QCD in the cores of neutron stars

HOW PQCD CONSTRAINS THE EQUATION OF STATE AT NEUTRON STAR DENSITIES
KOMOLTSEV & AK, PRL128 (2022) 20, 2111.05350

AB-INITIO QCD CALCULATIONS IMPACT THE INFERENCE OF NEUTRON-STAR EQUATION OF STATE
GORDA, KOMOLTSEV & AK, ASTROPHYS.J. 950 (2023), 2204.118

STRONGLY INTERACTING MATTER EXHIBITS DECONFINED BEHAVIOR IN MASSIVE NEUTRON STARS
ANNALA, AK, ET AL 2303.11356, NATURE COMM. IN PRINT
Gravity:

\[ R \sim 2R_s \]

QCD:

\[ n \sim 5 - 8n_s \sim \frac{\text{baryon}}{\text{fm}^3} \]

\[ \epsilon \sim 1 \frac{\text{GeV}}{\text{fm}^3} \]
Phase diagram of QCD

Heavy-ion collisions

Matter made of hadrons

0.2 GeV

Matter made of quarks and gluons

Vacuum

Nuclear matter

Neutron stars

Baryon number chemical potential

0-0.94 GeV

0.94 - GeV

T

µ B
Neutron stars are femtoscopes:

Hydrostatic equilibrium couples **macroscopic** properties of neutron stars to **microscopic** properties of the matter.

\[
d\frac{P}{dr} = -\frac{G\epsilon(r)M(r)}{r^2} \left[ 1 + \frac{P(r)}{\epsilon(r)} \right] \left[ 1 + \frac{4\pi r^3 P(r)}{M(r)} \right] \left[ 1 - \frac{2GM(r)}{r} \right]^{-1}
\]

\[
d\frac{M}{dr} = 4\pi r^2 \epsilon(r)
\]

\[\epsilon(P) \Leftrightarrow R(M)\]
Properties of neutron stars reflect properties of dense matter
Properties of neutron stars reflect properties of dense matter

Demorest et al. Nature 467, 1081-1083 (2010);
Antoniadis et al., Science 240, 448 (2013)
Properties of neutron stars reflect properties of dense matter

\[ M_{J1614-2230} = 1.908(16) \]
\[ M_{J0348+0432} = 2.01(4) \]
\[ M_{J0740+6620} = 2.14(10) \]

Demorest et al. Nature 467, 1081-1083 (2010);
Antoniadis et al., Science 240, 448 (2013)
Properties of neutron stars reflect properties of dense matter

Demorest et al. Nature 467, 1081-1083 (2010);
Antoniadis et al., Science 240, 448 (2013);

Green bank telescope

Miller et al. APJL 918 (2021);
Riley et al. APJL 918 (2021).

Abbott+ (LIGO Scientific, Virgo) PRL 119 (2017);
PRL 121 (2018); PRX 9 (2019).

Abbott+ (LIGO Scientific, Virgo) PRL 119 (2017);
PRL 121 (2018); PRX 9 (2019).
What do we know about the equation of state?
Equation of state, theoretically:

\[ P \approx \text{perturbative QCD} \]
\[ \text{Chiral EFT} \]

Tews et al., PRL 110 (2013)
Hebeler, Lattimer et al., APJ 773 (2013)
Drischler, Furnstahl et al., PRL 125 (2020)
Keller, et al, PRL 130, 072701 (2023)
Equation of state, pQCD:

EoS \( P(\mu) = \text{effective potential with: } \mathcal{L}_{QCD} + \mu \bar{\psi} \gamma_0 \psi \)

\[
\frac{1}{p_0^2 + \vec{p}^2} \rightarrow \frac{1}{(p_0 + i \mu)^2 + \vec{p}^2}
\]

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots
\]
Equation of state, pQCD:

\[ \frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots \]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \ldots
\]

\[
+ s_2 \alpha_s^2 + s_3 \alpha_s^3 + \ldots
\]

\[+ \ldots\]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots
\]

\[
+ s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots
\]

Medium polarization:

\[
\Pi \sim \alpha_s \int_{\mu} d^3 p \frac{d^3 p}{p} \sim \alpha_s \mu^2
\]
Equation of state, pQCD:

\[ \frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots \]

\[ + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots \]

Medium polarization:

\[ \Pi \sim \alpha_s \int \frac{d^3p}{p} \sim \alpha_s \mu^2 \]

If \( k^2 \sim \alpha_s \mu^2 \) correction not small:
Equation of state, pQCD:

\[ \frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots \]

Medium polarization:

\[ \Pi \sim \alpha_s \int \frac{d^3p}{p} \sim \alpha_s \mu^2 \]

If \( k^2 \sim \alpha_s \mu^2 \) correction not small:

If \( k^2 \ll \mu^2 \): Hard-thermal loop EFT

\[ \cdots \]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots
\]

Medium polarization:

\[\Pi \sim \alpha_s \int \frac{d^3p}{p} \sim \alpha_s \mu^2\]

If \(k^2 \sim \alpha_s \mu^2\) correction not small:

\[\cdots\]

