UPPSALA
UNIVERSITET

KERR AMPLITUDES FROM HIGHER SPIN THEORY

QCD Meets Gravity, CERN

14th December 2023

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[2212.06120] & [2311.14668] & [2401.XXXXX] with

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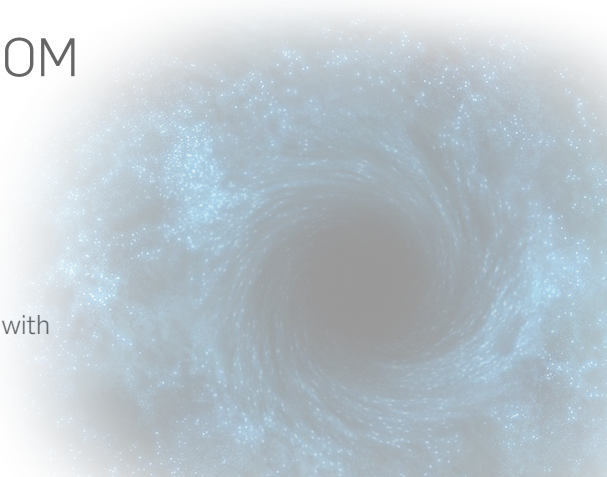


Structure

1. Introduction: 3pt amplitudes and Kerr

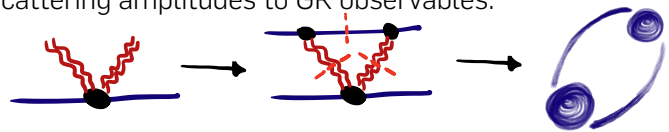
2. Construction of HS theories: non-chiral and chiral

3. Compton amplitudes: $\sqrt{\text{Kerr}}$ and Kerr

4. Classical analysis & fixing free contacts

Kerr from QFT

Pipeline from scattering amplitudes to GR observables:



inspired by [Buonanno, Khalil, O'Connell, Roiban, Solon, Zeng, Snowmass '22]

What are the scattering amplitudes relevant for Kerr BHs?

- 3pt Kerr amplitudes known [Arkani-Hamed, Huang, Huang '17; Guevara, Ochirov, Vines '18; Chung, Huang, Kim, Lee '18]

$$\mathcal{M}_{Kerr} = (\varepsilon \cdot p_1)^2 \left(\frac{\langle \mathbf{12} \rangle}{m} \right)^{2s} \quad \rightarrow \quad \text{lin. energy mom. tensor of Kerr BH}$$

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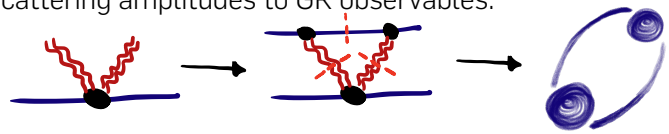

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$\mathcal{M}_{Kerr}$ from Higher Spin (HS) Theory [2212.06120]

HS constraints {massive gauge inv. + ...} **uniquely** predict 3pt Kerr & $\sqrt{\text{Kerr}}$ amps

Kerr from QFT at 3pts: Spin Variables

Express $\mathcal{M}_{Kerr}$ in spin variables [Maybee, O'Connell, Vines '19; Arkani-Hamed,Huang, O'Connell '19; Aoude, Haddad, Helset '20 '21]

$$\mathcal{M}_{Kerr} = (\varepsilon\cdot p_1)^2 \sum_{n=0}^{2s} \frac{1}{n!} \langle (q\cdot\hat{a})^n \rangle = (\varepsilon\cdot p_1)^2 \langle \exp(q\cdot\hat{a}) \rangle,$$

truncates at $2s$, **full quantum** amp.

1. particle 2 is **Lorentz boost** of particle 1: $p_2 = \Lambda p_1 = p_1 + q$,

2. re-express spinors $\rightarrow$ **expectation values of spin operator**

$$\langle \hat{a}^\mu \rangle = m^{-2s-1} \langle \bar{\mathbf{1}} |^{2s} \, \hat{S}^\mu \, | \mathbf{1} \rangle^{2s}$$

with Pauli-Lubanski op. $\hat{S}^\mu_{(s)} = \frac{1}{2m} \epsilon^{\mu\nu\rho\sigma} p_{1\nu} M_{(s)\rho\sigma}$,
$|\mathbf{1}\rangle = |1^a\rangle \, z_a$, $|\bar{\mathbf{1}}\rangle = |1^a\rangle \, \bar{z}_a$

Need infinite spin

1. for **full tower** of multipoles

2. to get correct **classical** spin multipoles

[Guevara, Ochirov, Vines '18 & '19; Arkani-Hamed,Huang, O'Connell '19] [LC, Pichini '22]

