Regge Trajectories from Pion Scattering

Alessandro Vichi

w/ B.McPeak, M.Venuti (2310.06888)
J.Albert, J.Henriksson, L.Rastelli (in progress)

QCD meets Gravity
December 11, 2023

INFN
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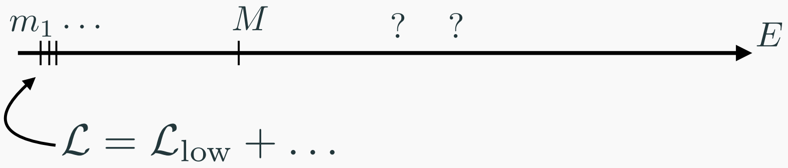
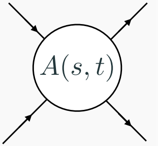
Effective Field theories

Consider a theory with a scale separation



In weakly coupled EFTs the $2 \to 2$ scattering amplitudes is



$\longrightarrow \begin{cases} \text{EFT: poles + integer powers in s,t} & IR \\ \text{agnostic: expand in partial waves} & UV \end{cases}$

Scattering Amplitudes - generalities

$$IR: \quad \mathcal{A}(s,t) = g_{0,1}(s+t) + g_{1,1}(s+t)(st) + g_{1,0}ts + g_{0,2}(s+t)^2 + \dots$$

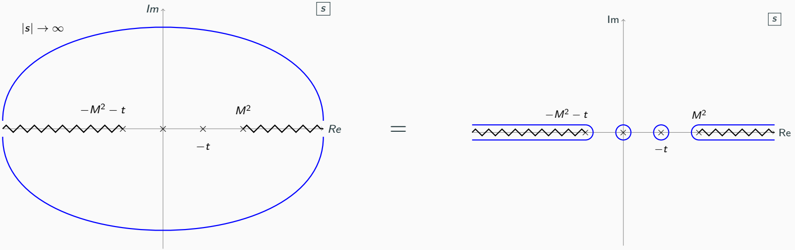
$$UV: \quad \mathcal{A}(s,t) = \sum_{J \text{ even}} n_J^{(d)} f_J(s) \mathcal{P}_J\left(1+\tfrac{2t}{s}\right), \qquad n_J^{(d)} : \text{normalizations}$$

Standard assumptions:

- Analyticity;
- Regge boundedness: $\lim_{|s|\to\infty} |\frac{\mathcal{A}(s,t)}{s^2}| = 0$ (fixed $t<0$);
- Weakly coupled theory (i.e. no cuts) below cut-off M;

Dispersion relations - review

$$\oint_\infty \frac{ds}{2\pi i s} \frac{\mathcal{A}(s,t)}{s^k (s+t)^\ell} = 0$$



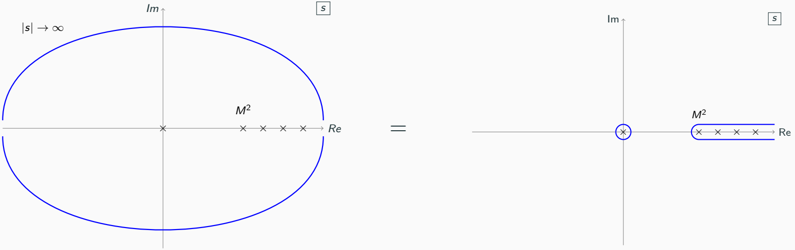
$$\Rightarrow \qquad \text{low energy data} = \mathcal{F}[\text{high energy data}]$$

Pion scattering at large-N ($\mathcal{N}_f = 2$)

$$\pi^a \pi^b \longrightarrow \pi^c \pi^d$$

Large-N limit:

- Three isospin channels $I = 0, 1, 2$ described by a single function $\mathcal{A}(s,t)$
- Only $\bar{q}q$-mesons contribute: poles only for $I = 0, 1$.
- $\mathcal{A}(s,t)$ has only poles on the real positive $s-$axis. (fixed t)
- Pomeron suppressed at large-N: Regge intercept ~ 0.5



