Caltech BURKE INSTITUTE

EFFECTIVE FIELD THEORY FOR EXTREME MASS RATIOS (PART 2)

QCD Meets Gravity 2023, CERN

11 Dec 2023

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based on: C. Cheung, J. Parra-Martinez, I. Z. Rothstein, N. Shah, J.W.-G. 2308.14832, 2312.XXXXX

J.W.-G. 2310.17731, 24XX.XXXXX

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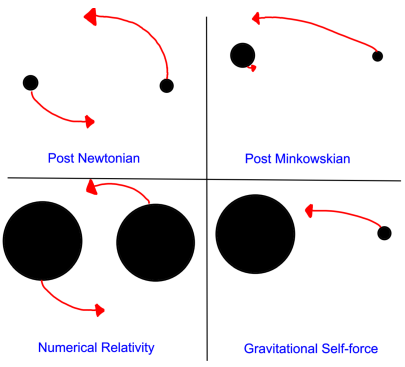
STRUCTURE

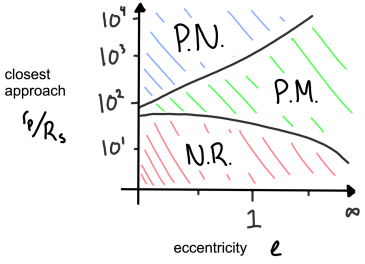
1. Introduction
2. Amplitudes/Radial Action
3. Computations/Checks
4. New results with additional fields

INTRODUCTION

BINARY DYNAMICS

Typical approaches:





NR: Keep everything (costly)
GSF: Expand in $\lambda = \frac{m_L}{m_H}$ (typically numerical, somewhat costly)
PN: Expand in $G \sim v^2$
PM: Expand in G to $\mathcal{O}(v^\infty)$

JUMPING TO THE PUNCHLINE

New classical heavy particle EFT:

- Use background field method (QFT in curved space)
- Systematically compute finite mass-ratio corrections
  - eg. backreaction on BFM
- Expand exact curved spacetime expressions perturbatively in G
- morally = perturbative Self-Force

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New results through 2-loops $\mathcal{O}(G^3)$:

- Dyons in Maxwell theory
- Black holes (one with scalar & electric charge)
- $\uparrow$ non-minimally coupled scalar
- Black holes in:
  - Einstein-Maxwell
  - Einstein-dilaton-Gauss-Bonnet
  - dynamical Chern-Simons gravity
  - Einstein-Hilbert $+Riem^3$

GRAVITATIONAL SELF-FORCE

Relativists already study large mass ratio limit:

Barack and Pound (2019), Mathisson (1937), Infeld and Schild (1949) Papapetrou (1952), Dixon (1974), D'Eath (1975), Thorne and Hartle (1985), Mino Sasaki and Tanaka (1997), Quinn and Wald (1997), Gralla and Wald (2008)

- Take a black hole solution and expand: $g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{m_L}{m_H} h^{(1)}_{\mu\nu} + ...$
- Linearize Einstein's equations about $\bar{g} \to \delta G_{\mu\nu}[h^{(1)}] = 8\pi T^{(1)}_{\mu\nu}$.
- Excise a small worldvolume containing the light body, give effective description of body via $h^{(1)}$ boundary conditions on surface of worldvolume
- Zeroth order Einstein's equation with these b.c.'s **implies** geodesic motion of light body, $\frac{D^2}{d\tau^2} x^\mu = 0$
- First order Einstein's equation implies

$$\frac{D^2}{d\tau^2} x(\tau) = \frac{m_L}{m_H} \dot{x}(\tau) \dot{x}(\tau) \int d\tau' \dot{x}(\tau') \dot{x}(\tau') \nabla G(\tau, \tau')$$

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***Challenges: How to compute the Green's function?**
How to regularize/renormalize self-force?
Horizon b.c.'s and zero modes? (Gauge reconstruction problem)

EMR EFT

Minimal point particle EFT: $S = -m_L \int \sqrt{g \dot{x}_L \dot{x}_L} - m_H \int \sqrt{g \dot{x}_H \dot{x}_H} + S_{EH}$

- Expand about **exact** solution in $m_L/m_H \to 0$ limit:

$$x_H = u\tau + \delta x_H, \quad g = \bar{g} + \delta g, \quad x_L = \bar{x}_L + \delta x_L.$$

$$\begin{aligned} S =& \bar{S} + \cancel{\frac{\delta S}{\delta x_L}\Big|\delta x_L} + \cancel{\frac{\delta S}{\delta x_H}\Big|\delta x_H} + \left(\frac{\delta S_L}{\delta g} + \cancel{\frac{\delta S_H}{\delta g}} + \cancel{\frac{\delta S_{EH}}{\delta g}}\right)\Big|\delta g + \cdots \\ =& \underbrace{\frac{\delta S_L}{\delta g}\Big|\delta g}_{\text{power counts as } m_L^2} + \cdots \end{aligned}$$

Total 1SF effective action is above tadpole + quadratic order in δx_H and δg

We can trivially integrate out δx_H.

*In EFT we can make sense of $\bar{g}_{\mu\nu}(\bar{x}_H)$.

