Parton showering meets perturbative QCD

Gregory Soyez
mostly based on work within PanScales:


IPhT, CNRS, CEA Saclay

QCD Meets Gravity, CERN, December 11-15 2023
PanScales
A project to bring logarithmic understanding and accuracy to parton showers

Former members

Silvia Zanoli
Oxford

Frédéric Dreyer

Rok Medves

Scarlett Woolnough
What makes them so successful/useful?

From fundamental theory...

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} D\psi + \cdots \]

\[ = \left\langle \frac{\langle ij \rangle^4}{\langle 12 \rangle \langle 23 \rangle \cdots \langle n1 \rangle} \right\rangle \]

...to a spectrum of applications

Basic idea: getting practical numbers

Applications:
- pheno studies (“run Pythia to test a pheno idea”)
- measurements (compare data/theory)
- modelling (systematic uncertainties)
- searches (estimate backgrounds)
- AI training (e.g. supervised classification)
- ...
What makes them so successful/useful?

Benchmark feature: versatility

- ranges from “fixed-order” parton-level to realistic full-event simulations (incl. detector)
- wide range of applications
- can compute any observable, fiducial cuts, ...

Precision challenge

Precision is increasingly required for LHC physics (and future colliders)
- Get precise background estimates
- Search for tiny deviations/rare processes
- Get precise predictions and small uncertainties
- Avoid AI picking up spurious effects

This requires control over the full chain: from the amplitude to the detector
What makes them so successful/useful?

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Anatomy of a high-energy collision

Simulating a high-energy collision requires several ingredients

- A hard process

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Anatomy of a high-energy collision

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- Parton shower (initial and final-state)
Anatomy of a high-energy collision

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Anatomy of a high-energy collision

Simulating a high-energy collision requires several ingredients:

- A hard process
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- Multi-parton interactions

Anatomy of a high-energy collision
Anatomy of a high-energy collision

Simulating a high-energy collision requires several ingredients

- A hard process
- Parton shower (initial and final-state)
- Hadronisation
- Multi-parton interactions

perturbatively “calculable”

non-pert. “modelled”
Basic message #2: physics at all scales

\[ Q \equiv 100 \text{ GeV} \rightarrow 1 \text{ TeV} \]

\[ Q \gg \mu_{NP} \]

\[ \mu_{NP} \sim 1 \text{ GeV} \]

\[ m_{\pi} \]

\[ m_c \]

\[ m_b \]

\[ m_H \]

\[ m_{W/Z} \]

Physics probed across many scales

Hard process, matching

Parton shower

Hadronisation

\[ \alpha_s \frac{\log Q}{\mu_{NP}} \]

\[ \alpha_s \log^2 \frac{Q}{\mu_{NP}} \]

\[ \alpha_s \log^3 \frac{Q}{\mu_{NP}} \]

\[ \alpha_s \log^4 \frac{Q}{\mu_{NP}} \]

Double, single,... logs to resum

Shower accuracy means logarithmic LL, NLL, N2LL, ...

Well-defined & systematically improvable

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Basic message #2: physics at all scales

\[ Q \equiv 100 \text{ GeV} \rightarrow 1 \text{ TeV} \]

BSM

Hard process, matching

\[ \alpha_s(Q) f_1(v) + \alpha_s^2(Q) f_2(v) + \alpha_s^3(Q) f_3(v) + \ldots \]

"Standard" perturbative expansion

\[ \text{LO NLO NNLO} \]

Parton shower

\[ \alpha_s \log^2 Q/\mu_{NP}, \alpha_s \log Q/\mu_{NP} \]

(double, single,...) logs to resum

\[ Q \gg \mu_{NP} \]

\[ \mu_{NP} \sim 1 \text{ GeV} \]

\[ m_{\pi} \]

Physics probed across many scales

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Basic message #2: physics at all scales, a shower resumes logs

\[ Q \equiv 100 \text{ GeV} \rightarrow 1 \text{ TeV} \]

\[ Q \gg \mu_{NP} \]

\[ \mu_{NP} \sim 1 \text{ GeV} \]

Parton shower

\[ \text{Hard process, matching} \]

\[ \text{“Standard” perturbative expansion} \]

\[ \alpha_s(Q)f_1(\nu) + \alpha_s^2(Q)f_2(\nu) + \alpha_s^3(Q)f_3(\nu) + \ldots \]

LO \quad NLO \quad NNLO

expect logs between disparate scales

\[ \alpha_s \log^2 Q/\mu_{NP}, \alpha_s \log Q/\mu_{NP} \]

(double, single,...) logs to resum

shower accuracy means logarithmic

LL, NLL, N^2LL, ...

well-defined & systematically improvable

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### Selected Collider-QCD Accuracy Milestones

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This talk: improve on this

Many of today's widely-used showers only LL@leading-colour.
An “easy” graphical representation

