Gravitational Waves and Dark Matter in G2HDM

TC Yuan, IOP Academia Sinica, Taiwan

Invited plenary talk at the Joint Workshop, UNSW, Sydney (December 9-13, 2024)

Based on 2408.05167 (Michael J Ramsey-Musolf, Van Que Tran, TCY)

Known Facts/Arguments

- To achieve FOEWPT in SM requires Higgs mass less than ~ 70 GeV. [Kajantie, *et al.*, NPB 466 (1996)]
- With a 125 GeV Higgs boson, SM has a smooth *crossover* transition at $T_c = 159.5 \pm 1.5\,\mathrm{GeV}$, as supported by lattice simulation.
- Non-perturbative sphaleron effects from SM does provide baryon number violation, if washout effects are negligible.
- Nevertheless, CP phase in the CKM matrix is *not* sufficient to provide matter-antimatter asymmetry.
- Moreover, for $\mu/T < 3$, LQCD predicts QCD phase transition is also a smooth *crossover*. [Stephanov, PoS(LAT2006)024 (2006), hep-lat/0701002]

→ **BSM with extended scalar sector and new CP violation sources to implement Electroweak Baryogenesis**

Outline

- A Succinct Review of Minimal G2HDM & Constraints
- First Order Electroweak Phase Transition (FOEWPT)
- Gravitational Wave Signals &
  Dark Matter Direct Detection
- Conclusions & Outlook

Gauged 2HDM (G2HDM)

Huang, Tsai, TCY, 1512.00229
Ramos, Tran, TCY, 2109.03185

- Main idea is to group H_1 and H_2 in 2HDM into a 2-dim irrep. of an hidden $SU(2)_H$ gauge group
- Aesthetically, we proposed a hidden replica of SM-like gauge sector
$$SU(2)_H \otimes U(1)_X$$
- A hidden Higgs doublet Φ_H (augmented by a Stueckelburg $U(1)_X$ scalar S) is also needed to break the hidden gauge group (to give masses to new gauge bosons $\gamma', Z', \mathscr{W}'^{(p,m)}$)
- No *ad hoc* discrete symmetry (like Z_2 in I2HDM, R-parity in MSSM, T-parity in Littlest Higgs model, KK-parity in extra dim models … for DM candidates). Instead there is an accidental discrete symmetry (*h*-parity) in the model!
- $\mathscr{W}'^{(p,m)}$ is h-parity odd and hence a DM candidate!

Scalar Sector in Minimal G2HDM

Ramos, Tran, Yuan, 2101.07115, 2109.03185

Scalar	$SU(2)_L$	$SU(2)_H$	$U(1)_Y$	$U(1)_X$	h-parity
$H = (H_1 \;\; H_2)^{\rm T}$	2	2	$\frac{1}{2}$	$\frac{1}{2}$	$(+,-)$
$\Phi_H = (\Phi_1 \;\; \Phi_2)^{\rm T}$	1	2	0	$\frac{1}{2}$	$(-,+)$
S	1	1	0	0	$+$

Table II. Higgs scalars in the minimal G2HDM and their quantum number assignments.

Inert Higgs

SM Higgs

Hidden doublet

Stueckelburg scalar

Emerges "naturally"! No need to impose *ad hoc* by hand!

$\Delta_H(1,3,0,0)$

Scalar Potential in G2HDM

$H_{\alpha i}$

$SU(2)_H$ $SU(2)_L$

$(\Phi_H)_\alpha$

- The scalar potential is (*no ad hoc Z_2 imposed!*)

$$V = -\mu_H^2 \left(H^{\alpha i} H_{\alpha i} \right) + \lambda_H \left(H^{\alpha i} H_{\alpha i} \right)^2 + \frac{1}{2} \lambda'_H \epsilon_{\alpha\beta} \epsilon^{\gamma\delta} \left(H^{\alpha i} H_{\gamma i} \right) \left(H^{\beta j} H_{\delta j} \right)$$
$$- \mu_\Phi^2 \Phi_H^\dagger \Phi_H + \lambda_\Phi \left(\Phi_H^\dagger \Phi_H \right)^2 + \lambda_{H\Phi} \left(H^\dagger H \right) \left(\Phi_H^\dagger \Phi_H \right) + \lambda'_{H\Phi} \left(H^\dagger \Phi_H \right) \left(\Phi_H^\dagger H \right) ,$$

- Invariant under $SU(2)_L \otimes U(1)_Y \otimes SU(2)_H \otimes U(1)_X$
- Same number (7) of parameters as I2HDM despite we have a total of 3 Higgs doublets! (Note that a general 3HDM scalar potential has 54 parameters - 12 real and 21 complex!)
- Special-tailored 3HDM
- Each term is self-hermitian, all couplings are real, hence no CP violation in the scalar sector of G2HDM

Symmetry Breaking

- As in I2HDM, we assume *h*-parity is *not* spontaneously broken

$$H_1 = \begin{pmatrix} G^+ \\ \frac{v+h_{\rm SM}}{\sqrt{2}} + i\frac{G^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ H_2^0 \end{pmatrix}, \quad \Phi_H = \begin{pmatrix} G_H^p \\ \frac{v_\Phi+\phi_H}{\sqrt{2}} + i\frac{G_H^0}{\sqrt{2}} \end{pmatrix}$$

SM Higgs doublet — Inert Higgs doublet $\langle H_2 \rangle = 0$ — Hidden Higgs doublet

- $(h_{\rm SM}, \phi_H) \to (h_1, h_2)$ with mixing angle θ_1.

$$\begin{pmatrix} h_{\rm SM} \\ \phi_H \end{pmatrix} = \mathcal{O}^S \cdot \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_1 & \sin\theta_1 \\ -\sin\theta_1 & \cos\theta_1 \end{pmatrix} \cdot \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

  $h(125) = h_1$ or h_2 is very much SM-like $\Longrightarrow$ Small effect to ΔM_W

  $|\sin\theta_1| < 0.25$ [ATLAS + CMS (2021)]

- $(H_2^0, G_H^m) \to (D, \tilde{G})$ with mixing angle θ_2.

  $\Delta M_W, T \Longrightarrow$ Constrains on θ_2 and mass splitting $(m_D - m_{H^\pm})$

Phenomenological and Theoretical Constraints

- Vacuum stability - Scalar potential is bounded from below ✓
- Perturbative unitarity constraints via $(S_1 S_2 \to S_3 S_4)$ ✓
- Signal strengths for $h_{\rm SM} \to \gamma\gamma$, $h_{\rm SM} \to VV^*(V = W, Z)$, $h_{\rm SM} \to \tau^+\tau^-$ from LHC ✓

$$\mu^{\gamma\gamma}_{ggh} = 0.96 \pm 0.14\,(\text{ATLAS 2020}),\ \mu^{\tau\tau}_{ggh} = 1.05^{+0.53}_{-0.47}\,(\text{CMS 2019})$$

$$\mu^{WW^*}_{ggh} = 1.13^{+0.13}_{-0.12},\ \mu^{ZZ^*}_{ggh} = 0.95^{+0.11}_{-0.11}\,(\text{ATLAS 2022})$$

- Higgs invisible width (*light* dark matter scenario):
  $\text{Br}(h \to \text{invisible}) < 0.13$ (ATLAS 2020) ✓
- Electroweak precision data from LEP: Z mass shift ✓
- Oblique parameters (ρ parameter, W mass shift, …) ✓
- Dark photon γ' searches (beam dump, BelleII, …) ✓
- Z' searches (High invariant mass dilepton searches) ✓
- Dark matter relic density: $\Omega_\chi h^2 = 0.120 \pm 0.001$ (PLANCK 2018) ✓
- Dark matter direct searches ($\sigma^{\rm SI}_{\chi p}$) from CRESST III, DarkSide-50, XENON1T, PandaX-4T, LZ, CDEX, NEWS-G, SuperCDMS *etc.* ✓

First Order EW Phase Transition

Bubble collisions,
sound waves,
turbulence →
Stochastic GWs

Laser Interferometers

P. Athron et al.   *Progress in Particle and Nuclear Physics 135 (2024) 104094*

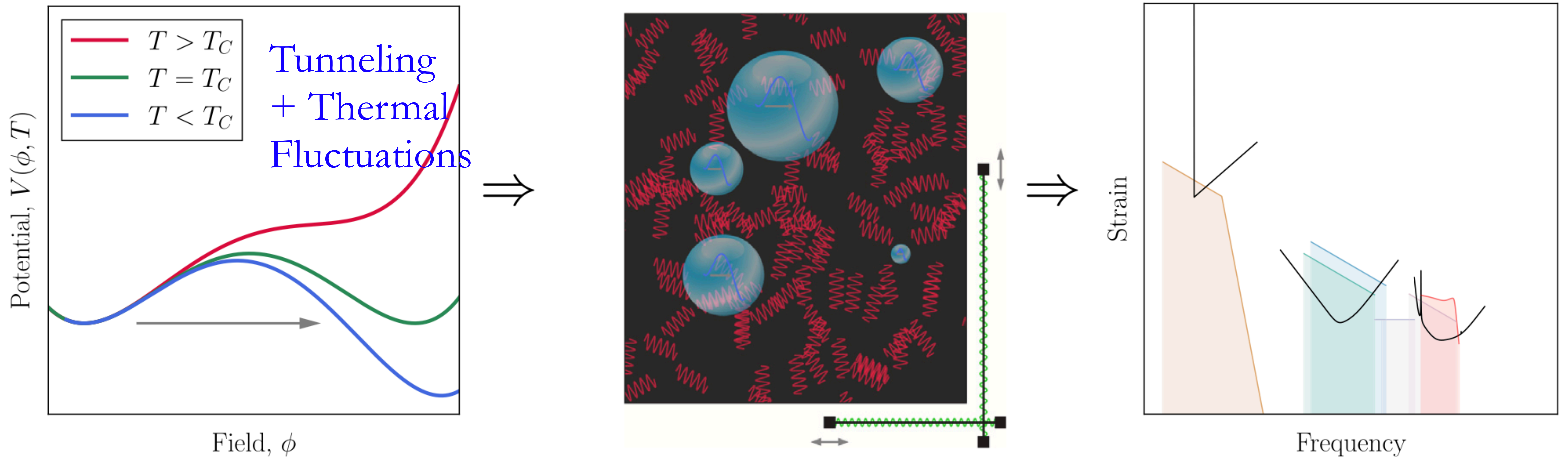


Fig. 1. As the Universe cools, the potential develops a new minima away from the origin (left). A first-order phase transition occurs through bubbles, which appear spontaneously and expand in the thermal plasma (center). The GWs from bubble collisions and the plasma may be measured using a laser interferometer, resulting in a stochastic GW background spectrum (right).

