Neutrino flavor oscillations in supernovae

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NSTC 國家科學及技術委員會 National Science and Technology Council

NCTS

Outline of this talk

- Introduction & motivation
- Quantum kinetic transport & collective neutrino fast flavor conversions (FFC)
- FFC in a local sub-grid: depolarization & coarse graining
- Global simulations of FFC and effective model

MRW, M. George, C.-Y. Lin, Z. Xiong, 2108.09886
Z. Xiong, MRW, S. Abbar, et al, 2307.11129
Z. Xiong, MRW, M. George, et al, 2402.19252
Z. Xiong, MRW, M. George, C.-Y. Lin, 2403.17269

Core-collapse supernovae (SNe)

– the death of massive stars $\gtrsim 8\ M_\odot$

– important multimessenger source
($E_\gamma \sim 10^{49}$ erg, $E_{\rm expl} \sim 10^{51}$ erg,
$E_\nu \sim 10^{53}$ erg, $E_{\rm GW} \sim 10^{45-48}$ erg)

– producing nature's elements after BBN

– excellent "astro lab" to probe fundamental physics:

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(From AAO website) SN1987a

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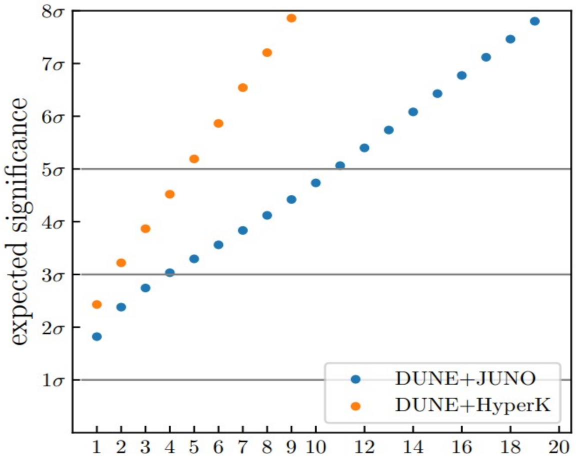
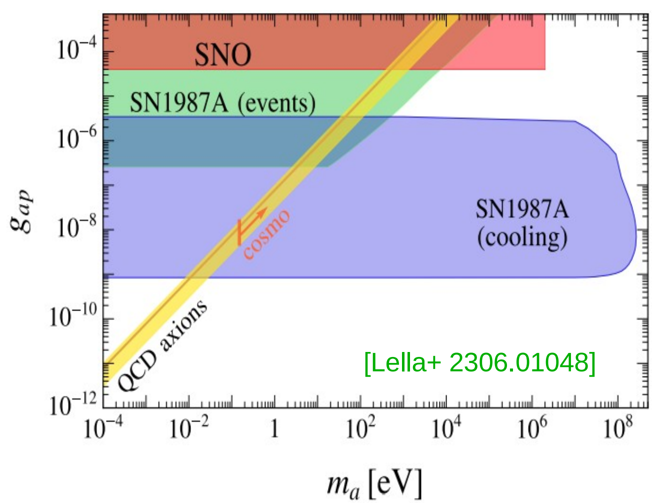
- excellent "astro lab" to probe fundamental physics:

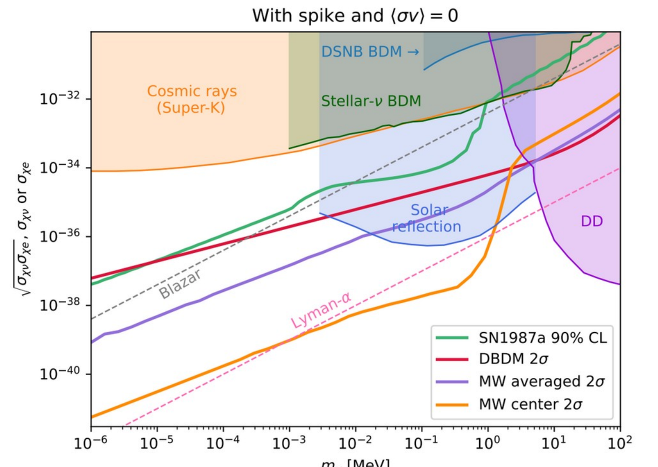
ν mass ordering; absolute ν mass; light sterile ν; non-standard ν interaction; light bSM particle emission; ν-DM interaction; ...



[Brdar+2204.13135]

For axion-like particles





[Lin & MRW, 2404.08528]

(see Yen-Hsun Lin's talk on Tuesday)

Need robust theory prediction for $\mathcal{O}(10^4)$ ν events from the next galactic SN

SN explosion and ν transport

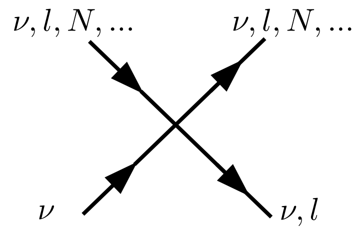
In SN explosions: $E_\nu \sim E_{\mathrm{G,NS}} \sim 10^{53}$ erg $\gg E_{\mathrm{expl}} \sim 10^{51}$ erg

