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Double Copies from Recursion

QCD Meets Gravity IX

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Plan of Talk

Introduction
- Motivation
- Spoiler
- Recursion Review

Gravity Recursion
- Building Blocks
- Color-Kinematic "*Denominators*"

A 5-pt example

Conclusion

Motivation

- Calculate Scattering Amplitudes.
- Perturbative QFT with Feynman Diagrams.
  - $gg \to gg$ # of diagrams $= 4$
  - $gg \to 10g$ # of diagrams $= 140,880,765,025$
  - $gg \to 15g$ # of diagrams $= 127,418,482,316,125$
- Scattering involving gravitons notoriously more complicated. (DeWitt 1967)
- Apply BCFW recursion (BCFW 2005, Heslop and Lipstein 2016)
- Double Copy (BCJ 2010, 2008)

Spoiler

Main Result

$$\mathcal{A}^{\mathrm{YM}}(1,\cdots,n) = \sum_{\vec{a}\in\mathfrak{S}(A)} \sum_{\Gamma} \frac{c^{\vec{a}}_{\alpha\beta}\; n\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)}{D\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)} \;\delta^{2\times 2}\left(\lambda\cdot\tilde{\lambda}\right)$$

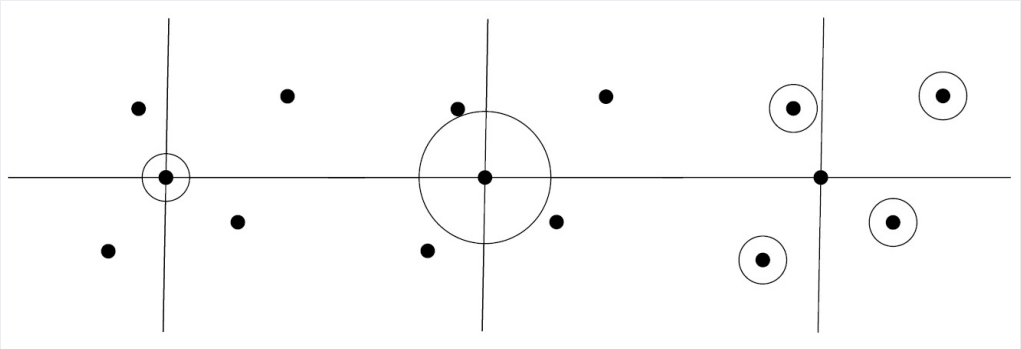
$$\mathcal{A}^{\mathrm{GR}}(1,\cdots,n) = \sum_{\vec{a}\in\mathfrak{S}(A)} \sum_{\Gamma} \frac{n\left(\Gamma^{\vec{a}}_{\alpha\beta}\right) n\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)}{D\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)} \delta^{2\times 2}\left(\lambda\cdot\tilde{\lambda}\right)$$

On-shell Recursion

Complex Shift and Cauchy's Theorem

$$\hat{p} = p + z \times r$$

$$\hat{P}_I^2 = P_I^2 + zP_I \cdot R_I$$



$$\mathcal{A}_n = \sum_{poles} \text{Res}\left(\frac{\hat{\mathcal{A}}_n(z)}{z}\right)_{poles}$$

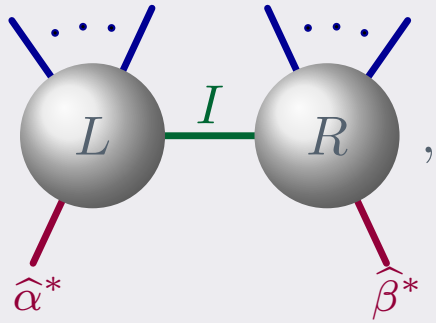
- Shifted Amplitude factorizes!
- A variation of the shift: BCFW $[i, j\rangle$

BCFW shift

- $p_\alpha \to \hat{p}_\alpha(z) = p_\alpha + z\lambda_\alpha\tilde{\lambda}_\beta$ and $p_\beta \to \hat{p}_\beta(z) = p_\beta - z\lambda_\alpha\tilde{\lambda}_\beta$
- $\tilde{\eta}_\alpha \to \tilde{\eta}_\alpha + z\tilde{\eta}_\beta$

Factorization

$$\mathcal{A}_L(\widehat{\alpha}^*,\cdots,I)\frac{1}{p_L^2}\mathcal{A}_R(I,\cdots,\widehat{\beta}^*) = \frac{1}{p_L^2}$$