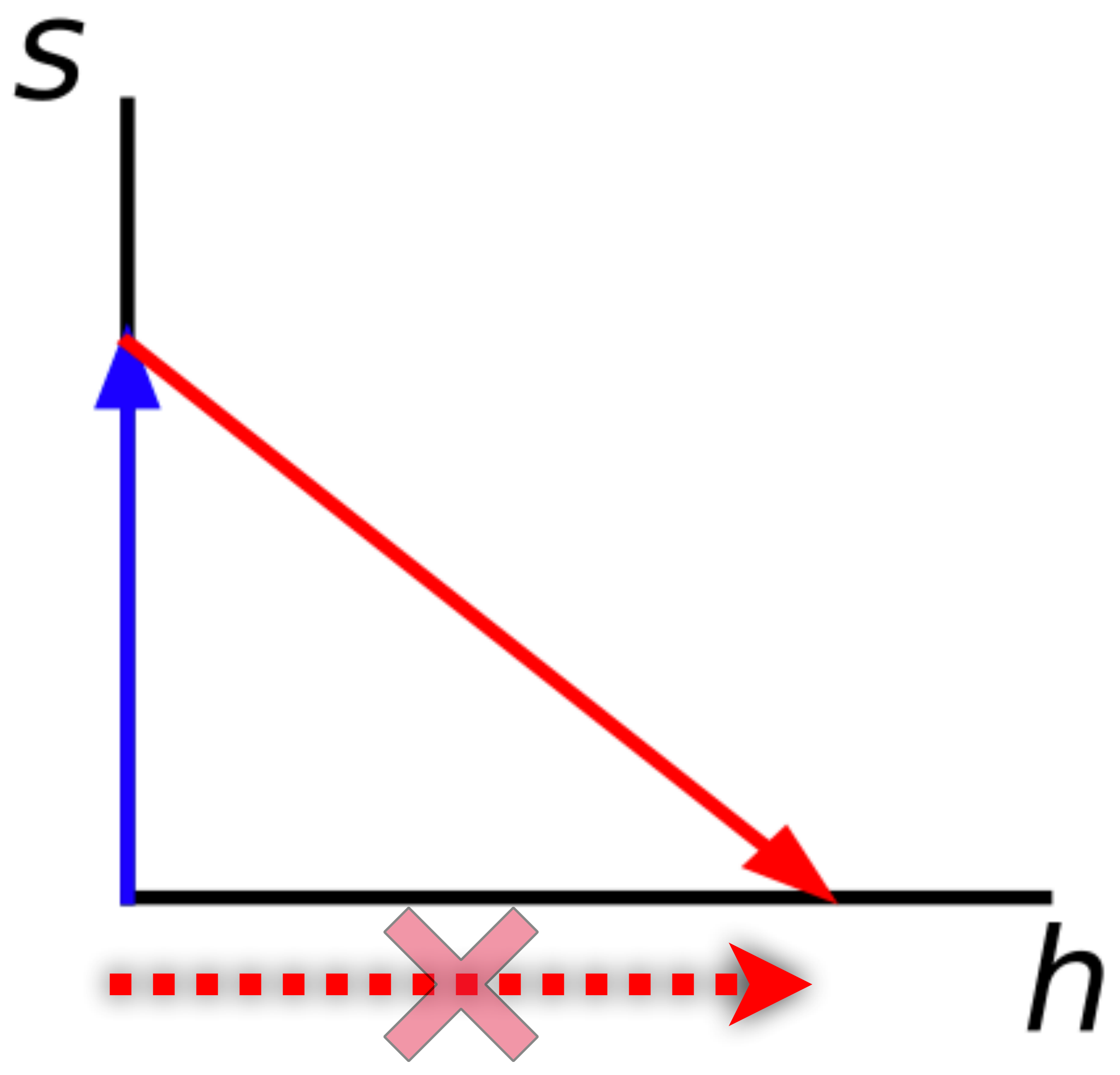
SM+Real Singlet Scalar and 2-step FOEWPT

With Z_2 symmetry

Requires $M_S \lesssim 700$ GeV due to $T_1 > T_2 \sim T_{\rm EW}$

(Ramsey-Musolf, 1912.07189 [hep-ph];

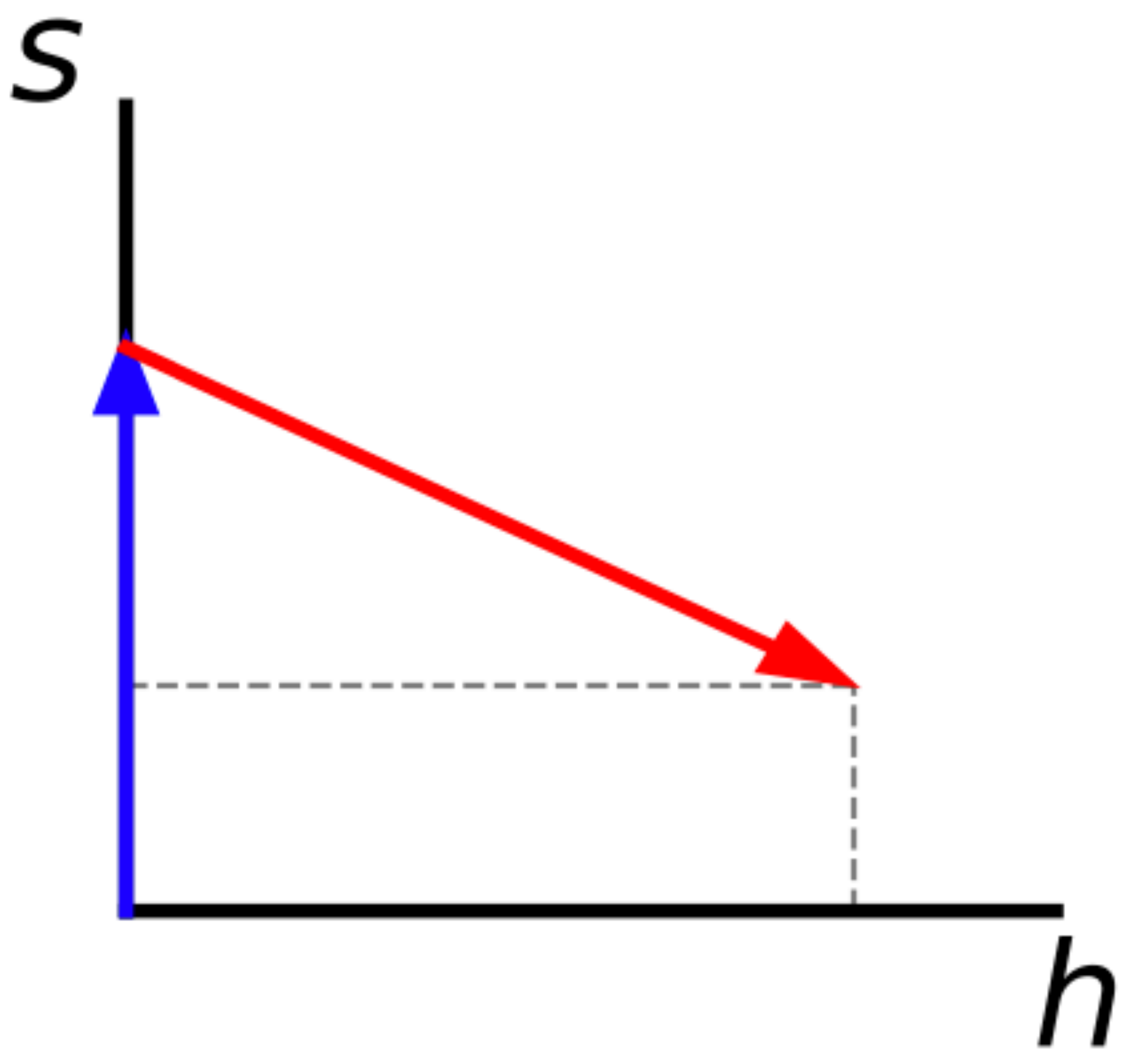
S can be DM candidate.
However, unless
$65\,\text{GeV} \lesssim M_S \lesssim 200\,\text{GeV}$,
this DM candidate has been excluded by direct detections already!
(Ghorbani, 2010.15708 [hep-ph])



(a)

Spontaneous Z_2 symmetry breaking

Requires $M_S \lesssim 50$ GeV



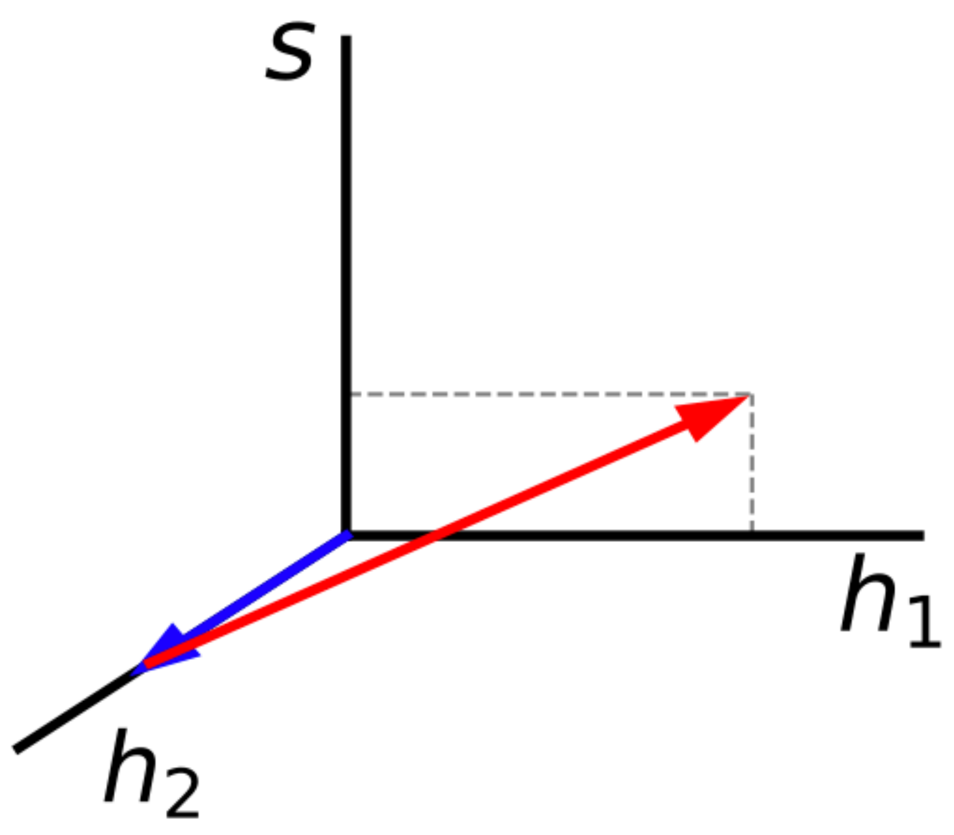
(b)

GW signals too weak to be detected at future detectors.

(Carena, Liu, Wang, 1911.10206 [hep-ph])

For SM+complex singlet, see Chiang, Ramsey-Musolf, Senaha, Phys. Rev. D 97, 015005

Effective Potential (Tree level)



$$\hat{\phi}(T) = \{h_{1c}(T), h_{2c}(T), \phi_{Hc}(T) \equiv S\}$$

$$H_1 = \begin{pmatrix} h_1^+ \\ H_1^0 = \frac{h_{1c}+h_{\rm SM}}{\sqrt{2}} + i\frac{G_1^0}{\sqrt{2}} \end{pmatrix}, \quad H_2 = \begin{pmatrix} h_2^+ \\ H_2^0 = \frac{h_{2c}+h_2^0}{\sqrt{2}} + i\frac{G_2^0}{\sqrt{2}} \end{pmatrix},$$

$$\Phi_H = \begin{pmatrix} G_H^p = \frac{G_H^1+iG_H^2}{\sqrt{2}} \\ \Phi_H^0 = \frac{\phi_{Hc}+\phi_H}{\sqrt{2}} + i\frac{G_H^0}{\sqrt{2}} \end{pmatrix},$$

$T \neq 0$

$$V_0(h_{1c}, h_{2c}, \phi_{Hc}) = \frac{1}{4}\Big[-2\mu_H^2\left(h_{1c}^2+h_{2c}^2\right) + \lambda_H\left(h_{1c}^2+h_{2c}^2\right)^2 - 2\mu_{\Phi_H}^2\phi_{Hc}^2 + \lambda_\Phi\phi_{Hc}^4 + \lambda_{H\Phi}\left(h_{1c}^2+h_{2c}^2\right)\phi_{Hc}^2 + \lambda'_{H\Phi}h_{2c}^2\phi_{Hc}^2\Big].$$

$T = 0$ $\quad (h_{1c} = v_1 = v, h_{2c} = v_2 = 0, \phi_{Hc} = v_\Phi)$

$$V_0(v,0,v_\Phi) = \frac{1}{4}\left[-2\mu_H^2 v^2 + \lambda_H v^4 - 2\mu_{\Phi_H}^2 v_\Phi^2 + \lambda_\Phi v_\Phi^4 + \lambda_{H\Phi} v^2 v_\Phi^2\right]$$

Provides tree level barrier. Larger $\lambda_{H\Phi}$ is preferred!