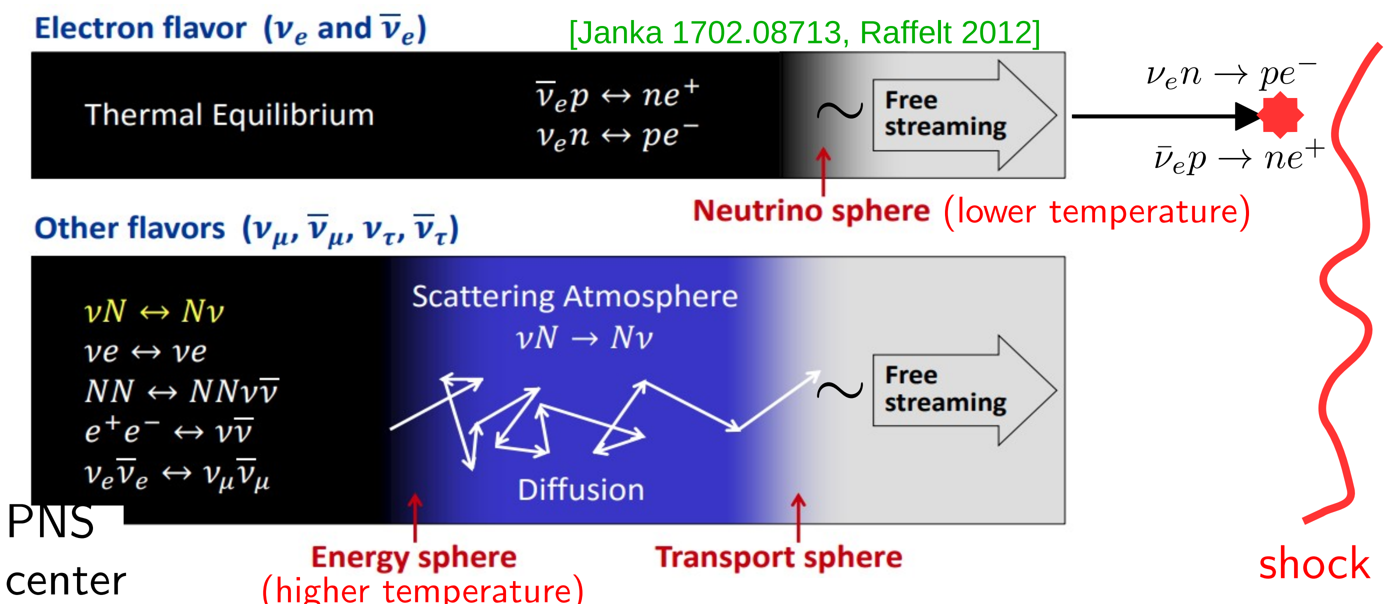
$\rightarrow$ a few percent of neutrino energy deposition is enough to deliver the explosion

$\rightarrow$ needs to model the neutrino transport accurately

State-of-the-art general relativistic (magneto-) hydrodynamic simulations of supernovae include approximate treatment of classical Boltzmann transport of neutrinos

$$(\partial_t + \mathbf{v}\cdot\partial_{\mathbf{x}} + \mathbf{F}\cdot\partial_{\mathbf{p}}) f_{\nu_\alpha}(\mathbf{x},\mathbf{p},t) = \mathcal{C}$$





ν_e and $\bar{\nu}_e$ dominate the energy deposition to the shock

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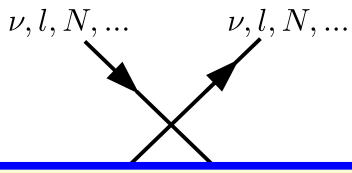
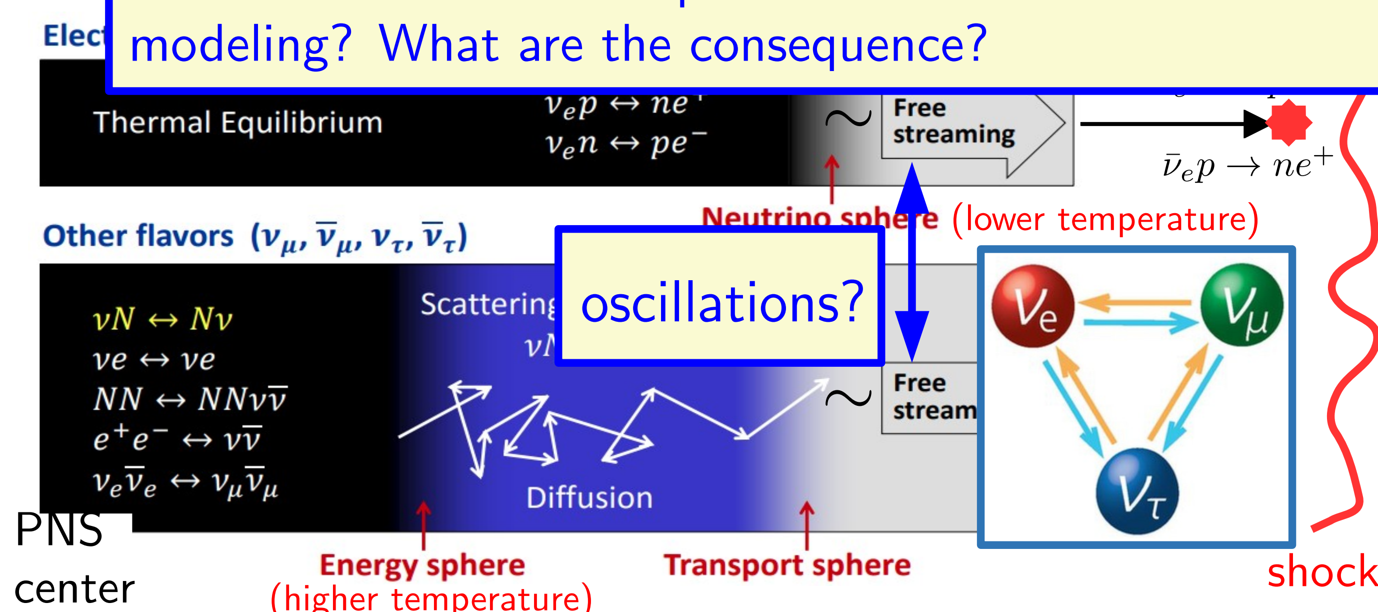


Are flavor oscillations important? How to include them in theoretical modeling? What are the consequence?