3pt amp. any theory

$$\mathcal{M} = (\varepsilon\cdot p_1)^2 \sum_{n=0}^{2s} c_n(s)\langle(q\cdot\hat{a})^n\rangle,$$

Leading Regge Superstring [LC, Pichini '22]

$c_0^{(s)} = c_1^{(s)} = 1,\ c_2^{(s)} = \frac{4s^2-7s+4}{(2s)(2s-1)}\cdots$

Classical matching only for

$$(\varepsilon\cdot p_1)^2 \sum_{n=0}^{\infty} c_n^{(\infty)}\langle(q\cdot\hat{a})^n\rangle \sim \varepsilon_{\mu\nu}\, T^{\mu\nu}_{\text{cl. string}}$$

where classical coefficients are $c_n^{(\infty)} := \lim_{s\to\infty} c_n^{(s)}$

Kerr

spin multipole coeffs.
$c_n^{(s)} = \frac{1}{n!}$

spin universality $c_n^{(\infty)} = c_n^{(s)}$

Higher Spin Constraints: Spin 1 example

start from spontaneously broken gauge theory

$$\mathcal{L}_{SSB} = -\frac{1}{4}F_{\mu\nu}^2 - 2|D_\mu W_\nu|^2 + |m W_\mu - D_\mu \phi|^2 - ieF_{\mu\nu} W^\mu \bar{W}^\nu + \dots$$

Current constraint

$\Longrightarrow$ softer high energy behaviour at tree level

$$p_1 \cdot \frac{\partial}{\partial \epsilon_1} V(W\bar{W}A)\Big|_{(2,3)} = \mathcal{O}(m)$$

where $V(W\bar{W}A) =$ offshell 3pt vertex

Massive gauge symmetry

$\mathcal{L}_{SSB}$ invariant under massive gauge symmetry

$$\delta W_\mu = \partial_\mu \xi + \dots, \qquad \delta\phi = m\xi + \dots$$

Ward identities:

$$m V(\phi \bar{W} A) - i p_1 \cdot \frac{\partial}{\partial \epsilon_1} V(W\bar{W}A)\Big|_{(2,3)} = 0$$

Bottom up construction of HS theories

Free theory for massive spin$-s$ [Zinoviev '01]

- d.o.f massless spin$-s \to \begin{cases} s+1 \text{ fields } \Phi^k & \Phi^k := \Phi^{\mu_1\cdots\mu_k} \\ \delta\Phi^k = \partial^{(1}\xi^{k-1)} + m\xi^k \dots & \xi^k := \xi^{\mu_1\cdots\mu_k} \end{cases}$
- free Lag. $\mathcal{L}_0 = -\sum_{k=0}^{s} \frac{(-1)^k}{2}\left[\Phi^k(\Box + m^2)\Phi^k - \frac{k(k-1)}{4}\tilde{\Phi}^k(\Box+m^2)\tilde{\Phi}^k\right] + \mathcal{L}_{\text{off-diag.}}$

Constraining interactions $V_{\Phi^k\bar{\Phi}^s A^h}$ [2212:06120] $\quad h = \begin{cases} 1, \text{ gauge th.} \\ 2, \text{ gravity} \end{cases}$

(MC) minimal coupling extension of $\mathcal{L}_0$

(PC) power-counting bound $\quad V_{\Phi^k\bar{\Phi}^s A^h} \sim \partial^{k+s-2h}\left(F_{\mu\nu}\right)^h$

(WI) massive Ward identities $\quad m V_{\Phi^k\bar{\Phi}^s A^h} - p\cdot\frac{\partial}{\partial\epsilon} V_{\Phi^{k+1}\bar{\Phi}^s A^h} + \left(\frac{\partial}{\partial\epsilon}\right)^2 V_{\Phi^{k+2}\bar{\Phi}^s A^h}\big|_{(2,3)} = 0$

(CC) current constraint $\quad p\cdot\frac{\partial}{\partial\epsilon} V_{\Phi^s\bar{\Phi}^s A^h}\big|_{(2,3)} = \mathcal{O}(m)$

(ND) Near-diagonal interactions: no interaction if $|s_1 - s_2| > 1$

Results from HS constraints

3pt amps. from Higher Spin (HS) Theory [2212.06120]

constraints {**(MC)**,**(PC)**,**(WI)**,**(CC)**} **uniquely** predict $\mathcal{M}_{Kerr}$
{...+ **(ND)**} **uniquely** predict $\sqrt{\text{Kerr}}$ (tested $s \leq 6$)