Pitfalls of 1-loop Effective Potential $V_{\rm eff}(\phi_{\rm cl}) = -\Gamma_{\rm 1PI}(\phi_{\rm cl})/{\rm Vol}$

- Gauge dependence - Exact effective potential is gauge invariant only at the extrema while the locations of extrema are gauge dependent. Nielsen-Fukuda-Kugo identity.
  $\Longrightarrow$ $\hbar$ expansion (Patel and Ramsey-Musolf = PRM), …
- Scale dependence - Explicit μ^2 dependence in the one-loop Coleman-Weinberg potential.
  $\Longrightarrow$ RG improved effective potential, …
- IR divergencies - Zero Matsubara modes of bosonic d.o.f. at high temperature enhances coupling $g^2 \to g^2 \frac{T}{m}$, hence perturbation expansion breaks down for $T \gg m$. [Weinberg, Jackiw & Dolan, …] [Linde, …]
  $\Longrightarrow$ Daisy resummation, …, 3d EFT, lattice simulations.

Take a simple approach: $\longrightarrow$ High-T Approximation (LO)

$$V_{\rm eff}^{\rm HT}(h_{1c}, h_{2c}, \phi_{Hc}, T) = V_0(h_{1c}, h_{2c}, \phi_{Hc}) + \frac{1}{2}\Pi_{H_1}(T)h_{1c}^2 + \frac{1}{2}\Pi_{H_2}(T)h_{2c}^2 + \frac{1}{2}\Pi_{\Phi_H}(T)\phi_{Hc}^2 ,$$

Tree level potential barrier

where $\Pi_{H_1,H_2,\Phi_H}(T)$ are *gauge invariant* thermal masses .

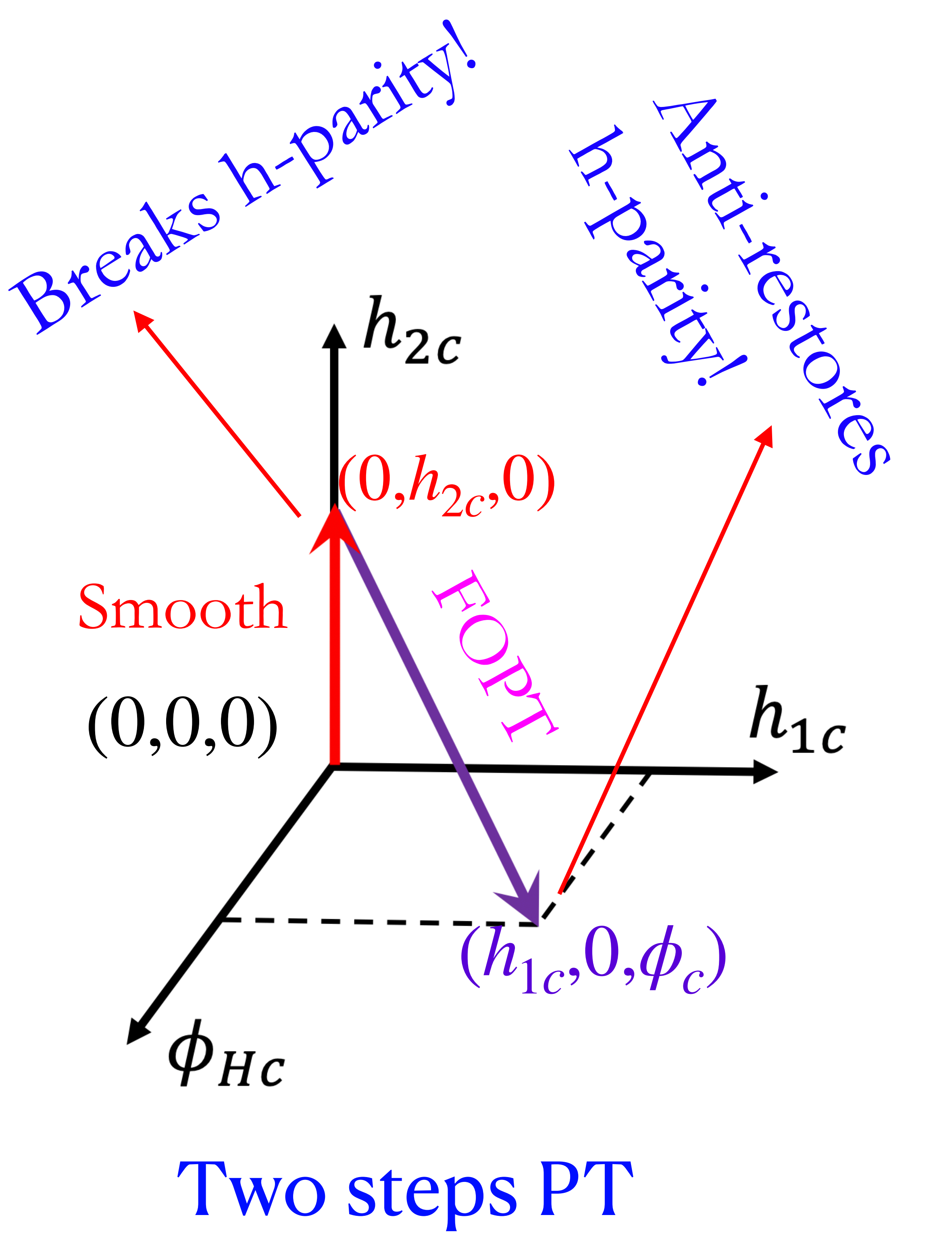
⋆ More rigorous approach is to use $\hbar$ expansion, 3d EFT, including higher loop corrections, non-perturbative methods, …

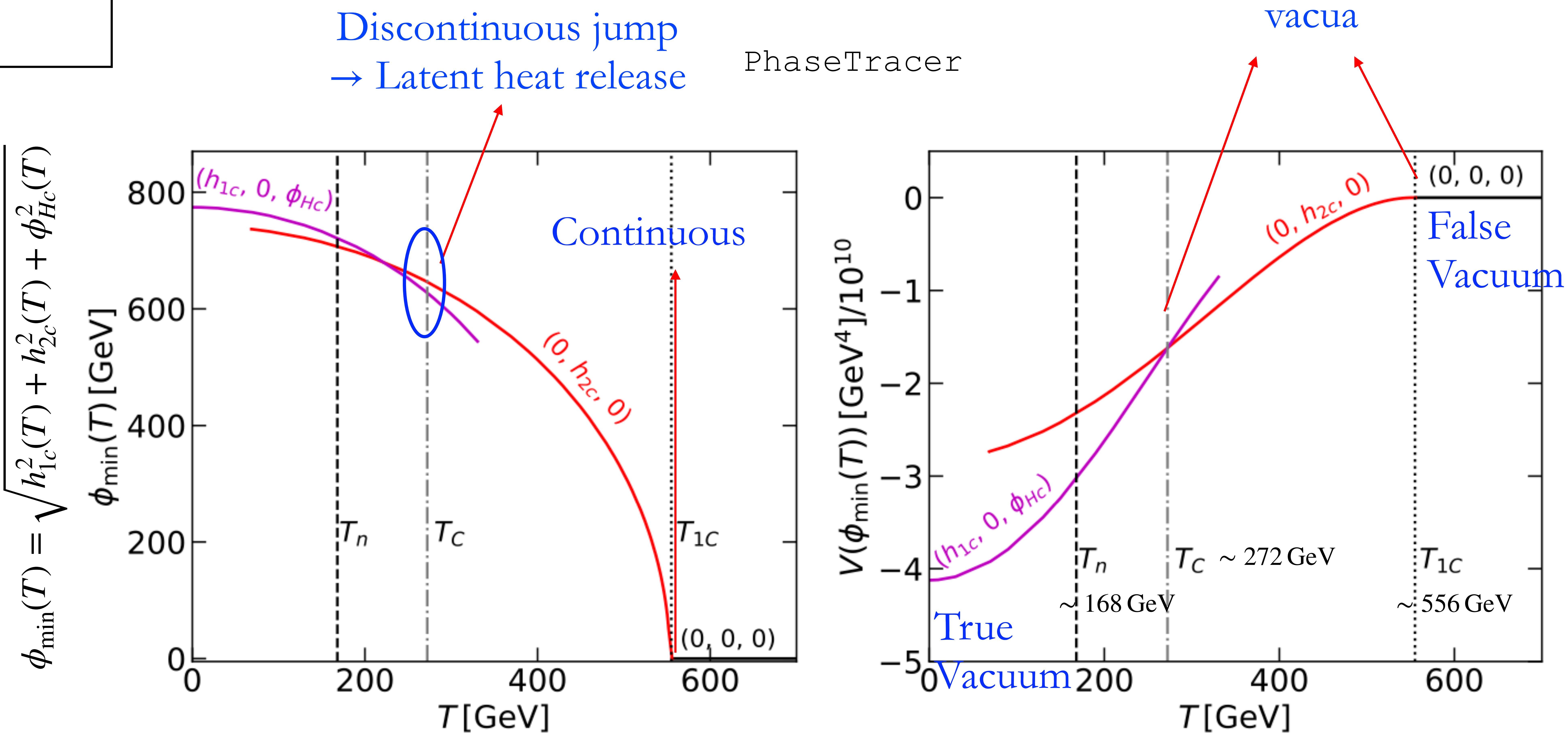
General Questions Posted

- Can upper limit on new scalar boson mass due to 2-step FOEWPT requirement be relaxed?
  — SM with 125 GeV Higgs boson provides only smooth crossover PT.
  — SM+Real Singlet S with Z_2 symmetry that supports a 2-step FOEWPT requires $m_S \lesssim 700\,\mathrm{GeV}$.
- Can the model yield a realistic DM candidate satisfying relic abundance and direct detection constraints?
- Can GW signals generated by FOEWPT detectable at next generation of GW detectors? And how does it interplay with DM direct detection signals?

Strong 2-Step FOEWPT in Minimal G2HDM (@ 1 bench mark point BM ★)

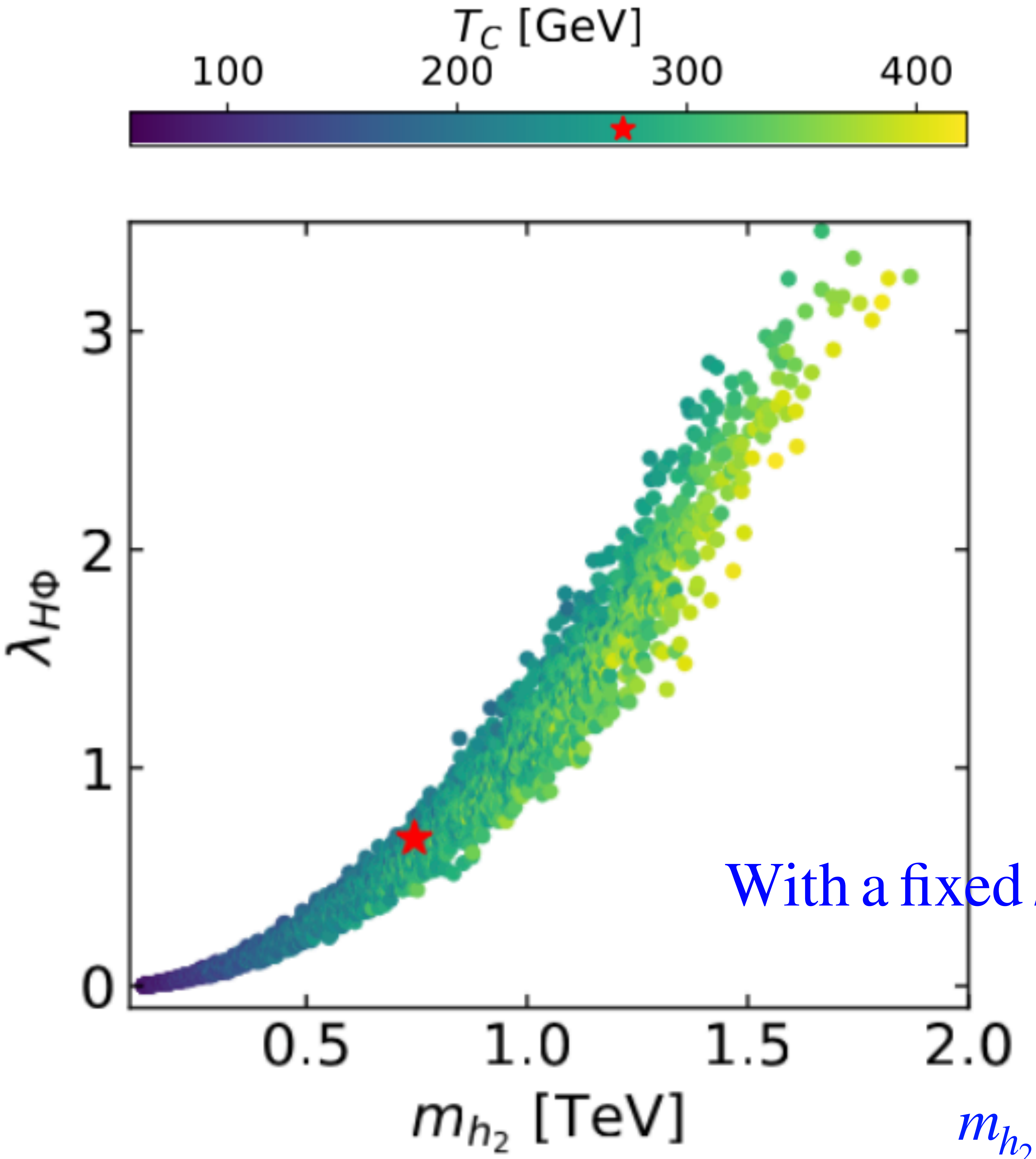
1-step FOPT (0,0,0) → (v,0,v_Φ) is impossible due to conflicting requirements of positivity and minimization conditions at zero temperature. (Ghorbani, 2010.15708)