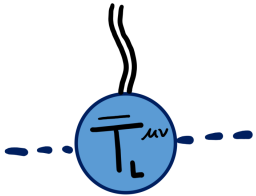
1SF ACTION

$$S_{1SF} = -\frac{1}{2}\int d^D x\sqrt{\bar{g}}\bar{T}^{\mu\nu}_L \delta g_{\mu\nu} - \frac{m_H}{2}\int d\tau \left[u^a u^b \delta\Gamma^{\mu}_{ab}\right]\frac{1}{\partial_\tau^2}\left[u^c u^d \delta\Gamma_{\mu\, cd}\right]$$
$$+\frac{1}{2\kappa^2}\int d^D x\sqrt{\bar{g}}\left[\bar{\nabla}_a \delta g_{bc}\bar{\nabla}^a \delta g^{bc} + \cdots\right]$$

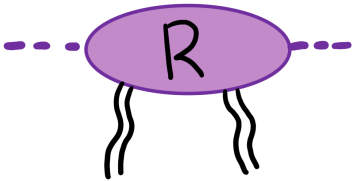
- Theory of free gravitons in a background black hole spacetime
- Sourced by light-particle geodesic
- Modified by a gauge-invariant, non-local in time **recoil operator**, localized to $\bar{x}_H$

FEYNMAN VERTICESX THROUGH 2SF

1 SF:

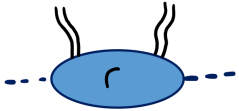


Light particle geodesic source

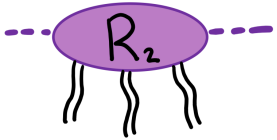


Heavy particle recoil

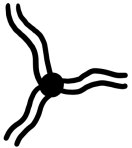
2 SF:



Geodesic deviation



Consecutive recoil



Fluctuation interactions

AMPLITUDES/RADIAL ACTION

WHAT DO WE ACTUALLY COMPUTE?

- Gauge-invariant "Amplitude-action relation"

$$i\mathcal{M}(q) = \int_J (e^{iI_r(J)} - 1) \qquad \chi(J) = -\frac{dI_r}{dJ}$$

- We can directly compute $I_r(J)$

$$\frac{I_r(b,\sigma)}{m_H m_L} = G\log(b) I^{(1)}_{0SF} + \frac{G^2}{b}\left[m_H I^{(2)}_{0SF} + m_L I^{(2)}_{1SF}\right] + \frac{G^3}{b^2}\left[m_H^2 I^{(3)}_{0SF} + m_H m_L I^{(3)}_{1SF} + m_L^2 I^{(3)}_{2SF}\right] + \cdots$$

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Organize by mass ratio:

$$\frac{I_r}{G m_H m_L} = \lambda^0 I_{0SF} + \lambda^1 I_{1SF} + \cdots$$

$$I_{nSF} = \left(\frac{Gm_H}{b}\right)^n I^{(n+1)}_{nSF} + \left(\frac{Gm_H}{b}\right)^{n+1} I^{(n+2)}_{nSF} + \cdots$$

RADIAL ACTION AT 0SF

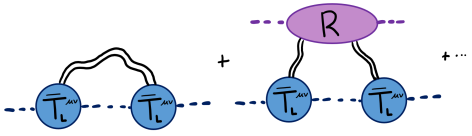
- 0SF **is** probe motion
- It's "solved" by computing geodesics

Using worldlines it's simple:
Solve for p_r from curved space on-shell condition, and integrate $I_r(J) = \int p_r dr$

$$I_{0SF} = -\frac{2\left(2\sigma^2-1\right)\left(\epsilon\log(b)-1\right)}{\sqrt{\sigma^2-1}\epsilon} + \frac{R_s}{b}\frac{3\pi\left(5\sigma^2-1\right)}{4\sqrt{\sigma^2-1}} + \frac{R_s^2}{b^2}\frac{\left(64\sigma^6-120\sigma^4+60\sigma^2-5\right)}{3\left(\sigma^2-1\right)^{5/2}} + \mathcal{O}((R_s/b)^3)$$

RADIAL ACTION AT 1SF

To all orders in G, I_{1SF} is a sum of recoil operator insertions into a self-energy diagram:



- Can evaluated in PM by expanding about flat space:
  $\bar{x}_L = b + u_L\tau + x^{(1)} + x^{(2)} + \cdots$
  $\mathscr{D} = D + D\bar{R}D + D\bar{R}D\bar{R}D + \cdots$
- 2PM (3PM) integrand needs just $x^{(0)}$ ($x^{(1)}$)

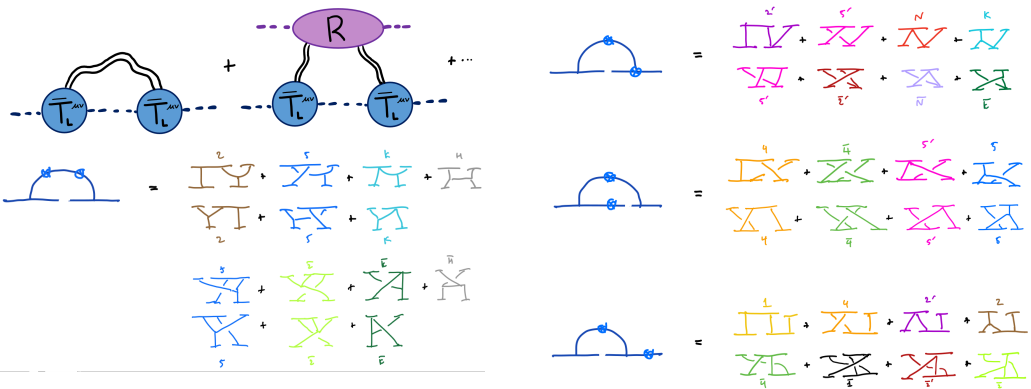
$$I_{1SF,s.e.} = \frac{i}{8}\int_{\ell_1\ell_2} d\tau_1 d\tau_2\, e^{-i\ell_1\bar{x}(\tau_1)-i\ell_2\bar{x}(\tau_2)}\,\dot{\bar{x}}^{\mu}(\tau_1)\dot{\bar{x}}^{\nu}(\tau_1)\dot{\bar{x}}^{\lambda}(\tau_2)\dot{\bar{x}}^{\rho}(\tau_2)\mathscr{D}_{\mu\nu\lambda\rho}(\ell_1,\ell_2)$$

