Implications of a new SU(2) flavour group in early-universe phase transitions

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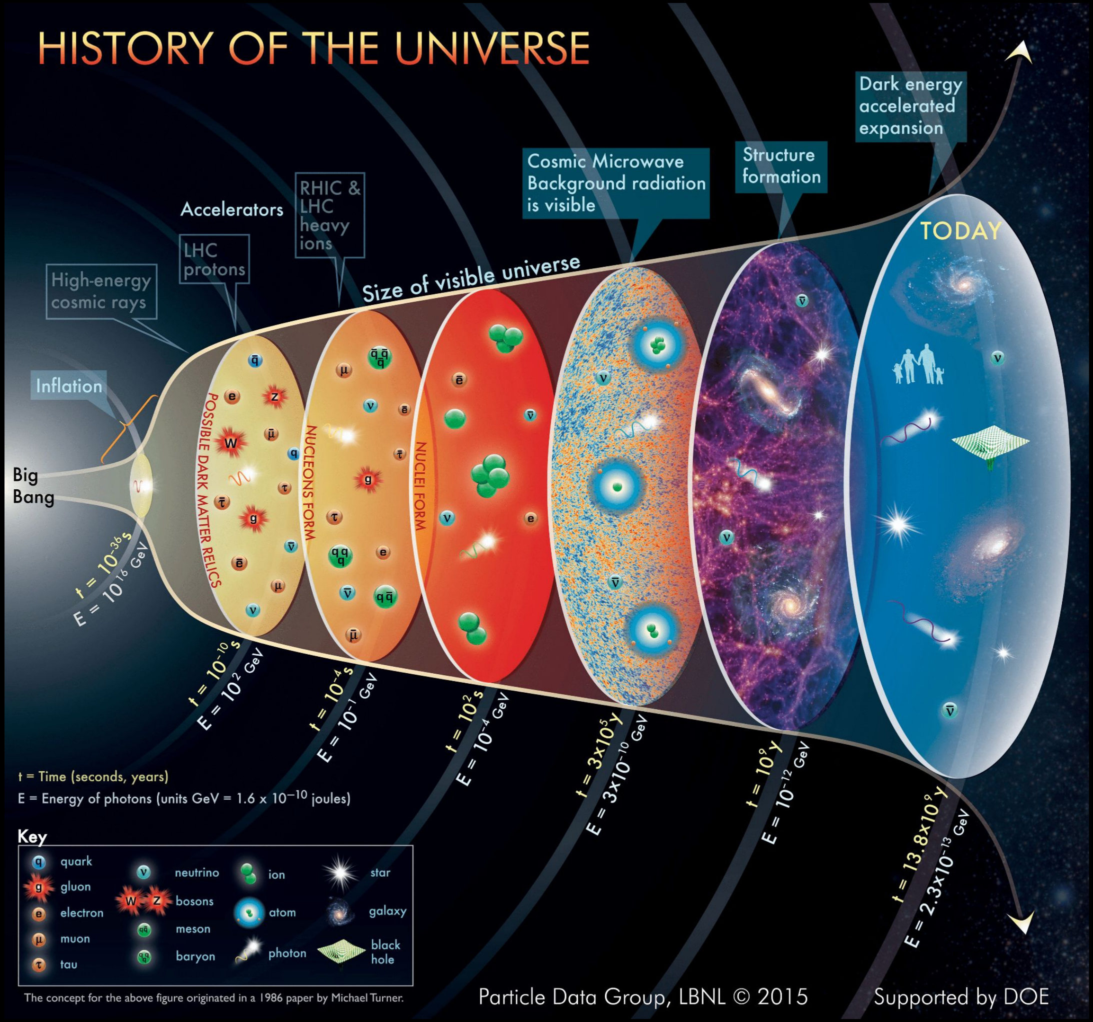
Overview

- Particle physics in the GW era: prospects & procedures
  - Understanding the first order phase transition
- (Breaking) a new horizontal SU(2) flavour symmetry
- Building the finite-temperature effective potential
  - “Parwani”/Truncated Full Dressing
  - Dimensional Reduction
- Is there a first-order phase transition?

What do gravitational waves have to offer particle physicists?

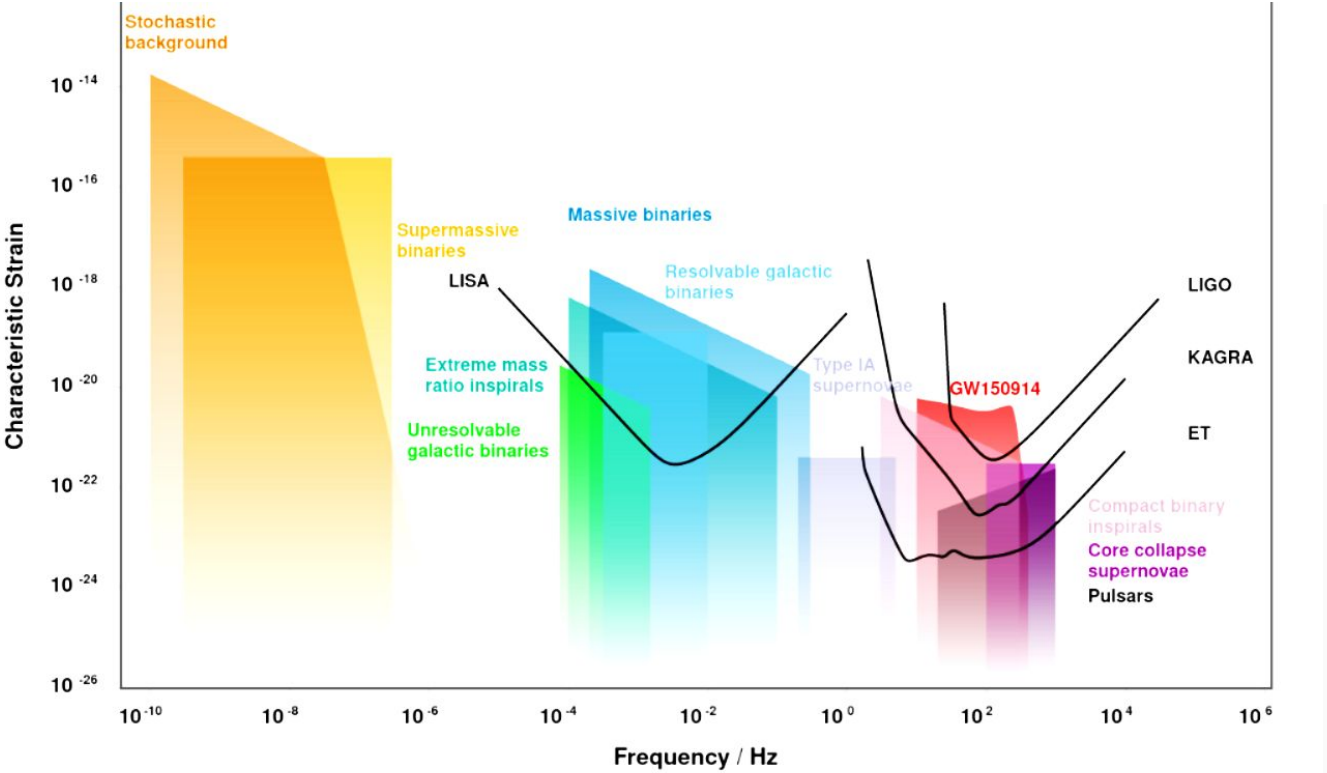

HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Particle Data Group, LBNL © 2015
Supported by DOE

