



中国科学院大学  
University of Chinese Academy of Sciences



ICTP-AP  
International Centre  
for Theoretical Physics Asia-Pacific  
国际理论物理中心-亚太地区

# Calculating and Detecting Gravitational Waves from the Sound in Cosmological Phase Transitions

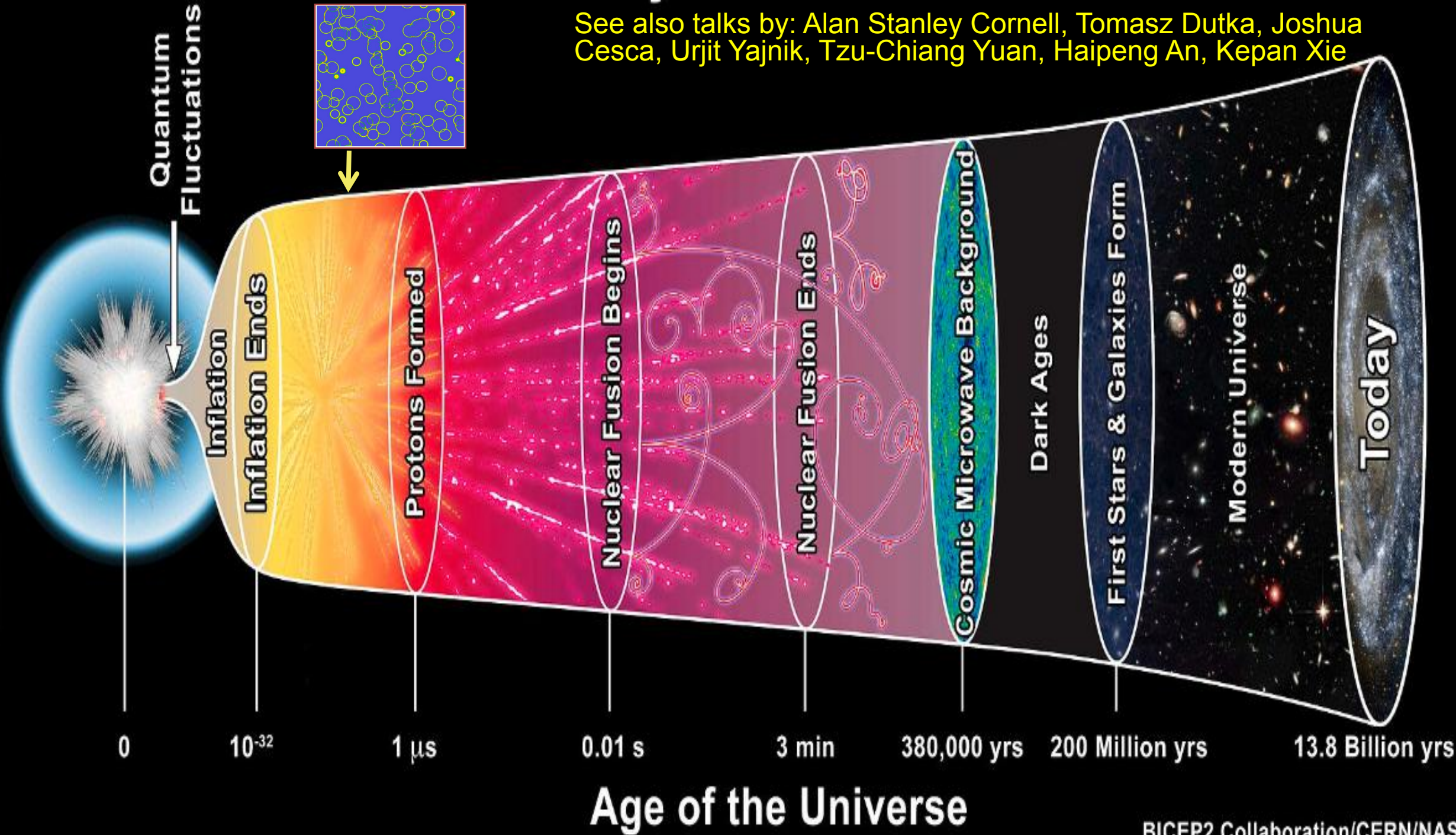
Huaike Guo

Dec. 11, 2024

International Joint Workshop on the Standard Model and Beyond 2024  
& 3rd Gordon Godfrey Workshop on Astroparticle Physics



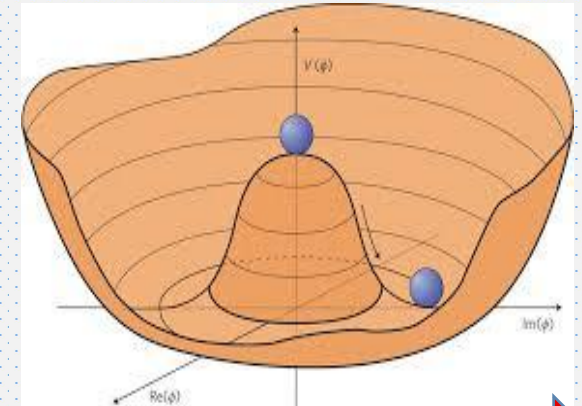
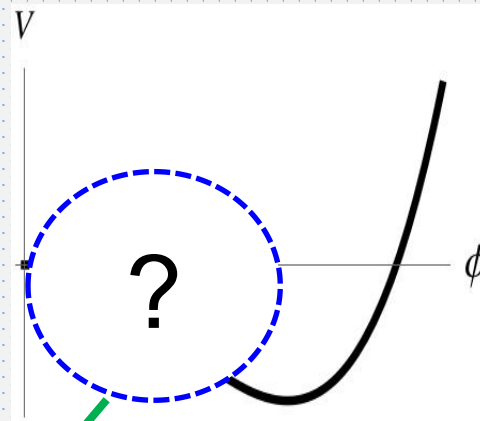
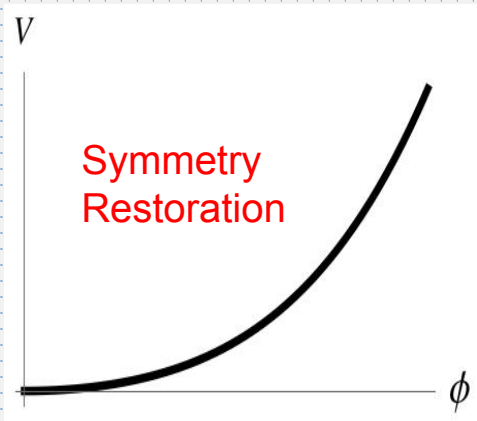
Radius of the Visible Universe



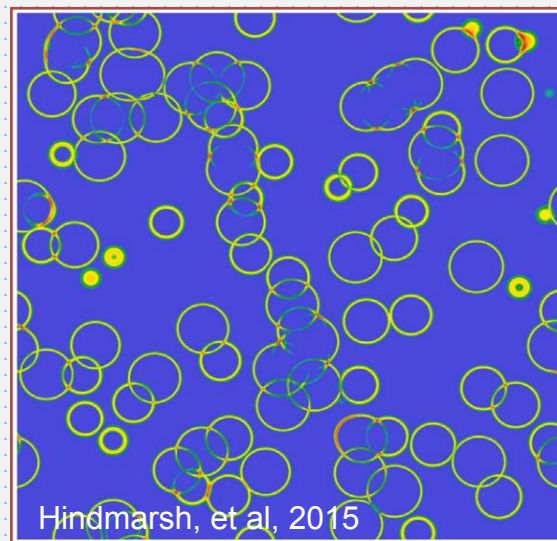
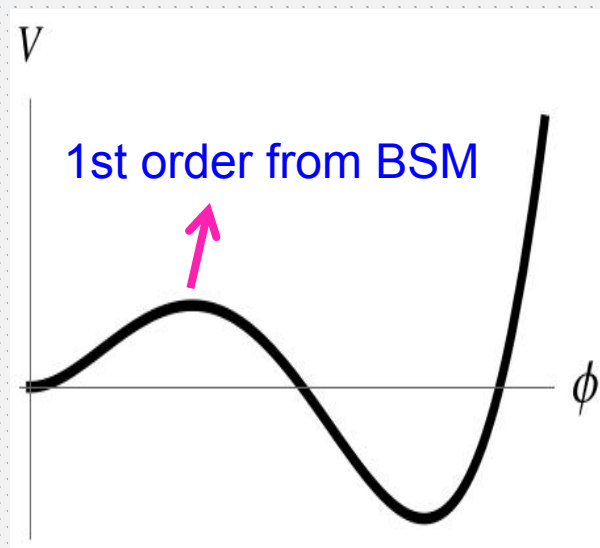
See also talks by: Alan Stanley Cornell, Tomasz Dutka, Joshua Cesca, Urjit Yajnik, Tzu-Chiang Yuan, Haipeng An, Kepan Xie



# Electroweak Phase Transition



Temperature drops

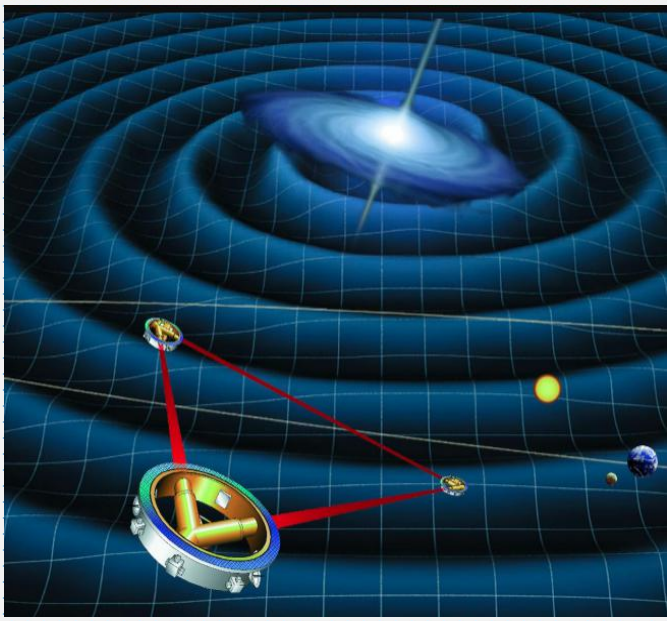


Electroweak Baryogenesis

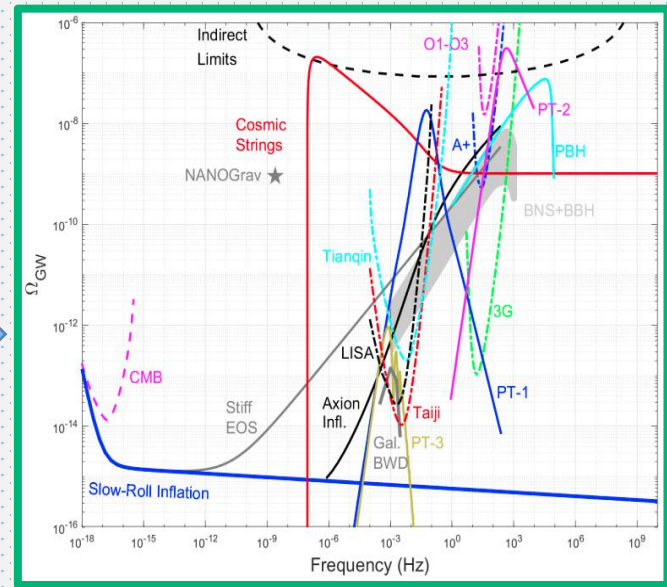
- Modified Higgs potential (Higgs physics, GW)
- Extra CP-violation (EDM, LHC)
- New particles, symmetries (LHC, GW)

Morrissey, Ramsey-Musolf [1206.2942]

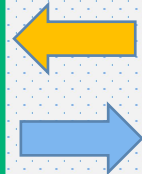
# From Theory to Experiment



LIGO, LISA/Taiji/Tianqin, PTA, ...