$$I_{1SF,rec.} = \frac{m_H}{8}\int_{\ell_1\ell_2\ell_3\ell_4} d\tau_1 d\tau_2\, e^{-i\ell_1\bar{x}(\tau_1)-i\ell_2\bar{x}(\tau_2)}\,\dot{\bar{x}}^{\mu}(\tau_1)\dot{\bar{x}}^{\nu}(\tau_1)\dot{\bar{x}}^{\lambda}(\tau_2)\dot{\bar{x}}^{\rho}(\tau_2)u^a u^b u^c u^d$$
$$\times\, \ell_{3\,e}\ell_{4\,g}\eta^{fh}\Gamma^{g\,\alpha\beta}_{h\,ab}\Gamma^{e\,\sigma\delta}_{f\,cd}\frac{\hat{\delta}(ul_3+u\ell_4)}{(ul_3)^2}\mathscr{D}_{\mu\nu\alpha\beta}(\ell_1,\ell_4)\mathscr{D}_{\lambda\rho\sigma\delta}(\ell_2,\ell_3)$$

RADIAL ACTION AT 1SF

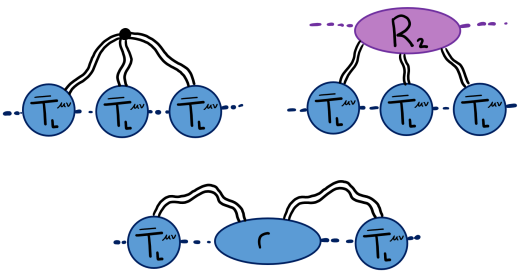
To all orders in G, I_{1SF} is a sum of recoil operator insertions into a self-energy diagram:

*Extra R insertions vanish in potential region



RADIAL ACTION AT 2SF

To all orders in G, I_{2SF} is a sum of three diagrams types:



- Also sum over recoil operator insertions
- At 3PM need only $x^{(0)}$ and first two types

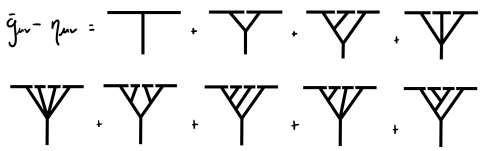
Similar SF graphs have appeared in literature before, but for us:

- It's an EFT $\Longrightarrow$ divergences are systematically understood
- No horizon $\Longrightarrow$ no horizon boundary condition matching

THE POWER OF EMREFT: SKELETONIZING GRAVITON TREES

- Simplification: Worldline EFT manifests matter line cuts

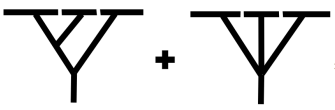
Worldlines+background field method **resums** subgraphs with consecutive cuts



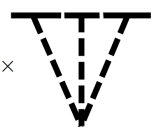
$$= A(r)\eta_{\mu\nu} + (B(r) - A(r))u_\mu u_\nu$$

$$A(r) = \left(\frac{2r+Gm_H}{2r}\right)^4 \quad B(r) = \left(\frac{2r+Gm_H}{2r-Gm_H}\right)^2$$

$$r = \sqrt{(x\cdot u)^2 - x^2}$$



$$= 2(4\pi G m_H)^3 \left(u_\mu u_\nu - \frac{1}{4}\eta_{\mu\nu}\right)\delta(u\cdot q)\times$$



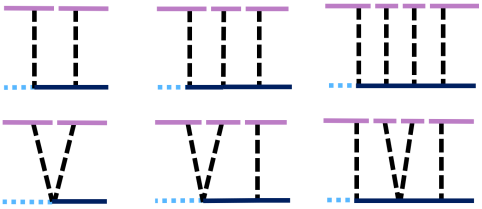
THE POWER OF EMREFT: SKELETONIZING PERTURBATIVE GEODESIC MOTION

Perturbative probe motion has matter propagator structures such as:

- We instead take exact geodesics $\bar{x}_L(\tau)$, unpack τ dependence.
- Number of indep. denominators is reduced, and numerators have been partially tensor reduced.

eg.

$$\begin{aligned}\operatorname{asinh}\hat{\tau} &= \frac{1}{\partial_{\hat{\tau}}^2}\partial_{\hat{\tau}}^2 \operatorname{asinh}\hat{\tau} = \frac{1}{\partial_{\hat{\tau}}^2}\left(-\frac{\hat{\tau}}{(1+\hat{\tau}^2)^{3/2}}\right) \\ &= \frac{1}{\partial_{\hat{\tau}}}\partial_{\hat{\tau}}\operatorname{asinh}\hat{\tau} = \frac{1}{\partial_{\hat{\tau}}}\left(\frac{1}{(1+\hat{\tau}^2)^{1/2}}\right) \\ &\sim \int_\ell e^{i\ell x_0}\frac{\delta(u_H\ell)}{u_L\ell}\frac{1}{\ell^2}\end{aligned}$$



