Spinning Waveforms from KMOC at Leading Order

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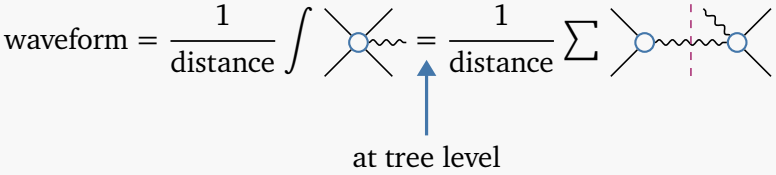
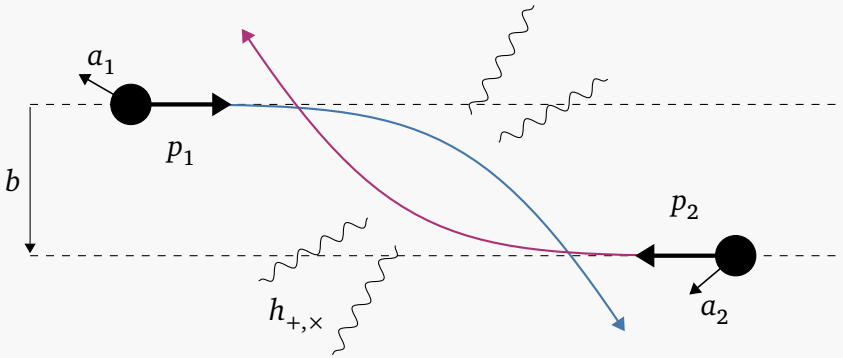
Motivation

- Waveform predictions for GW experiments (detection, parameter estimation, …)
- Amplitude-based approach is highly effective
- One of the challenges is to simplify the extraction of observables:
  1. precision demands $\sim \mathcal{O}(G^7)$
  2. unknown or hard-to-compute ingredients

Takeaway



Setup



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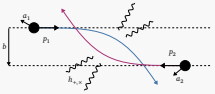


r

$\vec{n}$

[by DALL·E]

$$h_+ + \mathrm{i}\,h_\times \approx \frac{1}{r} f(t) \overset{\text{Fourier}}{\longleftrightarrow} \frac{1}{r} f(\omega)$$

Waveforms from amplitudes

[Cristofoli, Gonzo, Kosower, O'Connell 2022]

[Callum, Fei, Mathieu, Rodolfo's talks]

$$|\text{in}\rangle = \int_{p_1,p_2} \phi(p_1,p_2)\,|p_1p_2\rangle$$

$$f(\omega) \sim R_{\mu\nu\rho\sigma} = \lim_{\hbar\to 0}\langle \text{out}|\mathbb{R}_{\mu\nu\rho\sigma}|\text{out}\rangle = \lim_{\hbar\to 0}\langle \text{in}|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}S|\text{in}\rangle$$

$$= \lim_{\hbar\to 0}\int_{p_i,p_i'}\langle p_1'p_2'|S^\dagger\mathbb{R}_{\mu\nu\rho\sigma}S|p_1p_2\rangle \sim \lim_{\hbar\to 0}\int \mathscr{M}_5(p_1p_2\to p_1'p_2'k)$$

$$a(k)k_\mu\varepsilon_\nu k_\rho\varepsilon_\sigma$$

Classical limit

[Kosower, Maybee, O'Connell 2019]

"Classical" wavepacket $|\text{in}\rangle = \int_{p_1,p_2} \phi(p_1,p_2)\,|p_1 p_2\rangle$:

$$\begin{matrix}\text{Compton}\\ \text{wavelength}\end{matrix} \ll \begin{matrix}\text{wavepacket}\\ \text{spread}\end{matrix} \ll \begin{matrix}\text{particle}\\ \text{separation}\end{matrix}$$

Expand in $\hbar$ and keep only the leading term:

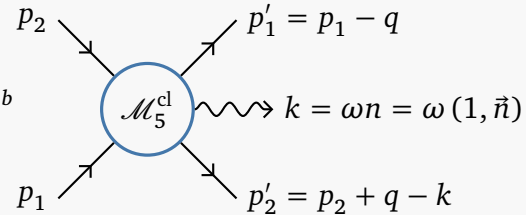
$$\kappa \sim \hbar^{-1/2} \qquad p_i - p_i' \sim \hbar \qquad k \sim \hbar \qquad a \sim \hbar^{-1}$$

$$\int_p \phi(p)\,(\dots) \to (\dots)\Big|_{p=p_{\text{classical}}}$$

Waveform at tree level

$$f(t) \sim \int \mathrm{d}\omega\, e^{-\mathrm{i}\omega t} \int \mathrm{d}\Phi(q)\, e^{-\mathrm{i}q\cdot b}$$

on-shell measure
fixes $q \cdot p_i$



$\mathscr{M}_5^{\mathrm{cl}}\,(q, \omega)$

Frequency dependence (non-spinning case)

