Dilaton Scattering and Regge Trajectories

[F. Acanfora, A. Guerrieri, K. Häring, DK; 2023]

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Overview of the talk

Part 1: Defining the problem ← Scattering of Goldstone bosons in 4d (dilatons, pions, branons)

Part 2: Presenting the results ← Bounds on dimensionless ratios of Wilson coefficients

Amplitudes on the boundary

Regge trajectories

Part 3: Summary and outlook

Part 1: definitions

Scattering amplitude:

$$\mathcal{T}(s,t,u)$$

$$(s+t+u=0)$$ ← **Mandelstam variables**

$$x \equiv \cos\theta$$ ← **scattering angle**

Partial amplitudes:

$$t = -\frac{s}{2}(1-x), \quad u = -\frac{s}{2}(1+x)$$

Legendre polynomial

$$\mathcal{T}_\ell(s) \equiv \frac{i}{32\pi}\int_{-1}^{+1} dx\, P_\ell(x)\mathcal{T}(s,t(x),u(x))$$

angular momentum: $\ell = 0,2,4,6,\ldots$

Unitarity and positivity:

$$s \geq 0, \quad \ell = 0,2,4,6,\ldots$$

$$Im\mathcal{T}_\ell(s) \geq 0$$ ← **positivity**

[B. Bellazzini, J. Elias Miró, R. Rattazzi, M. Riembau, F. Riva; 2021]
[S. Caron-Huot, V. Van Duong; 2021]
[A. Tolley, Z.-Y. Wang, S.-Y. Zhou; 2021]

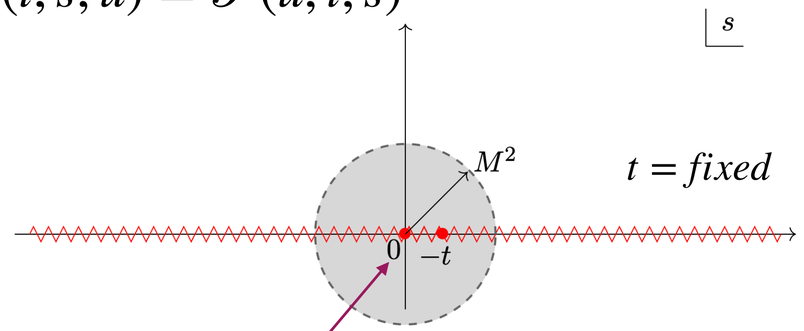
$$Im\mathcal{T}_\ell(s) \geq \frac{1}{16}|\mathcal{T}_\ell(s)|^2$$ ← **non-linear unitarity**

[M. Paulos, J. Penedones, J. Toledo, B. van Rees, P. Vieira; 2016]

Part 1: assumptions

1. Crossing: $\mathcal{T}(s,t,u) = \mathcal{T}(t,s,u) = \mathcal{T}(u,t,s)$

2. Maximal analyticity:



3. Unitarity: $Im\mathcal{T}_\ell(s) \geq \frac{1}{16}|\mathcal{T}_\ell(s)|^2$

4. Low energy expansion:

$$\mathcal{T}(s,t,u) = g_2(s^2+t^2+u^2) + g_3\, stu + g_4\left(s^2+t^2+u^2\right)^2$$
$$-\frac{g_2^2}{480\pi^2}\left(s^2(41s^2+t^2+u^2)\log\left(-s\sqrt{g_2}\right) + (s\leftrightarrow t) + (s\leftrightarrow u)\right) + O(s^5)$$

cut-off

finite coefficients

weak coupling

$$g_n = k_n M^{-2n}\alpha$$
$$\bar{g}_3 = \frac{k_3}{k_2^{3/2}}\frac{1}{\sqrt{\alpha}}, \quad \bar{g}_4 = \frac{k_4}{k_2^2}\frac{1}{\alpha}$$