UNPACKING TRAJECTORIES - I

EM

$$\hat{x}_0^\mu = \hat{b}^\mu + \frac{\hat{\tau}}{v}\check{u}_H^\mu + \frac{\hat{\tau}\sqrt{1-v^2}}{v}\check{u}_L^\mu$$

$$\hat{x}_1^\mu = \hat{\alpha}\left(\operatorname{asinh}\hat{\tau}\check{u}_H^\mu - \frac{\hat{r}_0(\hat{\tau})}{v}\hat{b}^\mu\right)$$

$$\hat{x}_2^\mu = \hat{\alpha}^2\left(\frac{\operatorname{asinh}\hat{\tau}}{v\hat{r}_0(\hat{\tau})}\check{u}_H^\mu + \frac{\hat{\tau}-v^2\operatorname{atan}\hat{\tau}}{2v\sqrt{1-v^2}}\check{u}_L^\mu - \frac{2\hat{\tau}\operatorname{asinh}\hat{\tau}+\hat{r}_0(\hat{\tau})\left(\hat{\tau}v^2\operatorname{atan}\hat{\tau}+v^2-1\right)}{2v^2\hat{r}_0(\hat{\tau})}\hat{b}^\mu\right)$$

GR *same functions of τ, just high v powers

$$\hat{x}_1^\mu = \frac{2Gm_H}{bv^2}\left(-\frac{\left(v^2+1\right)\hat{r}_0(\hat{\tau})}{2}\hat{b}^\mu + v\operatorname{asinh}\hat{\tau}\check{u}_H^\mu + \frac{v\left(v^2+1\right)\operatorname{asinh}\hat{\tau}}{2\sqrt{1-v^2}}\check{u}_L^\mu\right)$$

$$\hat{x}_2^\mu = \left(\frac{2Gm_H}{bv^2}\right)^2\left(\left[\frac{3v^3\operatorname{atan}\hat{\tau}}{2} - \frac{v\left(v^2-1\right)\operatorname{asinh}\hat{\tau}}{2\hat{r}_0(\hat{\tau})}\right]\check{u}_H^\mu + \left[\frac{\sqrt{1-v^2}v\left(v^2+1\right)\operatorname{asinh}\hat{\tau}}{4\hat{r}_0(\hat{\tau})} + \frac{2\hat{\tau}v\left(v^2+1\right)^2+3\left(v^2+4\right)v^3\operatorname{atan}\hat{\tau}}{16\sqrt{1-v^2}}\right]\check{u}_L^\mu\right.$$
$$\left. + \left[\frac{\hat{\tau}\left(v^4-1\right)\operatorname{asinh}\hat{\tau}}{4\hat{r}_0(\hat{\tau})} - \frac{3\hat{\tau}\left(v^2+4\right)v^2\operatorname{atan}\hat{\tau}+v^4+8v^2-2}{16}\right]\hat{b}^\mu\right)$$

UNPACKING TRAJECTORIES - II

GR - Time domain

$$\hat{x}_1^\mu = \frac{2Gm_H}{bv^2}\left(-\frac{\left(v^2+1\right)\hat{r}_0(\hat{\tau})}{2}\hat{b}^\mu + v\operatorname{asinh}\hat{\tau}\,\check{u}_H^\mu + \frac{v\left(v^2+1\right)\operatorname{asinh}\hat{\tau}}{2\sqrt{1-v^2}}\check{u}_L^\mu\right)$$

$$\hat{x}_2^\mu = \left(\frac{2Gm_H}{bv^2}\right)^2\left(\left[\frac{3v^3\operatorname{atan}\hat{\tau}}{2} - \frac{v\left(v^2-1\right)\operatorname{asinh}\hat{\tau}}{2\hat{r}_0(\hat{\tau})}\right]\check{u}_H^\mu + \left[\frac{\sqrt{1-v^2}v\left(v^2+1\right)\operatorname{asinh}\hat{\tau}}{4\hat{r}_0(\hat{\tau})} + \frac{2\hat{\tau}v\left(v^2+1\right)^2+3\left(v^2+4\right)v^3\operatorname{atan}\hat{\tau}}{16\sqrt{1-v^2}}\right]\check{u}_L^\mu + \left[\frac{\hat{\tau}\left(v^4-1\right)\operatorname{asinh}\hat{\tau}}{4\hat{r}_0(\hat{\tau})} - \frac{3\hat{\tau}\left(v^2+4\right)v^2\operatorname{atan}\hat{\tau}+v^4+8v^2-2}{16}\right]\hat{b}^\mu\right)$$