In the classical limit, the amplitude is homogeneous in (q, k) of degree -2:

$$\mathscr{M}_5^{\text{cl}}(\lambda q, \lambda k) = \lambda^{-2} \mathscr{M}_5^{\text{cl}}(q, k)$$

$k = \omega n$, so rescale $q \to \omega q$:

$$\begin{aligned} f(t) &\sim \int \mathrm{d}\omega \, e^{-\mathrm{i}\omega t} \int \omega^2 \mathrm{d}\Phi(q) e^{-\mathrm{i}\omega q \cdot b} \omega^{-2} \mathscr{M}_5^{\text{cl}}(q, n) \\ &= \int \mathrm{d}\Phi(q) \, \delta(t + q \cdot b) \, \mathscr{M}_5^{\text{cl}}(q, n) \end{aligned}$$

Phase-space integration

One-dimensional integral remains:

- on-shell conditions fix $q \cdot p_1$ and $q \cdot p_2$
- frequency integration fixes $q \cdot b$

$$q = c_1 p_1 + c_2 p_2 + c_3 b + z q_\perp \quad \rightarrow \quad f(t) \sim \int_{-\infty}^{\infty} \mathrm{d}z \, \mathscr{M}_5^{\mathrm{cl}}(q(z))$$

Discovery of the complex plane

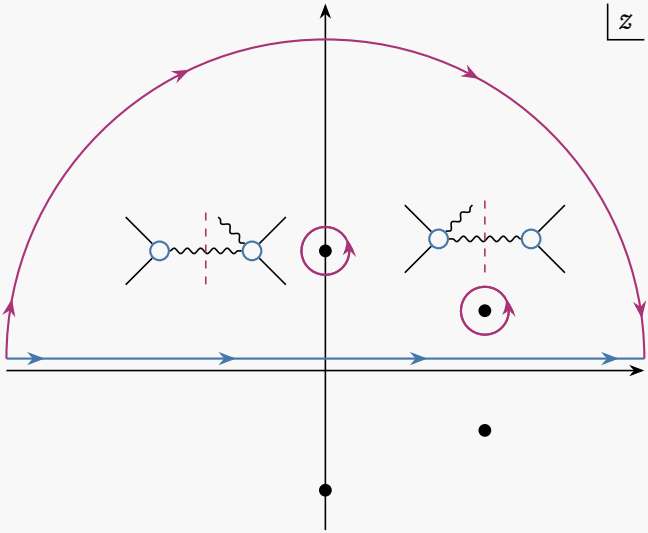
"One of the most remarkable discoveries in elementary particle physics has been that of the existence of the complex plane."

[Eden, Landshoff, Olive, Polkinghorne 1966]

Discovery of the complex plane

$$f(t) \sim \int_{-\infty}^{\infty} \mathrm{d}z\, \mathcal{M}_5^{\mathrm{cl}}(q(z))$$

$$= \sum_{\mathrm{UHP}} \mathrm{Res} + \text{arc at infinity}$$



Behavior at infinity (non-spinning case)

Homogeneity of degree -2: $\mathscr{M}_5^{\mathrm{cl}}(\lambda q, \lambda k) = \lambda^{-2}\mathscr{M}_5^{\mathrm{cl}}(q,k)$

large $z \sim$ large $q \sim$ small $k \sim$ soft limit: $\mathscr{M}_5^{\mathrm{cl}}(z) \sim k^{-1}q^{-1} \sim z^{-1}$

1. Integral diverges, should be understood as PV
2. Contribution from the arc at infinity is

$$\frac{1}{2}\operatorname*{Res}_{z=\infty} = -\frac{1}{2}\left(\sum_{\mathrm{UHP}}\mathrm{Res} + \sum_{\mathrm{LHP}}\mathrm{Res}\right)$$