Lund plane(s)
Basic features of QCD radiation

Take a gluon emission from a $(q\bar{q})$ dipole

\[ p_q \rightarrow \tilde{p}_q \]

Emission $(\tilde{p}_q \tilde{p}_{\bar{q}}) \rightarrow (p_q k)(k\bar{p}_{\bar{q}})$:

\[ k^\mu \equiv z_q \tilde{p}_q^\mu + z_{\bar{q}} \tilde{p}_{\bar{q}}^\mu + k_\perp^\mu \]

3 degrees of freedom:

- Rapidity: \( \eta = \frac{1}{2} \log \frac{z_q}{z_{\bar{q}}} \)
- Transverse momentum: \( k_\perp \)
- Azimuth: \( \phi \)

In the soft-collinear approximation

\[
dP = \frac{\alpha_s(k_\perp) C_F}{\pi^2} d\eta \frac{dk_\perp}{k_\perp} d\phi
\]
Basic features of QCD radiations: the Lund plane

Lund plane: natural representation uses the 2 “log” variables $\eta$ and $\log k_{\perp}$

$$\eta = -\log \tan(\theta/2)$$

![Diagram showing the Lund plane with variables $\eta$, $\log k_t$, $q$ side, $\bar{q}$ side, $E_k \leq \frac{1}{2} m_{q\bar{q}}$]
Basic features of QCD radiations: the Lund plane

**Lund plane**: natural representation uses the 2 “log” variables $\eta$ and $\log k_\perp$

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Lund plane: natural representation uses the 2 “log” variables $\eta$ and $\log k_\perp$

\[ \eta = -\log \tan(\theta/2) \]

\[ E_k \leq \frac{1}{2} m_{q\bar{q}} \]

- $q$ side
- $\bar{q}$ side
- $k$ side
- $k_\perp$ side
- $k$ hard collinear
- soft & colinear
- soft (large angle)
Basic features of QCD radiations: the Lund plane

Lund plane: natural representation uses the 2 “log” variables $\eta$ and $\log k_\perp$

\[ \eta = -\log \tan(\theta/2) \]

\[ E_k \leq \frac{1}{2} m_{\bar{q}q} \]

\[ q, \bar{q}, k, \bar{k} \]

Soft & Colinear
Hard collinear
Soft (large angle)
Basic features of QCD radiations: the Lund plane

Lund plane: natural representation uses the 2 “log” variables $\eta$ and $\log k_\perp$

$$\eta = -\log \tan(\theta/2)$$

![Diagram of the Lund plane showing the regions of soft & collinear, hard collinear, and soft (large angle) radiation.](image)
Multiple emissions in the Lund plane

\[ \eta = -\log \tan(\theta/2) \]

\[ E_k \lesssim \frac{1}{2} m_{\bar{q}q} \]

\[ \bar{q} \text{ side} \]

\[ q \text{ side} \]

primary plane

secondary plane(s)

ternary plane(s)
(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum $k_t$

$\eta = -\log \tan(\theta/2)$

Start with $k_t = Q$

one $q\bar{q}$ dipole
(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum $k_t$

$\eta = -\log \tan(\theta/2)$

$q$ side

$\bar{q}$ side

Generate $k_{t1} < Q$

(using Sudakov proba)

\[ E_k < \frac{1}{2} m_{q\bar{q}} \]
(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum $k_t$

$$\eta = -\log \tan(\theta/2)$$

$\bar{q}$ side

$q$ side

Generate $\eta_1$

& split dipoles

$$(q\bar{q}) \rightarrow (qg_1) + (g_1\bar{q})$$

$E_k \leq \frac{1}{2} m_{q\bar{q}}$

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(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum $k_t$

$$\log k_t \quad \eta = -\log \tan(\theta/2)$$

$q$ side \hspace{1cm} $\bar{q}$ side

$E_k < \frac{1}{2} m_{q\bar{q}}$

Generate $k_{t2} < k_{t1}$ (now from 2 dipoles)

$q$ side \hspace{1cm} $\bar{q}$ side
(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum $k_t$

\[ \eta = - \log \tan(\theta/2) \]

\[ \log k_t \]

$q$ side

$\bar{q}$ side

$E_k \leq \frac{1}{2} m_{q\bar{q}}$

Generate $\eta_2$

&split dipoles

$ (g_1 \bar{q}) \rightarrow (g_1 g_2) + (g_2 \bar{q}) $
(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum $k_t$

$\log k_t \quad \eta = -\log \tan(\theta/2)$

$\bar{q}$ side

$q$ side

$E_k \leq \frac{1}{2} m_{\bar{q}q}$

Iterate
(Dipole) parton shower in the Lund plane

Ordering variable: transverse momentum $k_t$

\[ \eta = -\log \tan(\theta/2) \]