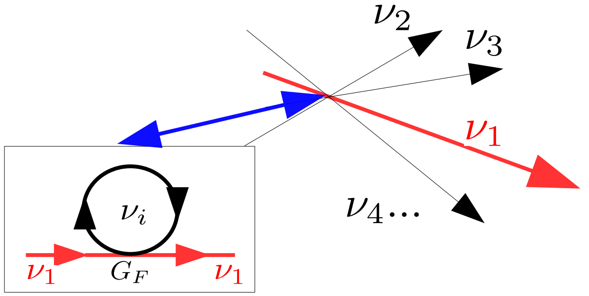
$\bar{\nu}_e$ and ν_e dominate the energy deposition to the shock

Collective fast neutrino oscillations

Oscillations (quantum nature of ν) $\rightarrow$ density matrix $\varrho(t, \mathbf{x}, \mathbf{p}) = \begin{bmatrix} f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho^*_{e\mu} & f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho^*_{e\tau} & \varrho^*_{\mu\tau} & f_{\nu_\tau} \end{bmatrix}$

$\rightarrow$ requires solving neutrino quantum kinetic equation (νQKE) (extended Boltzmann equation):

$$(\partial_t + \mathbf{v} \cdot \partial_{\mathbf{x}} + \mathbf{F} \cdot \partial_{\mathbf{p}})\varrho(\mathbf{x}, \mathbf{p}, t) = -i[H_{\text{vac}} + H_{\text{m}} + H_{\nu\nu}, \varrho(\mathbf{x}, \mathbf{p}, t)] + \mathcal{C}(\varrho)$$



$$H_{\nu\nu}(\mathbf{x}, \mathbf{p}, t) = \frac{\sqrt{2}G_F}{(2\pi)^3} \int d^3q(1 - \hat{\mathbf{p}} \cdot \hat{\mathbf{q}})[\varrho - \bar{\varrho}^*]$$

$\rightarrow$ a new scale associated with $H_{\nu\nu}$, $\sim \sqrt{2}G_F n_\nu \gtrsim \mathcal{O}(1)$ cm^{-1}

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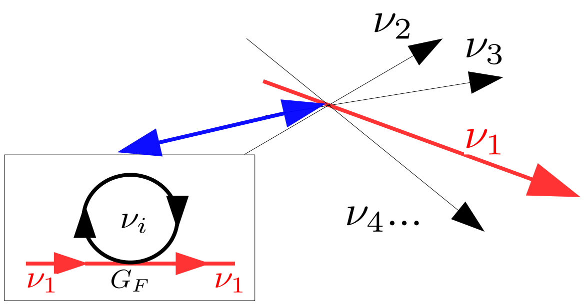

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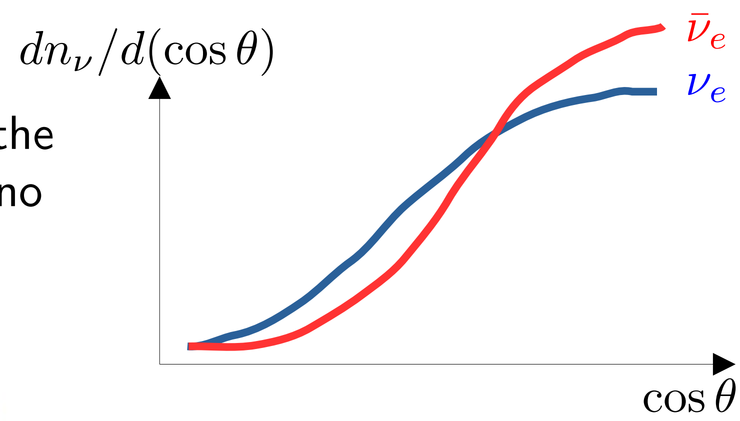
- $G_F n_\nu \gg \delta m^2/(2E_\nu)$ $\rightarrow$ "strong" coupling in flavor space $\rightarrow$ collective system
- $(G_F n_\nu)^{-1} \ll \tau_{\rm hydro}, \tau_{\rm advection}, \tau_{\rm collision}$ and the associated length scales

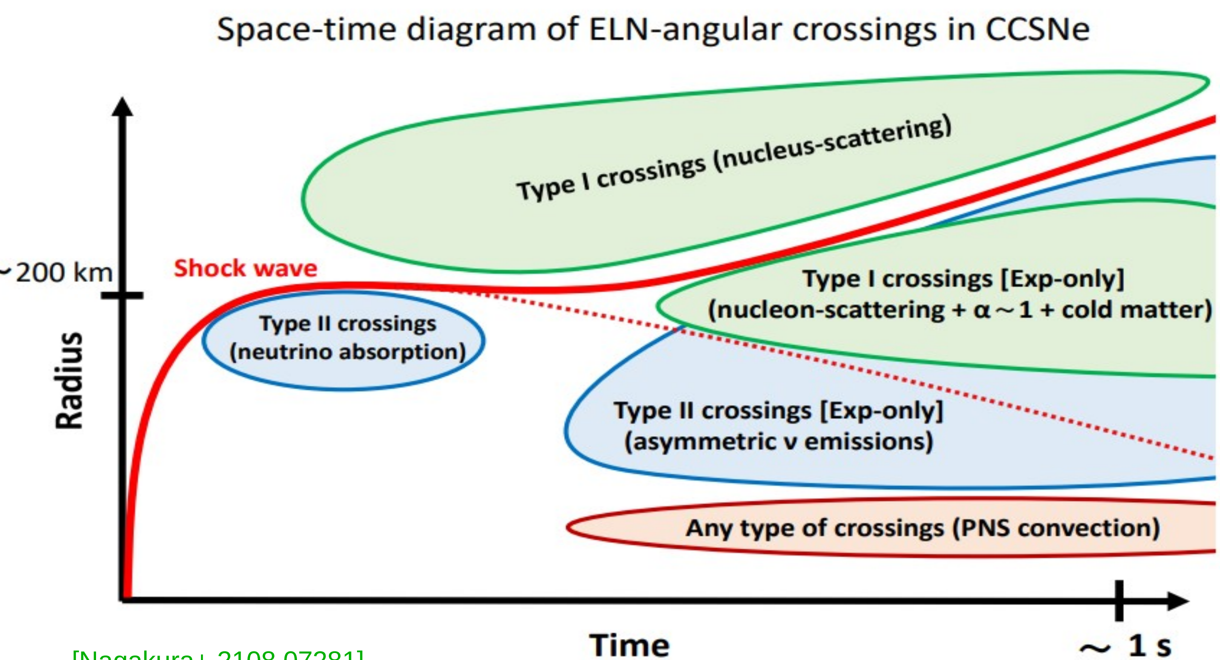
Can flavor oscillations of such a time scale happen? If so, does it require ultra-high spatial resolution (sub-cm) and tiny time step (sub-ns) to fully solve νQKE?

