EARLY UNIVERSE PHASE TRANSITIONS IN A CLASSICALLY SCALE INVARIANT STANDARD MODEL

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MOTIVATION FOR SCALE INVARIANCE

THE HIERARCHY PROBLEM

- Formulate the Standard Model in terms of a cutoff energy scale for new physics, Λ.

- Physical mass m_{phys} of the Higgs differs from the bare mass m_{bare}

$\boldsymbol{G_F(x-y)} =$

$+ \ldots$

$$m_{h\,phys}^2 = m_{h\,bare}^2 + A\,\Lambda^2 + B\,m_{h\,bare}^2 \log\frac{\Lambda^2}{m_{h\,bare}^2} + \cdots$$

$$m_{h\,phys} \approx 125\ \text{GeV} \qquad \Lambda > 10^4\ \text{GeV}$$

- No symmetry protecting the Higgs

MOTIVATION FOR SCALE INVARIANCE

CLASSICAL SCALE INVARIANCE AS A SOLUTION

- Scaling transformation:
  - Energy: $E \rightarrow \lambda\, E$
  - Momentum: $p \rightarrow \lambda\, p$
  - Distance: $x \rightarrow \lambda^{-1}\, x$
  - Time: $\mathrm{t} \rightarrow \lambda^{-1}\, \mathrm{t}$
  - Scalar field amplitude: $\varphi \rightarrow \lambda\, \varphi$
  - Fermion field amplitude: $\psi \rightarrow \lambda^{3/2}\, \psi$
- Classical scale invariance: $S \rightarrow S$ $\mathcal{L} \rightarrow \lambda^4 \mathcal{L}$
- Spontaneously broken by a quantum anomaly

Technical naturalness

Classical scale invariance in the UV is broken by the anomaly → Radiative corrections to $m_{h\,phys}$ are ∝ the anomaly and can remain small

Λ - Energy cutoff scale
λ_0- Cosmological constant
λ_h - Higgs self-interaction parameter
v_h - Higgs vacuum expectation value
λ_χ - Dilaton self-interaction parameter
v_χ - Dilaton vacuum expectation value
ξ – Higgs-dilaton coupling strength
α – ratio between Λ and v_χ

HOW SCALE INVARIANCE IS REALISED

PROMOTION OF DIMENSIONFUL PARAMETERS

- Replace all dimensionful parameters (m_h, Λ, λ_0) with χ and a dimensionless parameter

$$V(H) = \lambda_h(\Lambda)\,(\mathrm{H}^\dagger H\ - v_h^2(\Lambda))^2 + \lambda_0(\Lambda)$$

$$\Lambda \to \alpha\chi$$

$$V(H,\chi) = \lambda_h(\alpha\chi)\,(\mathrm{H}^\dagger H\ - \frac{\xi(\alpha\chi)}{2}\chi^2)^2 + \lambda_\chi(\alpha\chi)\chi^4$$

- χ develops a vacuum expectation value v_χ (dimensional transmutation)

- All other scales are set by v_χ

$$\Lambda = \alpha v_\chi$$

$$m_h^2 = \lambda_h(\alpha v_\chi)\xi(\alpha v_\chi)v_\chi^2$$

$$\lambda_0 = \lambda_\chi(\alpha v_\chi)v_\chi^4$$

EXISTENCE OF A MINIMUM

- Minimisation conditions

$$\frac{\partial V}{\partial \chi}_{\chi=v_\chi, h=v_h} = 0\,, \frac{\partial V}{\partial h}_{\chi=v_\chi, h=v_h} = 0$$

- Vanishing cosmological constant

$$V(v_h, v_\chi) = 0$$

- Experimental top mass:

$$m_t = 172.52 \pm 0.14 \pm 0.30$$

- Borderline but modified by other new physics

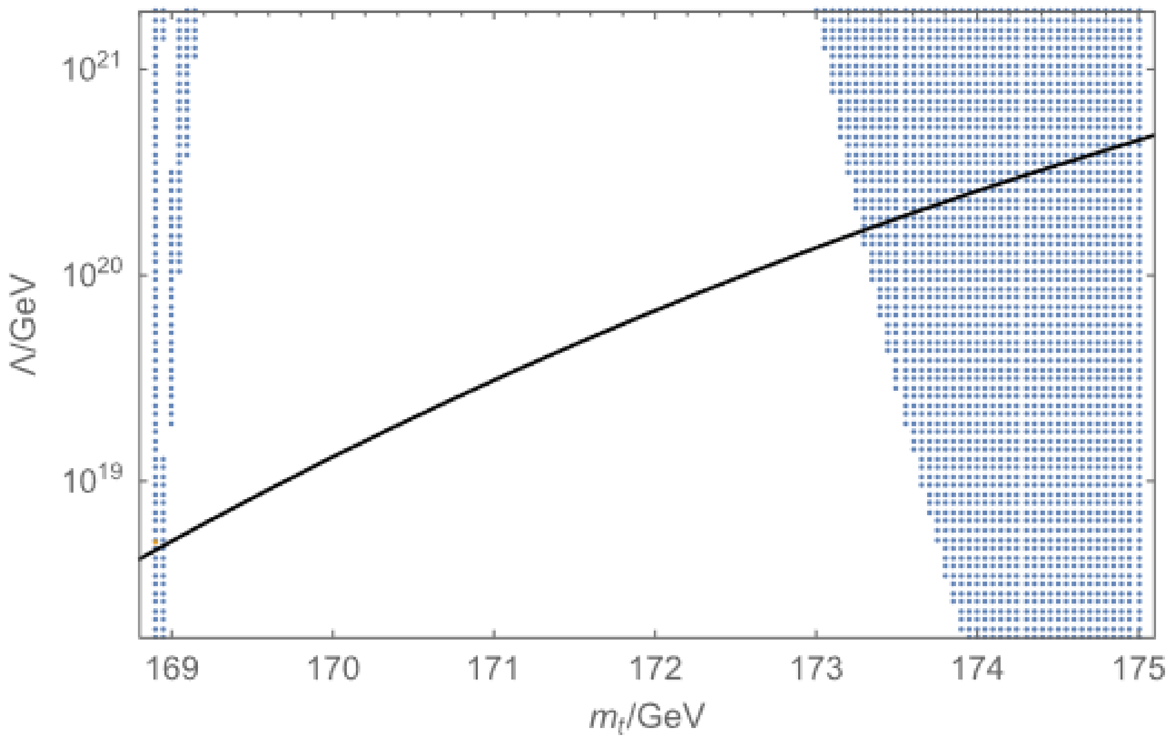


Fig. 1. Plot of the allowed range of parameters (shaded region) with $m_\chi^2(v_{ew}) > 0$, i.e., the electroweak vacuum being a minimum. The solid line displays the cut-off scale Λ as function of the top-quark mass m_t for which the conditions in Eq. (6) are satisfied.

Arunasalam et al., 2017

INITIAL IMPLICATIONS AND VIABILITY OF SCALE INVARIANCE

COUPLING AND MASS OF THE DILATON

- Assuming $\alpha \equiv \frac{\Lambda}{v_\chi} \sim 1$ and $\Lambda \sim 10^{19}\,\mathrm{GeV}$
- Then for $m_h \approx 125$ GeV, $\xi(\Lambda) = 2\frac{v_h^2}{v_\chi^2} \sim 10^{-34}$ so a very weak coupling between h and χ
- Dilaton develops a mass at second loop level (assuming a vanishing cosmological constant):

$$m_\chi^2 = \frac{\beta'_{\lambda\chi}(\Lambda)}{4\,\xi(\Lambda)} v_h^2(\Lambda) \sim (10^{-8}\,\mathrm{eV})^2$$

- Makes it a light dark matter candidate!

INITIAL IMPLICATIONS AND VIABILITY OF SCALE INVARIANCE

FIFTH FORCE AND EQUIVALENCE-PRINCIPLE CONSTRAINTS

- $\mathcal{L} \supset \frac{\chi}{\Lambda} d_{m_e} m_e \bar{\psi}\psi$
- No tree-level coupling to electron mass so expect $d_{m_e} \sim 10^{-2}$