Wilson coefficients

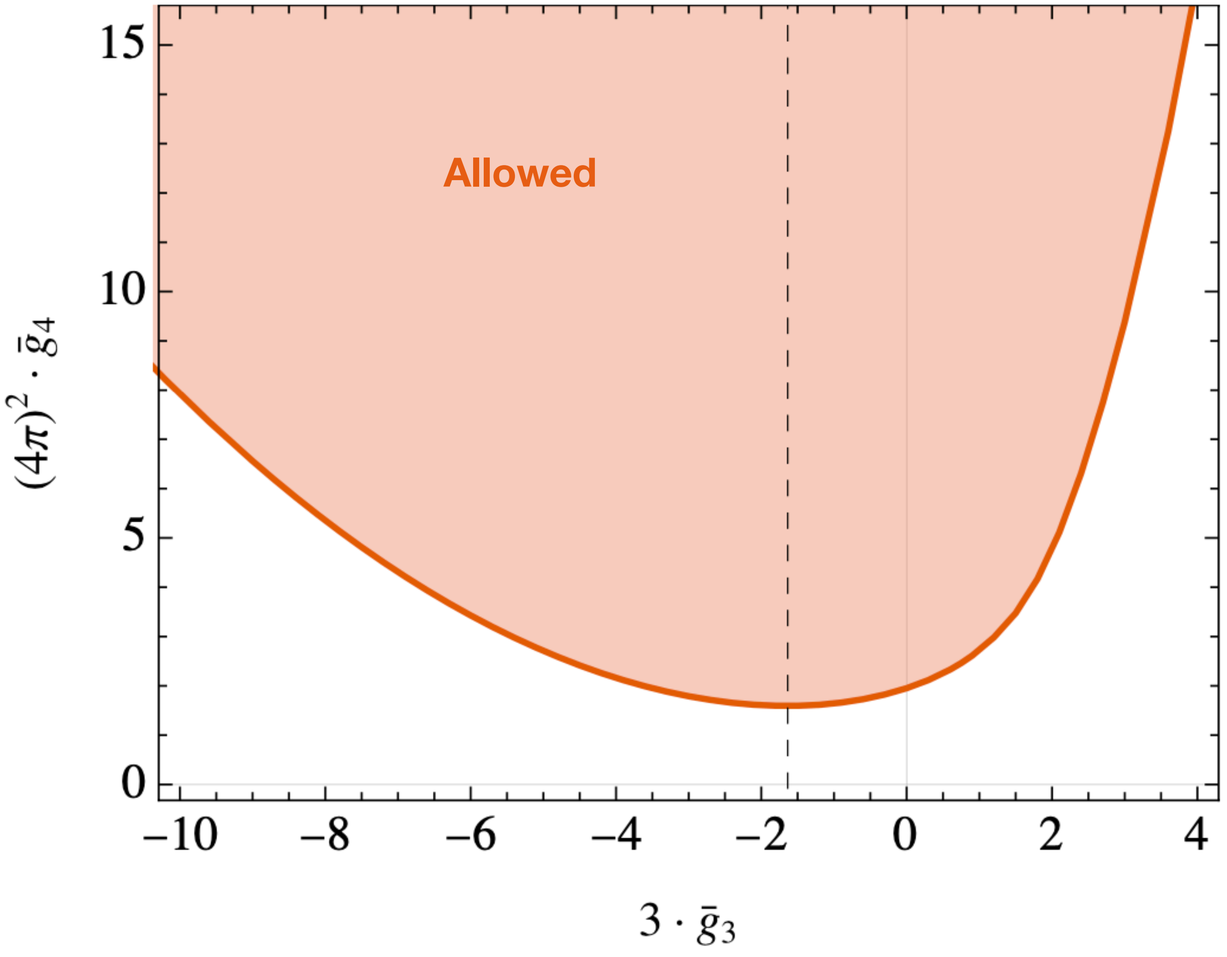
example

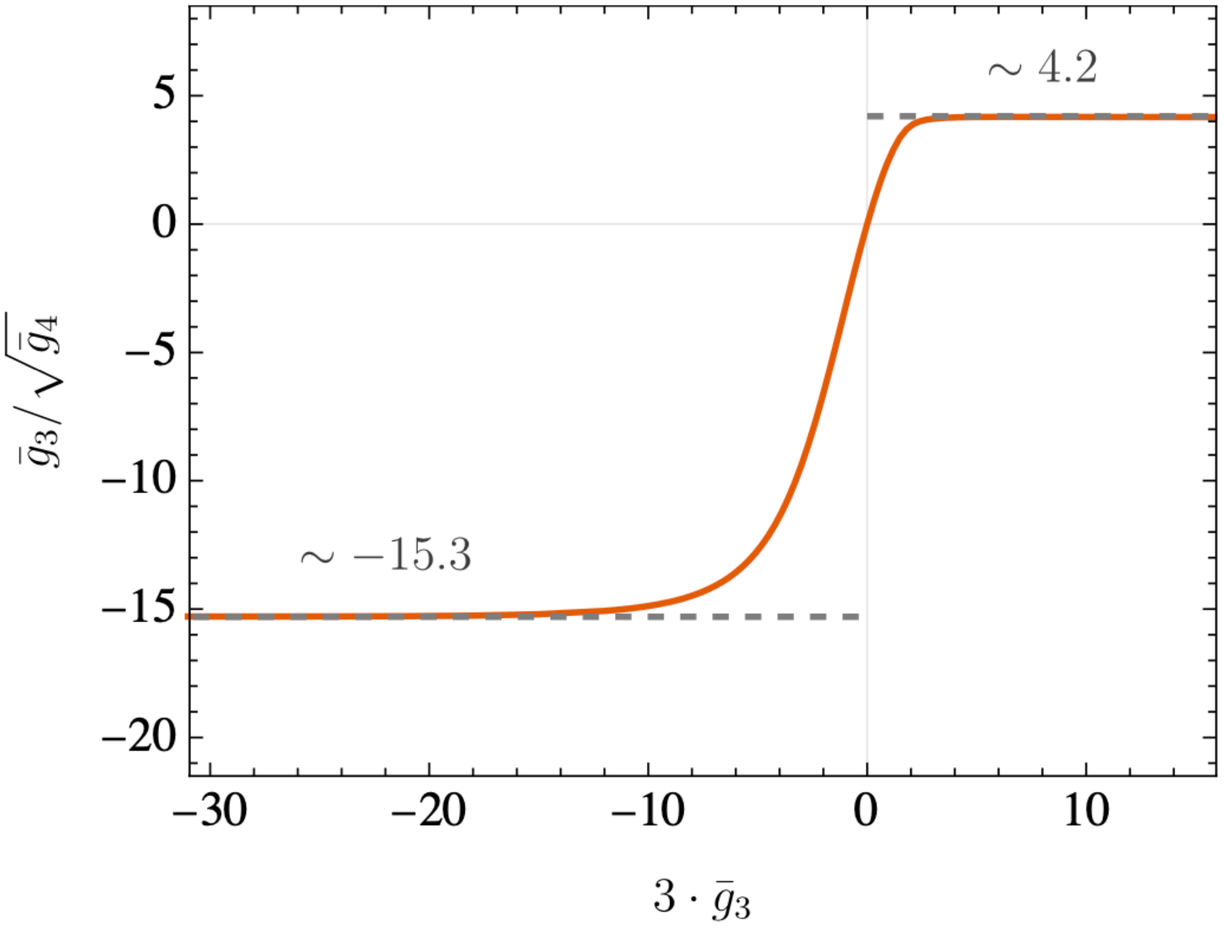
scale of the problem

$$\bar{g}_3 \equiv \frac{g_3}{g_2^{3/2}}, \quad \bar{g}_4 \equiv \frac{g_4}{g_2^2}$$

