Differential Equations for Homological Correlators

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“Kinematic Flow and the Emergence of Time”

“Differential Equations for Cosmological Correlators”

Appeared Today!!
Lots of people to acknowledge

(see the papers)

+ Lots of previous important work

(see the references in the papers)
The Main Result
(of short paper)
Kinematic Flow...

\[ \frac{dY}{\varepsilon} = \begin{cases} 
\circ & (Y - F) + \\
\quad & (Y - \tilde{F}) + \\
\quad & F + \\
\quad & \tilde{F} 
\end{cases} \]

\[ dF = \varepsilon \left[ \begin{array}{c}
\circ & (F - Z) + \\
\quad & Z 
\end{array} \right] \]

\[ d\tilde{F} = \varepsilon \left[ \begin{array}{c}
\circ & (\tilde{F} - Z) + \\
\quad & Z 
\end{array} \right] \]

\[ dZ = \varepsilon \left[ \begin{array}{c}
\circ & 2Z 
\end{array} \right] \]

...describes time evolution in power-law cosmology!
Kinematic Flow...

$4^{n_v-1}$ functions
The Main Result
(of long paper)
\[ a(\eta) = \eta \]

\[ \varphi \]

\[ \psi(\eta_1, \eta_2, \eta) \]

\[ (1 + \varepsilon) \]

\[ dZ = \varepsilon A \]

Gravity

Time integrals
Differential Equation Method
Twisted Cohomology
Master Integrals
Canonical forms

QCD
\[ c_i \begin{cases} \text{No folded singularity} \\ \text{factorization} \end{cases} \]
Motivation
Time Evolution $\Rightarrow$ Boundary Differential Equation

Time without time
Cosmological Bootstrap

In dS$_4$ (inflation), with & without boost symmetry
Locality, Unitarity, Perturbative & non-Perturbative

...
\[ \Psi (X_1, X_2, Y) \]

\[
\begin{align*}
[\Delta_{X_1} + \left(\frac{M}{\hbar}\right)^2 - 2] \Psi &= \frac{g^2}{X_1 + X_2} \\
[\Delta_{X_2} + \left(\frac{M}{\hbar}\right)^2 - 2] \Psi &= \frac{g^2}{X_1 + X_2}
\end{align*}
\]

\[
X_1 = k_1 + k_2 \\
X_2 = k_3 + k_4 \\
Y = |\vec{k}_1 + \vec{k}_2|
\]

Singular at \([X_1 + Y = 0, X_2 + Y = 0, X_1 + X_2 = 0]\).

Hypergeometric (2\textsuperscript{nd} order) op.

Shouldn't be at \([X_1 - Y = 0, X_2 - Y = 0]\).
Where do the differential equations come from?
Can we derive them (and solve them) systematically?
Model
**Prototype:** Conformal Scalars In Power Law Cosmologies

\[ \Psi(x_1, x_2, y) = \int_0^{\infty} \frac{\varepsilon \, dx_1 \, dx_2}{(x_1 + X_1 + y) (x_2 + X_2 + y) (x_1 + x_2 + X_1 + X_2)} \]

\[ ds^2 = \eta \left[ -d\eta^2 + \frac{d\xi^2}{\eta^2} \right] \]

\[ \varepsilon = \begin{cases} 0 & \text{Flat} \\ -1 & \text{Radiation} \\ -2 & \text{Matter} \end{cases} \]

How to find D.E. for \( \Psi \)?
Key Point

\[ F = \int \varepsilon \Omega \]

"twist"

Canonical Form for bounded region in \((x_1, x_2)\) plane (Cosmological Polytope)

Strategy: Exploit (twisted) cohomology of integrands
Consider \( F_{n_1, \ldots, n_5} = c_{n_1, n_2, n_3, n_4, n_5} \int_{0}^{\infty} \frac{dx_1 dx_2 (x_1 x_2)^{\varepsilon}}{T_1^{n_1} T_2^{n_2} L_1^{n_3} L_2^{n_4} L_3^{n_5}} \)

- \( T_1 : \chi_1 \)
- \( T_2 : \chi_2 \)
- \( L_1 : \chi_1 + \overline{X}_1 + Y \)
- \( L_2 : \chi_2 + \overline{X}_2 + Y \)
- \( L_3 : \chi_1 + \chi_2 + \overline{X}_1 + \overline{X}_2 \)

Basis size: \# of bounded regions
\[ d\Psi = \varepsilon \left[ \begin{array}{c} (\Psi - F) + \\
+ (\Psi - \tilde{F}) + \\
+ F + \\
+ \tilde{F} \end{array} \right] \]

\[ d\tilde{F} = \varepsilon \left[ \begin{array}{c} (\tilde{F} - Z) + \\
+ Z \end{array} \right] \]

\[ dZ = \varepsilon \left[ \begin{array}{c} 2Z \end{array} \right] \]

**A:** follow Kinematic Flow!
The Rules of the Flow ...
$Q_1$ 

Activation 

growth 

merge 

Absorption 

$\tilde{q}_3$ 

$\tilde{q}_2$
\[ dQ_1 = \varepsilon \left[ (Q_1 - q_1) + Q_1 + \tilde{q}_3 + (Q_1 - \tilde{q}_3) + (\tilde{q}_3 + \tilde{q}_2) - \tilde{q}_2 \right] \]
F.A.Q.
& Outlook
Q: Boundary Conditions?
A: No folded singularity + Single factorization limit

Q: What's the physical interpretation of \( I \)?
A: Flow: Bubbles \( \rightarrow ? \)

Bulk: Breaking propagator in 4 pieces

Q: Can you solve them?
A: To some extent. Beyond my wildest dreams for sure.

Q: What about (mass, spin, loops)?
A: Working on it (with brilliant students)
Works for sums of graphs!

\[ g^a \phi^b \phi^c \phi^a \]

Is there some structure that contains A in it???
Differential Equations May Be Entryway To New Framework For Homology!