Sum rules and null constraints

$$\left.\begin{array}{l} g_{0,1} = \langle \frac{1}{m^2} \rangle \\ \\ g_{1,1} = \dots \end{array}\right\} \leftarrow \text{sum rules} \qquad g_{0,1} > 0 \; (\sim \text{"central charge"})$$

$$\left.\begin{array}{l} 0 = \langle \frac{(J-2)J(J+1)(J+3)}{m^6} \rangle \\ \\ 0 = \dots \end{array}\right\} \leftarrow \text{null constraints } \mathcal{X}_{k,\ell}$$

Notation:

$$\langle F(m^2, J) \rangle \equiv \sum_{J \text{ even}} n_J^{(d)} \int_{M^2}^{\infty} \frac{dm^2}{\pi} \frac{m^{4-d}}{m^2} \rho_J(m^2) \left[F(m^2, J)\right].$$

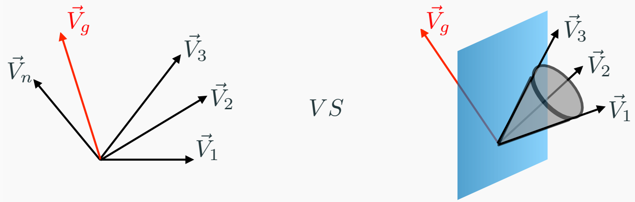
Unitarity $\Rightarrow \rho_J(s) \geq 0$

Bootstrap equations

Schematic form of equations:

$$\underbrace{\begin{pmatrix} g_{0,1} \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\color{red}{\vec{V}_g}} = \sum_X g^2_{\pi\pi X} \underbrace{\begin{pmatrix} \dots \\ \mathcal{X}_{3,1} \\ \dots \\ \mathcal{X}_{4,1} \\ \dots \end{pmatrix}}_{\vec{V}_X}$$

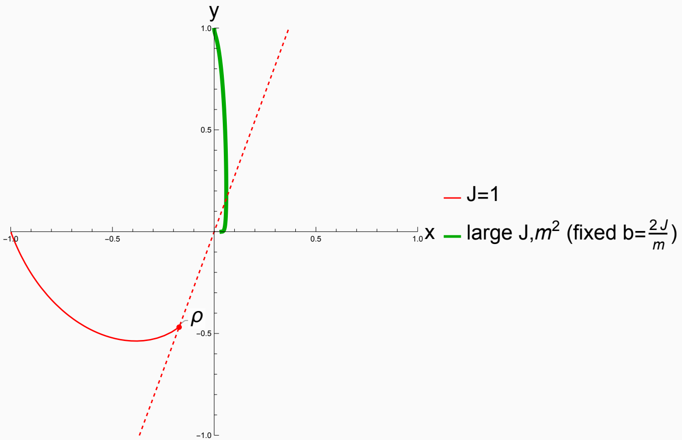
($X \equiv$ quantum numbers of states exchanged in $\pi\pi \to \pi\pi$)



Feasibility can be recast in a semi-definite positive problem and tested numerically

Consider two particular combinations of null constraints

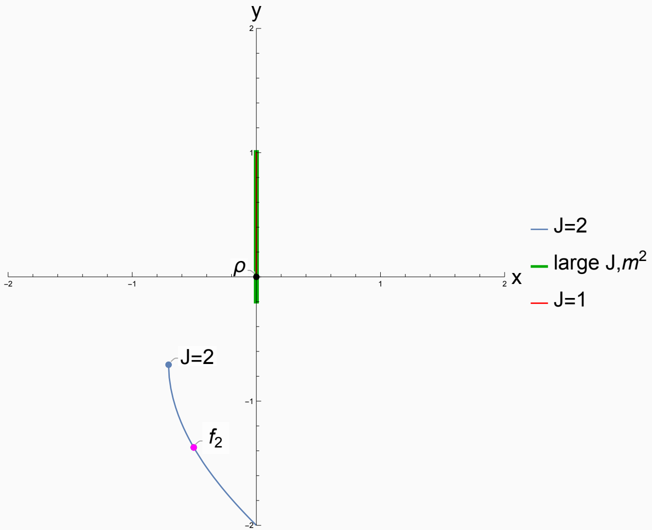
$$x : \vec{n}_1 \cdot \vec{V}_J(m^2)\,, \qquad y : \vec{n}_2 \cdot \vec{V}_J(m^2)$$