GR - Packaged for fourier transform

$$\hat{x}_1^\mu = \frac{2Gm_H}{bv^2}\left(-\frac{1}{2}\left(v^2+1\right)\frac{1}{\partial_{\hat{\tau}}^2}\left[\frac{\hat{b}^\mu}{\hat{r}_0^3}\right] + \left(v\check{u}_H^\mu + \frac{v\left(v^2+1\right)\check{u}_L^\mu}{2\sqrt{1-v^2}}\right)\frac{1}{\partial_{\hat{\tau}}}\left[\frac{1}{\hat{r}_0}\right]\right)$$

$$\hat{x}_2^\mu = \left(\frac{2Gm_H}{bv^2}\right)^2\left(\frac{1}{16}\left(v^4-12v^2-4\right)\frac{1}{\partial_{\hat{\tau}}}\left[\frac{\hat{b}^\mu}{\hat{r}_0}\;\frac{\hat{\tau}}{\hat{r}_0}\right] + \frac{\left(v^2+1\right)^2 v\check{u}_L^\mu}{8\sqrt{1-v^2}}\frac{1}{\partial_{\hat{\tau}}}\left[\frac{1}{\hat{r}_0}\;\hat{r}_0\right] + \frac{1}{4}\left(v^4-1\right)\frac{1}{\partial_{\hat{\tau}}}\left[\frac{\hat{b}^\mu\frac{1}{\partial_{\hat{\tau}}}\left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0^3}\right]\right.$$

$$-\frac{3}{16}\left(v^2+4\right)v^2\frac{1}{\partial_{\hat{\tau}}^2}\left[\frac{\hat{b}^\mu}{\hat{r}_0^2}\right] + \frac{1}{4}v\left(2\left(v^2-1\right)\check{u}_H^\mu - \sqrt{1-v^2}\left(v^2+1\right)\check{u}_L^\mu\right)\frac{1}{\partial_{\hat{\tau}}}\left[\frac{\hat{\tau}\frac{1}{\partial_{\hat{\tau}}}\left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0^3}\right]$$

$$\left.+\frac{1}{16}v\left(8\left(2v^2+1\right)\check{u}_H^\mu + \frac{\left(-v^4+12v^2+4\right)\check{u}_L^\mu}{\sqrt{1-v^2}}\right)\frac{1}{\partial_{\hat{\tau}}}\left[\frac{1}{\hat{r}_0^2}\right]\right)$$

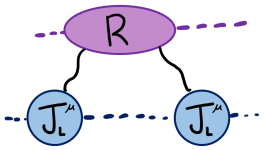
COMPUTATIONS/CHECKS

1SF SCALAR QED - I

Scalar QED with fixed charge-to-mass ratios z_H, z_L, and scale $\lambda = m_L/m_H \ll 1$.

- Theory has pert expansion in $(\alpha m_H z_H^2)/b$
- Theory also has SF expansion in λ
  - Radial action has only recoil diagrams

$$S_{1SF} = -\int d^D x \delta A_\mu \bar{J}_L^\mu - \frac{1}{4}\delta F_{\mu\nu}\delta F^{\mu\nu}$$
$$-\frac{z_H^2 m_H}{2}\int d\tau u^\mu \delta F_{\mu\nu}\frac{1}{\partial_\tau^2}\delta F^{\nu\rho}u_\rho$$



$$I_{1\mathrm{SF}} = \frac{z_H^2 z_L^2 m_H}{2} N_{\mu\nu}{}^{\alpha\beta}\int_{\ell_1\ell_2} d\tau_1 d\tau_2 e^{-i\ell_1\bar{x}_L(\tau_1)-i\ell_2\bar{x}_L(\tau_2)}\dot{\bar{x}}_L^\mu(\tau_1)\dot{\bar{x}}_L^\nu(\tau_2)\hat{\delta}(u\ell_1+u\ell_2)\frac{\ell_{1\,\alpha}\ell_{2\,\beta}}{\ell_1^2\ell_2^2(u\ell_1)^2}$$

1SF SCALAR QED - II

1SF Radial action integrand to (5PC) 4 loops:

$$I_{1\rm SF} = \frac{z_H^2 z_L^2 m_H}{2} N_{\mu\nu}{}^{\alpha\beta} \int_{\ell_1 \ell_2} \bar{J}_L^{\mu}(\ell_1) \bar{J}_L^{\nu}(\ell_2) \hat{\delta}(u\ell_1 + u\ell_2) \frac{\ell_{1\,\alpha} \ell_{2\,\beta}}{\ell_1^2 \ell_2^2 (u\ell_1)^2}$$

$$\bar{J}_L^{\mu}(\ell_1) = \int d\tau e^{-i\ell(b+u_L\tau)} e^{-i\ell(\bar{x}^{(1)}+\bar{x}^{(2)}+\bar{x}^{(3)}+\ldots)} \left(u_L + \dot{x}^{(1)\mu} + \dot{x}^{(2)\mu} + \dot{x}^{(3)\mu} + \ldots\right)$$

$$\bar{J}_L \bar{J}_L = J^{(0)} J^{(0)} + 2J^{(0)} J^{(1)} + \left(2J^{(0)} J^{(2)} + J^{(1)} J^{(1)}\right) + 2\left(J^{(0)} J^{(3)} + J^{(1)} J^{(2)}\right) + \left(2J^{(1)} J^{(3)} + J^{(2)} J^{(2)}\right)$$

$$\hat{x}_1^{\mu} = \hat{\alpha}\left(\check{u}_H^{\mu} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0}\right] - \frac{1}{v} \frac{1}{\partial_{\hat{\tau}}^2} \left[\frac{\hat{b}^{\mu}}{\hat{r}_0^3}\right]\right)$$

