Northwestern University

SLAC NATIONAL ACCELERATOR LABORATORY

# Bootstrapping the UV from the IR via Double Copy Consistency

Nic H. Pavao

December 11, 2023

Based on: [2310.06316] JJM Carrasco, NHP

QCD Meets Gravity IX, CERN

Can we bootstrap UV physics directly from IR principles of scattering amplitudes?

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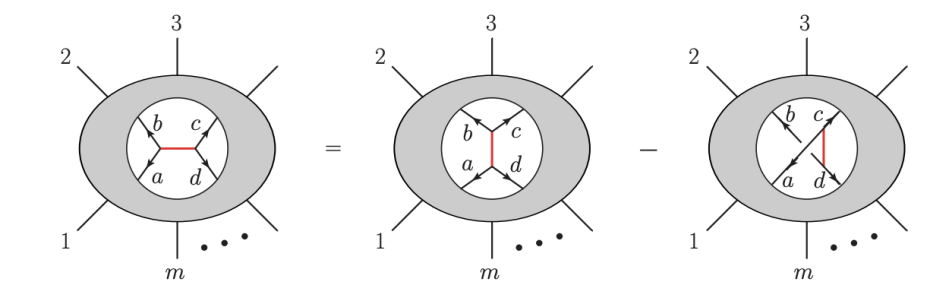
1. Color-Kinematics Duality [Bern,Carrasco,Johansson]




2. Consistent Factorization

$$\lim_{s_{123}\to 0} A_6^{(12)} = \frac{1}{s_{123}} \left[ \alpha^{12}\text{---}\alpha^{0} + \alpha^{9}\text{---}\alpha^{3} + \alpha^{6}\text{---}\alpha^{6} + \alpha^{3}\text{---}\alpha^{9} + \alpha^{0}\text{---}\alpha^{12} \right]$$

## Double Copy Consistency

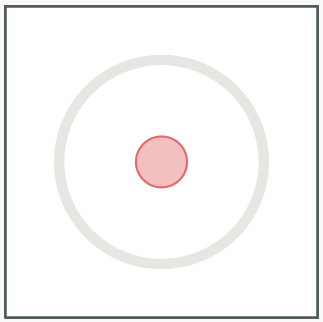
*When all tree-level amplitudes of the effective field theory are color-dual and exhibit consistent factorization down to lower point amplitudes.*

PRL 2023 [Carrasco, Lewandowski, **NHP**]

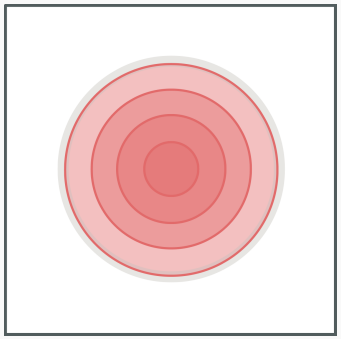
# Overview of this Talk



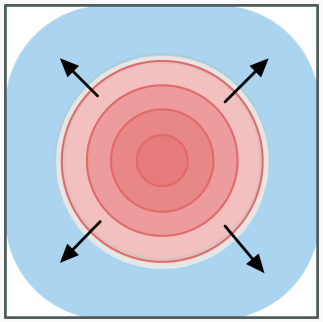
**Main Takeaway**: Color-dual EFTs exhibit incredibly rigid constraints



$$A^{\text{EFT}} = A^{\text{IR}} + \sum_{m,n}^{\infty} \alpha'^{n} X_{(m,n)}$$



$$A^{\text{UV}}_{\alpha' \to 0} = A^{\text{IR}} \prod_{n}^{\infty} \exp\left[\Omega_{(n)}\right]$$



$$A_4^{\text{UV}} = A_4^{\text{IR}} \prod_{n}^{\infty} \frac{P_n(\sigma_2,\sigma_3)}{(s-\mu_n)(t-\mu_n)(u-\mu_n)}$$

- Review and framing the setup
- Intro **Consistent Massive Resonance Double Copy** (CMRDC) Models
- Show **Double-Copy Consistency = Exponentiation**
- Gain insights about **UV** structure directly from **IR** principles
- Bridge from **IR** to **UV** with **Padé** extrapolation

# Double Copy Consistency of YM + $F^3$



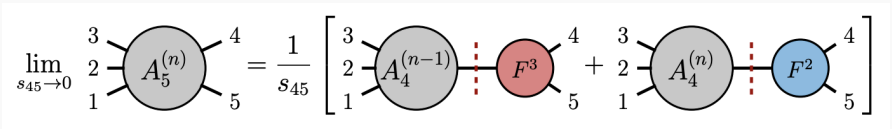
**Review**: first example upto $\mathcal{O}(\alpha'^2)$: [Broedel, Dixon]

$$\mathcal{L}^{\text{YM+F}^3} = -\frac{1}{4}(F^{\mu\nu})^2 + \frac{\alpha'}{3}F^{\mu\nu}F^{\nu\rho}F^{\rho\mu} + \cdots \Rightarrow \alpha'^2 F^4 + \mathcal{O}(\alpha'^3)$$

Requiring **Double-Copy Consistency** to all order $\mathcal{O}(\alpha'^n)$: [Carrasco, Lewandowski, **NHP**]

1. 5-point BCJ + 2. Factorization

All-order Tower from IR to UV!




$$\Rightarrow \quad \frac{A^{(n)}_{(1234)}}{u} = (F_1F_2)(F_3F_4)s_{12}^{n-1} + \text{cyc}(234)$$

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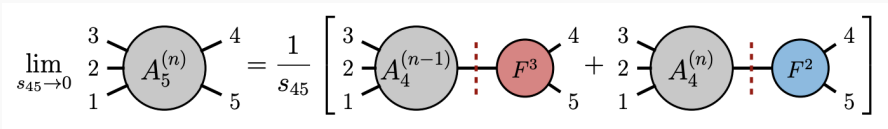


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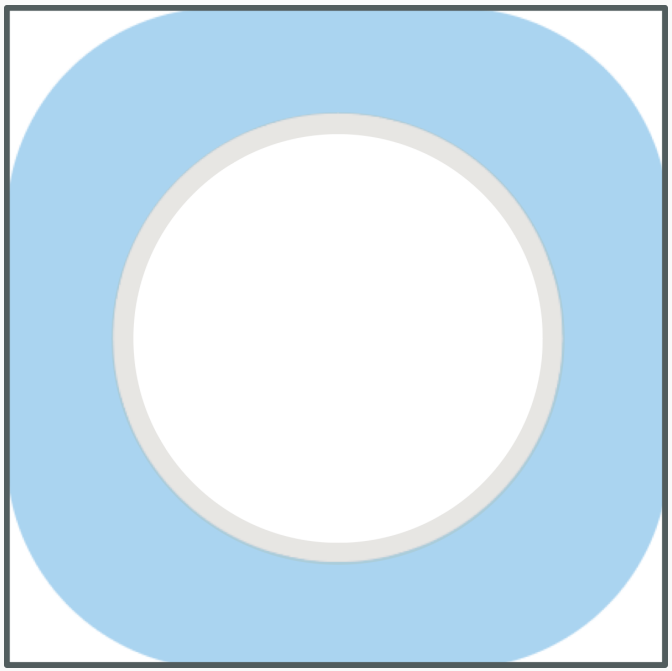
Tower of contacts resums to dimension-6 $\text{DF}^2 + \text{YM}$ theory: [Johansson, Nohle]

$$A_4^{\text{DF}^2+\text{YM}} = A_4^{\text{YM}} + \alpha' A_4^{F^3} + \alpha'^2 u \left[\frac{(F_1F_2)(F_3F_4)}{(\alpha' s - 1)s} + \text{cyc}(234)\right]$$