- Spin-1 alone are inconsistent
- A Spin-1 can be "fixed" by adding resonances at ∞

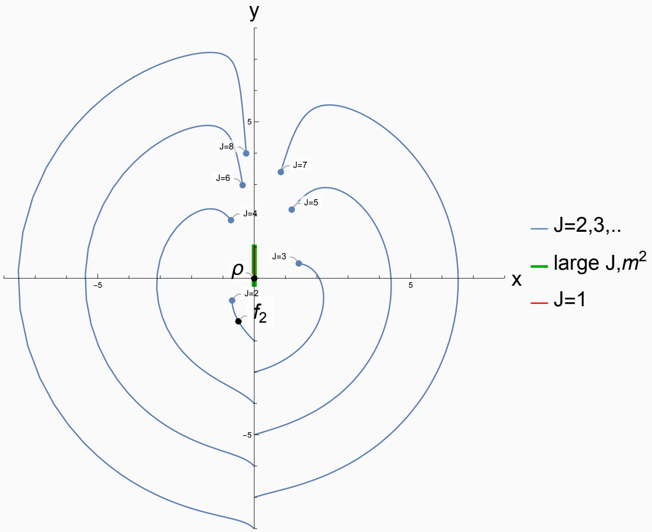
Geometry of null constraints - 2

Consider a *different* combination



- A Spin-2 cannot be "fixed" by adding resonances at ∞ or vectors

Geometry of null constraints - 2



- A Spin-2 cannot be "fixed" by adding resonances at ∞ or vectors
- A Spin-2 requires higher (odd) spins

General lessons

- $J = 1$ must be present at finite mass
- $J \geq 2$ do not need to be present at finite mass
- If we introduce a $J > 1$ spin, it must come with a whole tower of states of increasing spin ($\sim$ *CEMZ*)

Intermezzo: importance of Regge behavior

Complex scalar+gravity+gauge

In absence of gravity one can obtain "forward sum rules"

$$g_{0,2} \sim < \frac{1}{m^4} > \qquad \text{positive!}$$

Graviton exchange
$\Rightarrow$ $1/t$ pole in the 2-Subtracted Dispersion Relation (2SDR), regular in 1SDR

Photon exchange
$\Rightarrow$ $1/t$ pole in 1SDR, doesn't contribute to 2SDR

Loss of positivity in 2SDR: [Caron-Huot,Mazac,Rastelli,Simmons-Duffin '21]

$$g_{0,2} \geq -\# G \,,$$

Adding subtractions

- Gravity implies softer Regge behaviour ⇒ can impose 1SDRs [Haring,Zhiboedov '22]
- Problem: 1SDR are not sign definite at large mass, spin
  (charge 0,1 and charge 2 contribute in opposite ways)
- Generically this would make 1SDR useless.
- Let us restrict to (large-N)-like theories and assume (t-channel dominance)

$$\rho_J^{Q=2}(s) = 0 \qquad \text{no states in charge 2-sector}$$

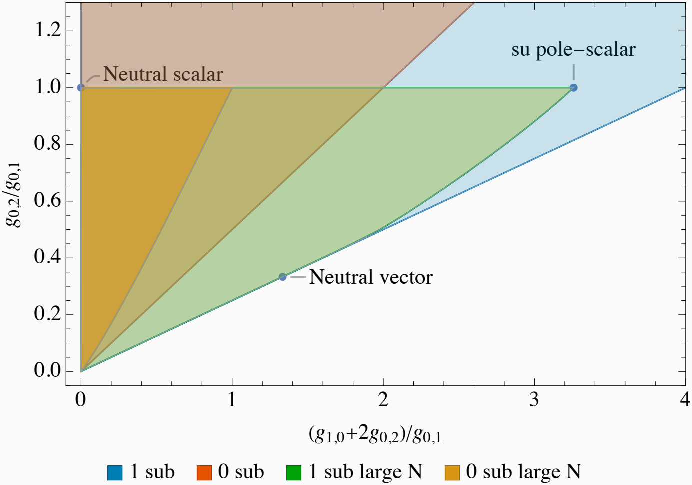
- with this assumption 1SDR can be used:

$$g_{0,2} \geq -\# e^2 \quad \text{(independent of } G\text{!)}$$

  In the limit $e^2 \to 0$ we recover positivity!
  [McPeak, Venuti, AV '23]