Beyond 3pt: Compton amp. [2212.06120, 2311.14668]

diag. propagator: $\Delta(\epsilon,\bar{\epsilon}) = \sum_{s=0}^{\infty} (\epsilon)^s \cdot \Delta^{(s)} \cdot (\bar{\epsilon})^s = \frac{1}{p^2 - m^2 + i0} \frac{1 - \frac{1}{4}\epsilon^2\bar{\epsilon}^2}{1 + \epsilon\cdot\bar{\epsilon} + \frac{1}{4}\epsilon^2\bar{\epsilon}^2}$

- gauge th. spin-2 : fixing 3pt data + 4pt **(WI)**
  $\implies$ **3 free contact terms** in $A_{\sqrt{\text{Kerr}}}(\Phi_1^2\Phi_2^2 A_3^- A_4^+)$
- studied $s = 3$ gauge th. & gravity

- not yet unique – missing 4pt **(CC)** or other constraint?
- computationally intense for $s \geq 3$

Chiral Formalism for Higher Spins

[Ochirov, Skvortsov '22]

Trade $\mathrm{SO}(1,3)$ tensors $\Phi_{\mu_1\ldots\mu_s} \longrightarrow \mathrm{SL}(2,\mathbb{C})$ chiral symmetric tensors $\Phi_{\alpha_1\ldots\alpha_{2s}}$

- $|\Phi\rangle := \Phi_{\alpha_1\ldots\alpha_{2s}} \to$ **correct dof's** for a massive spin-s field: $2s+1$
- simple free Lagrangian:

$$\mathcal{L}_0^{(s)} = \langle \partial_\mu \Phi | \partial^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle \,,$$

- chiral formulation maps spinor-helicity formulae to Lagrangians
  $\implies$ external wavefunctions $|\Phi\rangle \to \frac{1}{m^s} |\mathbf{1}\rangle^{\odot 2s}$
- $\mathcal{L}_0$ not parity invariant $\to$ needs to be enforced

Chiral Lagragians

$\sqrt{\text{Kerr}}$ Lagrangian [2311.14668]

$$\mathcal{L}_{\sqrt{\text{Kerr}}} = \langle D_\mu \Phi | D^\mu \Phi \rangle - m^2 \langle \Phi | \Phi \rangle - \underbrace{g \sum_{k=0}^{2s-1} \frac{1}{m^{2k}} \langle \Phi | \left\{ \left(|\overleftarrow{D}|\overrightarrow{D}| \right)^{\odot k} \odot |F^-| \right\} |\Phi\rangle}_{\text{non-min. term fixed by parity}} + \mathcal{L}_4$$

resummed non-minimal term:

$$\frac{1}{m^{4s-2}} \langle \Phi | \left\{ \frac{m^{4s} - |\overleftarrow{D}|\overrightarrow{D}|^{\odot 2s}}{m^2 - |\overleftarrow{D}|\overrightarrow{D}|} \odot |F^-| \right\} |\Phi\rangle$$

$\mathcal{L}_4 \to$ general 4pt operators in indep. helicity sectors:

- $\mathcal{L}_4^{++} = 0$
- $\mathcal{L}_4^{-+} \propto |F^-| \odot |\overleftarrow{D}|F^+|\overrightarrow{D}|$
- $\mathcal{L}_4^{--} \propto |F^-| \odot |F^-|$

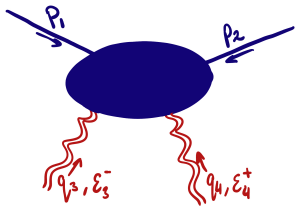
General spin $\sqrt{\text{Kerr}}$ color-ordered amplitudes

$\sqrt{\text{Kerr}}$ abelian Compton Amplitudes [2311.14668]

$$A^{\text{abel.}}(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^2}{m^{4s}t_{13}t_{14}}P_1^{(2s)} - \frac{\langle \mathbf{13}\rangle\langle 3|1|4][\mathbf{24}]}{m^{4s}t_{13}}P_2^{(2s)} + \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s}}P_2^{(2s-1)} - \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s-2}}\langle \mathbf{12}\rangle[\mathbf{12}]P_4^{(2s-1)} + C^{(s)},$$

- matches $s \leq 3$ results from massive gauge inv.
- simple polynomials $P_n^{(k)}$ encode general spin
- contact terms shown generated by $\mathcal{L}_3$ & consistent with non abel.
- $\mathcal{L}_4 \to C^{(s)}$ taken to be $C^{(s)}(P_n^{(k)})$

$$U = \tfrac{1}{2}\left(\langle \mathbf{1}|4|\mathbf{2}] - \langle \mathbf{2}|4|\mathbf{1}]\right) - m[\mathbf{12}]$$
$$V = \tfrac{1}{2}\left(\langle \mathbf{1}|4|\mathbf{2}] + \langle \mathbf{2}|4|\mathbf{1}]\right),\ t_{ij} = 2p_i\cdot p_j$$