# Framing: Bootstrapping the UV from the IR



**Normally**: from UV to the IR



Causality, Unitarity, UV completion

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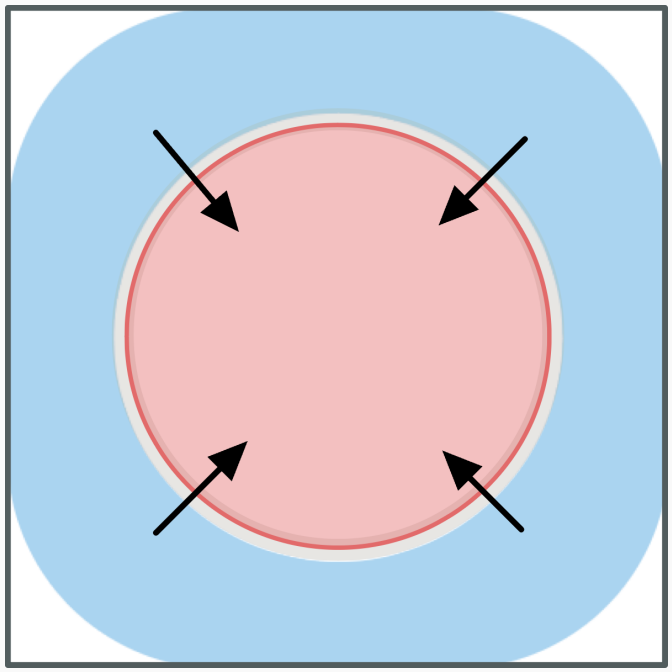

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Dispersion relations, EFT matching

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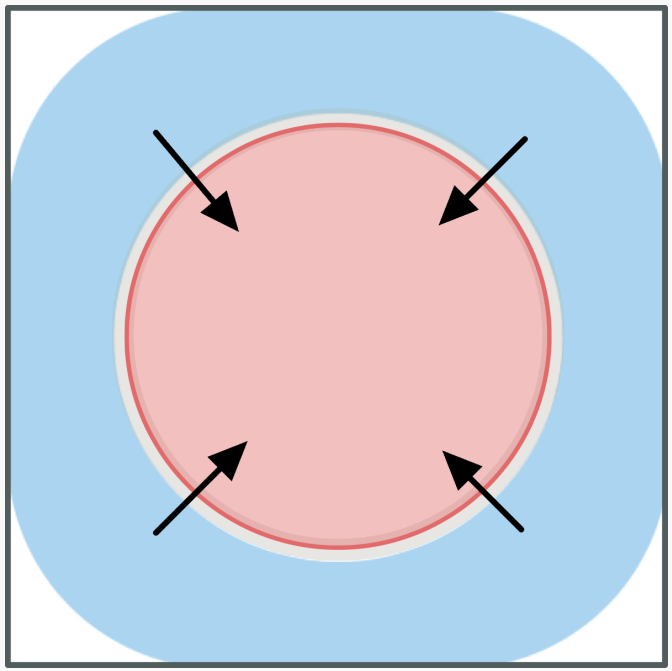

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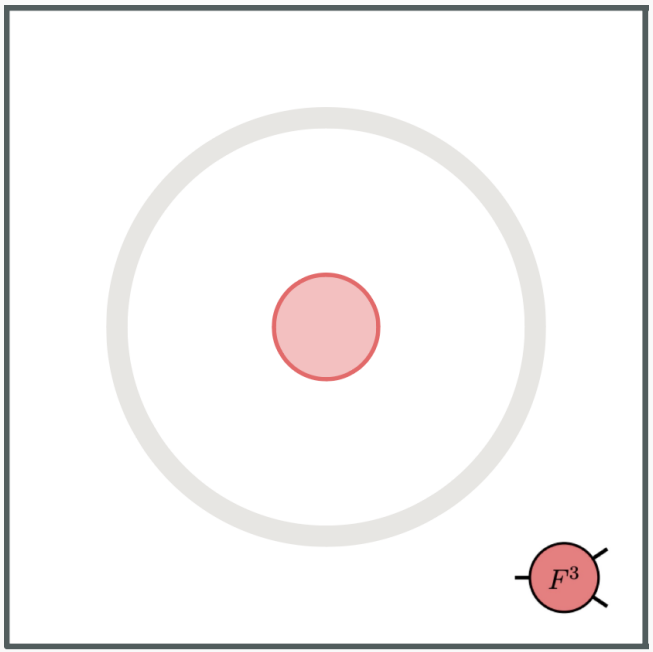


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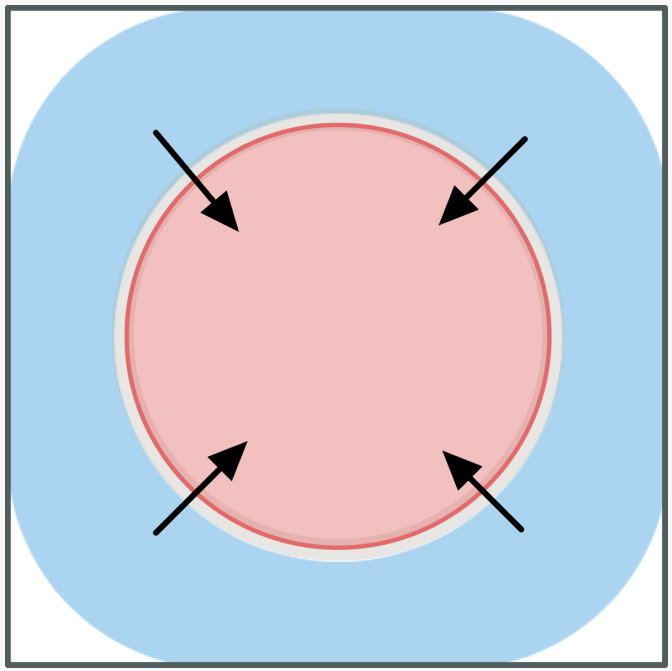

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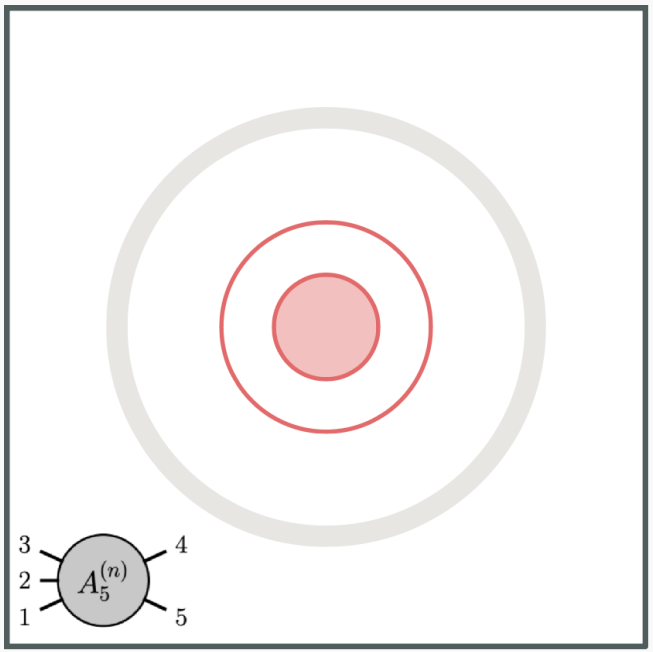


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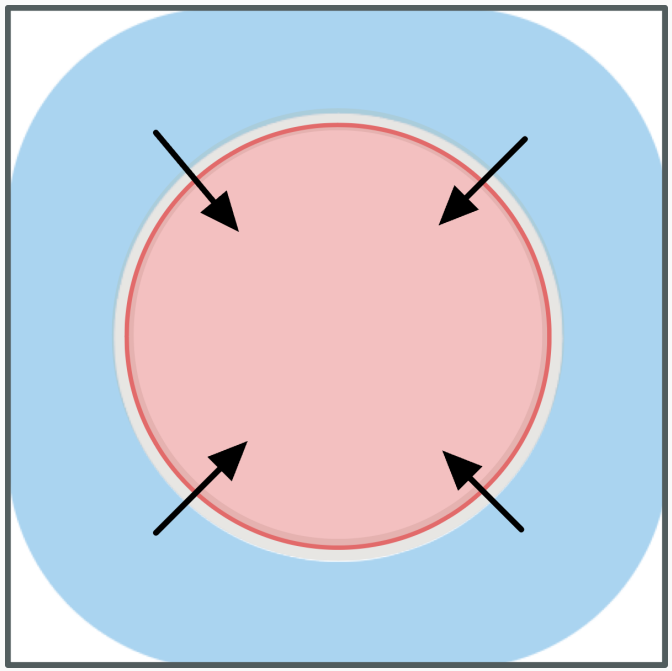