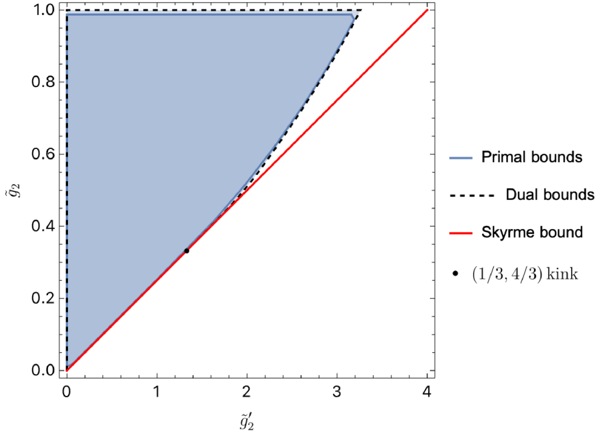
Back to pions

Regge behaviour & subtractions



[Albert, Rastelli '22] [Fernandez, Pomarol, Riva, Sciotti '22]

[McPeak, Venuti, AV '23]

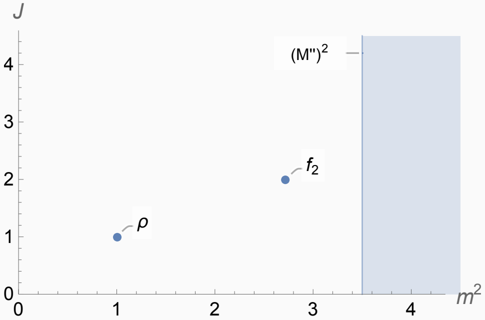


[Li '23] [Elias Miro, Guerrieri, Gumus '22]

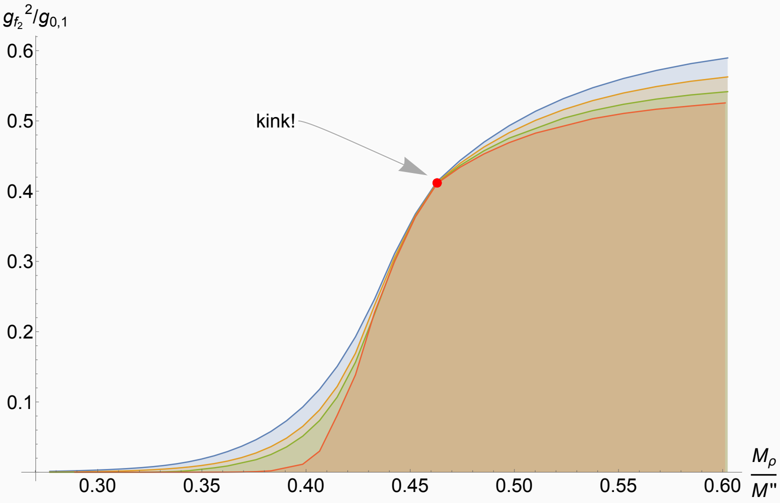
Forcing the f_2

Hot to force the presence of a spin $J \geq 2$ in the system?

Maximize the residue of a spin-2 resonance (aka f_2)

