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# The MCSM

**(The Minimal Cosmological Standard Model)**

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(with **G. Barenboim** & **P. Ko**)

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# Motivation

- **Big theoretical/cosmological/phenomenological puzzles**
  - The origin of dimensionful parameters ($m_P = 1/\sqrt{G}$ & $\mu_h = m_h/\sqrt{2}$)

  - Inflation ( $n_s \simeq 1 - 0.03,\ r_T < 0.063$)
  - Matter-antimatter asymmetry ($Y_B \approx 10^{-10}$)
  - Dark matter ($\Omega_{DM} \sim 0.27$)
  - $H_0$ tension ($\left(H_0^{SN} - H_0^{CMB}\right)/H_0^{CMB} \sim 0.1$ ($\gtrsim 4\sigma$ dev.))
  - Cosmological constant ($\rho_\Lambda^{obs}/\rho_\Lambda^{th} \lesssim \rho_\Lambda^{obs}/\Lambda_{QCD}^4 \sim 10^{-43}$)

  - Hierarchy problem ($\left(m_{ew}/M_P\right)^2 = \mathcal{O}(10^{-34})$)
  - Neutrino mass ($m_\nu/m_{ew} = \mathcal{O}(10^{-13})$)
  - Strong CP-problem ($\theta_{CP} < \mathcal{O}(10^{-10})$)

  - … (?)

mostly being separately tackled!

- **A dream (or a goal)**
  - A simple unified BSM framework for all(or most) of them?

  (The philosophy: nature works in the most efficient way if possible!)

- **Simple guiding principles for the goal (?)**

(i) Scale symmetry ⇒ Dynamical generation of scales w/o add-hoc mass parameters.
(ii) $U(1)_{\rm PQ}$ symmetry (at least in the matter sector) ⇒ Axion-solution
(iii) Minimality ⇒ Unification of PQ & seesaw sectors ( ⇒ "Axi-Majoron")

< Minimal DFSZ vs minimal KSVZ >

✓ **DFSZ**:
One extra Higgs-doublet (w/ the danger of the domain-wall problem)

**KSVZ**:
A pair of extra quark-triplets (w/o the domain-wall problem)

# Global Weyl scaling [Weyl, 1921]

**Transformation:**

$$g_{\mu\nu} \equiv \Omega^{-1}\tilde{g}_{\mu\nu} \ , \quad \left(\Omega \equiv e^{-2\sigma/f}\right)$$

$$\begin{cases} \phi \equiv \Omega^{1/2}\tilde{\phi} \\ \psi \equiv \Omega^{3/4}\tilde{\psi} \\ e^{\mu}_{\alpha} \equiv \Omega^{1/2}\tilde{e}^{\mu}_{\alpha} \ \ (\gamma^{\mu} = \gamma^{\alpha} e^{\mu}_{\alpha}) \text{ : vielbein} \\ R = \Omega\left[\tilde{R} - \frac{3}{2}\left(\partial \ln \Omega\right)^2 + 3\tilde{\nabla}^2 \ln \Omega\right] \text{ : Ricci scalar} \end{cases}$$

**Scale invariant action** ($d = 4$ operators w/o dimensionful parameters):

$$S = S_G + S_M \ ,$$

$$S_G \equiv -\frac{1}{2}\int d^4x\sqrt{-g}R\mathcal{F}(\{\varphi_i^2\}) \ ; \ \mathcal{F} \equiv \sum_i \xi_i \varphi_i^2 \ \ (\xi_i > 0)$$

→ Nonminimal gravitational int.

$$S_M = \text{Matter-action without mass-parameters}$$

**Noether current & its conservation:**

$$K_{\mu} \equiv \sum_i \kappa_i \varphi_i \partial_{\mu}\varphi_i \ \ (\kappa_i \equiv 1 + 6\xi_i)$$

$$D_{\mu}K^{\mu} = 4V - \sum_i \varphi_i \frac{\partial V}{\partial \varphi_i} = 0$$

→ global Weyl scale-invariant!

Shaponikov & Zenhousern, PLB671 (2009) 162
Ferreira, Hill & Ross, PRD95 (2017) 043507

## Quantum scale-symmetry

- Trace anomaly: Due to a fixed input scale $\mu_0$ for RG-running.

$$V = \frac{\lambda}{4}\varphi^4 + \frac{\beta_\lambda}{4}\varphi^4 \ln\left(\frac{\varphi}{\mu_0}\right)$$

$$\Rightarrow \quad D_\mu K^\mu = 4V - \varphi\frac{\partial V}{\partial \varphi} = -\frac{\beta_\lambda}{4}\varphi^4 \neq 0$$

$\Rightarrow$ Scale-invariant only if $\beta_\lambda = 0$

- Restoration of scale-sym.: Replacing $\mu_0$ to a Weyl co-variant field such as $\chi$.

$$V = \frac{\lambda}{4}\varphi^4 + \frac{\beta_\lambda}{4}\varphi^4 \ln\left(\frac{\varphi}{\chi}\right)$$

$$\Rightarrow \quad D_\mu K^\mu = 4V - \varphi\frac{\partial V}{\partial \varphi} - \chi\frac{\partial V}{\partial \chi} = 0$$

$\Rightarrow$ Scale-invariant even though $\beta_\lambda \neq 0$

# Scale generation

**The Weyl-current, a kernel & its asymptote** [Ferreira, Hill & Ross, PRD95, 043507 (2017)]

$$K_\mu = \sum_i \kappa_i \varphi_i \partial_\mu \varphi_i = \partial_\mu K \quad \left( K \equiv \sum_i \kappa_i \varphi_i^2/2, \;\; \kappa_i \equiv 1 + 6\xi_i \right)$$

$$D^\mu K_\mu = 0 \;\Rightarrow\; \ddot{K} + 3H\dot{K} = 0$$

$$\Rightarrow\; K = c_1 + c_2 \int \frac{dt}{a^3(t)} \xrightarrow{t\to\infty} c_1 \;\; \text{(a const.!)}$$

A scale appears!

⇨ Dynamic scale-generation!
Potential causes a field-dynamics, but with $K = \text{const.}$

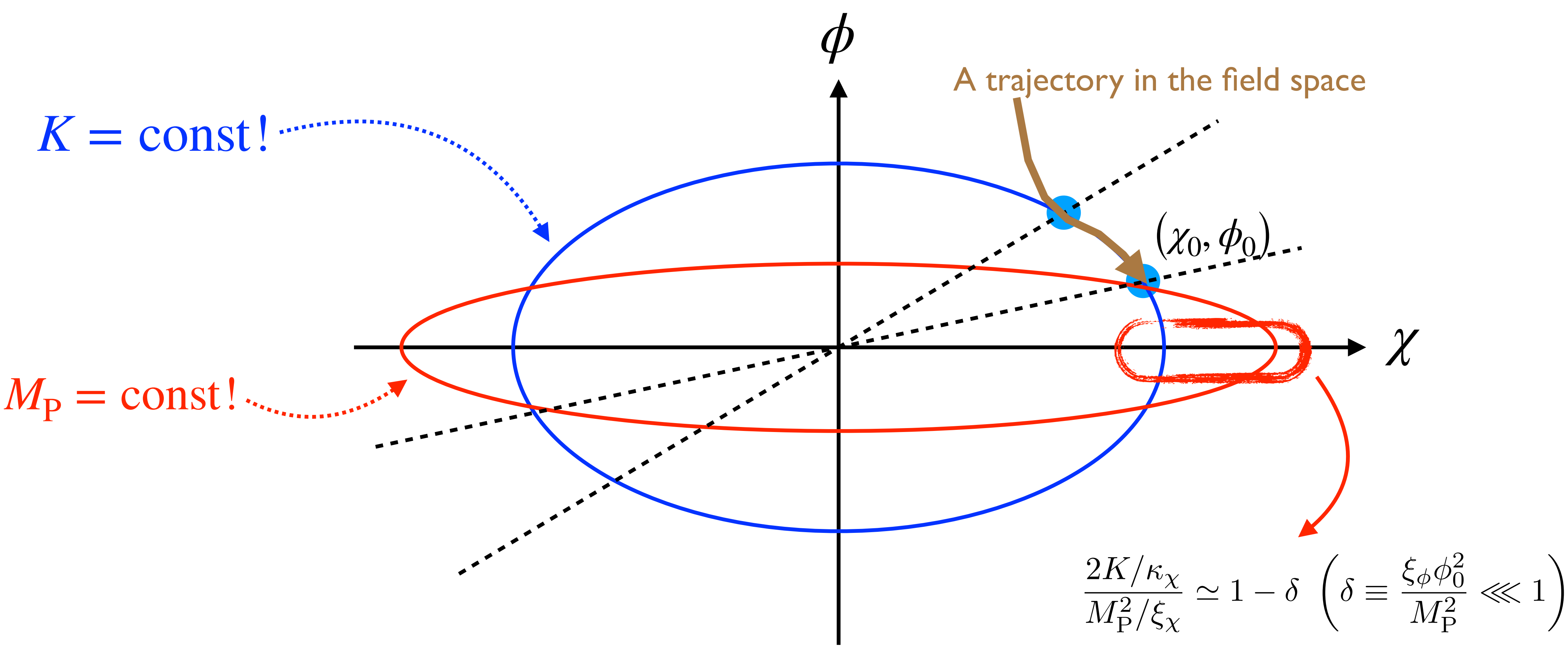
**A simple two-field picture (with $\xi_\chi \lll 1$):** [Ferreira, Hill & Ross, PRD95, 043507 (2017)]

$$K = \kappa_\chi \chi^2 + \kappa_\phi \phi^2 \simeq \chi^2 \left(1 + \frac{\kappa_\phi \phi^2}{\chi^2}\right) \to \text{const}\,.$$

$$M_{\rm P}^2 = \xi_\chi \chi^2 \left(1 + \frac{\xi_\phi \phi^2}{\xi_\chi \chi^2}\right) \xrightarrow{\xi_\phi^{1/2}\phi \lll \xi_\chi^{1/2}\chi} \xi_\chi \chi^2 \simeq M_{\rm P,0}^2$$

$$\sqrt{\kappa_\phi} \ll \frac{\chi_{\rm ini}}{\phi_{\rm ini}} \ll \sqrt{\xi_\phi/\xi_\chi} \;\Rightarrow\; \begin{cases} K \simeq \chi_{\rm ini}^2 & : \text{ always} \\ M_{\rm P}^2 \simeq \xi_\phi \phi_{\rm ini}^2 \gg M_{\rm P,0}^2 & : \sim \text{the SM Higgs inflation} \end{cases}$$

$\Rightarrow$ A large-field inflation can be realized in the same manner as the SM Higgs inflation.