Gravitational Wave Spectrum



$\alpha$   
 $\beta$   
 $v_w$   
 $T_*$   
 $g_s$   
 ...

Phase Transition Parameters



### Standard Model of Elementary Particles

	three generations of matter (fermions)			interactions / force carriers (bosons)	
	I	II	III		
mass	$\approx 2.2 \text{ MeV}/c^2$	$\approx 1.28 \text{ GeV}/c^2$	$\approx 173.1 \text{ GeV}/c^2$	0	$\approx 124.97 \text{ GeV}/c^2$
charge	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0	0
spin	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	0
	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>g</b> gluon	<b>H</b> higgs
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b><math>\gamma</math></b> photon	<b>BSM</b>
	<b>e</b> electron	<b><math>\mu</math></b> muon	<b><math>\tau</math></b> tau	<b>Z</b> Z boson	<b>GAUGE BOSONS</b> VECTOR BOSONS
	<b><math>\nu_e</math></b> electron neutrino	<b><math>\nu_\mu</math></b> muon neutrino	<b><math>\nu_\tau</math></b> tau neutrino	<b>W</b> W boson	
					<b>SCALAR BOSONS</b>



Particle Physics Model





# Phenomenological Studies

## Detection of early-universe gravitational-wave signatures and fundamental physics

Robert Caldwell, Yanou Cui, Huai-Ke Guo , Vuk Mandic, Alberto Mariotti, Jose Miguel No, Michael J. Ramsey-Musolf, Mairi Sakellariadou , Kuver Sinha, Lian-Tao Wang, Graham White, Yue Zhao, Haipeng An, Ligong Bian, Chiara Caprini, Sebastien Clesse, James M. Cline, Giulia Cusin, Bartosz Fornal, Ryusuke Jinno, Benoit Laurent, Noam Levi, Kun-Feng Lyu, Mario Martinez, Andrew L. Miller, Diego Redigolo, Claudia Scarlata, Alexander Sevrin, Barmak Shams Es Haghi, Jing Shu, Xavier Siemens, Danièle A. Steer, Raman Sundrum, Carlos Tamarit, David J. Weir, Ke-Pan Xie, Feng-Wei Yang & Siyi Zhou — Show fewer authors

*General Relativity and Gravitation* **54**, Article number: 156 (2022) | [Cite this article](#)

arXiv > hep-ph > arXiv:2203.08206

High Energy Physics - Phenomenology

[Submitted on 15 Mar 2022]

### Probing the Electroweak Phase Transition with Exotic Higgs Decays

Marcela Carena, Jonathan Kozaczuk, Zhen Liu, Tong Ou, Michael J. Ramsey-Musolf, Jessie Shelton, Yikun Wang, Ke-Pan Xie

arXiv > hep-ph > arXiv:2203.10046

High Energy Physics - Phenomenology

[Submitted on 18 Mar 2022]

### Scalar-mediated dark matter model at colliders and gravitational wave detectors -- A White paper for Snowmass 2021

Jia Liu, Xiao-Ping Wang, Ke-Pan Xie

Snowmass 2021 White papers

Models	Strong 1 <sup>st</sup> order phase transition	GW signal	Cold DM	Dark Radiation and small scale structure
<b>SM charged</b>				
Triplet [20–22]	✓	✓	✓	✗
complex and real Triplet [23] (Georgi-Machacek model)	✓	✓	✓	✗
Multiplet [24]	✓	✓	✓	
2HDM [25–30]	✓	✓		✗
MLRSM [31]	✓	✓	✗	✗
NMSSM [32–36]	✓	✓	✓	✗
<b>SM uncharged</b>				
$S_\nu$ (xSM) [37–49]	✓	✓	✗	✗
2 $S_\nu$ 's [50]	✓	✓	✓	✗
$S_c$ (cxSM) [49, 51–54]	✓	✓	✓	✗
$U(1)_D$ (no interaction with SM) [55]	✓	✓	✓	✗
$U(1)_D$ (Higgs Portal) [56]	✓	✓	✓	
$U(1)_D$ (Kinetic Mixing) [57]	✓	✓	✓	
Composite SU(7)/SU(6) [58]	✓	✓	✓	
$U(1)_L$ [59]	✓	✓	✓	✗
$SU(2)_D \rightarrow$ global $SO(3)$ by a doublet [60–62]			✓	✗
$SU(2)_D \rightarrow U(1)_D$ by a triplet [63–65]			✓	✓
$SU(2)_D \rightarrow Z_2$ by two triplets [66]			✓	✗
$SU(2)_D \rightarrow Z_3$ by a quadruplet [67, 68]			✓	✗
$SU(2)_D \times U(1)_{B-L} \rightarrow Z_2 \times Z_2$ by a quintuplet and a $S_c$ [69]			✓	✗
$SU(2)_D$ with two dark Higgs doublets [70]	✓	✓	✗	✗
$SU(3)_D \rightarrow Z_2 \times Z_2$ by two triplets [62, 71]			✓	✗
$SU(3)_D$ (dark QCD) (Higgs Portal) [72, 73]	✓	✓	✓	
$G_{SM} \times G_{D,SM} \times Z_2$ [74]	✓	✓	✓	
$G_{SM} \times G_{D,SM} \times G_{D,SM} \dots$ [75]	✓	✓	✓	
<b>Current work</b>				
$SU(2)_D \rightarrow U(1)_D$ (see the text)	✓	✓	✓	✓

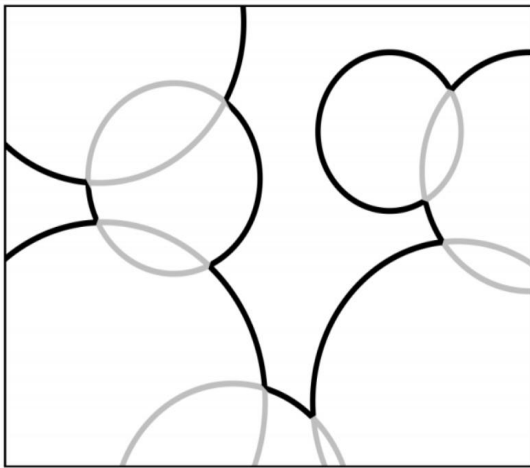
Ghosh, HG, Han, Liu, JHEP [2012.09758]

# Gravitational Wave Sources

The current understanding:

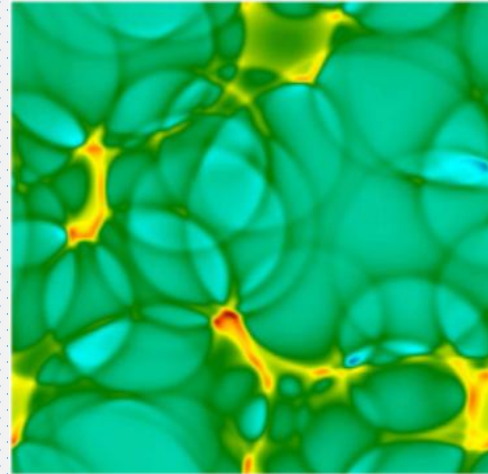
$$\square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

energy near the wall



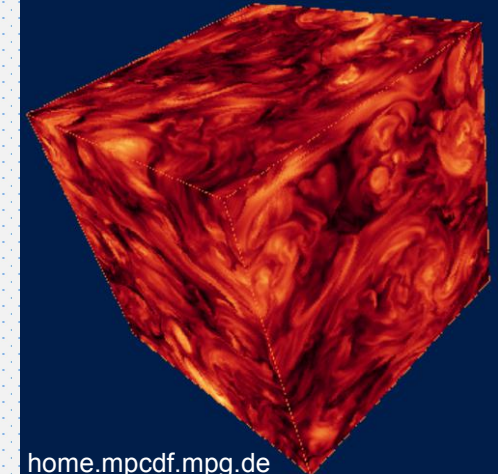
Bubble Collisions

fluid kinetic energy



Sound Waves

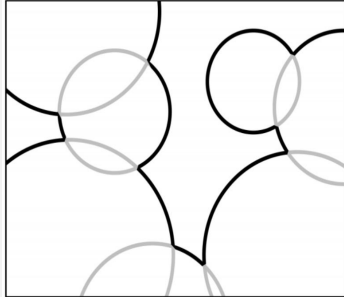
turbulent fluid + magnetic field



Magnetohydrodynamic Turbulence

# The GW Observable

bubble collision

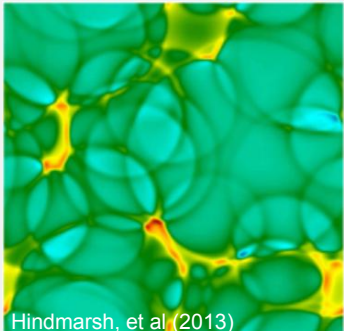


$$\Omega_{\text{coll}}(f)h^2 = 1.67 \times 10^{-5} \Delta \left( \frac{H_{\text{pt}}}{\beta} \right)^2 \left( \frac{\kappa_{\phi} \alpha}{1 + \alpha} \right)^2 \times \left( \frac{100}{g_*} \right)^{1/3} S_{\text{env}}(f),$$

Energy density Spectrum

$$\Omega_{\text{GW}}(f) = \frac{d\rho_{\text{GW}}}{\rho_c d \log f}$$

sound waves



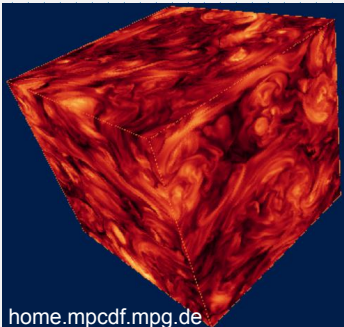
Hindmarsh, et al (2013)

$$\Omega_{\text{sw}}(f)h^2 = 2.65 \times 10^{-6} \left( \frac{H_{\text{pt}}}{\beta} \right) \left( \frac{\kappa_{\text{sw}} \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_*} \right)^{1/3} \times v_w \left( \frac{f}{f_{\text{sw}}} \right)^3 \left( \frac{7}{4 + 3(f/f_{\text{sw}})^2} \right)^{7/2} \Upsilon(\tau_{\text{sw}}),$$

$$\Upsilon = 1 - (1 + 2\tau_{\text{sw}} H_{\text{pt}})^{-1/2} \quad (\text{RD})$$