BM ★ : $\{m_{(h_2,H^\pm,m_D,m_{W'},m_X,m_{fH})}=(745,374,320,0.115,0.25,10^3)\,\text{GeV};\ (\theta_1,\theta_2)=(0.235,0.32)\,\text{rad};\ g_X=1.17\times10^{-4}\}$

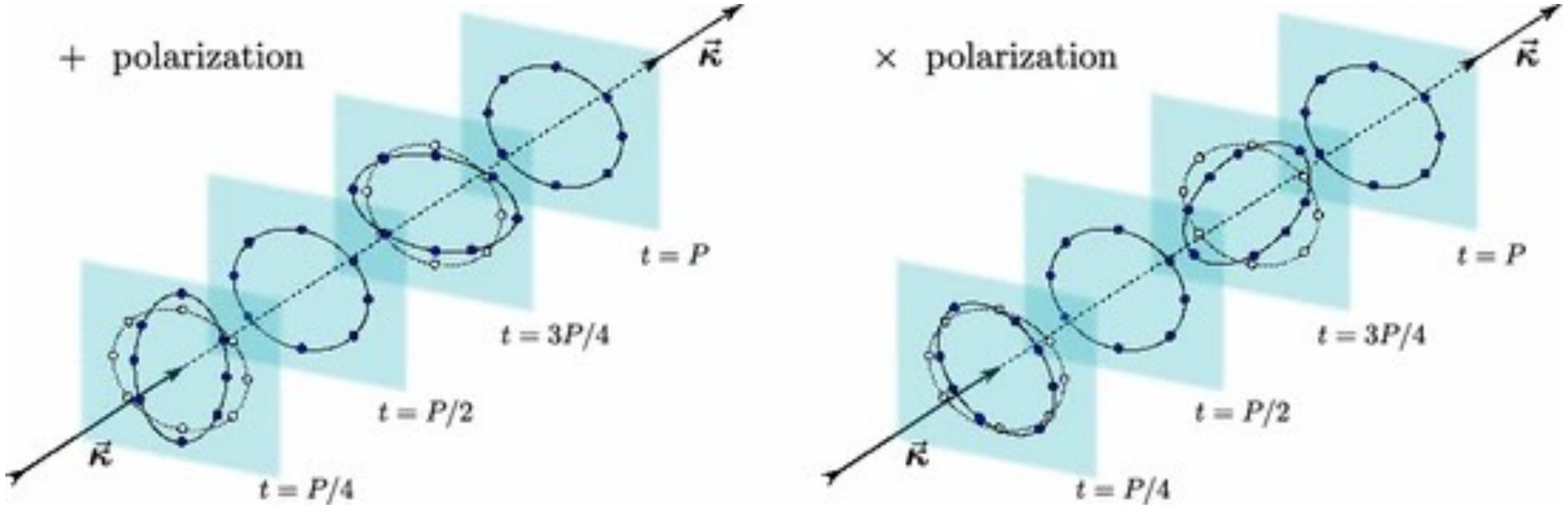
2-Step FOPT Viable Parameter Space (2σ)



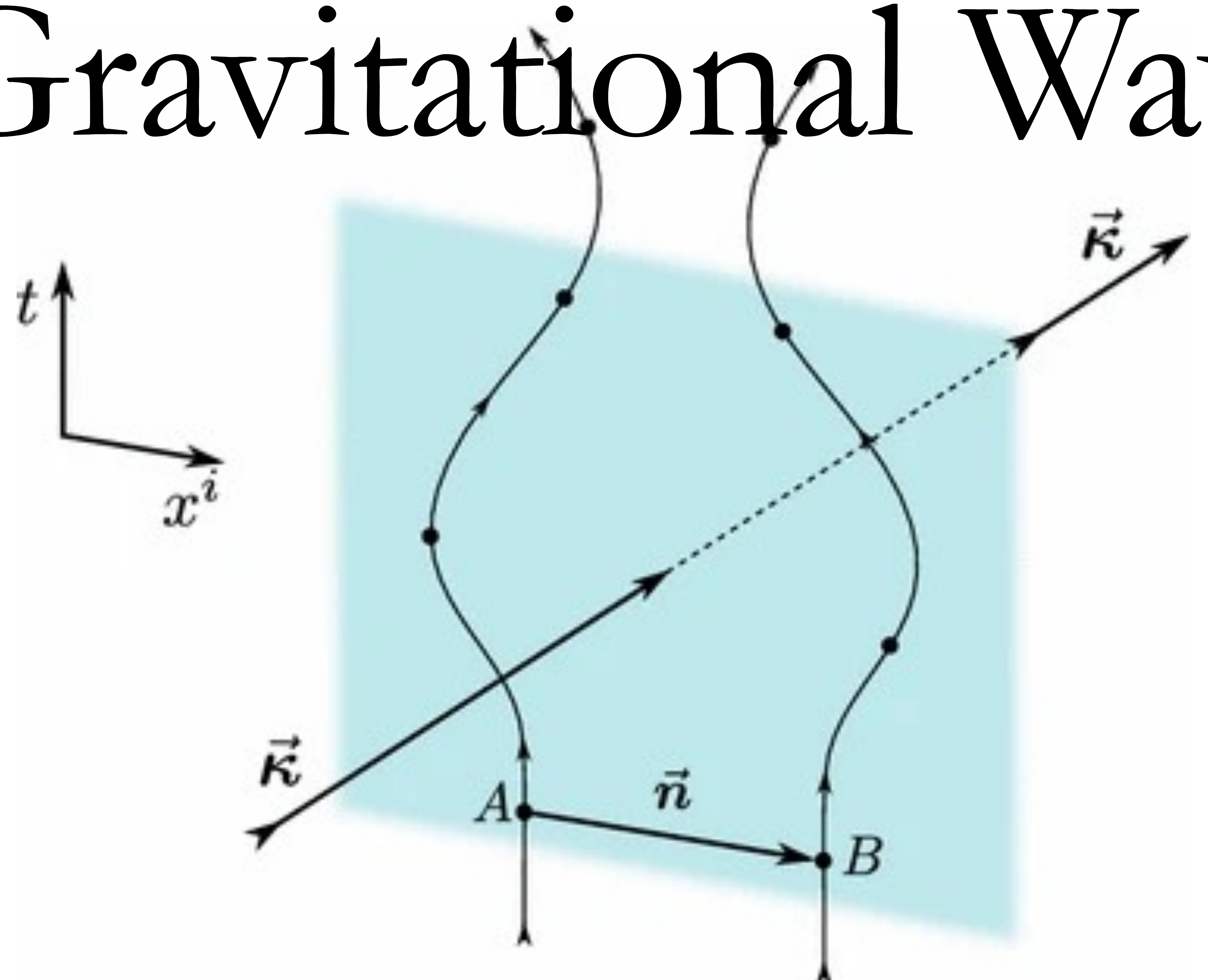
PhaseTracer

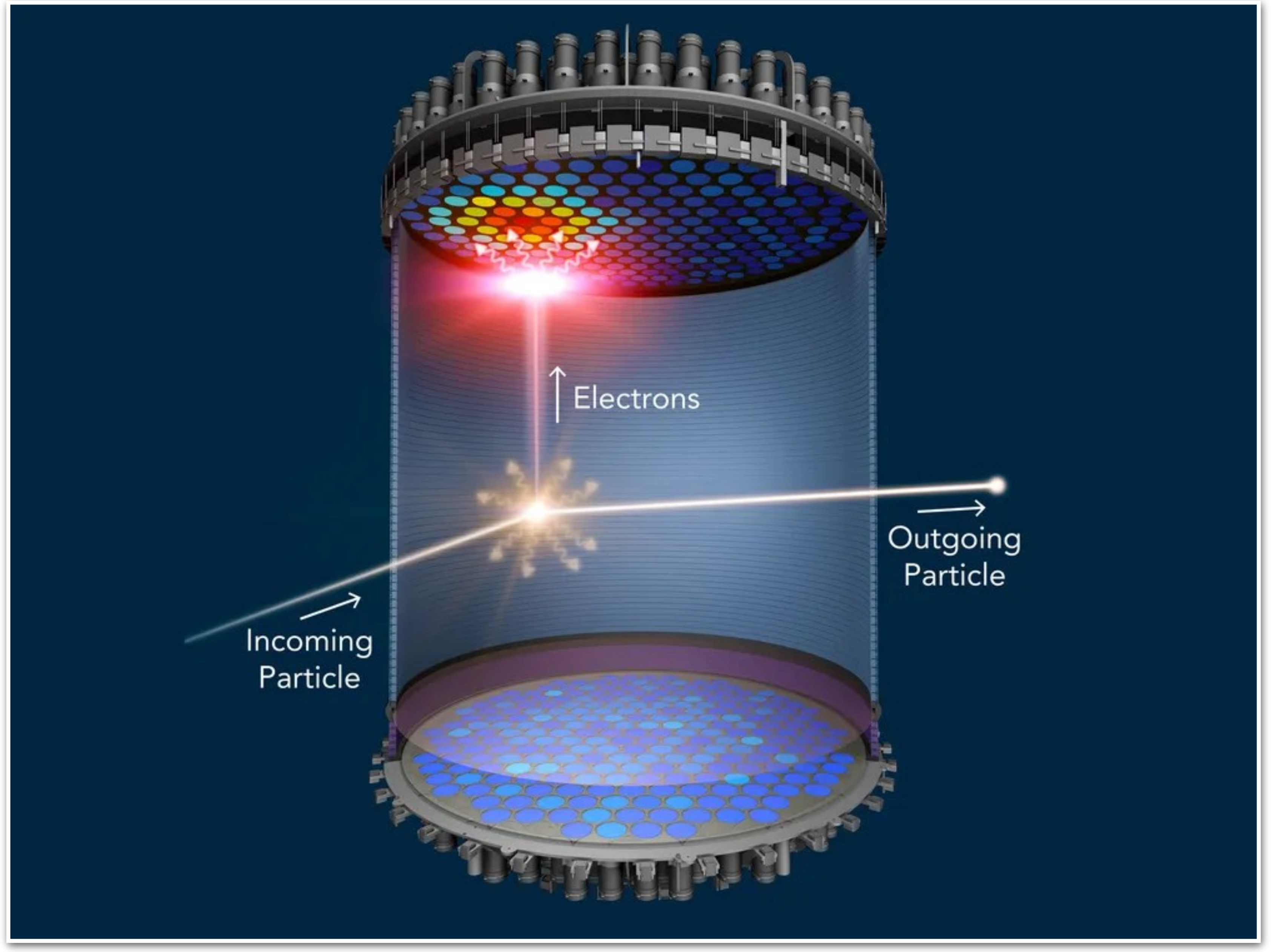
With a fixed m_{h_2}, $\lambda_{H\Phi} \uparrow \Leftrightarrow T_C \downarrow$

m_{h_2} can be at TeV scale!