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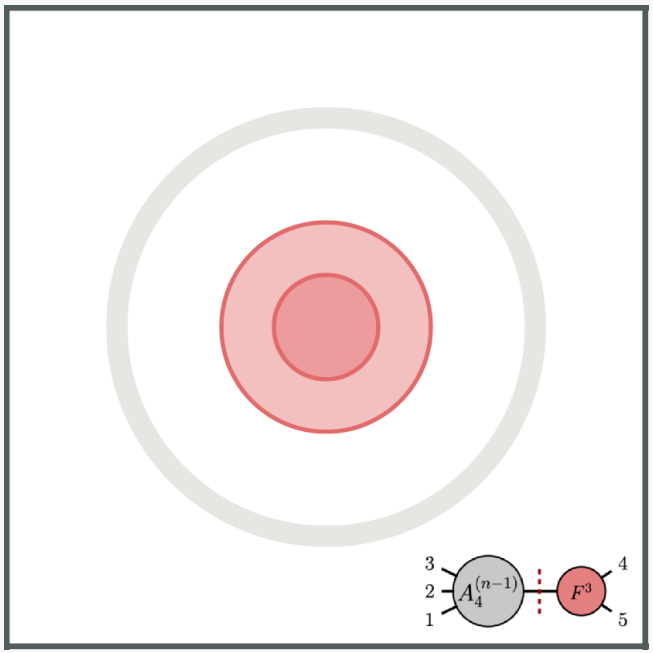


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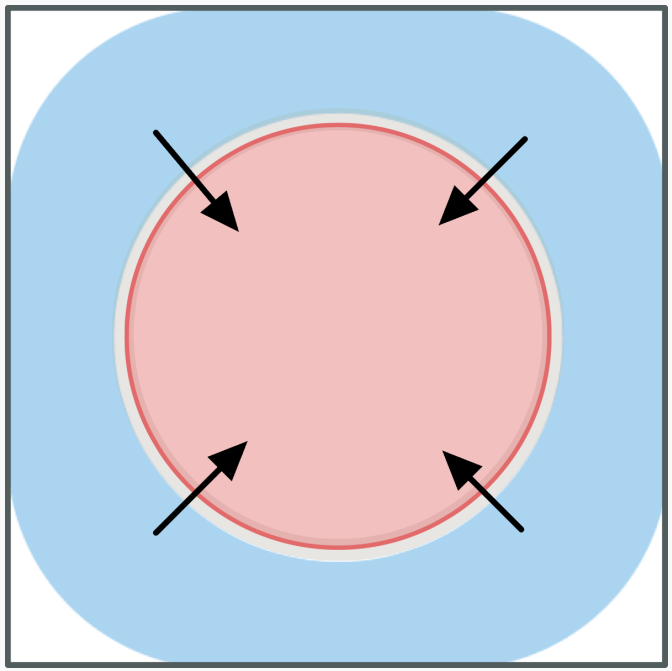

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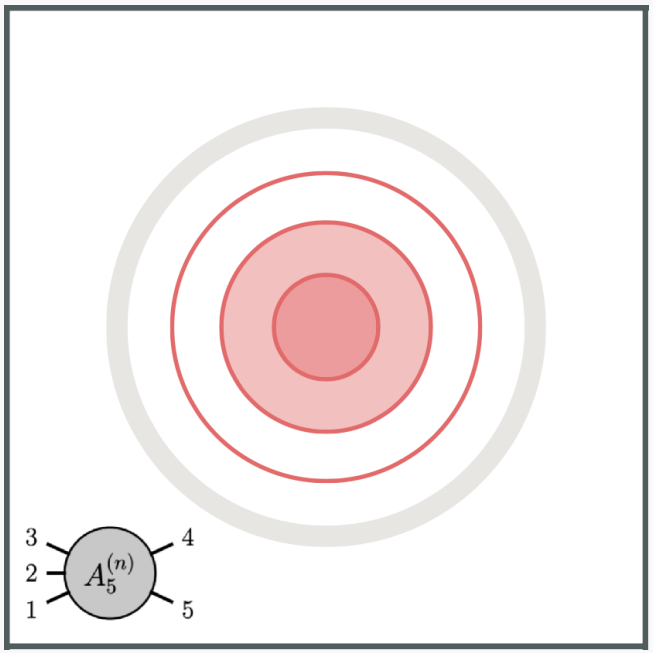


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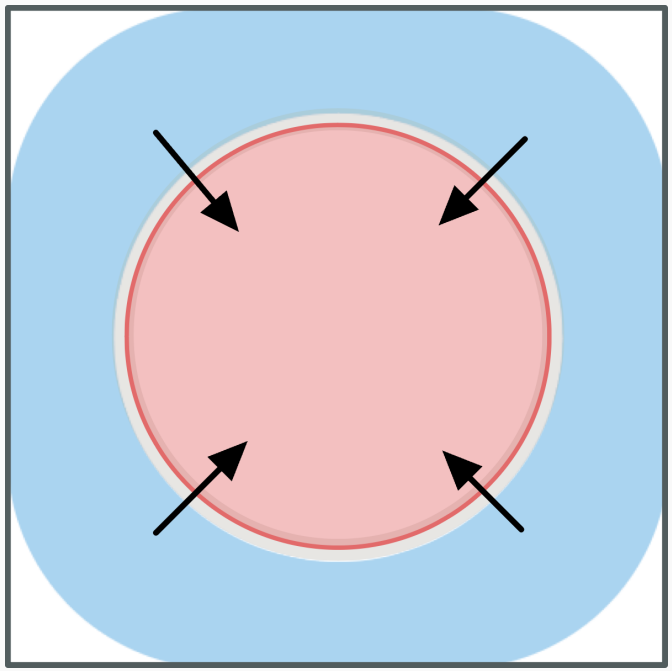

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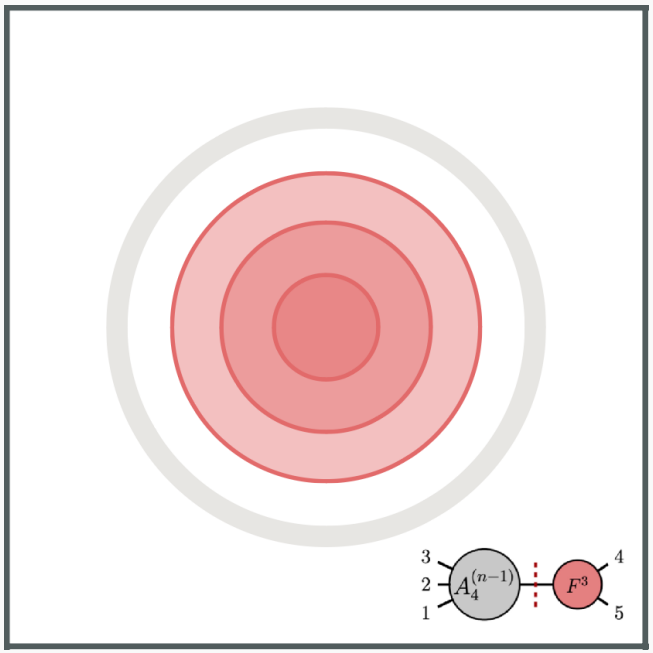