HG, Sinha, Vagie, White, JCAP [2007.08537]

MHD



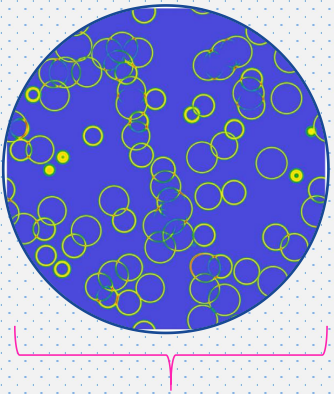
home.mpcdf.mpg.de

$$h^2 \Omega_{\text{turb}}(f) = 3.35 \times 10^{-4} \left( \frac{H_*}{\beta} \right) \left( \frac{\kappa_{\text{turb}} \alpha}{1 + \alpha} \right)^{\frac{3}{2}} \left( \frac{100}{g_*} \right)^{1/3} v_w S_{\text{turb}}(f)$$

Chiara Caprini et al JCAP [1512.06239]



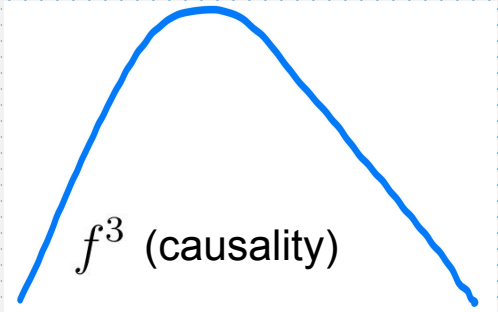
# Basic Properties



Hubble size:  $1/H^*$

$$f_{\text{now}} = 1.65 \times 10^{-5} \left( \frac{f_{\text{PT}}}{\beta} \right) \left( \frac{\beta}{H_*} \right) \left( \frac{T_*}{100\text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6} \text{ Hz}$$

~100-1000



Cai, Pi, Sasak, PRD [1909.13728]

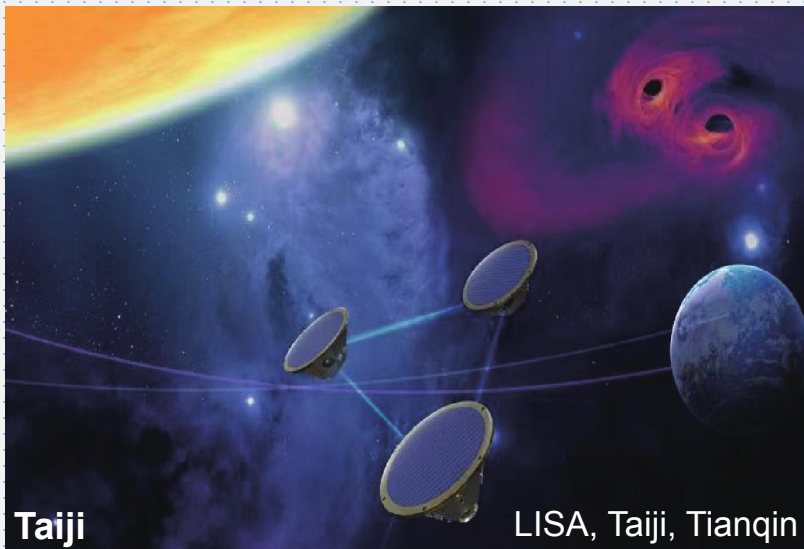
nHz (~100MeV) QCD scale

~mHz : (~100GeV) weak scale

~100Hz (~PeV - EeV) high scale



中国脉冲星测时阵列 (CPTA)



Taiji

LISA, Taiji, Tianqin



ligo.caltech.edu



# Multiband Searches

NANOGrav, ApjL [2306.16219 ]

EPTA [2306.16227]

Xue,Bian,Shu,Yuan,Zhu, et al, PRL [2110.03096]

Bian et al [2307.02376]

Wu, Chen, Huang [2307.03141]

...

Boileau et al, MNRAS [2105.04283]

LISA: Caprini et al [2403.03723]

Network: Wang, Han, PRD [2108.11151] ...

TDI optimization: Wang, Li, Xu, Fan, PRD [2201.10902]

...

Romero,Martinovic,Callister,HG,Martínez,Sakellaria

dou, Yang,Zhao, PRL [2102.01714]

Badger, ..., HG, ..., PRD [2209.14707]

Jiang, Huang, JCAP [2203.11781]

Yu, Wang, PRD [2211.13111]

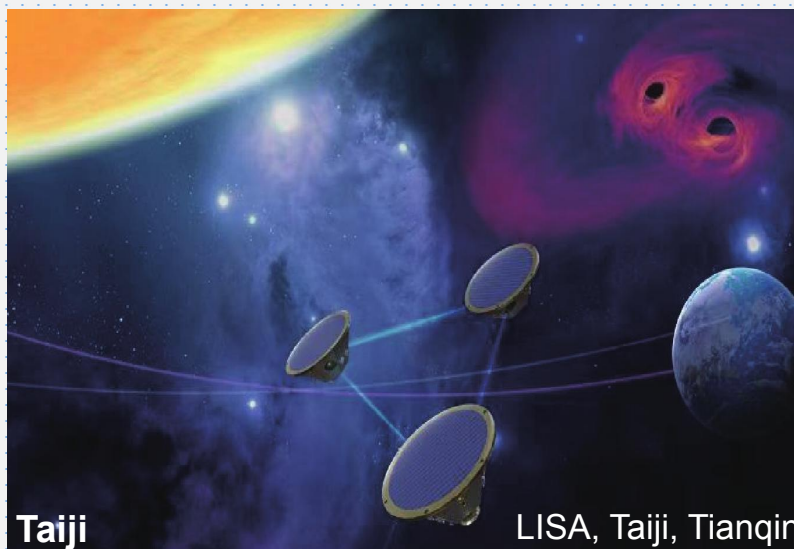
nHz (~100MeV) QCD scale

~mHz : (~100GeV) weak scale

~100Hz (~PeV - EeV) high scale



中国脉冲星测时阵列 (CPTA)



Taiji

LISA, Taiji, Tianqin



LIGO

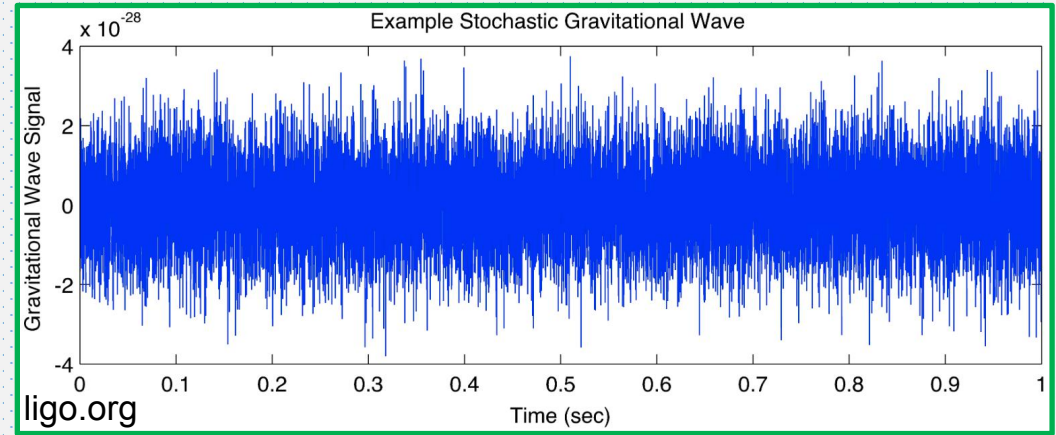
ligo.caltech.edu

# Detection at LIGO

- ✓ Gaussian, Stationary, Isotropic, Unpolarized

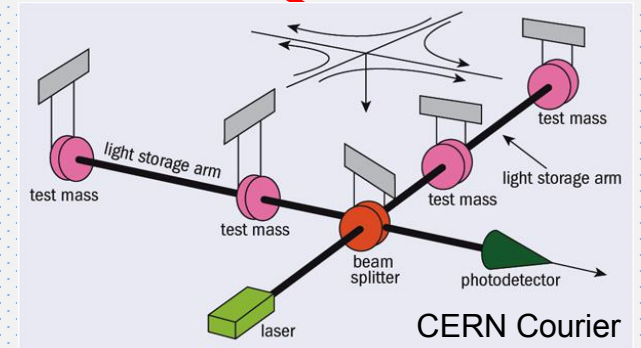
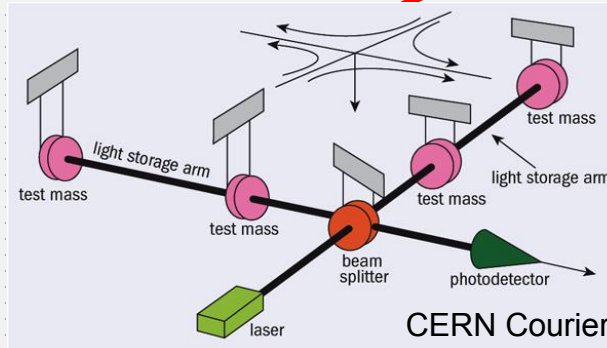
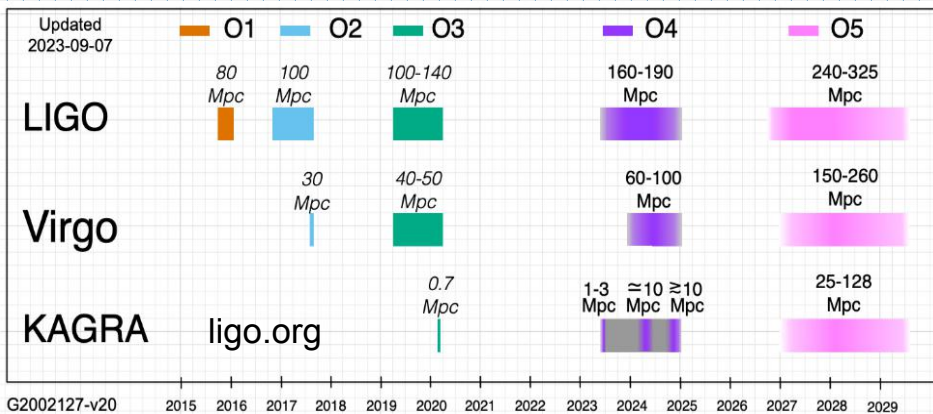
$$\langle h_A^*(f, \Omega) h_{A'}(f', \Omega') \rangle = \frac{3H_0^2}{32\pi^3} \delta^2(\Omega, \Omega') \delta_{AA'} \delta(f - f') f^{-3} \Omega_{GW}(f)$$

stochastic GWs: noise-like



solution: cross-correlation

O1+O2+O3@LIGO (H1, L1), Virgo





# Detection at LIGO

## O1+O2+O3@LIGO (H1, L1), Virgo

- No Evidence for Broken Power Law Signal
- No Evidence for Bubble Collision Domination Signal
- No Evidence for Sound Waves Domination Signal