# The MCSM

**(The Minimal Cosmological Standard Model)**

[Barenboim, Ko & **Park**, 2403.05390; 2403.08675]

## • Underlying symmetries

- Scale-invariance
- $U(1)_{\rm PQ}$ (but broken in the gravity sector)

## • Minimal field contents & charges

$$\begin{cases} \chi = \text{A real scalar} : \ \langle\chi\rangle \to M_{\rm P} \\ \Phi = \text{the Peccei-Quinn field(a complex scalar)} : \ \langle\Phi\rangle \to U(1)_{\rm PQ}\text{-breaking} \\ H_2 = \text{an additional Higgs doublet} : \ \text{DFSZ-axion model} \\ \nu_{R_i} = \text{three right-handed neutrino fields} : \ \text{Seesaw mechanism} + \text{reheating} \end{cases}$$

| Charge \ Field | $Q_L$ | $u_R$ | $d_R$ | $\ell_L$ | $e_R$ | $\nu_R$ | $H_1$ | $H_2$ | $\Phi$ |
|---|---|---|---|---|---|---|---|---|---|
| $q_{\rm PQ}$ | 3/2 | 1/2 | 1/2 | 3/2 | 1/2 | 1/2 | 1 | -1 | 1 |

- **The model (Axi-majoron + non-minimal grav.-coupling)**

Key terms of our scenario!

$$\mathcal{L} = \mathcal{L}_{\rm g} + \mathcal{L}_{\rm m}$$

$$\mathcal{L}_{\rm g} = -\frac{1}{2}\tilde{R}\left[\xi_\chi \chi^2 + \xi_\Phi |\Phi|^2 + \xi_+ \left(\Phi^2 + {\rm c.c.}\right) - i\xi_- \left(\Phi^2 - {\rm c.c.}\right) + \mathcal{F}(H_1, H_2)\right] + {\rm G.B.}$$

$$\mathcal{F} = \xi_{H_1} |H_1|^2 + \xi_{H_2} |H_2|^2 + \xi_+^H \left(H_1^\dagger H_2 + {\rm c.c.}\right) - i\xi_-^H \left(H_1^\dagger H_2 - {\rm c.c.}\right)$$

$$\mathcal{L}_{\rm m} \supset \mathcal{L}_{\rm ss} - \tilde{V}_S$$

**Simplified version with only one Higgs doublet:**

$$\tilde{V}_S = \frac{\lambda_\chi}{4}\chi^4 + \lambda_h |H|^4 + \lambda_\phi |\Phi|^4$$
$$-\frac{1}{2}\lambda_{\chi h}\chi^2 |H|^2 - \frac{1}{2}\lambda_{\chi\phi}\chi^2|\Phi|^2 - \lambda_{h\phi}|H|^2|\Phi|^2$$

$$\mathscr{L}_{\rm ss} = \frac{1}{2} y_N \Phi^* \overline{\nu_R^c}\nu_R + y_\nu \bar{\ell}_L \tilde{H} \nu_R + {\rm h.c.}\ .$$ (⇒ “Axi-majoron” model)

- Dynamically relevant three fields with $\langle H \rangle \sim 0$:

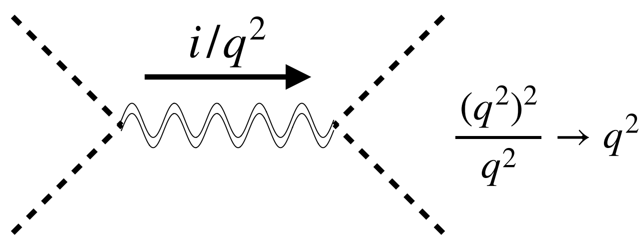
$$\varphi_\alpha = (\chi,\ \phi_r,\ \phi_i)\ \left(\text{for } \Phi \equiv (\phi_r + i\phi_i)/\sqrt{2} = \phi e^{i a_\phi/\langle\phi\rangle}/\sqrt{2}\right)$$

## • Axion quality problem(?)

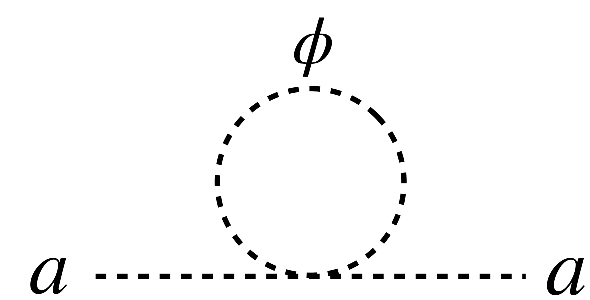
**Grav. non-pert. effect:** [Kallosh, Linde, Linde & Suskind, PRD 52 (1995) 912]
Gauss-Bonnet term may retain the corrections negligible.

**Perturbative effect:** [Hill and Ross, PRD102, 125014(2020)]
Impact of the dangerous graviton exchanges caused by the sym.-breaking terms:

$$S=\int\sqrt{-g}\left(\frac{1}{2}M_P^2R(g_{\mu\nu})+\frac{1}{2}F(\phi_i)R(g_{\mu\nu})+L(\phi_i)\right)$$




$$S_{CT}=-\int d^4x\,\frac{3}{4M_P^2}F(\phi_i)\,\partial^2F(\phi_i)+\int d^4x\,\frac{1}{2M_P^2}F(\phi_i)\,T(\phi_i)$$




$$\frac{\Delta m_a^2}{m_{a,\mathrm{QCD}}^2}\sim\alpha\xi_\phi^2\left(\frac{m_\phi^4}{m_{a,\mathrm{QCD}}^2M_{\mathrm{P}}^2}\right)\ln\left(\frac{\chi^2}{m_\phi^2}\right)\gg 1$$

$$m_\phi\sim\sqrt{\lambda_\phi}\phi_0\gtrsim\mathcal{O}(10^7)\mathrm{GeV}$$

**A cure**

$$\xi_a\to\xi_a=\xi_{a,0}e^{-c_a\chi/|\varphi_a|}\qquad\begin{cases}c_a\chi\ll|\varphi| & :\ \text{during inflation}\\ c_a\chi\gg|\varphi_0| & :\ \text{after inflation}\end{cases}$$

- **The origin of scales**

$\chi$ is nearly fixed, once the kernel is fixed

$$\tilde{V}_S(\chi, h, \phi) = \frac{\lambda'_\chi}{4}\chi^4 + \frac{\lambda'_\phi}{4}\left(\phi^2 - \zeta_{\phi\chi}\chi^2\right)^2 + \frac{\lambda_h}{4}\left(h^2 + \zeta_{h\phi}\phi^2 - \zeta_{h\chi}\chi^2\right)^2$$

$$\Downarrow \quad (\xi_\chi \chi^2 \to M_{\rm P}^2)$$

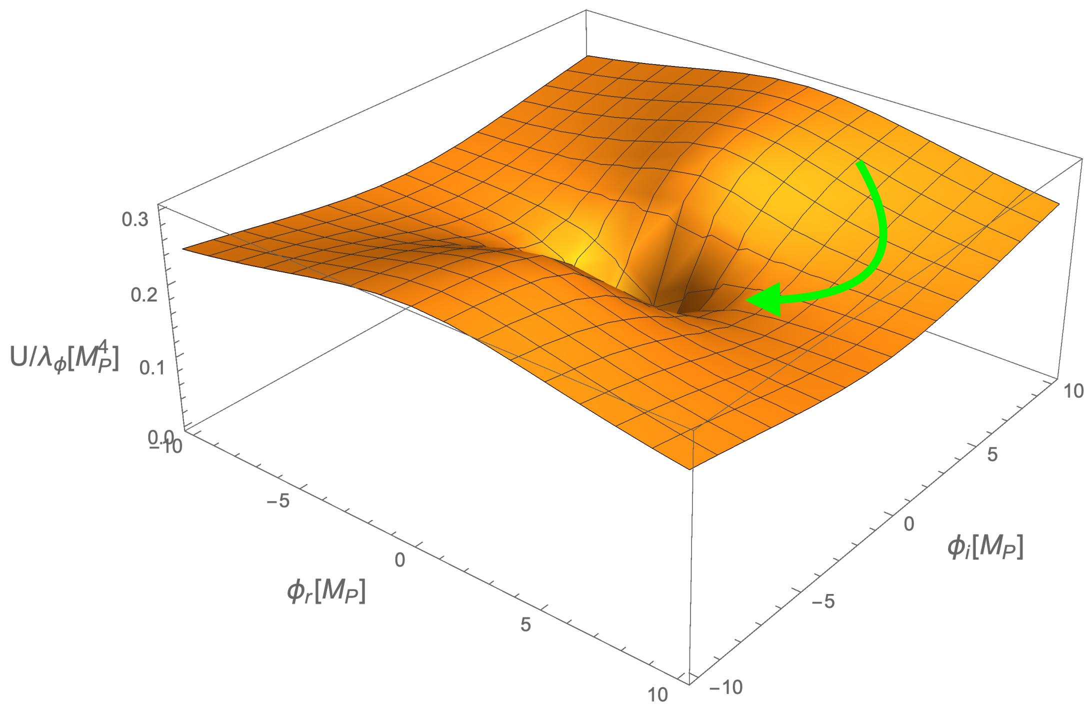
$$V(\chi, \phi, h) \equiv \frac{\tilde{V}_S}{\Omega^2} \approx V_0 + \frac{\lambda'_\phi}{4}\left(\phi^2 - \phi^2_{0,\chi}\right)^2 + \frac{\lambda_h}{4}\left(h^2 + \zeta_{h\phi}\phi^2 - h^2_{0,\chi}\right)^2$$