Gravitational Wave and Dark Matter





GW from Strong FOPT

FOPTs proceed through bubble nucleations and release of latent energy.

Three Mechanisms: $h^2\Omega_{\rm GW} \simeq h^2\Omega_\phi + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$

- Bubble Collisions - Breaks spherical symmetry to evade the shell theorem in classical dynamics. Dominated if $v_w \sim 0.99$ for runaway solutions, i.e. no terminal speed.

- Sound Waves - Plasma waves surrounding the walls accelerated by the bubble wall are propagated along with the bubble wall. This can lead to bulb motions and create GWs prior to bubble wall collisions; create both subsonic deflagrations 爆燃 ($v_w < c_{s,-}$), supersonic detonation 爆炸 ($v_w > c_{\rm CJ}$) and hybrid ($c_{s,-} < v_w < v_{\rm CJ}$).

$$v_{\rm CJ} = \frac{1+\sqrt{\alpha_+(2+3\alpha_+)}}{(1+\alpha_+)\sqrt{3}}$$

- Turbulences - May be developed after bubble collisions. Massive energy release at particular length scales with large Reynolds number ($\sim 10^{13}$). Characterized by irregular eddy motions, and typically modeled by classical Kolmogorov's theory (K41).

Simulations for a non-linear system of relativistic hydrodynamics couples to a scalar field for the bubble wall in a linearized gravity with a cosmological background.

Many nice reviews: E.g.
(1) Allen, arXiv:gr-qc/9604033v3
(2) Croon, TASI 2022, https://pos.sissa.it/439/003/pdf
(3) Athron, Balazs, Fowlie, Morris and Wu, https://arxiv.org/pdf/2305.02357

FOPT and Thermal Parameters

Semi-classical method captures both quantum tunneling through the barrier and temperature fluctuations over the barrier.

3-d Euclidean action (Bounce):

$$\mathcal{S}_3 = \int_0^\infty \mathrm{d}r r^2 \left[\frac{1}{2}\left(\frac{\mathrm{d}\hat{\phi}(r,T)}{\mathrm{d}r}\right)^2 + V^{\mathrm{eff}}(\hat{\phi},T)\right]$$

$$\hat{\phi}(r,T) = \{h_{1c}(r,T), h_{2c}(r,T), \phi_{Hc}(r,T)\}$$

Semi-classical EOM:

$$\frac{\mathrm{d}^2\hat{\phi}}{\mathrm{d}r^2} + \frac{2}{r}\frac{\mathrm{d}\hat{\phi}}{\mathrm{d}r} = \frac{\mathrm{d}V^{\mathrm{eff}}(\hat{\phi},T)}{\mathrm{d}r}$$

$$\left.\frac{\mathrm{d}\hat{\phi}(r)}{\mathrm{d}r}\right|_{r=0} = 0\,, \qquad \lim_{r\to\infty}\hat{\phi}(r) = 0\,.$$

CosmosTransitions
FindBounce

Tunneling rate: (Saddle point method)

$$\Gamma(T) \simeq T^4 \left(\frac{\mathcal{S}_3}{2\pi T}\right)^{3/2} \exp(-\mathcal{S}_3/T)$$

Nucleation temperature T_n:

$$\int_{T_n}^\infty \frac{\mathrm{d}T}{T}\frac{\Gamma(T)}{H(T)^4} \simeq 1$$

v_w = Bubble wall velocity

Solve the bubble nucleation condition

$\mathcal{S}_3(T_n)/T_n \backsimeq 140$

$$\alpha = \frac{\rho_{\mathrm{vac}}}{\rho^*_{\mathrm{rad}}} = \frac{1}{\rho^*_{\mathrm{rad}}}\left[\frac{1}{4}T\frac{\partial}{\partial T}\Delta V(T) - \Delta V(T)\right]\Bigg|_{T_*} \quad \text{(Bag Model)}\,, \quad \rho^*_{\mathrm{rad}} = g_*\frac{\pi^2 T_*^4}{30}$$

Ratio of latent heat to total radiation at $T_*^{GW} \sim T_n$.

$$\beta/H_* = T_*\frac{d(\mathcal{S}_3/T)}{dT}\Bigg|_{T_*} = \text{Inverse duration}$$

$$\kappa_v = \frac{\rho_v}{\rho_{\mathrm{vac}}}\,,\quad \kappa_\phi = \frac{\rho_\phi}{\rho_{\mathrm{vac}}}\,,\ R_*\,, etc\,.$$

Thermal Parameters

Stochastic GW Spectrum from FOPT

GW: $ds^2 = -dt^2 + a(t)^2 g_{ij} dx^i dx^j \simeq -dt^2 + a(t)^2(\delta_{ij} + h_{ij}) dx^i dx^j$

$$h_{ij}(t,x) = \sum_{I=+,\times} \int_{-\infty}^{+\infty} df \int_{S^2} d^2\Omega_k \, e_{ij}^I \, h_I(f,k) \, \exp[2\pi i f(t - k \cdot x/c)], \quad (i,j = 1,2,3)$$

Strain power spectrum

- *Stochastic* - stationary in time, approximately Gaussian, and isotropic; most likely unpolarized.

$$\langle h_I^*(f,\hat{n}) h_{I'}(f',\hat{n}') \rangle = \delta_{AA'} \delta(f - f') \frac{\delta^2(\hat{n},\hat{n}')}{4\pi} \frac{1}{2} S_h(f)$$

$$\langle h_{ij} h^{ij} \rangle = 4 \int_0^\infty df \, S_h(f) = 2 \int_0^\infty d\log f \, h_c^2(f) \to S_h = \frac{h_c^2}{2f}$$

- Spectrum of GW, $\Omega_{\rm GW}(f)$, can be obtained by fittings to detailed simulations and analysis of the spectrum from each source:
(1) bubble wall collisions;
(2) sound waves in the plasma;
(3) long-term MHD turbulences in the plasma.

$$\rho_{\rm GW} = \frac{1}{32\pi G_N} \langle \dot{h}_{ij}^2(t,x) \rangle = \int_0^\infty d\log f \frac{d\rho_{\rm GW}}{d\log f} \equiv \rho_{\rm cr} \int_0^\infty d\log f \, \Omega_{\rm GW}(f)$$

$$h^2\Omega_{\rm GW} \simeq h^2\Omega_\phi + h^2\Omega_{\rm sw} + h^2\Omega_{\rm turb}$$

$$\Omega_{\rm GW}(f) \sim f^2 \cdot h_c(f) \sim f^3 S_h(f)$$

- Signal-to-noise (SNR) can then be computed for future space-based GW interferometers.

$$\text{SNR} = \sqrt{\mathcal{T} \int_{\rm min}^{\rm max} df \left[\frac{h^2\Omega_{\rm GW}(f)}{h^2\Omega_{\rm exp}(f)} \right]^2}$$

Commonly accepted value of SNR is 10

$\mathcal{T}$ is the duration of the mission × duty cycle

GW Power Spectrum and Thermodynamics Variables

- Assuming *sound wave* spectrum dominates, an *ansatz* of GW spectrum consistent with simulations is

Empirical
$$h^2\Omega_{\rm sw}(f) = 1.19\times 10^{-6}\left(\frac{100}{g_*}\right)^{1/3}\kappa_f(\alpha, v_w)^2\frac{\alpha^2}{(1+\alpha)^2}\left(\frac{H_*}{\beta}\right)\left(1-\frac{1}{\sqrt{1+2H_*t_{\rm sw}}}\right)S_{\rm sw}(f)$$

Fluid efficiency
$$\kappa_f(\alpha, v_w) \simeq \begin{cases} \dfrac{\alpha}{0.73+0.083\sqrt{\alpha}+\alpha}, & v_{\rm w}\sim 1, \\ \dfrac{v_{\rm w}^{6/5}6.9\alpha}{1.36-0.037\sqrt{\alpha}+\alpha}, & v_{\rm w}\lesssim 0.1. \end{cases}$$

$$S_{\rm sw} = \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3\left(\frac{f}{f_{\rm sw}}\right)^2}\right]^{7/2}$$
Spectra shape

$$h^2\Omega_{\rm exp} = \frac{4\pi^2}{3H_0^2}f^3 S_{\rm noise}(f)$$

$$f_{\rm sw} = 8.9\,\mu{\rm Hz}\,\frac{1}{v_{\rm w}}\left(\frac{\beta}{H_*}\right)\left(\frac{z_{\rm p}}{10}\right)\left(\frac{T_*}{100\,{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6},$$
Approx. peak freq.

$z_{\rm p}$ is a simulation-derived factor

$$H_*t_{\rm sw} = 2(8\pi)^{1/3}\left(\frac{H_*}{\beta}\right)\left(\frac{1+\alpha}{3\alpha\kappa_f}\right)^{1/2}$$
Time scale

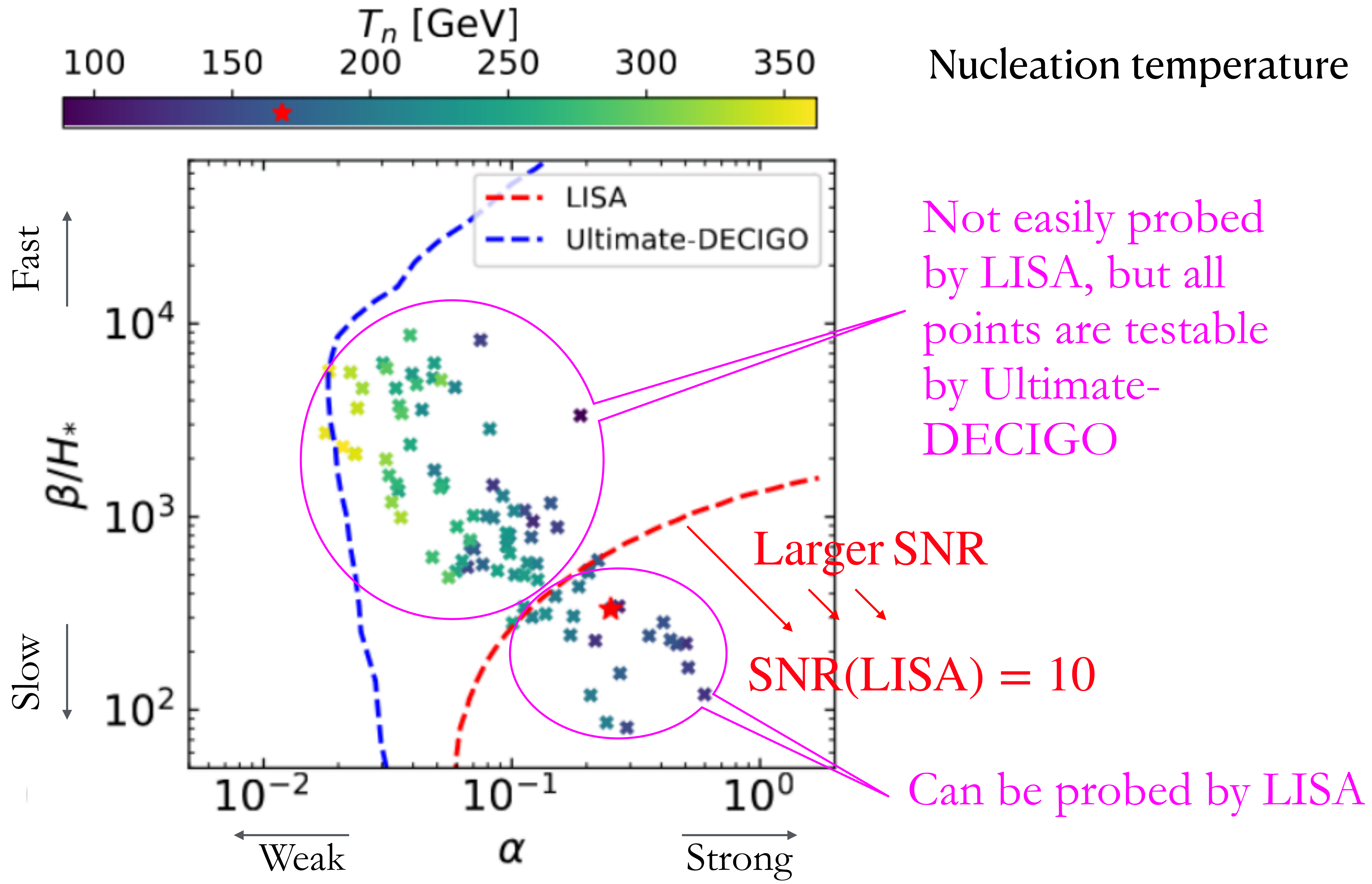
Hindmarsh, Huber, Rummukanien, and Weir, PRD96 (2017) 103520