The allure of gravitational waves

- ➔ *explore TeV-scale physics*
- ➔ *indirect constraints from gravitational wave experiments (complement collider experiments)*
- ➔ *early universe phase transitions ⇒ insights into fundamental physics e.g. symmetry breaking*
- ➔ *Phase transitions: QCD (~ 100 MeV), EW (~100 GeV)*

  Baryogenesis + baryon asymmetry, ***EWSB FOPT ↔ BSM***
- ➔ *Inflation (~ 10^{13} TeV)*
- ➔ *Exotic: cosmic strings, primordial black holes, Planck scale*
- ➔ *Plus tests of GR : > 2 polarisation states, modified dispersion relation, sub- or super-luminal propagation, etc.*

GW experiments



C. Moore, R. Cole, & C. Berry's GWplotter

Algorithm

model → **action** → **spectrum** → **SNR**

Build $V_{eff}(\mu,T)$

Determine field content, dof, etc.
Potential: zero-T + finite-T
Find degenerate min{$V_{eff}(\mu,T)$}

Is the PT first order ?

$$\frac{\phi_c}{T_c} \geq 1$$

Compute PT parameters

Compute Euclidean / 3d action
Extract phase transition parameters:

> PT strength α
> Inverse of PT duration β/H_*
> Bubble wall speed v_w

GW spectrum

Compute energy density of GWs as a function of frequency, based on PT parameters

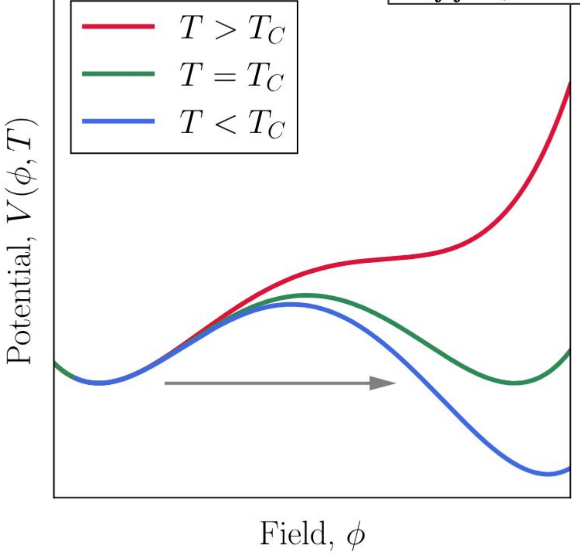
$$h^2\Omega_{\rm GW}(f;H_*,\alpha,\beta,v_w)$$

Sensitivity of detectors

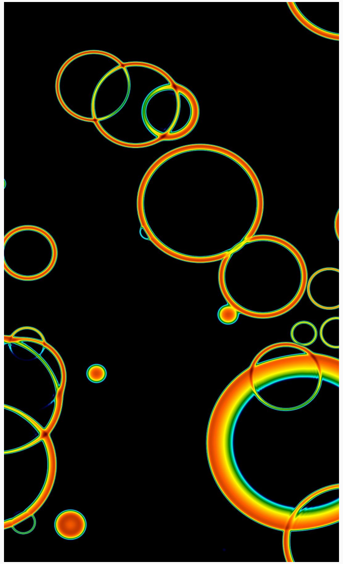
Compare GW power spectrum against detector sensitivity curve