### Bubble Collision

95% CL UL with fixed  $T_{pt}$  and  $\beta/H_{pt}$

Phenomenological model (bubble collisions)

$\Omega_{coll}^{95\%}(25 \text{ Hz})$

$\beta/H_{pt} \setminus T_{pt}$	$10^7 \text{ GeV}$	$10^8 \text{ GeV}$	$10^9 \text{ GeV}$	$10^{10} \text{ GeV}$
0.1	$9.2 \times 10^{-9}$	$8.8 \times 10^{-9}$	$1.0 \times 10^{-8}$	$7.2 \times 10^{-9}$
1	$1.0 \times 10^{-8}$	$8.4 \times 10^{-9}$	$5.0 \times 10^{-9}$	...
10	$4.0 \times 10^{-9}$	$6.3 \times 10^{-9}$	...	...

no sensitivity

### Broken Power Law

95% CL UL (CBC+BPL)

$$\Omega_{ref} = 6.1 \times 10^{-9}$$

$$\Omega_* = 5.6 \times 10^{-7}$$

$$\Omega_{BPL}(25 \text{ Hz}) = 4.4 \times 10^{-9}$$

### Sound Waves

95% CL UL

$$\Omega_{sw}(25 \text{ Hz}) = 5.9 \times 10^{-9}$$

$$\beta/H_{pt} < 1 \text{ and } T_{pt} > 10^8 \text{ GeV}$$

# Detection at LISA/Taiji/Tianqin

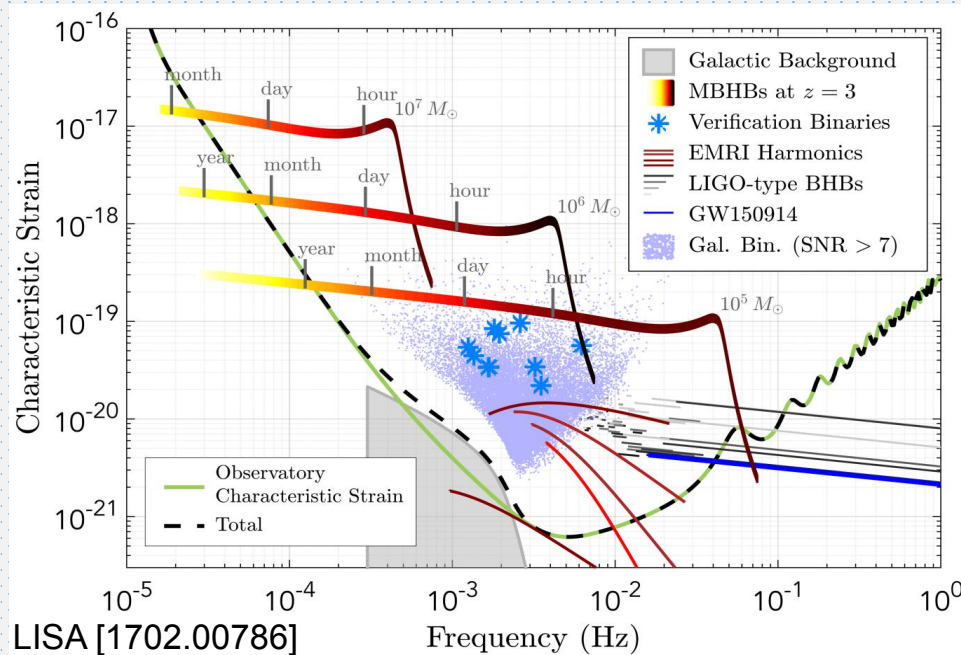
## Detection with a single detector

- Complicated, and correlated noise
- Complications from time-delay interferometry
- Solution: null channel method, or with a network

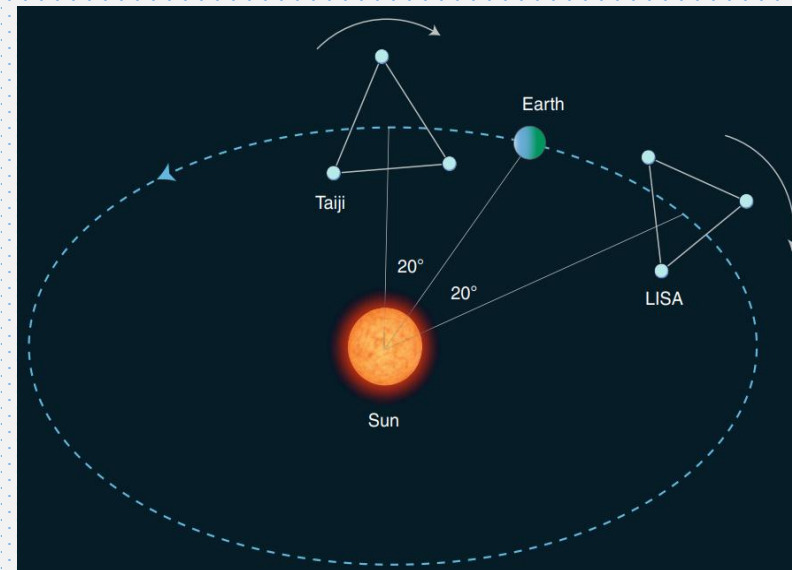
galactic foreground + astro background + cosmic background

SGWB detectable down to  $\Omega_{GW} \sim \mathcal{O}(10^{-13})$

Boileau et al, MNRAS [2105.04283]



The LISA–Taiji network



Ruan, Liu, Guo, Wu, Cai, Nature Astron [2002.03603]  
Cai et al [2305.04551]

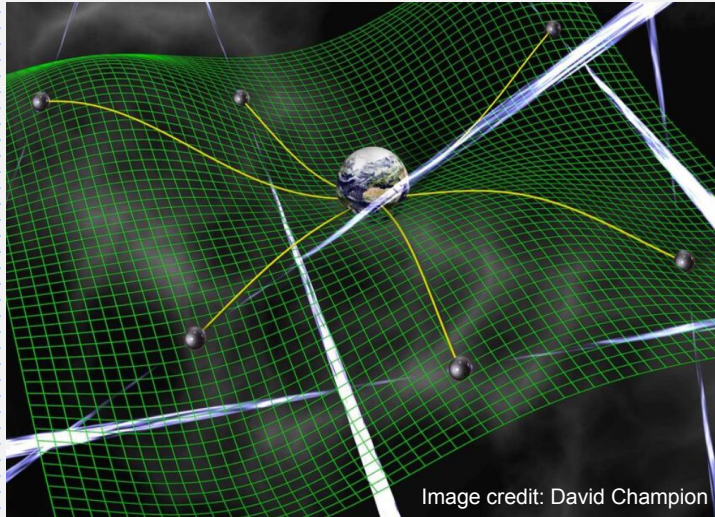
Gowling et al, JCAP [2209.13551, 2106.05984]

LISA: Caprini et al [2403.03723]

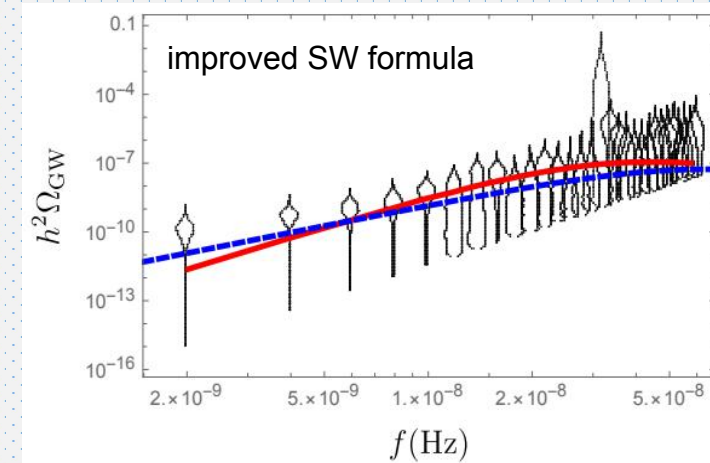
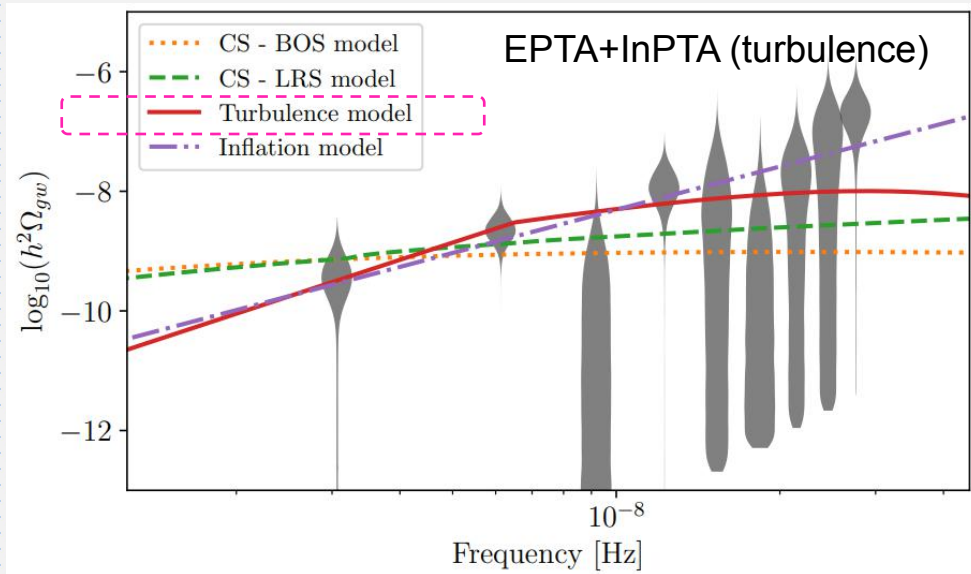
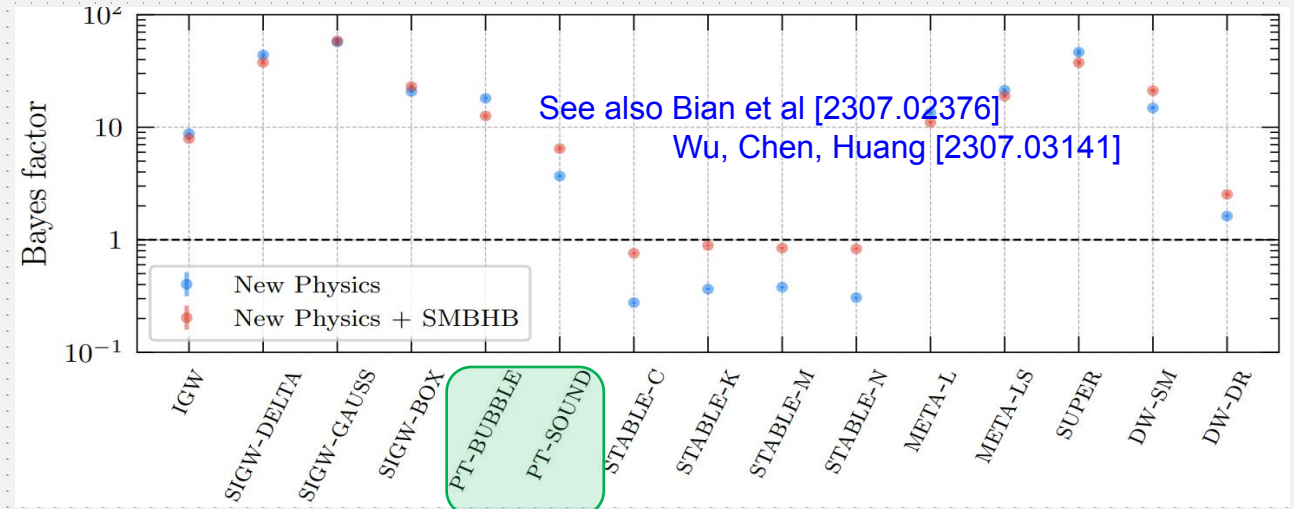
TDI optimization: Wang, Li, Xu, Fan, PRD [2201.10902]