Kerr Variables and Polynomials

- 4 local helicity-indep. spin-$1/2$ variables

$$\varsigma_1 = U+V, \quad \varsigma_2 = U-V, \quad \varsigma_3 = -m\langle\mathbf{12}\rangle, \quad \varsigma_4 = -m[\mathbf{12}],$$

- **polynomials $P_n^{(k)}$ of degree $k-n+1$**

$$P_n^{(k)} = \frac{\varsigma_1^k}{(\varsigma_1-\varsigma_2)(\varsigma_1-\varsigma_3)\dots(\varsigma_1-\varsigma_n)} + \text{perm}(\varsigma_1,\varsigma_2,\dots\varsigma_n)\,.$$

-$n=2$ example:

$$P_2^{(2s)} = \frac{\varsigma_1^{2s}}{\varsigma_1-\varsigma_2} + \frac{\varsigma_2^{2s}}{\varsigma_2-\varsigma_1} = \sum_{i=0}^{2s-1} \varsigma_1^i \varsigma_2^{2s-1-i}$$

- complete homogeneous symmetric polynomials
- naturally generated by $\mathcal{L}_{\sqrt{\text{Kerr}}}$
- generalises to Kerr

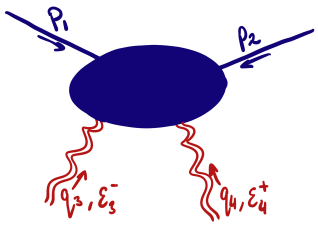
we will construct general spin contact terms out of $P_n^{(k)}$

General spin Kerr amplitudes

Kerr Compton [2401.XXXXX]

$$M(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|1|4]^4}{m^{4s}s_{12}t_{13}t_{14}}P_1^{(2s)} + \frac{\langle 3|1|4]^3\langle \mathbf{13}\rangle[\mathbf{42}]}{m^{4s}s_{12}t_{13}}P_2^{(2s)} + \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s}s_{12}}\langle 3|1|4]^2 P_2^{(2s-1)}$$
$$+ \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]\langle 3|1|4]\langle 3|\rho|4]}{m^{4s-2}s_{12}}\left(P_2^{(2s-2)} - m^2\langle \mathbf{12}\rangle[\mathbf{12}]P_4^{(2s-2)}\right)$$
$$+ \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]\langle 3|\rho|4]^2}{m^{4s-4}s_{12}}P_4^{(2s-1)} + C^{(s)}$$

- matches known $s \leq 2$ Compton amplitudes
  [Arkani-Hamed, Huang, Huang '17]
- matches previous $s = 5/2$ result
  [Chiodaroli, Johansson, Pichini '21]
- obtained from 3pt vertices
- $C^{(s)} = C^{(s)}(P_n^{(k)})$ for $s \geq 3$

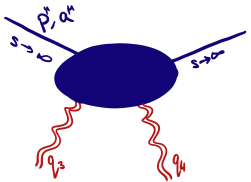


Classical Limits at 4pt

classical scaling: small deflection $\implies$ $q, q_\perp \sim \mathcal{O}(\hbar)$

Large spin limit

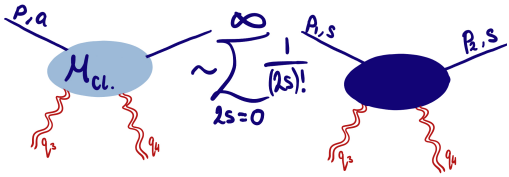
[Guevara, Ochirov, Vines '18 '19;
Arkani-Hamed,O'Connell, Huang '19]



- $s \to \infty$ s.t. $\hbar s \sim \mathcal{O}(1)$
- note $m|a| = m\sqrt{-\langle \hat{a} \rangle^2} = \hbar s$
- cl. ring radius: $a^\mu \dots a^\nu = \langle \hat{a}^\mu \dots \hat{a}^\nu \rangle$

Coherent spin states

[Aoude, Ochirov '22]



- classical amp.
  $\sum_{2s=0}^{\infty} e^{-|z|^2} \frac{1}{(2s)!} \mathcal{M}(\mathbf{1}^s, \mathbf{2}^s, 3^-, 4^+)$
- scaling: $z^a, \bar{z}^a \sim \frac{1}{\sqrt{\hbar}}$ and $|z|^2 = \frac{2m^2|a|}{\hbar}$

the two approaches generate the same classical amplitude!

