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Interplay between $(g-2)_{e_a}$ anomalies and lepton flavor violating decays in models beyond the Standard Model with inverse seesaw neutrinos

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Outline

- Motivation
- General formulas for Lepton flavor violating (LFV) partial decay widths and rates
- General Lagrangian, 1-loop Feynman diagrams, Feynman rules
- One-loop analytical formulas to LFV decay amplitudes in the unitary gauge
- Models beyond the standard model with inverse seesaw neutrinos
- Conclusions

Experimental constraints

- $(g-2)$ anomaly of a charged lepton $e_a = e, \mu, \tau$: $a_{e_a} \equiv \frac{(g-2)_{e_a}}{2}$.
  Deviations $\equiv$ experiment [PRL 131 (2023) 16, 161802] - the SM [Phys. Rept. 887, 1-166 (2020)]. For muon:

$$\Delta a_\mu^{\rm NP} \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM} = (2.49 \pm 0.48) \times 10^{-9} \text{(2023)[our work used]},$$
$$\Delta a_\mu^{\rm NP} = (1.13 \pm 0.35) \times 10^{-9} \text{[PDG 2024]},$$
$$\Delta a_\mu^{\rm NP} = (2.08 \pm 0.41) \times 10^{-9} \text{[PDG 2024]}$$

- For electron: $|\Delta a_e^{\rm NP}| \propto \mathcal{O}(10^{-13})$ [Science 360 (2018), 191; Phys. Rev. Lett. 130 (2023) no.7, 071801; Nature 588 (2020) no.7836, 61]
- LFV decays: cLFV: $e_b \to e_a\gamma$, LFVh: $h \to e_b e_a$, LFVZ: $Z \to e_b e_a$.
  Recent experimental contraints for LFV rates (Br):

$$\text{Br}(\tau \to \mu\gamma) < 4.2 \times 10^{-8},\ \text{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8},\ \text{Br}(\mu \to e\gamma) < 3.1 \times 10^{-13},$$
$$\text{Br}(h \to \tau\mu) < 1.5 \times 10^{-3},\ \text{Br}(h \to \tau e) < 2 \times 10^{-3},\ \text{Br}(h \to \mu e) < 4.4 \times 10^{-5},$$
$$\text{Br}(Z \to \tau\mu) < 6.5 \times 10^{-6},\ \text{Br}(Z \to \tau e) < 5.0 \times 10^{-6},\ \text{Br}(Z \to \mu e) < 2.62 \times 10^{-7}.$$

Our idea

- Neutrino oscillation $\rightarrow$ a real LFV source in the neutral lepton sector $\rightarrow$ a reason to hope that LFV decays relating to charged leptons (e, μ, τ) will appear.
- Our study: models beyond the SM (BSMs) with LFV sources from only (inverse seesaw and active) neutrino couplings are chosen to successfully explain the experimental data of $(g-2)_{e,\mu}$ anomalies and neutrino oscillation.
- Question: $(g-2)_{e,\mu}$ anomalies and LFV decay rates both come from loop contributions relating to the similar neutrino couplings to lepton and scalar/gauge bosons $\rightarrow$ large $\Delta a_\mu \propto 10^{-9}$ and $|\Delta a_e| \propto \mathcal{O}(10^{-13})$ result in large LFV decay rates: Will they be ruled out by experimental constraints or not ?$\rightarrow$ Numerical investigations in the two 3-3-1 and left-right models will address this question.
- Our second discussion: Constructing general analytic formulas for one-loop contributions to the LFVZ decay rates.

General Lagrangian for LFV sources and $(g-2)_{e_a}$ anomalies [1807.11484]:

$$\mathcal{L}_{FeS} = \sum_{F,S}\sum_{a=1}^{3} \overline{F}(g^L_{aFS}P_L + g^R_{aFS}P_R)e_a S + \text{h.c.}, \quad (1)$$

$$\mathcal{L}_{FeV} = \sum_{F,V}\sum_{a=1}^{3} \overline{F}\gamma^\mu(g^L_{aFV}P_L + g^R_{aFV}P_R)e_a V_\mu + \text{h.c.}, \quad (2)$$

$$Q_B = Q_F - 1 \text{ with } B = S, V.$$

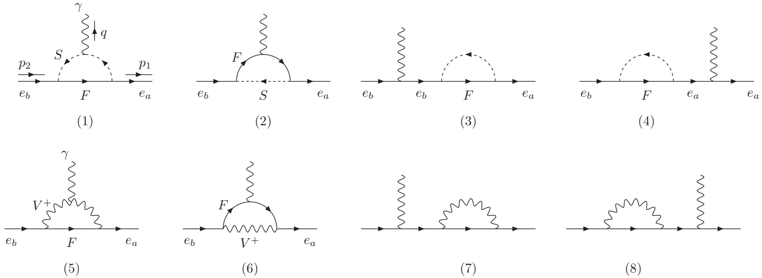
- $F = e_a \to (g-2)_{e_a}$ anomaly
- $F \neq e_a \to$ LFV couplings.

Feynmanrules for photon couplings:

Vertex	Coupling	Vertex	Couplings
$A^\mu(p_0)V^\nu(p_+)V^{*\lambda}(p_-)$	$-ieQ_V\Gamma_{\mu\nu\lambda}(p_0,p_+,p_-)$	$A^\mu S(p_+)S^*(p_-)$	$ieQ_S(p_+-p_-)_\mu$
$A^\mu\overline{F}F$	$ieQ_F\gamma_\mu$		

$$\Gamma_{\mu\nu\lambda}(p_0,p_+,p_-) = g_{\mu\nu}(p_0-p_+)_\lambda + g_{\nu\lambda}(p_+-p_-)_\mu + g_{\lambda\mu}(p_- - p_0)_\nu.$$

One-loop contributions in the unitary gauge



$$f_\Phi(x) = 2\tilde{g}_\Phi(x) = \frac{x^2 - 1 - 2x\ln x}{4(x-1)^3}, \quad g_\Phi = \frac{x - 1 - \ln x}{2(x-1)^2},$$

$$\tilde{f}_\Phi(x) = \frac{2x^3 + 3x^2 - 6x + 1 - 6x^2\ln x}{24(x-1)^4}, \quad f_V(x) = \frac{x^3 - 12x^2 + 15x - 4 + 6x^2\ln x}{4(x-1)^3},$$

$$g_V(x) = \frac{x^2 - 5x + 4 + 3x\ln x}{2(x-1)^2}, \quad \tilde{f}_V(x) = \frac{-4x^4 + 49x^3 - 78x^2 + 43x - 10 - 18x^3\ln x}{24(x-1)^4},$$

$$\tilde{g}_V(x) = \frac{-3(x^3 - 6x^2 + 7x - 2 + 2x^2\ln x)}{(x-1)^3}.$$

cLFV decays $e_b \to e_a\gamma$ and $(g-2)_{e_a}$ anomalies

One-loop form factors:

$$c^X_{(ab)R} \equiv \frac{e}{16\pi^2 m_X^2}\left\{g^{L*}_{aFX} g^{R}_{bFX} m_F \left[f_X(t_X) + Q_F g_X(t_X)\right] + \left[m_b g^{L*}_{aFX} g^{L}_{bFX} + m_a g^{R*}_{aFX} g^{R}_{bFX}\right]\left[\tilde{f}_X(t_X) + Q_F \tilde{g}_X(t_X)\right]\right\},$$

where $X = S, V_\mu$, $t_X \equiv m_F^2/m_X^2$ $(m_X^2 \gg m_{e_a}^2)$:

- a_{e_a}: $a_{e_a} = -\frac{4m_a}{e}\sum_{F,X} \mathrm{Re}\left[c^X_{(aa)R}\right]$
- electric dipole moment (EDM): $d_{e_a} = -2\sum_{F,X} \mathrm{Im}[c^X_{(aa)}]$.
- $\mathrm{Br}(e_b \to e_a\gamma) = \frac{48\pi^2}{G_F^2 m_b^2}\left(|c_{(ab)R}|^2 + |c_{(ba)R}|^2\right)\mathrm{Br}(e_b \to e_a\overline{\nu_a}\nu_b)$, with $c_{(ab)R} = \sum_{X,F} c^X_{(ab)R}$, $c_{(ba)R} = \sum_{X,F} c^X_{(ba)R}$, $\mathrm{Br}(\mu \to e\overline{\nu_e}\nu_\mu) \simeq 1$, $\mathrm{Br}(\tau \to e\overline{\nu_e}\nu_\tau) \simeq 0.1782$, and $\mathrm{Br}(\tau \to \mu\overline{\nu_\mu}\nu_\tau) \simeq 0.1739$.

LFVh and LFVZ decays

$$i\mathcal{M}(Z \to e_b^+ e_a^-) = \frac{ie}{16\pi^2} \overline{u}_a \left[\not{\varepsilon} \left(\bar{a}_l P_L + \bar{a}_r P_R \right) + (p_1.\varepsilon) \left(\bar{b}_l P_L + \bar{b}_r P_R \right) \right] v_b,$$

$$\Gamma(Z \to e_b^+ e_a^-) = \frac{\sqrt{\lambda}}{16\pi m_Z^3} \times \left(\frac{e}{16\pi^2} \right)^2 \left(\frac{\lambda M_0}{12 m_Z^2} + M_1 + \frac{M_2}{3 m_Z^2} \right),$$

$$M_0 = (m_Z^2 - m_a^2 - m_b^2) \left(|\bar{b}_l|^2 + |\bar{b}_r|^2 \right) - 4 m_a m_b \mathrm{Re} \left[\bar{b}_l \bar{b}_r^* \right]$$
$$- 4 m_b \mathrm{Re} \left[\bar{a}_r^* \bar{b}_l + \bar{a}_l^* \bar{b}_r \right] - 4 m_a \mathrm{Re} \left[\bar{a}_l^* \bar{b}_l + \bar{a}_r^* \bar{b}_r \right],$$

$$M_1 = 4 m_a m_b \mathrm{Re} \left[\bar{a}_l \bar{a}_r^* \right],$$

$$M_2 = \left[2 m_Z^4 - m_Z^2 \left(m_a^2 + m_b^2 \right) - \left(m_a^2 - m_b^2 \right)^2 \right] \left(|\bar{a}_l|^2 + |\bar{a}_r|^2 \right),$$

$$\lambda = m_Z^4 + m_b^4 + m_a^4 - 2(m_Z^2 m_a^2 + m_Z^2 m_b^2 + m_a^2 m_b^2);$$

[eprint: 1607.05257, 2107.14207, 2312.11427]

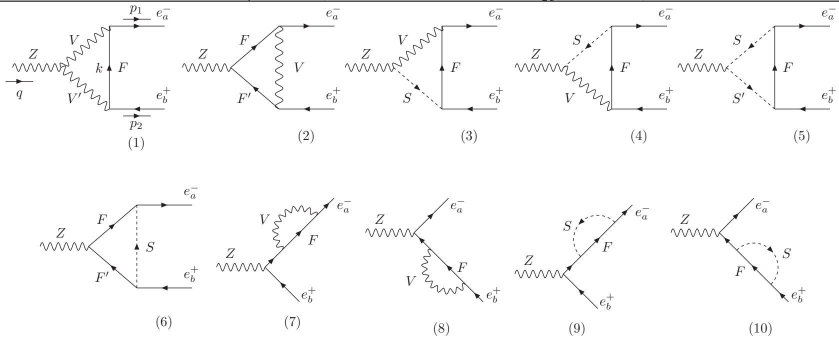
$$\mathcal{L}^{\mathrm{LFV}h} = h \left(\Delta_L^{(ab)} \overline{e_a} P_L e_b + \Delta_R^{(ab)} \overline{e_a} P_R e_b \right) + \mathrm{H.c.},$$ [eprint: 0407302]

$$\Gamma(h \to e_a e_b) \equiv \Gamma(h \to e_a^- e_b^+) + \Gamma(h \to e_a^+ e_b^-) \simeq \frac{m_h}{8\pi} \left(|\Delta_L^{(ab)}|^2 + |\Delta_R^{(ab)}|^2 \right). \quad (3)$$

$\bar{a}_{l,R}$, $\bar{b}_{l,R}$, $\Delta_{L,R}^{(ab)}$: loop corrections needed to calculate in a certain BSM.

LFVZ : Feynman rules and one-loop diagrams in the unitary gauge

Vertex	Coupling
$Z^\mu(p_0)V^\nu(p_+)V'^{*\lambda}(p_-)$	$-ieg_{ZVV'}\Gamma_{\mu\nu\lambda}(p_0,p_+,p_-)$
$Z^\mu S'^*(p_-)S(p_+)$,	$ieg_{ZS'^*S}(p_+-p_-)_\mu$
$SV^{\mu*}Z^\nu$, $S^*V^\mu Z^\nu$	$ieg_{SZV}g_{\mu\nu}$, $ieg^*_{SZV}g_{\mu\nu}$
$Z^\mu\overline{F}F'$, $Z^\mu\overline{F'}F$	$ie\gamma_\mu\left(g^L_{ZFF'}P_L+g_{ZFF'^R P_R}\right)$, $ie\gamma_\mu\left(g^{L*}_{ZFF'}P_L+g_{ZFF'^{R*} P_R}\right)$



Dirac vs Majorana fermions

[H. K. Dreiner, H. E. Haber and S. P. Martin, Phys. Rept. **494** (2010)]

- Dirac fermion $F \neq (F)^c$:

$$\begin{aligned} \mathcal{L}^D_{Zff} =& e \sum_{F,F'} \left[\overline{F} \gamma_\mu \left(g^L_{ZFF'} P_L + g^R_{ZFF'} P_R \right) F' Z^\mu + \text{h.c.} \right], \\ \mathcal{L}^D_{hff} =& - \sum_{F,F'} \left[\overline{F} \left(g^L_{hFF'} P_L + g^R_{hFF'} P_R \right) F' h + \text{h.c.} \right]. \end{aligned} \tag{4}$$

- Majorana fermion $F = (F)^c$ and $F' = (F')^c$:

$$\begin{aligned} \mathcal{L}^M_{Zff} =& \frac{e}{2} \sum_{F,F'} \overline{F} \gamma_\mu \left(g^L_{ZFF'} P_L - g^{L*}_{ZFF'} P_R \right) F' Z^\mu, \\ \mathcal{L}^M_{hff} =& - \frac{1}{2} \sum_{F,F'} \overline{F} \left(g^L_{hFF'} P_L + g^{L*}_{hFF'} P_R \right) F' h, \end{aligned} \tag{5}$$

where $g^R_{hFF'} = g^{L*}_{hFF'}$ and $g^R_{ZFF'} = -g^{L*}_{ZFF'} = -g^L_{ZF'F}$.