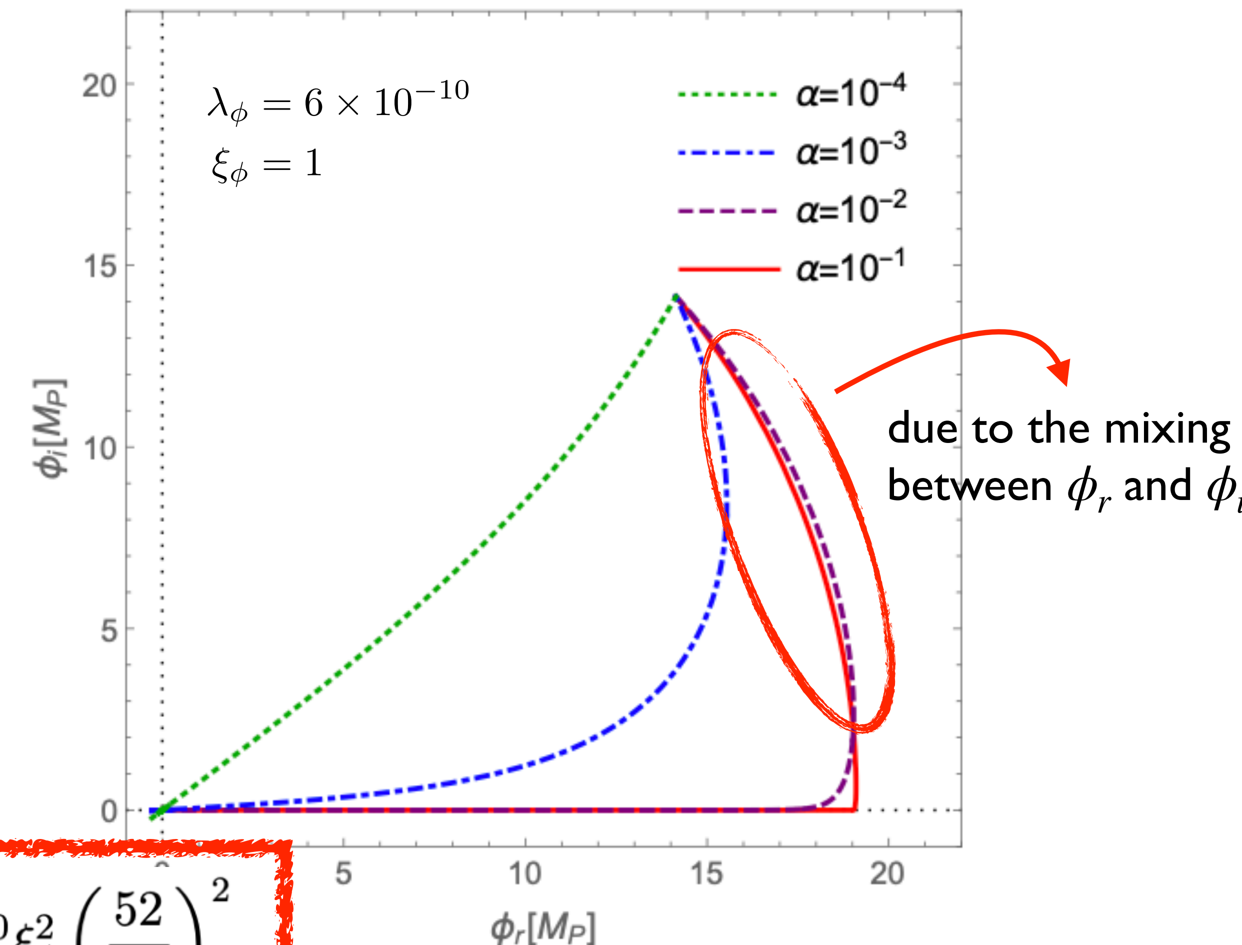
$$V_0 = \frac{\lambda'_\chi}{4\xi_\chi^2} M_{\rm P}^4 + \text{1-loop corrections}$$

1. $\chi$ is responsible for all of mass scales (except the dimensional transmutation).
2. C.C. appears after the spontaneous breaking of the scale-sym. at least due to $\lambda'_\chi$-term.
3. C.C. problem is now of the choice of $\lambda'_\chi/\xi_\chi^2$.
   (see however PLB671 (2009) 162; 187 (unimodular gravity) & T. Kugo's talk at SI2009)