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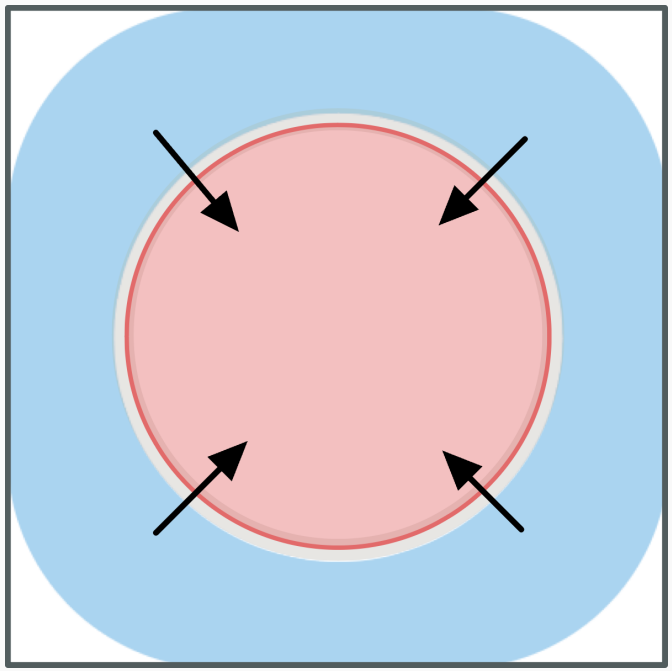

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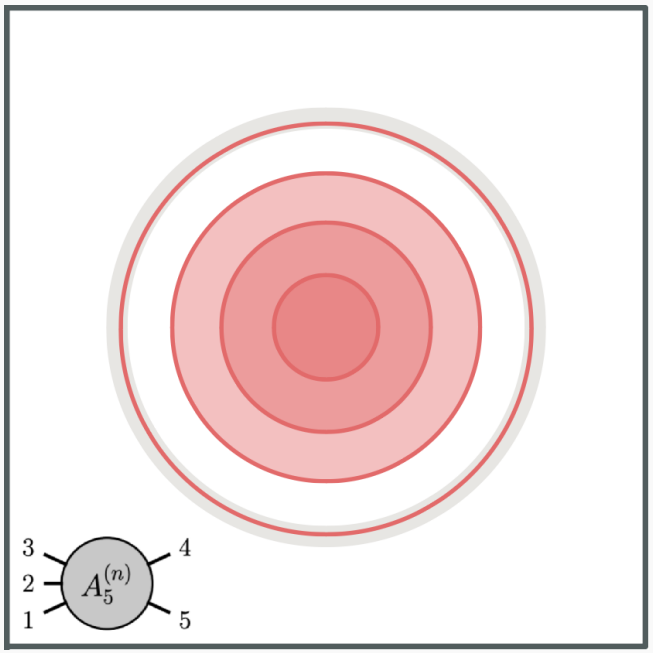


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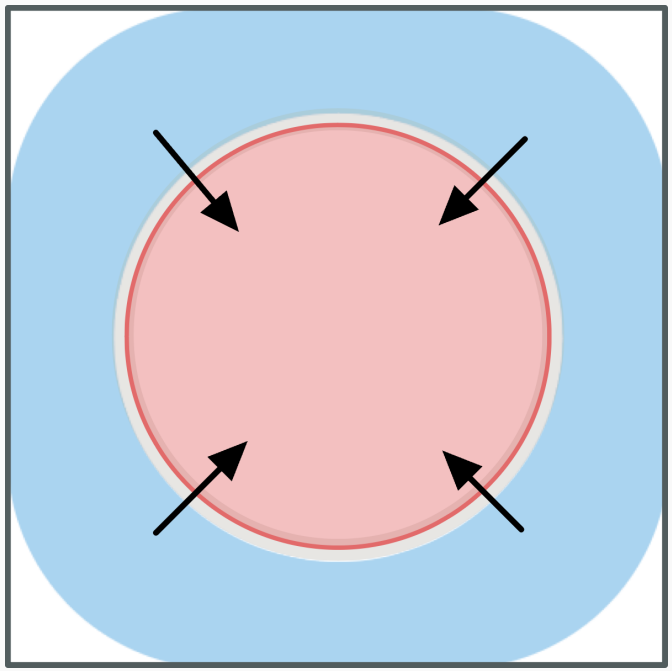

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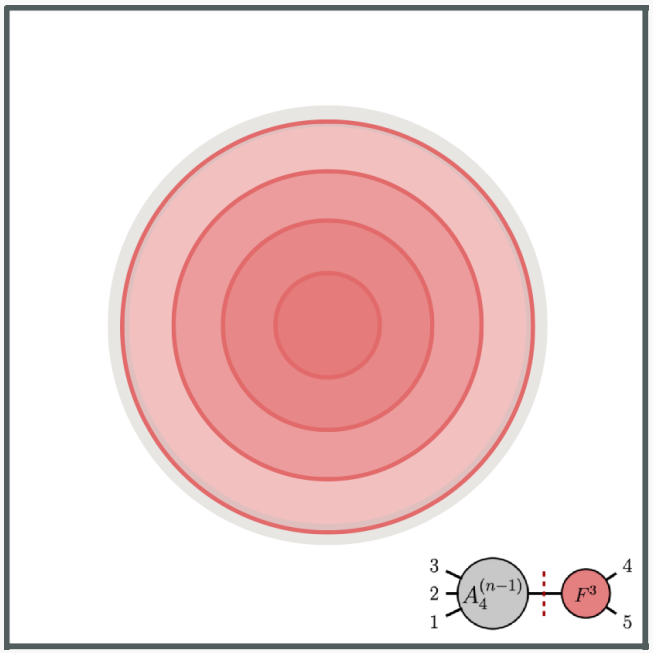


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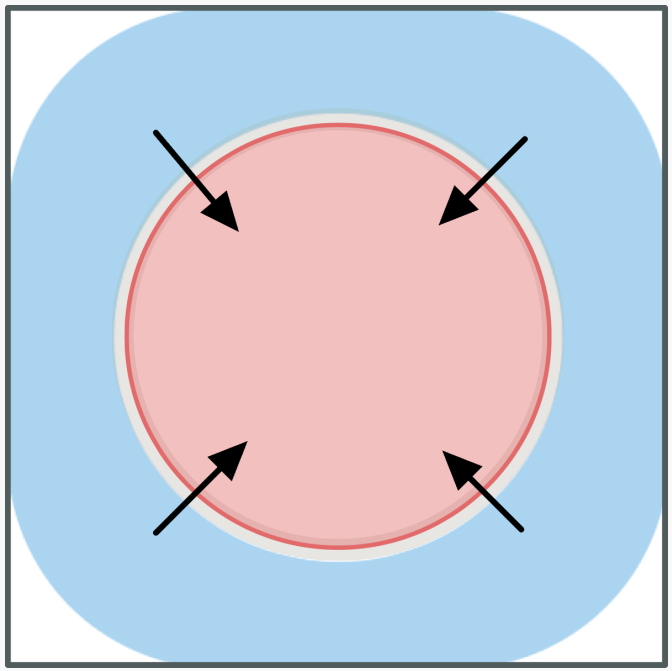

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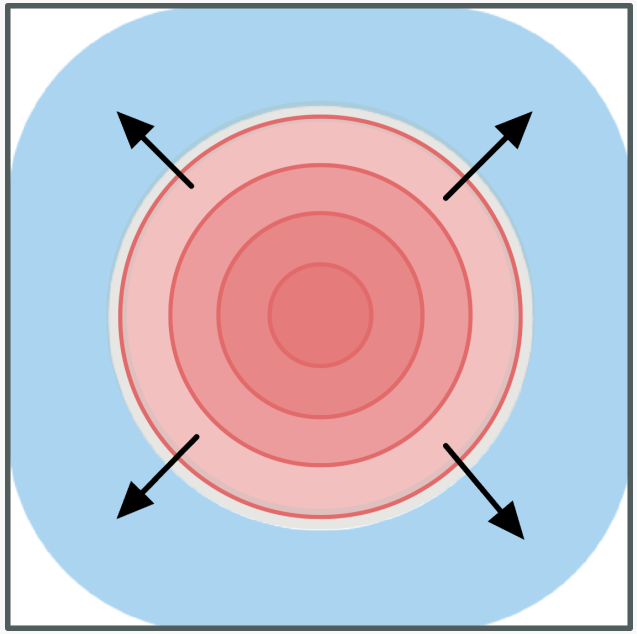

Double Copy Consistency

Resummation/Padé

UV Massive modes

# Growing Body of Evidence



Not just a special property of YM+$F^3$ vector theories!