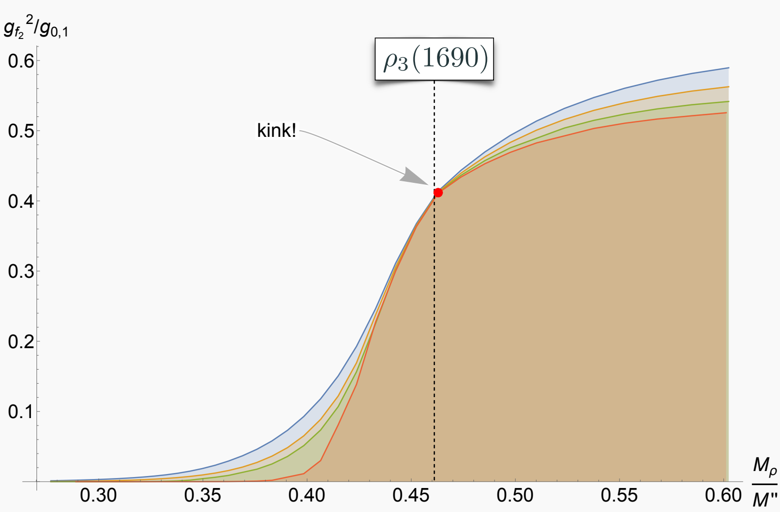


Forcing the f_2



(# null constraints = 56, 79, 106, 137)

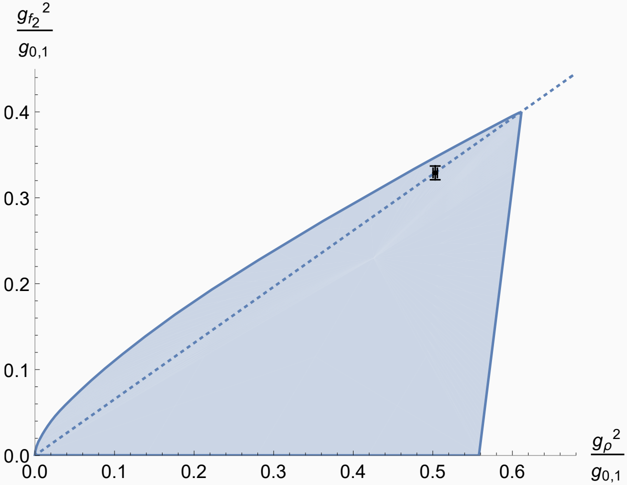
Forcing the f_2



(# null constraints = 56, 79, 106, 137)

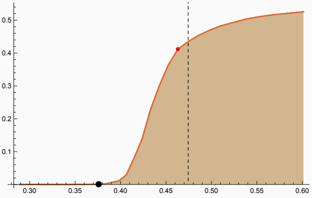
Can it be large-N QCD?

Fix the cut-off $M'' = M_{\text{kink}}$: $\rho-$coupling vs f_2-coupling

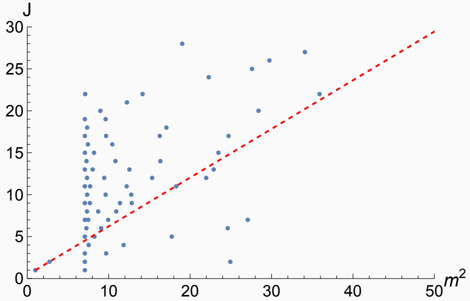


Ratio $\frac{g_{f_2}^2}{g_\rho^2}$ close to real word QCD!

Chew-Frautschi Plot

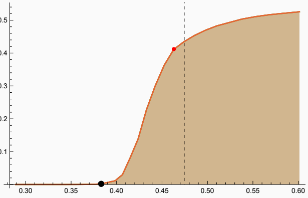


$$\frac{M_\rho}{M''} = 0.376$$

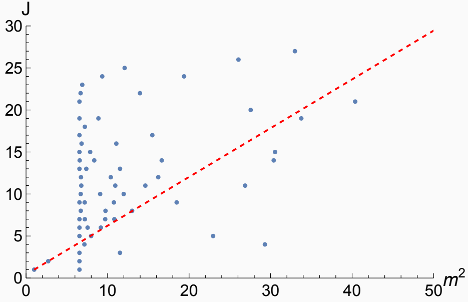


Kink seems to be related to the formation of a Regge trajectory

Chew-Frautschi Plot



$$\frac{M_\rho}{M''} = 0.391$$



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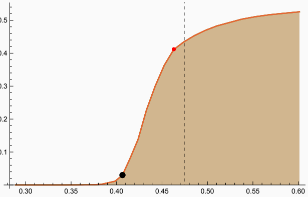
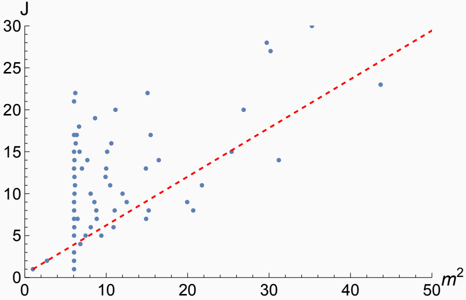