Steps of One-loop calculations

- Deriving $\bar{a}_{l,r}$ and $\bar{a}_{l,r}$ directly for every one-loop diagrams, using the unitary gauge for gauge boson:

$$\Delta_V^{(u)\mu\nu} = \frac{-i}{k^2 - m_V^2}\left(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_V^2}\right), \; \Delta_{G_V}^{(u)} = 0.$$

- Using the Form package [J. A. M. Vermaseren, arXiv:math-ph/0010025 [math-ph]] to crosscheck the results of contractions of the products of Dirac matrices.
- One-loop formulas are expressed in terms of the Passarino-Veltman functions [Nucl.Phys.B 160 (1979) 151], using the LoopTools conventions [Comput. Phys. Commun. 118 (1999), 153] (easy for numerical evaluations using LoopTools packgage):

$$A_0(m^2) = \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int \frac{d^d k}{k^2 - m^2 + i\delta},$$

$$B_{\{0,\mu\}}(p_i^2, M_0^2, M_i^2) = \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int \frac{d^d k \times \{1, k_\mu\}}{D_0 D_i}, \; i = 1, 2,$$

$$C_{\substack{0,\mu,\mu\nu \\ 11}} \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int \frac{d^d k\{1, k_\mu, k_\mu k_\nu\}}{D_0 D_1 D_2},$$

Steps of One-loop calculations

- Simple conventions:

$$B_0^{(12)} \equiv B_0(q^2; M_1^2, M_2^2) = \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int \frac{d^dk}{D_1 D_2},$$

$$B_\mu^{(12)} \equiv \frac{(2\pi\mu)^{4-d}}{i\pi^2}\int \frac{d^dk \times k_\mu}{D_1 D_2} = B_1^{(12)} q_\mu + B_0^{(12)} p_{1\mu},$$

where $B_1^{(12)} \equiv B_1(q^2; M_1^2, M_2^2)$.

- Notations appearing in many important formulas:

$$X_0 \equiv C_0 + C_1 + C_2,\ X_1 \equiv C_{11} + C_{12} + C_1\ X_2 \equiv C_{12} + C_{22} + C_2,$$

$$X_3 \equiv C_1 + C_2 = X_0 - C_0,\ X_{012} \equiv X_0 + X_1 + X_2,\ X_{ij} = X_i + X_j.$$

The divergent parts of the PV-functions:

$$\mathrm{div}[C_0] = \mathrm{div}[C_i] = \mathrm{div}[C_{ij}] = 0;\ i,j = 1,2,$$

$$\mathrm{div}[C_{00}] = \frac{C_{UV}}{4},\ \mathrm{div}[B_0^{(1)}] = \mathrm{div}[B_0^{(2)}] = \mathrm{div}[B_0^{(12)}] = C_{UV},$$

$$\mathrm{div}[B_1^{(1)}] = \mathrm{div}[B_1^{(2)}] = \mathrm{div}[B_1^{(12)}] = -\frac{C_{UV}}{2}.$$

⇒ easily to derive the divergent parts in one-loop formulas

- LFVZ decays: One-loop form factors.

Diagram (1) with $g^{XY} \equiv g^{XY}_{FVV'} = g^{X*}_{aFV} g^{Y}_{bFV'}$ with $X, Y = L, R$:

$$\begin{aligned}
\bar{a}_L^{FVV'} =& g_{ZVV'} \Big\{ g^{LL} \Big[\left(2(2-d) + m_F^2 f\right) C_{00} + 2(m_Z^2 - m_a^2 - m_b^2) X_3 \\
& - \left(f(m_V^2 + m_{V'}^2) + 4\right) (B_0^{(12)} + m_F^2 C_0) \\
& + \frac{1}{m_V^2} \left(A_0(m_V) + m_F^2 B_0^{(1)} + m_a^2 B_1^{(1)} - (m_V^2 - m_{V'}^2 + m_Z^2) m_a^2 C_1\right) \\
& + \frac{1}{m_{V'}^2} \left(A_0(m_{V'}) + m_F^2 B_0^{(2)} + m_b^2 B_1^{(2)} - (-m_V^2 + m_{V'}^2 + m_Z^2) m_b^2 C_2\right) \\
& + g^{RR} m_a m_b \left[f\left(C_{00} + m_{V'}^2 C_2 + m_V^2 C_1\right) - 2X_3 \right] \\
& - g^{RL} m_a m_F \left[f C_{00} + (2 - f m_{V'}^2) C_0 + f(m_V^2 - m_{V'}^2) C_1 + \frac{B_0^{(1)} + B_1^{(1)}}{m_V^2} \right] \\
& - g^{LR} m_b m_F \left[f C_{00} + (2 - f m_V^2) C_0 - f(m_V^2 - m_{V'}^2) C_2 + \frac{B_0^{(2)} + B_1^{(2)}}{m_{V'}^2} \right] \Big] \Big\},
\end{aligned} \tag{6}$$

$$\bar{a}_R^{FVV'} = \bar{a}_L^{FVV'} \left[g^{LL} \to g^{RR}, g^{RL} \to g^{LR},\ g^{LR} \to g^{RL} \right], \tag{7}$$

- LFVZ decays: One-loop form factors:

$$\begin{aligned}
\bar{b}_L^{FVV'} =& g_{ZVV'}\Big\{ g^{LL} m_a \left[4(X_3 - X_1) + f(m_F^2 X_{01} + m_b^2 X_2) - 2(m_{V'}^2 f + 2)C_2\right] \\
& + g^{RR} m_b \left[4(X_3 - X_2) + f(m_F^2 X_{02} + m_a^2 X_1) - 2(m_V^2 f + 2)C_1\right] \\
& - g^{RL} m_F \left[f\left(m_F^2 X_0 + m_a^2 X_1 + m_b^2 X_2 - 2m_V^2 C_1 - 2m_{V'}^2 C_2\right) + 4X_3\right] \\
& - g^{LR} m_a m_b m_F f X_{012}\Big\}, \qquad (8) \\
\bar{b}_R^{FVV'} =& \bar{b}_L^{FVV'}\left[g^{LL} \to g^{RR}, g^{RR} \to g^{LL}, g^{RL} \to g^{LR},\ g^{LR} \to g^{RL}\right], \\
f =& \frac{(m_Z^2 - m_V^2 - m_{V'}^2)}{m_V^2 m_{V'}^2}.
\end{aligned}$$

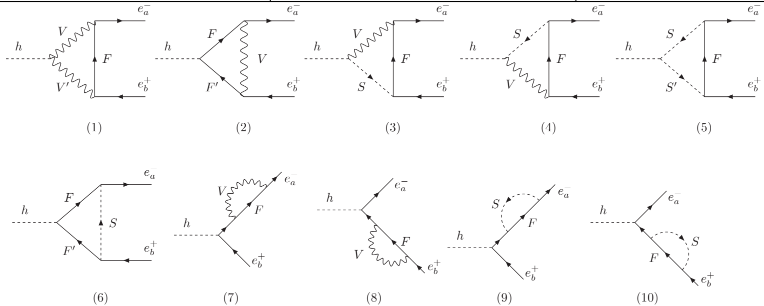
- Results: in agreement with [JHEP 03 (2022), 106], restricted to the two Higgs doublet model with seesaw neutrinos and only $g^{LL} \neq 0$. One-loop formulas are derived in the 't Hooft-Feynman gauge.