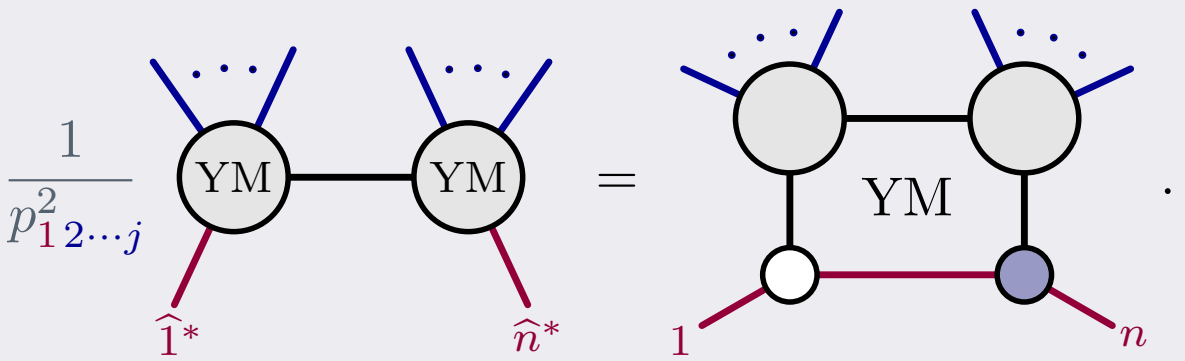


- $z^* = \frac{p_L^2}{\langle\alpha|(p_L)|\beta]}$

On-shell Diagrams/BCFW Bridge

- For color ordered diagrams:

Recursion Diagrammatics



- Recursively:

$$A^{YM}(1,2,...,n) = \sum_{\Gamma} \hat{f}_{\Gamma} \delta^{2\times 2}(\lambda \cdot \tilde{\lambda}) = \sum_{\Gamma} \frac{1}{\mathcal{J}(\Gamma)} \prod_{v} A_v$$

- $\mathcal{A}(1, \vec{\alpha}, n) = c^{\vec{\alpha}}_{1,n} A(1, \vec{\alpha}, n)$ (DDM 2000)

Building Blocks

3-point amplitudes (DSVW 2009, HL 2016)

MHV

$$\mathcal{A}(1,2,3) = \frac{\delta^{2\mathcal{N}}\left(\eta_1\lambda_1 + \eta_2\lambda_2 + \eta_3\lambda_3\right)}{\left(\langle 12\rangle\langle 23\rangle\langle 31\rangle\right)^{\mathcal{N}/4}}$$

$\overline{MHV}$

$$\bar{\mathcal{A}}(1,2,3) = \frac{\delta^{\mathcal{N}}\left(\eta_1\,[23] + \eta_2\,[31] + \eta_3\,[12]\right)}{\left([12]\,[23]\,[31]\right)^{\mathcal{N}/4}}$$

Double Copy and Color-Kinematic Duality

- As long as we represent (QCD) YM amplitudes in terms of color-kinematic dual numerators (meets) gravitational scattering amplitudes (Gravity) may be represented as "double copies" of the former.

- BCJ 2008, 2010

$$c_i + c_j + c_k = 0 \iff n_i + n_j + n_k = 0$$

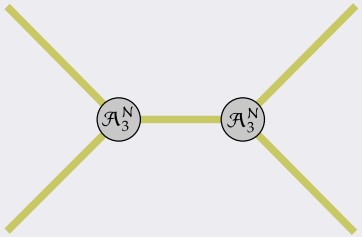
- Maldacena 1998 AdS/CFT correspondence

- KLT 1986

$$\mathcal{A}_{closed} \sim (\mathcal{A}_{open})^2$$

Double Copy and Recursion

A 4-point sketch



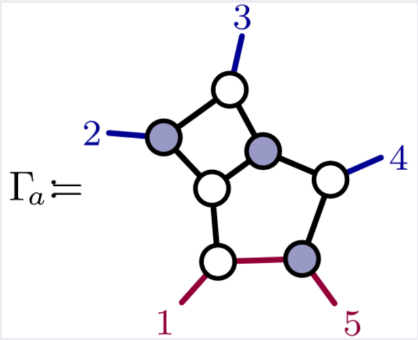
$$f_\Gamma^{YM} = \frac{1}{\mathcal{J}(\Gamma)} \mathcal{A}_3^{YM} \mathcal{A}_3^{YM}$$

$$f_\Gamma^{GR} = \frac{1}{\mathcal{J}(\Gamma)} \mathcal{A}_3^{GR} \mathcal{A}_3^{GR}$$