Fixing $\sqrt{\text{Kerr}}$ Contact terms

Fixing $C^{(s)}$ by [2311.14668]

1. ansatz using $P_2^{(k)}$ & $P_4^{(k)}$
2. require finite $A^{\text{abel.}}$ in classical limit (for $C^{(s)} = 0$ it diverges)
3. compatible with HS symm. at $s \leq 3$
4. match exponential up to $\mathcal{O}(a^2)$ [Guevara, Ochirov, Vines '18 '19;Chung, Huang, Kim, Lee '18; Aoude, Haddad, Helset '20 '21]

Note: polynomials $P_2^{(k)}$ generate exponentials in classical limit, eg

$$\sum_{2s=0}^{\infty} \frac{1}{(2s)!} P_2^{(2s)} = \frac{e^{\varsigma_1}}{\varsigma_1 - \varsigma_2} - \frac{e^{\varsigma_2}}{\varsigma_1 - \varsigma_2}$$

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$\sqrt{\text{Kerr}}$ abelian contact term

$$C^{(s)} = \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{2m^{4s-3}}(\langle \mathbf{12}\rangle + [\mathbf{12}])\left(P_4^{(2s)} - P_2^{(2s-2)}\right)$$

Not unique for quantum contacts but **unique in classical limit** given our ansatz space

Classical $\sqrt{\text{Kerr}}$ amplitudes

$$\mathcal{A}_{\text{cl}}=-(p\cdot\chi)^2\left(\frac{[T^{c_3},T^{c_4}]}{q^2\,p\cdot q_\perp}+\frac{\{T^{c_3},T^{c_4}\}}{2(p\cdot q_\perp)^2}\right)\left(e^x\cosh z-we^x\text{sinhc}z+\frac{w^2-z^2}{2}E(x,y,z)\right)$$
$$+(p\cdot\chi)^2\frac{[T^{c_3},T^{c_4}]}{q^2\,(p\cdot q_\perp)^2}\frac{w^2-z^2}{2}i\epsilon(q_\perp,p,q,a)\tilde{E}(x,y,z)$$

Compton kinematics:

$q=p_3+p_4,\ \ q_\perp=p_4-p_3,$
+ helicity dep. vector $\chi=\langle 3|\sigma|4]$
$\Longrightarrow$ 4 spin-dep. vars.

$$x=a\cdot q_\perp\,,\ y=a\cdot q\,,$$
$$z=|a|\frac{p\cdot q_\perp}{m}\,,\ w=\frac{a\cdot\chi\,p\cdot q_\perp}{p\cdot\chi}.$$

Entire functions:

$$E(x,y,z)=\frac{e^y-e^x\cosh z+(x-y)e^x\,\text{sinhc}\,z}{(x-y)^2-z^2}\ +\ (y\to -y)$$

$$\tilde{E}(x,y,z)=\frac{4xe^x\cosh z-2(x^2-y^2+z^2)e^x\,\text{sinhc}\,z}{\big((x-y)^2-z^2\big)\big((x+y)^2-z^2\big)}$$
$$-\frac{4x\cosh y+2(x^2+y^2-z^2)\,\text{sinhc}\,y}{\big((x-y)^2-z^2\big)\big((x+y)^2-z^2\big)}.$$

Classical $\sqrt{\text{Kerr}}$ amplitude

Mulitpole expansion of $\mathcal{A}_{\text{cl}}$

$$\mathcal{A}_{\text{cl}}^{\text{abel.}} = \mathcal{A}_0^{\text{abel.}} \Big(\underbrace{1 + (x-w) + \frac{1}{2}(x-w)^2}_{\text{matches low order exponential}} + \frac{1}{3!}(x-w)(2w^2 - 4wx + 2x^2 - y^2) + \mathcal{O}(a^4) \Big)$$

Compton kinematics:
$q = p_3 + p_4,\ q_\perp = p_4 - p_3,$
+ helicity dep. vector $\chi = \langle 3|\sigma|4]$
$\implies$ 4 spin-dep. vars.

$$x = a \cdot q_\perp \,, y = a \cdot q \,,$$
$$z = |a| \frac{p \cdot q_\perp}{m} \,, w = \frac{a \cdot \chi \, p \cdot q_\perp}{p \cdot \chi}.$$

- $\mathcal{O}(a^3)$ break of spin shift symmetry
  $a \to a + q \implies x \to x, y \to y + q^2, w \to w,$
  $z^2 \to z^2 + \frac{(p \cdot q_\perp)^2}{m^2}(2y + q^2)$
  [Bern, Kosmopoulos, Luna, Roiban, Teng '22; Aoude, Haddad, Helset '22]
- even in $z \implies$ no $|a|$ terms
- recent work by [Scheopner, Vines '23]

Classical Kerr amplitudes

Classical Kerr amplitude

$$\mathcal{M}_{\text{Kerr}}(\mathbf{1},\mathbf{2},3^-,4^+) = \frac{(p\cdot\chi)^4}{q^2\,(p\cdot q_\perp)^2}\left(e^x\cosh z - w\,e^x \text{sinhc}\, z + \frac{w^2-z^2}{2}E(x,y,z)\right)$$
$$+\frac{(p\cdot\chi)^3}{q^2\,(p\cdot q_\perp)^2}\frac{w^2-z^2}{2}i\epsilon(\chi,p,q,a)\tilde{E}(x,y,z) + C^{(s)}$$

