \qquad\qquad W_1 = \bar{\chi} \otimes \chi_1 \qquad\qquad \bar{W}_1 = \chi \otimes \bar{\chi}_1 \\ A^{i+1} &= A \otimes \phi^i \qquad\qquad W_2 = \bar{\chi} \otimes \chi_2 \qquad\qquad \bar{W}_2 = \chi \otimes \bar{\chi}_2 \end{aligned}$$

Parameter counting

GT1 masses for hypermultiplets: m_1, m_2
mass for W bosons: m_W

$$\boxed{m_1 - m_2 = m_W \qquad \text{(Higgs mechanism)}}$$

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GT2 masses for fermions: $\tilde{m}_1, \tilde{m}_2$
mass for scalars $\tilde{m}_\varphi$
trilinear couplings: λ

$$\tilde{m}_1 + \tilde{m}_2 = \frac{\lambda}{2} \qquad \text{(C/K duality constraint)}$$

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mass for scalars $\tilde{m}_\varphi$
trilinear couplings: λ

$$\tilde{m}_1 + \tilde{m}_2 = \frac{\lambda}{2} \qquad \text{(C/K duality constraint)}$$

- Supergravity fields correspond to double copies with conjugate gauge-theory representations
- Masses between the two gauge theories can be matched up to a sign

Option one: only massive vectors

Rep.	GT 1	mass 1	GT 2	mass 2	SUGRA fields
Adj.	$\mathcal{V}_{\mathcal{N}=2}$	0	$A_\mu \oplus \phi$	0	$\mathcal{H}_{\mathcal{N}=2} \oplus 2\mathcal{V}_{\mathcal{N}=2}$
$(N_1, \bar{N}_2)$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$	$m_W = u_1 - u_2$	φ	$\tilde{m}_\varphi = -m_W$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(\bar{N}_1, N_2)$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$	$-m_W$	$\bar{\varphi}$	$-\tilde{m}_\varphi$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(N_1, 1)$	$\Phi_{\mathcal{N}=2}$	$m_1 = m + u_1$	χ_1	$\tilde{m}_1 = -m_1$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(\bar{N}_1, 1)$	$\bar{\Phi}_{\mathcal{N}=2}$	$-m_1$	$\bar{\chi}_1$	$-\tilde{m}_1$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(1, N_2)$	$\Phi_{\mathcal{N}=2}$	$m_2 = m + u_2$	χ_2	$\tilde{m}_2 = -m_2$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(1, \bar{N}_2)$	$\bar{\Phi}_{\mathcal{N}=2}$	$-m_2$	$\bar{\chi}_2$	$-\tilde{m}_2$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$

Fermionic masses are matched with opposite sign

Fun with Heisenberg algebras (take $u_1 = u_2$)

⋆ Inspection of three-point amplitudes reveals unbroken $U(2)$ gauge group with gluons A^1, A^2, A^3, A^4

⋆ W and $\bar{W}$ transform as $SU(2)$ doublets and with opposite $U(1)$ charge

⋆ There is an extra nonzero amplitude

$$\mathcal{M}_3\left(1\bar{W}, 2W, 3A^0\right) = im\langle W\overline{W}A\rangle_{\overline{W}\cdot F\cdot W}$$

⋆ Reverse-engineering of structure constants reveals the appearance of the central charge in the Heisenberg group $\mathcal{H}_5$

This is a $U(2) \ltimes \mathcal{H}_5$ gauging!

