Tsallis Distribution in High-Energy Collisions

(arXiv:1101.3023, accepted in EPL)

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in collaboration

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High-pT Physics for the LHC 2011, Utrecht 4-7 April 2011

OUTLINE

- Motivation
  - High & low-p_T hadron spectra
  - Necessary bad: Tsallis–Pareto distribution
- Test of the Tsallis–Pareto
  - from Nucleus-Nucleus (AA), proton-proton (pp), to e^+e^-
- Tsallis–Pareto based model for e^+e^-
  - Canonical Tsallis–Pareto
  - Microcanonical Tsallis–Pareto
  - Microcanonical Tsallis–Pareto at low-x

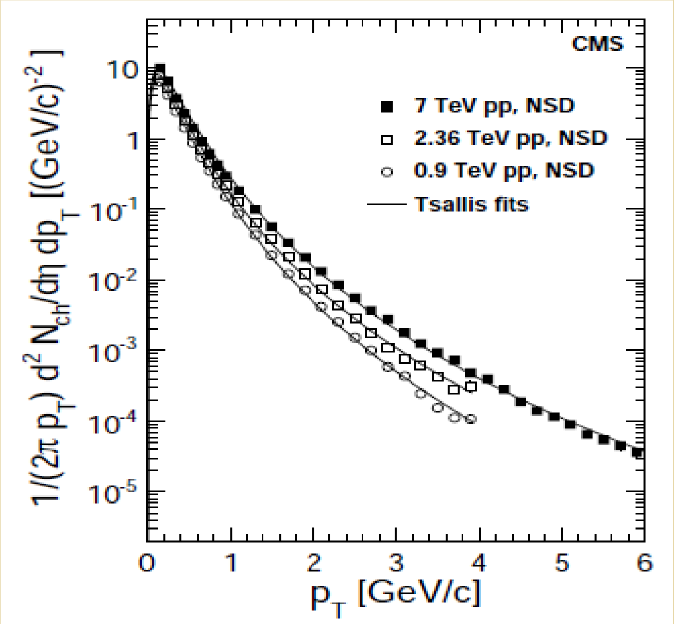
MOTIVATION

- New LHC pp data (CMS)

  CMS: JHEP 1002:041(2010)

  fitted Tsallis distribution for p_T spectra:

$$E\frac{d^3N_{ch}}{dp^3} = \frac{1}{2\pi p_T}\frac{E}{p}\frac{d^2N_{ch}}{d\eta dp_T} = C(n,T,m)\frac{dN_{ch}}{dy}\left(1+\frac{E_T}{nT}\right)^{-n}$$



  Parameters:

  0.9 TeV T= 130 MeV, q=1.13

  2.36 TeV T= 140 MeV, q=1.15 $n := (q-1)^{-1}$

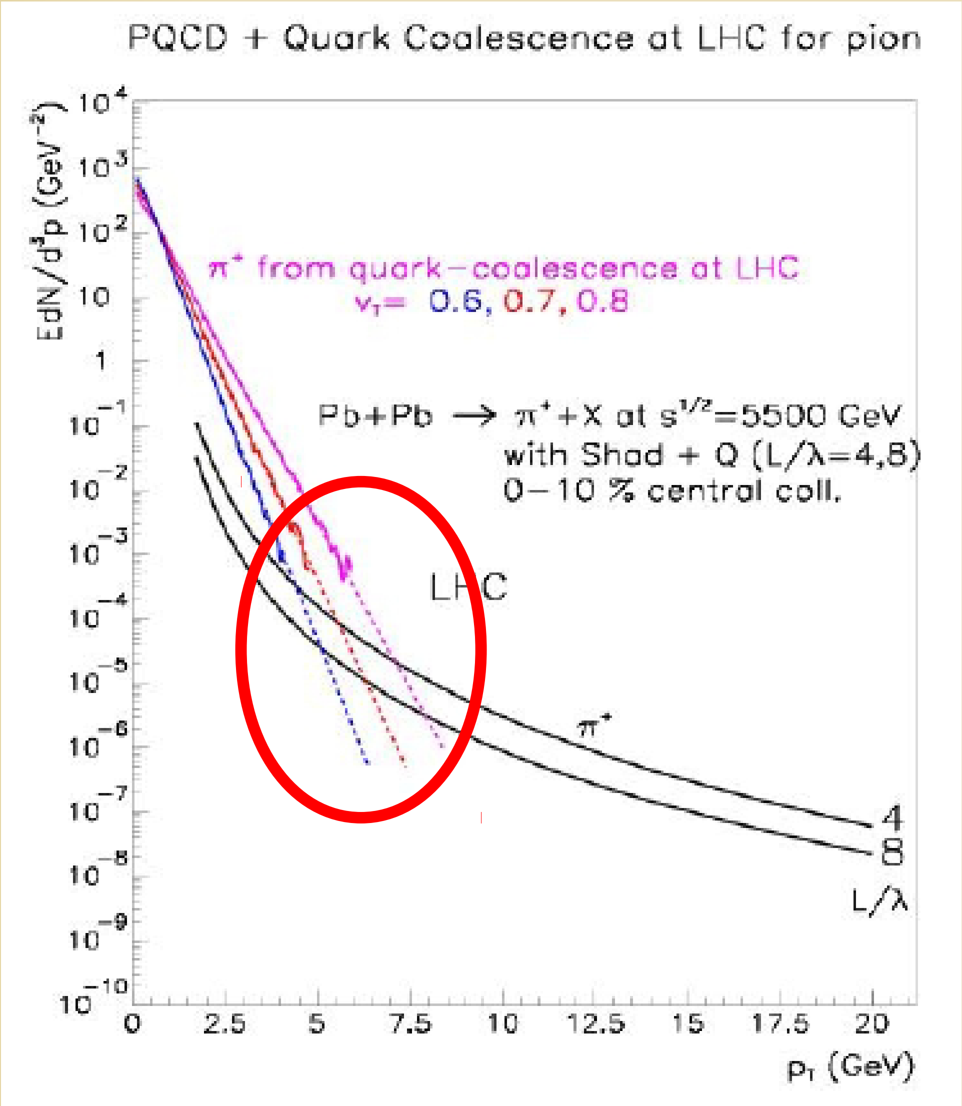
- RHIC analysis on AuAu data (y=0)

  Cooper-Frye model: K. Ürmössy, T.S. Bíró: PL B689 14 (2010)

  Parameters: $f(E) = A[1+(q-1)E/T]^{-1/(q-1)}$

  200 GeV T= 51 MeV, q= 1.062 (fit for p_T < 6 GeV/c)

MOTIVATION

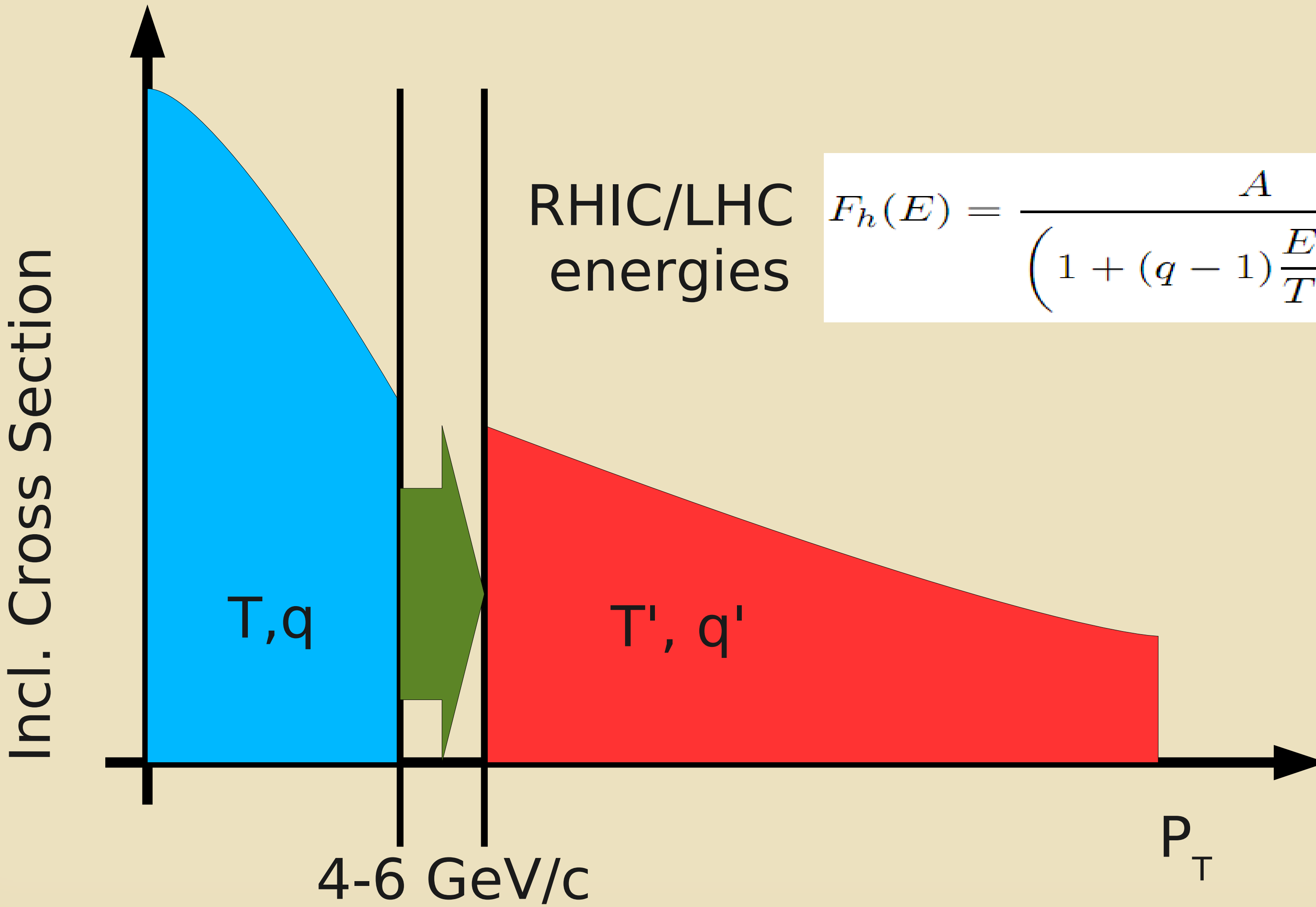


- pQCD based parton model:
  - QCD at T → 0 temperature
  - power law distribution
  - strong dependence on FF
  - good for high-p_T hadrons
- Quark-coalescence model
  - Thermal, finite temperature
  - exponential distribution $e^{-m / T}$
  - parton-hadron duality
  - good for high-p_T hadrons

P. Lévai, GGB, G. Fai: JPG35, 104111 (2008)

On the way to find a proper distribution

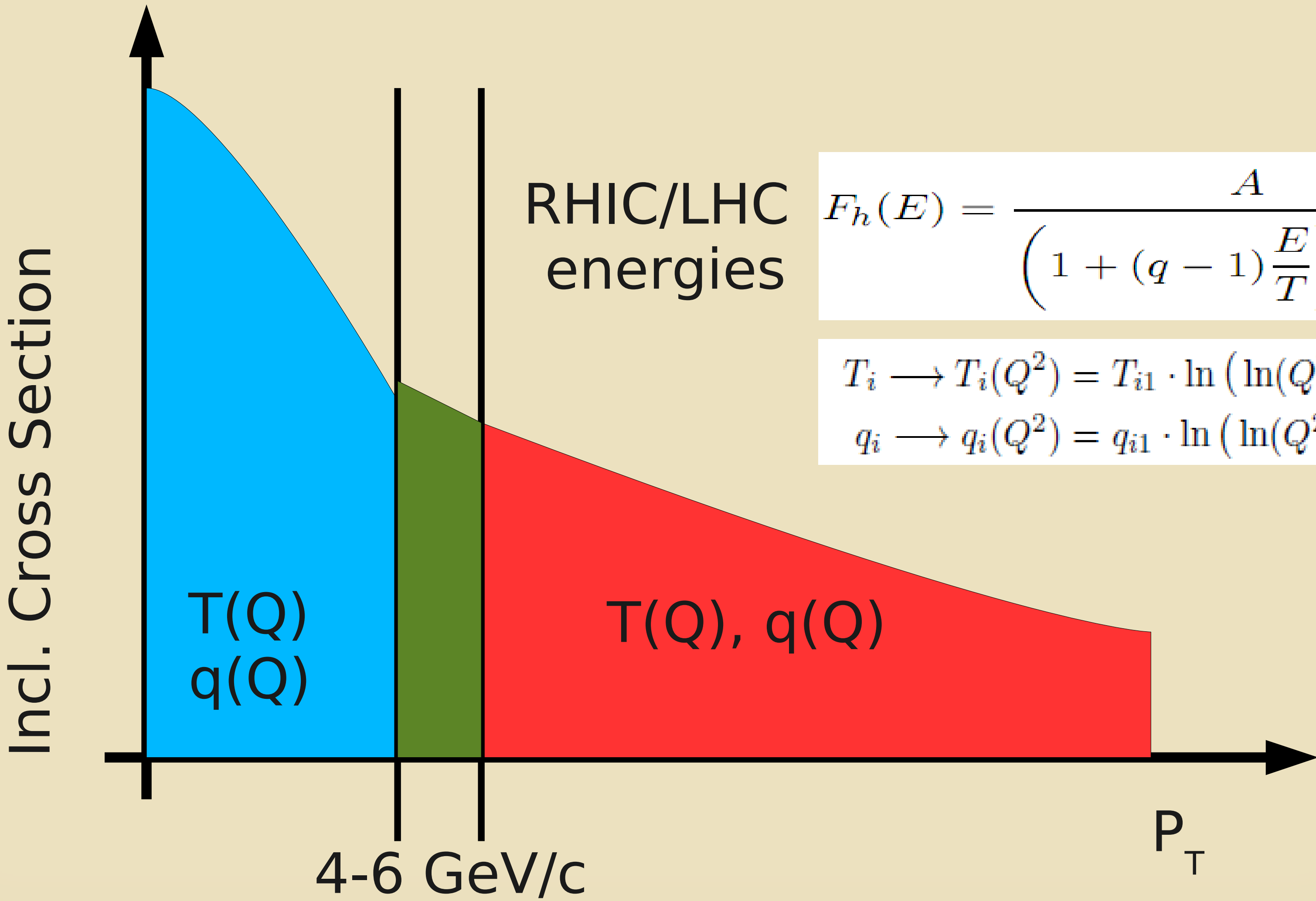
- Original proposed way: Tsallis–Pareto



$$F_h(E) = \frac{A}{\left(1 + (q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

… and to find a (good) theory

- A new proposed way: Tsallis–Pareto + evolution



$$F_h(E) = \frac{A}{\left(1 + (q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

$$T_i \longrightarrow T_i(Q^2) = T_{i1} \cdot \ln\left(\ln(Q^2)\right) + T_{i0}\ ,$$
$$q_i \longrightarrow q_i(Q^2) = q_{i1} \cdot \ln\left(\ln(Q^2)\right) + q_{i0}\ .$$

Basics of non-extensive thermodynamics

Non-extensive thermodynamics (Based on: T.S. Biró: EPL84, 56003,2008)
associative composition rule, (non-additive) :

$$h(h(x,y),z) \;=\; h(x,h(y,z))$$

Then should exist a strict monotonic function, X(x) 'generalised logarithm' (an entropy-like quantity), for which:

$$h(x,y) = X^{-1}\left(X(x) + X(y)\right) \qquad X(h(x,y)) = X(x) + X(y).$$

Examples: (i) Classical Boltzmann–Gibbs thermodynamics:

$$f(E) = \; e^{-\beta E}/Z \qquad h(x,y) = x + y.$$

(ii) Tsallis–Pareto-like distribution with $a = q - 1$:

$$f(E) = \frac{1}{Z} e^{-\frac{\beta}{a}\ln(1+aE)} = \frac{1}{Z}\left(1+aE\right)^{-\beta/a} \qquad h(x,y) \;=\; x + y + axy$$

$$S = \int f \, \frac{e^{-a\ln(f)} - 1}{a} \;=\; \frac{1}{a}\int \left(f^{1-a} - f\right).$$

Hadronization via non-extensive way

Our program:

i) Search and fit Tsallis distribution
to data from AA, pp, ee.

ii) Test: can a BFKL / DGLAP-like
evolution equation be obtained?