Weir, Phil. Trans. Roy. Soc. Long. A376 (2018) 20170126

Caprini *et al.*, J. Cosmo. Astropart. Phys. 04 (2016) 001; 03 (2020) 024

Nucleation Viable Parameter Space (2σ)

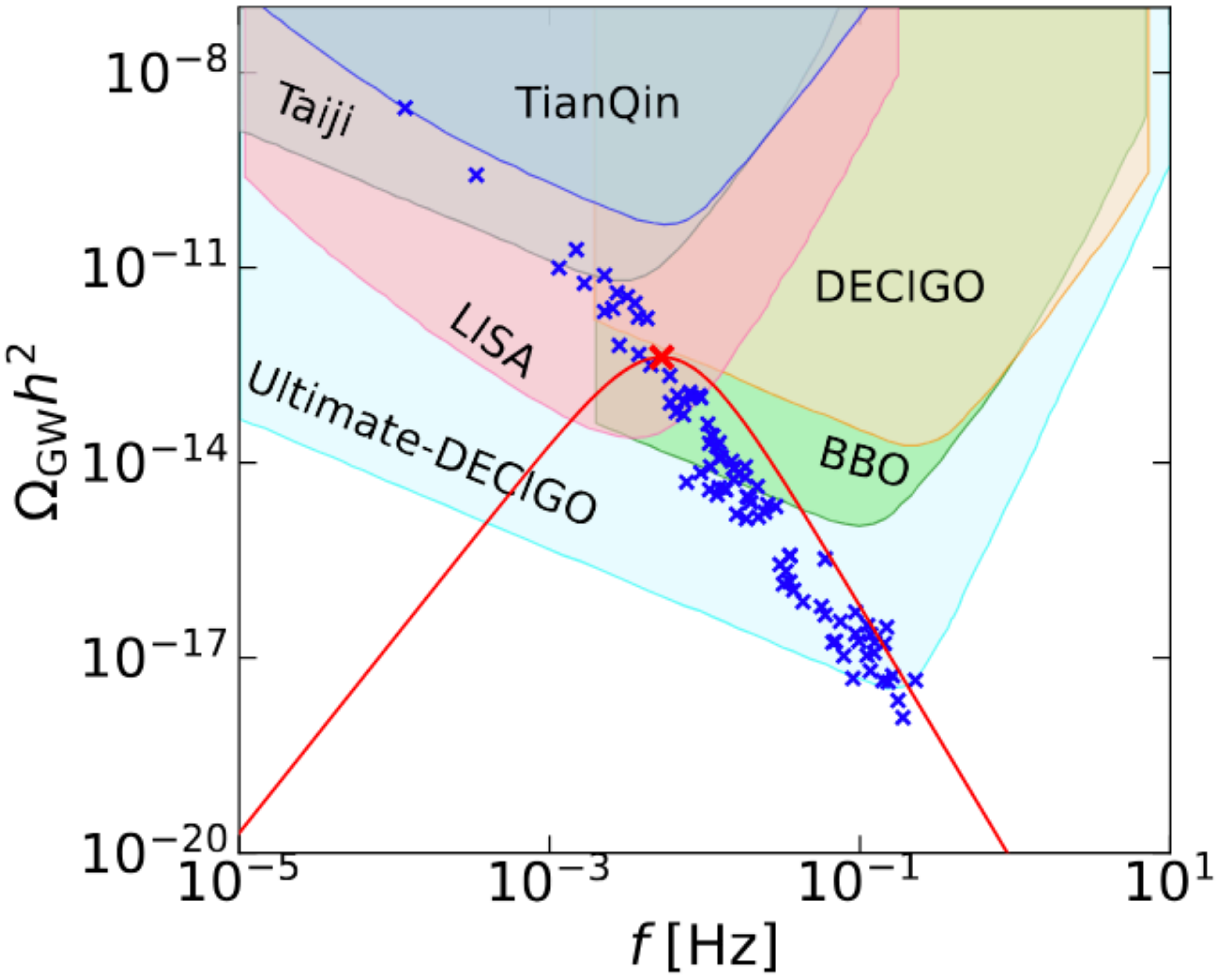
For the benchmark point ★ : $\{T_n, \alpha, \beta/H_n, \text{SNR}; v_w\} = \{168\ \text{GeV}, 0.43, 332, 144; 0.95\}$



CosmoTransitions + PTPlot packages

Stochastic GW Signal Spectrum

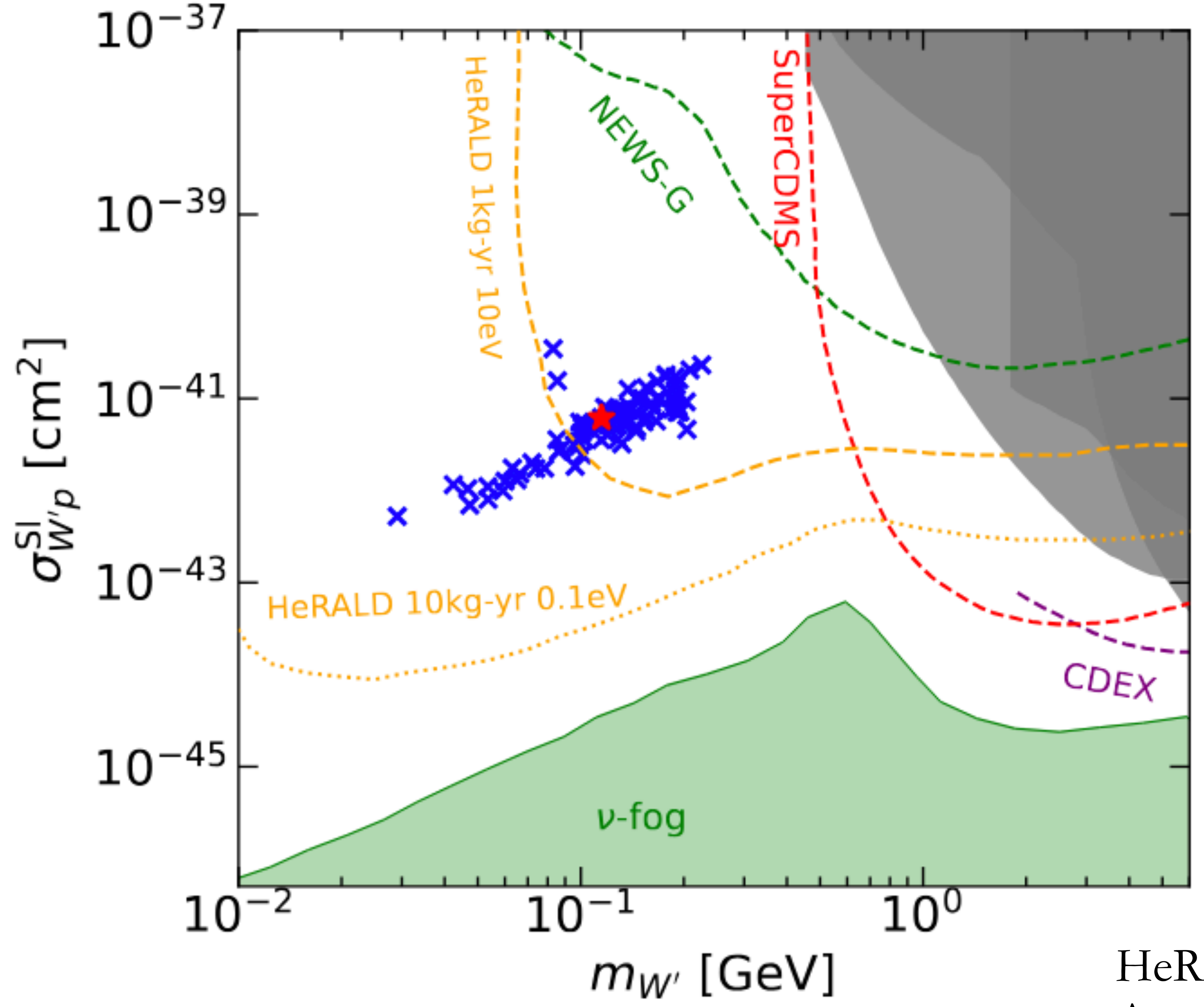
$\propto$ strain sensitivity



× represent peaks of signal spectra of other BM points

PTPlot package → GW spectrum and SNR

DM Direct Detection



micrOMEGAs package → $\Omega_{W'}h^2, \sigma^{SI}_{W'p}$

HeRALD : Helium Roton Apparatus for Light Dark matter
https://tesseract.lbl.gov/herald/

Conclusions & Outlook

- A 2-step PT $(0,0,0) \rightarrow (0,h_{2c},0) \rightarrow (h_{1c},0,\phi_{Hc})$ is possible.
  — Using the high-T approximation in the 1-loop effective potential that manifests gauge invariance, we found that the 1st step is 2nd PT, while the 2nd step is 1st order.
  — The 1st step spontaneously breaks h-parity, while the 2nd step *anti-restores* it to provide a dark matter candidate.

- Unlike SM+real singlet extension with Z_2 symmetry, masses of new scalar bosons can be relaxed to reach $\mathcal{O}(1\,\text{TeV})$.
  — $m_{h_2} \lesssim 1.8\,\text{TeV}$ (h-parity even) and $m_D, m_{H^\pm} \lesssim 500\,\text{GeV}$ (h-parity odd). Collider search.
  — In addition, $0.5\,\text{TeV} \lesssim v_\Phi \lesssim 1.4\,\text{TeV}$ and $\lambda_{H\Phi}$ has to be *sizable* for heavier h_2 to provide the tree-level potential barrier.

- Predicted GW power spectrum in the frequency range of $10^{-4} \sim 0.2$ hertz with peak yields at $10^{-18} \sim 10^{-9}$, which can be probed by next-generation GW detectors, including BBO, LISA, TianQin, Taiji and (Ultimate-)DECIGO.

- Interesting and importantly, parameter space probed by next generation GW detectors can also be searched for in the future sub-GeV dark matter direct detection experiments, in particular the superfluid-He target detectors.

- *Caveats*: Results are derived in the high-T approximation in the one-loop effective potential. Higher order corrections, 3-dim EFT plus lattice simulation are needed to address gauge-invariance, scale dependence, IR issue, boundary between smooth crossover versus FOEWPT, *etc* for more *accurate* theoretical FOPT as well as GW predictions and other phenomenological implications like *electroweak baryogengesis* and *collider physics* in the model.

Thank you!

Backup Slides

(-: LHC Inverse Problem for G2HDM :-)

- Determine fundamental parameters from LHC data

$$\lambda_H = \frac{1}{2v^2}\left(m_{h_1}^2 \cos^2\theta_1 + m_{h_2}^2 \sin^2\theta_1\right) \; ,$$

$$\lambda_\Phi = \frac{1}{2v_\Phi^2}\left(m_{h_1}^2 \sin^2\theta_1 + m_{h_2}^2 \cos^2\theta_1\right) \; ,$$

$$\lambda_{H\Phi} = \frac{1}{2vv_\Phi}\left(m_{h_2}^2 - m_{h_1}^2\right)\sin\left(2\theta_1\right) \; ,$$

$$\lambda'_{H\Phi} = \frac{2m_D^2}{v^2+v_\Phi^2} \; ,$$

$$\lambda'_H = \frac{2}{v^2}\left(\frac{m_D^2 v_\Phi^2}{v^2+v_\Phi^2} - m_{H^\pm}^2\right) \; .$$

Recall

$$\lambda_{\rm SM} = \frac{m_{h_{\rm SM}}^2}{2v^2}$$

$$g_H = \frac{2m_{W'}}{\sqrt{v^2+v_\Phi^2}} \; .$$

Parameter Scanning Ranges

$$
\begin{aligned}
m_{h_2}/\mathrm{GeV} &\in (130,\ 2000)\,, \\
m_{H^\pm}/\mathrm{GeV} &\in (80,\ 2000)\,, \\
m_D/\mathrm{GeV} &\in (10,\ 2000)\,, \\
m_{W'}/\mathrm{GeV} &\in (0.01,\ 50)\,, \\
\theta_{1,2}/\mathrm{rad} &\in \left(-\frac{\pi}{2},\ \frac{\pi}{2}\right)\,, \\
g_X &\in (10^{-6},\ 10^{-2})\,, \\
M_X/\mathrm{GeV} &\in (10^{-3},\ 10^{2})\,,
\end{aligned}
$$

and we fix $m_{f^H} = 1$ TeV.