scale of the problem

Part 2: results





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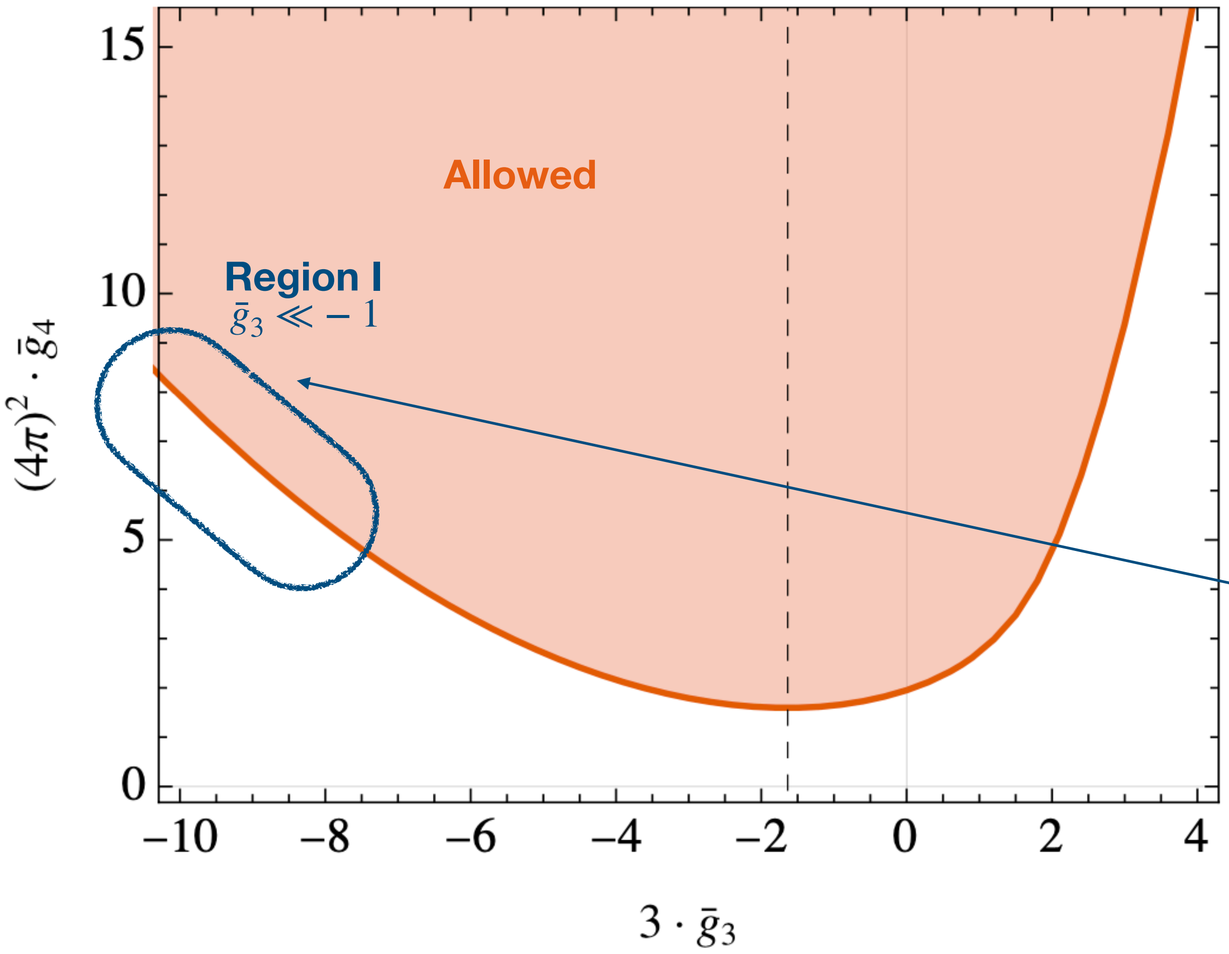