Fast flavor conversions (FFC)

Fast flavor conversions can occur at regions where the "$e-\mu$ lepton number crossing" exists in the neutrino angular distribution, causing $\nu_e \leftrightarrow \nu_{\mu,\tau}$ $(\bar{\nu}_e \leftrightarrow \bar{\nu}_{\mu,\tau})$

[Sawyer+, Izaguirre+, Dasgupta+,...many others]





[Nagakura+ 2108.07281]

(based on simulation results w/o oscillations)

$\rightarrow$ generally happens in regions below the shock, not only affect ν signals, but also the shock revival, i.e., the SN dynamics [see e.g., Ehring+ 2305.11207]

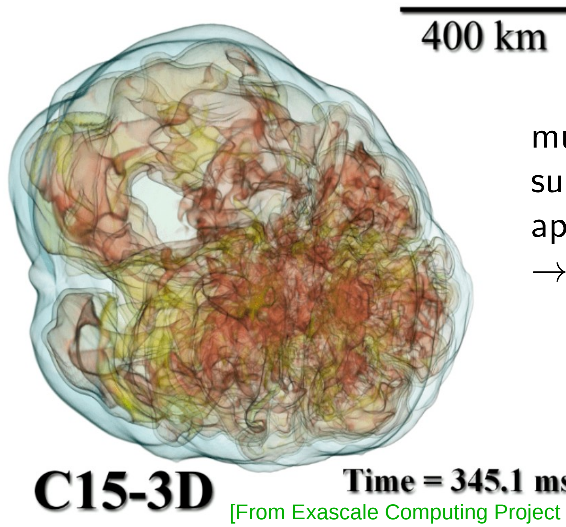
$\rightarrow$ CANNOT avoid dealing with flavor oscilltions in modeling supernovae

A big challenge in practice

Can we directly solve the extended ν transport equation in supernova simulations?

$$(\partial_t + \mathbf{v}\cdot\partial_{\mathbf{x}} + \mathbf{F}\cdot\partial_{\mathbf{p}})\varrho(\mathbf{x},\mathbf{p},t) = -i[H_{\rm vac} + H_{\rm m} + {\color{red}H_{\nu\nu}}, \varrho(\mathbf{x},\mathbf{p},t)] + \mathcal{C}(\varrho)$$

Unfortunately this is NOT possible as it'd require resolving length scale of sub-centimeters

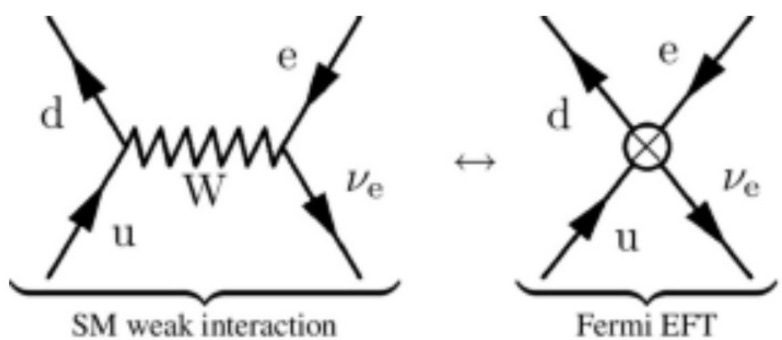


[From Exascale Computing Project website]

multi-dimensional+
sub km spatial resolution +
approximate ν transport
$\rightarrow \gtrsim$ millions CPU hours

Can we leverage the scale separation nature of the problem?

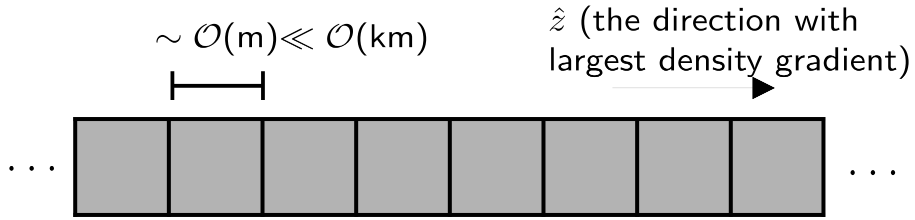
$\rightarrow$ needs to develope effective (sub-grid) model



[From Toni Mäkelä]

Fast flavor conversion (FFC) within a sub-grid

Assuming within length scale $\ll \mathcal{O}$(km) the system is nearly homogeneous, one may use periodic boundary condition to study how fast flavor conversions evolves within the box mimicking a tiny volumn inside a supernova



[Bhattacharyya+, Richers+, MRW+, Martin+; see Richers+ 2205.06282 for detailed comparison of simulation results]