Effective Potential (1-loop + Finite Temperature Correction)

(Landau gauge)

T>0

$$V_{\rm eff}(h_{1c},h_{2c},\phi_{Hc},T) = V_0(h_{1c},h_{2c},\phi_{Hc}) + V_1(h_{1c},h_{2c},\phi_{Hc},T)\ ,$$

$$V_1(h_{1c},h_{2c},\phi_{Hc},T) \;=\; V_{\rm CW}(h_{1c},h_{2c},\phi_{Hc}) + \Delta V_1(h_{1c},h_{2c},\phi_{Hc},T) + \Delta V_{\rm C.T.}(h_{1c},h_{2c},\phi_{Hc})\ .$$

Coleman-Weinberg:

$$V_{\rm CW}(h_{1c},h_{2c},\phi_{Hc}) = \frac{1}{64\pi^2}\sum_i (-1)^{2s_i} n_i m_i^4(h_{1c},h_{2c},\phi_{Hc}) \times \left\{\log\left(\frac{m_i^2(h_{1c},h_{2c},\phi_{Hc})}{\mu^2}\right) - C_i - C_{\rm UV}\right\}$$

$C_i = 3/2\ (S=0,1/2),\ 5/6\ (S=1)$

$\overline{\rm MS}$: $\quad C_{\rm UV} = \frac{2}{\epsilon} - \gamma_E + \log 4\pi\ ,$

Finite temperature correction: Arnold-Espinosa scheme

$$\Delta V_1(h_{1c},h_{2c},\phi_{Hc},T) = \frac{T^4}{2\pi^2}\sum_i (-1)^{2s_i} n_i \mathcal{J}_i\left(\frac{m_i(h_{1c},h_{2c},\phi_{Hc})}{T}\right)$$

$$\mathcal{J}_i\left(\frac{m_i}{T}\right) = \begin{cases} J_B(\frac{m_i}{T}) - \frac{\pi}{6}\left(\frac{\overline{m}_i^3}{T^3} - \frac{m_i^3}{T^3}\right), & i\in\{S_1,S_2,S_3,S_4,P_1,P_2,P_3,P_4,G^\pm,H^\pm, W_L^\pm, W_L'^2, Z_{1L},Z_{2L},Z_{3L},Z_{4L},\gamma_L\}\,; \\ J_B(\frac{m_i}{T}), & i\in\{W_T^\pm, W_T'^2, Z_{1T},Z_{2T},Z_{3T},Z_{4T},\gamma_T\}\,; \\ J_F(\frac{m_i}{T}), & i\in\{l,\nu,l^H,\nu^H,q,q^H\}. \end{cases}$$

Daisy

$$J_{B/F}(x) \equiv \int_0^\infty {\rm d}y\, y^2 \log\left[1 \mp \exp\left(-\sqrt{x^2+y^2}\right)\right]$$

Thermal Masses

$$\begin{aligned}
\Pi_{H_1}(T) &\equiv \Pi_{h_1}(T) = \Pi_{G_1^0}(T) = \Pi_{h_1^\pm}(T) = \tilde{\Pi}_{H_1} T^2 \\
&= \frac{1}{12}\left(10\lambda_H - \lambda'_H + 2\lambda_{H\Phi} + \lambda'_{H\Phi}\right) T^2 \qquad \text{(B1)} \\
&\quad + \frac{1}{16}\left(3g^2 + g'^2 + 3g_H^2 + g_X^2\right) T^2 + \frac{1}{4} y_t^2 T^2 ,
\end{aligned}$$

$$\begin{aligned}
\Pi_{H_2}(T) &\equiv \Pi_{h_2}(T) = \Pi_{G_2^0}(T) = \Pi_{h_2^\pm}(T) = \tilde{\Pi}_{H_2} T^2 \\
&= \frac{1}{12}\left(10\lambda_H - \lambda'_H + 2\lambda_{H\Phi} + \lambda'_{H\Phi}\right) T^2 \\
&\quad + \frac{1}{16}\left(3g^2 + g'^2 + 3g_H^2 + g_X^2\right) T^2 , \qquad \text{(B2)}
\end{aligned}$$

$$\begin{aligned}
\Pi_{\Phi_H}(T) &\equiv \Pi_{\phi_H}(T) = \Pi_{G_H^0}(T) = \Pi_{G_H^{(p,m)}}(T) = \tilde{\Pi}_{\Phi_H} T^2 \\
&= \frac{1}{6}\left(3\lambda_\Phi + 2\lambda_{H\Phi} + \lambda'_{H\Phi}\right) T^2 + \frac{1}{16}\left(3g_H^2 + g_X^2\right) T^2 + \frac{1}{4} y_t'^2 T^2 . \qquad \text{(B3)}
\end{aligned}$$

$$\begin{aligned}
\Pi_W(T) &\equiv \Pi_{W^i}(T) = 2g^2 T^2 , \quad (i = 1, 2, 3) \qquad \text{(B4)} \\
\Pi_B(T) &= \frac{14}{3} g'^2 T^2 . \qquad \text{(B5)}
\end{aligned}$$

$$\begin{aligned}
\Pi_{W'}(T) &\equiv \Pi_{W'^i}(T) = \frac{19}{6} g_H^2 T^2 , \quad (i = 1, 2, 3) \qquad \text{(B6)} \\
\Pi_X(T) &= \frac{17}{2} g_X^2 T^2 . \qquad \text{(B7)}
\end{aligned}$$

Critical Temperatures at High-T Approximation

$$\left.\frac{\partial V_{\rm eff}^{\rm HT}}{\partial \hat{\phi}(T)}\right|_{\rm min} = 0 \implies \{h_{1c}(T), h_{2c}(T), \phi_{Hc}(T)\}_{\rm min}$$

First Step $(0,0,0) \to (0,h_{2c},0)$: (Vacuum degenerate condition)

$$V_{\rm eff}^{\rm HT}(0,0,0,T_{1C}) = V_{\rm eff}^{\rm HT}(0,h_{2c},0,T_{1C}) \quad \Rightarrow \quad T_{1C} = \sqrt{\mu_H^2/\tilde{\Pi}_{H_2}}$$

where $\mu_H^2 = \lambda_H v^2 + \lambda_{H\Phi} v_\Phi^2/2 \quad \tilde{\Pi}_{H_2} = \frac{1}{12}(10\lambda_H - \lambda'_H + 2\lambda_{H\Phi} + \lambda'_{H\Phi}) + \frac{1}{16}(3g^2 + g'^2 + 3g_H^2 + g_X^2)$

Second Step $(0,h_{2c},0) \to (h_{1c},0,\phi_{Hc})$: (Vacuum degenerate condition)

$$V_{\rm eff}^{\rm HT}(0,h_{2c},0,T_C) = V_{\rm eff}^{\rm HT}(h_{1c},0,\phi_{Hc},T_C) \quad \Rightarrow \quad T_C = \sqrt{\frac{(\lambda_{H\Phi}^2 - 4\lambda_H\lambda_\Phi)\, v_\Phi^4}{2\left(\lambda_{H\Phi}\tilde{\Pi}_{H_2} - 2\lambda_H\tilde{\Pi}_{\Phi_H}\right) v_\Phi^2 - \lambda_H y_t^2 v^2 - \kappa}}\,,$$

where

$$\begin{aligned}\kappa &= y_t \lambda_H^{1/2}\Big\{\left[4\lambda_{H\Phi}\tilde{\Pi}_{\Phi_H} - \lambda_\Phi(8\tilde{\Pi}_{H_2} + y_t^2)\right] v_\Phi^4 \\ &\quad + 4\left(2\lambda_H\tilde{\Pi}_{\Phi_H} - \lambda_{H\Phi}\tilde{\Pi}_{H_2}\right) v^2 v_\Phi^2 + \lambda_H y_t^2 v^4\Big\}^{1/2}\,, \\ \tilde{\Pi}_{\Phi_H} &= \frac{1}{6}(3\lambda_\Phi + 2\lambda_{H\Phi} + \lambda'_{H\Phi}) + \frac{1}{16}(3g_H^2 + g_X^2) + \frac{1}{4}y_t'^2\,.\end{aligned}$$

Nielsen-Fukuda-Kugo (NFK) Identity - I

NFK Identity:
NPB 101, 173 (1975);
PRD 13, 3469 (1976).

Zero Temperature!

$$\frac{\partial V_{\rm eff}(\varphi)}{\partial \xi} = -C(\varphi,\xi)\frac{\partial V_{\rm eff}(\varphi)}{\partial \varphi}\,, \qquad \text{(Exact)} \qquad \text{(D1)}$$

Patel & Ramsey-Musolf,
JHEP 07, 029 (2011).

$$V_{\rm eff}(\varphi) = V_0(\varphi) + \hbar V_1(\varphi) + \hbar^2 V_2(\varphi) + \cdots\,, \qquad \text{(D2)}$$

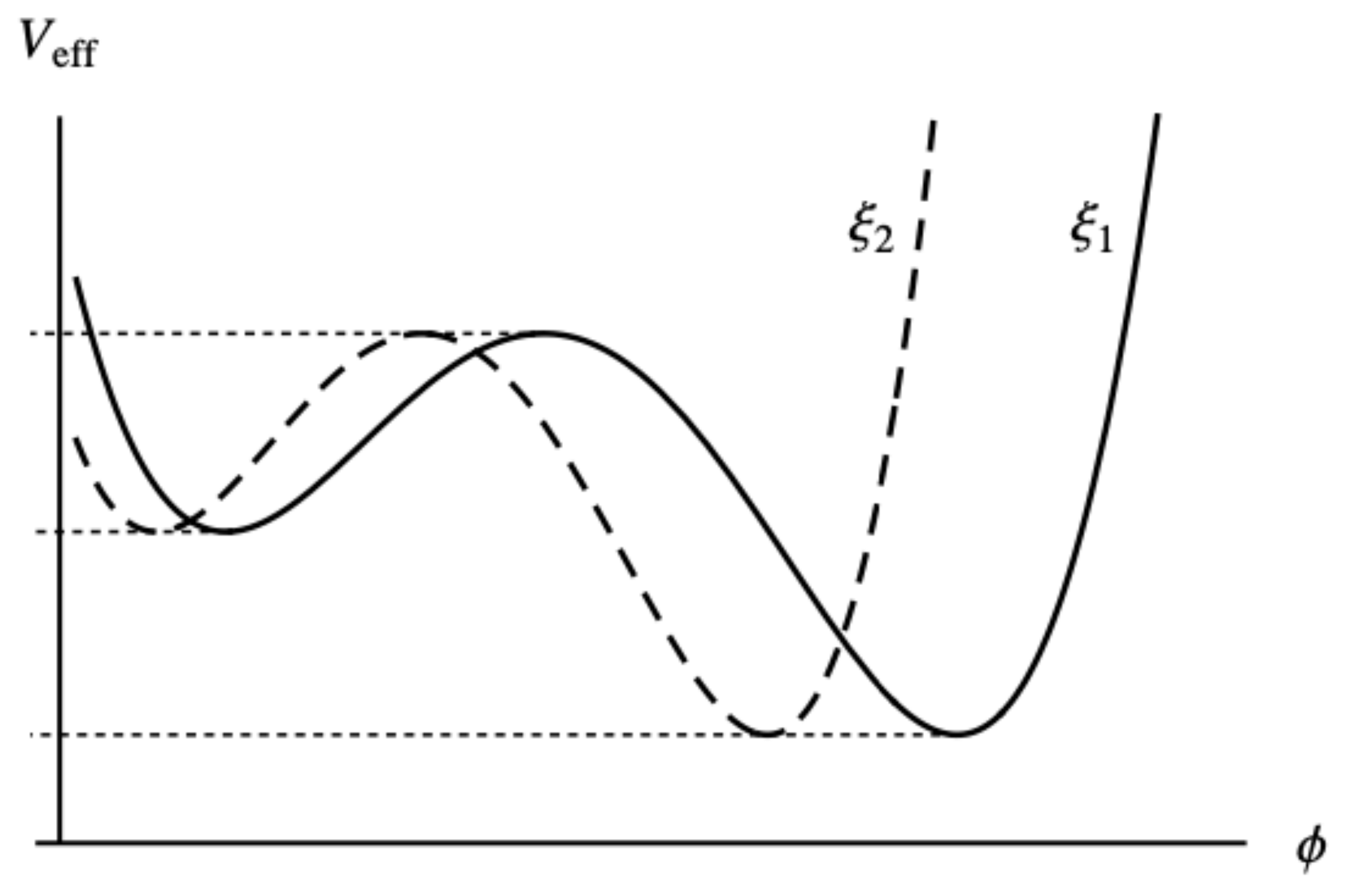
$$C(\varphi,\xi) = c_0 + \hbar c_1(\varphi) + \hbar^2 c_2(\varphi) + \cdots\,, \qquad \text{(D3)}$$