Compton kinematics:

$q = p_3 + p_4,\ \ q_\perp = p_4 - p_3,$

+ helicity dep. vector $\chi = \langle 3|\sigma|4]$

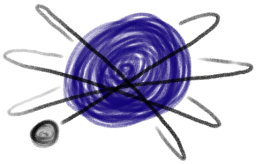
$\Longrightarrow$ 4 spin-dep. vars.

$$x = a\cdot q_\perp\,,\ y = a\cdot q\,,$$
$$z = |a|\frac{p\cdot q_\perp}{m}\,,\ w = \frac{a\cdot\chi\, p\cdot q_\perp}{p\cdot\chi}.$$

- same E, $\tilde{E}$ as $\sqrt{\text{Kerr}}$
- naive classical double copy: 1st line ✓
  2nd line ✗
- even in z
- free contact terms $C^{(s)}$

Fixing Kerr contact terms

1. ansatz $C^{(s)}(P_n^{(k)})$
2. compatible with massive gauge invariance
3. input low order data from classical GR calculations:

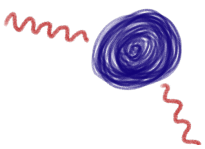


self-force $\implies$ data up to $\mathcal{O}(a^4)$

[Siemonsen, Vines '19]

[GOV '18 '19; Chung et al. '18, Aoude et al. '20 '21]

4. compare to classical GR calculations:



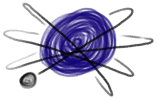
black hole perturbation theory (BHPT):

[Bautista, Guevara, Kavanagh, Vines '22] $\implies$ data up to $\mathcal{O}(a^6)$

[Bautista, Bonelli, Iossa, Tanzini, Zhou '23] $\implies$ data up to $\mathcal{O}(a^8)$

Fixing $\mathcal{O}(a^4)$

self-force calc.
[Siemonsen, Vines '19]



$$\implies \mathcal{M}_{\text{exp}} = \mathcal{M}_0\, e^{x-w} \text{ for } \mathcal{O}(a^{\leq 4})$$
$$= \mathcal{M}_0\big(1 + (x-w) + \frac{1}{2}(x-w)^2 + \frac{1}{3!}(x-w)^3 + \frac{1}{4!}(x-w)^4 + \mathcal{O}(a^5)\big)$$

ansatz $C_1^{(s)}$ has **1 free parameter** : fixed using $\mathcal{M}_{\text{Kerr}} - \mathcal{M}_{\text{exp}} = \mathcal{O}(a^5)$

$$x = a\cdot q_\perp, y = a\cdot q, z = |a|\tfrac{p\cdot q_\perp}{m}, w = \tfrac{a\cdot\chi}{p\cdot\chi}p\cdot q_\perp, \xi^{-1} = \tfrac{q^2m^2}{(p\cdot q_\perp)^2}$$

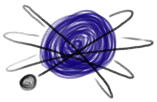
Proposal for Kerr contact terms

$$C_1^{(s)} = \Big(\langle 3|\rho|4]^2 - \langle 3|\bar{\rho}|4]^2\Big)^2\Big(P_5^{(2s)}[\varsigma_1] + P_5^{(2s)}[\varsigma_2] - 2P_4^{(2s-1)} - P_3^{(2s-2)}[\varsigma_1] - P_3^{(2s-2)}[\varsigma_2] + 2P_2^{(2s-1)}\Big)$$

$P_n^{(2s)}[\varsigma'] := \lim_{\varsigma_n\to\varsigma'} P_n^{(2s)}$

Fixing $\mathcal{O}(a^4)$

self-force calc.
[Siemonsen, Vines '19]



$$\implies \mathcal{M}_{\text{exp}} = \mathcal{M}_0\, e^{x-w} \text{ for } \mathcal{O}(a^{\leq 4})$$
$$= \mathcal{M}_0\big(1 + (x-w) + \frac{1}{2}(x-w)^2 + \frac{1}{3!}(x-w)^3 + \frac{1}{4!}(x-w)^4 + \mathcal{O}(a^5)\big)$$

ansatz $C_1^{(s)}$ has **1 free parameter** : fixed using $\mathcal{M}_{\text{Kerr}} - \mathcal{M}_{\text{exp}} = \mathcal{O}(a^5)$

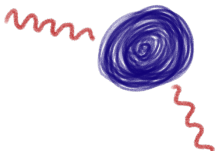
$$x = a\cdot q_\perp, y = a\cdot q, z = |a|\tfrac{p\cdot q_\perp}{m}, w = \tfrac{a\cdot\chi}{p\cdot\chi}p\cdot q_\perp, \xi^{-1} = \tfrac{q^2m^2}{(p\cdot q_\perp)^2}$$