- Results: consistent with [Eur.Phys.J.C 83 (2023) 6, 494], calculated in both unitary and 't Hooft-Feynman gauges, but for a simple SM extension with seesaw neutrinos. Only $g^{LL} \neq 0$.

- For more one-loop formulas, see our paper, arix:2409.01390, EPJC.

LFVh : Feynman rules and one-loop diagrams in the unitary gauge

Vertex	Coupling
$hV^{\mu}V'^{*\nu}$	$ig_{hVV'}g_{\mu\nu}$
hSS'^{*}	$-i\lambda_{hSS'}$
$V^{\mu}S^{*}(p_-)h(p_0)$, $V^{*\mu}S(p_+)h(p_0)$	$ig_{VSh}(p_0-p_-)_{\mu}$, $ig^{*}_{VSh}(p_+-p_0)_{\mu}$
$h\overline{F}F'$, $h\overline{F'}F$	$-i\left(g^{L}_{hFF'}P_L+g^{R}_{hFF'}P_R\right)$, $-i\left(g^{L*}_{hFF'}P_R+g_{hFF'R*}P_L\right)$



LFVh decays: one-loop form factors.

$$
\begin{aligned}
\Delta_L^{FVV'} = &-\frac{g_{hVV'}g^{LL}m_a}{16\pi^2}\Bigg\{2C_1 - \frac{1}{m_V^2}\left[B_1^{(12)} - (m_F^2 - m_a^2)(C_0 + C_1) - (B_0^{(2)} + m_V^2 C_0)\right] \\
&\quad - \frac{1}{m_{V'}^2}\left[B_1^{(12)} + B_0^{(12)} - (m_F^2 - m_b^2)C_1\right] \\
&\quad - \frac{1}{2m_V^2 m_{V'}^2}\Big[A_0(m_{V'}) + m_F^2\left(B_0^{(1)} + B_0^{(2)} + B_1^{(1)}\right) + m_b^2 B_1^{(2)} \\
&\quad + (m_V^2 + m_{V'}^2 - q^2)\left(m_F^2(C_0 + C_1) + m_b^2 C_2 - B_1^{(12)}\right) \\
&-\frac{g_{hVV'}g^{RL}m_F}{32\pi^2 m_V^2 m_{V'}^2}\Big\{4m_V^2 m_{V'}^2 C_0 + A_0(m_V) + A_0(m_{V'}) + (m_F^2 - 2m_V^2)B_0^{(1)} \\
&\quad + (m_F^2 - 2m_{V'}^2)B_0^{(2)} + m_a^2 B_1^{(1)} + m_b^2 B_1^{(2)} \\
&\quad + (m_V^2 + m_{V'}^2 - q^2)(B_0^{(12)} + m_a^2 C_1 + m_b^2 C_2 + m_F^2 C_0)\Big\} \\
&-\frac{g_{hVV'}g^{LR}m_a m_b m_F}{32\pi^2 m_V^2 m_{V'}^2}\left[B_0^{(1)} + B_0^{(2)} + B_1^{(1)} + B_1^{(2)} + (m_V^2 + m_{V'}^2 - q^2)X_0\right] \\
&-\frac{g_{hVV'}g^{RR}m_b}{16\pi^2}\Bigg\{2C_2 + \frac{1}{m_V^2}\left[B_1^{(12)} + (m_F^2 - m_a^2)C_2\right] \\
&\quad + \frac{1}{m_{V'}^2}\left[B_1^{(12)} + B_0^{(12)} + (m_F^2 - m_b^2)(C_0 + C_2) + B_0^{(1)} + m_{V'}^2 C_0\right] \\
&\quad - \frac{1}{2m_V^2 m_{V'}^2}\Big[A_0(m_V) + m_F^2\left(B_0^{(1)} + B_0^{(2)} + B_1^{(2)}\right) + m_a^2 B_1^{(1)}
\end{aligned}
$$

The left-right model with ISS neutrinos (LRIS)

The gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times Z_2$:
[arxiv: 2207.05828, PRD; 2101.08255,PRD]

$$
\begin{aligned}
Q_L &= \begin{pmatrix} u \\ d \end{pmatrix}_L \sim \left(3, 2, 1, \frac{1}{3}, +\right), \; L_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \sim (1, 2, 1, -1, +), \\
Q_R &= \begin{pmatrix} u \\ d \end{pmatrix}_R \sim \left(3, 1, 2, \frac{1}{3}, +\right), \; L_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R \sim (1, 1, 2, -1, +), \\
& S_1 \sim (1, 1, 1, 0, -), \; S_2 \sim (1, 1, 1, 0, +), \\
& \phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \sim (1, 2, 2, 0, +), \; \chi_R = \begin{pmatrix} \chi_R^+ \\ \chi_R^0 \end{pmatrix} \sim (1, 1, 2, 1, +).
\end{aligned} \tag{10}
$$

Yukawa Lagrangian for leptons

$$-\mathcal{L}_Y = \sum_{i,j=1}^{3} \left[y^L_{ij}\overline{L_{R_i}}\phi^\dagger L_{Lj} + \tilde{y}^L_{ij}\overline{L_{R_i}}\tilde{\phi}^\dagger L_{Lj} + y^s_{ij}\overline{L_{R_i}}\tilde{\chi}_R (S_{2j})^c + \text{H.c.}\right],$$

$$\tilde{\phi} = \sigma_2\phi^*\sigma_2, \text{ and } \tilde{\chi}_R = i\sigma_2\chi^*, \ \langle\phi\rangle = \text{diag}\left(\frac{v_1}{\sqrt{2}}, \ \frac{v_2}{\sqrt{2}}\right), \ \langle\chi^0_R\rangle = \frac{v_R}{\sqrt{2}}$$

$$\begin{aligned} -\mathcal{L}^\nu_{mass} =& \overline{\nu_R} m_D \nu_L + \overline{\nu_R} M_R^T (S_2)^c + \frac{\mu_s}{2}\overline{S_2}(S_2)^c + \text{h.c.} \\ =& \frac{1}{2}\left(\overline{(\nu_L)^c}, \ \overline{\nu_R}, \overline{S_2}\right)\mathcal{M}^\nu \left(\nu_L, \ (\nu_R)^c, (S_2)^c\right)^T + \text{h.c.}, \end{aligned} \quad (11)$$

$$\nu_L = (\nu_1, \nu_2, \nu_3)^T_L, \ \nu_R = (\nu_1, \nu_2, \nu_3)^T_R, \ S_2 = (S_{21}, S_{22}, S_{23})^T.$$

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Lepton masses and mixing

$$\mathcal{M}^{\ell} = (y^L c_\beta + \tilde{y}^L s_\beta) v/\sqrt{2},$$

$$\mathcal{M}^{\nu} = \begin{pmatrix} \mathcal{O}_{3\times3} & m_D^T & \mathcal{O}_{3\times3} \\ m_D & \mathcal{O}_{3\times3} & M_R^T \\ \mathcal{O}_{3\times3} & M_R & \mu_s \end{pmatrix}, \; m_D = \frac{v}{\sqrt{2}} \left(y^L s_\beta + \tilde{y}^L c_\beta \right), \; M_R = \frac{y^{sT}}{\sqrt{2}} v_R, \quad (12)$$

$$U^{\nu T} \mathcal{M}^{\nu} U^{\nu} = \hat{\mathcal{M}}^{\nu} = \mathrm{diag}(m_{n_1},\, m_{n_2},\, ..., m_{n_9}) = \mathrm{diag}(\hat{m}_\nu, \hat{M}_N),$$
$$n'_L = U^{\nu} n_L,\; n'_R = U^{\nu *} n_R = U^{\nu *} (n_L)^c, \quad (13)$$

$$U^{\nu} \simeq \begin{pmatrix} \left(I_3 - \frac{R_0 R_0^\dagger}{2} \right) U_3^{\nu} & \frac{i R_0}{\sqrt{2}} & \frac{R_0}{\sqrt{2}} \\ \mathcal{O}_{3\times3} & -\frac{i I_3}{\sqrt{2}} & \frac{I_3}{\sqrt{2}} \\ -R_0^\dagger U_3^{\nu} & \frac{i}{\sqrt{2}} \left(I_3 - \frac{R_0^\dagger R_0}{2} \right) & \frac{1}{\sqrt{2}} \left(I_3 - \frac{R_0^\dagger R_0}{2} \right) \end{pmatrix}, \quad (14)$$