- Higher derivative vector constraints
  [Carrasco, Lewandowski, **NHP**],[Carrasco, **NHP**]
- Generalized KLT bootstraps
  [Chi, Elvang, Herderschee, Jones, Paranjape], [Chen,Elvang,Herderschee]
- Mixed vector-scalar bootstraps
  [Carrasco, Lewandowski, **NHP**]
- Scalar BCJ bootstraps
  [Brown,Kampf,Oktem,Paranjape,Trnka],[Li,Roest,ter Veldhius]

Higher derivative constraints on EFTs appear to be universal!

**Question**:

*What UV completions are double-copy consistent?*

# Emergence of Massive Residues



Taking a closer look at $\mathrm{DF}^2 + \mathrm{YM}$ theory...

$$A_4^{\mathrm{DF}^2+\mathrm{YM}} = A_4^{\mathrm{YM}} + \alpha' A_4^{F^3} + \alpha'^2 u \left[ \frac{(F_1 F_2)(F_3 F_4)}{(\alpha' s - 1)s} + \mathrm{cyc}(234) \right] = \sum_g \frac{c_g n_g^{\mathrm{UV}}}{d_g}$$

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**Lesson**: Emergent Modes do not need to be color-ordered

$$n_g^{\mathrm{UV}} = n_g^{\mathrm{YM}} + \frac{\alpha' n_g^{F^3} + \cdots + \alpha'^4 n_g^{D^4F^4}}{(1-\alpha' s)(1-\alpha' t)(1-\alpha' u)}$$

$$A^{\mathrm{UV}} = A^{\mathrm{IR}} \prod_n \frac{P_n(\sigma_2, \sigma_3)}{(s-\mu_n)(t-\mu_n)(u-\mu_n)}$$

$$n_g^{\mathrm{IR}} = n_g^{\mathrm{YM}}(1-\alpha' s)(1-\alpha' t)(1-\alpha' u) + \alpha' n_g^{F^3} + \cdots + \alpha'^4 n_g^{D^4F^4}$$

# Consistent Massive Resonance



Traditionally, massive double copy sums over massive propagators:

**Hard Coded**: $$\mathcal{A} = \sum_{g \in \Gamma^3} \frac{c_g n_g}{d_g - m_{IR}^2} \quad \rightarrow \quad \mathcal{M} = \sum_{g \in \Gamma^3} \frac{\tilde{n}_g n_g}{d_g - m_{IR}^2}$$

[Carillo-Gonzalez,Momeni,Rumbutis],[Johnson, Jones,Paranjape],[Moynihan], [Han,He,Shen],[Chiordaroli,Günaydin,Johansson,Roiban]

New way of thinking about massive modes in double copy theories!

**Emergent**: $$\mathcal{A} = \sum_{g \in \Gamma^3} \frac{c_g N_g^{\text{UV}}}{d_g} \quad \rightarrow \quad \mathcal{M} = \sum_{g \in \Gamma^3} \frac{\tilde{n}_g N_g^{\text{UV}}}{d_g}$$

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**Introducing**: Consistent Massive Resonance Double-Copy (CMRDC) Models

$$N_g^{\text{UV}} = n_g + \sum_x (\alpha')^x c_{(x,y)} n_g^{(x,y)} \overset{\text{4-point}}{\Rightarrow} n_g \prod_i \frac{P_i(\sigma_2, \sigma_3)}{(s-\mu_i)(t-\mu_i)(u-\mu_i)}$$

# Color-dual Bootstrap: from the IR to the UV



**Setup the bootstrap**: start by reinstating scalar kinematic freedom:

$$A_4^{\text{EFT}} = A_4^{\text{IR}} + \sum_{m\geq 1,n} c_{(m,n)} X_4^{(m,n)} \qquad X_4^{(m,n)} \equiv \sigma_3^m \sigma_2^n A_4^{\text{IR}}$$

where $\sigma_3 = stu, \sigma_2 = (s^2 + t^2 + u^2)$.

**Define**: higher-point EFT ansatz:

$$A^{\text{EFT}}_{(12P5)} = A^{\text{IR}}_{(12P5)} + \sum_m \alpha'^{3m+2n} \Omega^{(m,n)}_{(P|Q)} A^{\text{IR}}_{(12Q5)}$$

- $P, Q \in \text{perm}(3, 4, ..., n-1)$
- In general, $\Omega$, are $(n-3)! \times (n-3)!$ higher derivative matrices with appropriate mass dimension of mandelstams.

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**Goal**: Bootstrap the promotion operators, such that

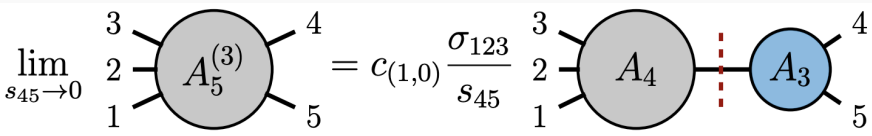
$$\text{Res}[\Omega^{(m,n)} A_5^{\text{IR}}] \to A_3^{\text{IR}} \times (\sigma_3^m \sigma_2^n A_4^{\text{IR}})$$

# Recurrent Structures at 5-point and 6-point



**Example**: Bootstrapping $\Omega^{(m,0)}$ operators that factor down to $\sigma_3^m A_4^{\text{IR}}$

1. 5-point BCJ relations
2. Factorization




$$\Rightarrow A^{(1,0)}_{1P45} = c_{(1,0)} \Omega^{(1,0)}_{(P|Q)} A^{\text{IR}}_{(1Q5)}$$

$$\Rightarrow A^{(2,0)}_{1P45} = c_{(2,0)} \Omega^{(2,0)}_{(P|Q)}$$

$$= \boxed{c_{(2,0)} \Omega^{(1,0)}_{(P|Q)} \Omega^{(1,0)}_{(P|Q)} A^{\text{IR}}_{(1Q5)}}$$

70 params $\Rightarrow$ unique $\Omega^{(1,0)}$

420 params $\Rightarrow$ unique $\Omega^{(2,0)}$

# Recurrent Structures at 5-point and 6-point



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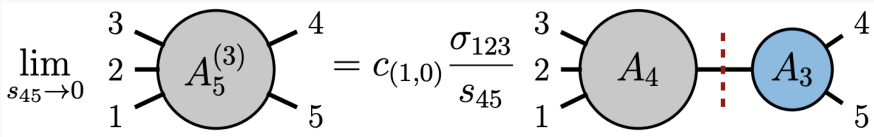
1. 5-point BCJ relations
2. Factorization




$$\Rightarrow A^{(1,0)}_{1P45} = c_{(1,0)}\Omega^{(1,0)}_{(P|Q)}A^{\text{IR}}_{(1Q5)}$$

$$\Rightarrow A^{(2,0)}_{1P45} = c_{(2,0)}\Omega^{(2,0)}_{(P|Q)}$$

$$= \boxed{c_{(2,0)}\Omega^{(1,0)}_{(P|Q)}\Omega^{(1,0)}_{(P|Q)}A^{\text{IR}}_{(1Q5)}}$$

70 params $\Rightarrow$ unique $\Omega^{(1,0)}$

420 params $\Rightarrow$ unique $\Omega^{(2,0)}$

**Similar iterative structure at 6-point!**

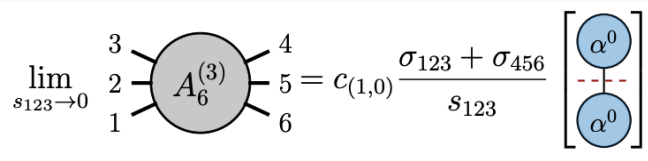
**Conjecture**: verified through $\mathcal{O}(\alpha'^{12})$

$$A^{(n,0)}_{1P56} = c_{(n,0)}(\Omega^{(1,0)}\Omega^{(1,0)}\cdots\Omega^{(1,0)})_{(P|Q)}A^{(0,0)}_{1Q56} + 5,6 \text{ contacts}$$