Proposal for Kerr contact terms

$$C_1^{(\infty)} = \frac{(w^2-z^2)^2}{2\xi}\Big(\frac{e^{-y}(x+y)}{y((x+y)^2-z^2)^2} + \frac{e^{x+z}}{2z}\Big(\frac{1}{(x+z)^2-y^2} - \frac{2x+2z}{((x+z)^2-y^2)^2}\Big)\Big) + (y\to -y)$$

Comparing to BHPT Results

$$\mathcal{M}_{\text{BHPT}} = \mathcal{M}_0\Big(e^{x-w}+f(x,y,w,\xi)\Big)$$



using BHPT techniques f fixed up to:

- $\mathcal{O}(a^6)$ for generic α, η
  [Bautista, Guevara, Kavanagh, Vines '22]
- $\mathcal{O}(a^8)$ for $\alpha = 1, \eta = 0$
  [Bautista, Bonelli, Iossa, Tanzini, Zhou '23]

$\mathcal{M}_{\text{BHPT}}$ in [Bautista, Guevara, Kavanagh, Vines '22] has four contributions

	$\alpha = 0$	$\propto \alpha$
$\eta = 0$ conservative	✓	$C_2^{(s)}, C_3^{(s)} \dots$
$\propto \eta$ non-conservative	$\tilde{C}_1^{(s)}$	$\tilde{C}_2^{(s)}, \tilde{C}_3^{(s)} \dots$

Comparing to BHPT: $\alpha = 0$ contribution

- $\mathcal{M}_{\text{Kerr}}$ (with $C_1^{(s)}$) $\Longrightarrow$ **predicts** $\mathcal{M}_{\text{BHPT}}|_{\alpha,\eta=0}$ **up to** $\mathcal{O}(a^7)$
- constructing $\tilde{C}_1^{(s)}$ using $\mathcal{O}(a^5)$ $\Longrightarrow$ **predicts** $\mathcal{M}_{\text{BHPT}}|_{\alpha,\eta=1}$ term **up to** $\mathcal{O}(a^7)$

[Bautista, Guevara, Kavanagh, Vines '22]

Mismatch when $C^{(s)} = 0$ (for $\alpha = 0$)

$$\mathcal{M}_{\text{Kerr}} - \mathcal{M}_{\text{BHPT}}\Big|_{\alpha=0} = -\mathcal{M}_0 \xi^{-1}\Big[(w^2-z^2)^2\Big(\tfrac{1}{24} + \tfrac{1}{40}x + \tfrac{1}{720}(6x^2+y^2+2z^2)\Big)$$
$$+ \eta z (w^2-z^2)(w-x-y)(w-x+y)\Big(\tfrac{1}{120} + \tfrac{1}{180}x\Big) + \mathcal{O}(a^7)\Big]$$

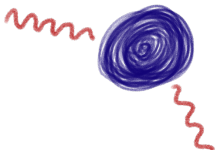
$$x = a\cdot q_\perp, y = a\cdot q, z = |a|\tfrac{p\cdot q_\perp}{m}, w = \tfrac{a\cdot\chi}{p\cdot\chi} p\cdot q_\perp, \xi^{-1} = \tfrac{q^2 m^2}{(p\cdot q_\perp)^2}$$

Summary of Classical Kerr Results

properties of our $\mathcal{M}_{\text{Kerr}}$

- constructed from entire functions $\implies$ all orders in spin
- spurious pole free (ie. no $w^{>4}$ terms)
- matches known BHPT results

$\mathcal{M}_{\text{Kerr}}$ contrib.	$\alpha = 0$	$\propto \alpha$
$\eta = 0$ conservative	**predictive**	✓
$\propto \eta$ non-conservative	**predictive**	✓



Open Qs:

- physical meaning of $\alpha = 0$? [Bautista, Bonelli, Iossa, Tanzini, Zhou '23]
- are there finite number of C_i's that capture $\alpha \neq 0$
- can we capture non-perturbative contributions

Review & Outlook

In this talk

- HS theory + ... $\Longrightarrow$ **uniqueness** of 3pt, not yet 4pt
- identify **general spin structures of quantum amps.** for Kerr that is compatible with HS symm.
- proposal all-spin $\sqrt{\text{Kerr}}$ & Kerr Compton amps: **Quantum** + **Classical**
- **predict Kerr** for $\alpha = 0$
- polynomial basis $\Longrightarrow$ can match $\alpha \neq 0$ results in [Bautista, Guevara, Kavanagh, Vines '22] & [Bautista, Bonelli, Iossa, Tanzini, Zhou '23]