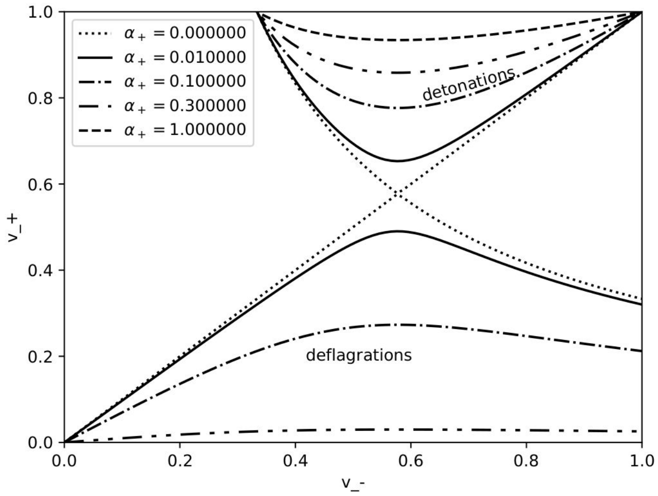
Algorithm

$$V_{eff}\,(\mu, T) = V_{tree} + V_{CW} + V_T$$



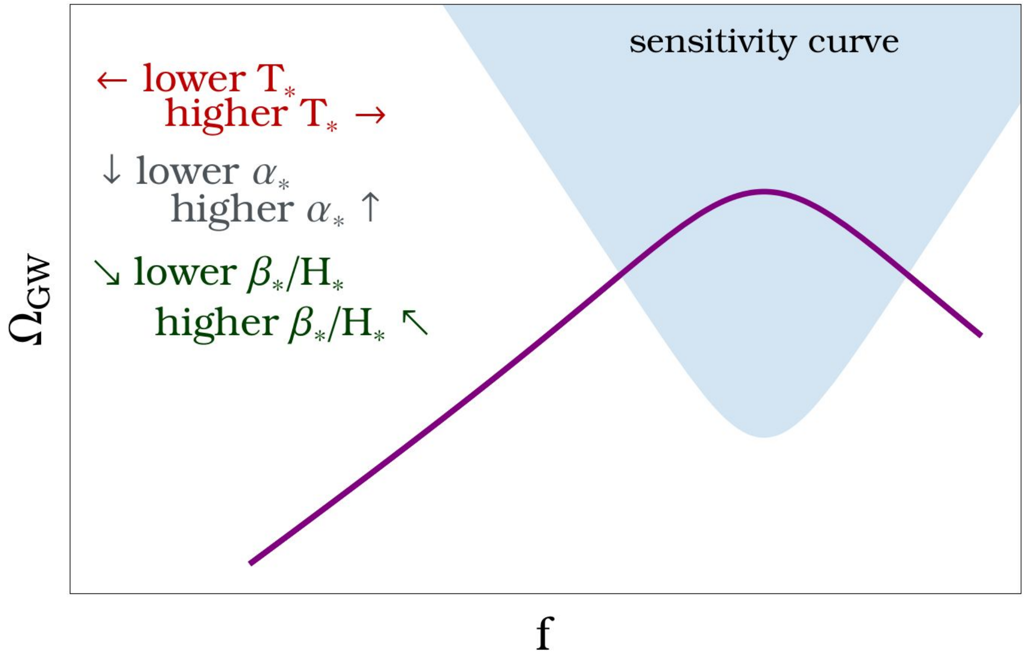
$$\frac{\phi_c}{T_c} \geq 1$$



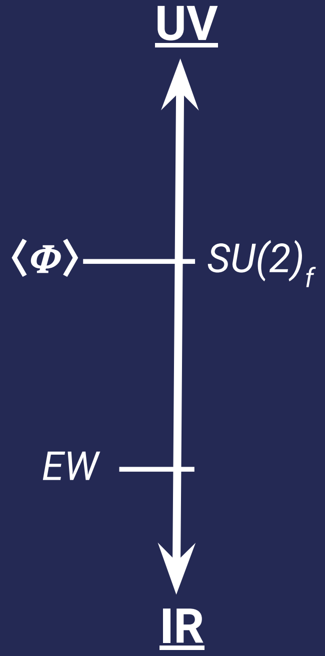


GW spectrum

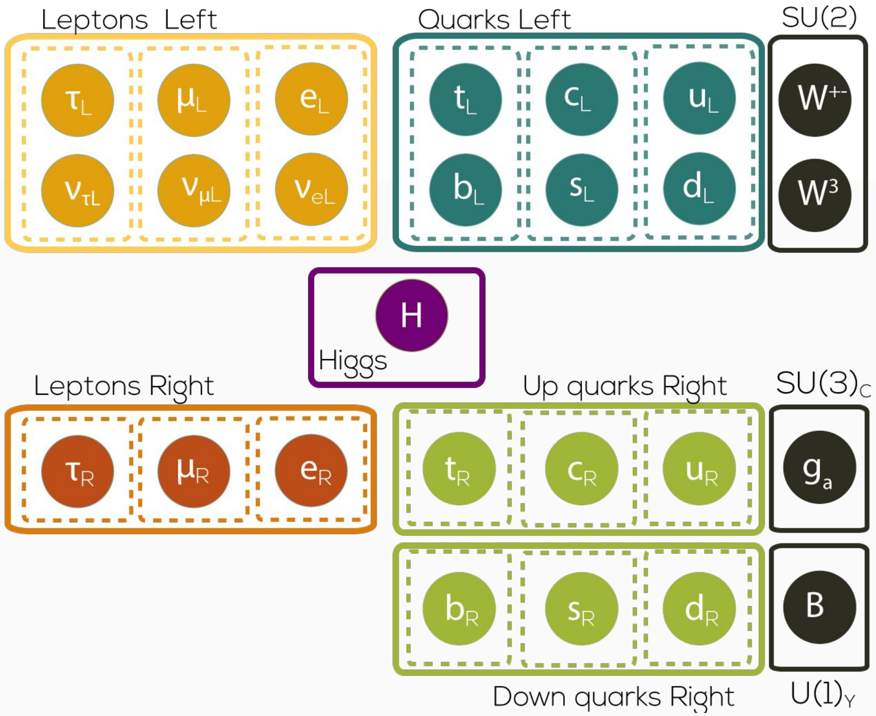
Piecewise function: broken power law joined at f_{peak}



Breaking a new « horizontal gauge symmetry » in the flavour sector



Horizontal flavour gauge group



The SM has a large global $U(3)^5$ symmetry group

→broken by the Yukawa interactions

$$\mathcal{L}_Y = -Y^d_{ij}\,\overline{Q^I_{Li}}\,\phi\, d^I_{Rj} - Y^u_{ij}\,\overline{Q^I_{Li}}\,\epsilon\,\phi^* u^I_{Rj} + \text{h.c.},$$

We can gauge a subset of this group ?

→U(1) case: Frogatt-Nielsen constructions, $L_\mu - L_\tau$, flavons, etc…

The non-abelian case has been sparsely studied.

→In any case the new gauge coupling is a free parameter

SU(2) flavour gauge groups

Starting point: add a new SU(2) gauge group in the SM, acting on flavour space

→ The « charged» SM fermion can be either part of a doublets or a triplet

→ Only the mixed $SU(2)_f^2 \times U(1)_Y$ anomaly is non-zero

$$A = ([C(Q_i) - C(L_i)] - [2C(u_{R,i}) - C(d_{R,i}) - C(e_{Ri})])$$

Gauge boson masses are free parameters!

 Even with a large VEV, small gauge couplings (required by flavour constraints imply light new states

For instance: left-handed scenario with $(12)_\ell(12)_{Q_L}$ interactions

→ Reduce the number of fundamental fermions

→Couples both to LH leptons and LH quarks

3 new « W-like » gauge bosons carrying a « flavour-charge »

$$M_{V_1}^2 = M_{V_2}^2 = M_{V_3}^2 = \frac{g_f}{2}\sum_i v_\phi^2$$

+ *rotation matrices to mass basis: V_{uL}, V_{dL}, ...*

Masses and textures

The presence of $SU(2)_f$ implies that the fermion mass matrices have a structure: let us focus on a left-handed model with Q_i, L_i

→We introduce δY_i , a $SU(2)_f$ spurion

→In the most generic case, this does not distinguish first and second generation

$\delta Y_i = (\delta Y, 0)$

$$L \supset y_d^\alpha \delta Y_i \, \bar{Q}^i \cdot H \, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H \, d_{R,\alpha} + Y_{3,d} \bar{Q}_3 \cdot H \, b_R$$

$$L \supset \delta Y(\bar{Q}^1 \cdot H \,(y_d^\alpha d_{R,\alpha}) \\ -\delta Y \,(\bar{Q}^2 \cdot H \,(\tilde{y}_d^\alpha d_{R,\alpha}) \\ +Y_{3,d} \bar{Q}_3 \cdot H \, b_R$$

α are generation indices but NOT gauge indices

i,j are $SU(2)_f$ gauge indices

We use the $U(3)_f$ global reparametrisation for $d_{R,\alpha}$

Arranging $\delta Y \ll Y_3$ still leads to the same mass scale for first and second generation

L. Darmé, A. Deandrea, F. Mahmoudi's *Gauge SU(2)$_f$ flavour transfers*

Masses and textures

How can we generate a hierarchy between 1st and 2nd generation ?