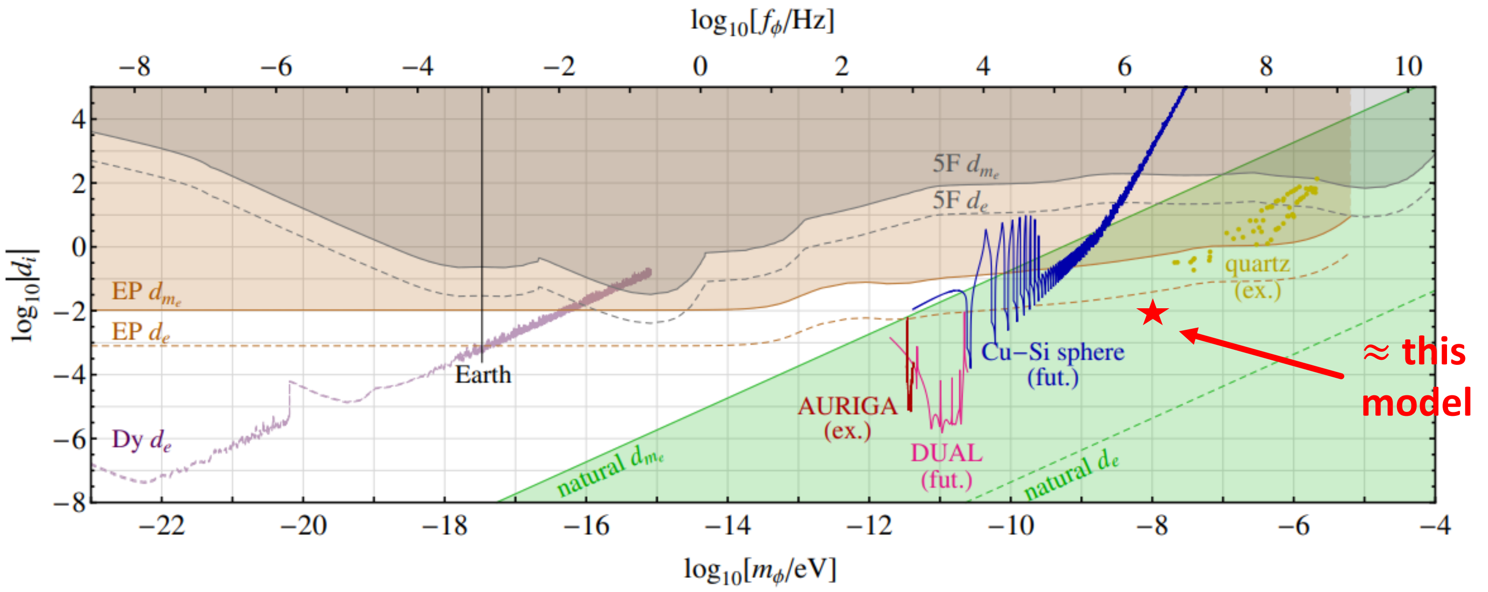


FIG. 1. Scalar field parameter space, with mass m_ϕ and corresponding DM oscillation frequency $f_\phi = m_\phi/2\pi$ on the bottom and top horizontal axes, and couplings of both an electron mass modulus ($d_i = d_{m_e}$) and electromagnetic gauge modulus ($d_i = d_e$) on the vertical axis. Natural parameter space for a 10 TeV cutoff is depicted in green, while the other regions and dashed curves represent 95% CL limits from fifth-force tests ("5F", gray), equivalence-principle tests ("EP", orange), atomic spectroscopy in dysprosium ("Dy", purple), and low-frequency terrestrial seismology ("Earth", black). The blue curve shows the projected SNR = 1 reach of a proposed resonant-mass detector—a copper-silicon (Cu-Si) sphere 30 cm in radius—after 1.6 y of integration time, while the red curve shows the reach for the current AURIGA detector with 8 y of recasted data. Rough estimates of the 1-y reach of a proposed DUAL detector (pink) and several harmonics of two piezoelectric quartz resonators (gold points) are also shown.