Lepton masses and mixing parameters

Casas-Ibarra parameterisation for m_D [hep-ph/0103065, NPB]:

$$m_D = x_0^{\frac{1}{2}} M_R \hat{\mu}_s^{-\frac{1}{2}} \xi \hat{x}_\nu^{\frac{1}{2}} U_3^{\nu\dagger}, \tag{15}$$

$$R_0 \equiv x_0^{\frac{1}{2}} U_3^{\nu} \hat{x}_\nu^{\frac{1}{2}} \xi^\dagger \hat{\mu}_s^{-\frac{1}{2}}, \tag{16}$$

$$M_R = \hat{M}_R = \mathrm{diag}(M_1, M_2, M_3),\; \hat{M}_N = \mathrm{diag}(\hat{M}_R, \hat{M}_R),$$

$$x_0 \equiv \frac{\max[m_{n_1}, m_{n_2}, m_{n_3}]}{|(\mu_s)_{22}|} \ll 1,$$

$$\hat{\mu}_s \equiv \frac{\mu_s}{|(\mu_s)_{22}|},\; \hat{x}_\nu \equiv \frac{\hat{m}_\nu}{\max[m_{n_1}, m_{n_2}, m_{n_3}]}. \tag{17}$$

$$U_R^{\ell\dagger} \mathcal{M}^\ell U_L^\ell = \hat{\mathcal{M}}^\ell = \mathrm{diag}(m_e, m_\mu, m_\tau),\; e_{L,R} \to U_{L,R}^\ell e_{L,R},$$

$$U_{\mathrm{PMNS}} = U_L^{\ell\dagger} U_3^\nu. \tag{18}$$

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Gauge boson

$$\mathcal{L}_k^H = \mathrm{Tr}\left[(D_\mu\phi)^\dagger(D^\mu\phi)\right] + (D_\mu\chi)^\dagger\,(D^\mu\chi)\,. \tag{19}$$

$$W^\pm_{L,R\mu} \equiv (W^1_{L,R\mu} \mp iW^2_{L,R\mu})/\sqrt{2},$$

$$\begin{pmatrix} W^\pm_{L\mu} \\ W^\pm_{R\mu} \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} W^\pm_\mu \\ W'^\pm_\mu \end{pmatrix},$$

$$m_W^2 = \frac{g^2v^2}{4} \times (1 - t_\theta s_{2\beta})\,,\ m_{W'}^2 = \frac{g^2v_R^2}{4} \times \frac{c_\theta^2\,(s_{2\beta}+t_\theta)}{c_{2\theta}s_{2\beta}}, \tag{20}$$

$$\begin{pmatrix} W^3_{R\mu} \\ B'_\mu \\ W^3_{L\mu} \end{pmatrix} = \begin{pmatrix} c_\zeta c_\varphi - s_W s_\zeta s_\varphi & -c_\varphi s_\zeta - c_\zeta s_W s_\varphi & c_W s_\varphi \\ -c_\varphi s_W s_\zeta - c_\zeta s_\varphi & s_\zeta s_\varphi - c_\zeta c_\varphi s_W & c_W c_\varphi \\ c_W s_\zeta & c_W c_\zeta & s_W \end{pmatrix} \begin{pmatrix} Z'_\mu \\ Z_\mu \\ A_\mu \end{pmatrix},$$

$$m_Z^2 \simeq \frac{m_W^2}{c_W^2},\ m_{Z'}^2 \simeq \frac{m_{W'}^2}{c_\varphi^2}. \tag{21}$$

Higgs sector

$$V_h = \mu_1^2 \mathrm{Tr}(\phi^\dagger\phi) + \mu_2^2\left[\mathrm{Tr}(\phi^\dagger\tilde{\phi}) + \mathrm{Tr}(\phi\tilde{\phi}^\dagger)\right] + \lambda_1\left[\mathrm{Tr}(\phi^\dagger\phi)\right]^2 + \lambda_2\left[\left(\mathrm{Tr}(\phi^\dagger\tilde{\phi})\right)^2 + \left(\mathrm{Tr}(\phi\tilde{\phi}^\dagger)\right)^2\right]$$
$$+ \lambda_3 \mathrm{Tr}(\phi^\dagger\tilde{\phi})\mathrm{Tr}(\phi\tilde{\phi}^\dagger) + \lambda_4 \mathrm{Tr}(\phi^\dagger\phi)\left[\mathrm{Tr}(\phi^\dagger\tilde{\phi}) + \mathrm{Tr}(\phi\tilde{\phi}^\dagger)\right] + \mu_3^2\left(\chi_R^\dagger\chi_R\right) + \lambda_5\left(\chi_R^\dagger\chi_R\right)^2$$
$$+ \alpha_1 \mathrm{Tr}(\phi^\dagger\phi)\left(\chi_R^\dagger\chi_R\right) + \alpha_2(\chi_R^\dagger\phi^\dagger\phi\chi_R) + \alpha_3(\chi_R^\dagger\tilde{\phi}^\dagger\tilde{\phi}\chi_R) + \alpha_4\left[(\chi_R^\dagger\phi^\dagger\tilde{\phi}\chi_R) + \mathrm{H.c.}\right]. \quad (22)$$

$$\begin{pmatrix}\phi_1^\pm\\ \phi_2^\pm\\ \chi_R^\pm\end{pmatrix} = C_{h^\pm}^T\begin{pmatrix}G_1^\pm\\ G_2^\pm\\ H^\pm\end{pmatrix},\quad C_{h^\pm}^T = \begin{pmatrix} s_\beta s_\xi & c_\beta & c_\xi s_\beta\\ c_\beta s_\xi & -s_\beta & c_\beta c_\xi\\ c_\xi & 0 & -s_\xi\end{pmatrix},\quad m_{H^\pm}^2 = \alpha_{32}\left(c_{2\beta}v^2 + v_R^2/(2c_{2\beta})\right).$$

$$\begin{pmatrix}a_1\\ a_2\\ a_R\end{pmatrix} = C_a\begin{pmatrix}G_1^0\\ G_2^0\\ A^0\end{pmatrix},\quad C_a = \begin{pmatrix} 0 & -s_\beta & c_\beta\\ 0 & c_\beta & s_\beta\\ 1 & 0 & 0\end{pmatrix},\quad m_A^2 = 2v^2(\lambda_3 - 2\lambda_2) + \frac{v_R^2(\alpha_3-\alpha_2)}{2c_{2\beta}}$$
$$C_1\mathcal{M}_H^2 C_1^T = \mathcal{M}_{H,0}^2,\ C_1 = \begin{pmatrix} s_\beta & c_\beta & 0\\ c_\beta & -s_\beta & 0\\ 0 & 0 & 1\end{pmatrix},$$
$$(r_1, r_2^T, r_R)^T = C_1^T(h, h_1, h_2)^T \quad (23)$$