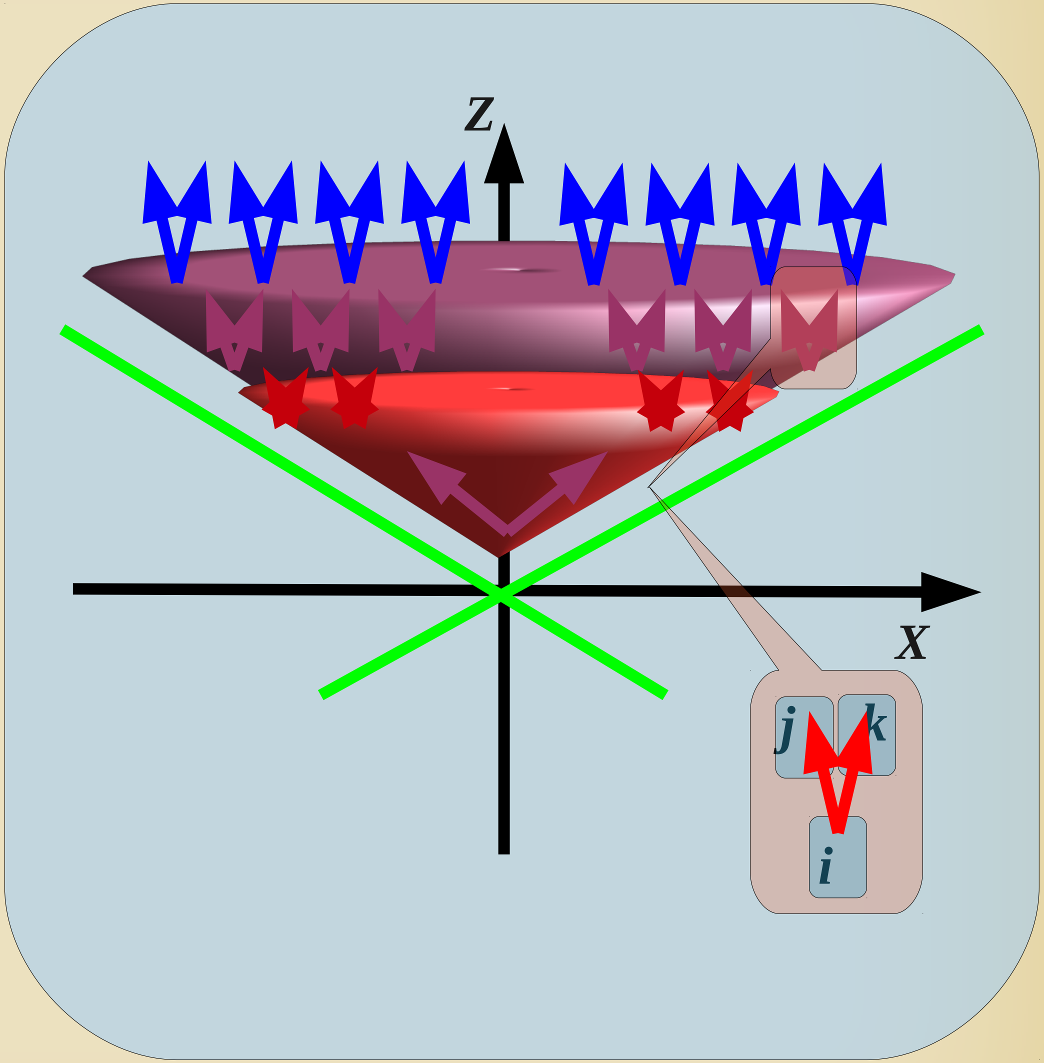
$$D(x,Q^2) \sim f(E,T,q) * f(\ln(Q^2))$$

$$D(x,Q^2) \sim f(E,T(\ln(Q^2)),q(\ln(Q^2)))$$

iii) Build up a simple theory to test.

iv) Search for physical meaning of
T and q parameters.

→ This is a hard thing...



From AA and pp to ee

an inverse story…

AA: Generalization of BG to Tsallis–Pareto

Find a distribution for low & high momentum spectra:

$$e^{-\beta E} \longrightarrow F_h(E) = \frac{A}{\left(1 + (q-1)\frac{E}{T}\right)^{1/(q-1)}}$$

In AA collisions particle energy modified by the flow:

$$E = \gamma(m_t - v_{flow} p_t), \quad m_t = \sqrt{m^2 + p_t^2}$$

Slope for particle spectra can be fitted:

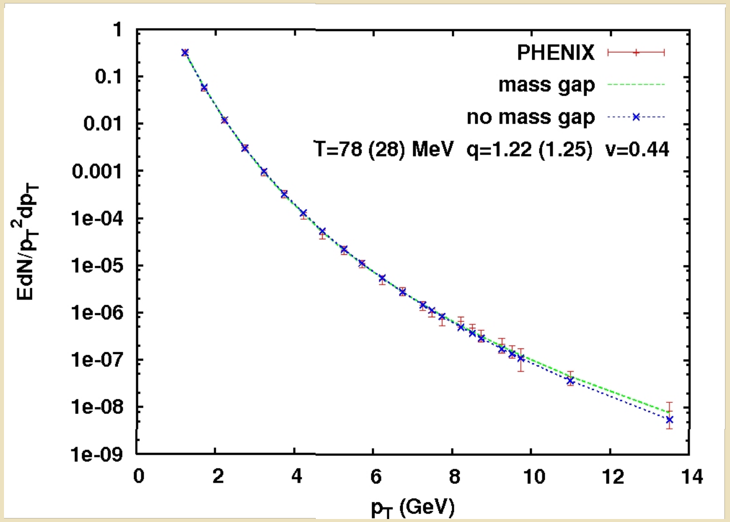
$$T_{slope} = -\frac{\partial E}{\partial ln(F_h(E))} = T(1 + aE) = T + (q-1)E$$

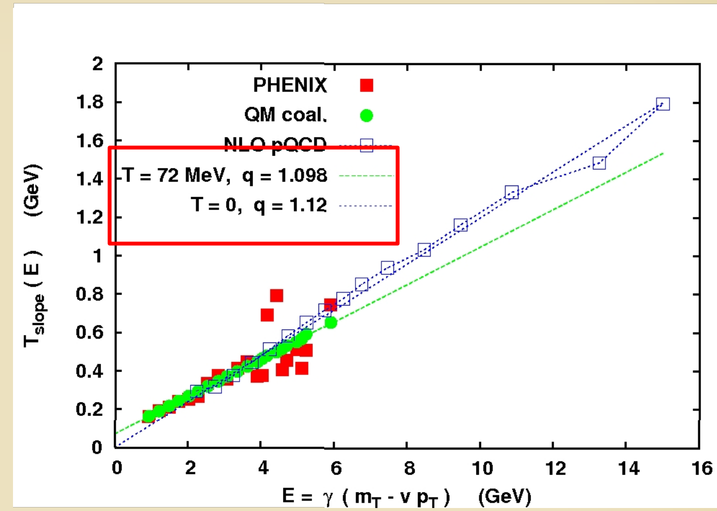
furthermore, if $E \gg m$, then $T \rightarrow T\sqrt{(1+v)/(1-v)}$

Ref: K. Ürmössy, TS Biró, GGB, J.Phys. G35 044012 2008

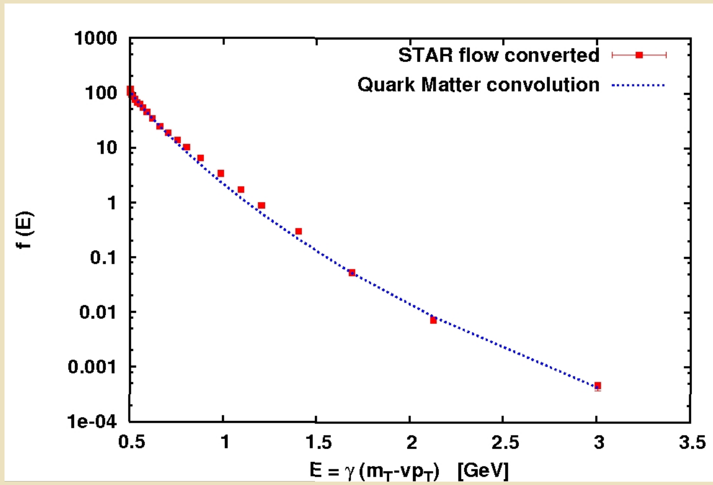
AA: Recombination & pQCD in AuAu @ RHIC

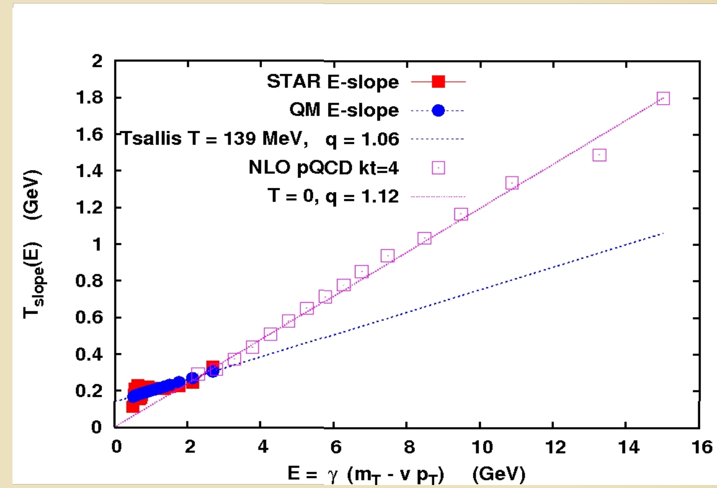
Pion spectrum





Kaon spectrum



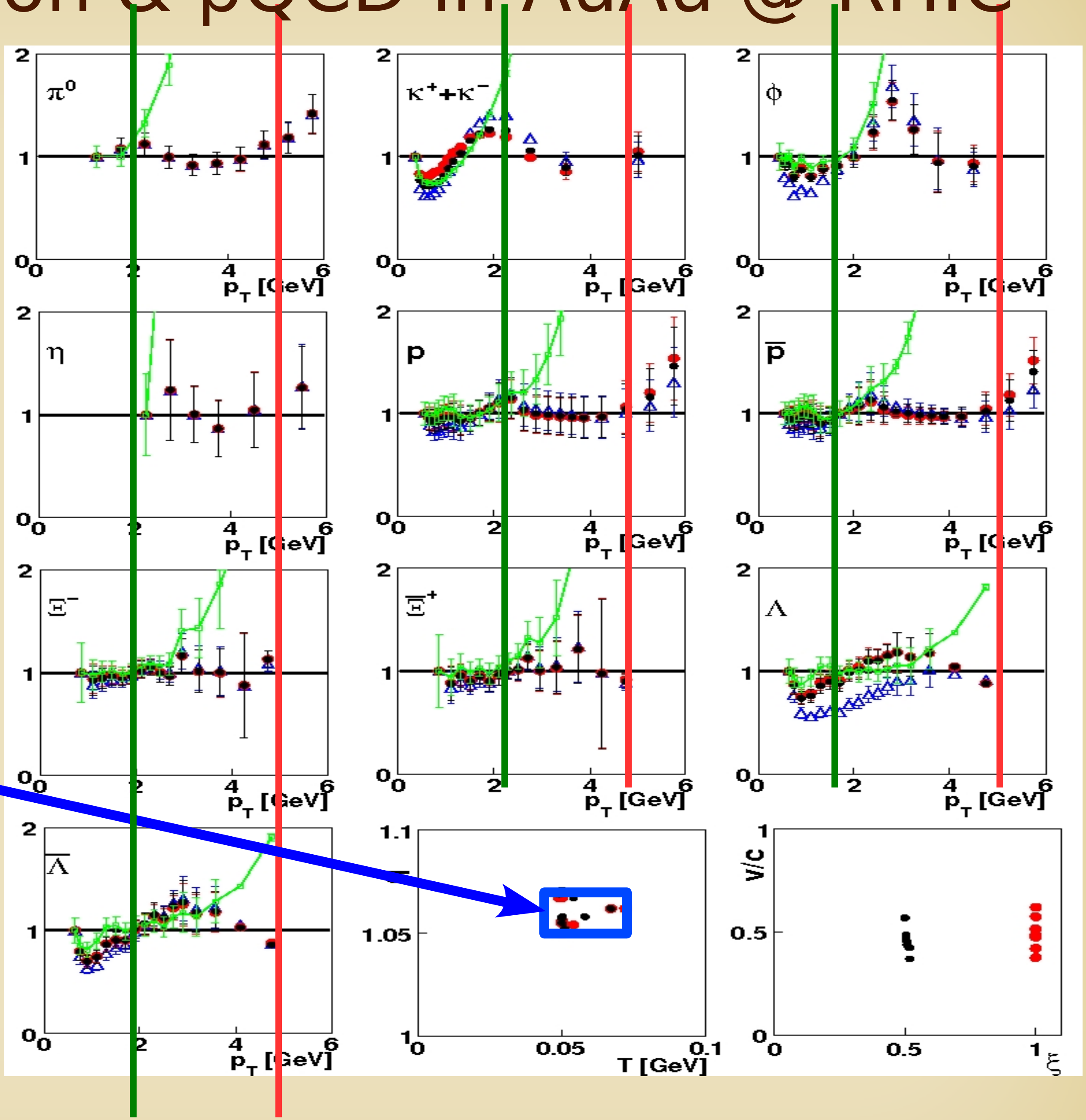


Fitted T & q Tsallis parameters at RHIC energies: (recombination and NLO pQCD+AKK FF, *v=0.5* for kaons): T.S. Bíró, K. Ürmössy, GGB: JPG35, 044012 (2008)