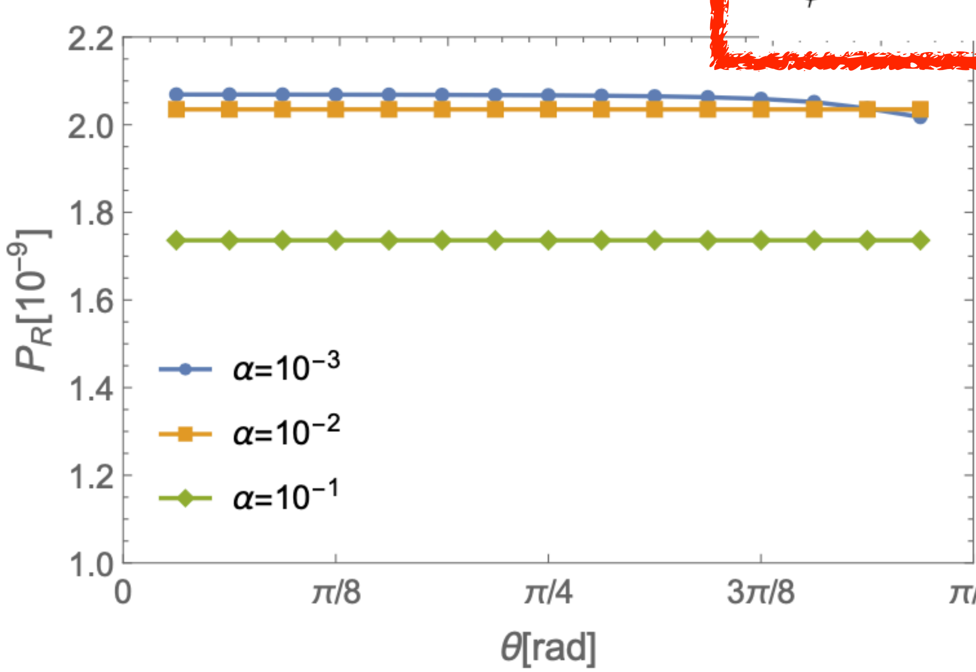
# Cosmology

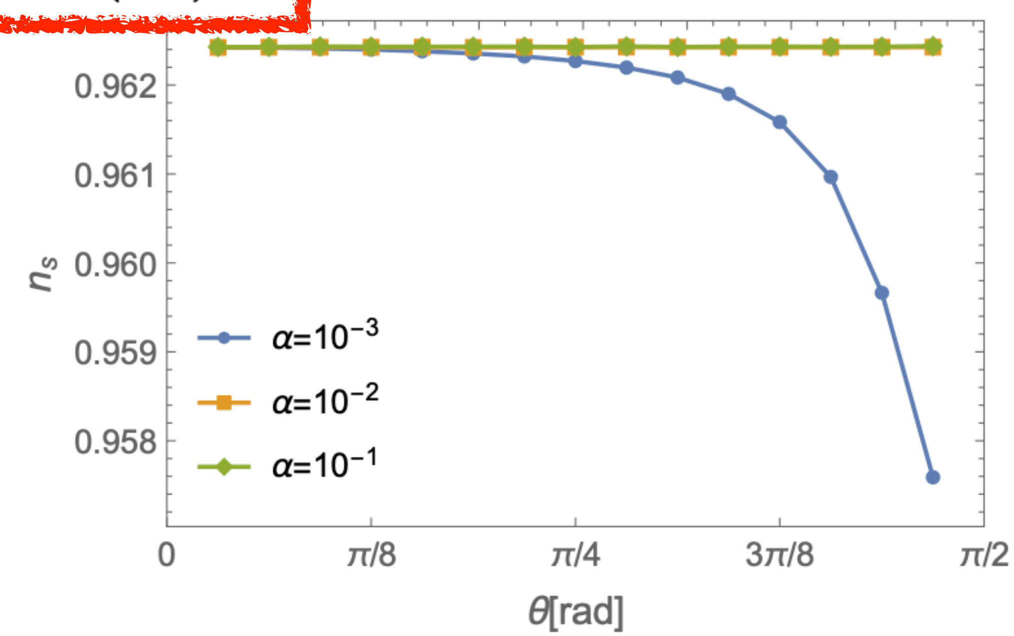
- **Axi-Majoron Hybrid Inflation**




$$\lambda_\phi \sim 6\times10^{-10}\xi_\phi^2\left(\frac{52}{N_e}\right)^2$$

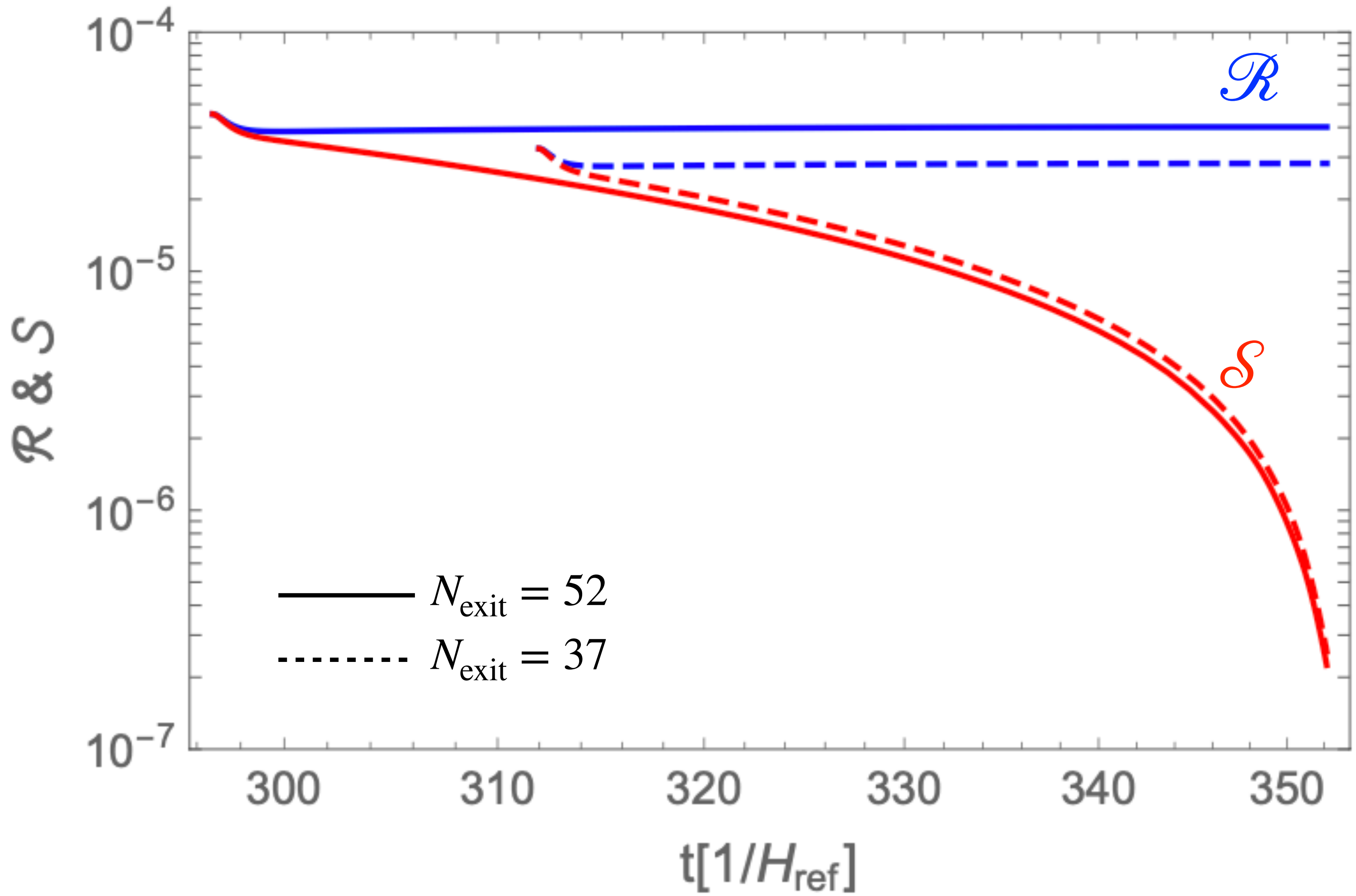


## Iso-curvature perturbations:

- Evolution of $\phi$ causes a suppression ($\dot{\phi}$ increases toward the end of inflation):

$$S \equiv \frac{H}{\dot{\phi}} (\delta\phi^a)_\perp \;\; \& \;\; (\delta\phi^a)_\perp \sim \text{const.} \;\Rightarrow\; \frac{P_I}{P_R} \propto \frac{\epsilon_{\rm exit}(k)}{\epsilon_{\rm end}} \sim \epsilon_{\rm exit}(k) \sim \mathcal{O}(N_{\rm exit}^{-2}(k))$$



- **Reheating after inflation**

**Relevant interactions:**

$$\mathcal{L} \supset \left(\theta - \frac{c_a a_\phi}{\phi_0}\right) \frac{g_s^2}{32\pi^2} G\tilde{G} - V$$

$$V \supset \frac{\lambda_\phi}{4}\left(\phi^2 - \phi_0^2\right)^2 + \left(\frac{1}{2} y_N \Phi^* \overline{\nu_R^c} \nu_R + y_\nu \bar{\ell}_L \tilde{H} \nu_R + \text{h.c.}\right)$$

$$m_\phi = \begin{cases} \sqrt{3\lambda_\phi}\phi & : \ \phi \gg \phi_0 \\ \sqrt{2\lambda_\phi}\phi_0 & : \ |\delta\phi \equiv \phi - \phi_0| \ll \phi_0 \end{cases}$$

(i) $\phi$ decays dominantly to $a_\phi$s(axi-majorons) via the kinetic term.

(ii) It should be able to decay to RHNs ( $\Rightarrow y_{N_i} < \sqrt{\lambda_\phi}$ ), too.

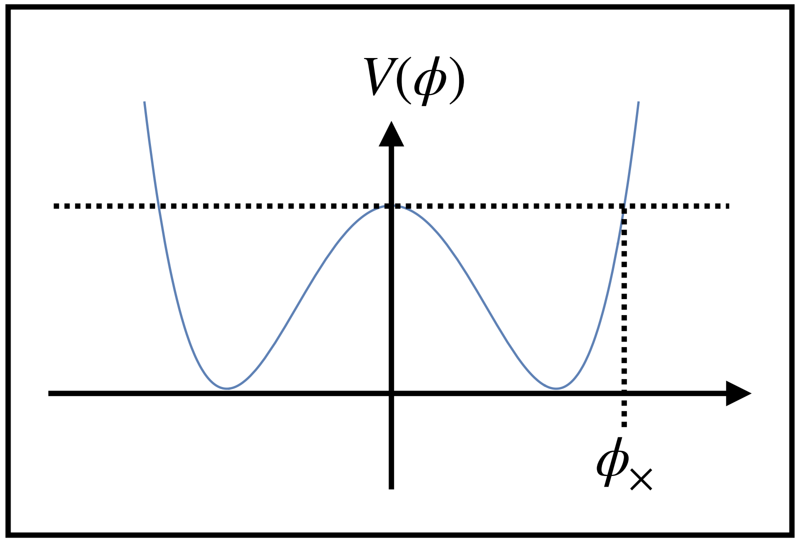
(iii) There would be effects of preheating by axi-majoron-dynamics (under investigation!)

(iv) Recovering the SM thermal bath requires a period of MD era due to long-lived RHNs.

$$\Rightarrow \quad \frac{m_{\nu_1}}{m_\nu} < 2 \times 10^{-4} \times \left(\frac{\Delta N_{\text{eff}}^{\text{obs}}}{0.5}\right)^{3/2} \left(\frac{\xi_\phi}{10}\right) \left(\frac{\text{B}_1}{0.1}\right)^2 \left(\frac{\phi_0}{\phi_0^{\text{ref}}}\right)$$

$$\left(m_\nu \equiv 0.05\text{eV}, \ \phi_{\text{ref}} \equiv 10^{12}\text{GeV}\right)$$

## • Symmetry non-restoration of $U(1)_{\rm PQ}$




**Preheating:**
[Greene et al, PRD56 (1997) 6175; Greene & Kofman, PLB448 (1999) 6]

(i) To $\phi$-quanta (due to $\lambda_\phi$-term): $\Delta\rho_\phi = \mathcal{O}(10^{-3})\rho_\phi$

(ii) To $\nu_R$s: $\Delta\rho_N = \mathcal{O}(0.1) \times b_N^{5/4}\lambda_\phi\rho_\phi \lll \rho_\phi$

$\Rightarrow$ Energy transfer to $\nu_R$ via preheating is inefficient!