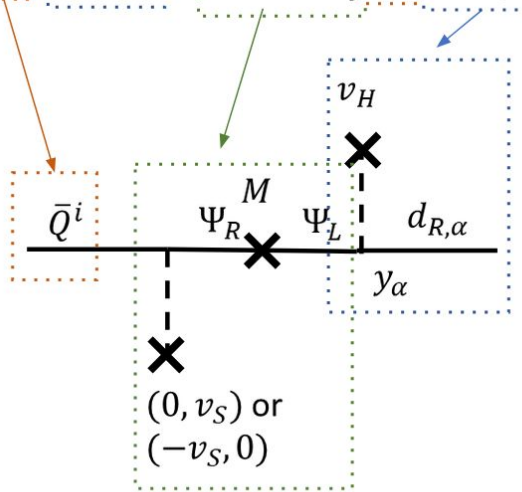
→ Standard approach: add another U(1) factor distinguishing 1st and 2nd

and therefore a new spurion…

→ We take a step back and realise that y_d^α and $\tilde{y}_d^\alpha$ are not necessarily independent parameters

→ Let's consider a simple model with a $SU(2)_f$ breaking scalar S_i and a VL quark

$y_d^\alpha \delta Y_i \bar{Q}^i \cdot H\, d_{R,\alpha} + \tilde{y}_d^\alpha \delta Y^{\dagger,i} \epsilon_{ij} \bar{Q}^j \cdot H\, d_{R,\alpha}$



A simple VL-driven example

$$L \supset \delta Y(\bar{Q}^1 \cdot H\,(y_d^\alpha d_{R,\alpha}) - \delta Y\,(\bar{Q}^2 \cdot H\,(\tilde{y}_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

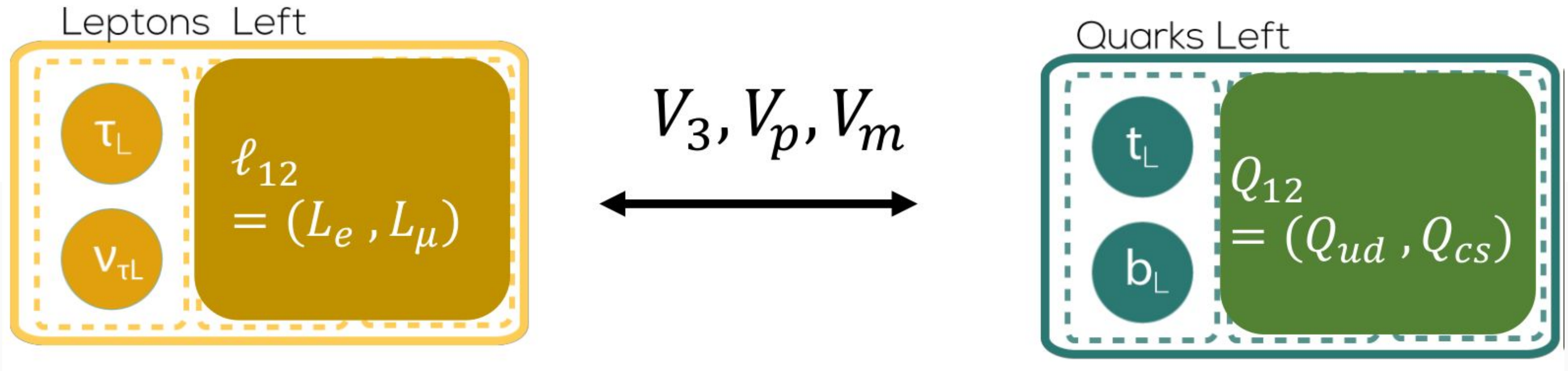
Leads to $y_d^\alpha \propto \tilde{y}_d^\alpha$

→ The down-quark mass matrix is only rank 2

$$L \supset \delta Y(\bar{\tilde{Q}}^2 \cdot H\,(y_d^\alpha d_{R,\alpha}) + Y_{3,d} \bar{Q}_3 \cdot H\, b_R$$

□ Repeat for the third generation

The « flavour-transfer » mechanism



rather than break flavour, the new gauge bosons transfer flavour from one fermionic sector to another

A flavour-violating transition ΔF_f in one fermionic sector is pairwise related to $\Delta F'_f$ in another
Four-fermion operators arising from flavour gauge boson exchanges satisfy $\Delta F_f + \Delta F'_f = 0$

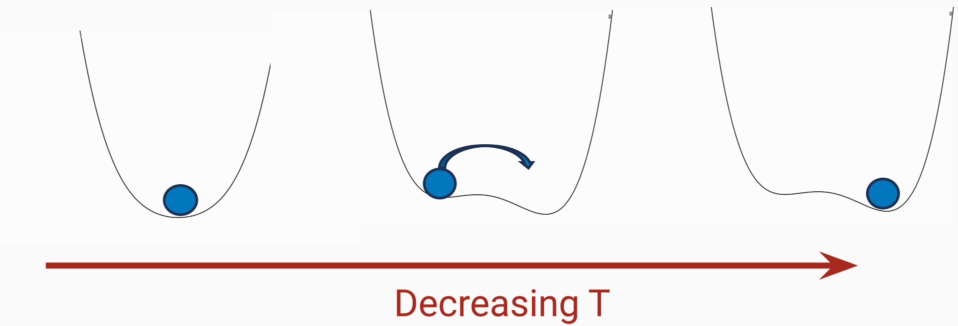
- ➔ Ensures overall balance in the flavour structure.
- ➔ Only the $SU(2)_f \times SU(2)_f \times U(1)_Y$ mixed anomaly is non-zero

True and false vacua

➔ To break the flavour gauge symmetries we need the appearance of a VEV for the new scalars

This occurs in the early universe at temperatures close to the VEV

➔ Flavour constraints point towards 100 TeV scale for the complete flavourful theory



$SU(2)_f$ and $SU(2)_W \times U(1)_Y$ symmetric theory

$\sim$ 100 TeV

$SU(2)_f$ breaking by new scalar Φ

flavour bosons

$SU(2)_W \times U(1)_Y$ symmetric theory

$\sim$ 0.2 TeV

EW breaking

EW bosons

$U(1)_{em}$ symmetric theory

T

Building the finite-temperature effective potential: Truncated Full Dressing vs Dimensional Reduction

TFD: $V_{eff}(\mu, T) \rightarrow V_{eff}(\mu + \pi T, T)$

DR: $V_{4deff}(\mu, T) \rightarrow V_{3deff}(\mu_3, T)$

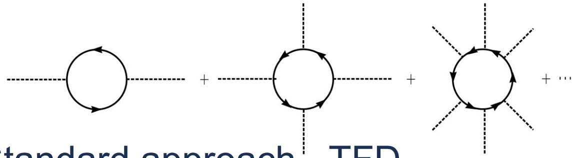
Thermal corrections : TFD vs DR

Quiros 1999,
Curtin 2006

How to compute the effective thermal potential ?