LFV Couplings

$$
\begin{aligned}
\mathcal{L}^{\ell\ell H} =& \left[-\frac{h}{2v}\overline{n_i}\left[\lambda_{ij}P_L+\lambda^*_{ij}P_R\right]n_j-\left(1+\frac{h}{v}\right)\overline{e_R}\mathcal{M}^\ell e_L+\text{h.c.}\right]\\
&-\sum_{i=1}^{9}\overline{n_i}\left\{\left[U^{\nu*}_{ai}\left(y^{L\dagger}s_\beta-\tilde{y}^{L\dagger}c_\beta\right)_{ab}c_\xi+U^{\nu*}_{(a+6)i}y^{s\dagger}_{ab}s_\xi\right]P_R\right.\\
&\left.\qquad+U^{\nu}_{(a+3)i}\left(y^{L}c_\beta-\tilde{y}^{L}s_\beta\right)_{ab}c_\xi P_L\right\}e_b H^+ +\text{h.c.}+\ldots, \qquad (24)\\
\mathcal{L}^{\ell\ell V} =& -eA_\mu\overline{e_a}\gamma^\mu e_a\\
&+\left[\frac{g_2}{\sqrt{2}}W^{+\mu}\sum_{i=1}^{9}\overline{n_i}\gamma^\mu\left(c_\theta U^{\nu*}_{ai}P_L+s_\theta U^{\nu}_{(a+3)i}P_R\right)e_a+\text{h.c.}\right]\\
&+\left[\frac{g_2}{\sqrt{2}}W'^{+\mu}\sum_{i=1}^{9}\overline{n_i}\gamma^\mu\left(-s_\theta U^{\nu*}_{ai}P_L+c_\theta U^{\nu}_{(a+3)i}P_R\right)e_a+\text{h.c.}\right]\\
&+eZ^\mu\overline{e_a}\left[\left(c_\zeta t^{\text{SM}}_L-\frac{s_\zeta s_W}{2c_\varphi c_W^2}\right)P_L+\left(c_\zeta t^{\text{SM}}_R+\frac{s_\zeta(c_\varphi^2c_W^2-s_W^2)}{2s_Wc_\varphi c_W^2}\right)P_R\right]e_a\\
&+\frac{e}{2}Z^\mu\sum_{i,j=1}^{2}\overline{n_i}\left[g^L_{Zij}P_L-g^L_{Zji}P_R\right]n_j+\ldots, \qquad (25)
\end{aligned}
$$

LFV Couplings

$$\lambda_{ij}=\lambda_{ji}=\sum_{c=1}^{3}\left(m_{n_i}U^{\nu *}_{ci}U^{\nu}_{cj}+m_{n_j}U^{\nu *}_{cj}U^{\nu}_{ci}\right)\to g^{R*}_{hij}=g^{L}_{hij}=\frac{g\delta_{hee}}{2m_W}\lambda_{ij},$$

$$g^{L}_{Zij}=\sum_{c=1}^{3}\left[U^{\nu *}_{ci}U^{\nu}_{cj}\left(\frac{c_\zeta}{2c_W s_W}-\frac{s_\zeta s_W}{2c_\varphi c_W^2}\right)+U^{\nu *}_{(c+3)i}U^{\nu}_{(c+3)j}\frac{s_\zeta\left(c_\varphi^2 c_W^2+s_W^2\right)}{2c_\varphi c_W^2 s_W}\right], \qquad (26)$$

$$g^{L}_{aiH+}=\frac{\sqrt{2}c_\xi}{vc_{2\beta}}\sum_{c=1}^{3}\left[U^{\nu}_{(c+3)i}\left(\hat{\mathcal{M}}^{\ell}-m_D s_{2\beta}\right)_{ca}\right],$$

$$g^{R}_{aiH+}=\frac{\sqrt{2}c_\xi}{vc_{2\beta}}\sum_{c=1}^{3}\left[U^{\nu *}_{ci}\left(\hat{\mathcal{M}}^{\ell\dagger}s_{2\beta}-m_D^{\dagger}\right)_{ca}+U^{\nu *}_{(c+6)i}(\hat{M}_R)_{ca}t_\xi^2\right],$$

$$g^{L}_{aiW}=\frac{g_2}{\sqrt{2}}c_\theta U^{\nu *}_{ai},\ g^{R}_{aiW}=\frac{g_2}{\sqrt{2}}s_\theta U^{\nu}_{(a+3)i},$$

$$g^{L}_{aiW'}=-\frac{g_2}{\sqrt{2}}s_\theta U^{\nu *}_{ai},\ g^{R}_{aiW'}=\frac{g_2}{\sqrt{2}}c_\theta U^{\nu}_{(a+3)i}, \qquad (27)$$

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Feynman rules

Vertex	Coupling:	Vertex	Coupling
$g_{hW^+W^-}$	$g m_W (1 - s_{2\beta} s_{2\theta}) \delta_{hee}^{-1}$	$g_{hW'^+W'^-}$	$g m_W (s_{2\beta} s_{2\theta} + 1) \delta_{hee}^{-1}$
$g_{hW^+W'^-}$	$g m_W s_{2\beta} \left(s_\theta^2 - c_\theta^2\right) \delta_{hee}^{-1}$		
$g_{W^+H^-h}$	$\frac{g}{2} c_\xi s_\theta \left(c_\beta^2 - s_\beta^2\right)$	$g_{W'^+H^-h}$	$\frac{g}{2} c_\theta c_\xi \left(c_\beta^2 - s_\beta^2\right)$

Vertex	Coupling
$g_{ZH^+H^-}$	$-\frac{c_\zeta \left(s_\xi^2 + 2 s_W^2 - 1\right)}{2 s_W c_W} + \frac{s_\zeta \left(\left(s_\xi^2 + 2\right) s_W^2 - 1\right)}{2 s_W c_W \sqrt{1 - 2 s_W^2}}$
$g_{H^-W^+Z}$	$\frac{c_{2\beta} c_\xi m_W s_\theta}{s_W c_W} \left(c_\zeta - \frac{s_\zeta s_W^2}{\sqrt{1 - 2 s_W^2}}\right)$
$g_{H^-W'^+Z}$	$\frac{c_{2\beta} c_\theta c_\xi m_W}{s_W c_W} \left(c_\zeta - \frac{s_\zeta s_W^2}{\sqrt{1 - 2 s_W^2}}\right)$
$g_{ZW^+W^-}$	$\frac{c_\zeta c_\theta^2}{t_W} - s_\theta^2 \left(c_\zeta t_W + s_\zeta \sqrt{1 - t_W^2} / s_W\right)$
$g_{ZW'^+W'^-}$	$c_\zeta s_\theta^2 t_W^{-1} - c_\theta^2 \left(c_\zeta t_W + s_\zeta \sqrt{1 - t_W^2} / s_W\right)$
$g_{ZW'^+W^-}, g_{ZW^+W'^-}$	$-c_\theta s_\theta \left(\frac{c_\zeta}{s_W c_W} + s_\zeta \sqrt{1 - t_W^2} / s_W\right)$