[MC, Günaydin, Johansson, Roiban, to appear]

Option two: only massive tensors

Rep.	GT 1	mass 1	GT 2	mass 2	SUGRA fields
Adj.	$\mathcal{V}_{\mathcal{N}=2}$	0	$A_\mu \oplus \phi$	0	$\mathcal{H}_{\mathcal{N}=2} \oplus 2\mathcal{V}_{\mathcal{N}=2}$
$(N_1, \bar{N}_2)$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$	$m_W = u_1 - u_2$	φ	$\tilde{m}_\varphi = -m_W$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(\bar{N}_1, N_2)$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$	$-m_W$	$\bar{\varphi}$	$-\tilde{m}_\varphi$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(N_1, 1)$	$\Phi_{\mathcal{N}=2}$	$m_1 = m + u_1$	χ_1	$\tilde{m}_1 = m_1$	$\mathcal{T}^{(m)}_{\mathcal{N}=2}$
$(\bar{N}_1, 1)$	$\bar{\Phi}_{\mathcal{N}=2}$	$-m_1$	$\bar{\chi}_1$	$-\tilde{m}_1$	$\mathcal{T}^{(m)}_{\mathcal{N}=2}$
$(1, N_2)$	$\Phi_{\mathcal{N}=2}$	$m_2 = m + u_2$	χ_2	$\tilde{m}_2 = m_2$	$\mathcal{T}^{(m)}_{\mathcal{N}=2}$
$(1, \bar{N}_2)$	$\bar{\Phi}_{\mathcal{N}=2}$	$-m_2$	$\bar{\chi}_2$	$-\tilde{m}_2$	$\mathcal{T}^{(m)}_{\mathcal{N}=2}$

Gauge group is $U(2) \to U(1)^2$, unbroken when $u_1 = u_2$. Massive tensors transform as doublets of $SU(2)$. This is a $U(2)$ gauging with tensors

Option three: two vectors and two tensors

Rep.	GT 1	mass 1	GT 2	mass 2	SUGRA fields
Adj.	$\mathcal{V}_{\mathcal{N}=2}$	0	$A_\mu \oplus \phi$	0	$\mathcal{H}_{\mathcal{N}=2} \oplus 2\mathcal{V}_{\mathcal{N}=2}$
$(N_1, \bar{N}_2)$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$	$m_W = u_1 - u_2$	φ	$\tilde{m}_\varphi = -m_W$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(\bar{N}_1, N_2)$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$	$-m_W$	$\bar{\varphi}$	$-\tilde{m}_\varphi$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(N_1, 1)$	$\Phi_{\mathcal{N}=2}$	$m_1 = m + u_1$	χ_1	$\tilde{m}_1 = -m_1$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(\bar{N}_1, 1)$	$\bar{\Phi}_{\mathcal{N}=2}$	$-m_1$	$\bar{\chi}_1$	$-\tilde{m}_1$	$\mathcal{V}^{(m)}_{\mathcal{N}=2}$
$(1, N_2)$	$\Phi_{\mathcal{N}=2}$	$m_2 = m + u_2$	χ_2	$\tilde{m}_2 = m_2$	$\mathcal{T}^{(m)}_{\mathcal{N}=2}$
$(1, \bar{N}_2)$	$\bar{\Phi}_{\mathcal{N}=2}$	$-m_2$	$\bar{\chi}_2$	$-\tilde{m}_2$	$\mathcal{T}^{(m)}_{\mathcal{N}=2}$

$$\frac{\lambda}{2} = \tilde{m}_1 + \tilde{m}_2 = u_1 - u_2$$

Double-copy construction for $U(1,1) \ltimes \mathcal{H}_3$ gauging with tensors

Exotic matter couplings: one tensor can turn into a vector (!)
[Bergshoeff, Cucu, de Wit, Gheerardyn, Vandoren, Van Proeyen, 2004]

$$\mathcal{M}_3(1\bar{B}, 2W, 3A) = \langle \mathbf{12} \rangle \langle \mathbf{1} | \varepsilon_3 | \mathbf{2}]$$

Conclusions

⋆ We have presented several examples of YME theories in 5D. Non-compact gauge groups and massive tensors can be obtained from the double copy. Signs matter!

⋆ These constructions rely on a simple massive deformation of the gauge theories involved – can this be studied with different approaches?

⋆ The appearance of the Heisenberg algebra and of the tensor-vector-vector couplings is a surprise. Known in SUGRA community, but regarded as exotic

⋆ On the other side, realizing more "conventional" gaugings such as $SU(3)$ or $SU(2,1)$ for the complex magical theory as double copies remains an open problem

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