AA: Recombination & pQCD in AuAu @ RHIC

Analysing
Meson & Baryon
specta in
AuAu collisions

- Ratio of theoretical or experimental p_T spectra in y=0, AuAu collisions fitted by Tsallis distribution.
- T=50-70MeV, q=1.06-1.07
- Here the fit is only for p_T < 6 GeV/c.
- K. Ürmössy, T.S. Bíró: PLB689:14 (2010)



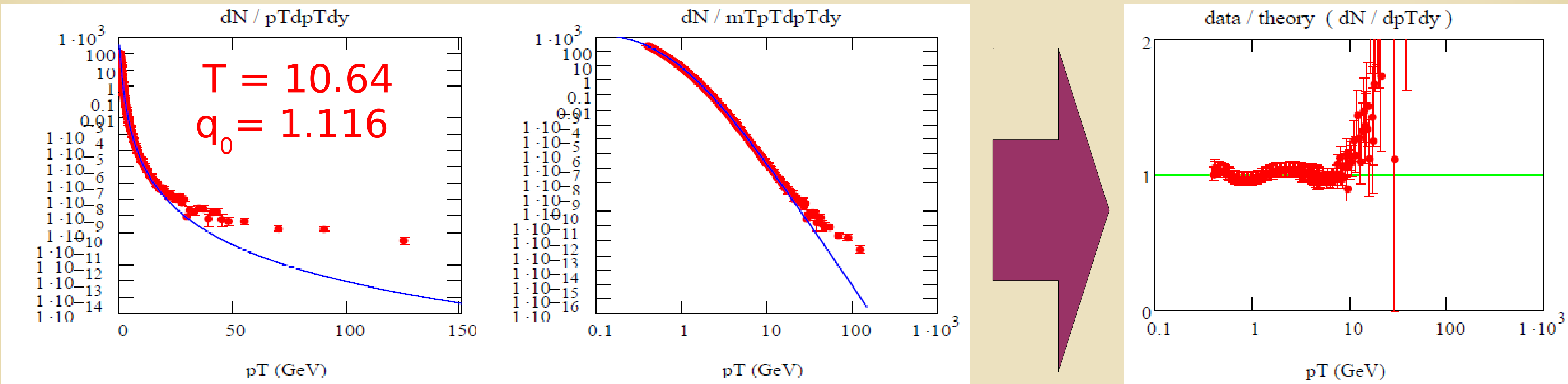
…as you might tried:

Tsallis–Pareto on pp

pp: Need extra → Tsallis–Pareto

- TEST on CDF ch. hadron data in pp @ 1.96 TeV |y|<1

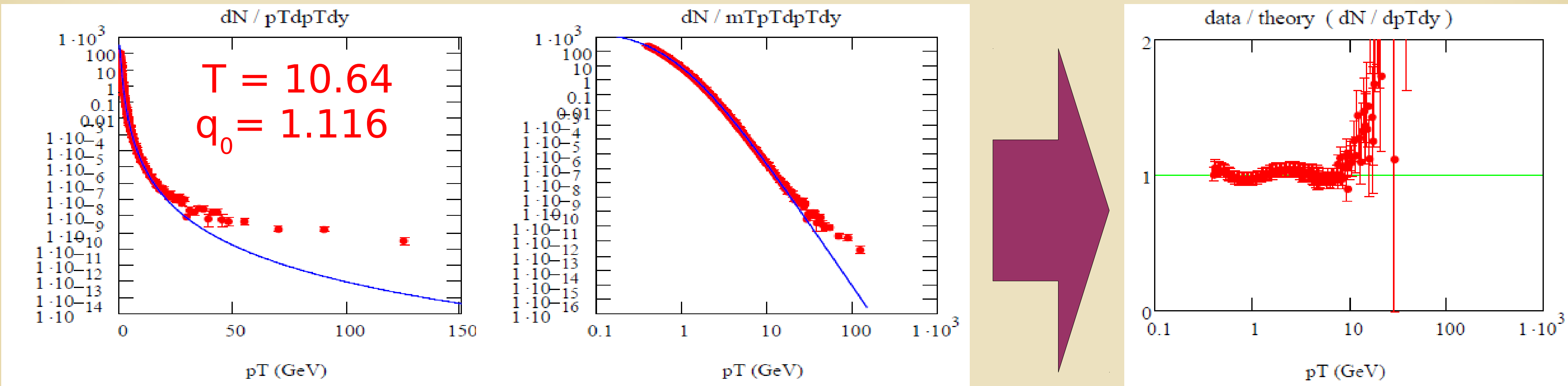
NO evolution



pp: Need extra → Tsallis–Pareto + evolution

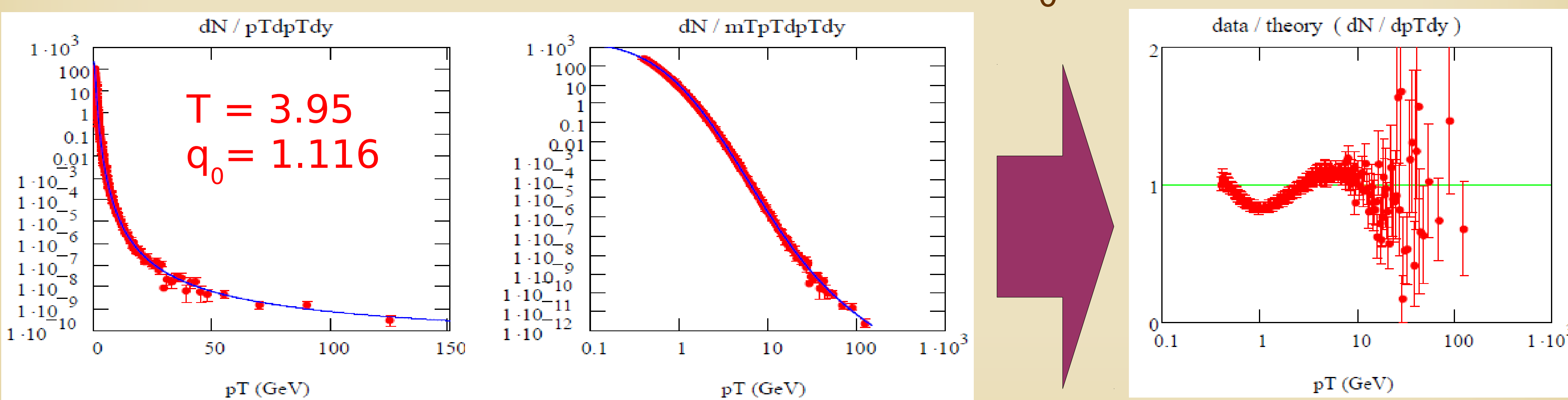
- TEST on CDF ch. hadron data in pp @ 1.96 TeV |y|<1

NO evolution



- DGLAP motivated evolution: $n = (q_0-1)^{-1} - 2*\log(\log(Q))$

With evolution



pp: Tsallis–Pareto fits from 0.2-7 TeV

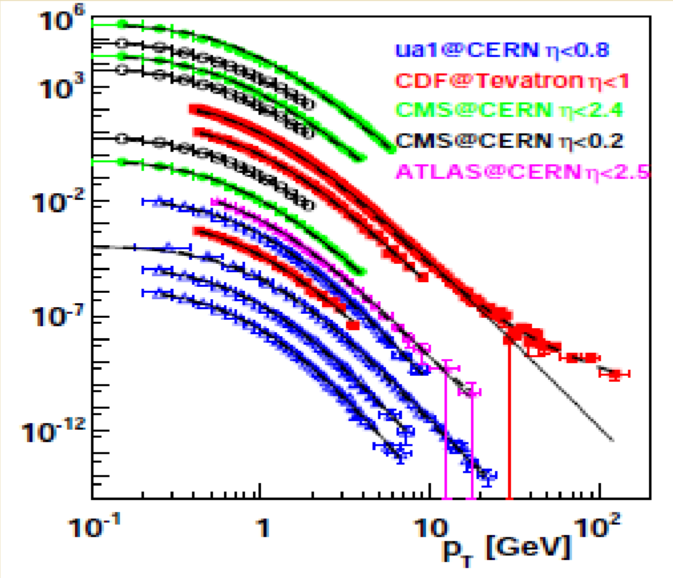
Data used for fits

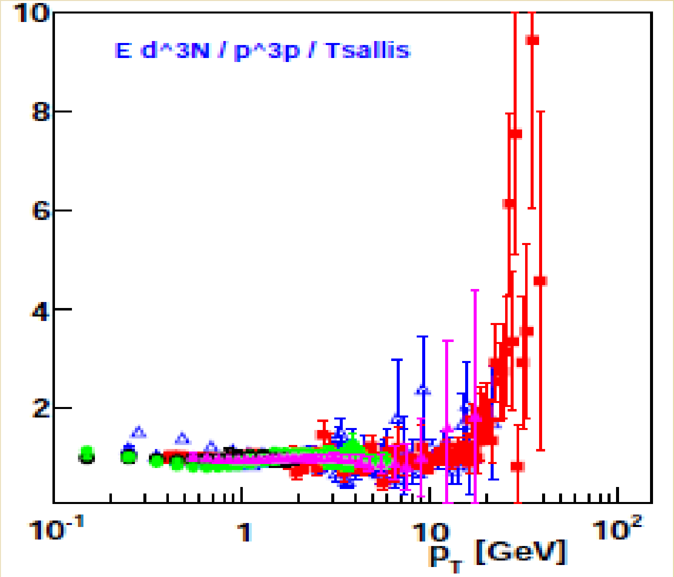
- Khachatryan V et al [CMS] 2010 JHEP02(2010)041, CMS-QCD-10-006, CERN-PH-EP-2010-009, FERMILAB-PUB- 10-170-CMS
- Aaltonen T et al [CDF] 2009 PRD 79 112005
- Adare A et al [PHENIX] 2010 arXiv:1005.3674 [hep-ex]
- Albajar C et al [UAl] 1990 Nucl. Phys. B 335 261
- Bocquet G et al [UAl] 1996 Phys. Lett. B 366 434
- Abe F et al [CDF] 1988 Phys. Rev. Lett. 61 1819
- Aad G et al [ATLAS] 2010 Phys. Lett. B 688 21

Ref: GGB, K. Ürmössy, TS Biró: J.Phys. CS 270 012008 2011

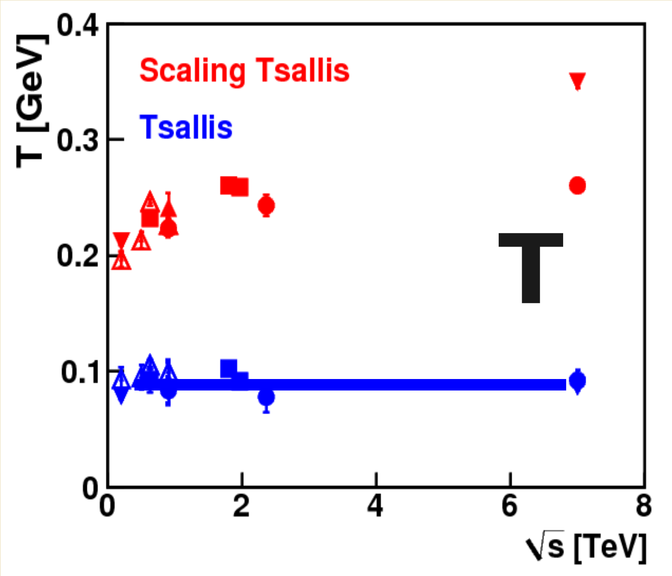
pp: Tsallis–Pareto with evolution in pp

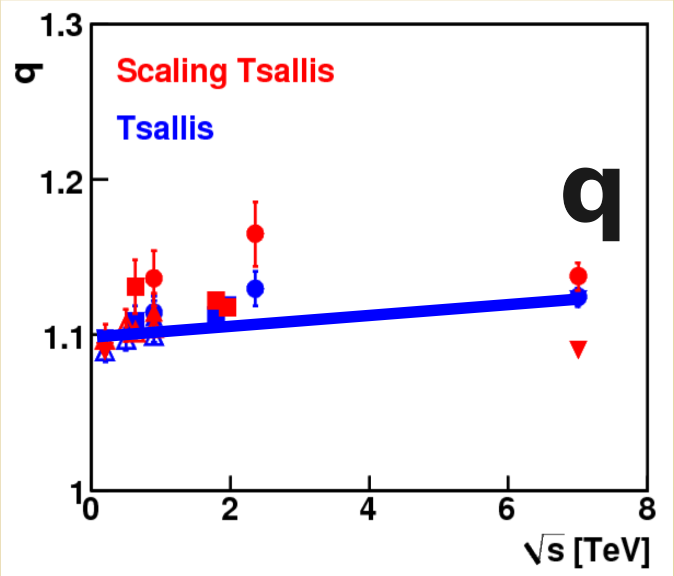
- More TEST: 0.2 - 7 TeV midrapidity data