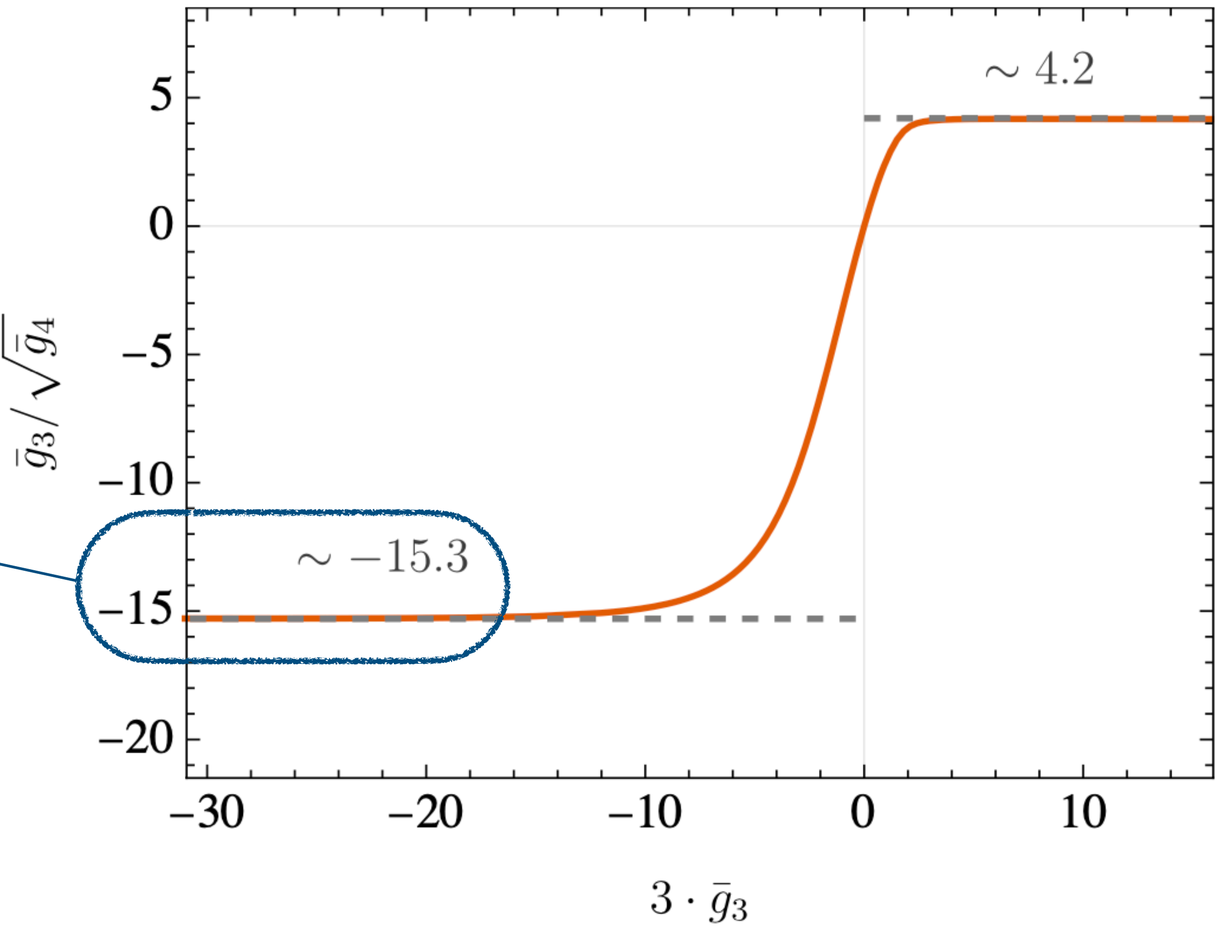

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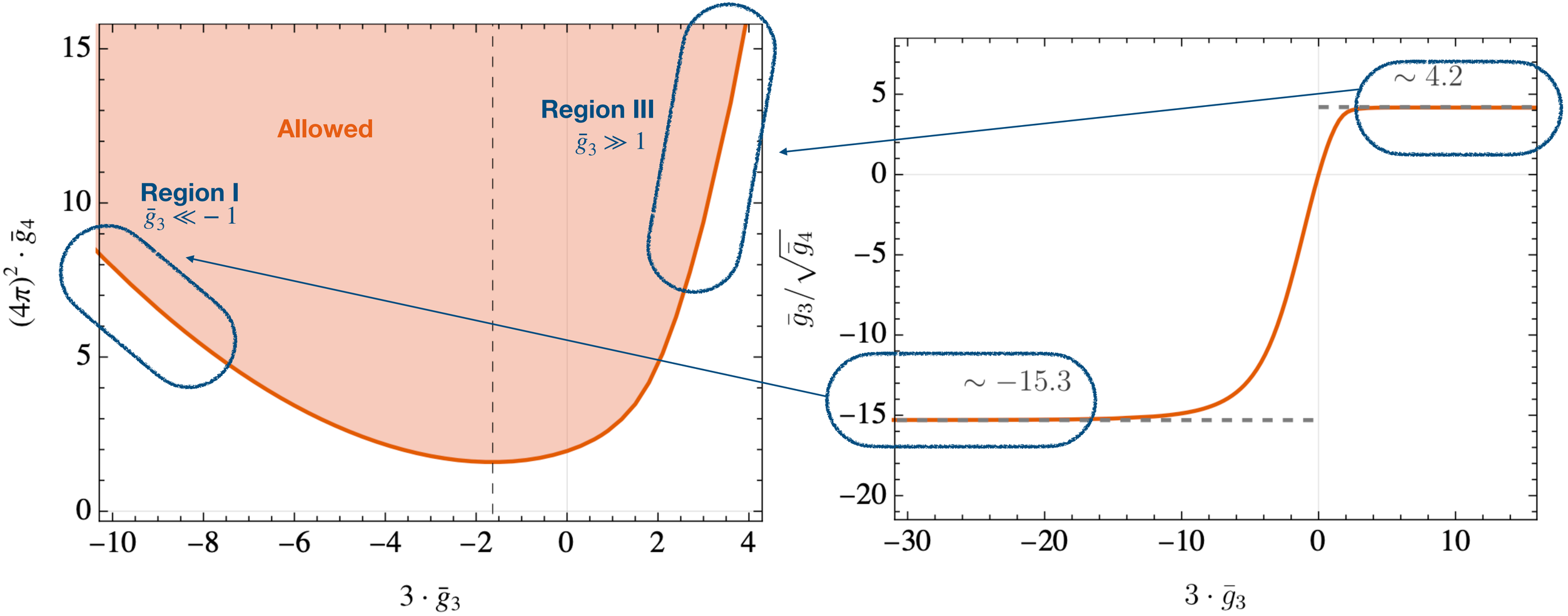