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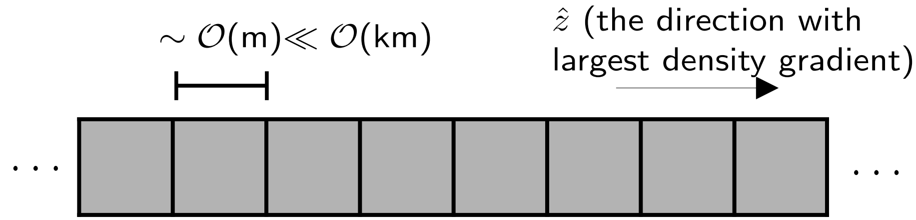
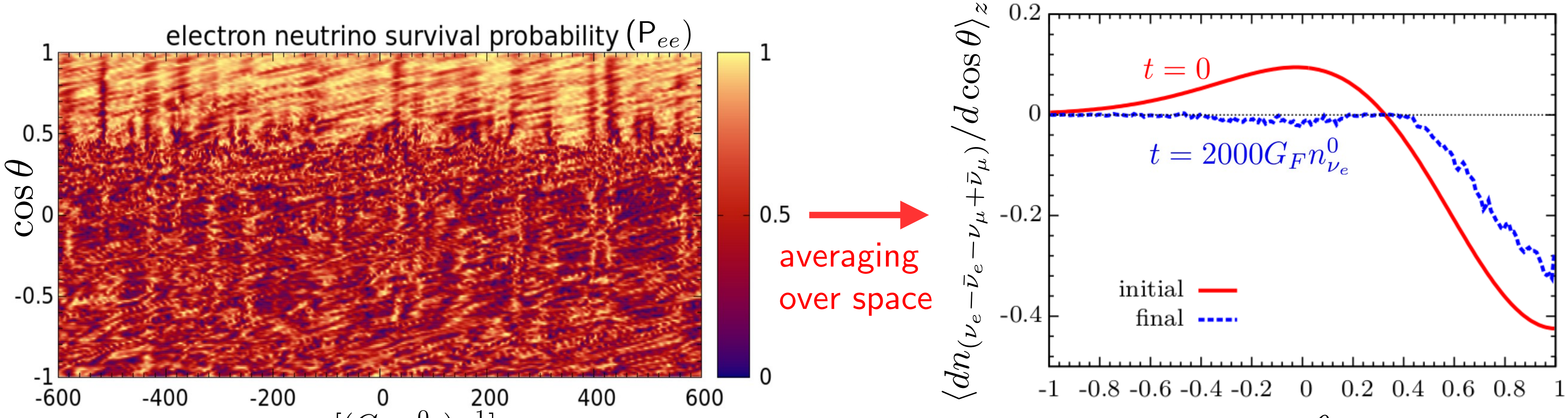


[Bhattacharyya+, Richers+, MRW+, Martin+; see Richers+ 2205.06282 for detailed comparison of simulation results]

$\rightarrow$ FFC occurs and drives the system evolving toward a quasi-stationary state

$\rightarrow$ In the quasi-stationary state, "flavor depolarization" happens, leading to coarse-grained flavor equipartition on one side of the $e-\mu$ crossing



[MRW+ 2108.09886]

$$\left\langle \frac{dn_{\nu_e}}{d\cos\theta}\right\rangle_z \simeq \left\langle \frac{dn_{\nu_\mu}}{d\cos\theta}\right\rangle_z,\ \left\langle \frac{dn_{\bar{\nu}_e}}{d\cos\theta}\right\rangle_z \simeq \left\langle \frac{dn_{\bar{\nu}_\mu}}{d\cos\theta}\right\rangle_z \text{ for } \cos\theta \lesssim 0.35$$

Fast flavor conversion (FFC) within a sub-grid

Taking perodic boundary condtion, the equation of motion demands the conservation of the $e - \mu$ lepton number in the entire simulation domain:

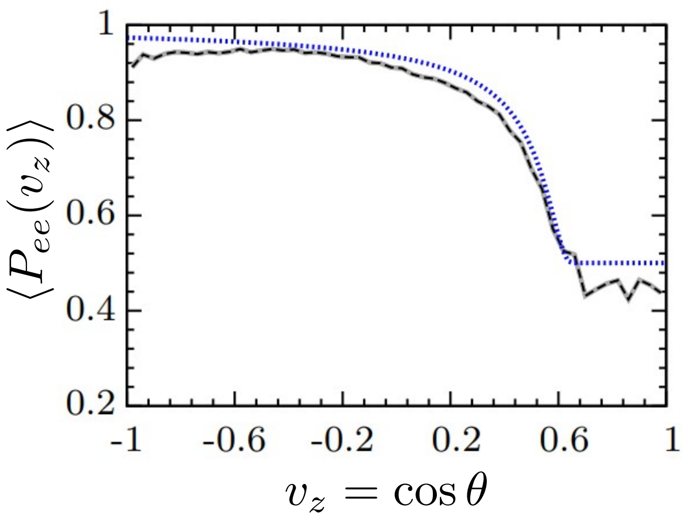
$$\int d(\cos\theta)\langle dn_{(\nu_e-\bar{\nu}_e-\nu_\mu+\bar{\nu}_\mu)}/d\cos\theta\rangle_z = \text{constant}$$

One can analytically parametrize the coarse-grained survival probability function $\langle P_{ee}\rangle$ as

– on the flavor-equilibration side: $P_{ee} = 0.5$

– on the other side: $P_{ee}(v_z) = 1 - 0.5h(|v_z - v_c|/a)$, where $h(x) = (x^2+1)^{-1/2}$

[Xiong+ 2307.11129]



→ gives $\lesssim 3\%$ errors in angular moments when averaged over $\sim 10^4$ different initial conditions

Beyond sub-grid – FFC in global transport

Although it is not possible to fully solve νQKE with required resolution, one can artifically quench $H_{\nu\nu}$ to solve the νQKE with static SN profiles

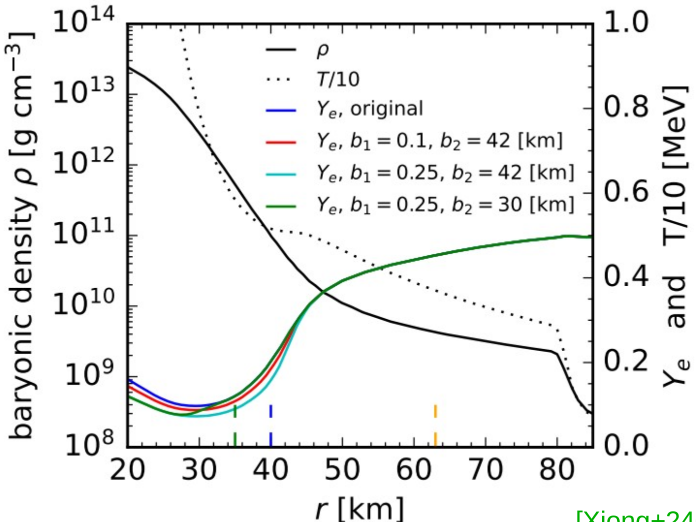
– Take SN matter background profiles $(\rho_b, T, Y_e, ...)$ from spherically symmetric SN simulations