One-loop form factors for LFVh decay rates

$$\Delta^{(1+2+7+8)}_{L,R} = \sum_{i=1}^{9}\left[\Delta^{iWW}_{L,R} + \Delta^{iW'W'}_{L,R} + \Delta^{iWW'}_{L,R} + \Delta^{iW'W}_{L,R} + \Delta^{iW}_{L,R} + \Delta^{iW'}_{L,R}\right]$$

$$+ \sum_{i,j=1}^{9}\left[\Delta^{Wij}_{L,R} + \Delta^{W'ij}_{L,R}\right],$$

$$\Delta^{(5+6+9+10)}_{L,R} = \sum_{i=1}^{9}\left[\Delta^{iH^+H^+}_{L,R} + \Delta^{iH^+}_{L,R}\right] + \sum_{i,j=1}^{9}\Delta^{H^+ij}_{L,R},$$

$$\Delta^{(3+4)}_{L,R} = \sum_{i=1}^{9}\left(\Delta^{iH^+W}_{L,R} + \Delta^{iWH^+}_{L,R} + \Delta^{iH^+W'}_{L,R} + \Delta^{iW'H^+}_{L,R}\right). \quad (28)$$

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One-loop form factors for LFVZ decay rates

$$
\begin{aligned}
\bar{a}_{L,R} &= \bar{a}^{(1+2+7+8)}_{L,R} + \bar{a}^{(5+6+9+10)}_{L,R} + \bar{a}^{(3+4)}_{L,R},\\
\bar{b}_{L,R} &= \bar{b}^{(1+2+7+8)}_{L,R} + \bar{b}^{(5+6+9+10)}_{L,R} + \bar{b}^{(3+4)}_{L,R},\\
\bar{a}^{(1+2+7+8)}_{L,R} &= \sum_{i=1}^{9}\left[\bar{a}^{iWW}_{L,R} + \bar{a}^{iW'W'}_{L,R} + \bar{a}^{iWW'}_{L,R} + \bar{a}^{iW'W}_{L,R} + \bar{a}^{iW}_{L,R} + \bar{a}^{iW'}_{L,R}\right]\\
&\quad + \sum_{i,j=1}^{9}\left[\bar{a}^{Wij}_{L,R} + \bar{a}^{W'ij}_{L,R}\right],\\
\bar{b}^{(1+2+7+8)}_{L,R} &= \sum_{i=1}^{9}\left[\bar{b}^{iWW}_{L,R} + \bar{b}^{iW'W'}_{L,R} + \bar{b}^{iWW'}_{L,R} + \bar{b}^{iW'W}_{L,R} + \bar{b}^{iW}_{L,R} + \bar{b}^{FW'}_{L,R}\right]\\
&\quad + \sum_{i,j=1}^{9}\left[\bar{b}^{Wij}_{L,R} + \bar{b}^{W'ij}_{L,R}\right],
\end{aligned} \tag{29}
$$

One-loop form factors for LFV Z decay rates

$$\bar{a}^{(5+6+9+10)}_{L,R} = \sum_{i=1}^{9} \left[\bar{a}^{iH^+H^-}_{L,R} + \bar{a}^{iH^+}_{L,R}\right] + \sum_{i,j=1}^{9} \bar{a}^{H^+ij}_{L,R},$$

$$\bar{b}^{(5+6+9+10)}_{L,R} = \sum_{i=1}^{9} \left[\bar{b}^{iH^+H^+}_{L,R} + \bar{b}^{iH^+}_{L,R}\right] + \sum_{i,j=1}^{9} \bar{b}^{H^+ij}_{L,R}, \quad (30)$$

$$\bar{a}^{(3+4)}_{L,R} = \sum_{i=1}^{9} \left(\bar{a}^{iH^+W}_{L,R} + \bar{a}^{iWH^+}_{L,R} + \bar{a}^{iH^+W'}_{L,R} + \bar{a}^{iW'H^+}_{L,R}\right),$$

$$\bar{b}^{(3+4)}_{L,R} = \sum_{i=1}^{9} \left(\bar{b}^{iH^+W}_{L,R} + \bar{b}^{iWH^+}_{L,R} + \bar{b}^{iH^+W'}_{L,R} + \bar{b}^{iW'H^+}_{L,R}\right), \quad (31)$$

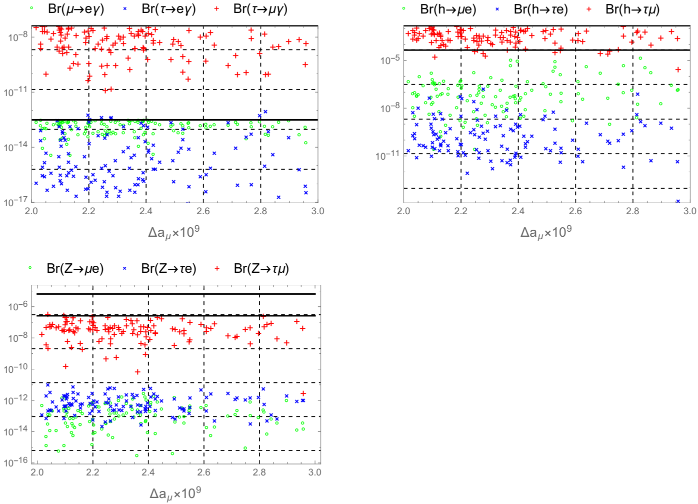
Setup parameters

The unknown parameters of the LRIS model will be scanned in the following ranges:

$$v_R \in [10,\ 100]\ [\text{TeV}];\ m_{H^\pm} \in [0.3,\ 5]\ [\text{TeV}];\ M_{1,2,3} \in [0.1,\ 10]\ [\text{TeV}];$$
$$m_{n_1} \in [10^{-3}, 0.035]\ [\text{eV}];\ t_\beta \in [0.02, 0.8];\ x_0 \in [10^{-6},\ 5\times 10^{-4}];\ (\hat{\mu}_s)_{11,33} \in [0.2, 50], \quad (32)$$

$$\Delta a_\mu^{\text{NP}} \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (2.49 \pm 0.48) \times 10^{-9}$$

Δa_μ vs LFV decays rates



Two black lines show the upper bounds of LFV decay rates for cLFV ($\tau \to \mu\gamma$, $\mu \to e\gamma$), LFVh ($h \to \tau\mu, \mu e$), and LFVZ ($Z \to \tau\mu, \mu e$), respectively

Numerical results

$$
\begin{aligned}
\mathrm{Br}(\tau \to e\gamma) &\leq 9 \times 10^{-13}, \\
\mathrm{Br}(h \to \tau e) &\leq 1.77 \times 10^{-7}, \\
\mathrm{Br}(h \to \mu e) &\leq 1.3 \times 10^{-5}, \\
\mathrm{Br}(Z \to \mu^+ e^-) &\leq 2.8 \times 10^{-12}, \\
\mathrm{Br}(Z \to \tau^+ e^-) &\leq 1.1 \times 10^{-11}, \\
\mathrm{Br}(Z \to \tau^+ \mu^-) &\leq 3.8 \times 10^{-7}.
\end{aligned} \tag{33}
$$

The 3-3-1 model with ISS neutrinos (331ISS)

Based on the gauge group: $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ [2404.05524, PRD].