# Double Copy Consistency = Exponentiation



Given iterative structure from (1) higher-point **BCJ** constraint:




$$\Rightarrow \lim_{s_{123}\to 0} A^{(n,0)}_{1P56} = \boxed{c_{(n,0)} \frac{(\sigma_{123}+\sigma_{456})^n}{s_{123}} A_{(123I)} A_{(I456)}}$$

where $\sigma_{ijk} = s_{ij}s_{jk}s_{ik}$

# Double Copy Consistency = Exponentiation



Given iterative structure from (1) higher-point **BCJ** constraint:

$$\lim_{s_{123}\to 0}\ A_6^{(3)}(3,2,1,4,5,6) = c_{(1,0)}\frac{\sigma_{123}+\sigma_{456}}{s_{123}}\left[\alpha^0 \cdots \alpha^0\right] \quad \Rightarrow \quad \lim_{s_{123}\to 0} A^{(n,0)}_{1P56} = \boxed{c_{(n,0)}\frac{(\sigma_{123}+\sigma_{456})^n}{s_{123}}A_{(123I)}A_{(I456)}}$$

where $\sigma_{ijk} = s_{ij}s_{jk}s_{ik}$, and (2) split **factorization** constraint:

$$\lim_{s_{123}\to 0} A_6^{(12)}(3,2,1,4,5,6) = \frac{1}{s_{123}}\left[\alpha^{12}\cdots\alpha^0 + \alpha^9\cdots\alpha^3 + \alpha^6\cdots\alpha^6 + \alpha^3\cdots\alpha^9 + \alpha^0\cdots\alpha^{12}\right]$$

$$\Rightarrow \quad \lim_{s_{123}\to 0} A^{(n,0)}_{1P56} = \boxed{\sum_{k=0}^{n} c_{(n-k,0)}c_{(k,0)}\frac{\sigma_{123}^{n-k}\sigma_{456}^{n}}{s_{123}}A_{(123I)}A_{(I456)}}$$

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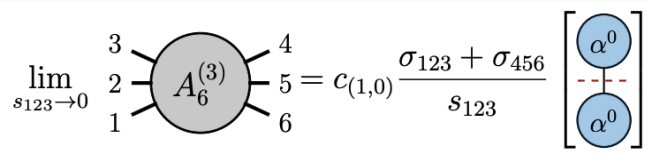


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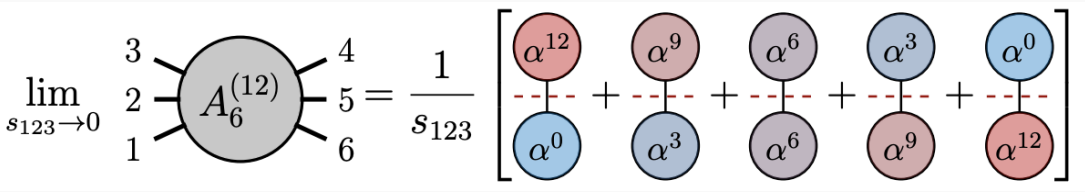



$$\Rightarrow \lim_{s_{123}\to 0} A^{(n,0)}_{1P56} = \boxed{\sum_{k=0}^{n} c_{(n-k,0)} c_{(k,0)} \frac{\sigma_{123}^{n-k}\sigma_{456}^{n}}{s_{123}} A_{(123I)} A_{(I456)}}$$

**Lesson**: Quadratic constraints from split factorization!

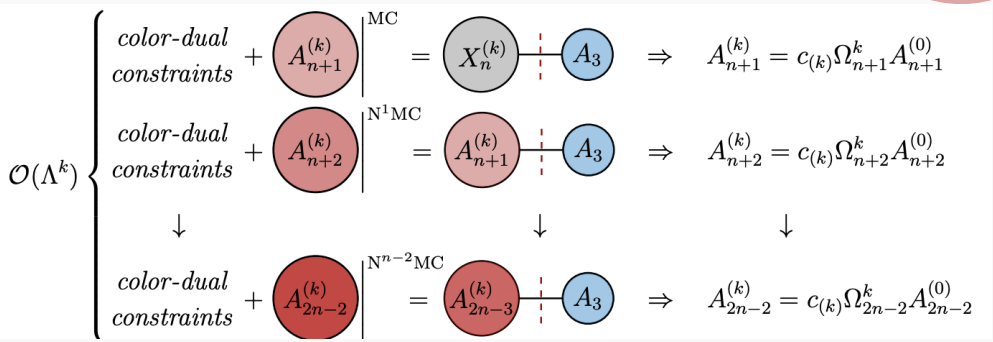
$$c_{(n,0)} \binom{n}{k} = c_{(n-k,0)} c_{(k,0)} \quad \Rightarrow \quad c_{(n,0)} = \frac{1}{n!} c_{(1,0)}$$

# All Multiplicity Structure

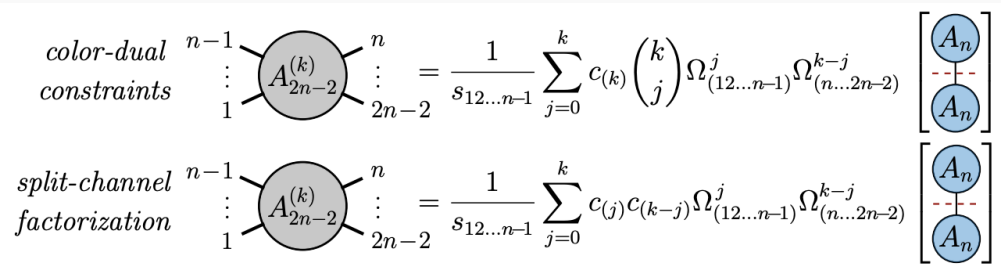


1. Start with n-point color-dual **contact** operator, $X_n \equiv \Omega_n A_n^{\text{IR}}$.

2. Iteratively apply **three-point** cut constraints




3. Fix on **split-channel** residue of (2n-2)-point amplitude




**Conjecture**: n-point color-dual contacts exponentiate

$$A^{\text{UV}} = \prod_k \exp\left[c_{(k)}\Omega_k^{\text{DCC}}\right] A^{\text{IR}}$$

where $\Omega_k$ are higher derivative $(n-3)! \times (n-3)!$ matrix operators that mix vector structure of $A^{\text{IR}}$ consistent with BCJ relations and factorization.

# What can this tell us about UV theories?



**String amplitudes** also exhibit the structure of CMRDC models

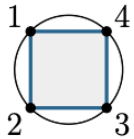
$$\mathcal{A}^{\text{OS}} = A^{\text{YM}} \otimes Z \qquad \mathcal{M}^{\text{CS}} = A^{\text{YM}} \otimes \langle\!\langle A^{\text{YM}} \rangle\!\rangle_{\text{SV}}$$

Where we define the **SV map** in terms of string KLT and Z-theory amplitudes:

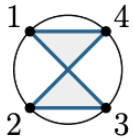
[Broedel, Schlotterer, Stieberger],[Carrasco, Mafra, Schlotterer],[Carrasco, Lewandowski, **NHP**]

$$\langle\!\langle X \rangle\!\rangle_{\text{SV}} = Z \overset{\alpha'}{\otimes} Z \otimes X$$

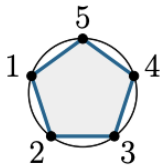
- All the UV massive structure lives in the $\langle\!\langle A^{\text{YM}} \rangle\!\rangle_{\text{SV}}$ of the **closed string**.




$$= \frac{\Gamma(-s_{12})\Gamma(-s_{23})}{\Gamma(-s_{12}-s_{23})}$$




$$= -\frac{\Gamma(1-s_{12})\Gamma(-s_{23})}{\Gamma(1-s_{12}-s_{23})}$$




$$= {}_3F_2\left[\begin{matrix}-s_{12},\, s_{14},\, -s_{34} \\ -s_{12}-s_{15},\, -s_{34}-s_{45}\end{matrix}; 1\right] \frac{\Gamma(-s_{15})\Gamma(-s_{12})\Gamma(-s_{34})\Gamma(-s_{45})}{\Gamma(-s_{12}-s_{15})\Gamma(-s_{34}-s_{45})}$$

- Can we understand the EFT structure of **SV map** at high multiplicity?