- ➔ Describe the correlation functions a QFT in a thermal bath, Greens functions can be computed by compactifying time along the imaginary direction
- ➔ Stability of the vacuum be estimated from this quantity (equivalent to free energy in thermodynamics)

Stay in 4D, every loop comes with an infinite sum from the modes in along the imaginary time direction



Standard approach - TFD

Integrate out the modes from the compactified dimension and match the 4D theory to a 3D theory
 Dimensional Reduction approach (EFT-like)

More modern approach, partially automatised through DRalgo

Effective potential: Truncated Full Dressing (Parwani)

$$V_{\text{tree}}(\phi) = -\frac{1}{2}\mu_\phi^2\phi^2 + \frac{1}{4}\lambda_\phi\phi^4 + \frac{1}{2}\mu_s^2|s|^2 + \frac{1}{4}\lambda_s|s|^4 + \frac{1}{2}\lambda_{\phi s}\phi^2|s|^2$$

Field	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$SU(2)_f$	DoF
χ	1	2	1/2	1	3
Φ	1	1	0	2	1
S	3	1	2/3	2	$3 \times 2 \times 2 = 12$
V	1	1	0	3	$3 \times 3 = 9$

$$V(\phi, T) = V_{tree}(\phi) + V_{CW}(\phi) + V_T(\phi, T)$$

$$V_{\text{CW}}(\phi) = \sum_{i=\phi,\chi,f,s} \pm \frac{n_i}{64\pi^2} m_i^4 \left[\log\left\{\frac{m_i^2}{\mu^2}\right\} - C_i\right]$$

Gives the usual log-like Coleman Weinberg terms

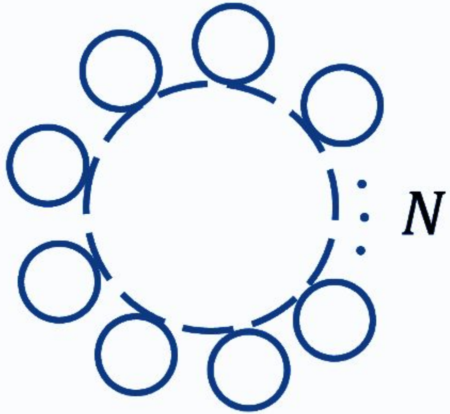
$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right)$$

$$J_{B/F}(a) = \pm\int_o^\infty dy y^2 \log\left[1 \mp e^{-\sqrt{y^2+a}}\right]$$

Effective potential: Truncated Full Dressing (Parwani)

$$m^2(\phi) = m^2_{\text{tree}}(\phi) + \Pi(\phi, T)$$

$\sim g^2\, T^2$



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$$\pi_\phi = \pi_\chi = \frac{\lambda_\phi}{2} + \frac{9}{48} g_f^2$$

$$\pi_{\text{f}}^L = \frac{3}{2} g_f^2$$

Effective potential: Truncated Full Dressing (Parwani)

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$\sim g^2\, T^2$

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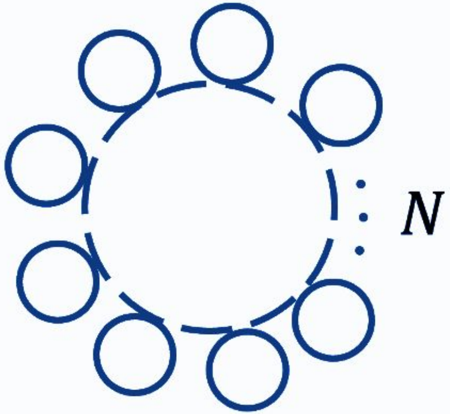

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$$\pi_{\text{f}}^L = \frac{3}{2} g_f^2$$

Effective potential: Truncated Full Dressing (Parwani)

Multiple sources of theoretical uncertainty :

- ➔ Nonperturbativity (IR modes at high T) [Linde 1980]
- ➔ Inconsistencies (non-negligible Im{V}) [Weinberg & Wu 1987; Weinberg 1992]
- ➔ higher-order perturbative corrections [Arnold & Espinosa 1992]
- ➔ gauge dependence [Laine 1994]
- ➔ renormalisation scale dependence [Farakos *et al.* 1994]

See Croon *et al.* JHEP 04 (2021) 055

3D EFT approach - mitigating errors

At zero temperature, the one-loop effective potential is renormalisation group invariant

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}(V_{\text{tree}} + V_{\text{1-loop}}) = 0.$$

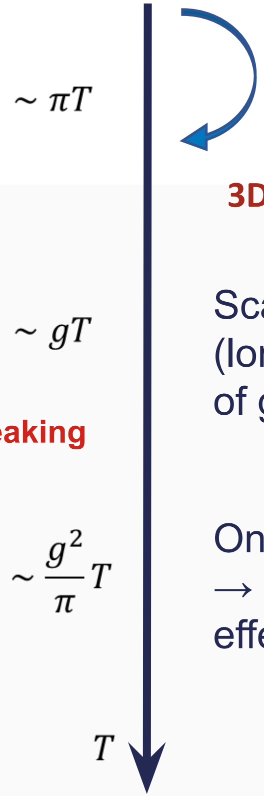
But, at high temperatures this fails, even at leading order

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}(V_{\text{tree}} + V_{\text{1-loop}}^{\text{thermal}}) \neq 0.$$

The problem can be traced to the scale hierarchy $\pi T \gg m$, and to

$$\frac{\mathrm{d}}{\mathrm{d}\log\mu}\left(\frac{1}{2}\Pi_T\phi^2\right).$$

4D theory



Decoupling of the towers of thermal modes

3D theory

Scalars + temporal (longitudinal) components of gauge bosons

Symmetry breaking

Only the lightest scalar → corresponds to the effective potential

Gould, Tenkanen, *JHEP* 06 (2021)

3D EFT approach - mitigating errors

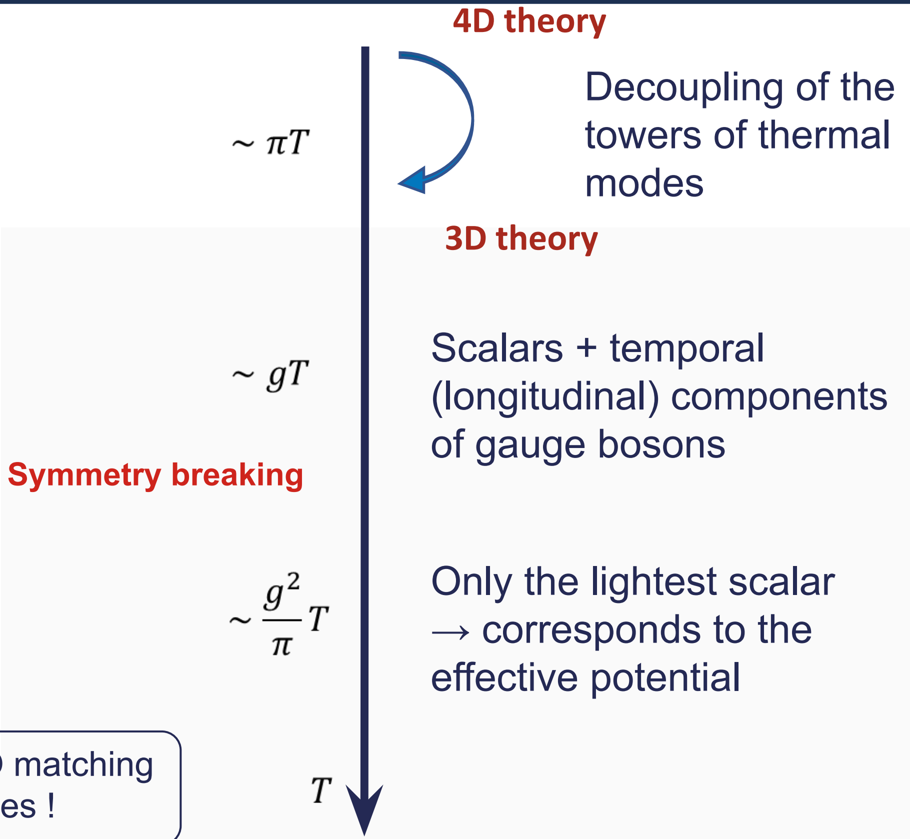
Step-by-step approach to decouple all thermal DoF