$E_k < \frac{1}{2} m_{q\bar{q}}$

$q$ side

$\bar{q}$ side

$\log k_t$

until $k_t = k_{t,\text{cut}}$

$E_k \leq \frac{1}{2} m_{q\bar{q}}$

$q$ side

$\bar{q}$ side
Physics result #1: an organising principle:

at a given (all-order) accuracy, what physics do we need to get right?
Accuray \leftrightarrow\text{reproducing sets of MEs}

- handles disparate scales
- all-order perturbative QCD

\[ \downarrow \]

minimum: get the ME for an arbitrary number of well-separated emissions

- If "log distance" \( \Delta \) emissions factorise up to \( \mathcal{O}(e^{-\Delta}) \) corrections
- this achieves NLL accuracy
  i.e. all-order NLL is like fixed-order LO
- In particular, in a parton showers, an emission should not be affected by subsequent distant emissions

\( \eta = \ln \tan \frac{\theta}{2} \)

Separation in any direction

Robust construction in pQCD
Systematically improvable
"only" a handful of ME at each order thanks to QCD factorisation

difficulty: the shower algorithm generates spurious terms one needs to avoid/correct for

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\[ \ln k_t \]

(only half the primary Lund plane for simplicity)

\[ \eta = \ln \tan \frac{\theta}{2} \]

mistake allowed at NLL
Accuracy ↔ reproducing sets of MEs

handles disparate scales
all-order perturbative QCD

minimum: get the ME for an arbitrary number of well-separated emissions (NLL!)

Beyond NLL

- At NNLL we also want an arbitrary number of pairs of emissions
- $N^3$LL also requires triplets, etc...

\[ \eta = \ln \tan \frac{\theta}{2} \]

(only half the primary Lund plane for simplicity)

any # pairs required at NNLL
Accuracy ↔ reproducing sets of MEs

Handles disparate scales
All-order perturbative QCD

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Beyond NLL
- At NNLL we also want an arbitrary number of pairs of emissions
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For resummation experts:
- N^kLL counts exponentiating observables: \( \Sigma(v) = e^{g_1(\alpha_s L) + g_2(\alpha_s L) + g_3(\alpha_s L)\alpha_s + \ldots} \)
- NNLL requires any number of well-separated pairs, triplets for N^3LL, ...
- N^kDL counts observable logs: \( \Sigma(v) = h_1(\alpha_s L^2) + h_2(\alpha_s L^2)\sqrt{\alpha_s} + h_3(\alpha_s L^2)\alpha_s + \ldots \)
- DL has only soft&collinear, NDL has a unique single-log,
  NNDL has a unique pair or two single-logs, etc...

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Accuracy ↔ reproducing sets of MEs

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Robust construction in pQCD
Systematically improvable
“only” a handful of ME at each order thanks to QCD factorisation
difficulty: the shower algorithm generates spurious terms one needs to avoid/correct for
Physics result #2: NLL-accurate showers
Novel approach for testing accuracy

Resummation regime: $\alpha_s \log(v) \sim 1$, $\alpha_s \ll 1$

Idea for NLL testing:

$$\frac{\Sigma_{MC}(\lambda=\alpha_s L, \alpha_s)}{\Sigma_{NLL}(\lambda=\alpha_s L, \alpha_s)} \quad \text{v. 1}$$

with $\lambda = \alpha_s L$

NLL deviations

or

subleading effects?

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Novel approach for testing accuracy

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with $\lambda = \alpha_s L$

NLL deviations

or

subleading effects?
Novel approach for testing accuracy

Resummation regime: $\alpha_s \log(v) \sim 1$, $\alpha_s \ll 1$

Idea for NLL testing:

$$\frac{\Sigma_{MC}(\lambda=\alpha_s L, \alpha_s)}{\Sigma_{NLL}(\lambda=\alpha_s L, \alpha_s)} \xrightarrow{\alpha_s \to 0} 1$$

at fixed $\lambda = \alpha_s L$

- NLL deviations
- or
- subleading effects?
Assessing accuracy: $y_{23}$

**NNLL if**

$$\frac{\sum_{MC}(\lambda = \alpha_s L, \alpha_s)}{\sum_{NLL}(\lambda = \alpha_s L, \alpha_s)} \xrightarrow{\alpha_s \rightarrow 0} 1$$

**Failure of standard dipole showers**

Pythia8, Dire(v1) deviate from NLL

**Reason:**

spurious recoil for commensurate-$k_t$
emissions at disparate angles
violates our NLL ME requirement
Assessing accuracy: $y_{23}$

NNLL if \[ \frac{\sum_{MC}(\lambda=\alpha_s L, \alpha_s)}{\sum_{NLL}(\lambda=\alpha_s L, \alpha_s)} \xrightarrow{\alpha_s \to 0} 1 \]

Failure of standard dipole showers

- Pythia8, Dire(v1) deviate from NLL

New series of NLL-accurate showers

- PanLocal($0 < \beta < 1$): local recoil (dipole or antenna)
- PanGlobal($0 \leq \beta < 1$): global recoil

Cam. $y_{23}$, ratio to NLL

\[ \frac{\text{MC}}{\text{NLL}}(s_0, \lambda) \]

Dipole(Py8)

Dipole(Dire v1)

PanLocal($\beta=\frac{1}{2}$, dip.)