**Thermal trapping?:**

(i) Scatterings of $\phi$ to $\nu_R$s are inefficient.

$$\left.\begin{array}{l}\Gamma_s \sim y_N^4 T \lesssim \lambda_\phi^2 T \\ H(T) \gtrsim T^2/M_{\rm P}\end{array}\right\} \Rightarrow \Gamma_s/H(T) \lesssim \lambda_\phi^2 M_{\rm P}/T \;\; (\text{at least for } T \gtrsim \phi_0)$$

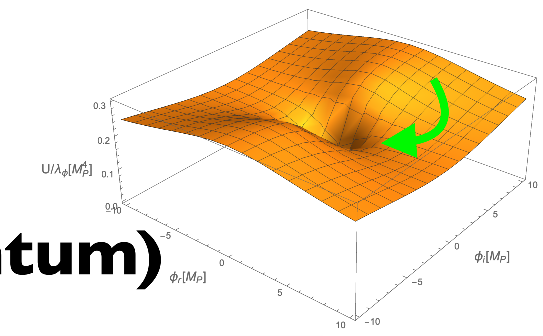
(ii) The effective mass-squared at the origin when $\phi \sim \phi_* \gtrsim \phi_\times$:

$$m_{\phi,\rm eff}^2(0) \approx \lambda_\phi\phi_0^2\left[-1 + c_T\lambda_\phi^{5/2}\left(\frac{9}{32\pi}\frac{M_{\rm P}}{\phi_0}\right)^2\sum_i b_i\right] < 0 \quad \Rightarrow \quad \xi_\phi \lesssim 680\left(\frac{0.1}{b_N}\right)^{1/5}\left(\frac{\phi_0}{\phi_0^{\rm ref}}\right)^{2/5}$$

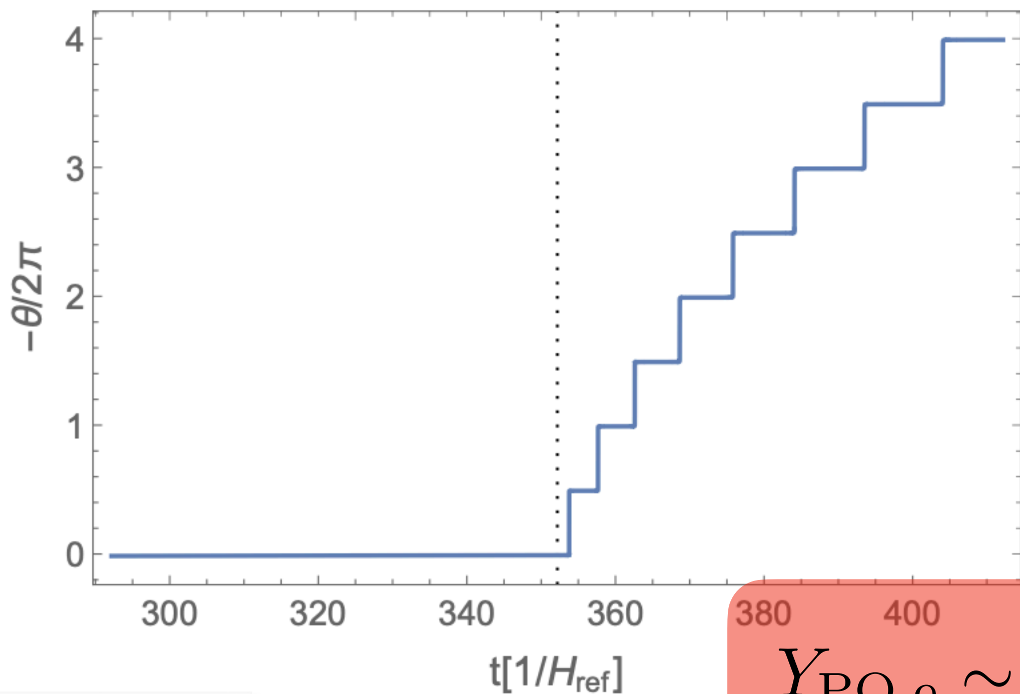
$\therefore$ It is possible to have the symmetry not restored.

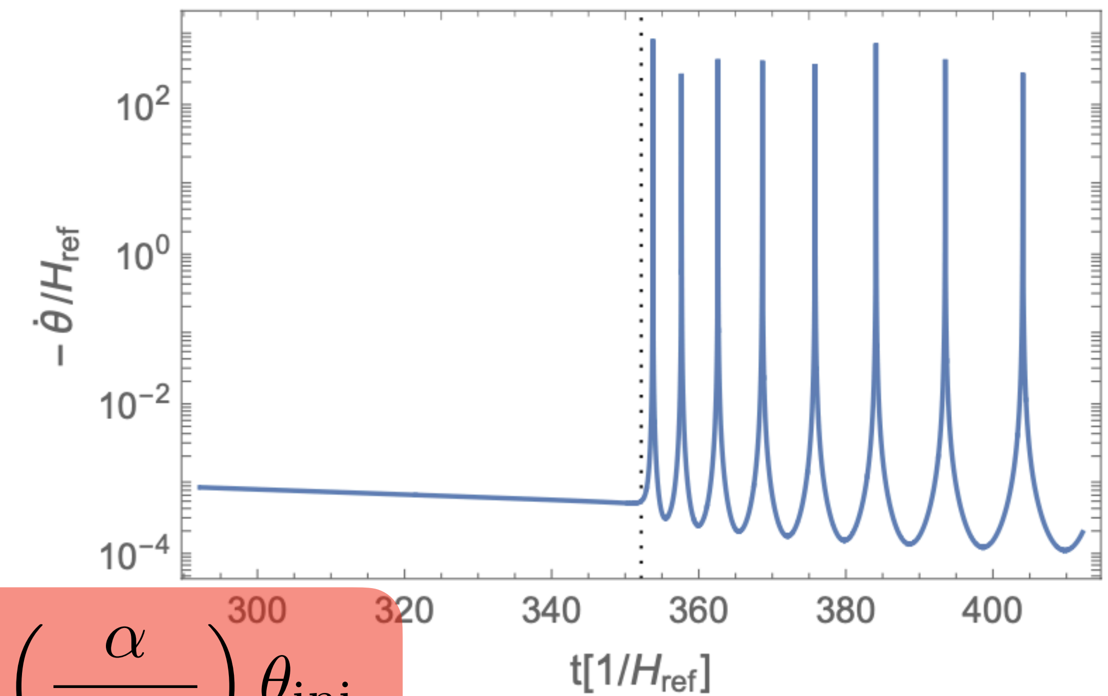
$\Rightarrow$ We can avoid the domain-wall problem of the minimal DFSZ axion model!