$$L'_{aL} = \begin{pmatrix} e'_a & -\nu'_a & E'_a \end{pmatrix}^T_L \sim \left(3^* \,, -\frac{1}{2} + \frac{\beta}{2\sqrt{3}}\right), \quad a = 1,2,3,$$

$$e'_{aR} \sim (1\,, -1)\,, \quad \nu_{IR},\ X_{IR} \sim (\,1\,, 0)\,, \quad E'_{aR} \sim \left(\,1\,, -\frac{1}{2} + \frac{\sqrt{3}\beta}{2}\right),$$

$$\chi = \begin{pmatrix} \chi_A & \chi_B & \chi^0 \end{pmatrix}^T \sim \left(3\,, \frac{\beta}{\sqrt{3}}\right),\ \rho = \begin{pmatrix} \rho^+ & \rho^0 & \rho^{-B} \end{pmatrix}^T \sim \left(3\,, \frac{1}{2} - \frac{\beta}{2\sqrt{3}}\right),$$

$$\eta = \begin{pmatrix} \eta^0 & \eta^- & \eta^{-A} \end{pmatrix}^T \sim \left(3\,, -\frac{1}{2} - \frac{\beta}{2\sqrt{3}}\right),\ h^+ \sim (1,1,1). \quad (34)$$

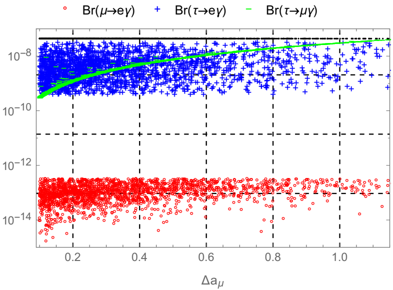
$$\text{vev: } \langle \chi^0 \rangle = \tfrac{v_3}{\sqrt{2}},\ \langle \rho^0 \rangle = \tfrac{v_2}{\sqrt{2}},\ \langle \eta^0 \rangle = \tfrac{v_1}{\sqrt{2}}.$$

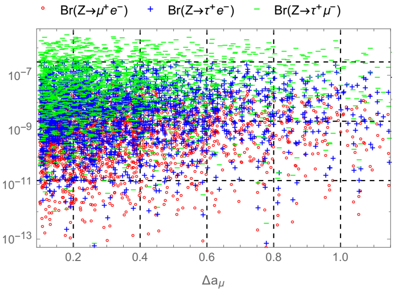
$$-\mathcal{L}^{\text{yuk}}_{\text{lepton}} = \overline{e'_R} Y^e \eta^T L'_L + \overline{E'_R} Y^E \chi^T L'_L + \overline{\nu_R} Y^\nu \rho^T L'_L + \overline{\nu_R} M_R (X_R)^c + \frac{1}{2}\overline{X_R}\mu_X (X_R)^c + \overline{(X_R)^c} Y^h e'_R h^+ + \text{h.c.}, \quad (35)$$

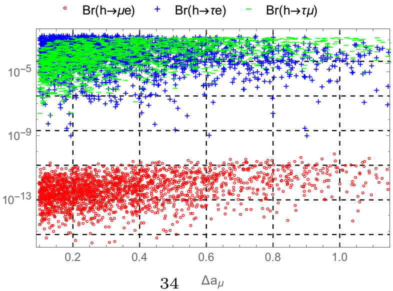
Feynman rules

Vertex	Coupling
$\overline{n}_i e_a H_k^+$	$-\frac{ig}{\sqrt{2}m_W}\left(\lambda_{ia}^{L,k}P_L+\lambda_{ia}^{R,k}P_R\right)$
$\overline{e_a} n_i H_k^-$	$-\frac{ig}{\sqrt{2}m_W}\left(\lambda_{ia}^{R,k*}P_L+\lambda_{ia}^{L,k*}P_R\right)$
$\overline{E}_c e_a H^{+A}$	$\frac{-ig(V_L^{E\dagger}U_L^e)_{ca}}{\sqrt{2}m_Y}\left(m_{E_c}t_{13}P_L+\frac{m_a}{t_{13}}P_R\right)$
$\overline{e}_a E_c H^{-A}$	$\frac{-ig(V_L^{E\dagger}U_L^e)^*_{ca}}{\sqrt{2}m_Y}\left(\frac{m_a}{t_{13}}P_L+m_{E_c}t_{13}P_R\right)$
$\overline{n_i} e_a W^{+\mu}$	$\frac{ig}{\sqrt{2}}(U_0^{\nu\dagger}U_L^e)_{ia}\gamma_\mu P_L$
$\overline{e_a} n_i W^{-\mu}$	$\frac{ig}{\sqrt{2}}(U_0^{\nu\dagger}U_L^e)^*_{ia}\gamma_\mu P_L$
$\overline{E}_c e_a Y^{+A\mu}$	$\frac{ig(V_L^{E\dagger}U_L^e)_{ca}}{\sqrt{2}}\gamma_\mu P_L$
$\overline{e_a} E_c Y^{-A\mu}$	$\frac{ig(V_L^{E\dagger}U_L^e)^*_{ca}}{\sqrt{2}}\gamma_\mu P_L$
$A^\lambda W^{+\mu} W^{-\nu}$	$-ie\Gamma_{\lambda\mu\nu}(p_0,p_+,p_-)$
$A^\lambda Y^{+A\mu} Y^{-A\nu}$	$-ieA\Gamma_{\lambda\mu\nu}(p_0,p_+,p_-)$
$A^\mu H_k^+ H_k^-$	$ie(p_+-p_-)_\mu$
$A^\mu H^{+A} H^{-A}$	$ieA(p_+-p_-)_\mu$
$A^\mu \overline{e_a} e_a,\ A^\mu \overline{E_a} E_a$	$-ie\gamma_\mu,\ ieB\gamma_\mu$

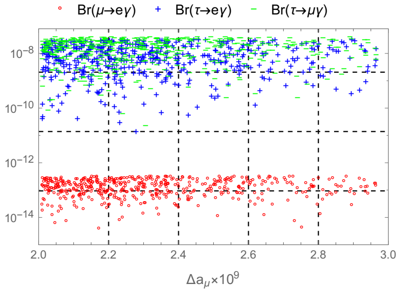
Numerical results for 4 ISS neutrinos

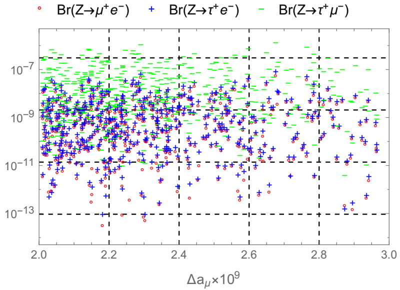


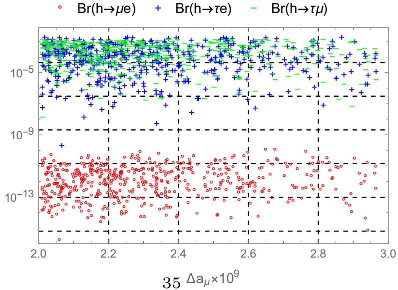




Numerical results for 6 ISS neutrinos







Conclusions

1. We completely introduce two classes of general master formulas expressing one-loop contributions to the LFVh and LFVZ decay amplitudes in the BSMs. The calculations were performed in the unitary gauge, independent of the couplings of nonphysical states such as Goldstone bosons.
2. The 3-3-1 and LRIS models with LFV sources coming only from the couplings of active and ISS neutrinos predict strong correlations between $(g-2)_{e_a}$ anomalies and LFV decay rates. However, there still exist allowed regions of the parameter space satisfying all of these experimental constraints.

Thank you for listening!