PanLocal($\beta=\frac{1}{2}$, ant.)

PanGlobal($\beta=0$)

PanGlobal($\beta=\frac{1}{2}$)

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Assessing accuracy: extensive observable list

\[ \text{PanLocal}(0 < \beta < 1) \text{ and PanGlobal}(0 \leq \beta < 1) \text{ get expected NLL (i.e. 0)} \]
**Physics:**

\[ \Delta \psi \] distribution due to spin correlations

Solution: adapt the Collins-Knowles alg.

build and update a spin correlation tree as shower progresses

\[ \vec{n}_1 \vec{n}_2 \]
\[ \Delta \psi_{12} \]
\[ P_1 \]
\[ P_2 \]
\[ \vec{p}_1 \]
\[ \vec{p}_2 \]
\[ \vec{p}_3 \]
\[ \vec{p}_4 \]
\[ \vec{p}_5 \]

**Tests:**

both hard & collinear

also EEEC v. analytics

soft + hard collinear

first all-order result

\[ \sigma_{\text{tot}} \]
\[ \frac{d\sigma}{d\Delta \psi_{12}} \times 10^{-2} \] All channels

\[ \mathcal{O}(\alpha_s^2) \cdot \langle S + C \rangle / \langle \mathcal{O}(\alpha_s^2) \rangle \]

No spin

Collinear spin

Soft + collinear spin

\[ \gamma^* \rightarrow q \bar{q} \]

Toy shower

PanGlobal \( \beta = 0 \)

PanLocal (dip.) \( \beta = 0.5 \)

PanLocal (ant.) \( \beta = 0.5 \)

Pythia 8

**Beyond large \( N_c \) (backup)**

(collinear & soft) spin correlations

hadronic collisions DIS/VBF (backup)
Physics result #3: towards NNLL-accurate showers
(NNLL) accuracy ↔ reproducing (extra) sets of MEs

**NNLL: include pairs of emissions**

\[ \eta = \ln \tan \frac{\theta}{2} \]

- Hard emission angle and \( k_t \) similar to "hard" Born
- Soft emission angle and \( k_t \) similar to earlier one (can be large angle)
NNLL: include pairs of emissions

Matching

Get exact 3-jet LO (2-jet NLO) ME
\equiv one hard emission (pair with the hard event)

Standard approaches work but require care to preserve NLL accuracy

\[ \eta = \ln \tan \frac{\theta}{2} \]

NNLL: include pairs of emissions

Matching


Double-soft corrections

Two soft emissions at commensurate angles and $k_t$
(not necessarily collinear)

- Correction spurious shower ME $\rightarrow$ correct ME
  watch out for flavour channels and colour flows
- Need to get the correct virtual contributions
  (done through a modified $K_{CMW}$)

Gain: state-of-the-art (next-to-single-log) non-global logs

Beyond NLL: double-soft corrections

NSL accuracy tests: energy in a slice

no double-soft

double-soft, $n_f^{\text{real}}=0$

double-soft

Successfully reproduce next-to-single (non-global) logs for emissions in a slice
Conclusions and perspectives

**Basics**

Parton showers are extensively relied upon and need to be brought to high accuracy

**PanScales**

- Recoil issue limits standard generators to LL (and large $N_c$)
- Fixed in our PanScales ($\text{PanLocal}(0 < \beta < 1)$ and $\text{PanGlobal}(0 \leq \beta < 1)$) showers
- Included at NLL: subleading-$N_c$, spin correlations, hadron collisions
- First NNLL ingredients: $(ee)$ 3-jet matching, double-soft corrections
- Prelim pheno effect: reduction of uncertainty at $Q \sim 100$ GeV (larger effects at $Q \sim 1$ TeV)

**Future**

- Beyond Drell-Yan ($pp$) and 3 jets ($ee$)
- Investigate phenomenology
- Add missing NNLL
- Provide public code
Open question

Can this have connections with “gravity”?

I am definitely not sure...

... but parton showers likely apply for weakly-coupled problems with widely separated scales.

Can that be helpful for extreme mass ratios? Is there some \( \log(\nu/c) \) enhancements?
Backup
What do Event Generators provide?

- Broad range of applications
- Searches
- Background (and signal) estimate

Example:

\[ H \rightarrow ZZ \rightarrow 4\ell \]

[CMS, arXiv:1207.7235]
What do Event Generators provide?