- **Matter-antimatter asymmetry**

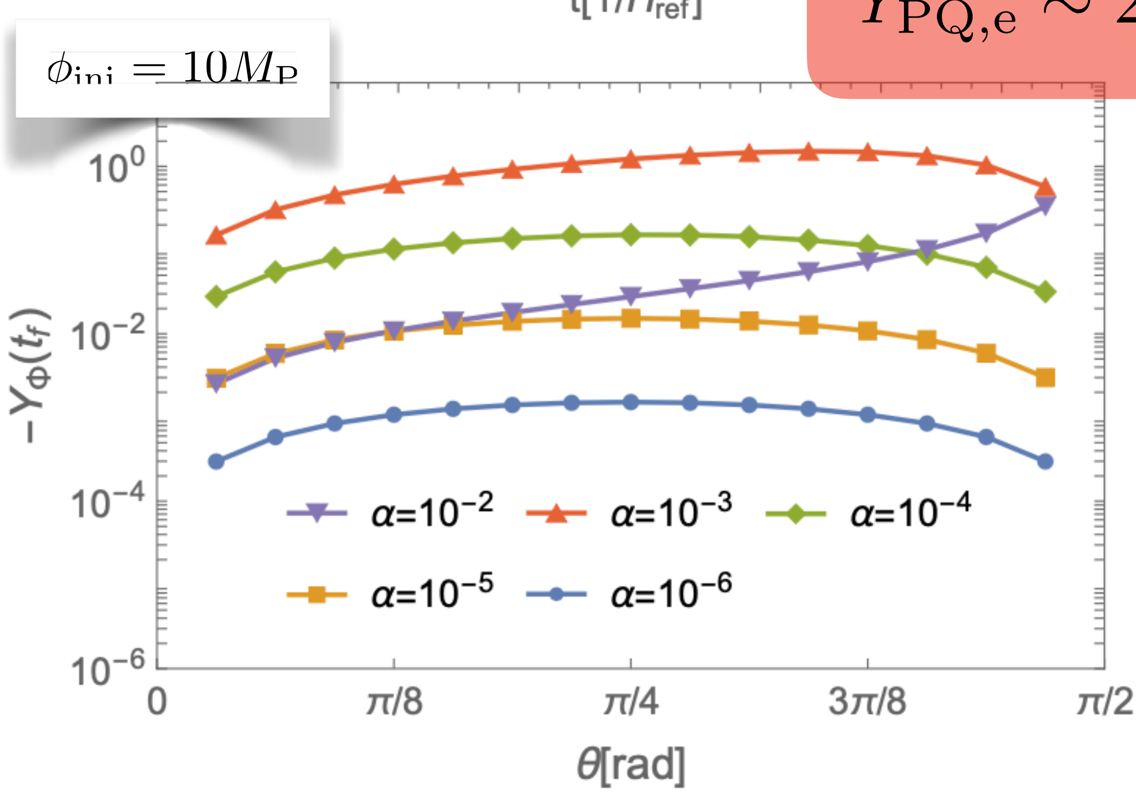


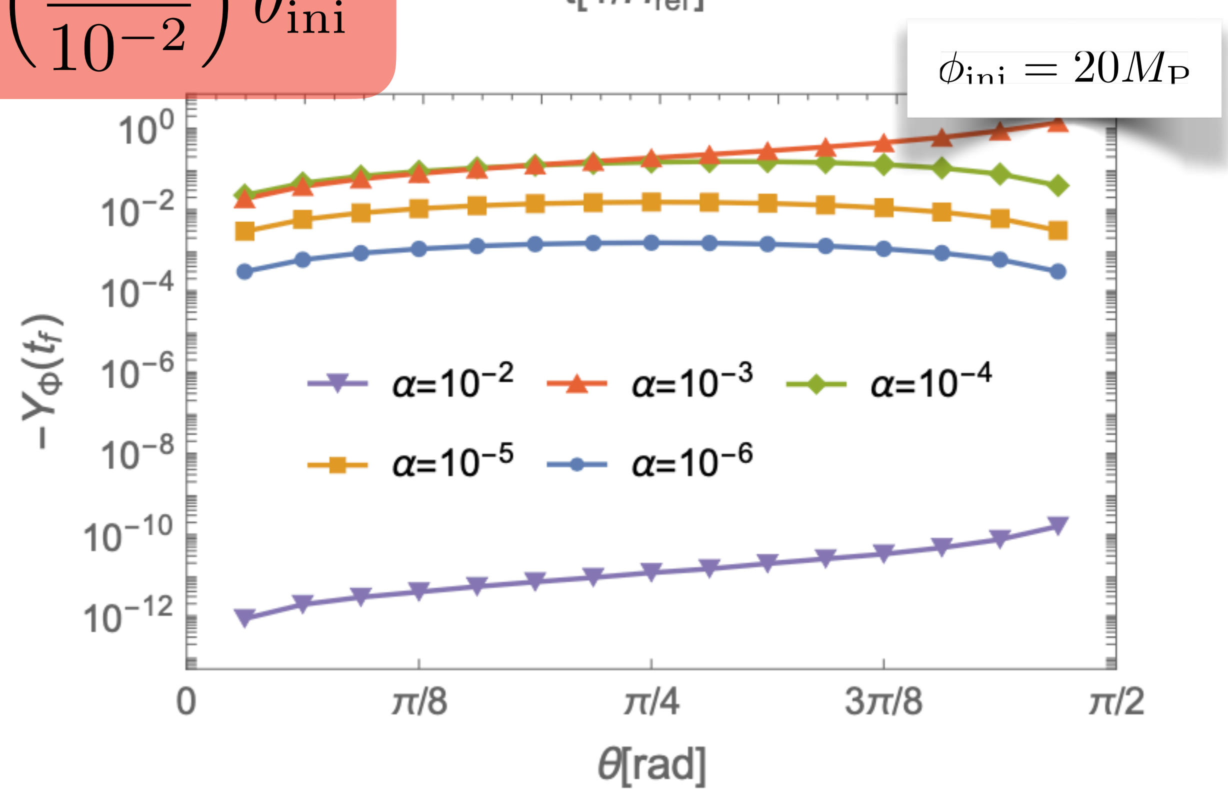

**Affleck-Dine PQ-# generation ( = non-zero angular momentum)**



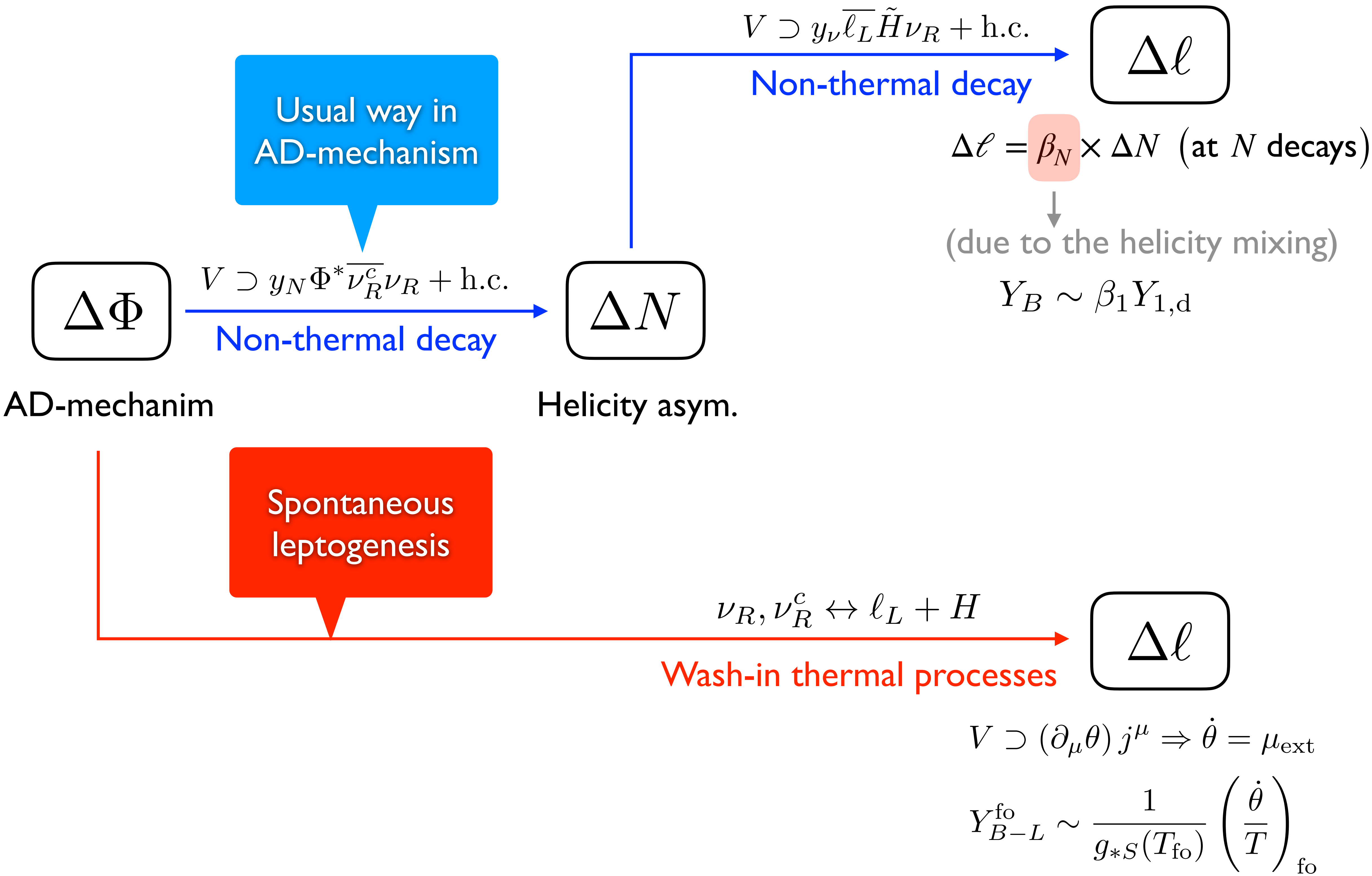




$$Y_{\rm PQ,e} \sim 2.5\left(\frac{\alpha}{10^{-2}}\right)\theta_{\rm ini}$$



## Transferring PQ-number to the SM sector

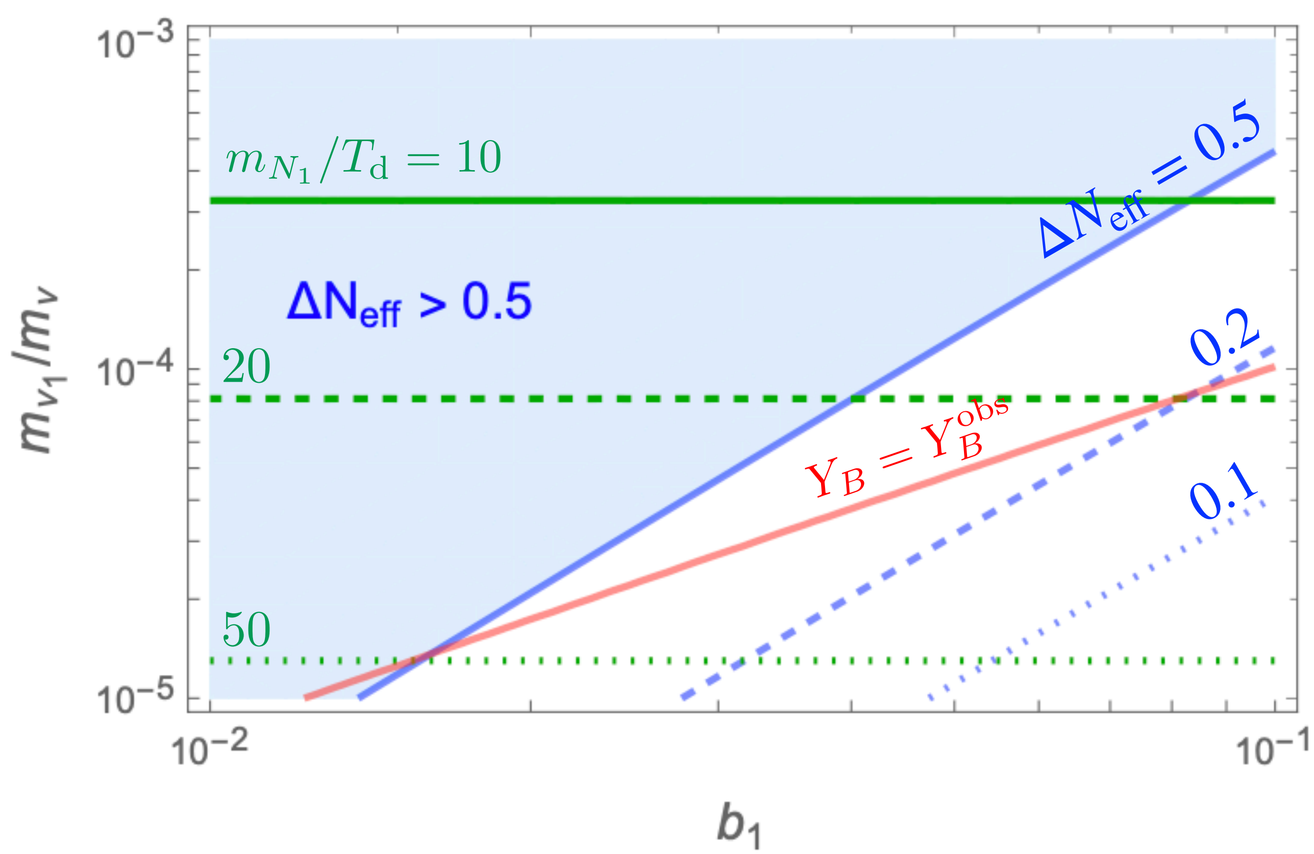


## Parametric dependence of the baryon # asymmetry

Non-thermal decay of RHNs (transfer of helicity asym.): $\Delta\Phi \to \Delta N \to \Delta\ell$

$$Y_{B,\mathrm{AD}}^{>} = \frac{12}{37} \times \beta_{1,\mathrm{d}}^{>} Y_{1,\mathrm{d}}^{>}$$
$$\simeq 6.2 \times 10^{-11} \times \left(\frac{0.1}{\mathrm{B}_1}\right)^{4/3} \left(\frac{\xi_\phi}{200}\right)^{7/6} \left(\frac{10^4 m_{\nu_1}}{m_\nu}\right)^{7/6} \left(\frac{Y_{\Phi,e}}{10^{-6}}\right)$$