1. RGE from μ_{ini} to μ_{hard}
2. Match 4d to 3d at « hard scale »

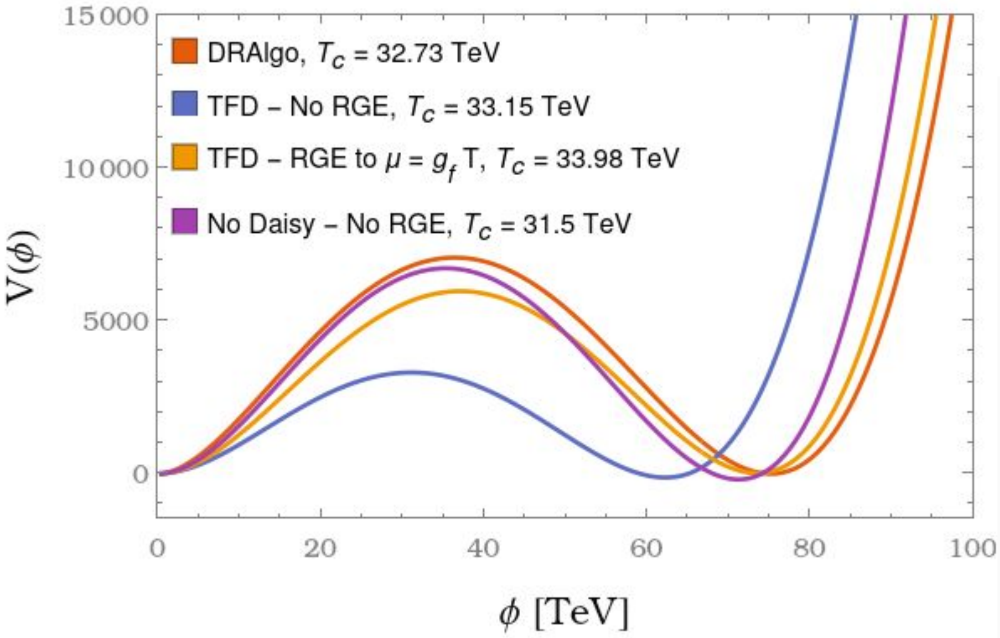
   $\mu_{hard} \sim \pi T$ (thermal mass of fermions + transverse gauge bosons)
3. Run gT in the 3d theory
4. Decouple remaining bosonic modes, except scalar field ϕ triggering the PT

Implement using DRalgo

Up to NNLO matching in some cases !



Compare against DRalgo (4d)



μ_{ini} = 50 TeV

$$\{\lambda_\phi, g_f^2, \lambda_s\} \sim \{0.0075, 0.7500, 0\}$$

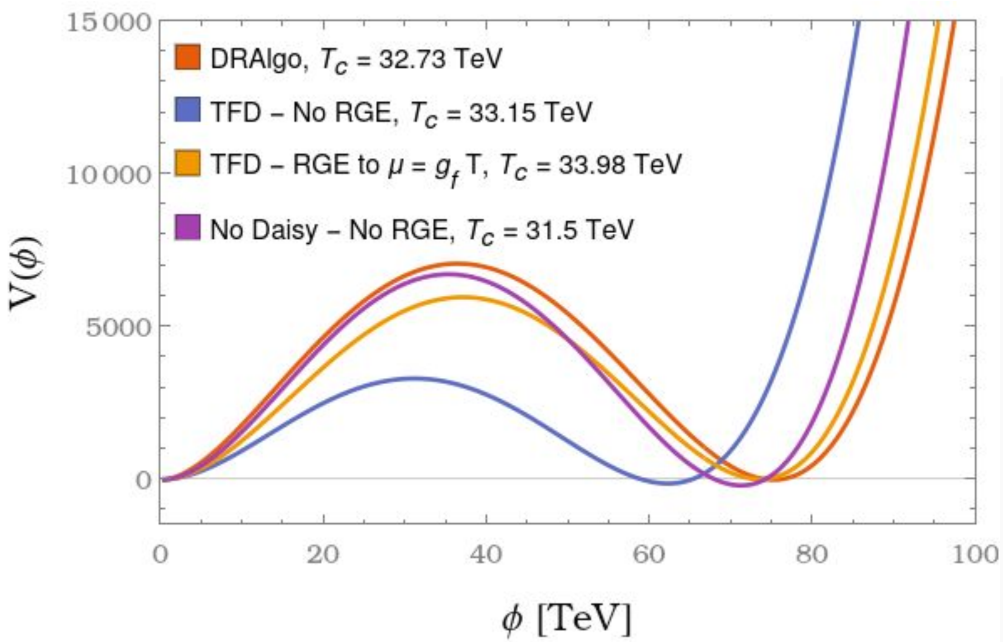
μ_{ini} = 30 TeV

$$\{\lambda_\phi, g_f^2, \lambda_s\} \sim \{0.0036, 0.7618, 0\}$$

$$V_{4d} = V_{3d} \times T$$

Compare against DRalgo (4d)

Preliminary results



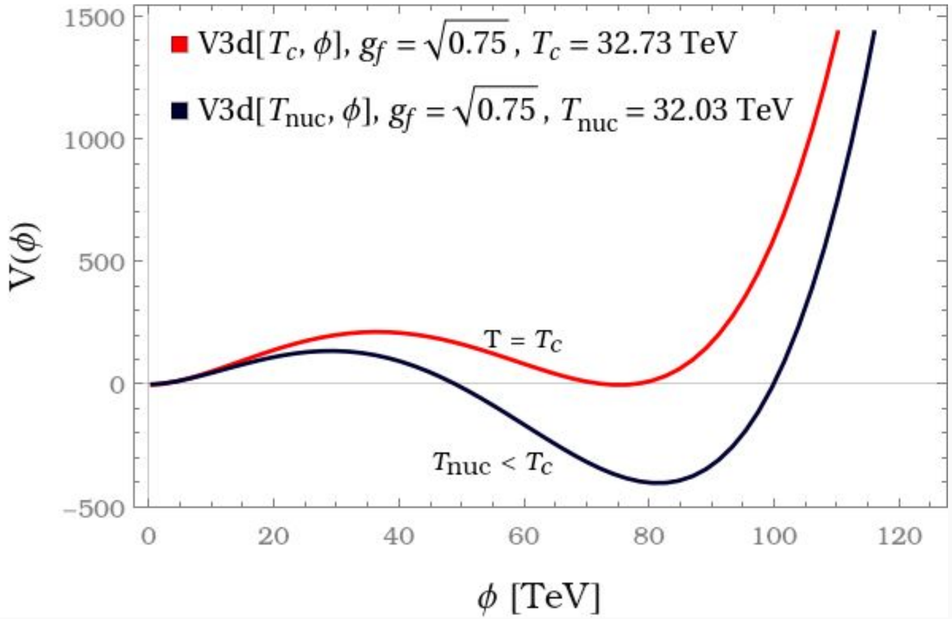
- $\frac{\phi_c}{T_c} \sim 2.30$
- $\frac{\phi_c}{T_c} \sim 1.88$
- $\frac{\phi_c}{T_c} \sim 2.16$
- $\frac{\phi_c}{T_c} \sim 2.26$