Arvanitaki, Dimopoulos, and Tilburg, 2016

INITIAL IMPLICATIONS AND VIABILITY OF SCALE INVARIANCE

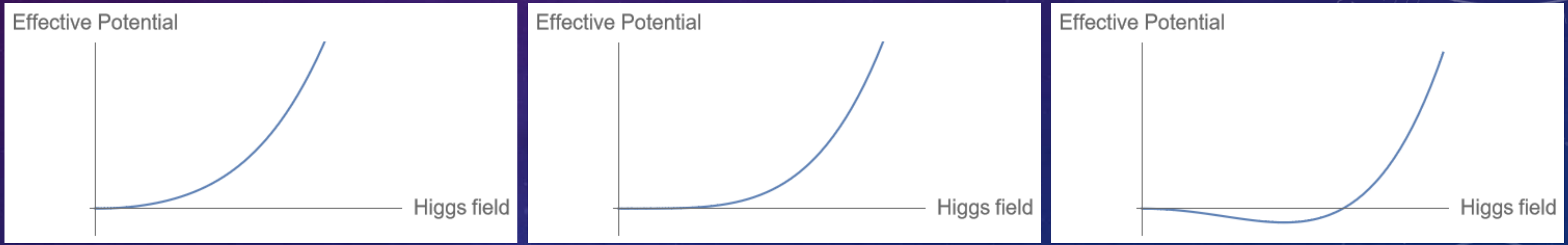
THE ELECTROWEAK PHASE TRANSITION

Increasing time

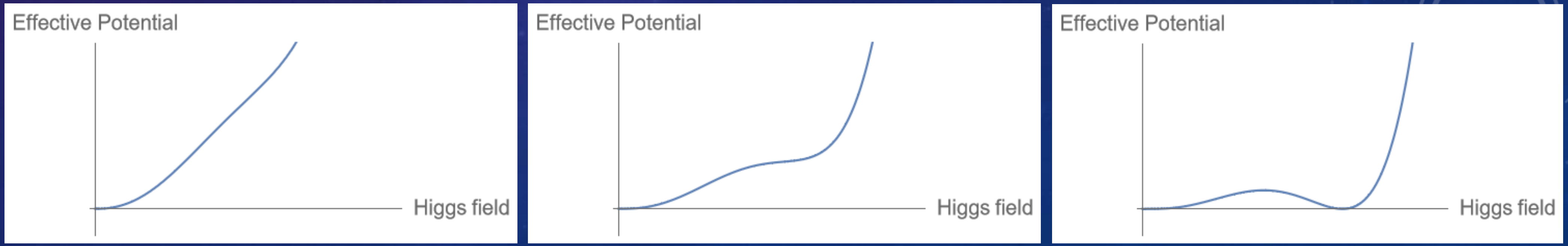
Decreasing temperature

T = 0

Standard Model:



Scale invariant Standard Model: (along $\frac{\partial V}{\partial \chi} = 0$)



T = 0?

INITIAL IMPLICATIONS AND VIABILITY OF SCALE INVARIANCE

THE CHIRAL PHASE TRANSITION

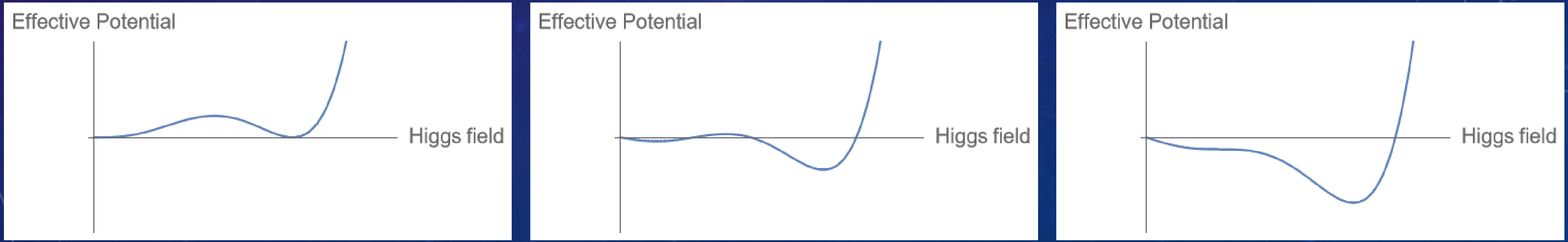
- Yukawa couplings: $V_{yukawa} = y_u^{ij}\ Q^i u^j \widetilde{H} + y_d^{ij} Q^i d^j H + h.c.$ are linear in the Higgs field

- When the quark condensate $\langle \bar{q} q \rangle \neq 0$, the Yukawa terms contribute a linear term to the Higgs effective potential

- Naively expect:

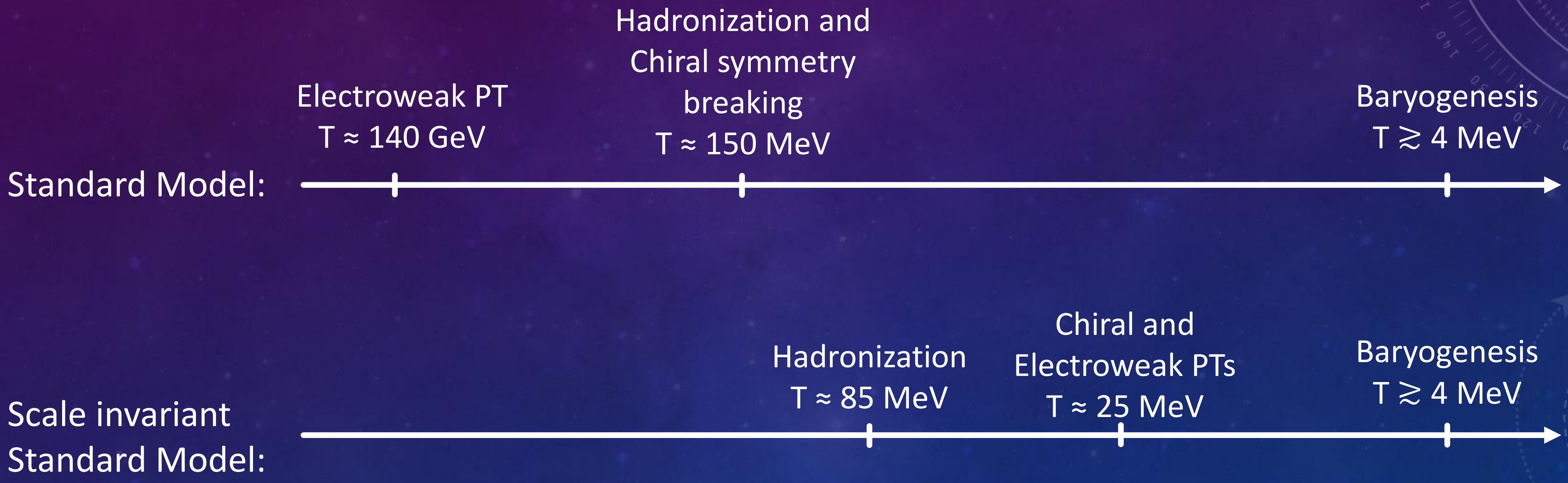
Increasing time

Decreasing temperature



INITIAL IMPLICATIONS AND VIABILITY OF SCALE INVARIANCE

COSMOLOGICAL TIMELINE



MODEL CHECKLIST

- Should be $SU(6)_V \times SU(6)_A$ symmetric (except for a mass term)
- Needs an order parameter σ and symmetry breaking potential $V(\sigma)$ to model the chiral symmetry breaking
- Integrating out σ should reduce the theory to the usual non-linear sigma model of pions

$$\mathcal{L} = \frac{f_\pi^2}{4} Tr\left(\partial_\mu U \partial^\mu U\right) + \kappa\, Tr(UM) + h.c.$$

$$U = e^{2i\frac{\Pi^a T^a}{f_\pi^2}}$$

- Should be classically scale invariant

σ – Chiral transition order parameter
Π^a - 35 pion fields
T^a - 35 SU(6) generators
y_i - Yukawa couplings
n_f - Number of flavours = 6
Λ_{QCD} - QCD confinement scale

LINEAR SIGMA MODEL OF CHIRAL AND ELECTROWEAK SYMMETRY BREAKING

LAGRANGIAN AND EFFECTIVE POTENTIAL

$$\mathcal{L} = Tr(\partial_\mu \Phi \partial^\mu \Phi) - \lambda_\sigma \left(Tr(\Phi^\dagger \Phi) - n_f v_\sigma^2\right)^2 + \lambda_\kappa Tr(\Phi^\dagger \Phi\Phi M) + h.c.$$

$$\Phi = \frac{\sigma}{\sqrt{2n_f}} e^{i\sqrt{2n_f}\frac{\Pi^a T^a}{\sigma^2}}$$

$$M = diag(y_u, y_d, y_c, y_c, y_b, y_t)\frac{h}{\sqrt{2}}$$

- Effective potential:

$$V_0(\sigma, h) = \frac{\lambda_\sigma}{4}\sigma^4 - \frac{\lambda_\kappa y_t}{2\, n_f^{3/2}}\sigma^3 h$$