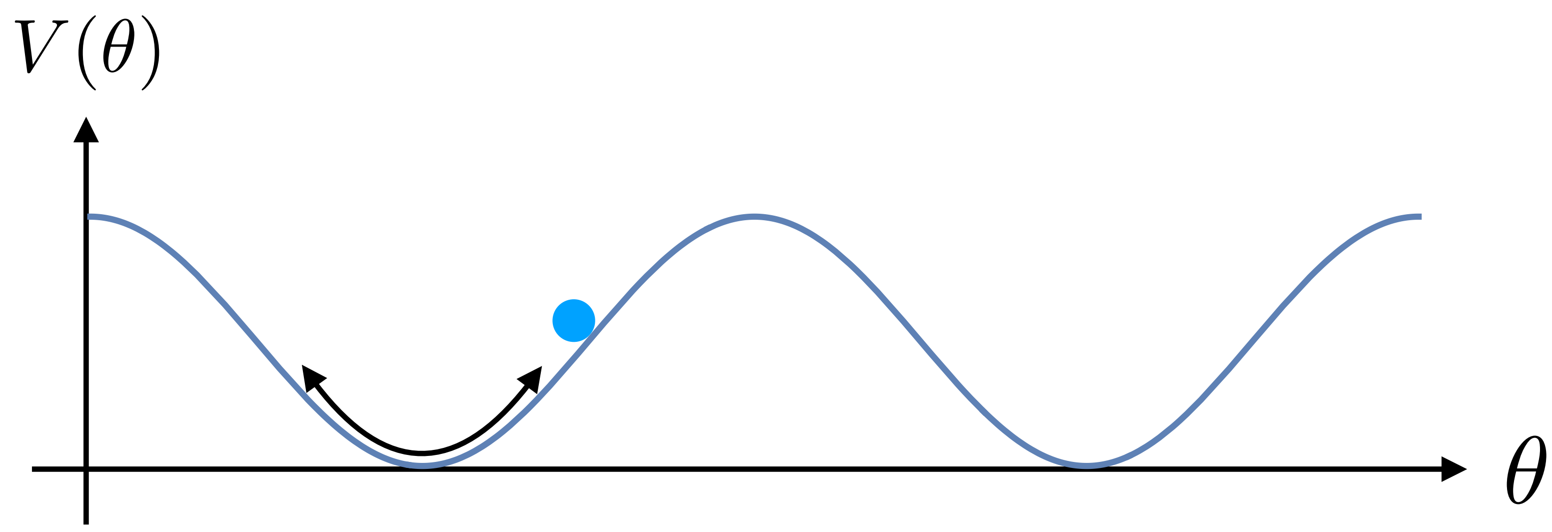
$$\left.\begin{aligned} \mathrm{B}_{2,3} &= 0.1 \\ \xi_\phi &= 200 \\ \phi_0 &= 10^{12}\mathrm{GeV} \\ Y_{\mathrm{PQ,e}} &= 10^{-6} \end{aligned}\right\}$$



## • Dark matter & dark radiation

### Dark matter

Cold axi-majorons from the misalignment only $\Rightarrow$ $\phi_0 = \mathcal{O}(10^{12})$ GeV




### Dark radiation

Hot axi-majorons from the decay of the inflaton:

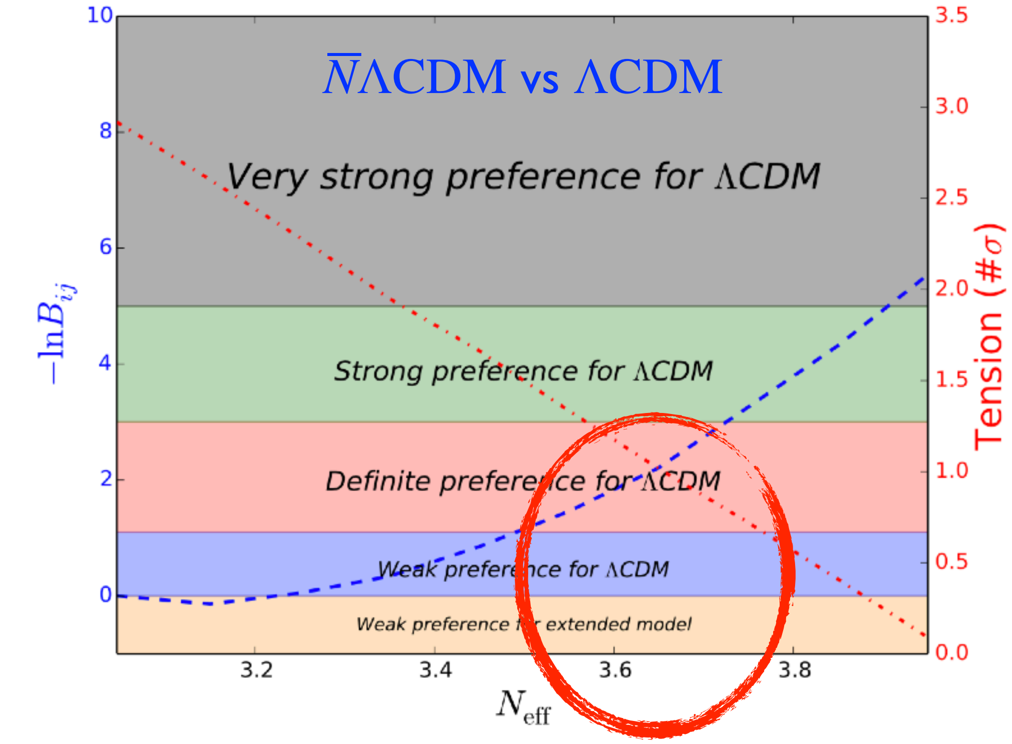
$$\Delta N_{\rm eff} \simeq 0.47 \left( \frac{10\Gamma_{N_1}}{H_{1,\rm eq}} \right)^{2/3}$$

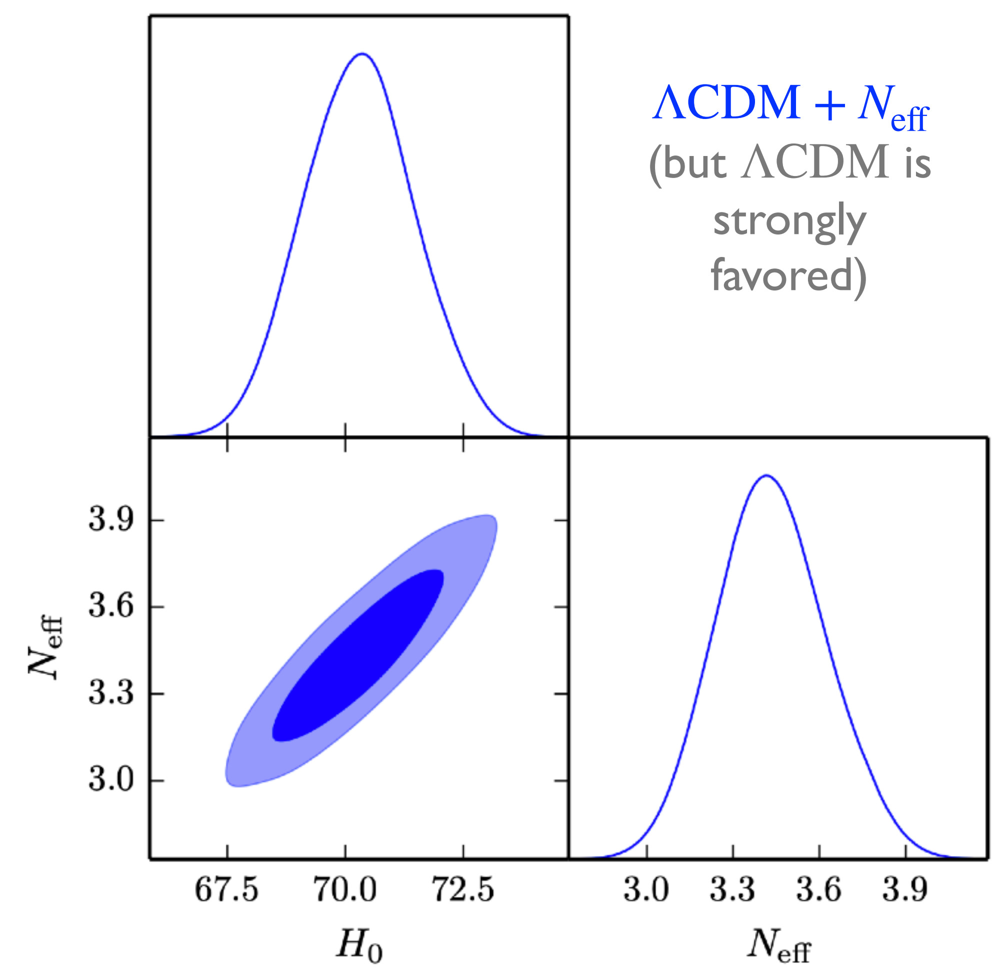
(It helps improve the Hubble tension!)