# Stringy Example: from the UV to the IR



## Question

To what extent is the low-energy EFT structure of $\langle\!\langle A_n^{\rm YM} \rangle\!\rangle_{\rm SV}$ determined by **double-copy consistency**?

Series expanding the disc integrals, one finds remarkable simplicity:

[Broedel, Schlotterer, Stieberger]

$$\langle\!\langle A_4^{\rm YM} \rangle\!\rangle_{\rm SV}^{\alpha'\to 0} = \prod_{k \text{ odd}} \exp(2\zeta_{2k+1}\sigma_{2k+1}) A_4^{\rm YM}$$

$$\langle\!\langle A_5^{\rm YM} \rangle\!\rangle_{\rm SV}^{\alpha'\to 0} = [\cdots \exp\left(\zeta_{3,3,5}[M_3[M_3, M_5]]\right)] \prod_{k \text{ odd}} \exp(\zeta_{2k+1} M_{2k+1}) A_5^{\rm YM}$$

where $\sigma_k = \frac{\alpha'^k}{k}(s^k + t^k + u^k)$ and $M_k$ are matrix valued acting on $(n-3)!$ basis of color-dual amplitudes.

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$$\boxed{M_3 \equiv \Omega^{(1,0)}, M_5 \equiv \Omega^{(1,1)}, M_7 \equiv \Omega^{(1,2)}, \ldots}$$

[Carrasco, **NHP**]

# From Masses to Wilson Coefficients



How do we extract CMRDC models from double-copy consistent EFTs?

$$A^{\text{UV}} = \prod_{n=1}^{\infty} \frac{(s+\mu_n)(t+\mu_n)(u+\mu_n)}{(s-\mu_n)(t-\mu_n)(u-\mu_n)} A^{\text{IR}}$$
$$= \prod_{x,y} \exp\left[c_{(x,y)}\sigma_3^x \sigma_2^y\right] A^{\text{IR}}$$

**Obstacle**: Wilson coefficients are sensitive to infinite d.o.f. of full UV theory!

$$c_{(1,m)} = \sum_{n=1}^{\infty} \frac{1}{\mu_n^{2m+1}}$$

**NB**: $\mu_n = n \Rightarrow c_{(1,m)} = \zeta_{2m+1}$

**How do we resolve this inverse problem?**

## Resurgence

Method for extracting the maximal amount of non-perturbative information from power series via analytic continuation.

**Padé Approximant**:

$$F(x) = \sum_n c_n x^n \quad \rightarrow \quad \mathcal{R}_{[m,n]}(x) = \frac{A_m(x)}{B_n(x)}$$

# From Wilson Coefficients to Masses



## Resurgence

Method for extracting the maximal amount of non-perturbative information from power series via analytic continuation.

**Padé Approximant**:

$$F(x) = \sum_n c_n x^n \quad \rightarrow \quad \mathcal{R}_{[m,n]}(x) = \frac{A_m(x)}{B_n(x)}$$

Match derivatives around IR fixed point $x = x_0$:

$$F^{(k)}(x_0) = \mathcal{R}^{(k)}_{[m,n]}(x_0) \quad \Leftrightarrow \quad A^{\text{EFT}}|_{s\to 0} = A^{\text{UV}}|_{s\to 0}$$

Padé is incredibly sensitive to small deviations in Wilson coefficients, and can provide information about full transseries ($g = \alpha' s$):

$$A^{\text{UV}} = \sum_{x,y,z} c_{(x,y,z)} g^x \log^y(g) e^{S_z/g}$$

# Resurgence for a Simple Resonance Model



**Example**: Take a simple scalar model of NLSM plus two resonant modes:

$$\mathcal{A}^{\text{UV}}_{\alpha'\to 0} = \mathcal{A}^{\text{NLSM}} \frac{1}{(s-1)(t-1)(u-1)} \frac{1}{(s-10)(t-10)(u-10)}$$

Describes pion scattering with massive resonance, $m_1^2 = 1/\alpha'$ and $m_2^2 = 10/\alpha'$.

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Describes pion scattering with massive resonance, $m_1^2 = 1/\alpha'$ and $m_2^2 = 10/\alpha'$.

At fixed angle scattering kinematics, $u = -s - t, t = -s/2$, expand to $\mathcal{O}(\alpha'^{10})$:

$$\begin{aligned}\mathcal{A}^{\text{UV}}_{\alpha'\to 0} = \mathcal{A}^{\text{IR}} \times \Big[&1.000 + 1.010(\alpha')^2\sigma_2 + 1.001(\alpha')^3\sigma_3 + 1.010(\alpha')^4\sigma_2^2 + 2.01(\alpha')^5\sigma_2\sigma_3 \\ &+ (\alpha')^6\left(1.010\sigma_2^3 + 1.001\sigma_3^2\right) + 3.02(\alpha')^7\sigma_2^2\sigma_3 + (\alpha')^8\left(1.010\sigma_2^4 + 3.01\sigma_2\sigma_3^2\right) \\ &+(\alpha')^9\left(4.03\sigma_2^3\sigma_3 + 1.001\sigma_3^3\right) + (\alpha')^{10}\left(1.01\sigma_2^5 + 6.03\sigma_2^2\sigma_3^2\right) + \mathcal{O}(\alpha')^{11}\Big]\end{aligned}$$

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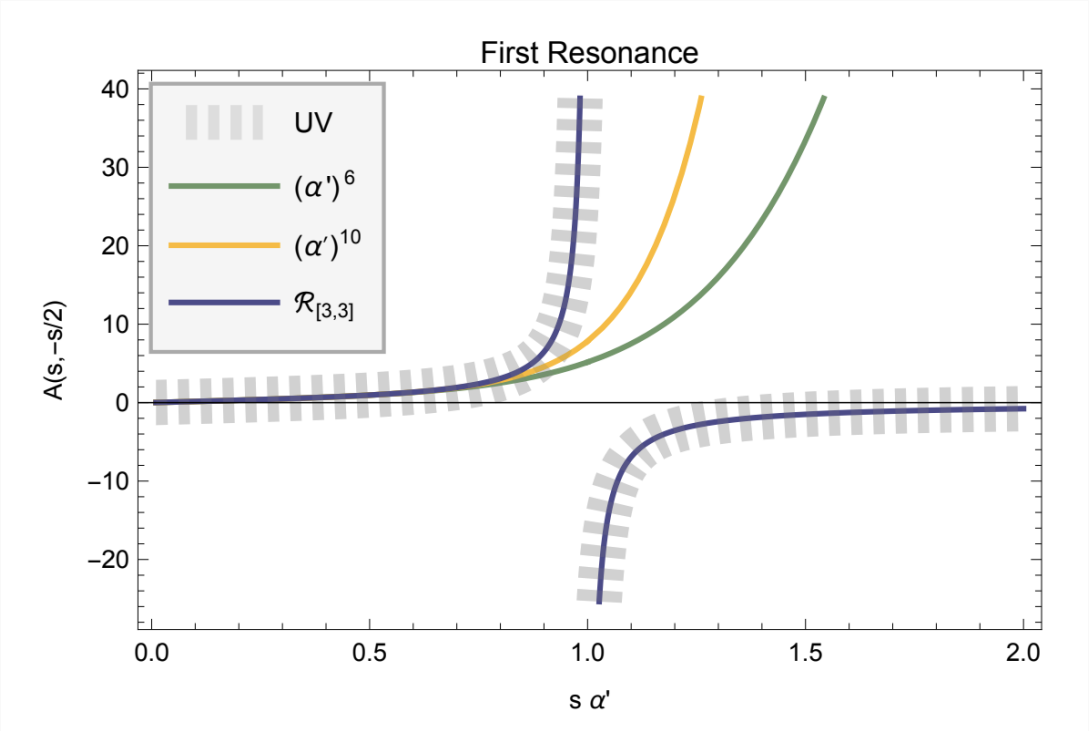