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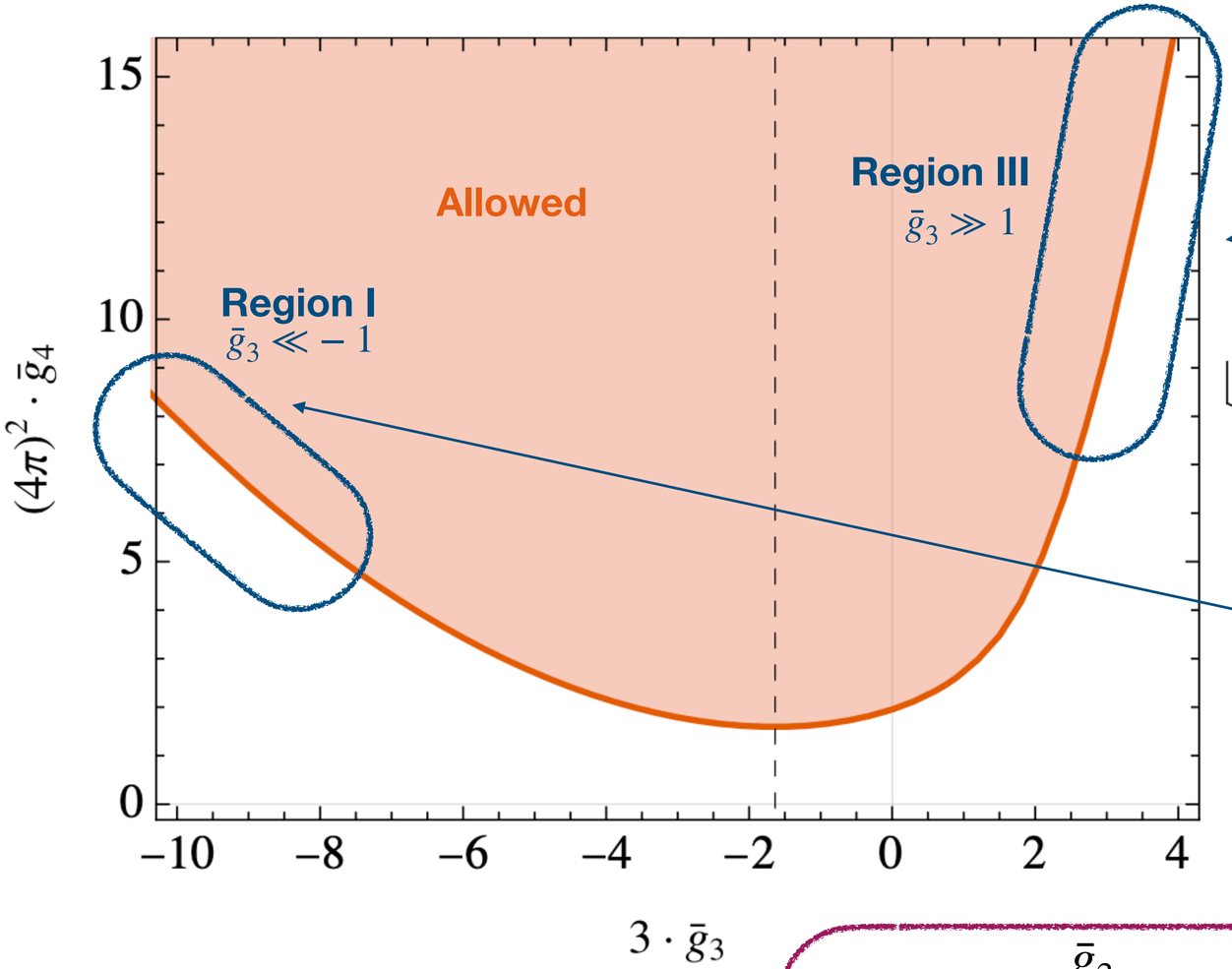
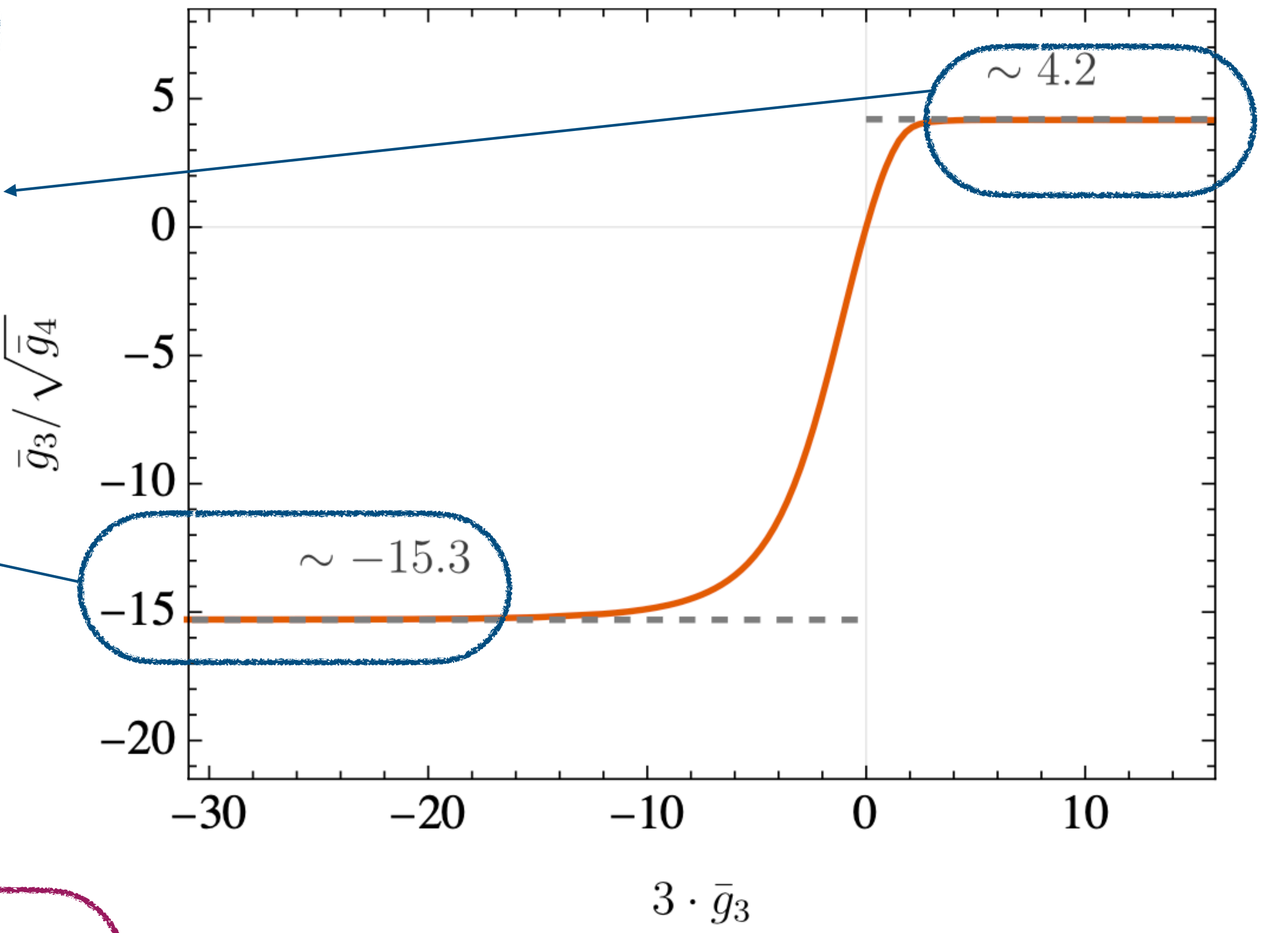