## * Hubble tension


[Class. Quantum Grav. 38, 153001 (2021)]

## < Impact of $\Delta N_{\rm eff}$ >

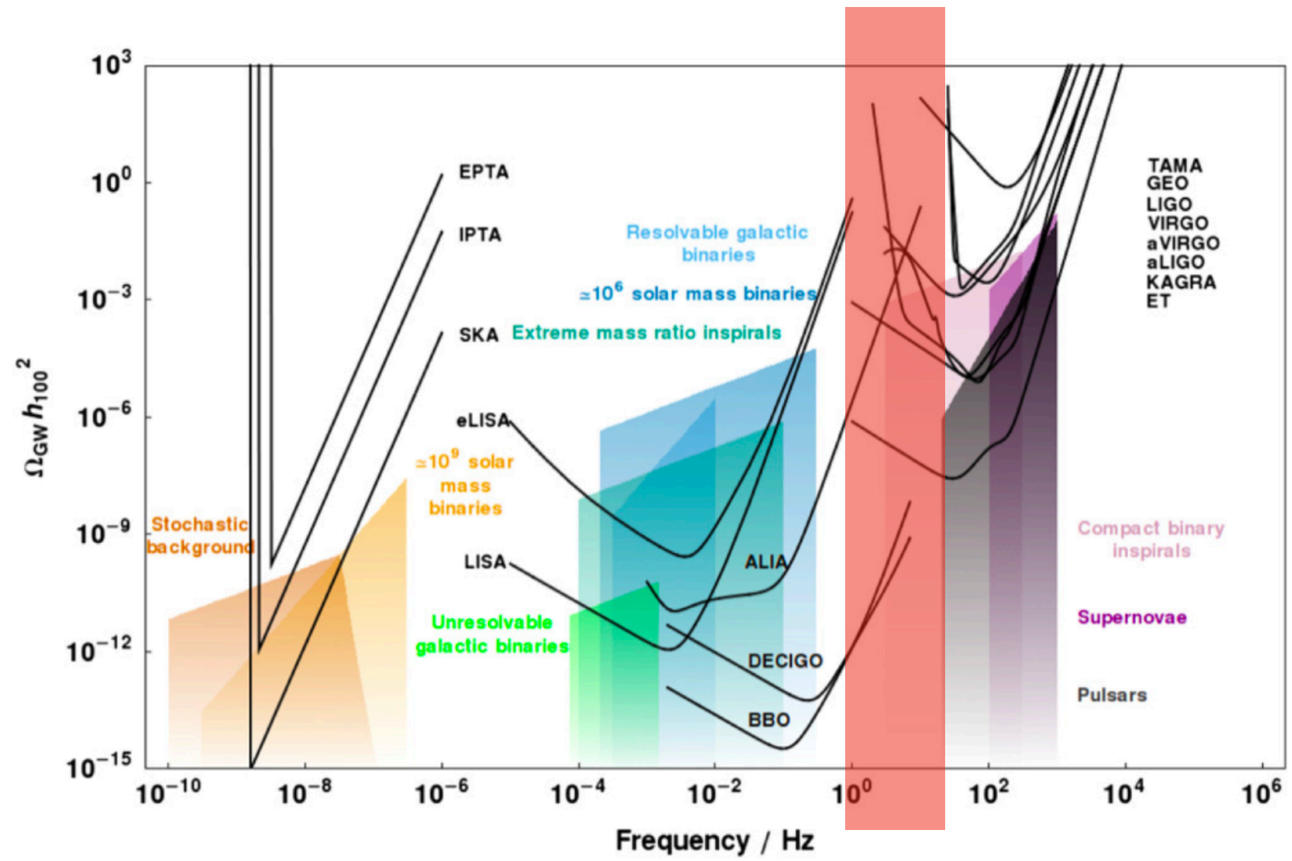


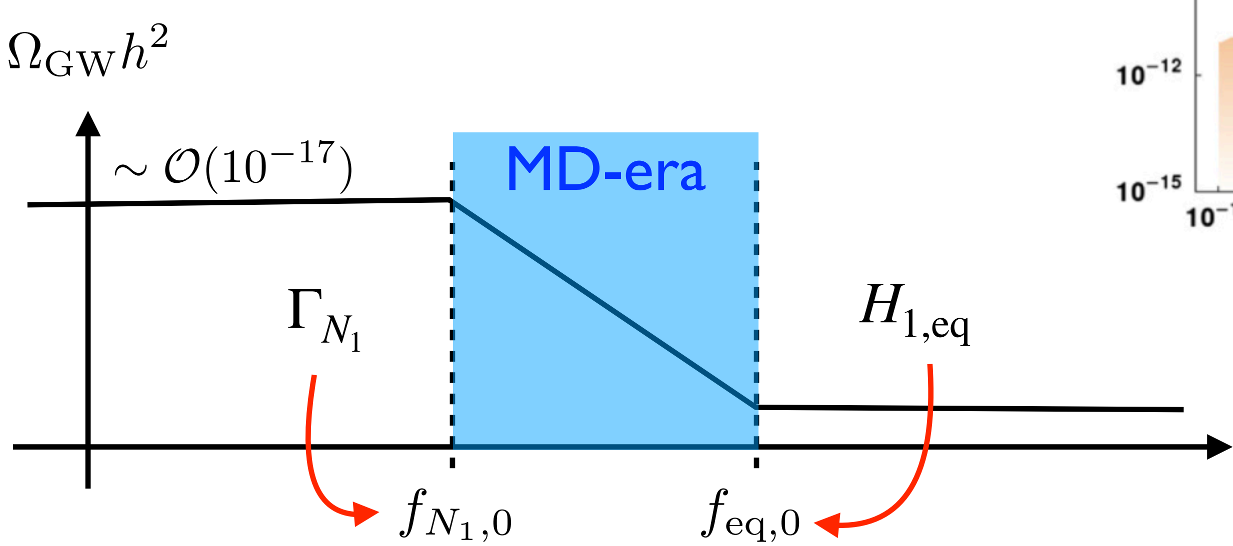

[S.Vagnozzi, Phys.Rev.D 102 (2020) 2, 023518]

# Signatures

- **Distortion of inflationary GWs**




Characteristic frequencies:




$$f_{\rm eq,0} \approx 35 \times \left[ \left(\frac{\rm B_1}{0.1}\right)^2 \left(\frac{100}{\xi_\phi}\right) \left(\frac{m_\nu}{10^4 m_{\nu_1}}\right) \right]^{2/3} f_{N_1,0}$$

$$f_{N_1,0} \approx 1{\rm Hz} \left[ \left(\frac{0.1}{b_1}\right) \left(\frac{\xi_\phi}{100}\right)^2 \left(\frac{10^4 m_{\nu_1}}{m_\nu}\right) \left(\frac{\phi_0}{10^{12}{\rm GeV}}\right)^2 \right]^{1/2}$$

Consistency check:

$$\Delta N_{\rm eff} \simeq 0.48 \times \left(\frac{100}{g_*(T_{\gamma,\rm d})}\right)^{1/3} \left(\frac{10\Gamma_{N_1}}{H_{1,\rm eq}}\right)^{2/3}$$

$$(\text{c.f. } H_{\rm d} = (2/3)\Gamma_{N_1})$$

- **Presence of axi-Majoron DR**

$$\Delta N_{\rm eff} \simeq 0.48 \times \left(\frac{100}{g_*(T_{\gamma,\rm d})}\right)^{1/3} \left(\frac{10\Gamma_{N_1}}{H_{1,\rm eq}}\right)^{2/3}$$

$\Rightarrow$ Duration of MD-era

$\Rightarrow$ Change of the total inflationary $e$-folds($N_e$)

- **Shift of $n_s(N_e)$ ( $\Leftarrow$ Shift of $N_e$)**

$$n_s \simeq 1 - 2N_e^{-1}, \quad r_T \simeq 12 N_e^{-2}$$

$$\Omega_{\rm GW} \xrightarrow{f_{N_1,0}} \Gamma_{N_1} \xrightarrow{\Delta N_{\rm eff}} H_{1,\rm eq} \xrightarrow[\Delta N_e]{\text{MD era}} n_s, r_T$$

- **Implication(s) on neutrinos physics**

The masses of two heavy states are nearly fixed (since $m_{\nu_1} \lll m_\nu \equiv 0.05\text{eV}$)

# Summary

- We constructed the simplest minimal BSM model under the scale-sym. and $U(1)_{\rm PQ}$-sym.

- *$U(1)_{\rm PQ}$-breaking terms may exist only in the gravity sector.*

- *The model dubbed as the MCSM can address simultaneously following big puzzles:*

  - the origin of scales
  - the primordial inflation
  - the matter-antimatter asymmetry
  - dark matter and its relic density
  - dark radiation
  - the strong CP-problem
  - the origin of the tiny neutrino-mass
  - ~~the C.C. problem(modulo tuning of $\lambda_\chi$)~~

  - Iso-curvature perturbations suppressed
  - No domain-wall problem
  - No axion-quality problem
  - No fifth force constraints
  - No hierarchy problem

- The MCSM provides the simplest unified framework for the history of the universe from inflation to the present universe (thanks to the father($\Phi$) and the mother($\chi$) of the universe)

Thank you!