Outlook

- 4pt non-uniqueness from HS symm - need new constraints
- comparisons to recent work [Scheopner, Vines '23]
- compare to higher orders in BHPT calc. $\mathcal{O}(a^{\geq 7})$
- constructing observables

General spin non-abelian $\sqrt{\text{Kerr}}$ amplitudes

color-ordered $\sqrt{\text{Kerr}}$ Compton

$$A(\mathbf{1}^s,\mathbf{2}^s,3^-,4^+) = \frac{\langle 3|\mathbf{1}|4]^2(U+V)^{2s}}{m^{4s}s_{12}t_{14}} - \frac{\langle \mathbf{13}\rangle\langle 3|\mathbf{1}|4][\mathbf{42}]}{m^{4s}s_{12}}P_2^{(2s)} + \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s}s_{12}}t_{13}P_2^{(2s-1)}$$
$$- \frac{\langle \mathbf{13}\rangle\langle \mathbf{32}\rangle[\mathbf{14}][\mathbf{42}]}{m^{4s-2}s_{12}}\left(t_{13}\langle \mathbf{12}\rangle[\mathbf{12}]P_4^{(2s-1)} + 2VP_4^{(2s)}\right) + C^{(s)}$$

Proposed contact terms

Proposal for Kerr contact terms to match $\eta = 0$ up to $\mathcal{O}(a^6)$ [2311.14668]

$$C_1^{(s)} = \Big(\langle 3|\rho|4]^2 - \langle 3|\bar{\rho}|4]^2\Big)^2 \Big(P_5^{(2s)}[\varsigma_1] + P_5^{(2s)}[\varsigma_2] - 2P_4^{(2s-1)} - P_3^{(2s-2)}[\varsigma_1] - P_3^{(2s-2)}[\varsigma_2] + 2P_2^{(2s-1)}\Big)$$

$$C_2^{(s)} = \alpha \langle 3|\rho|4]\langle 3|\bar{\rho}|4]\Big(\langle 3|\rho|4]^2 + 3\langle 3|\bar{\rho}|4]^2\Big)\Big(P_5^{(2s)}[\varsigma_1] - P_5^{(2s)}[\varsigma_2] - (\varsigma_1 - \varsigma_2)P_4^{(2s-2)}\Big)\,,$$

$$C_3^{(s)} = \alpha\Big(\langle 3|\rho|4]^4 + 12\langle 3|\rho|4]^2\langle 3|\bar{\rho}|4]^2 + 3\langle 3|\bar{\rho}|4]^4\Big)(\varsigma_1 - \varsigma_2)\big(P_6^{(2s)}[\varsigma_1, \varsigma_1] - P_6^{(2s)}[\varsigma_2, \varsigma_2]\big)\,.$$

Guiding constraints

$$P_n^{(2s)}[\varsigma_i] := \lim_{\varsigma_n \to \varsigma_i} P_n^{(2s)}, \quad P_n^{(2s)}[\varsigma_i, \varsigma_j] := \lim_{\substack{\varsigma_n \to \varsigma_j \\ \varsigma_{n-1} \to \varsigma_i}} P_6^{(2s)}$$

1. ansatz $C^{(s)}$ as $P_n^{(k)}$
2. unmodified for $s = 5/2$
3. matching to self-force calc. of $\mathcal{O}(a^4)$ [Siemonsen, Vines '19]
4. matching to Teukolsky results [Bautista, Guevara, Kavanagh, Vines '22]

Comparing to BHPT: conservative contributions

Mismatch to [Bautista, Guevara, Kavanagh, Vines '22] for $\eta = 0$

$$\mathcal{M}_{\text{cl}} - \mathcal{M}_{\text{BHPT}}\Big|_{\eta=0} = -\mathcal{M}_0\Big[\alpha\xi^{-1}\tfrac{wz^2}{30}(3w^2+z^2)(1+\tfrac{2}{3}x+\tfrac{1}{42}(10x^2+y^2+2z^2)+\dots) + \alpha\xi^{-1}\tfrac{z^2}{180}\big(3w^4+12w^2z^2+z^4\big)(1+\tfrac{5}{7}x+\dots) + \mathcal{O}(a^7)\Big]$$

$$x = a\cdot q_\perp,\ y = a\cdot q,\ z = |a|\tfrac{p\cdot q_\perp}{m},\ w = \tfrac{a\cdot\chi}{p\cdot\chi}p\cdot q_\perp,\ \xi^{-1} = \tfrac{q^2m^2}{(p\cdot q_\perp)^2}$$

- up to $\mathcal{O}(a^6)$: identify 2 more contact terms $C_2^{(s)} + C_3^{(s)}$
- can construct other polynomials
  can match [Bautista, Bonelli, Iossa, Tanzini, Zhou '23] due to $P_n^{(k)}$'s being complete basis