Broad range of applications

- Searches
- Measurements

Idea: data v. MC
- allows the use of MC as modelling tool
- helps developing better MC

What do Event Generators provide?

- Broad range of applications
  - Searches
  - Measurements & modelling
- Tool to estimate uncertainties

Example:
- top mass measurement
  
  [ATLAS-CONF-2019-046]
What do Event Generators provide?

- Broad range of applications
- Searches
- Measurements & modelling
- Phenostudies

Long list of applications:
- New tools & observables (incl. substructure)
- Comparison to analytics
- Comparison to data
- BSM models

Graph:
- LH17 2-prong tagger
- 1/N_{truth} dN/dv
- $p_{T,jet}>2000$ GeV
- Pythia8(4C), anti-$k_t$(1.0)
- $65<m<105$
- W, LH17
- q/g, LH17
What do Event Generators provide?

Broad range of applications

- Searches
- Measurements & modelling
- Pheno studies
- Machine learning

- Deep Learning increasingly used at the LHC
- Training often done on MCs
- Shows interesting performance
- Example: boosted $W \rightarrow q\bar{q}$ v. QCD jet

[plot from Frederic Dreyer]
Many showers (Pythia, Sherpa, Vincia, Dire, ...) are **dipole/antenna** showers (main exception: Herwig)
Many showers (Pythia, Sherpa, Vincia, Dire, ...) are dipole/antenna showers (main exception: Herwig)

Idea #1:

\[ (ij) \rightarrow (ik)(kj) \]

captures the soft/collinear limits

**key ingredient:** mapping

\[ \tilde{p}_i, \tilde{p}_j \rightarrow p_i, p_j, p_k \]

before split \hspace{1cm} after split

includes recoil & energy-mom conservation
Many showers (Pythia, Sherpa, Vincia, Dire, ...) are dipole/antenna showers (main exception: Herwig)

Idea #2:
iterate dipole splittings (populate the full phase space with multiple emissions)

Rooted in QCD factorisation

\[ P_{n+1}(v_{n+1}) = e^{-\Delta_n(v_0,v)} |M^2| (v) P_n(v_n) \]
Dipole/Antenna showers: ingredients

Many showers (Pythia, Sherpa, Vincia, Dire, ...) are dipole/antenna showers (main exception: Herwig)

ka\_np \equiv \text{iterate dipole splittings (populate the full phase space with multiple emissions)}

Rooted in QCD factorisation

\[ P_{n+1}(v_{n+1}) = e^{-\Delta_n(v_0,v)} |M^2(v)| P_n(v_n) \]

\text{Sudakov} \equiv \text{"no emissions" (virtuals)}

\text{real emission}
Many showers (Pythia, Sherpa, Vincia, Dire, ...) are dipole/antenna showers (main exception: Herwig)

Idea #2:
iterate dipole splittings (populate the full phase space with multiple emissions)

Several challenges:
- ordering variable ($\nu$)
- beyond large/leading-$N_c$
- treat recoil properly
- assess/improve accuracy
Different ordering variables...

... can lead to different emission orderings

$k_t$ (transv. mom.) ordering

$q$ (virtuality) ordering

$k_{ta} > k_{tb}$

$\Rightarrow a$ emitted before $b$

$q_b > q_s$

$\Rightarrow b$ emitted before $a$
Recent progress (for completeness)

Lots of progress in several key directions over the past years:

- **(subleading) 1 → 3 splitting functions** (example: Dire(v2)).
  
  See e.g. [Jadach et al,16], [Li,Skands,16], [Höche,Krauss,Prestel,17], [Höche,Prestel,17]

- **Subleading colour**
  
  - most showers are leading colour (even at leading-log)
  - complex soft-gluon patterns
  - see e.g. [Nagy,Soper,12], [Gieseke,Kirchgaesser,Plätzer,Siodmock,18], [Höche,Reichelt,20], [Forshaw,Holguin,Plätzer,20]

- **Amplitude-level showers**, see e.g. [Forshaw,Holguin,Plätzer,19]

- **Electroweak showers**
  
  - more involved splitting kernels than in QCD
  - explicit chirality/spin dependence
  - see e.g. [Kleiss,Verheyen,20], [Bauer,Ferland,Webber,17-18], [Bauer,DeJong,Nachman,Provasoli,19]
(Cumulative) distributions can (often) be written as

\[ P(\nu < e^{-L}) = \exp \left[ g_1(\alpha_s L) + g_2(\alpha_s L) + g_3(\alpha_s L)\alpha_s + \ldots \right] \]

leading log(LL)  next-to-leading log(NLL)  NNLL

Examples:

- **Thrust**  \( T = \max_{|\vec{u}|=1} \frac{\sum_i |\vec{p}_i \cdot \vec{u}|}{\sum_i |\vec{p}_i|} \)

- **Cambridge y_{23}** \( (\approx \text{largest } k_t \text{ in an angular-ordered clustering}) \)

- angularities

- ...