# Detection with PTA



NANOGrav, ApJL [2306.16219]



Ghosh, Ghoshal, HG, ..., JCAP [2307.02259]

# Problem: uncertainties

- Finite T effective potential calculations
- Phase transition parameter calculations (vw)
- GW spectra calculations (simulations, modellings)

$\Delta\Omega_{\text{GW}}/\Omega_{\text{GW}}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High- $T$ approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

Croon, Gould, Schicho, Tenkanen, White, JHEP [2009.10080]

Uncertainty	pre-factor1	pre-factor2	pre-factor3
$T_p$	0.003%	0.003%	0.002%
$\beta R^*$	8.1%	7.9%	5.9%
$N_{\text{tot}}$	11.4%	11.0%	9.8%
$f_{\beta R^*}^{\text{peak}}$	11.8%	12.0%	14.1%
$\Omega_{\text{GW}} h_{\beta R^*}^2$	37.6%	36.5%	28.9%
$f_{\text{sim}}^{\text{peak}}$	36.4%	36.4%	35.1%
$\Omega_{\text{GW}} h_{\text{sim}}^2$	334.0%	330.8%	336.7%

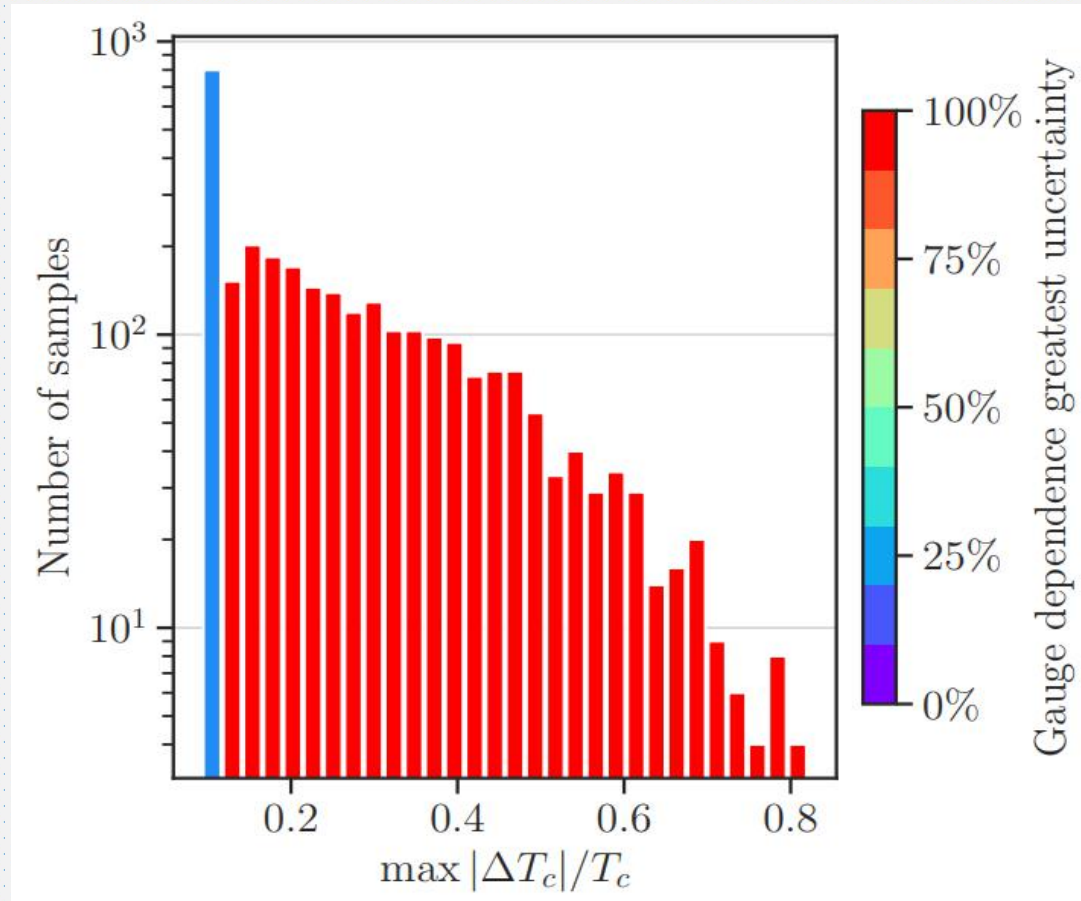
HG, Xiao, Yang, Zhang [2310.04654]

Effect (fixed wall velocity)	Range of error (medium)	Range of error (low)	Type of error
Transition temperature	$\mathcal{O}(10^{-4} - 10^1)$	$\mathcal{O}(10^{-1} - 10^0)$	Random
Mean bubble separation	$\mathcal{O}(0 - 10^{-1})$	$\mathcal{O}(10^{-1} - 10^0)$	Suppression
Fluid velocity	$\mathcal{O}(10^{-2} - 10^0)$	$\mathcal{O}(10^{-2} - 10^0)$	Random
Finite lifetime	$\mathcal{O}(10^{-3} - 10^{-1})$	$\mathcal{O}(10^1 - 10^3)$	Enhancement
Vorticity effects	$\mathcal{O}(10^{-1} - 10^0)$	–	Random

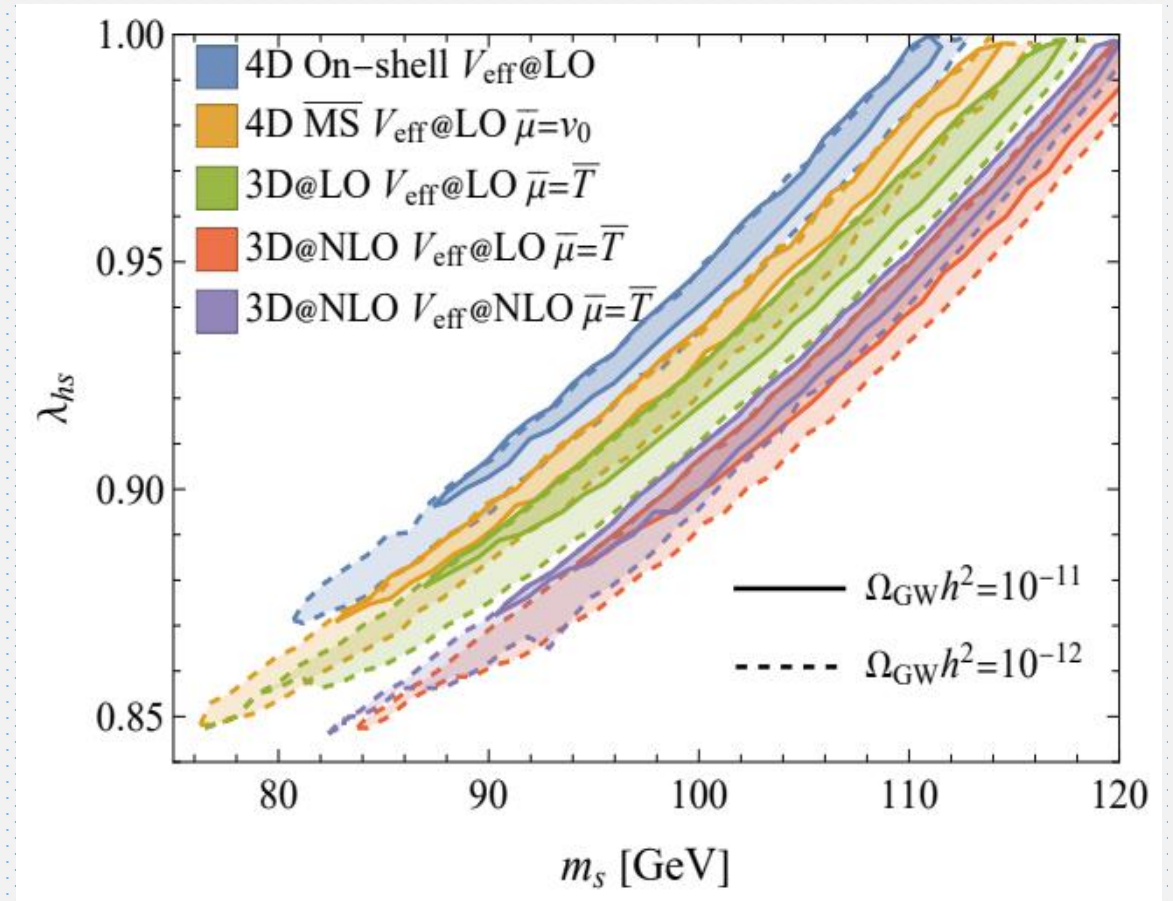
HG, Sinha, Vagie, White, JHEP [2103.06933]



# Problem: uncertainties



Athron, Balazs, Fowlie, Morris, White, JHEP [2403.03769]



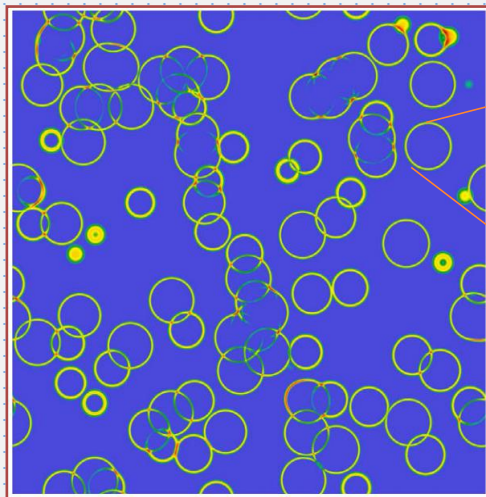
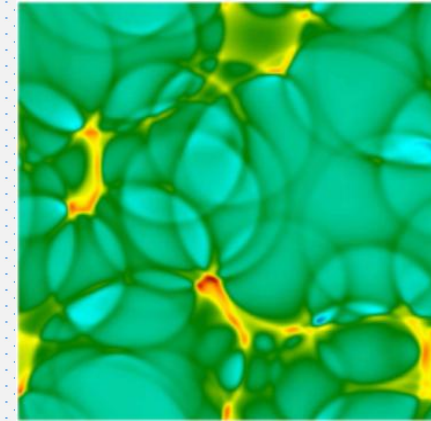
Lewicki, Merchand, Sagunski, Schicho, Schmitt, PRD [2403.03769]