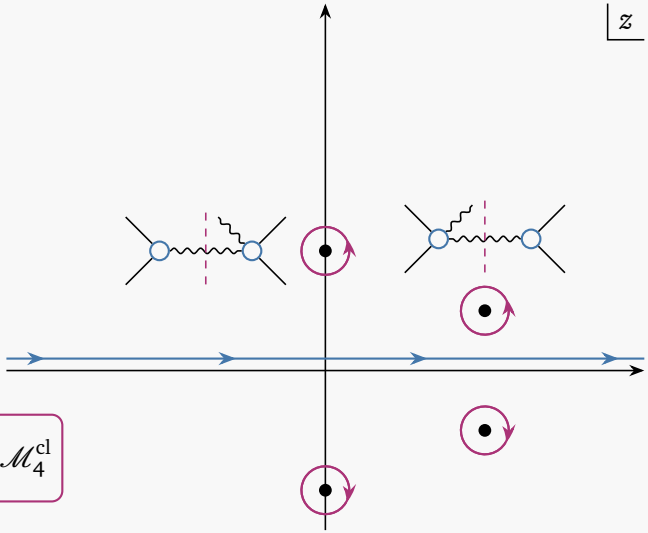
Discovery of the complex plane

$$f(t) \sim \int_{-\infty}^{\infty} \mathrm{d}z\, \mathcal{M}_5^{\mathrm{cl}}(q(z))$$

$$= \sum_{\mathrm{UHP}} \mathrm{Res} + \text{arc at infinity}$$

$$= \frac{1}{2}\left(\sum_{\mathrm{UHP}} \mathrm{Res} - \sum_{\mathrm{LHP}} \mathrm{Res} \right)$$

$$f(t) \sim \int \mathcal{M}_5^{\mathrm{cl}} \sim \sum \mathcal{M}_3^{\mathrm{cl}} \times \mathcal{M}_4^{\mathrm{cl}}$$



Spinning case

Expand $\mathcal{M}_5^{\rm cl}$ in spin, consider component of order $a_1^{s_1} a_2^{s_2}$:

1. Homogeneity in (q, k) of degree $\left(-2 + s_1 + s_2\right)$

2. Frequency dependence:

$$\mathcal{M}_5^{\rm cl} \sim \omega^{-2+s_1+s_2} \quad \Rightarrow \quad \int \mathrm{d}\omega \to \delta^{(s_1+s_2)}(t + q \cdot b)$$

3. Behavior at infinity:

$$\mathcal{M}_5^{\rm cl} \sim z^{-1+s_1+s_2} \quad \Rightarrow \quad \int \mathrm{d}\omega\, \mathcal{M}_5^{\rm cl} \sim z^{-1}$$

Kerr amplitudes

Three-point amplitude to $\mathcal{O}(a^\infty)$:

$$\sim (\varepsilon \cdot p)^2 \exp\left(\frac{\mathrm{i}\, m\varepsilon \cdot S \cdot k}{\varepsilon \cdot p}\right), \qquad \left(\frac{m\varepsilon \cdot S \cdot k}{\varepsilon \cdot p}\right)^2 = k \cdot S \cdot S \cdot k$$

Compton amplitude to $\mathcal{O}(a^4)$:

$$\sim \omega_0^2 \exp\left(\frac{\omega_1}{\omega_0}\right), \qquad \left(\frac{\omega_1}{\omega_0}\right)^2 = \frac{2\omega_2}{\omega_0}$$

[Lucile and Trevor's talks]

Checks

Our leading-order scattering waveform for two Kerr black holes up to $\mathcal{O}(a^4)$:

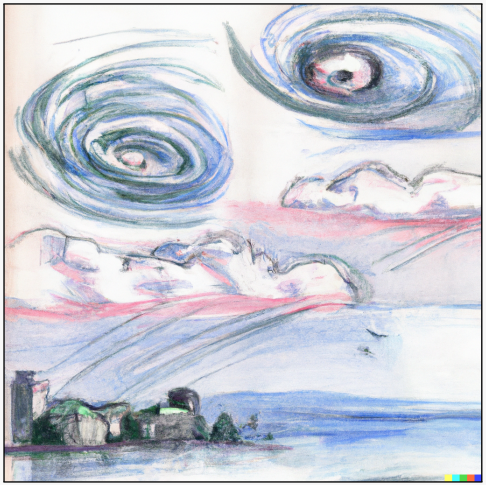
- matches the $\mathcal{O}(a^0)$ waveform [Kovacs, Thorne 1978]
- matches the $\mathcal{O}(a^2)$ worldline QFT result [Jakobsen, Mogull, Plefka, Steinhoff 2022] [Gustav's talk]
- was confirmed at $\mathcal{O}(a^4)$ [Aoude, Haddad, Heissenberg, Helset 2023; Brandhuber, Brown, Chen, Gowdy, Travaglini 2023]

Summary

1. Time-domain scattering waveform at leading order is algebraically related to the three-point amplitude and the Compton amplitude

2. We used this relation to compute the waveform for two Kerr black holes up to the fourth order in spin

3. This method may also simplify computations of other observables, and at higher orders



Thank you!