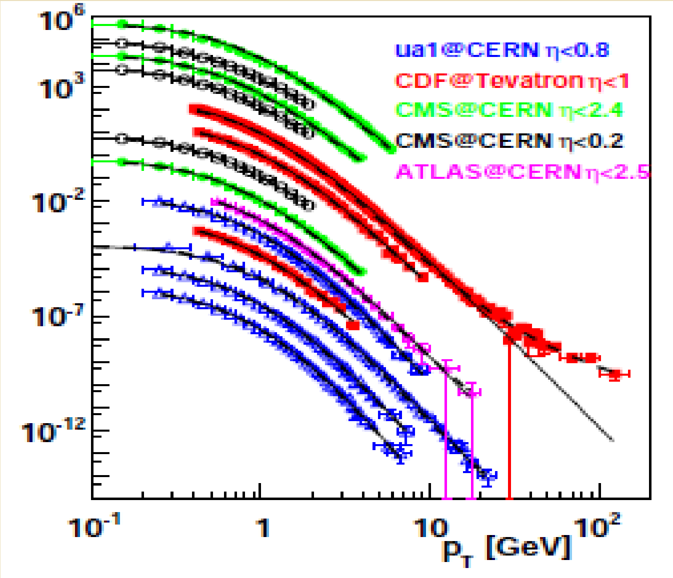
- C.m. Energy dependence of the T & q parameters

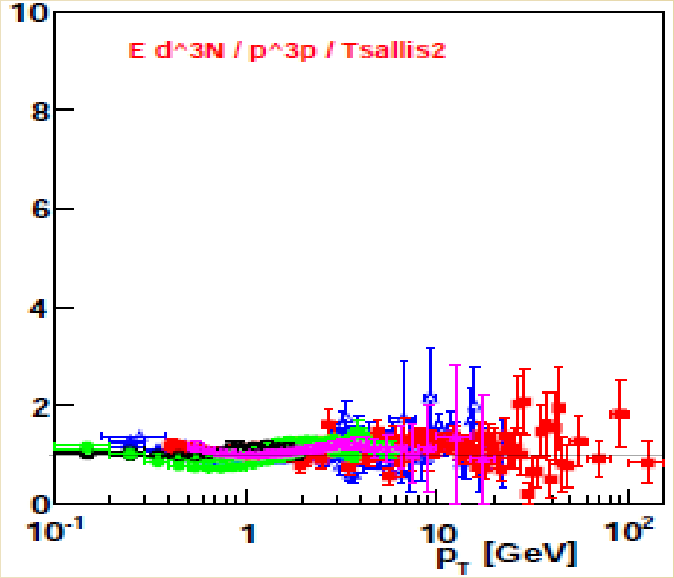




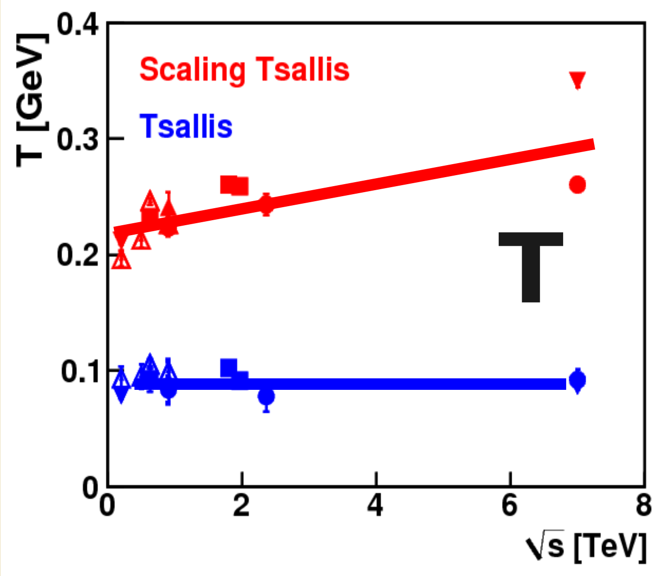
pp: Tsallis–Pareto with evolution in pp

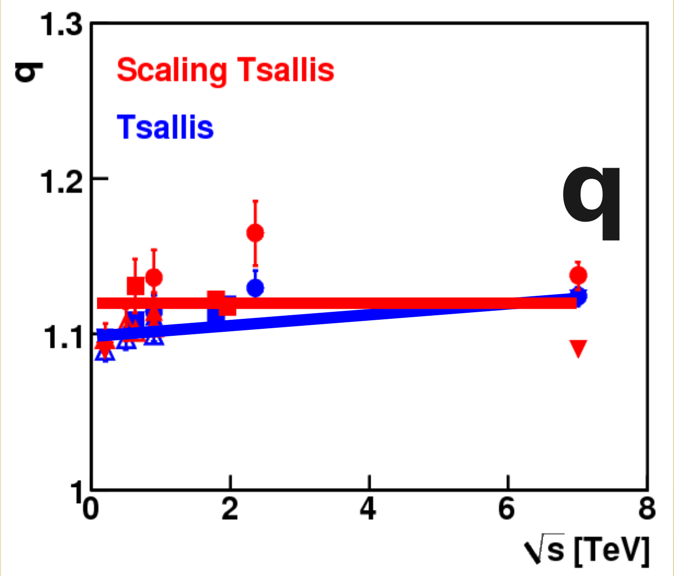
- More TEST: 0.2 - 7 TeV midrapidity data





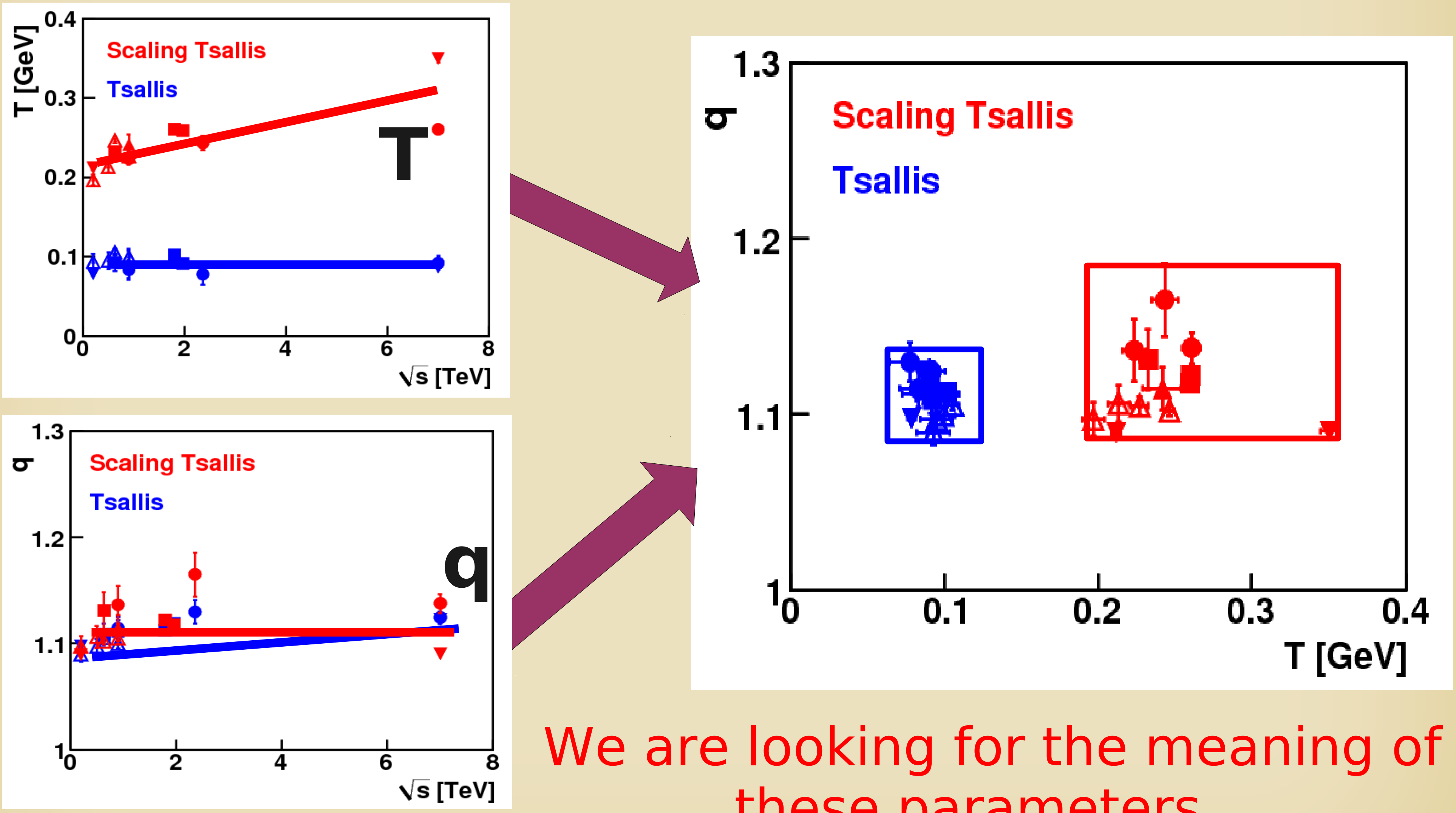
- C.m. Energy dependence of the T & q parameters





pp: T–q parameter space and evolution

- TEST on various midrapidity pp data @ 0.2-7 TeV



We are looking for the meaning of these parameters...

Looking for the 'unknown beauty'?

(simplest case: e^+e^-)

ee: Basic model assumptions for e^+e^-

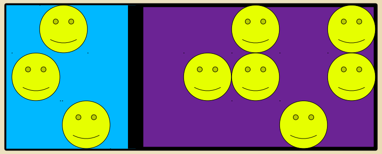
In case of a high-energy collisions (at high-p_T) we can expect:

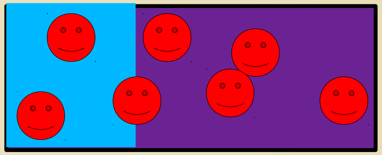
- Consider jets: narrow objects
- Narrow momentum distribution

} 1-dimensional object

- Events at $O(10^6)$ → Statistics
- At $\sqrt{s} \geq M_Z$ and ~90% of the events are 2-jet-events: $\sqrt{s}/2 = E$

- Energy-momentum conservation with $m_j = 0$ thus: $\epsilon_j = |\vec{p}|$
  - Micro-canonical: $\sum_j \epsilon_j = E$ (E conserv.)
  - Canonical case: $\sum_j \langle \epsilon_j \rangle = E$





Ref: K Ürmössy, GGB, TS Biró, arXiv:1101.3023 (2011)

ee: Canonical & microcanonical ensambles

Canonical case for TP:

- One-particle distribution (with multiplicity *N*): $f_N(\epsilon) = A_c\, e^{-\beta_N \epsilon}$
- Gamma distribution for multiplicity: $p(N) = A_m\, N^{\alpha-1} e^{-\beta N},$
- Momentum distribution (CTP): $\frac{\mathrm{d}\,\sigma}{\mathrm{d}\,^D p} = \sum p(N) N\, f_N(\epsilon) \approx \frac{\kappa_{D,E}}{\left(1+\frac{D}{\beta}x\right)^{\alpha+D+1}}$

Microcanonical generalization of TP

- One-particle distribution (with multiplicity *N*): $f_N(\epsilon) = A_{mc}\,(1-x)^{D(N-1)-1}$
- Shifted Gamma distribution for multiplicity (no to violate the KNO scaling, $N_0 = 1+2/D$): $p(N) = A_m\,(N-N_0)^{\alpha-1} e^{-\beta(N-N_0)}$
- Momentum distribution (μCTP): $\frac{d\sigma}{d^D p} \propto \frac{1-x}{\left(1-\frac{D}{\beta}\ln(1-x)\right)^{\alpha+D+1}}$

Ref: K Ürmössy, GGB, TS Biró, arXiv:1101.3023 (2011)

ee: Tsallis–Pareto fits from 14-201 GeV

Data used for fits

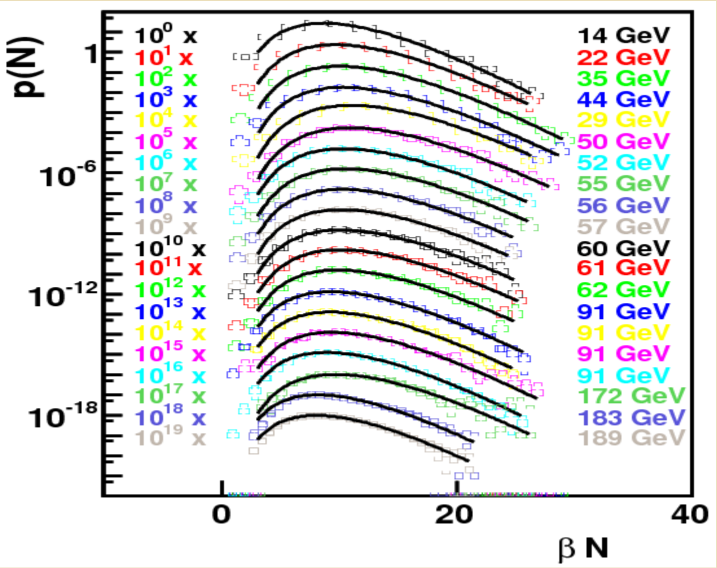
- Braunschweig W et al [TASSO] 1998 Z.Phys. C47 187; 1989 Z.Phys. C45 193
- Aihara H et al [TPC/Two Gamma] 1988 PRL 61 1263
- Abreu P et al [DELPHI] 1993 PL B311 408; 1991 Z.Phys. C50 185
- Akers R et al [OPAL] 1995 Z. Phys. C68 203
- Alexander G. et al [OPAL] 1996 Z.Phys. C72 191, 2000 Eur.Phys.J. C16 185, 2003 Eur.Phys.J. C27 467
- Derrick, M. et al 1986 Phys. Rev. D34 3304
- Zheng, H.W. et al. [AMY] 1990 Phys. Rev. D42 737
- Adeva, B. et al.[L3] 1992 Z.Phys. C55 39
- Acton, P.D. et al. [OPAL] 1992 Z.Phys. C53 539