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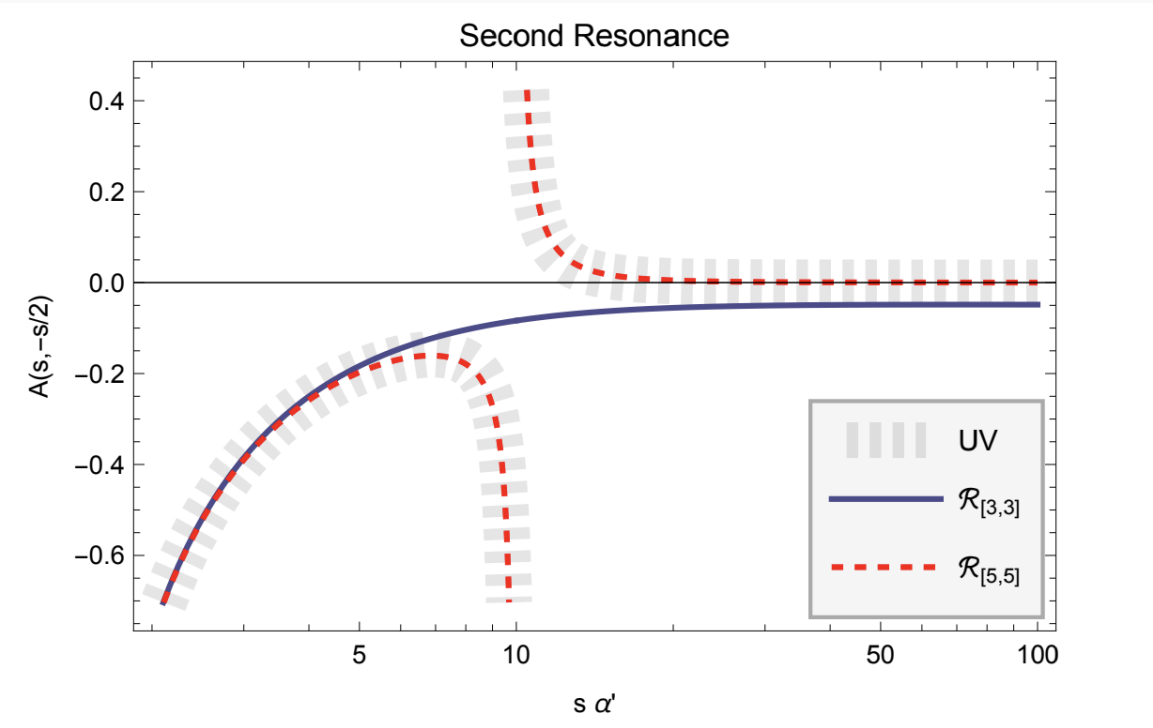


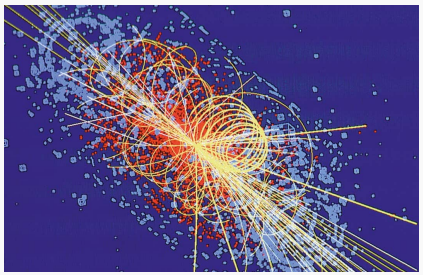
# Future Directions



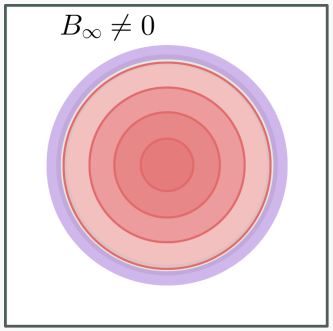
- Can we bootstrap Z-theory, dual resonance models from double-copy consistency by mixing color and kinematics?
  [Cheung, Remmen],[Geiser,Lindwasser],[Bhardwaj,De,Spradlin,Volovich],[Chi, Elvang, Herderschee, Jones, Paranjape], [Chen,Elvang,Herderschee]

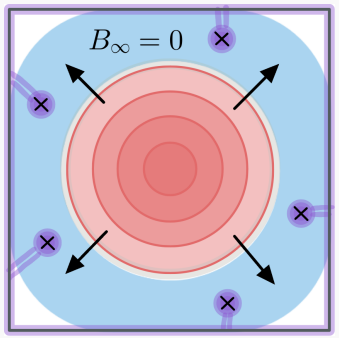
$$\mathcal{A}^{\text{OS}} = A^{\text{YM}} \otimes Z \quad \Rightarrow \quad Z = \sum_g \frac{c_g z_g(color, k_i k_j)}{d_g}$$

- Color-dual model building: hadronic resonances, UV completions?



- Are UV completions on-shell constructible from IR information?
  [Cheung, Kampf,Novotnoy,Shen,Trnka],[Cheung, O'Connel,Wecht]

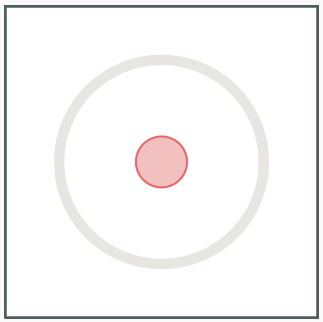


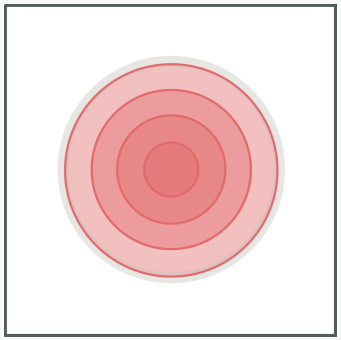
# Takeaways



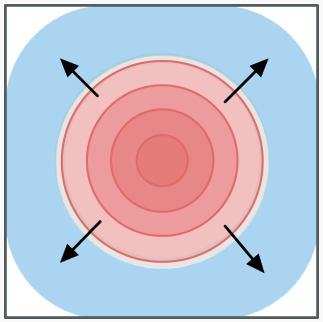
**Main Takeaway**: UV structure can be built from IR Double Copy Consistency



$$A^{\text{EFT}} = A^{\text{IR}} + \sum_{m,n}^{\infty} \alpha'^{n} X_{(m,n)}$$



$$A^{\text{UV}}_{\alpha' \to 0} = A^{\text{IR}} \prod_{n}^{\infty} \exp\left[\Omega_{(n)}\right]$$



$$A^{\text{UV}}_{4} = A^{\text{IR}}_{4} \prod_{n}^{\infty} \frac{P_n(\sigma_2,\sigma_3)}{(s-\mu_n)(t-\mu_n)(u-\mu_n)}$$

- New approach to massive double-copy with **CMRDC models**!
- IR **double copy consistency** demands **exponentiation**!
- Closed string **UV restrictions** can be understood from **IR principles**!
- **Absent infinite precision** of EFTs, we can approximate the location of **UV massive poles** with **Padé**!

# Backup Slides

# Single Value Selection from the IR

## Multiple-Zeta Values in string effective actions

In the **open string**, multiple-zeta values (MZVs) weight higher-point contacts:

$$A^{OS}_{\alpha' \to 0} = \cdots + \zeta_{3,5}[M_3, M_5]A^{\mathrm{YM}} + \cdots$$

However, in the **closed string**, first MZV to appear is, $\zeta_{3,3,5}$, due to selection rules from UV sphere integrals:

$$M^{CS}_{\alpha' \to 0} = \cdots + \zeta_{3,3,5}[M_3, [M_3, M_5]](A^{\mathrm{YM}})^2 + \cdots$$

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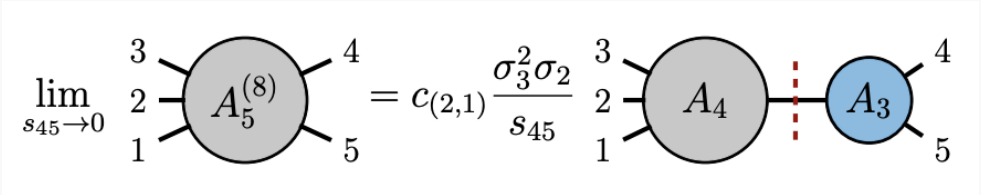
Alternatively, at order $\mathcal{O}(\alpha'^8)$ we find that $\Omega^{\text{OS}} = [M_3, M_5]$ is *incompatible*:




Thus, $\zeta_{3,5}$, doesn't appear in the **closed string** because the **operator** is excluded from our bootstrap by IR double copy consistency!