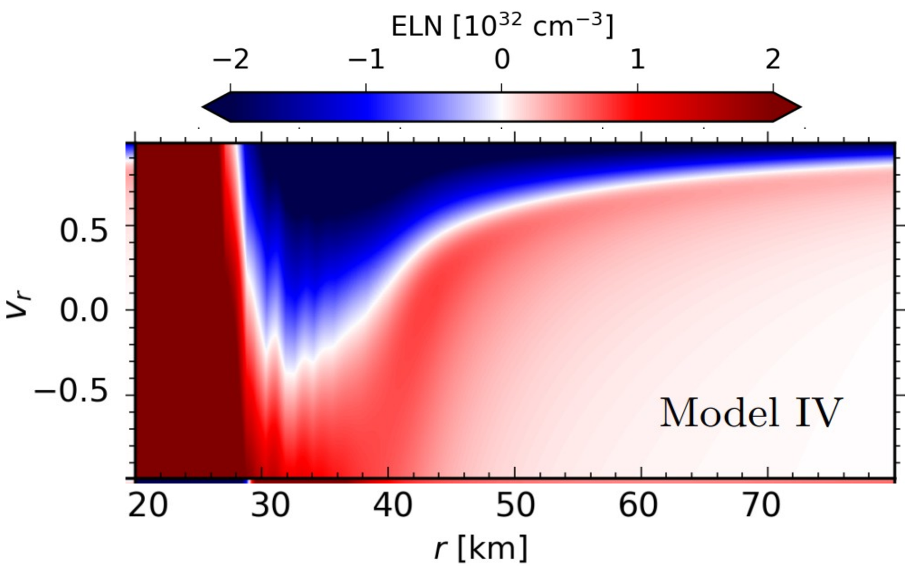
– Solve classical neutrino transport equations to obtain neutrino distributions without considering flavor oscillations

$$(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})f_\nu = \mathcal{C}$$

– Use the above steady-state solution as initial condition for νQKE simulations

$$(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})\varrho_\nu = -i[H_{\rm vac} + H_{\rm m} + aH_{\nu\nu}, \varrho_\nu] + \mathcal{C} \text{ with } a \sim 10^{-3}$$

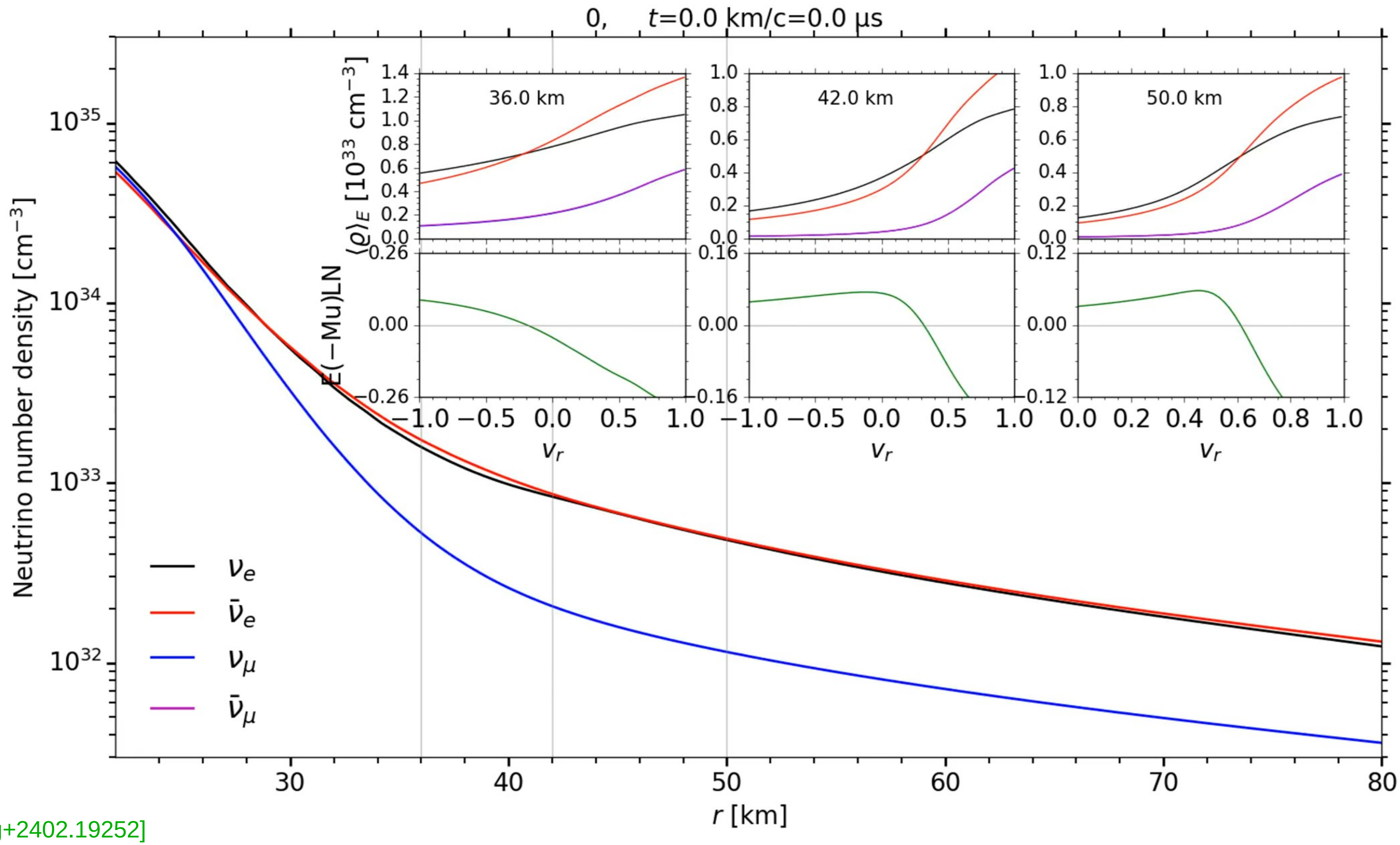




[Xiong+2402.19252]