$$\frac{M_\rho}{M''} = 0.407$$



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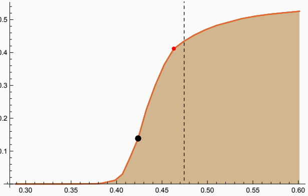
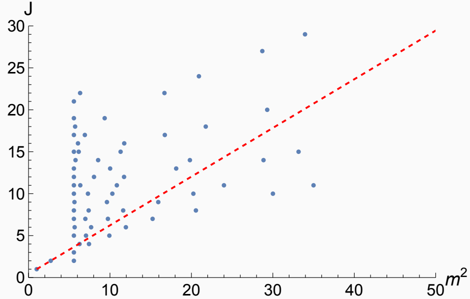


$$\frac{M_\rho}{M''} = 0.424$$



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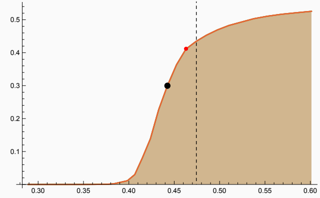
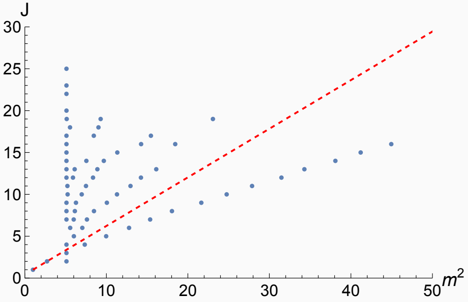


$$\frac{M_\rho}{M''} = 0.442$$



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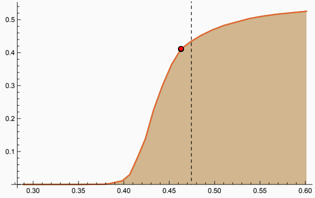
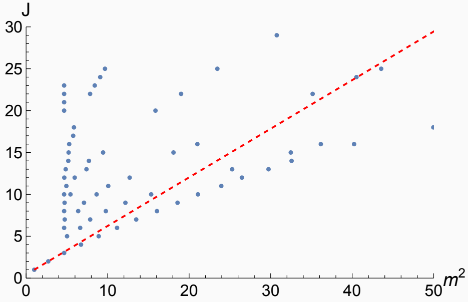


$\frac{M_\rho}{M''}=0.463$



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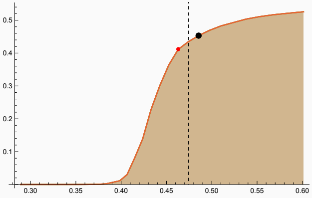
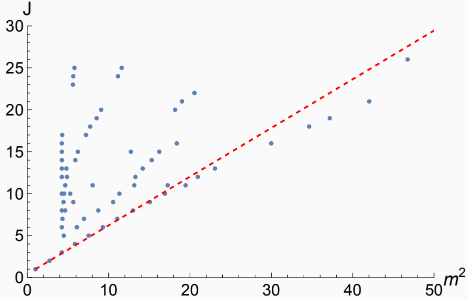


$$\frac{M_\rho}{M''} = 0.485$$



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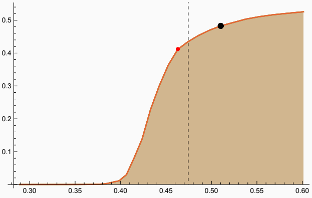
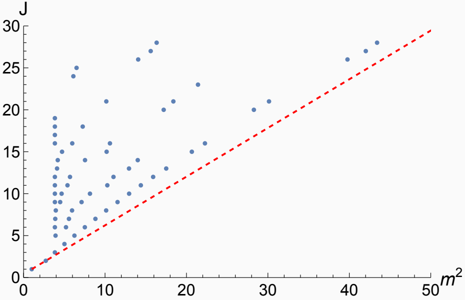