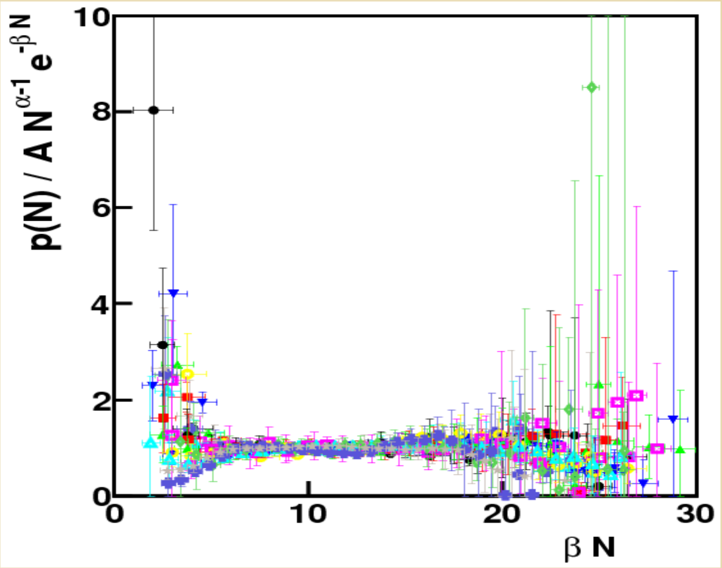
Ref: K. Ürmössy, GGB, TS Biró: arXiv:1101.3023 (2011)

ee: Multiplicity fluctuations with Gamma

Gamma distribution:

$$p(N) = A_m N^{\alpha-1} e^{-\beta N},$$

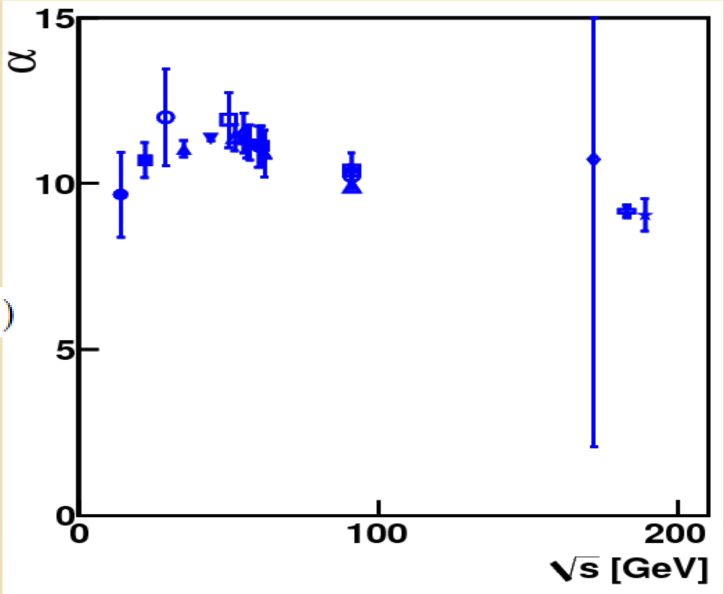


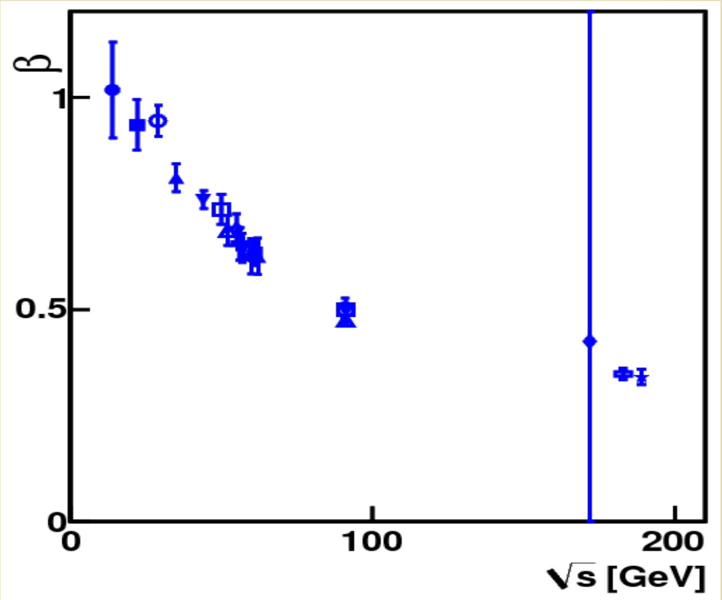


Parameters:

$$q = 1 + 1/(\alpha + D + 1)$$

$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$



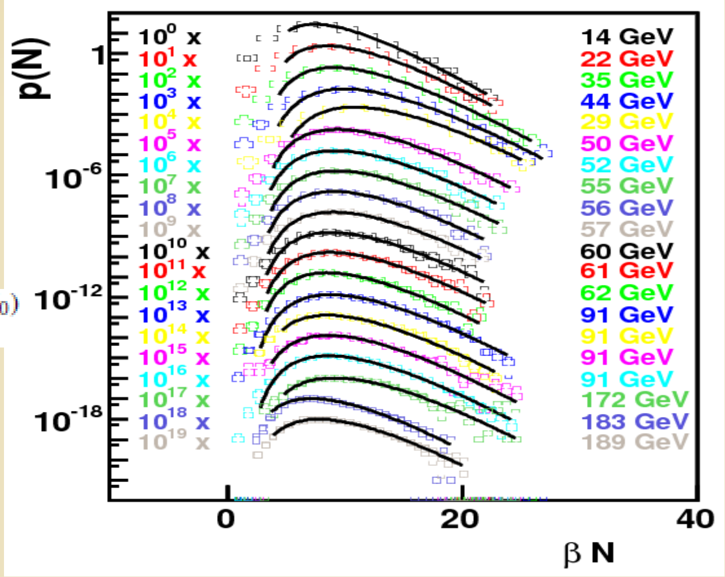


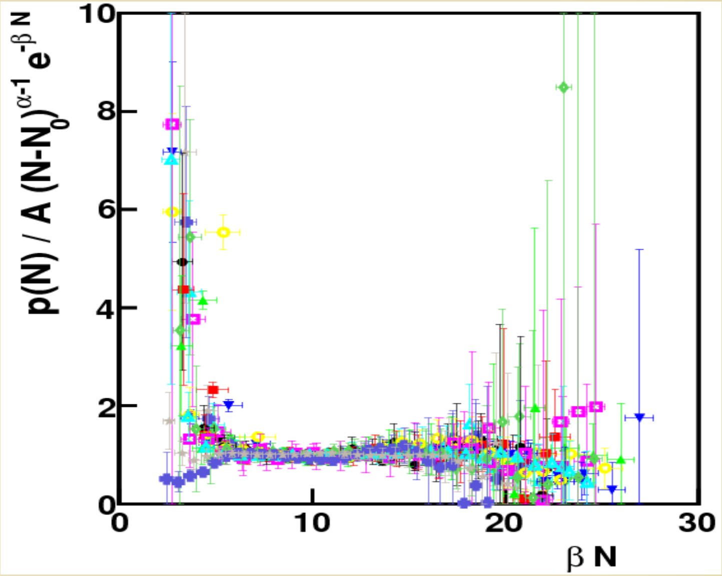
ee: Multiplicity fluctuations with Shift-Gamma

Shift-Gamma distribution:

$$p(N) = A_m\,(N - N_0)^{\alpha-1} e^{-\beta(N-N_0)}$$

$$N_0 = 1 + 2/D$$

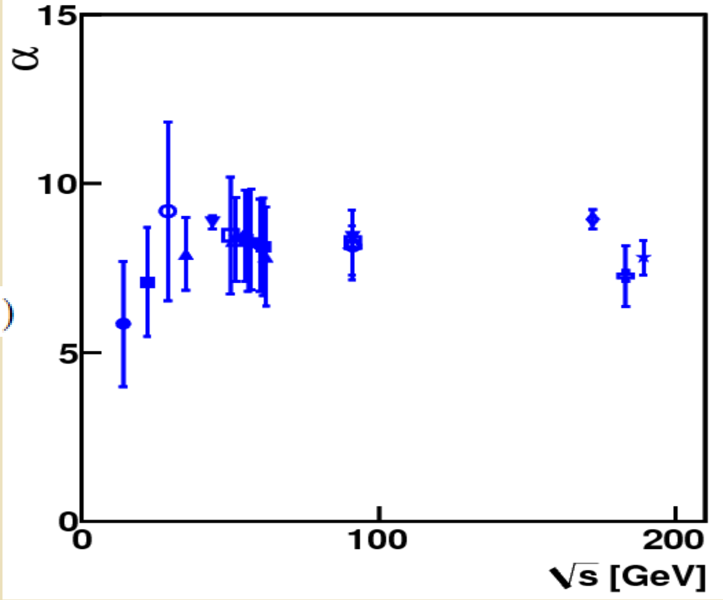


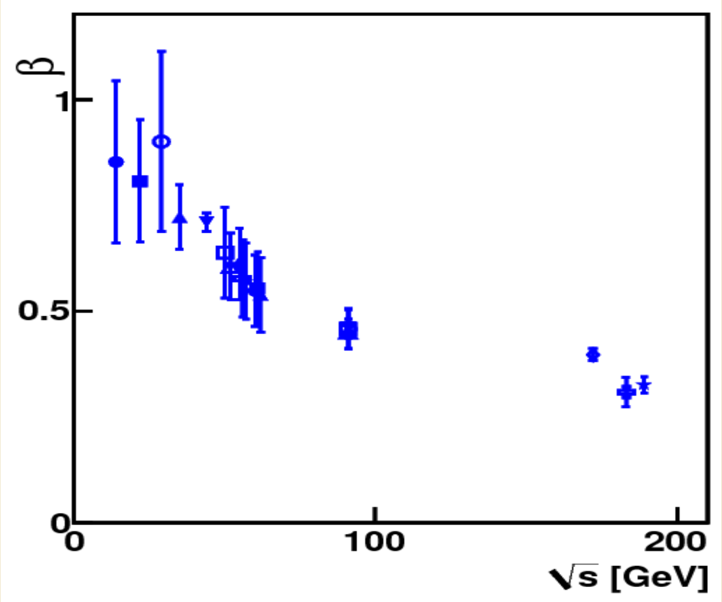


Parameters:

$$q = 1 + 1/(\alpha + D + 1)$$

$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$

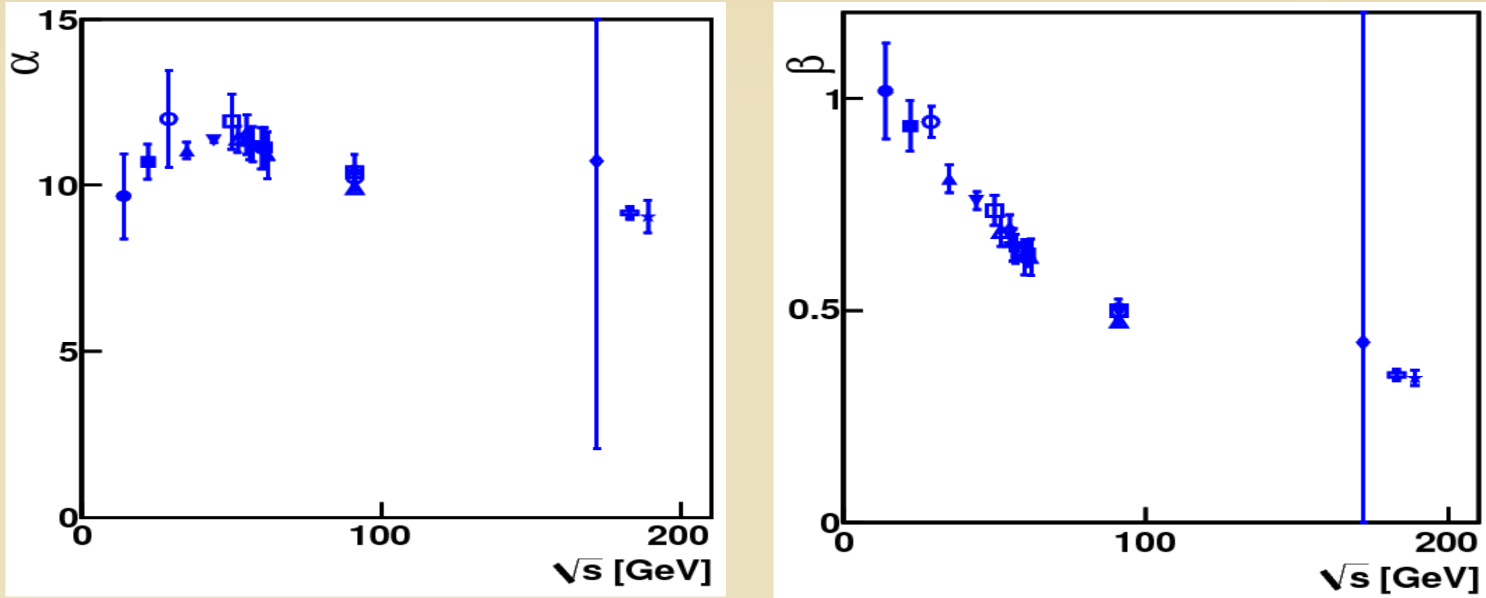




ee: Satisfied KNO scaling for both cases in e^+e^-

Gamma distribution:

$$p(N) = A_m N^{\alpha-1} e^{-\beta N},$$



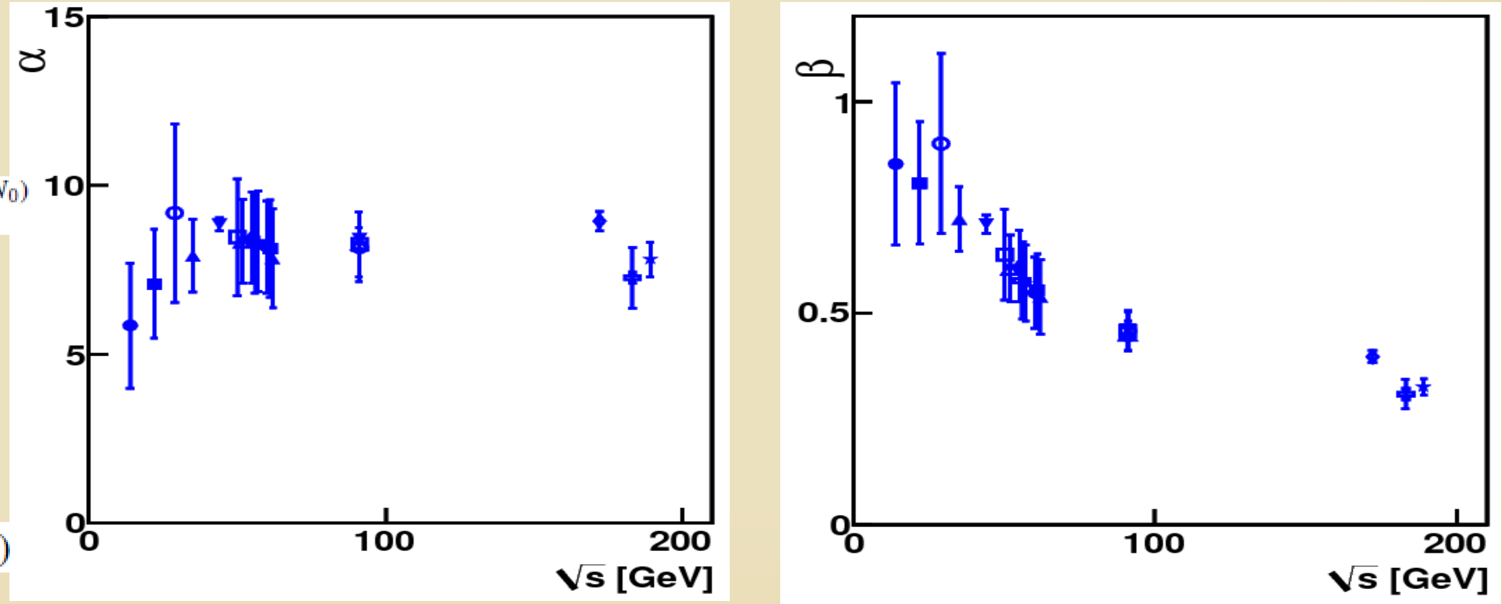
Shift-Gamma distribution:

$$p(N) = A_m (N - N_0)^{\alpha-1} e^{-\beta (N-N_0)}$$

Parameters:

$$q = 1 + 1/(\alpha + D + 1)$$

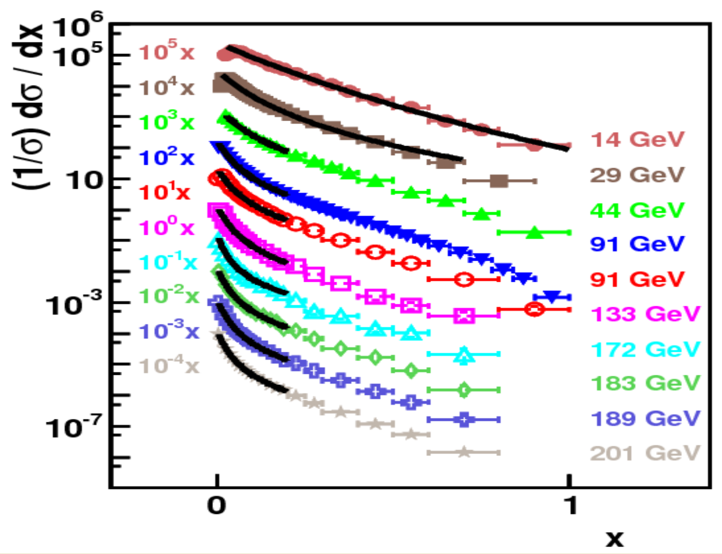
$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$

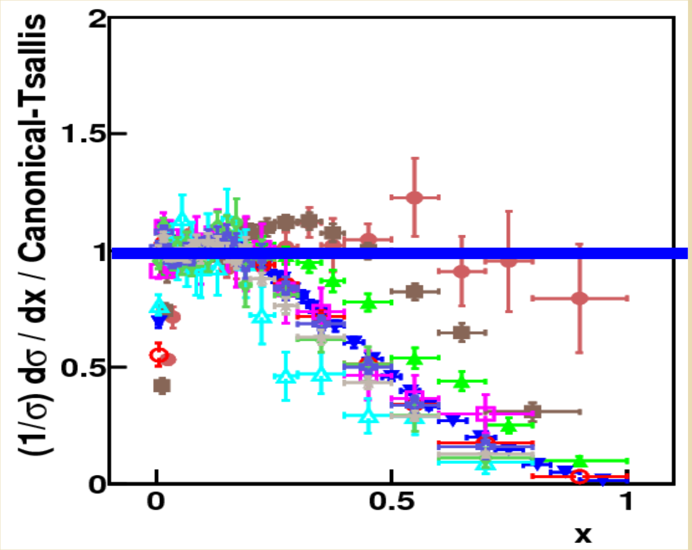


ee: Canonical Tsallis – Pareto in e^+e^-

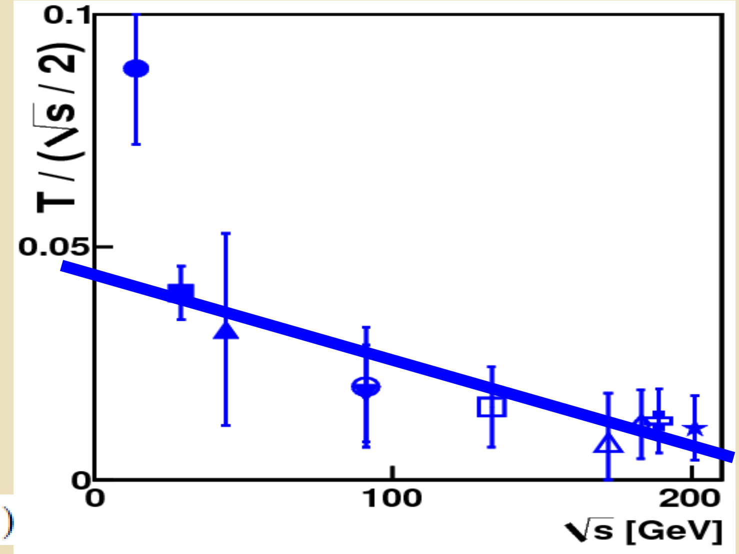
- Canonical Tsallis in e^+e^- coll.

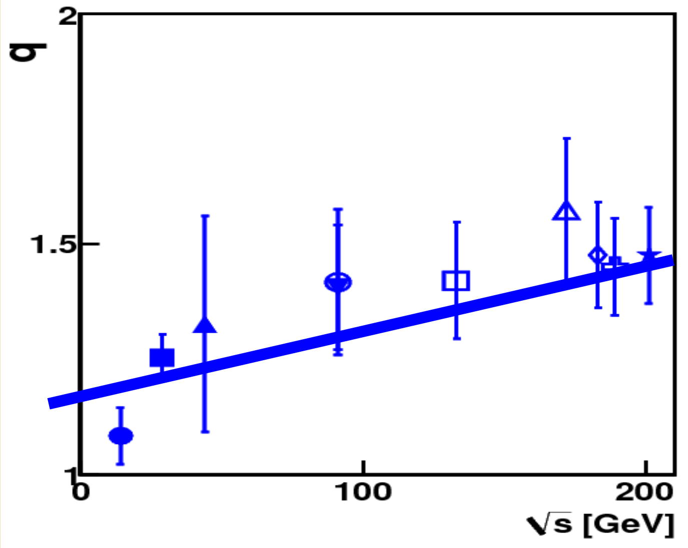
$$Ax^{D-1}\left(1+\frac{q-1}{T/(\sqrt{s}/2)}x\right)^{-1/(q-1)}$$





- T smaller
- q larger





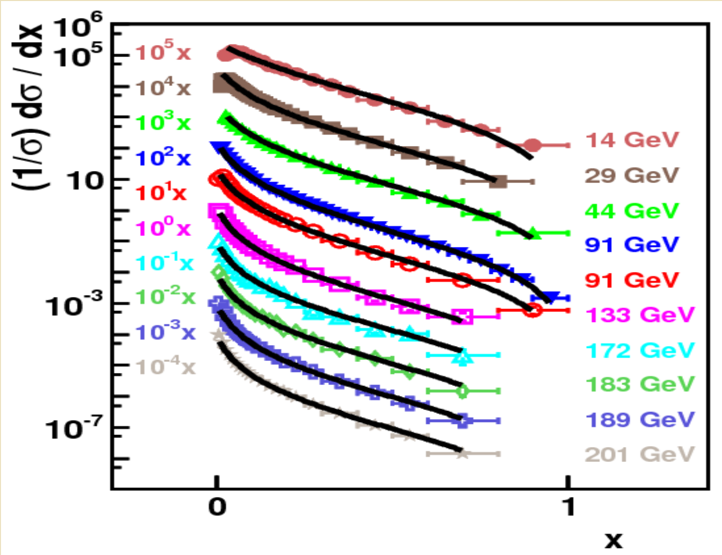
$$q = 1 + 1/(\alpha + D + 1)$$

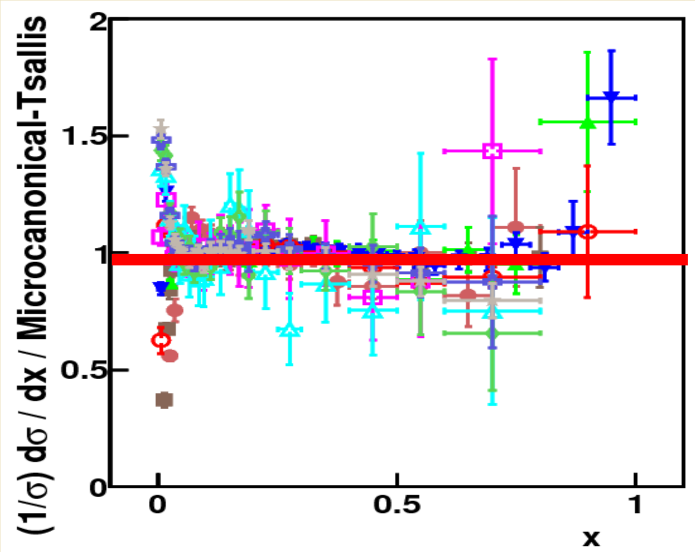
$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$

ee: Microcanonical Tsallis – Pareto in e^+e^-

- Micro-canonical Tsallis

$$\frac{A\,x^{D-1}(1-x)}{\left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$

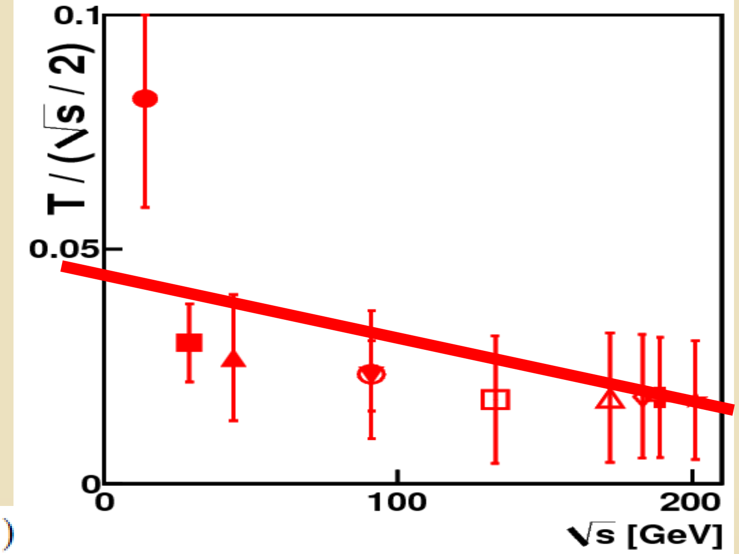


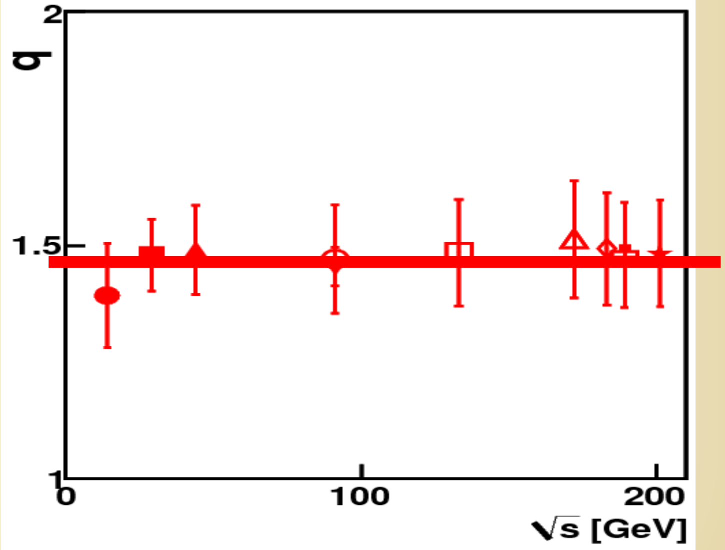


- T smaller
- q=const

$$q = 1 + 1/(\alpha + D + 1)$$

$$T = (\sqrt{s}/2)\beta/(D(\alpha + D + 1))$$

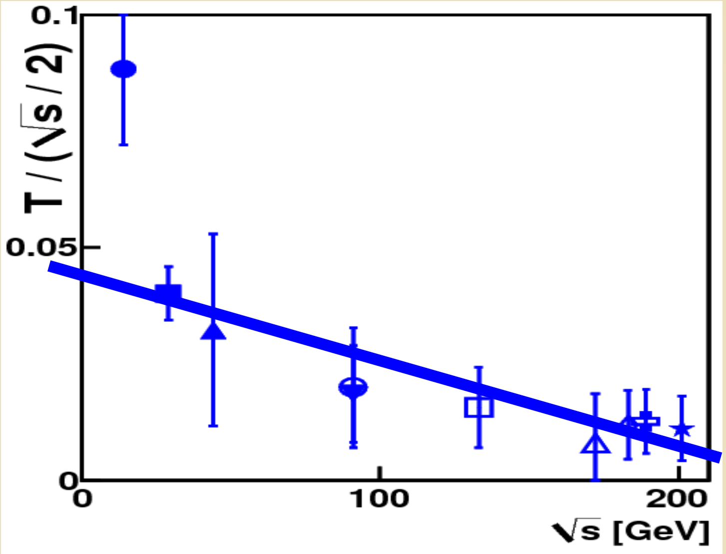


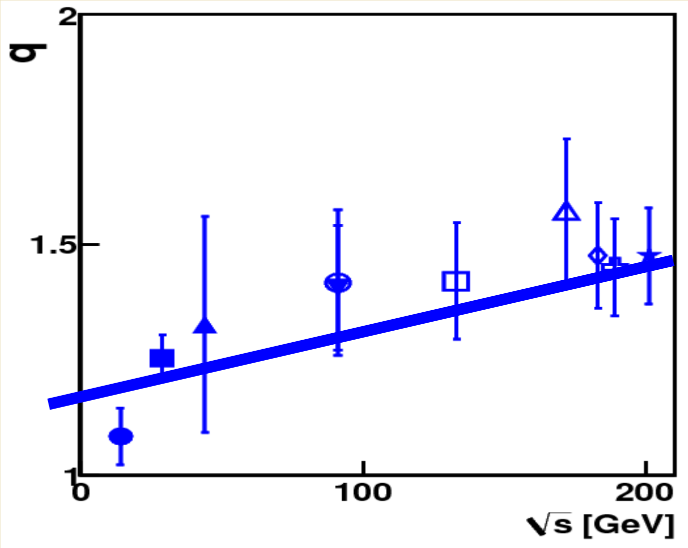


ee: T–q Tsallis parameters in e^+e^- collisions

- Canonical

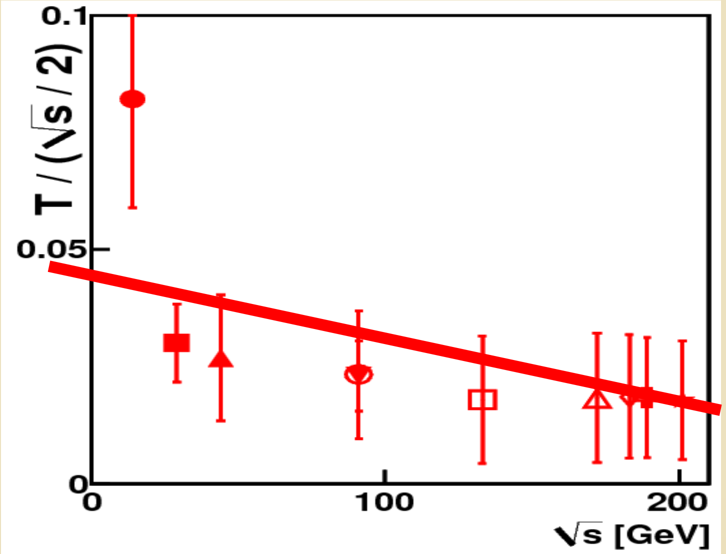
$$A x^{D-1}\left(1+\frac{q-1}{T/(\sqrt{s}/2)}\,x\right)^{-1/(q-1)}$$