$$\hat{x}_2^{\mu} = \hat{\alpha}^2\Bigg(\left(-\frac{1}{v^2} - \frac{1}{2}\right) \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{b}^{\mu}}{\hat{r}_0} \frac{\hat{\tau}}{\hat{r}_0}\right] + \frac{\check{u}_L^{\mu}}{2v\sqrt{1-v^2}} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0} \hat{r}_0\right] - \frac{1}{v^2} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{b}^{\mu} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0^3}\right]$$
$$-\frac{1}{2} \frac{1}{\partial_{\hat{\tau}}^2} \left[\frac{\hat{b}^{\mu}}{\hat{r}_0^3}\right] + \left(\frac{\check{u}_L^{\mu}}{v} - \frac{v\check{u}_L^{\mu}}{2\sqrt{1-v^2}}\right) \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0^2}\right] - \frac{\check{u}_L^{\mu}}{v} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{\tau} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0^3}\right]\Bigg)$$

$$\hat{x}_3^{\mu} = \hat{\alpha}^3\Big(-\frac{\left(3v^2+1\right)}{2v^3} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{b}^{\mu}}{\hat{r}_0} \frac{\hat{\tau}}{\hat{r}_0^2}\right] + \frac{\left(1-2v^2\right)}{2v^3} \frac{1}{\partial_{\hat{\tau}}} \left[\hat{b}^{\mu} \hat{r}_0 \frac{\hat{\tau}}{\hat{r}_0^2}\right] + \frac{\left(v^2+1\right)\check{u}_L^{\mu}}{2v^2\sqrt{1-v^2}} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0^2} \hat{r}_0\right] - \frac{1}{v^3} \frac{(\hat{b}^{\mu} + \hat{\tau} v \check{u}_H^{\mu})}{\hat{r}_0^3} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0}\right] - \frac{\left(v^2+2\right)}{v^3} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{b}^{\mu} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0^4}\right] + \frac{1}{v^3} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{b}^{\mu} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0^2}\right]$$
$$-\frac{1}{2v} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{b}^{\mu} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0^2}\right]}{\hat{r}_0}\right] + \left(\frac{\check{u}_L^{\mu}}{\sqrt{1-v^2}} - \frac{2\check{u}_H^{\mu}}{v^2}\right) \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{\tau} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0}\right]}{\hat{r}_0^3}\right] + \frac{\check{u}_L^{\mu}}{2\sqrt{1-v^2}} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{\hat{\tau} \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0^2}\right]}{\hat{r}_0}\right] + \left(\frac{1}{2}\left(\frac{1}{v^2}+1\right) \check{u}_H^{\mu} - \frac{\check{u}_L^{\mu}}{\sqrt{1-v^2}}\right) \frac{1}{\partial_{\hat{\tau}}} \left[\frac{1}{\hat{r}_0^3}\right]\Big)$$

1SF SCALAR QED - III

Integrated electric 1SF Radial action to 3PC

$$I_{1SF} = \lambda m_L r_c \frac{r_c}{b} \frac{\pi}{2\sqrt{\sigma^2-1}} - \lambda m_L r_c \left(\frac{r_c}{b}\right)^2 \frac{2\sigma^4-6\sigma^2+6}{3(\sigma^2-1)^{5/2}} \qquad r_c = -\frac{q_H q_L}{4\pi m_L}$$

*matches Bern et al. JHEP08(2022)131

1SF SCALAR QED - III

Integrated electric 1SF Radial action to 3PC

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*matches Bern et al. JHEP08(2022)131

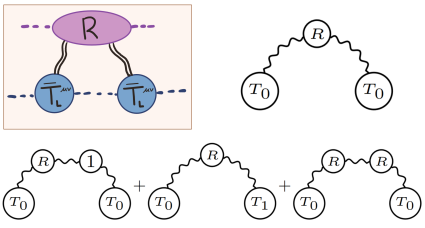
Integrated **dyonic** 1SF Radial action to 3PC $\quad (q_i \to q_i \cos\theta_i\,,\ \ g_i \to q_i \sin\theta_i)$

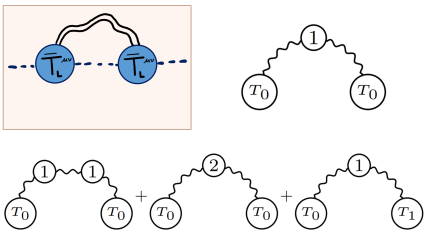
$$I_{1SF} = \lambda m_L r_c \frac{r_c}{b} \frac{\pi\left(\cos^2\theta_H+\sin^2\theta_H\right)\left(\cos^2\theta_L+\sin^2\theta_L\right)}{2\sqrt{\sigma^2-1}} - \lambda m_L r_c \left(\frac{r_c}{b}\right)^2 \frac{\cos\Delta\theta\left(\sigma^2\cos 2\Delta\theta+2\sigma^4-7\sigma^2+6\right)}{3(\sigma^2-1)^{5/2}}$$