$$\frac{M_\rho}{M''}=0.510$$



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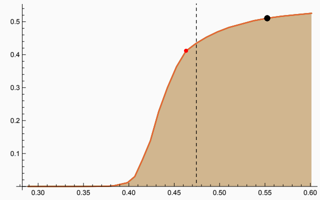
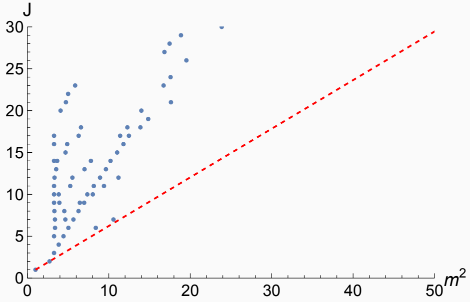


$$\frac{M_\rho}{M''} = 0.552$$



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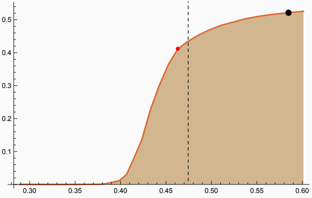
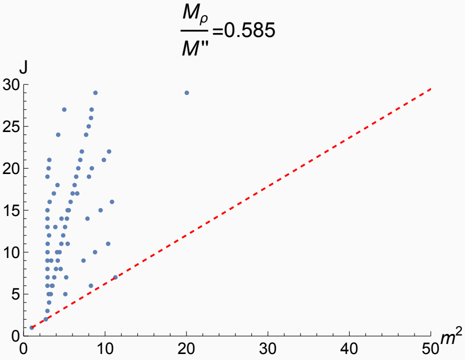
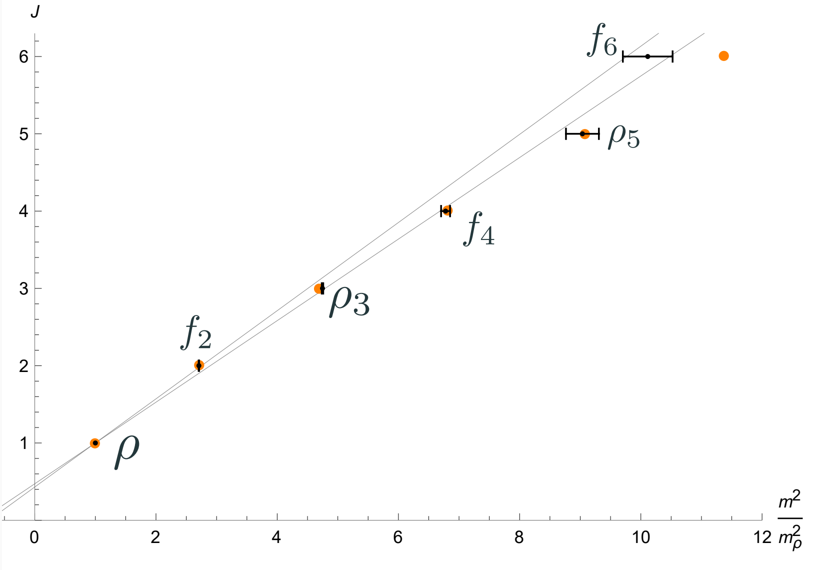




Kink seems to be related to the formation of a Regge trajectory

spectrum @ kink VS real world



Can it be large-N QCD?

	spectrum VS real-word	Asymptotically Linear Regge trajectories	daughter trajectories	degenerate ρ, f trajectories
Large-N QCD	?	expected	suppressed(?)	✓
Kink solution	✓	X	not seen	✓

Where do we go next?

Imposing the presence of higher spin resonances creates Regge-like trajectories

Similar kinks and spectra for $J = 3, 4$

Maximising the ratio $g_{f_2}^2/g_{0,1}$ pushes towards the right direction of parameter space. Perhaps the right answer is behind the corner?

To do better we need to impose more constraints $\Rightarrow$ mixed amplitudes

- mix π^a with the first scalar meson
- glueball scattering (also compare with simulations at large-N) [Guerrieri, Hebbar, van Rees '23] [Haring, Zhiboedov '23]
- mix π and ρ-vector [Albert, Henriksson, Rastelli, AV - in progress]

Stay tuned!