Note: substructure techniques (e.g. Lund-plane based) can help design more observables
(Cumulative) distributions can (often) be written as

\[ P(\nu < e^{-L}) = \exp \left[ g_1(\alpha_s L)L + g_2(\alpha_s L) + g_3(\alpha_s L)\alpha_s + \ldots \right] \]

in resummation regime:

\[ \alpha_s \ll 1, \quad L \gg 1, \quad \lambda \equiv \alpha_s L \sim 1 \]

*We should control at least \( \mathcal{O}(1) \) contributions*
Lund-plane representation: transverse recoil boundaries

\[ \eta = -\log \tan(\theta/2) \]

- Gluon \( a \) radiated at scale \( k_{ta} \) and angle \( \theta_a \)
- Gluon \( b \) radiated at scale \( k_{tb} \leq k_{ta} \)

Expected

\( a \) takes recoil iff \( \theta_{ab} < \theta_a \)
Lund-plane representation: transverse recoil boundaries

\[ \eta = -\log \tan(\theta/2) \]

- gluon \( a \) radiated at scale \( k_{ta} \) and angle \( \theta_a \)
- gluon \( b \) radiated at scale \( k_{tb} \leq k_{ta} \)

Expected

\( a \) takes recoil iff \( \theta_{ab} < \theta_a \)

standard dipole shower
e.g. Pythia8/Dire

\[ E_k < \frac{1}{2} m \bar{q} \]

\( \bar{q} \) recoils

\( q \) recoils

\( g \) recoils

\( g \) recoils

decided in dipole frame:
\( a \) takes recoil if
\[ \theta_{bg}^{(dip)} < \theta_{bq}^{(dip)} \]

WRONG!
Lund-plane representation: transverse recoil boundaries

\[ \eta = -\log \tan(\theta/2) \]

\[ E_k \leq \frac{1}{2} m \bar{q} q \]

- Gluon \( a \) radiated at scale \( k_{ta} \) and angle \( \theta_a \)
- Gluon \( b \) radiated at scale \( k_{tb} \leq k_{ta} \)

**Expected**

- \( a \) takes recoil iff \( \theta_{ab} < \theta_a \)

**PanLocal (step 1)**

- Decided in event frame:
  - \( a \) takes recoil if \( \theta_{bg} < \theta_{bq} \)
  - Better but still wrong!
Lund-plane representation: PanLocal evolution variable

\[ \eta = - \log \tan(\theta/2) \]

\[ k_t \text{ ordering} \]

\[ k_{tb} \text{ recoil from } q: \text{ OK} \]

\[ E_k \leq \frac{1}{2} m_{q\bar{q}} \]
Lund-plane representation: PanLocal evolution variable

\[ \eta = -\log \tan(\theta/2) \]

- \( \log k_t \)
- \( q \) side
- \( \bar{q} \) side

- \( k_t \) ordering
- \( k_{tb} \) recoil from \( a \): not OK

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Lund-plane representation: PanLocal evolution variable

\[ \eta = -\log \tan(\theta/2) \]

\[ v \propto k_t e^{-\beta|\eta|} \]

ordering \( k_{tb} \) recoil from \( q \): OK

commensurate \( k_t \) emissions generated from central to forward rapidities

\[ \Rightarrow \text{no recoil issue} \]
Kinematic map

(just to give an idea of what it takes)

\[ p_k = a_k \vec{p}_i + b_k \vec{p}_j + k_\perp \]
\[ p_i = a_i \vec{p}_i + b_i \vec{p}_j - f k_\perp \]
\[ p_j = a_j \vec{p}_i + b_j \vec{p}_j - (1-f)k_\perp \]

with (PanLocal(\(\beta\)), variables \(\nu\) and \(\tilde{\eta}\))

\[ |k_\perp| = \rho \nu e^{\beta |\tilde{\eta}|} \quad \rho = \left( \frac{2\vec{p}_i \cdot Q \vec{p}_j \cdot Q}{Q^2 \vec{p}_i \cdot \vec{p}_j} \right)^{\beta/2} \]
\[ a_k = \sqrt{\frac{\vec{p}_j \cdot Q}{2\vec{p}_j \cdot Q \vec{p}_i \cdot \vec{p}_j}} |k_\perp| e^{+\tilde{\eta}}, \]
\[ b_k = \sqrt{\frac{\vec{p}_i \cdot Q}{2\vec{p}_j \cdot Q \vec{p}_i \cdot \vec{p}_j}} |k_\perp| e^{-\tilde{\eta}}, \]

\(f \approx \Theta(\tilde{\eta})\) and E-mom conservation

\(f\) decides where to put recoil
- \(f \to 1\) when \(k \to i\)
- \(f \to 0\) when \(k \to j\)