Consistently first order, of similar strength

Phase transition parameters using DRalgo (3d)

Preliminary results

For $\lambda_\phi = 0.0075,\ M_\phi = 10\sqrt{2}\,\text{TeV},\ \mu_\phi = -M_\phi^2/2$



$$\frac{\beta_*}{H_*} \approx 173.20$$

$$\alpha \approx 0.046$$

$$v_w \sim 1$$

Phase transition parameters using DRalgo (3d)

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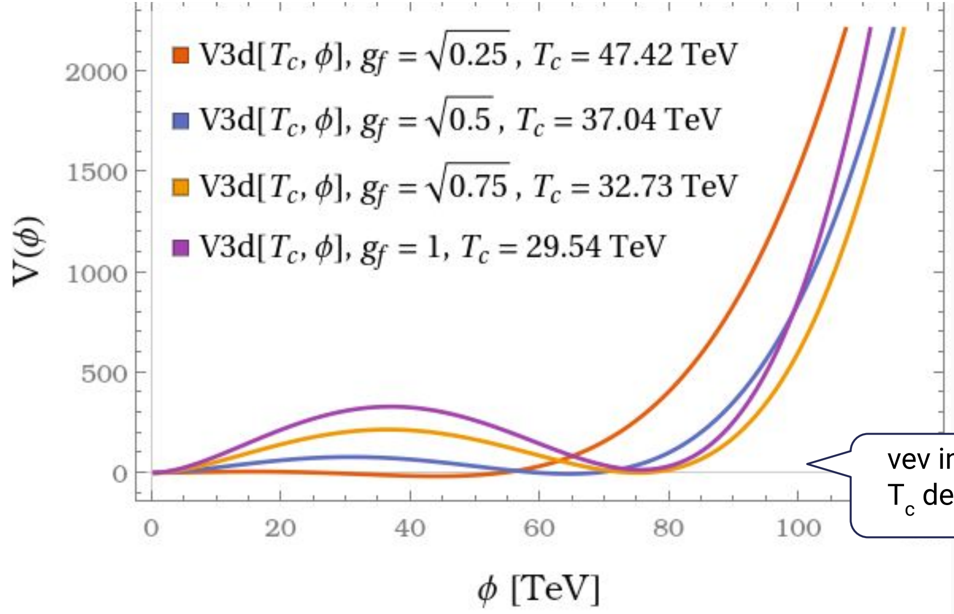


vev increases with g_f
T_c decreases with g_f

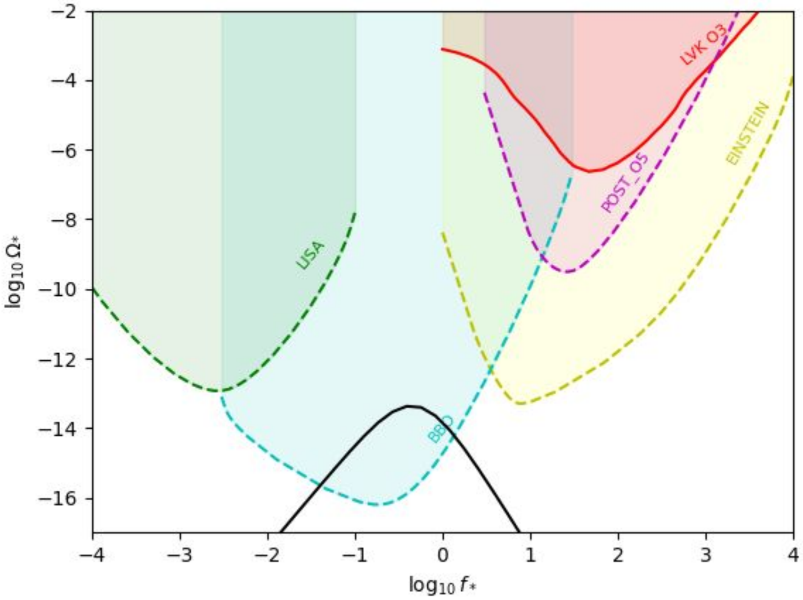
- (red) $\frac{\phi_c}{T_c} \sim 0.93$
- (blue) $\frac{\phi_c}{T_c} \sim 1.74$
- (yellow) $\frac{\phi_c}{T_c} \sim 2.30$
- (purple) $\frac{\phi_c}{T_c} \sim 2.56$

No FOPT if g_f is too small !

Expected GW spectrum

***Preliminary** results*

For $\lambda_\phi = 0.0075,\ M_\phi = 10\sqrt{2}\,\text{TeV},\ \mu_\phi = -M_\phi^2/2$



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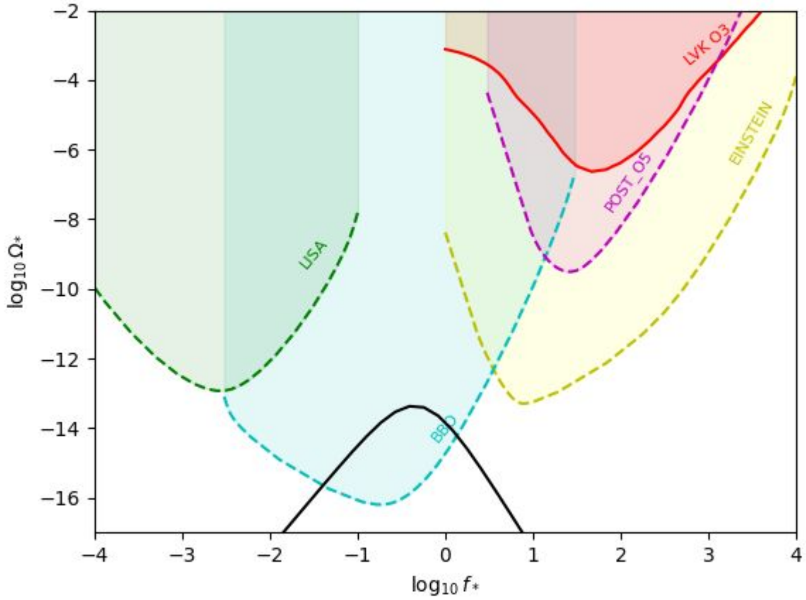


LIGO prospects : not great
Probable causes :
– very small phase transition strength
– large inverse-duration (i.e. short PT)