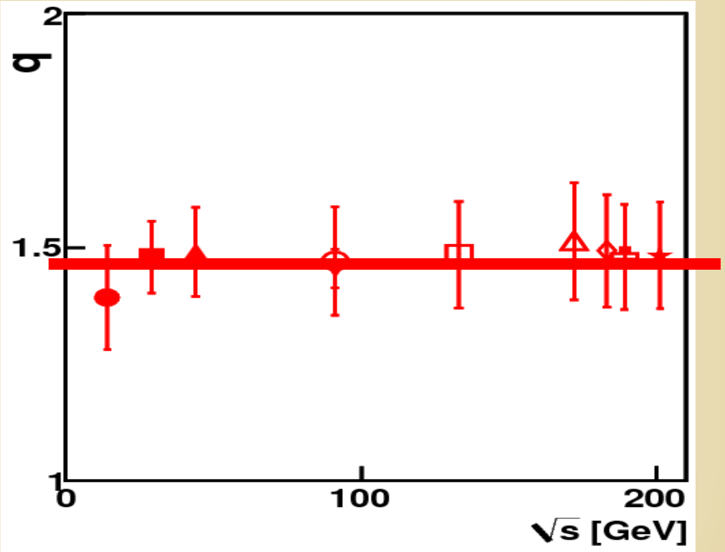




- Micro-canonical

$$\frac{A\,x^{D-1}\,(1-x)}{\left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$

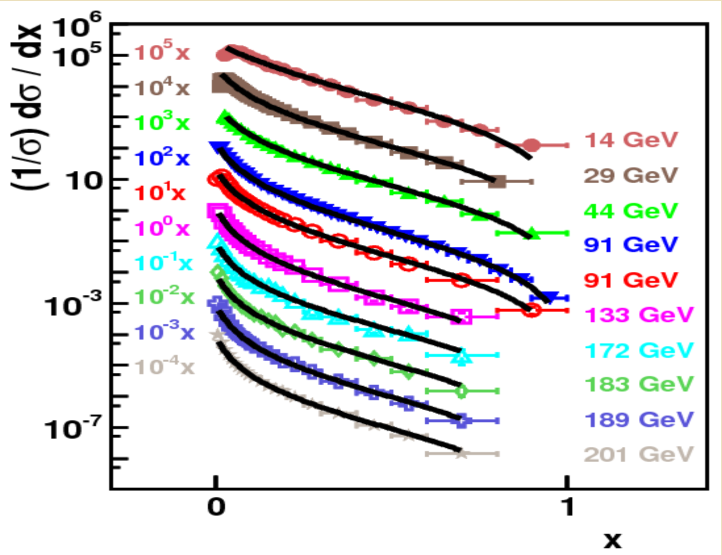


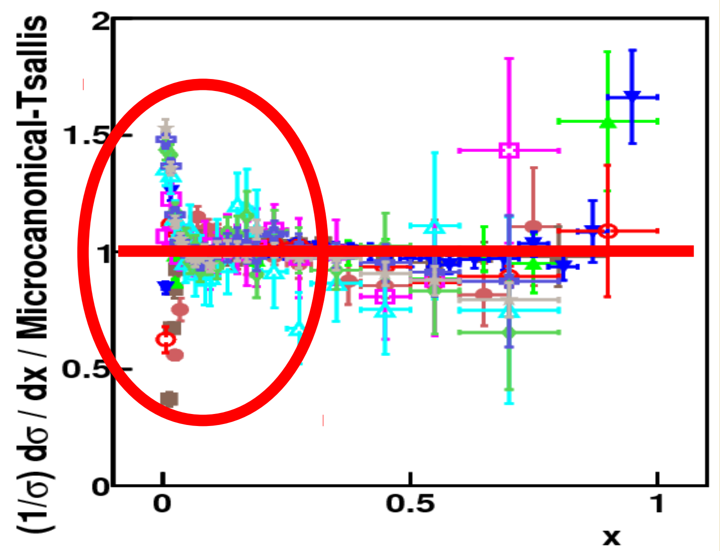


ee: Tsallis–Pareto with an increased dimension

- e^+e^- micro-canonical $D=1$

$$\frac{A\,x^{D-1}\,(1-x)}{\left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$

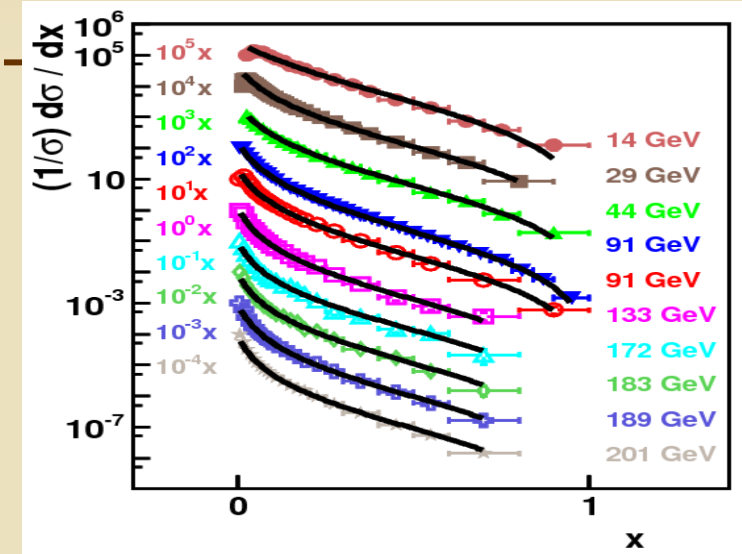


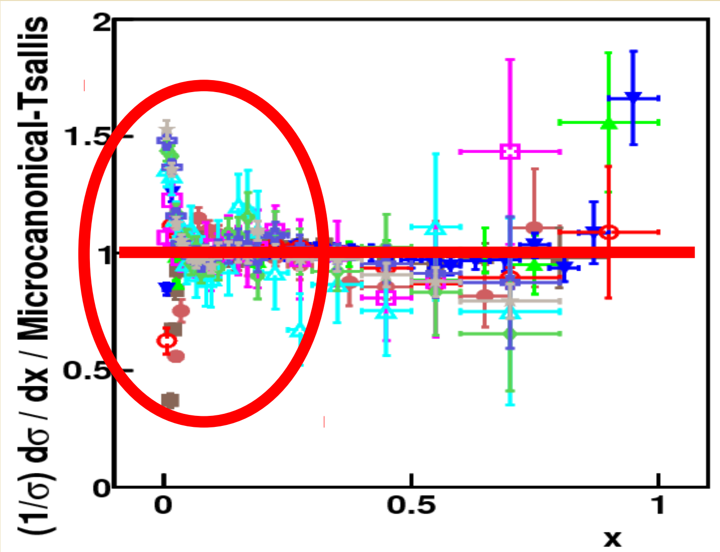


ee: Tsallis–Pareto with an increased dimension

- e^+e^- micro-canonical D=1

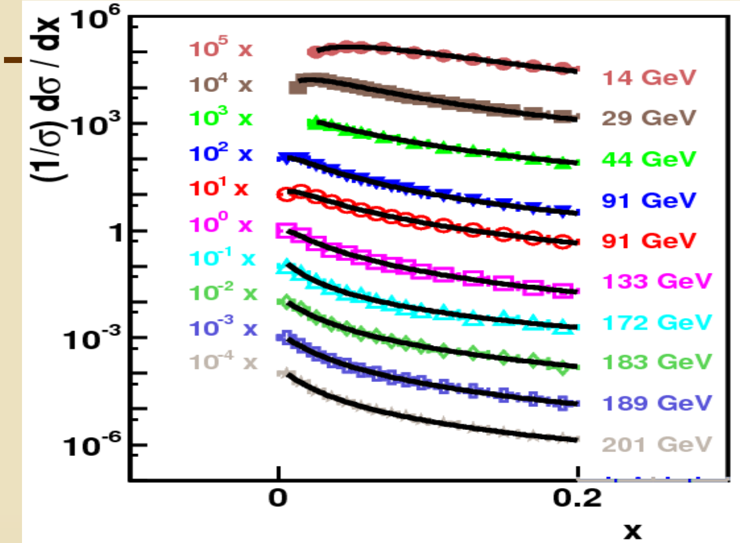
$$\frac{A\,x^{D-1}(1-x)}{\left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$

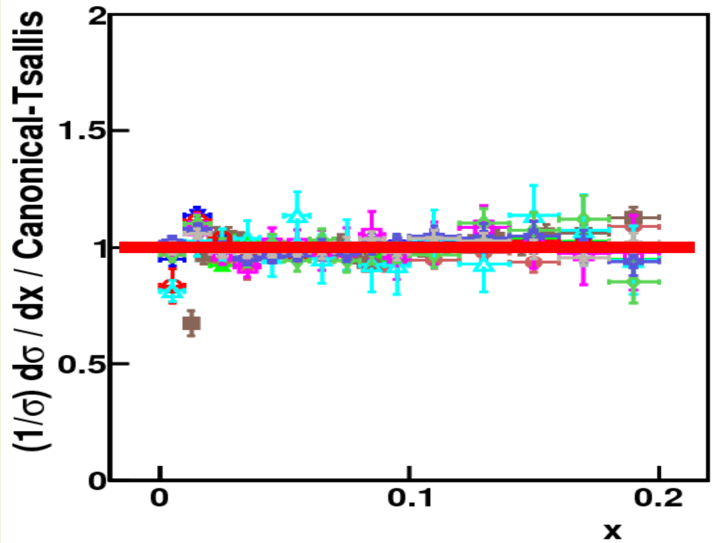




- e^+e^- micro-canonical D=3

$$\frac{A\,x^{D-1}(1-x)}{\left(1-\frac{q-1}{T/(\sqrt{s}/2)}\ln(1-x)\right)^{1/(q-1)}}$$





SUMMARY

- High & low p_T spectra has different distribution..
  - ...however hadronization should not work differently
  - a common model would be nice.
- Non-extensive (non-equilibrium) thermodynamic
  - Can be applied generally in AA and pp, but evolution ansatz need to introduce to obtain the best fit.
- Simplest case: Tsallis–Pareto in e^+e^-
  - High-p_T behavior is described by a TP-based model, after introducing microcanonical generalization of TP
  - Tsallis–Pareto is not enough, need a non-ext. model
  - Lower, intermediate p_T data also described by larger *D*
- Stay tuned! More coming soon for QM

BACKUP

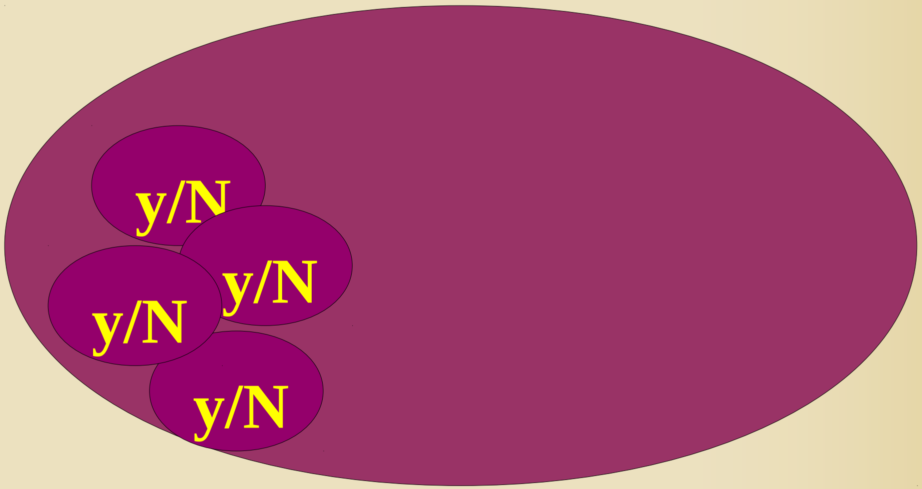
Associative composition ⇉ evolution eq.

Non-extensive Gibbs, generalised

logarithm: $f(x) = \frac{1}{Z} e^{-\beta X(x)}$.

Composition rule for sub-systems:

$$x_N(y) := \underbrace{h \circ \ldots \circ h}_{N-1} \left(\frac{y}{N}, \ldots, \frac{y}{N} \right)$$



Meanwhile satisfy: $\lim_{N \to \infty} x_N(y) < \infty$.