$$f_\Gamma^{GR} = \mathcal{J}(\Gamma) \left(f_\Gamma^{YM}\right)^2$$

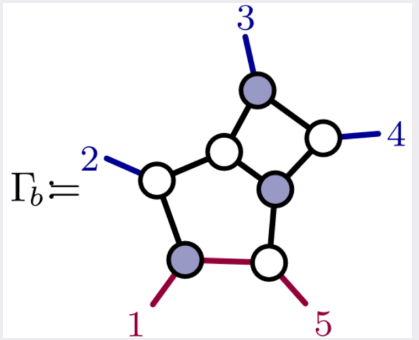
A 5-pt example

5-pt MHV on-shell diagram α



- $\mathcal{J}(\Gamma_\alpha) = \frac{[23]^2[45]}{\langle 14 \rangle}$

5-pt MHV on-shell diagram β



- $\mathcal{J}(\Gamma_\alpha) = \frac{[34]^2[12]}{\langle 25 \rangle}$

- Related by square moves and mergers (in YM)
- Gravitational diagrams only square moves

Color-Kinematic "*Denominators*" & Squaring

Gauge Theory

$$\mathcal{A}^{YM} = \sum_{\sigma} \sum_{\Gamma} \frac{c_\sigma n(\Gamma)}{D(\Gamma)} \delta^{2\times 2}(\lambda \cdot \tilde{\lambda})$$

- DDM basis: strip the color factor

$$A^{YM}(1,2,...,n) = \sum_{\Gamma} \hat{f}^{YM}_{\Gamma} \delta^{2\times 2}(\lambda \cdot \tilde{\lambda})$$

$$\hat{f}^{YM}_{\Gamma} = \frac{n}{D}$$

Gravity

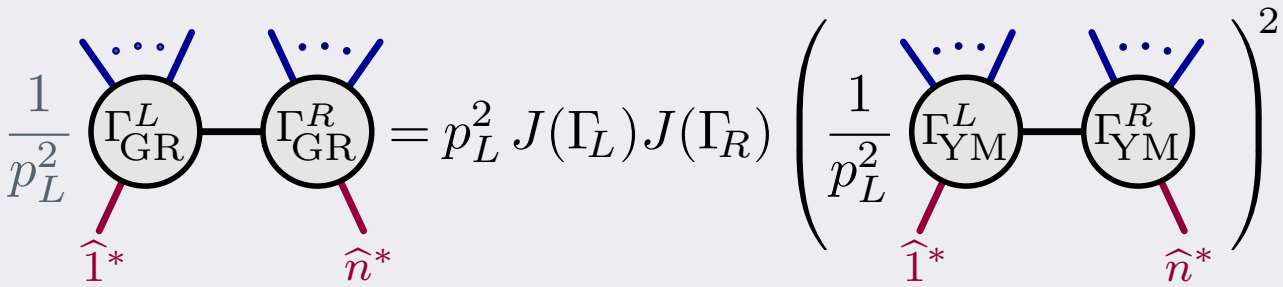
$$\mathcal{A}^{GR} = \sum_{\sigma} \sum_{\Gamma} \frac{n(\Gamma) n(\Gamma)}{D(\Gamma)} \delta^{2\times 2}(\lambda \cdot \tilde{\lambda})$$

- $n = D \hat{f}^{YM}_{\Gamma}$

$$\mathcal{A}^{GR} = \sum_{\sigma,\Gamma} D(\Gamma) \left(\hat{f}^{YM}_{\Gamma}\right)^2 \delta^{2\times 2}(\lambda \cdot \tilde{\lambda})$$

Denominators continued

Diagrammatically



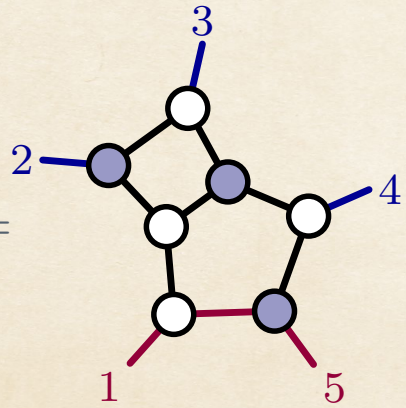
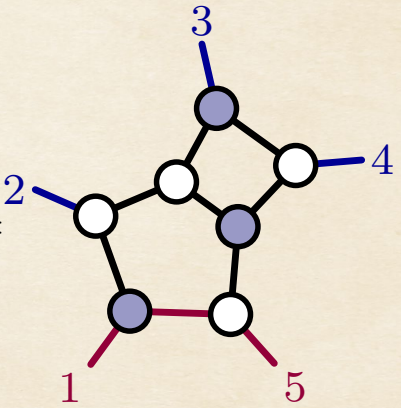
$$=: D(\Gamma)\left(\widehat{\mathfrak{f}}^{\rm YM}_\Gamma\right)^2 \delta^{2\times 2}\left(\lambda\cdot\widetilde{\lambda}\right).$$