$$-15.3 \lessapprox \frac{\bar{g}_3}{\sqrt{\bar{g}_4}} \lessapprox 4.2$$

agree with [S. Caron-Huot, V. Van Duong; 2021]

[M. Paulos, J. Penedones, J. Toledo, B. van Rees, P. Vieira; 2016]

Part 2: S-matrix bootstrap

Ansatz:

$$\mathcal{T}(s,t,u) = \sum_{a,b,c=0}^{N_{max}} \alpha_{(abc)}\, \rho^a(s)\rho^b(t)\rho^c(u)$$

size of the ansatz

real dimensionless coefficients

$\rho-$variables:

$$\rho(s) \equiv \frac{1-\sqrt{-s}}{1+\sqrt{-s}}$$

Low energy expansion:

$$\alpha_{(abc)} \longrightarrow (\bar{g}_3, \bar{g}_4)$$

linear relation

[A. Guerrieri, J. Penedones, P. Vieira; 2020]

- ✓ crossing
- ✓ analyticity
- ✓ low energy expansion

Impose unitarity using SDPB

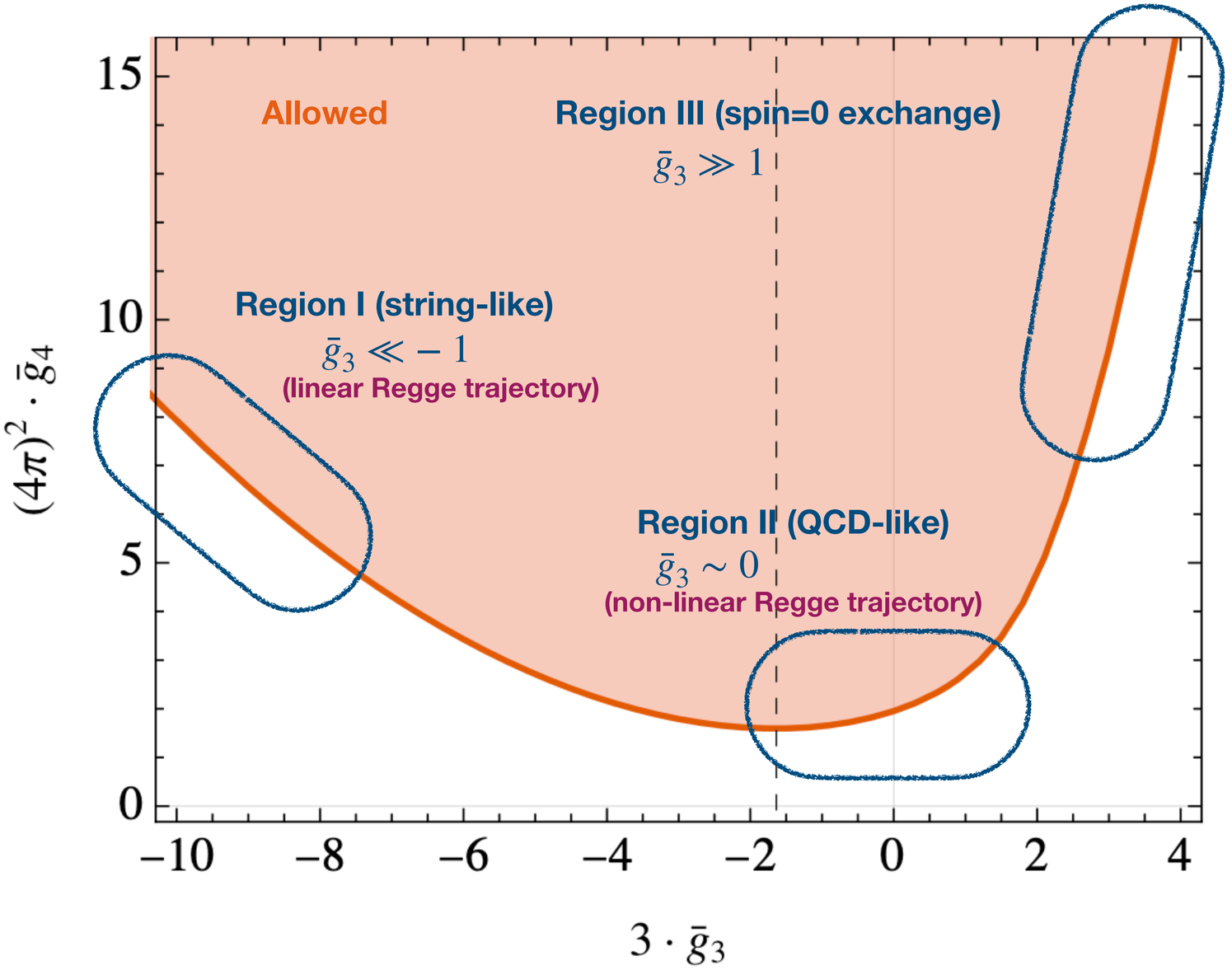
[D. Simmons-Duffin; 2015]

[W. Landry, D. Simmons-Duffin; 2019]

$$\bar{g}_3 = fixed$$
$$minimize(\bar{g}_4)$$

→ numerical values for $\alpha_{(abc)}$

Part 2: results

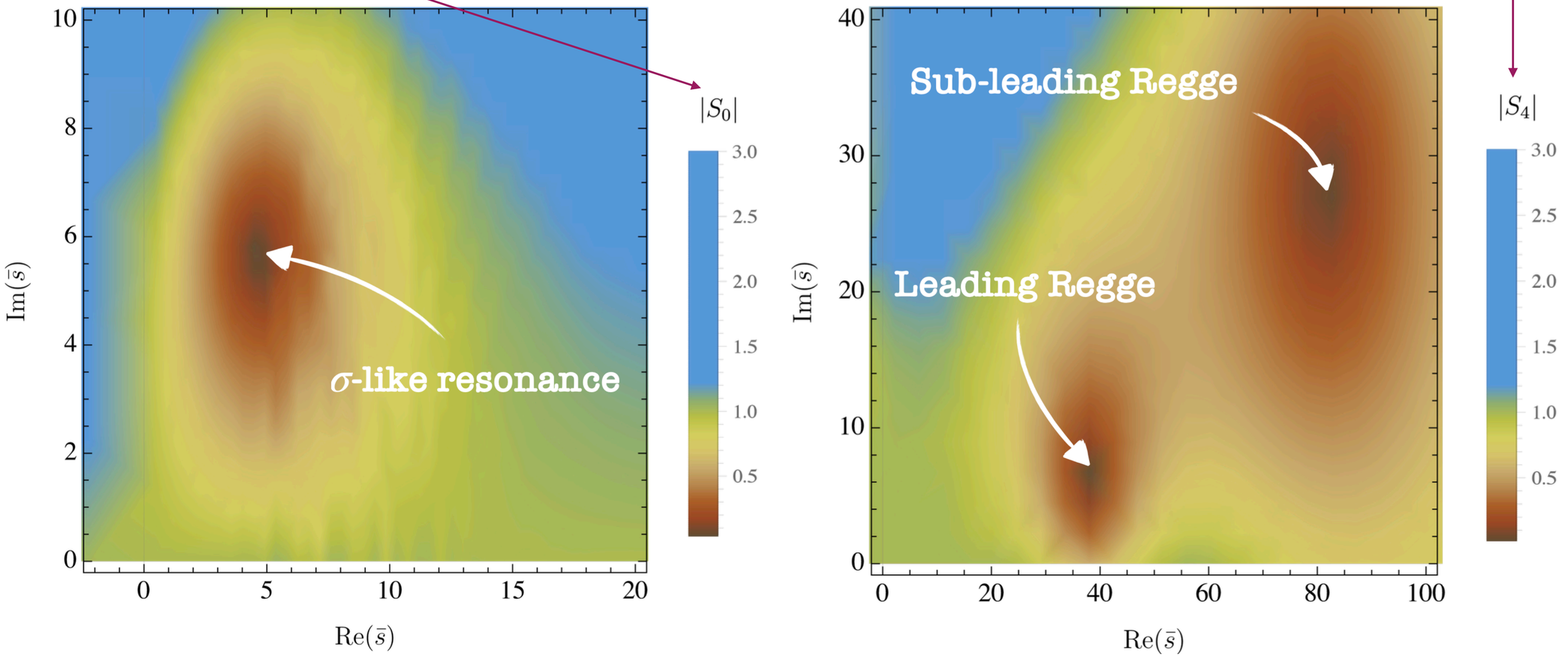


Part 2: zeros

$$\bar{g}_3 = -0.53$$

spin=0 partial amplitude

spin=4 partial amplitude



Zero on the first sheet = pole on the second sheet (for elastic amplitudes)