To zero order in $\hbar$: $c_0 = 0$

To first order in $\hbar$:

$$\frac{\partial V_1}{\partial \xi} = -\,c_1 \frac{\partial V_0}{\partial \varphi}$$

★ One-loop potential is gauge-independent only where the tree-level potential is extremized, not where the one-loop potential is extremized.



★ $\phi_{\rm min}$ is not gauge-invariant! So is the critical temperature T_C.

Nielsen-Fukuda-Kugo (NFK) Identity - II

- $\hbar$ Expansion: Patel & Ramsey-Musolf, JHEP 07, 029 (2011) [PRM]

At $T \neq 0$,
our goal is to minimize $\left.\frac{\partial V_{\rm eff}(\phi, T)}{\partial \phi}\right|_{\phi_{\rm min}} = 0.$

$$V_{\rm eff}(\phi, T) = V_0(\phi) + \hbar V_1(\phi, T) + \hbar^2 V_2(\phi, T) + \cdots$$
$$\phi_{\rm min} = \phi_0 + \hbar\phi_1(T,\xi) + \hbar^2\phi_2(T,\xi) + \cdots$$

Upon substitution into minimization condition,

$$\mathcal{O}(\hbar^0): \qquad 0 = \left.\frac{\partial V_0}{\partial \phi}\right|_{\phi_0}$$
$$\mathcal{O}(\hbar^1): \quad \phi_1(T,\xi) = -\left(\left.\frac{\partial^2 V_0}{\partial \phi^2}\right|_{\phi_0}\right)^{-1} \left.\frac{\partial V_1(T,\xi)}{\partial \phi}\right|_{\phi_0}$$

ϕ_0 is any one of the minima of the tree-level effective potential

Substitute back in $V_{\rm eff}$ and expand again, we have

$$V_{\rm eff}(\phi_{\rm min}(T), T) = V_0(\phi_0) + \hbar V_1(\phi_0, T) + \hbar^2\left[V_2(\phi_0, T, \xi) - \tfrac{1}{2}\phi_1^2(T,\xi)\frac{\partial^2 V_0}{\partial \phi^2}|_{\phi_0}\right] + \mathcal{O}(\hbar^3)$$

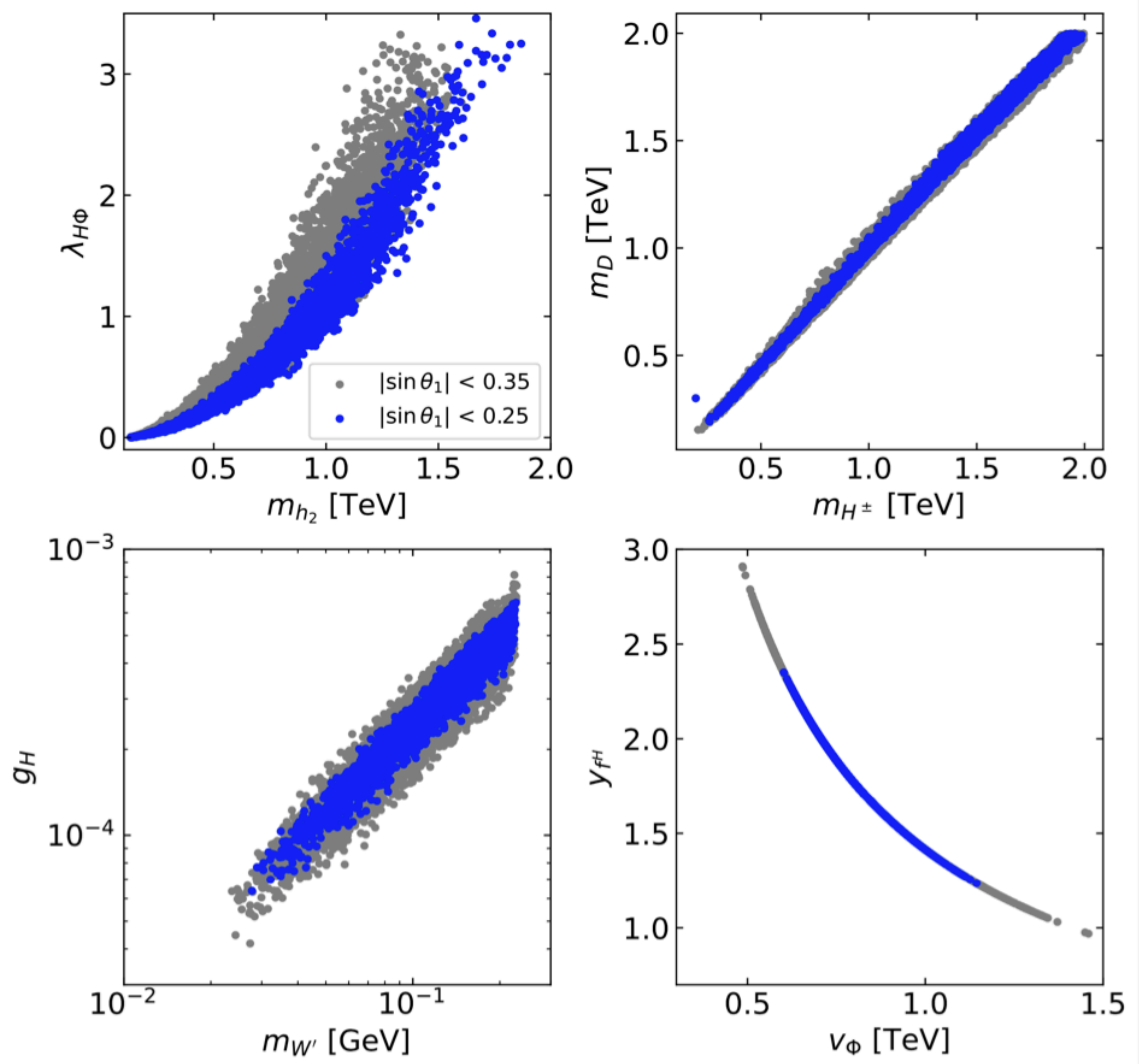
At each order in $\hbar$, this effective potential is gauge independent in accordance with Nielsen's identity, expecting these identities still hold at finite T. [PRM]

2-Step FOPT Viable Parameter Space (2σ)

Higgs Physics

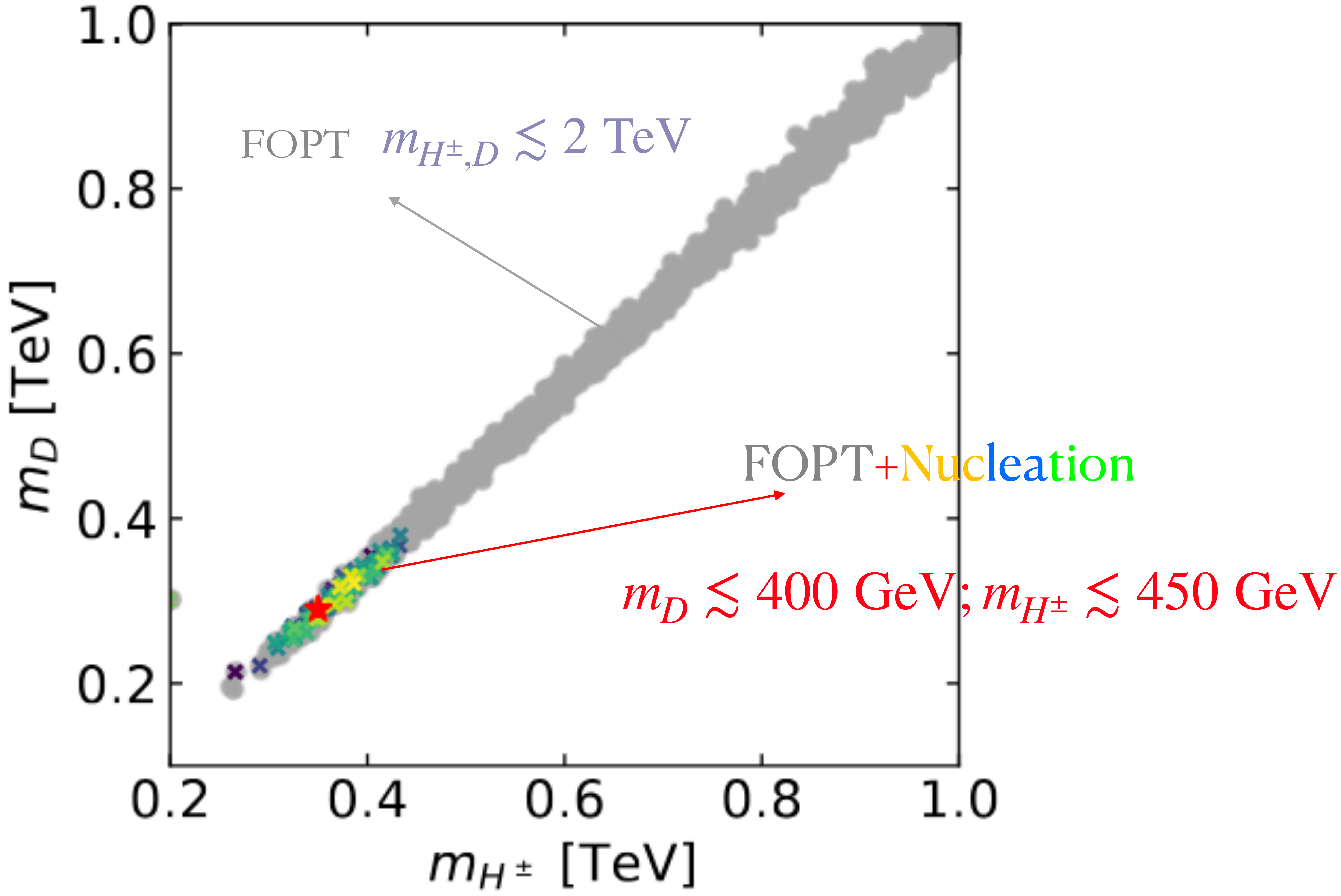
$|\sin\theta_1| \lesssim 0.35$ (CMS)

$|\sin\theta_1| \lesssim 0.25$ (CMS + ATLAS)



Nucleation Viable Parameter Space (2σ)

For the benchmark point ★ : $\{T_n, \alpha, \beta/H_n, \mathrm{SNR}; v_w\} = \{168\ \mathrm{GeV}, 0.43, 332, 144; 0.95\}$



`CosmoTransitions + PTPlot` packages