Where to put the transition?
- Pythia8/Dire: equal angles in dipole rest frame
- PanLocal: equal angles in event frame
A last example

- Look at angle $\Delta \psi_{12}$ between two hardest “emissions” in jet
  (defined through Lund declusterings)
A last example

- Look at angle $\Delta \psi_{12}$ between two hardest “emissions” in jet (defined through Lund declusterings)
- quite large NLL deviations in current dipole showers
- differences between quark and gluon jets

![Graph showing $\Delta \psi_{12}$ and various curves for different NLL, Dire(v1), quark, and Dire(v1), gluon. The x-axis represents $|\Delta \psi_{12}|$ ranging from $\pi/4$ to $\pi$, and the y-axis represents $\Sigma_{MC}/\Sigma_{NLL}(\Delta \psi_{12}, k_{t2}/k_{t1})$. The graph highlights the NLL, Dire(v1), quark, and Dire(v1), gluon curves with specific conditions: $-0.6 < \alpha_s \log \frac{k_{t1}}{Q} < -0.5$ and $0.3 < k_{t2}/k_{t1} < 0.5$. The graph also indicates that the differences between quark and gluon jets are significant.]
A last example

- Look at angle $\Delta \psi_{12}$ between two hardest “emissions” in jet (defined through Lund declusterings)
- Quite large NLL deviations in current dipole showers
- Differences between quark and gluon jets
- PanScales showers (here PanGlobal) get the correct NLL

![Graph showing $\Delta \psi_{12}$ vs. $|\Delta \psi_{12}|$ with NLL, Dire(v1), quark, Dire(v1), gluon, and PanGlobal lines]
A last example

- Look at angle $\Delta \psi_{12}$ between two hardest “emissions” in jet (defined through Lund declusterings)

- quite large NLL deviations in current dipole showers

- differences between quark and gluon jets

- PanScales showers (here PanGlobal) get the correct NLL

- ML could “wrongly/correctly” learn this
Beyond large $N_c$

**Physics:**

Keep track of the $C_F - C_A/2$ transitions

First generate assuming $C_A(/2)$, then correct in one of 2 ways:

1. segment
   - factor $2C_F/C_A$ if in quark segment
   - OK in the angular-ordered limit

2. NODS
   - (soft) $q\bar{q}g$ matrix-element correction
   - also OK for 2 emissions at $\sim$ angles

**Fixed-order tests:**

as in pythia

WRONG
similar to recoi earlier

perform as expected
Beyond large $N_c$

**Physics:**

Keep track of the $C_F - C_A/2$ transitions

First generate assuming $C_A(/2)$, then correct in one of 2 ways:

1. **segment**
   - factor $2C_F/C_A$ if in quark segment
   - OK in the angular-ordered limit

2. **NODS**
   - (soft) $q\bar{q}g$ matrix-element correction
   - also OK for 2 emissions at $\sim$ angles

**All-order tests:**

- **LL accuracy tests** - CFFE method

- **NLL accuracy tests** - NODS method

Non-global logs: large-$N_c$ + (full-$N_c$ at $O(\alpha_s^2)$)
(Collinear) spin correlations

**Physics:**

\[ \Delta \psi \] distribution due to spin correlations

Solution: adapt the Collins-Knowles alg.

build and update a spin correlation tree as shower progresses

**Tests:**

both hard & collinear

also EEEC v. analytics

soft + hard collinear

first all-order result

\[ \sigma_{\text{tot}} \] vs. \( \Delta \psi \)

- Toy shower
- PanGlobal \( \beta = 0 \)
- PanLocal (dip.) \( \beta = 0.5 \)
- PanLocal (ant.) \( \beta = 0.5 \)
- Pythia 8
**Hadronic collisions**

**Physics:**

- hadron collision  \( \Rightarrow \) initial-state radiation
- Consider Drell-Yan
- existing showers have the same recoil issue as for final state
  earlier emission takes recoil instead of the \( Z \)
- fix is essentially the same (modulo kinematic differences)
- includes colour and spin
- so far limited to colour singlet production

**Tests:**

- explicit test of DGLAP
- usual tests: \( Z \)-boson \( p_T \), event shapes
  + multiplicity, non-globals, beyond large-\( N_c \), spin

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Matching within PanScales

Matching = exact fixed-order generator + parton shower resumming logs

Physics

Focus on $e^+e^-$ collisions. We want

✓ exact $q\bar{q}g (\mathcal{O}(\alpha_s))$ distributions
✓ maintain NLL accuracy

Benefit: “NNDL” accuracy for event shapes

$$\Sigma(L) = h_1(\alpha_sL^2) + \sqrt{\alpha_s}h_2(\alpha_sL^2) + \alpha_s h_3(\alpha_sL^2) + \ldots$$