Beyond sub-grid – FFC in global transport

- FFC occurs promply and erase $e-\mu$ crossings across all radii
- neutrino emissions/absorptions inside the neutrinosphere replenish ν_e and $\bar{\nu}_e$, driving gradual evoluton of $e-\mu$ distributions, but no new $e-\mu$ crossings appear

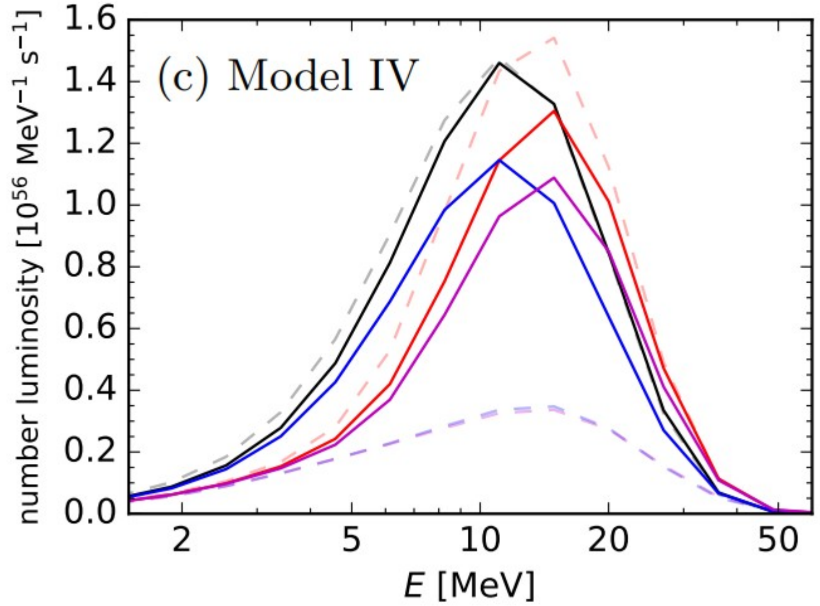


[Xiong+2402.19252]

Beyond sub-grid – FFC in global transport

• Globally FFC reduces the amount ν_e and $\bar{\nu}_e$, while enhances that of heavy-lepton flavors

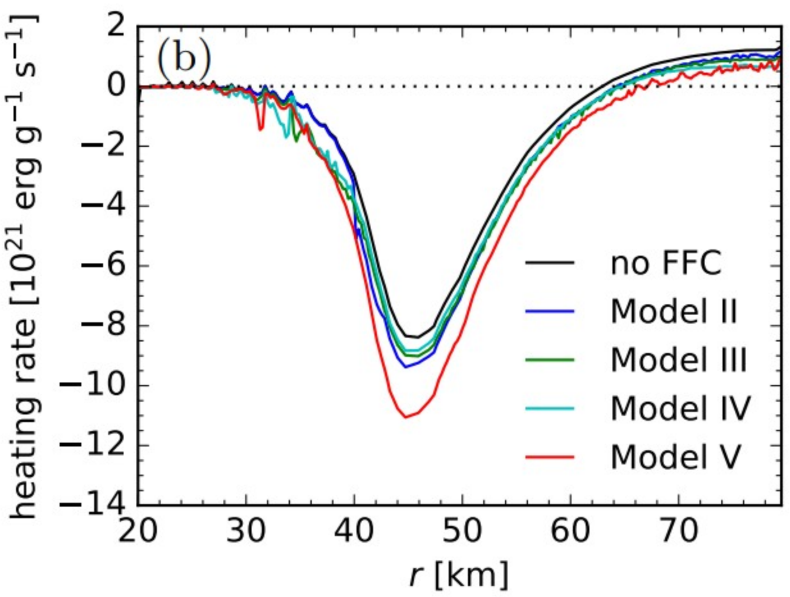
$\rightarrow$ emerging neutrino energy spectra of different flavors become more similar



[Xiong+2402.19252]

• reduce the net-heating rate between the ν-sphere and shock

$\rightarrow$ the exact consequence can only be understood when FFC is implemented consistently in SN simulations

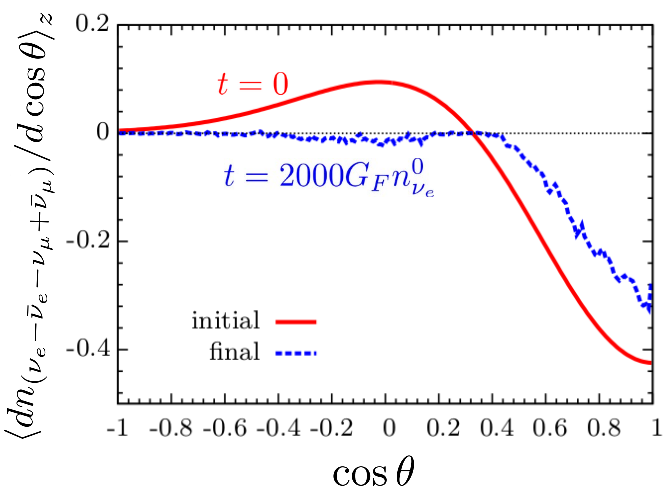


Effective classical transport model including FFC

(i) local periodic box simulations suggest the system quickly relaxed to coarse-grained stationary state, where the initial $e-\mu$ angular crossings are erased and can be parametrized analytically

(ii) global simulations seem to observe similar outcome across the simulation domain, with feedback effect coming from collisions that keep driving the flavor evolution

If we implement the analytical prescriptions learned from (i) into global classical transport simulations at everywhere that contains a $e-\mu$ crossing, can it work well as an effective model?