Need to understand fully to resolve

Einstein & future detectors still viable

Conclusions

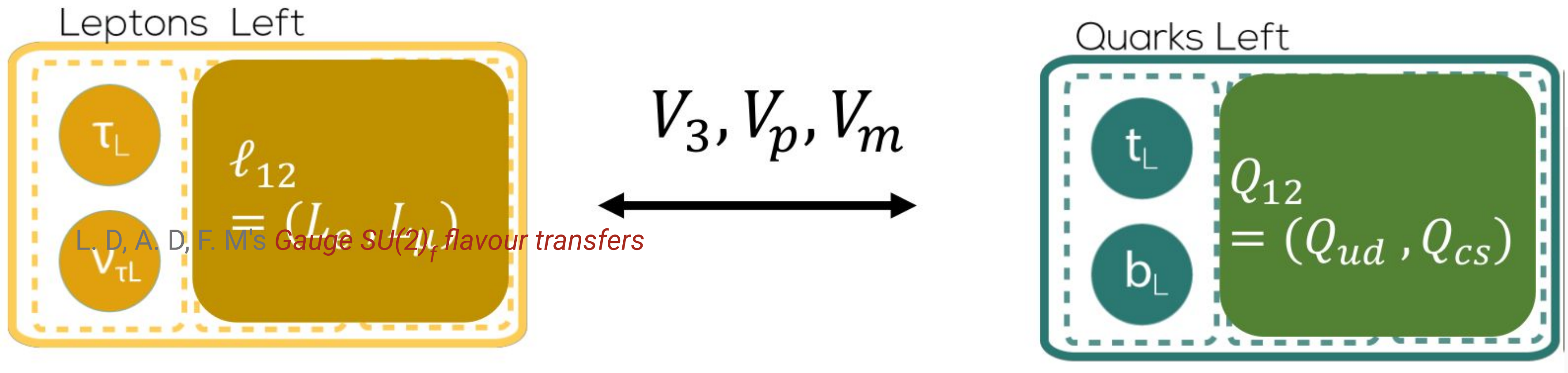
- Most models of flavour relies on broken symmetries to create the observed patterns in the SM-Higgs Yukawa couplings
- For flavour gauge symmetries, this means introducing new Higgs-like scalars, that can undergo first order phase transitions in the early universe
- Cooler phase transition for heavier flavour bosons
- Ongoing work: to finalise the effective potential based on two different approaches
  - Still discrepancies to be ironed out / understood
- The temperature range corresponding to actual flavour constraints matches the realm of LIGO/Einstein telescope range (if the PT can be made strongly-enough first order)
  - Remain: hydrodynamics simulation to improve GW spectrum predictions for our $SU(2)_f$ model

Thanks!

Any questions?

Or reach out via email using
acornell@uj.ac.za

The « flavour-transfer » mechanism



L. D, A. D, F. M's *Gauge $SU(2)_f$ flavour transfers*

rather than <u>break</u> flavour, the new gauge bosons <u>transfer</u> flavour from one fermionic sector to another

$$V_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad V_p = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad V_m = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

The corresponding generators in flavour space

Temperature corrections

$$V_T(\phi, T) = \sum_i \frac{n_i T^4}{2\pi^2} J_B\left(\frac{m_i^2}{T^2}\right) + \sum_i \frac{n_i T^4}{2\pi^2} J_F\left(\frac{m_i^2}{T^2}\right)$$

$$J_{B/F}(a) = \pm \int_o^\infty dy y^2 \log\left[1 \mp e^{-\sqrt{y^2+a}}\right]$$

$$J_{B,F}^{low}(a) \approx -\sqrt{\frac{\pi}{2}} a^{3/4} e^{-\sqrt{a}} \left(1 + \frac{15}{8} a^{-1/2} + \frac{105}{128} a^{-1}\right)$$

$$J_B^{high}(a) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12} a - \frac{\pi}{6} a^{3/2} - \frac{a^2}{32}\left(\log(a) - c_B\right)$$

$$J_B(a) \approx e^{-\left(\frac{a}{6.3}\right)^4} J_B^{high}(a) + \left(1 - e^{-\left(\frac{a}{6.3}\right)^4}\right) J_B^{low}(a)$$

Uncertainties

$\Delta\Omega_{\mathrm{GW}}/\Omega_{\mathrm{GW}}$	4d approach	3d approach
RG scale dependence	$\mathcal{O}(10^2 - 10^3)$	$\mathcal{O}(10^0 - 10^1)$
Gauge dependence	$\mathcal{O}(10^1)$	$\mathcal{O}(10^{-3})$
High-T approximation	$\mathcal{O}(10^{-1} - 10^0)$	$\mathcal{O}(10^0 - 10^2)$
Higher loop orders	unknown	$\mathcal{O}(10^0 - 10^1)$
Nucleation corrections	unknown	$\mathcal{O}(10^{-1} - 10^0)$
Nonperturbative corrections	unknown	unknown

Sources of theoretical uncertainty and relative importance quantified by the parameter ΔΩGW /ΩGW over the range M = {580 − 700} GeV in the SMEFT. Although we do not have reliable estimates for the uncertainties of the 4d approach due to higher loop orders and nucleation corrections, they are expected to be much larger than the corresponding uncertainties of the 3d approach

Power counting

To illustrate next-to-leading order dimensional reduction, we consider a schematic model with scalar mass parameter μ^2, scalar quartic coupling λ, and gauge coupling g. Given the power counting $\mu^2 \sim g^2T^2$, $\lambda \sim g^2$, the matching of the mass parameter is

$$\bar{\mu}_3^2 = \underbrace{\boxed{\overset{\text{tree-level}}{\mu^2} + \overset{\text{1-loop}}{\#g^2T^2}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2\mu^2} + \overset{\text{2-loop}}{\#g^4T^2}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)$$

$$+ \underbrace{\boxed{\overset{\text{1-loop}}{\#g^2m_{\rm D}}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\overset{\text{2-loop}}{\#g^4}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\ , \qquad (1.3)$$

where the first line (with even powers of g) results from the first step, and the second line (with odd power of g) from second step of the dimensional reduction. In practice, *full* $\mathcal{O}(g^4)$ contributions are included. Going to higher orders, requires a three-loop computation for both steps of the dimensional reduction. The situation is similar for the coupling:

Power counting

$$\bar{\lambda}_3 = \underbrace{\boxed{\begin{matrix}\text{tree-level}\\ T\lambda\end{matrix}}}_{\mathcal{O}(g^2)} + \underbrace{\boxed{\begin{matrix}\text{1-loop}\\ \# g^4\end{matrix}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^6)$$

$$+ \underbrace{\boxed{\begin{matrix}\text{1-loop}\\ \# \frac{g^4}{m_\mathrm{D}}\end{matrix}}}_{\mathcal{O}(g^3)} + \underbrace{\boxed{\begin{matrix}\text{2-loop}\\ \# \frac{g^6}{m_\mathrm{D}^2}\end{matrix}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\,. \qquad (1.4)$$

Power counting

$$V_{\text{eff}}^{\text{3d}} = \underbrace{V_{\text{tree}}^{\text{3d}}}_{\mathcal{O}(g^2)} + \underbrace{V_{\text{1-loop}}^{\text{3d}}}_{\mathcal{O}(g^3)} + \underbrace{V_{\text{2-loop}}^{\text{3d}}}_{\mathcal{O}(g^4)} + \mathcal{O}(g^5)\ . \qquad (1.5)$$