Implementation

Several possibilities:

- **simple multiplicative** matching (accept first emission with probability $P_{\text{exact}}/P_{\text{shower}}$)
- **MC@NLO-like** matching
- **POWHEG-like** matching (with $\beta$ scaling and careful veto to avoid double-counting when switching from POWHEG to the shower)

(*) Note: $N^k$LL expands $\ln \Sigma(\alpha_sL, \alpha_s)$ for “exponentiating” observables; $N^k$DL directly expands $\Sigma(\alpha_sL^2, \alpha_s)$

alternative viewpoint: NLL requires an arbitrary number of single-logs ($\alpha_sL^n$); NDL requires only one ($\alpha_sL(\alpha_sL^2)^n$)
Accuracy tests

\[ SD_z > 0.25, \beta_{SD} = 0 \ln k_t / Q, \sqrt{s} = 2 \text{ TeV} \]

- no matching
- wrong matching (no veto)
- correct matching

- visible effect at large \( k_t \) (right)
- spurious effect if not careful
- “correct” matching OK everywhere

\[ \lim_{s \to 0} \frac{\Sigma_{PS} - \Sigma_{NNDL}}{\Sigma_{NNDL}} \]

\[ \gamma^* \rightarrow q\bar{q}, \alpha_s \Lambda^2 = 1.296 \text{ (no matching)} \]

- PanLocal (\( \beta_{PS} = \frac{1}{2}, \text{dip.} \))
- PanLocal (\( \beta_{PS} = \frac{1}{2}, \text{ant.} \))
- PanGlobal (\( \beta_{PS} = 0 \))
- PanGlobal (\( \beta_{PS} = \frac{1}{2} \))

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Accuracy tests

- no matching ⇒ wrong NNDEL
- with matching ⇒ OK at NNDEL

visible effect at large \( k_t \) (right)
spurious effect if not careful
“correct” matching OK everywhere
Correct reproduction of the double-soft matrix elements
Extra double-soft results: multiplicity, $\delta K$

**NNDL accuracy tests: Lund multiplicity**

- **No double-soft**
  - $C_A = 2, C_F = \frac{3}{2}$
  - $N_{PS}, N_{NDL}$

- **With double-soft**
  - $C_A = 2, C_F = 3$
  - $n_f = 5$

Reproduces NNDL multiplicity

**Energy in a slice: $PG_{\beta = \frac{1}{2}}$**

- No double-soft
- Double-soft (only real)
- Double-soft (real + $\delta K$)

Requires the correct $K_{CMW}$ prescription
Extra double-soft results: multiplicity, $\delta K$

No large shift of central value but large reduction of the uncertainty estimates
Uncertainties:

- renormalisation scale variation:
  for NLL-accurate showers include compensation term to maintain 2-loop running for soft emissions
- factorisation scale variations (note: use of toy PDFs)
- term associated with lack of matching for $k_t \sim M_Z$
- for LL showers: a term associated with spurious recoil for commensurate $k_t$'s

Observations: Differences are relatively small except

- at very small $k_t$ for dipole-$k_t$ (esp. w global recoil)
- NLL brings significant uncertainty reduction
Example #2: $\Delta \psi_{12}$

Drell-Yan, $M_Z = 91.1876$ GeV

- Dipole-$k_t$ with global recoil (LL) quite off
- All others [local dipole-$k_t$(LL) and PanScales(NLL)] similar

PanGlobal($\beta_P = 0$) [NLL]
PanGlobal($\beta_P = 0.5$) [NLL]
PanLocal($\beta_P = 0.5$, dip.) [NLL]
PanLocal($\beta_P = 0.5$, ant.) [NLL]
Dipole-$k_t$(global) [LL]
Dipole-$k_t$(local) [LL]

PanScales [NLL]

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Example #2: \( \Delta \psi_{12} \)

**Drell-Yan, \( M_Z = 91.1876 \) GeV**

- Dipole-\( k_t \) with global recoil (LL) quite off
- All others [local dipole-\( k_t \) (LL) and PanScales (NLL)] similar

**Drell-Yan, \( M_{Z'} = 500 \) GeV**

- At higher scale: dipole-\( k_t \) (LL) \( \neq \) PanScales (NLL)
- DANGER: false sense of control from lower-energy info!
Details:
- PanLocal($\beta = 1/2$) dipole shower
- heavy quarks (preliminary, $m_c = 1.5$ GeV, $m_b = 4.8$ GeV)
- multiplicative matching
- extra $A_3 (\alpha_s \equiv \alpha_s^{(CMW)} + A_3 \alpha_s^3)$
- interfaced as a Pythia8 plugin
- hadronisation from Pythia8 (Vincia tune)

Observations:
- Promising start
- further tuning needed
- 4-jet matching would greatly help
- what about NNLL?