If \(k^2 \ll \mu^2\): Hard-thermal loop EFT

\[\Pi^{\mu\nu}(K) = \mathcal{P}_T^{\mu\nu}(\hat{K}) \Pi_T(K) + \mathcal{P}_L^{\mu\nu}(\hat{K}) \Pi_L(K),\]

\[\Pi_L(P) = 2m^2 \frac{P^2}{|p|^2} \left[1 - \frac{iP^0}{2|p|} \ln \frac{iP^0 + |p|}{iP^0 - |p|}\right],\]
Equation of state, pQCD:

\[
\frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots
\]

\[
+ s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots
\]

\[
\sim \int^{g\mu} d^3 p \sim \alpha_s^2
\]
Equation of state, pQCD:

\[ \frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots \]

\[ + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots \]

\[ \sim \int g^{\mu \nu} d^3 p p \sim \alpha_s^2 \]
Equation of state, pQCD:

\[ \frac{p}{p_0} = 1 + h_1 \alpha_s + h_2 \alpha_s^2 + h_3 \alpha_s^3 + \cdots \]

\[ + s_2 \alpha_s^2 + s_3 \alpha_s^3 + \cdots \]
Is pQCD of any use in Neutron Stars?
Equation of state, theoretically:

Freedman, McLerran, PRD 16 (1977)
Kurkela et al. PRD 81 (2010)
Kurkela, Vuorinen, PRL 117 (2016)
Gorda, Kurkela et al. PRL 121 (2018)
Gorda, Kurkela et al. PRD 104 (2021)
Gorda, Kurkela et al. PRL 127 (2021)
Gorda, Kurkela et al. PRD 107 (2023)
Robust EoS constraints:

General considerations:

- Mechanical stability: $c_s^2 > 0$
- Causality: $c_s^2 < 1$

Lope-Oter, Llanes-Estrada, EPJA 58 (2022)
Robust EoS constraints:

General considerations:

- Mechanical stability: $c_s^2 > 0$
- Causality: $c_s^2 < 1$
- Consistency:

\[ P(\varepsilon) \text{ vs. } \Omega(\mu) \]

Reduced EoS \hspace{1cm} Full EoS
Information of \{P, \varepsilon, n\}

Lope-Oter, Llanes-Estrada, EPJA 58 (2022)
EoS constraints:

- **Stability**
  \[
  \frac{\partial^2 \Omega(\mu)}{\partial \mu^2} \leq 0 \quad \Rightarrow \quad \frac{\partial \mu}{\partial n} n(\mu) \geq 0
  \]

- **Causality**
  \[
  c_s^{-2} = \frac{\mu \partial n}{n \partial \mu} \geq 1 \quad \Rightarrow \quad \frac{\partial \mu}{\partial n} n(\mu) \geq \frac{n}{\mu}
  \]

- **Consistency**
  \[
  \int_{\mu_{\text{CET}}}^{\mu_{\text{QCD}}} n(\mu) \, d\mu = p_{\text{QCD}} - p_{\text{CET}} = \Delta p
  \]
Constraints for fixed $n$ on $\epsilon - p$ -plane

Komoltsev & AK, PRL128 (2022)
pQCD and Equation-of-State inference
Radio astronomy

Gravitational waves

Radio astronomy

X-ray astronomy

Nuclear theory

low-energy nuclear experiments

Particle theory

Particle/HIC experiment

...
Gorda, Komoltsev, AK, ApJ. (2023)
Effect of QCD:

QCD input complements NS observations

GW170817

QCD

Pulsars

- 19% QCD
- 13% GW170817
- 4% GW170817
- 4% Pulsars
- 7% GW170817 & Pulsars
- 1% GW170817, Pulsars & QCD
- 29% Pulsars & QCD

Gorda, Komoltsev & AK APJ (2023)
QCD responsible for **softening**:

**Diagram 1:**
- **Pressure vs. Energy Density:**
  - **Pulsars+Λ+QCD**
  - **Pulsars+Λ**
  - **Prior**
  - **Maximal central densities:** 750 MeV/fm³

**Diagram 2:**
- **Sound Speed vs. Neutron Number Density:**
  - **Pulsars+Λ+QCD**
  - **Pulsars+Λ**
  - **Prior**
  - **Maximal central densities**

Is there quark matter in neutron stars?
EoS tells about phases of matter: HIC

Hadrons

Matter described by hadronic mass scale

Quarks and gluons

Approximately conformal matter, with large number of d.o.f.

Borsanyi et al. PLB 730, 99 (2014)
EoS tells about phases of matter:

Matter described by Hadronic mass scale

Approximately conformal matter, with large number of d.o.f.

A clear non-coformal to conformal transition within cores of neutron stars
Conclusions

• QCD meets gravity in neutron stars in a very concrete way

• Physics of neutron-star cores can be probed by pQCD computations.
• Neutron stars are non-perturbative but causality, stability consistency contrain the EoS

• pQCD responsible for the conformationalization that has natural interpretation as the onset of Quark Matter
• Improvement through completing the N3LO computation
Constraints in \( \{p, \varepsilon, n\} \) - space

Models from CompOSE database

Komoltsev & AK, PRL128 (2022)
QCD likelihood function:

- **Machine learning** based Bayesian interpretation of **Scale variation** and **Missing Higher Order** errors
- Perturbative series modelled as draw from statistical model that is trained with the available terms
Implementing pQCD to EoS inference:

• Bayesian inference setup:

\[
P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS}) \cdot P(\text{data} \mid \text{EoS})}{P(\text{data})}
\]
Implementing pQCD to EoS inference:

- **Gaussian-process** based inference:
  Similar to Landry & Essick PRD 99 (2019), but for function of $n$ instead of $\varepsilon$

$$P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS}) P(\text{data} \mid \text{EoS})}{P(\text{data})}$$
Setup:

\begin{align*}
P(EoS \mid data) &= \frac{P(EoS)P(data \mid EoS)}{P(data)} \\
P(data \mid EoS) &= P(QCD \mid EoS)P(Mass \mid EoS)P(NICER \mid EoS)P(\tilde{A}, BH \mid EoS).
\end{align*}
Implementing pQCD to EoS inference:

- Inference setup where QCD can be turned on/off
- Easily implemented to any other extrapolation setup

Extrapolate to here and use this area to condition the extrapolation