# The Picture

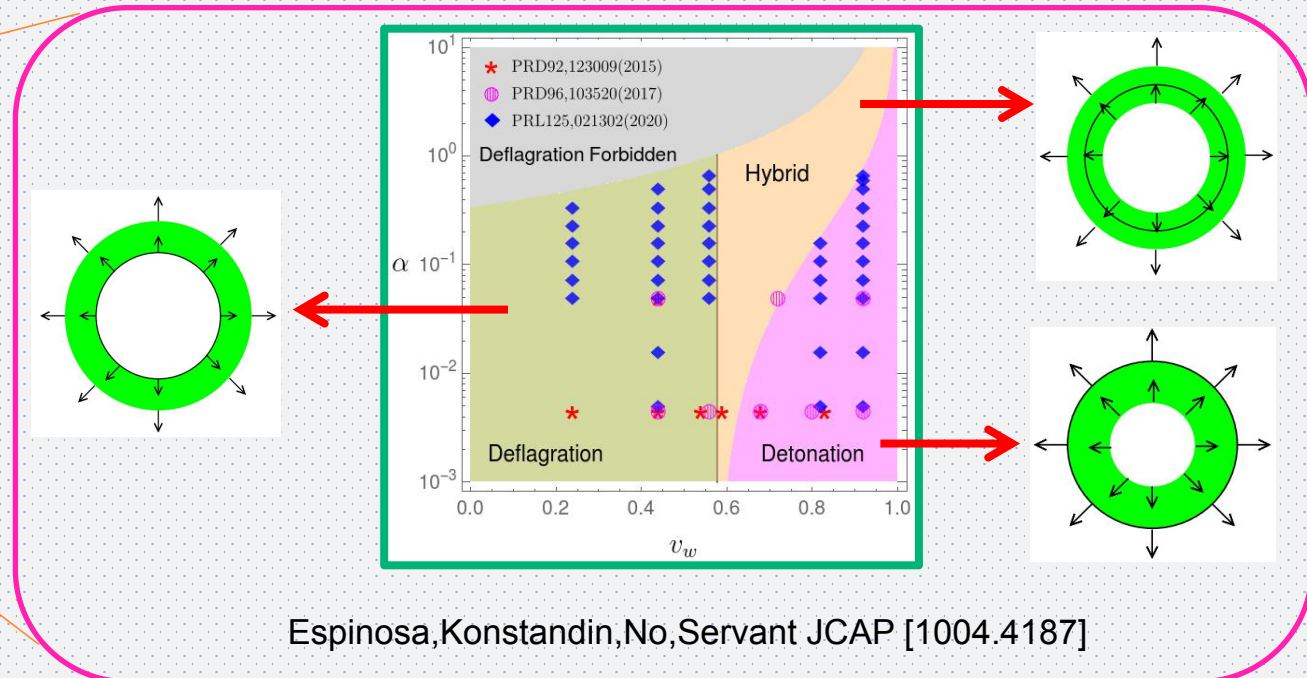
Precise calculation of PT parameters:

Minkowski spacetime: Hindmarsh, Hijazi, JCAP [1909.10040]

Expanding universe: HG, Sinha, Vagie, White, JCAP [2007.08537], JHEP [2103.06933]



Hindmarsh et al, 2015



Espinosa, Konstandin, No, Servant JCAP [1004.4187]

New phenomena?



# Sound Waves

## Analytical Modelling

- Refine the sound shell model
- Synergy with simulations

### Sound Shell Model

Hindmarsh, PRL [1608.04735]

Hindmarsh, Hijazi, JCAP [1909.10040]

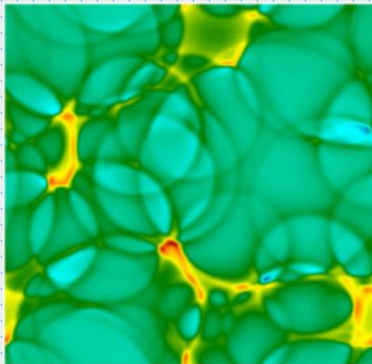
HG, Sinha, Vagie, White, JCAP [2007.08537]

Cai, Wang, Yuwen, PRD Letter [2305.00074]

Pol, Procacci, Caprini, PRD [2308.12943]

Sharma, Dahl, Brandenburg, Hindmarsh [2308.12916]

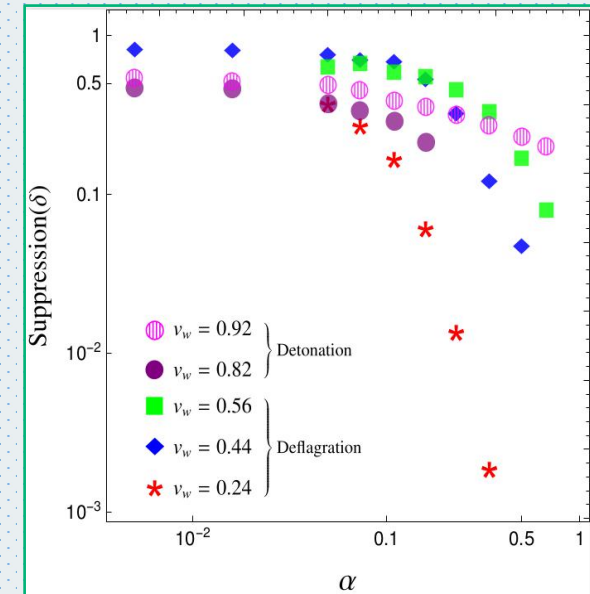
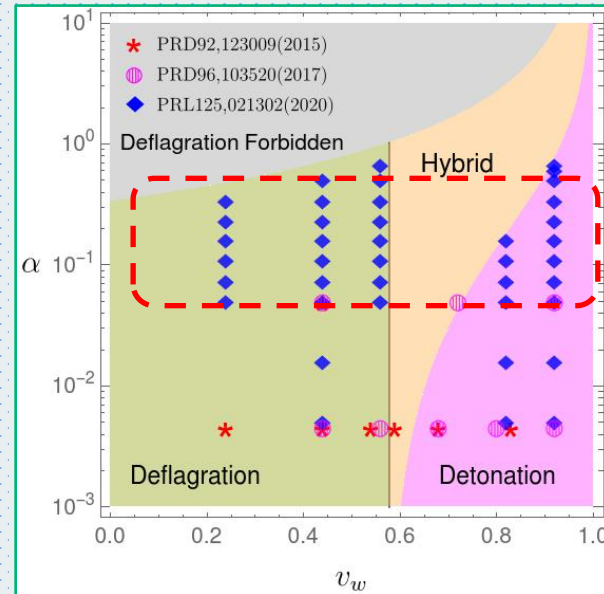
$$v_{\mathbf{q}}^i = \sum_{n=1}^{N_b} v_{\mathbf{q}}^{i(n)}$$



## Numerical Simulation

- Suppression found for strong transitions with small  $v_w$
- Need to cover more parameter space (very strong PT)

$$h^2 \Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left( \frac{100}{g_*} \right)^{\frac{1}{3}} \left( \frac{H_*}{\beta} \right) \left( \frac{\kappa_{\text{sw}} \alpha}{1 + \alpha} \right)^2 v_w S_{\text{sw}}(f) \Upsilon(\tau_{\text{sw}})$$



Cutting, Hindmarsh, Weir, PRL [1906.00480]

# Sound Waves

Hindmarsh, Huber, Rummukainen, Weir, PRL [1304.2433]

$$T^{ij} \propto (p + e)v^i v^j$$

$$h^2 \Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\text{sw}} \alpha}{1 + \alpha}\right)^2 v_w S_{\text{sw}}(f) \Upsilon(\tau_{\text{sw}})$$

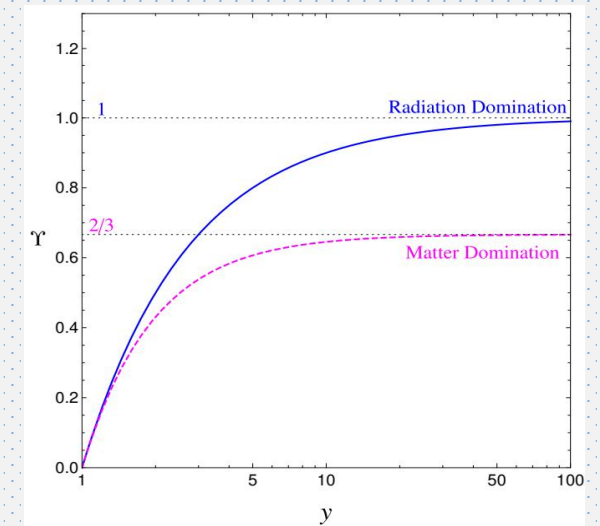
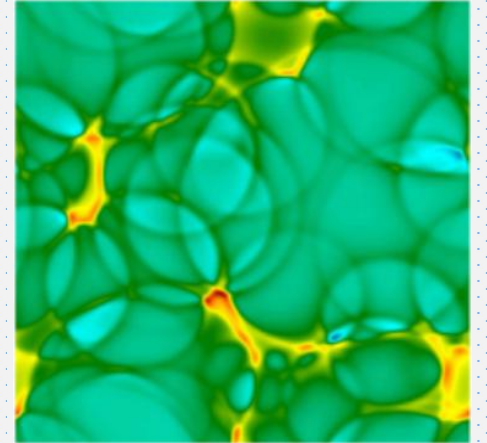
$$S_{\text{sw}}(f) = \left(\frac{f}{f_{\text{sw}}}\right)^3 \left[\frac{7}{4 + 3(f/f_{\text{sw}})^2}\right]^{7/2} \quad f_* = \frac{2\beta}{\sqrt{3}v_w} \approx \frac{3.4}{R_*}$$

Hindmarsh, Huber, Rummukainen, Weir, PRD [1504.03291]

Slight different fit obtained by the same group, PRD [1704.05871]

$$\Upsilon = 1 - (1 + 2\tau_{\text{sw}} H_{\text{pt}})^{-1/2} \quad (\text{radiation domination})$$

HG, Sinha, Vagie, White, JCAP [2007.08537]





# Sound Waves: the Upsilon factor

Why an overall factor (largely f-independent)?