Assimptotically, if $N_1, N_2 \to \infty$. :

$$x_{N_1+N_2} = \varphi(x_{N_1}, x_{N_2})$$

recursive equation can be given:

$x_n = h\left(x_{n-1}, \frac{y}{N}\right)$, where $h(x,0) = x$.

$$x_n - x_{n-1} = h\left(x_{n-1}, \frac{y}{N}\right) - h(x_{n-1}, 0).$$

Evolution equation can carry out:

$$\frac{dx}{dt} = \frac{y}{t_f} h_2'(x, 0^+)$$

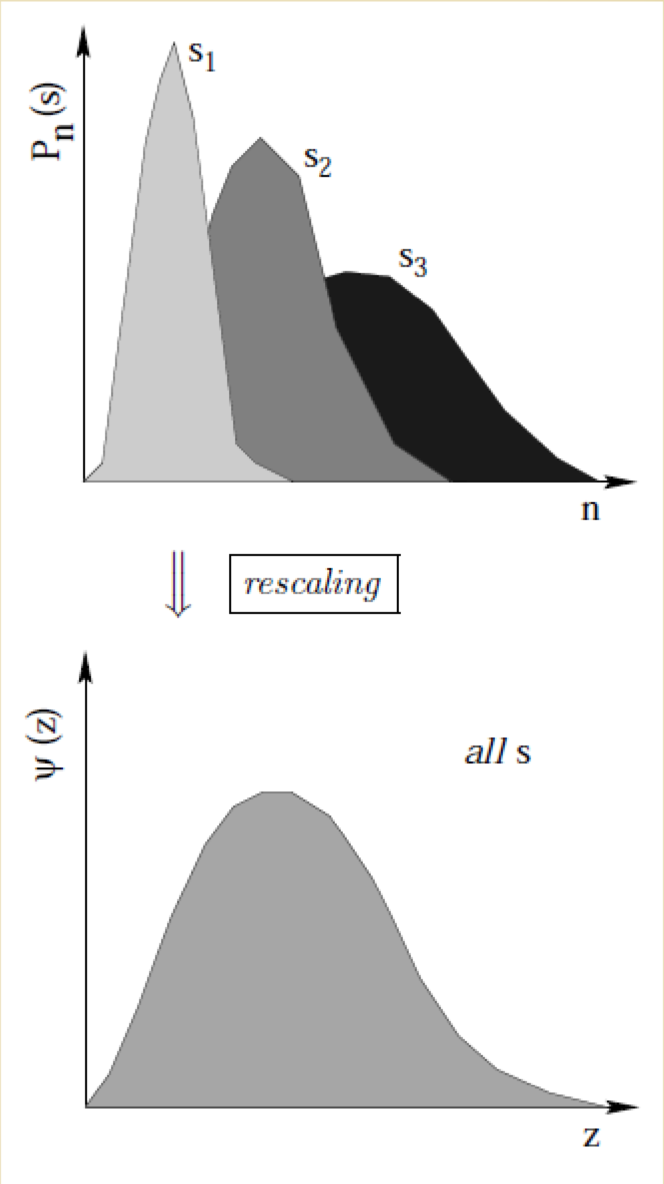
$$L(x) = \int_0^x \frac{dz}{h_2'(z, 0^+)} = y \frac{t}{t_f}.$$

Koba–Nielsen–Olesen (KNO) scaling

Refs:S Hegyi: Nucl. Phys. B 40 (1972) 317, and arXiv:0011301

Hypothesis by Polyakov and Koba–Nielsen–Olesen at very high collision energies, the probability distributions $P_n(s)$ for detecting n final state particles exhibit a scaling (homog-eneity) relation:

$$P_n(s) = \frac{1}{\langle n(s)\rangle}\,\psi\left(\frac{n}{\langle n(s)\rangle}\right)$$



As $s \to \infty$ with $<n(s)>$ being the average multiplicity of secondaries measured at collision energy s.

KNO: Simple rescaled multiplicity distributions are only a copy of an universal one, $\Psi(z)$ depending on scale $z=n/<n(s)>$ only,

Hadronization with parameter Evolution

Ref: GGB, G Kalmár, K Ürmössy, TS Biró, Proc. of Gribov 80. (2011):

Tsallis based hadronization for π:

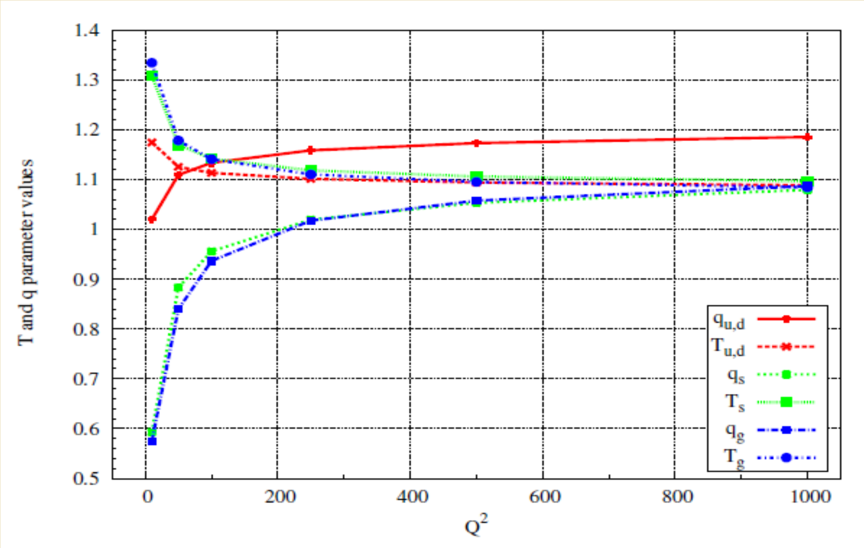
$$\sim \left(1 + (q_i - 1) \cdot \frac{z}{T_i}\right)^{-1/(q_i - 1)}$$

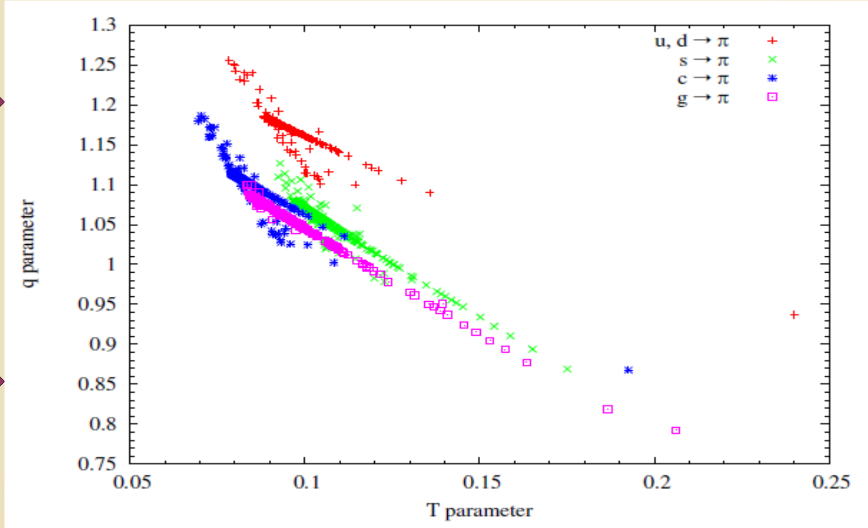
Tsallis–Pareto parameters can be extracted for hadronization:

parton, i	T_{i1}	T_{i0}	q_{i1}	q_{i0}
$u, \bar{u}, d, \bar{d}$	-0.057753	0.239825	0.124000	0.860351
$s, \bar{s}$	-0.093988	0.343175	0.265042	0.384453
$c, \bar{c}$	-0.048170	0.205408	-0.40198	2.142750
$b, \bar{b}$	-0.033599	0.156249	0.103565	0.803255
g	-0.118556	0.394749	0.318477	0.253205

Including the evolution-asatz:

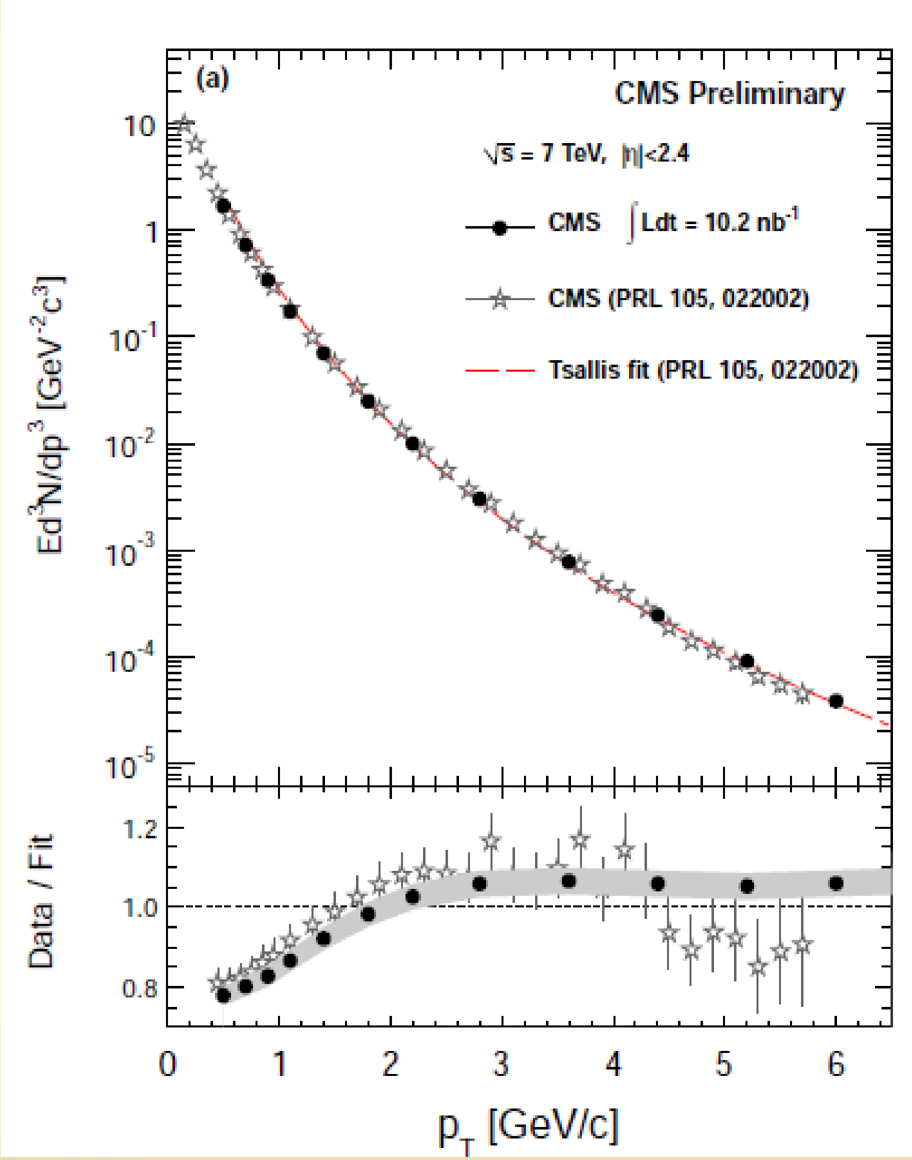
$$T_i \longrightarrow T_i(Q^2) = T_{i1} \cdot \ln\left(\ln(Q^2)\right) + T_{i0} \, ,$$
$$q_i \longrightarrow q_i(Q^2) = q_{i1} \cdot \ln\left(\ln(Q^2)\right) + q_{i0} \, .$$

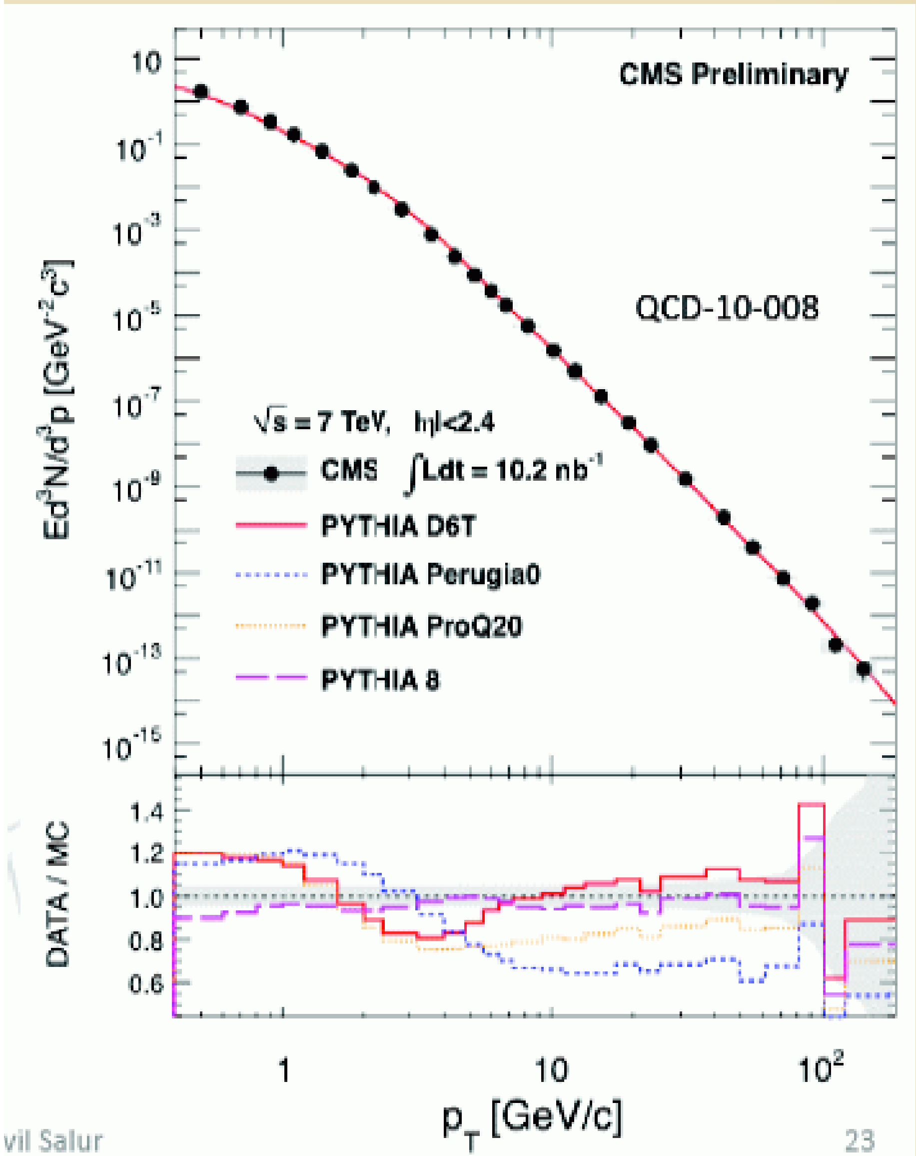




New (HOT) data at LHC energies – today

See: ALICE: Prague Jet workshop & CMS: QCD-10-008





Comparision in x_T: old/new data by Tevatron

See: CDF: PRD 79 112005 (2009) & CMS QCD-10-008