*2PM checked with Kol, O'Connell, Telem JHEP03(2022)141

3PM is new

1SF GRAVITY



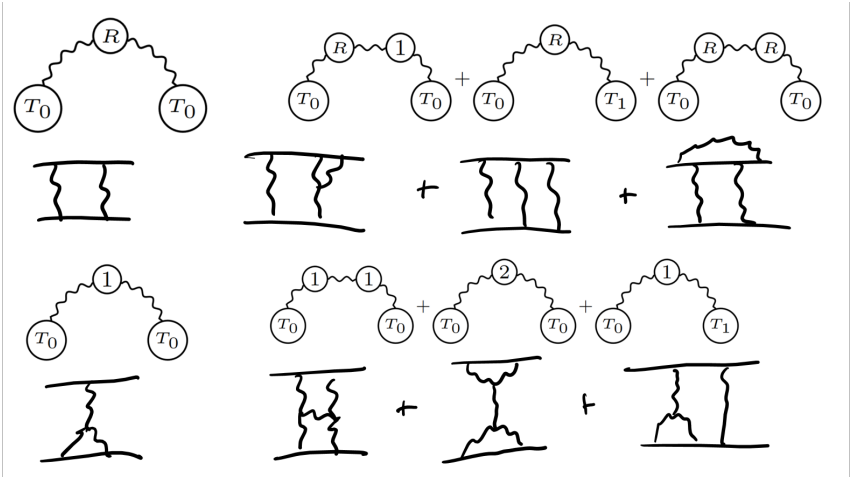


All of the 1SF diagrams to 3PM order.

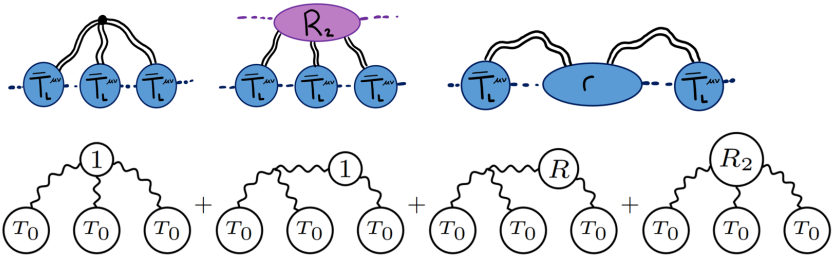
$$I_{1SF} = \lambda m_L R_g \frac{R_g}{b} \frac{3\pi\left(5\sigma^2-1\right)}{4\sqrt{\sigma^2-1}} + \lambda m_L R_g \frac{R_g^2}{b^2}\left(\frac{2\sigma\left(36\sigma^6-114\sigma^4+132\sigma^2-55\right)}{3\left(\sigma^2-1\right)^{5/2}} - \frac{4\left(4\sigma^4-12\sigma^2-3\right)\cosh^{-1}(\sigma)}{\sigma^2-1}\right)$$

*Agrees with Bern et al PRL 122, 201603 (2019)

1SF GRAVITY



2SF



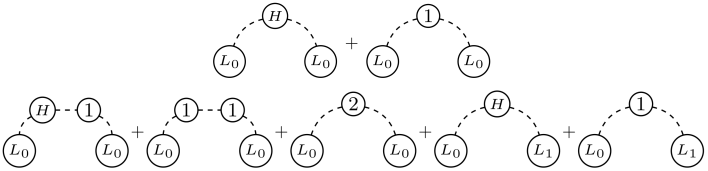
- **At 3PM:** Highly non-trivial check of EMREFT ($2SF = 0SF$)

$$I_{2SF}^{EM} = \lambda^2 m_L r_c \left(\frac{r_c}{b}\right)^2 \frac{\sigma\left(3-2\sigma^2\right)}{3\left(\sigma^2-1\right)^{5/2}}$$

$$I_{2SF}^{Grav.} = \lambda^2 m_L R_g \frac{R_g^2}{b^2} \frac{\left(64\sigma^6-120\sigma^4+60\sigma^2-5\right)}{3\left(\sigma^2-1\right)^{5/2}}$$

NEW RESULTS WITH ADDITIONAL FIELDS

DIAGRAMS AND GENERAL STRATEGY



Generalized recoil operator has mixing terms.

$$S_{\rm recoil} = -\frac{m_H}{2}\int d\tau\, \delta f^\mu(\bar{x}_H)\frac{1}{\partial_\tau^2}\delta f_\mu(\bar{x}_H)$$

$$\delta f^\mu_{\rm E.\text{-}M.} = u^\alpha u^\beta \delta\Gamma^\mu_{\alpha\beta}(\bar{x}_H) - \eta_H u^\alpha \delta F^\mu_\alpha(\bar{x}_H)$$

$$\begin{aligned}\delta f^\mu_{\rm EdGB} &= u^\alpha u^\beta \delta\Gamma^\mu_{\alpha\beta}(\bar{x}_H) \\ &\quad - d_H(u^\mu u^\alpha - \tfrac{1}{2}\eta^{\mu\alpha})\partial_\alpha \delta\varphi(\bar{x}_H)\end{aligned}$$

Models:

- Scalar and vector charge on light body
- Vector charge on both
- $(\nabla\varphi)^2 + \alpha_{GB}\varphi R_{abcd}R^{abcd}$
- $(\nabla\vartheta)^2 + \alpha_{dCS}\vartheta R_{abcd}\tilde{R}^{abcd}$
- $c_6 R_{ab}{}^{cd} R_{cd}{}^{ef} R_{ef}{}^{ab}$