- Gaussian
- Stationary
- Autocorrelation time much smaller than Hubble time ( $|y_-| \ll y_+$ )

$$h_{ij}(\tilde{y}, \mathbf{q}) = \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}' G(\tilde{y}, \tilde{y}') \frac{16\pi G a(\tilde{y}')^2 \pi_{ij}^T(\tilde{y}', \mathbf{q})}{q^2}$$

correlator

$$\mathcal{P}_{\text{GW}}(y, kR_{*c}) = \frac{[16\pi G (\tilde{\epsilon} + \tilde{p}) \bar{U}_f^2]^2}{24\pi^2 H^2 H_s^2} \frac{1}{y^4} (kR_{*c})^3$$

$$\times \int dy_- \tilde{\Pi}^2(kR_{*c}, \beta_c |\eta_1 - \eta_2|) \left[ \int dy_+ \frac{\mathcal{G}_2(\tilde{y}, \tilde{y}_1, \tilde{y}_2)}{\tilde{k}^2} \begin{Bmatrix} y_1^{-2} y_2^{-2} \\ y_1^{-3/2} y_2^{-3/2} \end{Bmatrix} \right]$$

factorization

$$\left[ \int dy_+ \dots \right] = \frac{1}{2} \Upsilon(y) \cos(\tilde{k} y_-)$$

$$h^2 \Omega_{\text{sw}}(f) = 2.65 \times 10^{-6} \left( \frac{100}{g_*} \right)^{\frac{1}{3}} \left( \frac{H_*}{\beta} \right) \left( \frac{\kappa_{\text{sw}} \alpha}{1 + \alpha} \right)^2 v_w S_{\text{sw}}(f) \Upsilon(\tau_{\text{sw}})$$

y: a/a\* (a: scale factor)

radiation domination:

$y_- = y_1 - y_2$

$y_+ = (y_1 + y_2)/2$

# Sound Waves: the Upsilon factor

A generic formula to describe a wide class of scenarios:

- Expansion dominated by radiation

$$\Upsilon_{\text{RD}} = 1 - \frac{1}{y}$$

- Expansion dominated by matter

$$\Upsilon_{\text{MD}} = \frac{2}{3} \left( 1 - \frac{1}{y^{3/2}} \right)$$

- Generic equation of state

$$\rho \propto a^{-3(1+w)}$$

$$\Upsilon = \frac{2[1 - y^{3(w-1)/2}]}{3(1-w)}$$

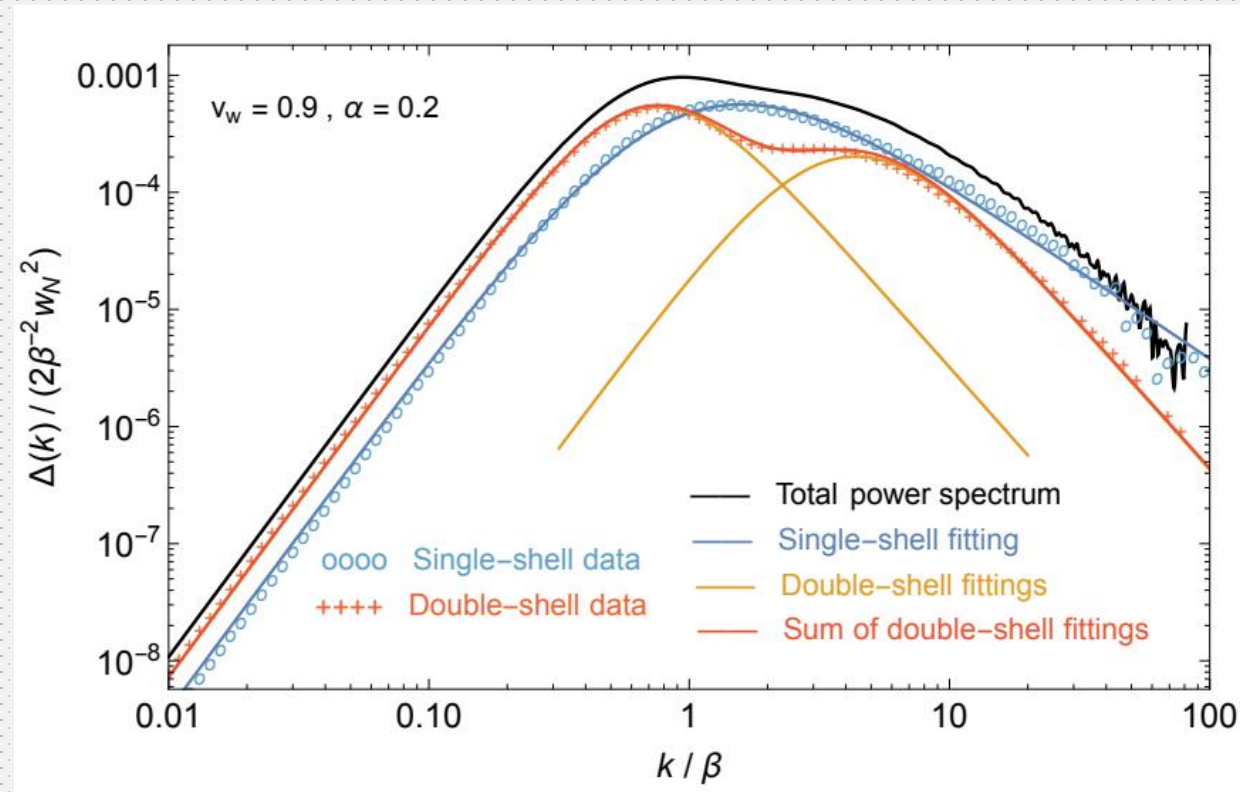
HG, Jiahang Hu, Yang Xiao, Jin Min Yang, Yang Zhang [2410.23666]

A universal factor in all similar stochastic sources.



# Sound Waves: Forced Collisions

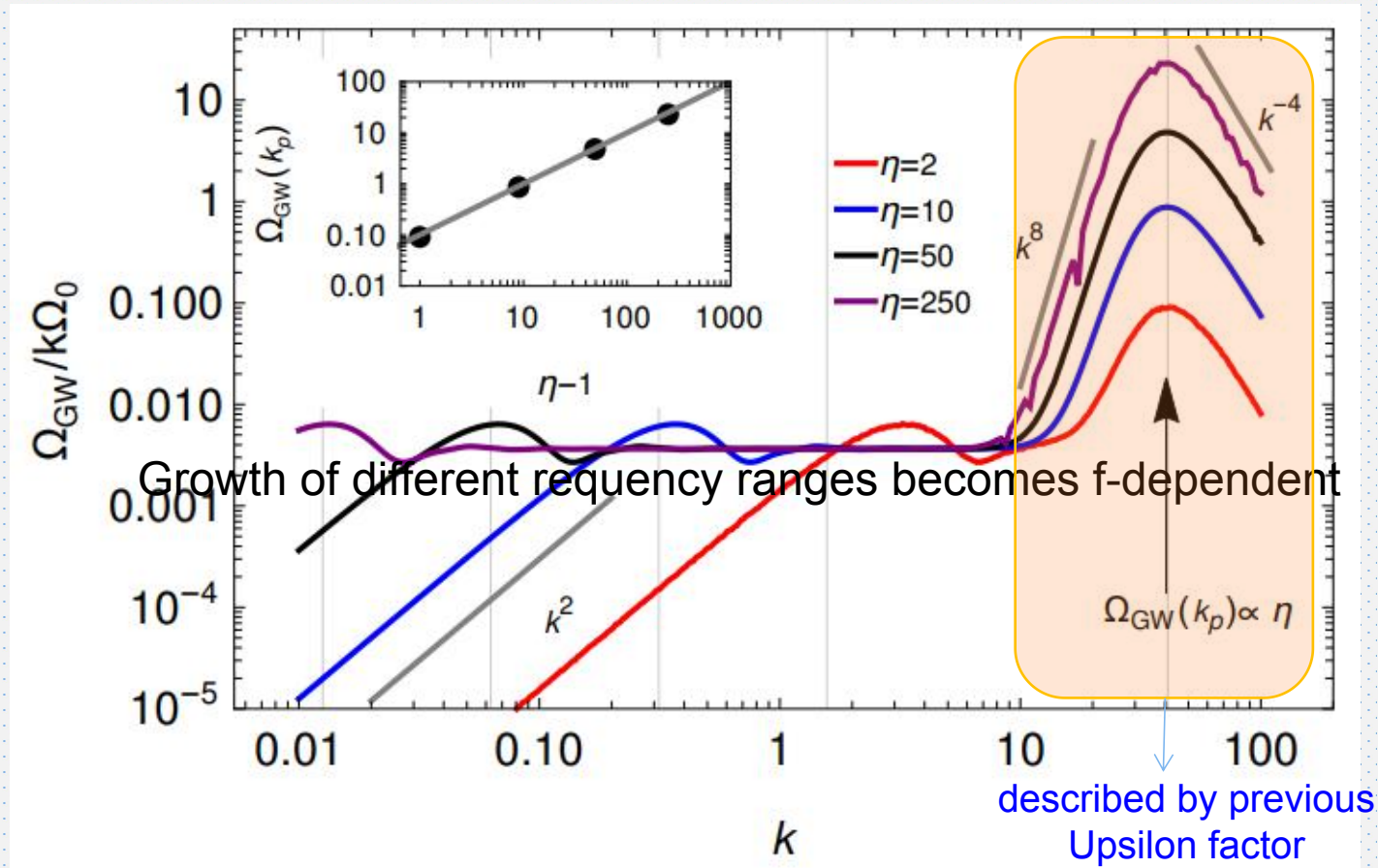
Cai, Wang, Yuwen, PRD Letter [2305.00074]



$$\left. \frac{d\Omega_{\text{GW}}}{d \ln k} \right|_{\text{SW}} = \left. \frac{d\Omega_{\text{GW}}}{d \ln k} \right|_{\text{SW}}^{\text{forced}} + \left. \frac{d\Omega_{\text{GW}}}{d \ln k} \right|_{\text{SW}}^{\text{free}}$$

# Sound Waves: Generalized Upsilon Factor?

- New slopes found for low frequency ranges ( $k^3 \rightarrow k \rightarrow k^9$ )
- Growth rate becomes f-dependent



# Dissipative Effects

GW depends on (large) bulk velocity of the system

$$h \sim 10^{-22} \frac{M/M_{\odot}}{r/100\text{Mpc}} \left(\frac{v}{c}\right)^2$$

Dissipative effects dissipate away the bulk kinetic energy (leaves imprint)

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p + \mu \nabla^2 \mathbf{v} + \left( \zeta + \frac{1}{3} \mu \right) \nabla (\nabla \cdot \mathbf{v})$$

Navier–Stokes equations



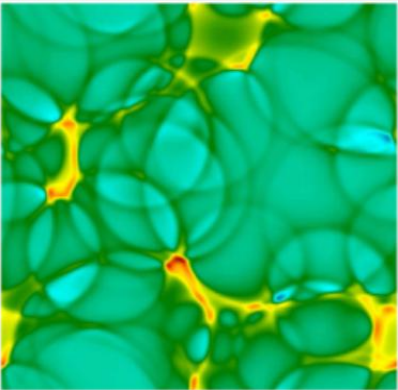
So far, it has largely been neglected, especially in connection with new physics.