How do we define those "Denominators"

- Choose the first and last labels for recursion.
- Diagram: $\Gamma[\hat{\alpha}^*, \vec{L}, I] \otimes \Gamma[I, \vec{R}, \hat{\beta}^*]$
- Denominators defined recursively: $D(\Gamma) = p^2_{\alpha L} D(\Gamma_L) D(\Gamma_R)$

A 5-pt example: The End

These denominators are not unique. Different recursion schemata lead to different denominators.

Γ_i	$\Gamma_a \coloneqq$ 	$\Gamma_b \coloneqq$ 
$J(\Gamma_i)$	$s_{51}\, s_{23}\, D(\Gamma_a)$	$s_{51}\, s_{34}\, D(\Gamma_b)$
$D(\Gamma_i)$	$s_{45}\, \dfrac{\langle 1\vert(2)\vert(3)\vert 4\rangle}{\langle 1\,4\rangle}$	$s_{12}\, \dfrac{\langle 2\vert(3)\vert(4)\vert 5\rangle}{\langle 2\,5\rangle}$
$n(\Gamma_i)$	$\dfrac{[2\,3][4\,5]}{\langle 1\,4\rangle\langle 2\,3\rangle\langle 5\,1\rangle}\,\delta^{2\times 4}(\lambda\cdot\widetilde{\eta})$	$\dfrac{[1\,2][3\,4]}{\langle 3\,4\rangle\langle 2\,5\rangle\langle 5\,1\rangle}\,\delta^{2\times 4}(\lambda\cdot\widetilde{\eta})$

$$\mathcal{A}_a^{GR} = \frac{\langle 1|(2)|(3)|4\rangle\,[45]\,\delta^{2\times 8}(\lambda\cdot\bar{\eta})}{\langle 14\rangle\langle 45\rangle\left(\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 51\rangle\right)^2}\delta^4 \qquad \mathcal{A}_b^{GR} = \frac{\langle 2|(3)|(4)|5\rangle\,[12]\,\delta^{2\times 8}(\lambda\cdot\bar{\eta})}{\langle 12\rangle\langle 25\rangle\left(\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle\right)^2}\delta^4$$

The End Continued

$$\sum_{\sigma(2,3,4)} \mathcal{A}_a^{GR}(1,2,3,4,5) = \sum_{\sigma(2,3,4)} \mathcal{A}_b^{GR}(1,2,3,4,5)$$

$$\sum_{\sigma(2,3,4)} \frac{\langle 1|(2)|(3)|4\rangle\,[45]\,\delta^{2\times 8}(\lambda\cdot\bar{\eta})}{\langle 14\rangle\langle 45\rangle\left(\langle 12\rangle\langle 23\rangle\langle 34\rangle\langle 51\rangle\right)^2}\delta^4 = \sum_{\sigma(2,3,4)} \frac{\langle 2|(3)|(4)|5\rangle\,[12]\,\delta^{2\times 8}(\lambda\cdot\bar{\eta})}{\langle 12\rangle\langle 25\rangle\left(\langle 23\rangle\langle 34\rangle\langle 45\rangle\langle 51\rangle\right)^2}\delta^4$$

- Although different expressions, the sum over the permutations of the other legs is the same (as should).

Conclusion

Result

$$\mathcal{A}^{\mathrm{YM}}(1,\cdots,n) = \sum_{\vec{a}\in\mathfrak{S}(A)} \sum_{\Gamma} \frac{c^{\vec{a}}_{\alpha\beta}\, n\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)}{D\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)} \delta^{2\times 2}(\lambda\cdot\tilde{\lambda})$$

$$\mathcal{A}^{\mathrm{GR}}(1,\cdots,n) = \sum_{\vec{a}\in\mathfrak{S}(A)} \sum_{\Gamma} \frac{n\left(\Gamma^{\vec{a}}_{\alpha\beta}\right) n\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)}{D\left(\Gamma^{\vec{a}}_{\alpha\beta}\right)} \delta^{2\times 2}(\lambda\cdot\tilde{\lambda})$$

- Non uniqueness of the "Denominators"
- These give rise to relations between partial amplitudes in YM. (BCJ Relations)
- At 6-points there is more variability. More relations for on-shell functions (A-H,C,K 2008)
- Color-kinematic dual numerators for higher order amplitudes?

Thank you!