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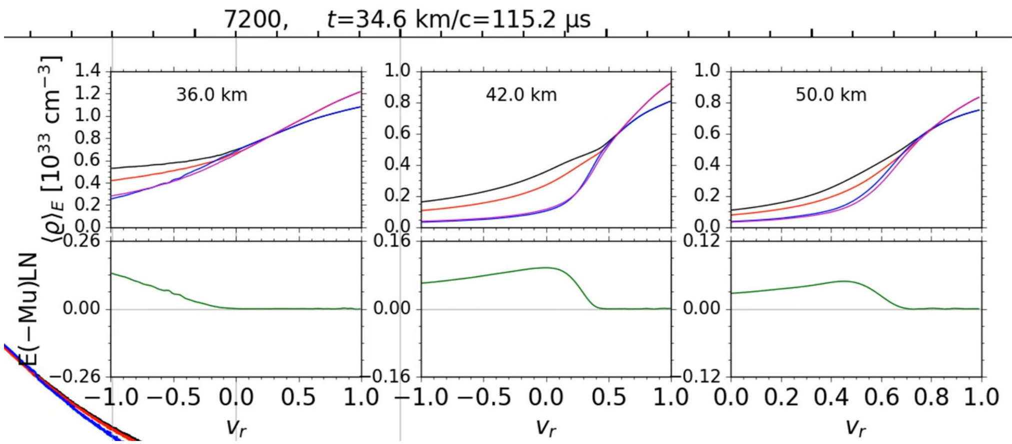


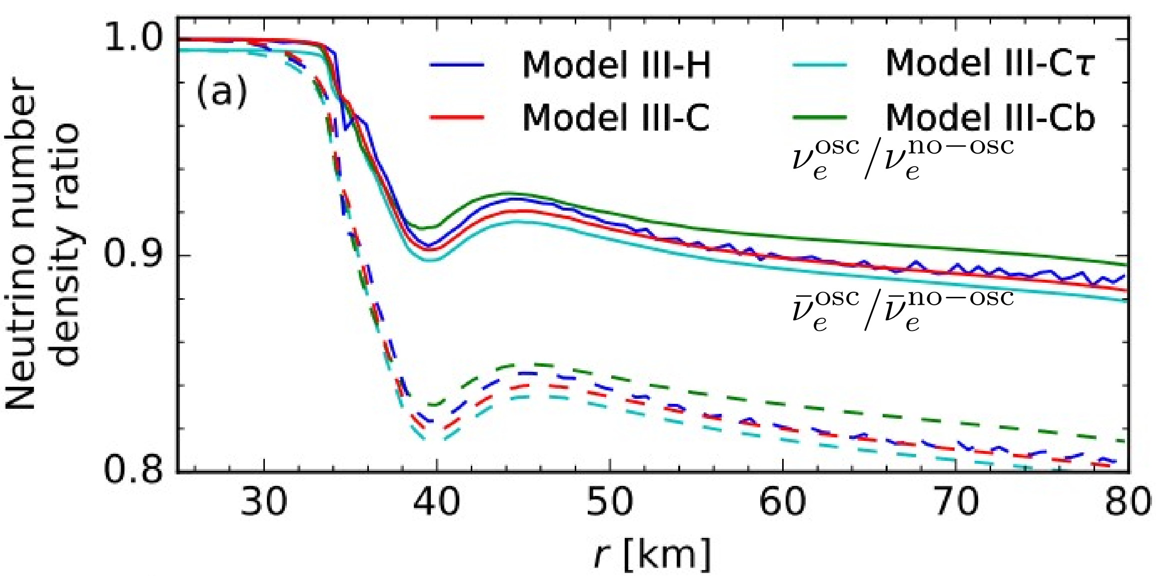
Effective classical transport model including FFC

- Model III-H: solves $(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})\varrho_\nu = -i[H_{\rm vac} + H_{\rm m} + H_{\nu\nu}, \varrho_\nu] + \mathcal{C}$

$$H_{\nu\nu}(\mathbf{x},\mathbf{p},t) = \frac{\sqrt{2}G_F}{(2\pi)^3}\int d^3q(1-\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})[\varrho - \bar{\varrho}^*], \quad \varrho(t,\mathbf{x},\mathbf{p}) = \begin{bmatrix} f_{\nu_e} & \varrho_{e\mu} & \varrho_{e\tau} \\ \varrho^*_{e\mu} & f_{\nu_\mu} & \varrho_{\mu\tau} \\ \varrho^*_{e\tau} & \varrho^*_{\mu\tau} & f_{\nu_\tau} \end{bmatrix}$$

- Model III-C: solves $(\partial_t + v_r\partial_r + \frac{1-v_r^2}{r}\partial_{v_r})f_\nu = \mathcal{C}$ with:

  f_ν replaced by $f_\nu^{\rm eq}$ computed using the analytical formula at each time and location where $e-\mu$ crossing is found



The effective model takes

~ 100 less radial grids

~ 10 bigger time step size

$\rightarrow$ speed-up factor $\gtrsim 10^3$

a promising step toward fully including FFC in SN simulations!

[Xiong+ 2403.17269]

Summary & outlook

• The collective flavor oscillations of neutrinos in SNe is a challenging issue that needs to be taken care of in SN modeling. In particular, fast flavor conversions (FFC) can happen deep inside a SN and can affect its evolution.

• Studies performing "local" simulations within a "sub-grid" found that the FFC evolves to a quasistationary state such that the $e-\mu$ lepton number angular crossing is erased in a coarse-grained sense. The outcome can be parametrized by analytical formulas subject to the conservaion law.

• Global νQKE transport simulations found similar features: $e-\mu$ crossings are erased, with important collisions feedback that continues to drive the evolution of the system.

• Effective classical transport models that utilize the scale separation nature of the problem can robustly reproduce the outcome from solving the global νQKE. This is done by implementing the coarse-grained outcome learned from the local sub-grid simulations in global classical transport simulations. This method adds very little computational burden to classical transport, suggesting that it may be directly implemented in SN simulations.

Summary & outlook

– for general cases without imposed symmetries? [George+ 2409.08833]

– does it work without knowing the full angular distribution of neutrinos? [Abbar+2311.15656, 2401.17424]

– limitation of the effective models? Do the separation of scales and coarse-graining always work?

– “thermodynamics” of neutrino oscillations? [Johns 2306.14982]