# Effects of Dissipation

- Disturbed fluid comes into rest eventually

$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3} [v_{\mathbf{q}}^i e^{-i\omega\eta + i\mathbf{q}\cdot\mathbf{x}} + c.c.]$$



$$v_{\mathbf{q}}^i(\eta) \propto \exp \left[ - \int \Gamma(\mu, \zeta, \xi) d\eta \right]$$

$$\Gamma \propto q^2$$

shear viscosity bulk viscosity

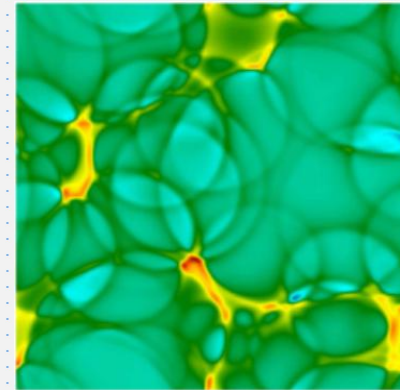
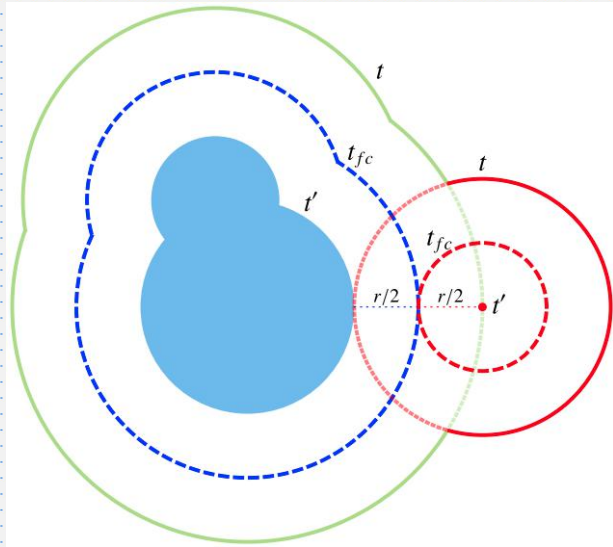
$$\Delta T^{ij} = -\mu \left( \frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{U} \right) - \zeta \delta_{ij} \nabla \cdot \mathbf{U},$$

$$\Delta T^{i0} = -\chi \left( \frac{\partial T}{\partial x^i} + T \dot{U}_i \right). \tag{1}$$

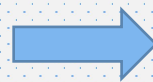
thermal conduction Weinberg, ApJ, 1971

Euler equation -> Navier–Stokes equations

# Effects of Dissipation with the Sound Shell Model



$$v_{\mathbf{q}}^i = \sum_{n=1}^{N_b} v_{\mathbf{q}}^{i(n)}$$

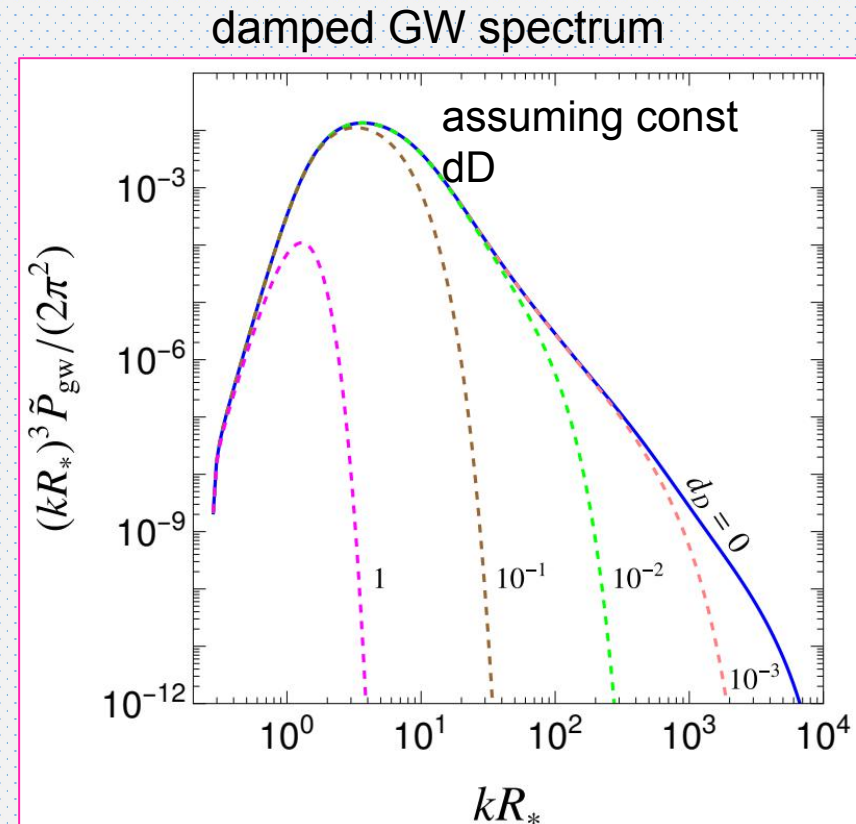
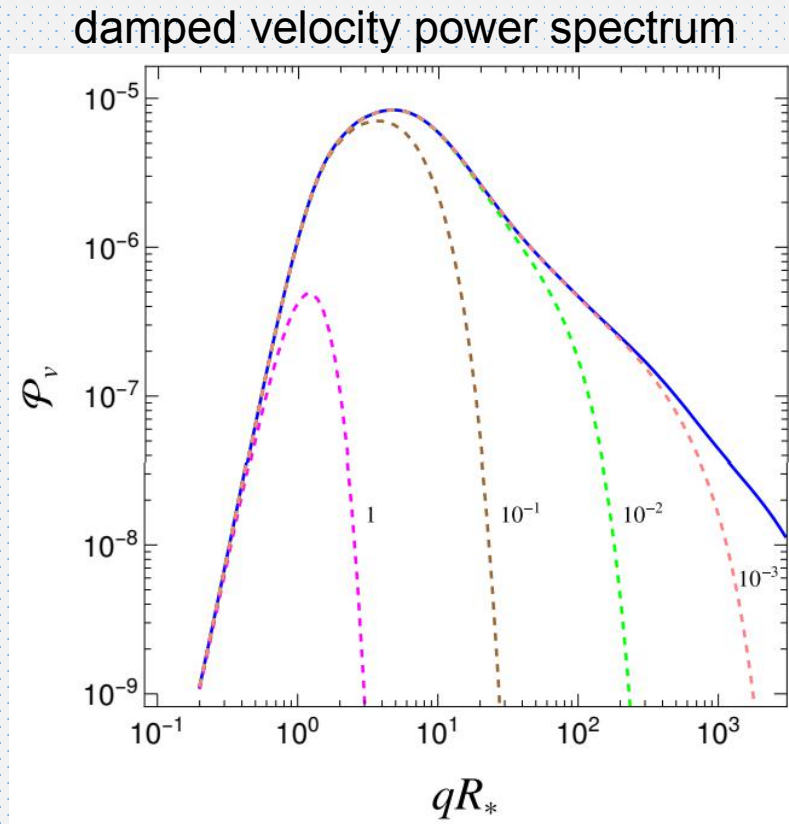


$$v_{\mathbf{q}}^i(\eta) = \sum_{n=1}^{N_b} v_{\mathbf{q}}^{i(n)} \exp \left[ - \int_{\eta_d^{(n)}}^{\eta} \Gamma d\bar{\eta} \right] \theta(\eta - \eta_d^{(n)})$$

bubble destruction time

# Effects of Dissipation: Damping of GW Spectrum

- Peak frequency in strongly dissipative systems
- Upsilon factor might not appear

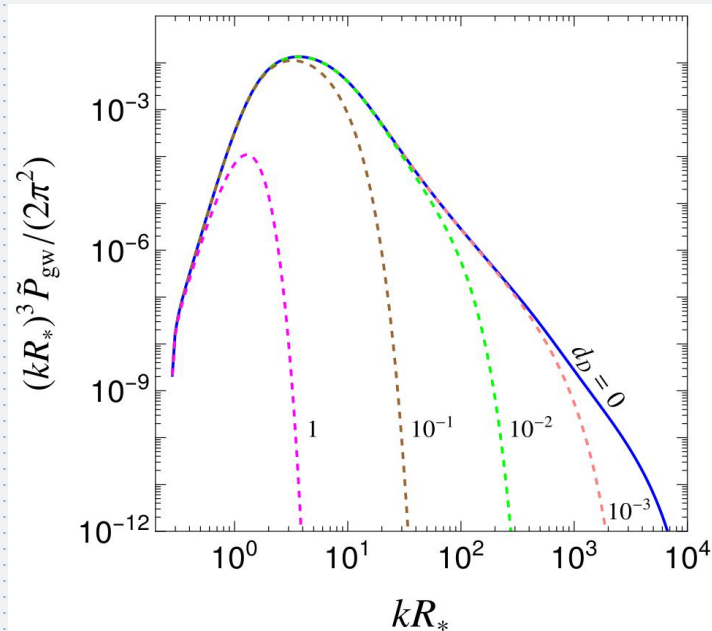




# Dissipative Effects as New Observables

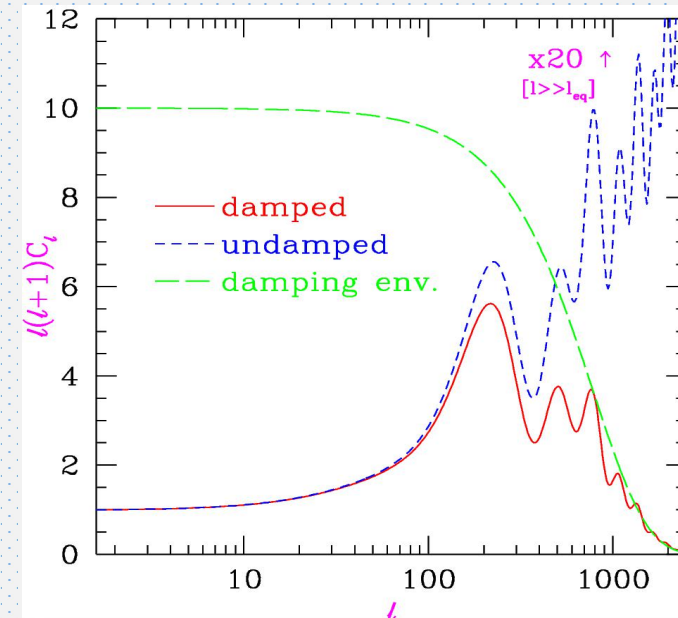
- Probe very weak interactions (analog: Silk damping)
- Break parameter degeneracy
- Can be searched for at LIGO, PTA, LISA/Taiji/Tianqin ...

damping of GW



HG [2310.10927]

Silk damping of CMB Anisotropy



Hu, White, ApJ [9609079]

# Dissipative Effects: Lifetime of Sound Waves

- Expansion of the universe provides an effective lifetime
- Dissipation effects, when strong, provide a shorter effective lifetime
- Onset of MHD turbulence serves as a cut-off (dissipation causes changes)

Realistic cases: intertwining of these effects (makes GW spectrum [model dependent](#))

Model dependent spectrum carries information about each model ([break parameter degeneracy](#))

# Summary

- Phase transition is an important target for all GW experiments
- Precision studies now become high priority
- Progress in GW spectrum from sound waves (slope, growth rate)
- Dissipative effects can serve as new observables



Thanks!