RESULTS: REISSNER-NORDSTRÖM SCATTERING

$$\frac{I_r}{Gm_Hm_L} = \log(b)I_{0,1}^{H,L} + \frac{G}{b}(m_H I_{0,2}^{H,L} + m_L I_{0,2}^{L,H}) + \frac{G^2}{b^2}(m_H^2 I_{0,3}^{H,L} + m_L^2 I_{0,3}^{L,H} + m_H m_L I_{1,3}^{H,L}) + \mathcal{O}(G^4)$$

$$I_{0,1}^{H,L} = \frac{2}{(\sigma^2-1)^{1/2}}\left(\eta_H\eta_L\sigma - 2\sigma^2 + 1\right) \qquad I_{0,2}^{H,L} = \frac{\pi}{4(\sigma^2-1)^{1/2}}\left(2\eta_H^2\eta_L^2 + \eta_H^2(1-3\sigma^2) - 12\eta_H\eta_L\sigma + 15\sigma^2 - 3\right)$$

$$I_{0,3}^{H,L} = \frac{1}{3\left(\sigma^2-1\right)^{5/2}}\left(\eta_H^3\eta_L^3\sigma\left(3-2\sigma^2\right) + 3\eta_H^3\eta_L\sigma\left(4\sigma^4-7\sigma^2+3\right) + 3\eta_H^2\eta_L^2\left(8\sigma^4-12\sigma^2+3\right)\right.$$
$$\left. + 3\eta_H^2\left(-8\sigma^6+16\sigma^4-9\sigma^2+1\right) + 3\eta_H\eta_L\sigma\left(-24\sigma^4+40\sigma^2-15\right) + 64\sigma^6 - 120\sigma^4 + 60\sigma^2 - 5\right)$$

$$I_{1,3}^{H,L} = \frac{1}{3(\sigma^2-1)^{5/2}}\left(2\eta_H^3\eta_L^3\left(\sigma^4-3\sigma^2+3\right) + \eta_H\eta_L\left(\eta_H^2+\eta_L^2\right)\left(8\sigma^6-6\sigma^4-9\sigma^2+7\right) + 6\eta_H^2\eta_L^2\sigma\left(4\sigma^4-4\sigma^2-1\right)\right.$$
$$+ \eta_H\eta_L\left(-64\sigma^6+96\sigma^4-42\sigma^2+16\right) + \left(\eta_H^2+\eta_L^2\right)\left(-8\sigma^7+36\sigma^5-51\sigma^3+23\sigma\right) + 2\sigma\left(36\sigma^6-114\sigma^4+132\sigma^2-55\right)\Big)$$
$$+ \frac{\operatorname{arccosh}(\sigma)}{\sigma^2-1}\left(\eta_H^2\eta_L^2\left(4-8\sigma^2\right) + 16\eta_H\eta_L\sigma\left(\sigma^2-2\right) - 4\left(2\sigma^2+1\right)\left(\eta_H^2+\eta_L^2\right) - 16\sigma^4 + 48\sigma^2 + 12\right).$$

RESULTS: "TESTING GR" SCATTERING

$$
\begin{aligned}
I_{1,3}^{H,L} = &-\frac{R_s^2}{b^2}\Bigg(\frac{\sigma\left(2\sigma^6\left(d_H^2+d_L^2\right)-3\sigma^4\left(d_H^2+10d_Hd_L+d_L^2\right)+d_H^2+42d_Hd_L\sigma^2-9d_Hd_L+d_L^2\right)}{24\left(\sigma^2-1\right)^{5/2}} \\
&\qquad + d_Hd_L\cosh^{-1}(\sigma)\Bigg) \\
&+\frac{\alpha_{GB}(d_H+d_L)R_s^2}{b^4}\left(\frac{\sigma\left(16\sigma^4-22\sigma^2-243\right)}{6\left(\sigma^2-1\right)^{3/2}}+\frac{\left(-8\sigma^4+38\sigma^2+10\right)\cosh^{-1}(\sigma)}{\left(\sigma^2-1\right)^2}\right) \\
&+\frac{\alpha_{GB}^2R_s^2}{b^6}\left(-\frac{16\sigma\left(44\sigma^6-94\sigma^4+356\sigma^2+351\right)}{3\left(\sigma^2-1\right)^{5/2}}+\frac{16\left(8\sigma^8-8\sigma^6+36\sigma^4+166\sigma^2+17\right)\cosh^{-1}(\sigma)}{\left(\sigma^2-1\right)^3}\right) \\
&+\frac{\alpha_{dCS}^2R_s^2}{b^6}\left(\frac{2\sigma\left(22\sigma^6-47\sigma^4+124\sigma^2+126\right)}{3\left(\sigma^2-1\right)^{5/2}}-\frac{2\left(4\sigma^8-4\sigma^6+9\sigma^4+60\sigma^2+6\right)\cosh^{-1}(\sigma)}{\left(\sigma^2-1\right)^3}\right) \\
&-\frac{c_6R_s^2}{b^6}\left(-\frac{32\sigma\left(2\sigma^6-7\sigma^4+2\sigma^2-42\right)}{\left(\sigma^2-1\right)^{5/2}}-\frac{288\left(4\sigma^2+1\right)\cosh^{-1}(\sigma)}{\left(\sigma^2-1\right)^3}\right)
\end{aligned}
$$

SUMMARY

- New EFT for Extreme Mass Ratios, tailored for classical physics
- Leverages background field method and exact solutions
- 1SF: Background field + “recoil operator” localized on heavy body
- Radial action computed and checked at 1SF and 2SF through 2 loops for scalar QED w/ dyons and gravity w/ extra fields + higher curvature

Next steps:

- Add finite size effects: spin, tides, absorption
- Classical double copy: Coulomb/Schwarzschild, $\sqrt{\text{Kerr}}$/Kerr, Dyon/ Taub-NUT
- Utilizing known quasinormal mode eigenfunctions
- Numerical integration
- Radiation/waveforms