Part 2: zeros

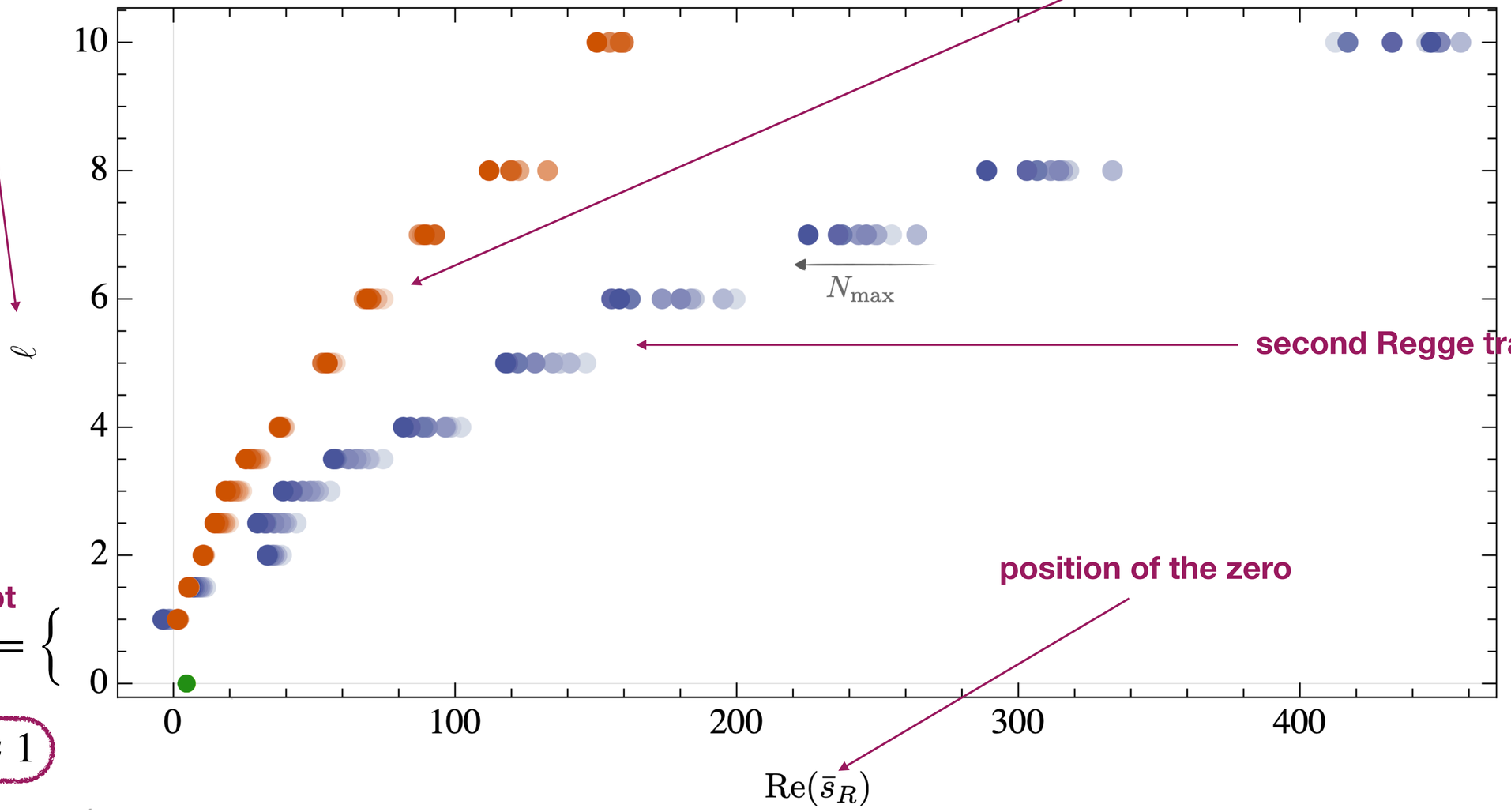
[A. Bose, A. Sinha and S. S. Tiwari; 2020]

[A. Guerrieri, H. Murali, J. Penedones, P. Vieira; 2022]

$\bar{g}_3 = -0.53$

spin of the partial amplitude

first Regge trajectory



second Regge trajectory

position of the zero

intercept

$$\ell(0) = \begin{cases} \\ \\ \end{cases}$$

$\ell(0) \approx 1$

Part 3: summary

1. Obtained bounds on Wilson coefficients (using non-linear unitarity)

2. Our bounds are compared to the positivity ones

3. Amplitudes on the boundary are extracted and investigated

4. Regge trajectories are discovered

Part 3: outlook

[A. Guerrieri, J. Penedones, P. Vieira; 2018]

[Y. He, M. Kruczenski; 2023]

1. Consider the scattering of massive pions with SO(3) symmetry

2. Inject some QCD experimental data about pion scattering

3. Determine the QCD amplitude with the minimal experimental input

4. Perform spectral analysis of the resonances (appearing for complex energies)

Thank you!