- With 1-loop radiative and thermal corrections:

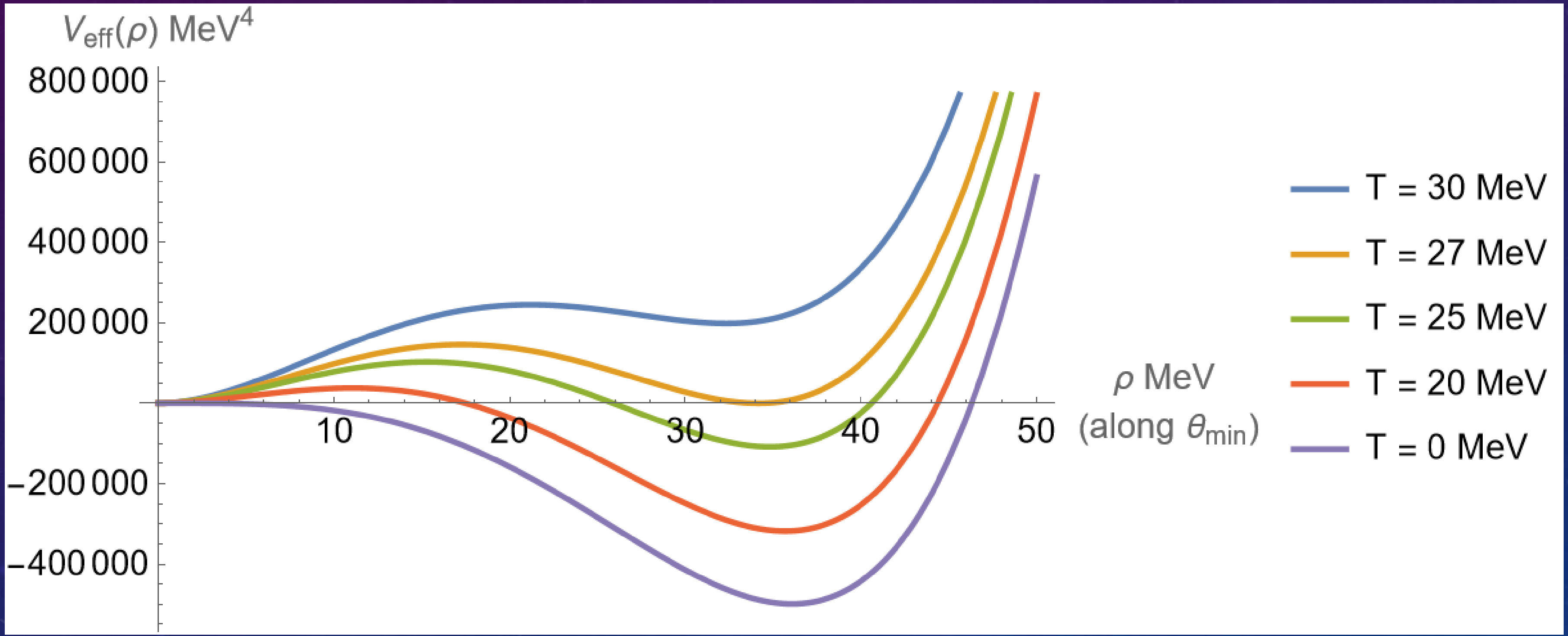
$$V_{CW}(\sigma, \mathrm{h}) = \sum_i g_i \frac{m_i^4(\sigma,h)}{64\,\pi^2} \log\frac{m_i^2(\sigma,h)}{\Lambda_{QCD}^2}$$

$$V_T(\sigma, \mathrm{h}, \mathrm{T}) = \sum_i g_i \frac{T^4}{2\pi^2} J_B\left(\frac{m_i^2(\sigma,h)}{T^2}\right)$$

Dominant contribution from:
$i = W^\pm, Z, \pi_{25}, \pi_{26}, \dots, \pi_{35}$

IMPLICATIONS OF THE MODEL

THE EFFECTIVE POTENTIAL



Sigma field:

$$\sigma = \rho \sin \theta$$

Higgs field:

$$h = \rho \cos \theta$$

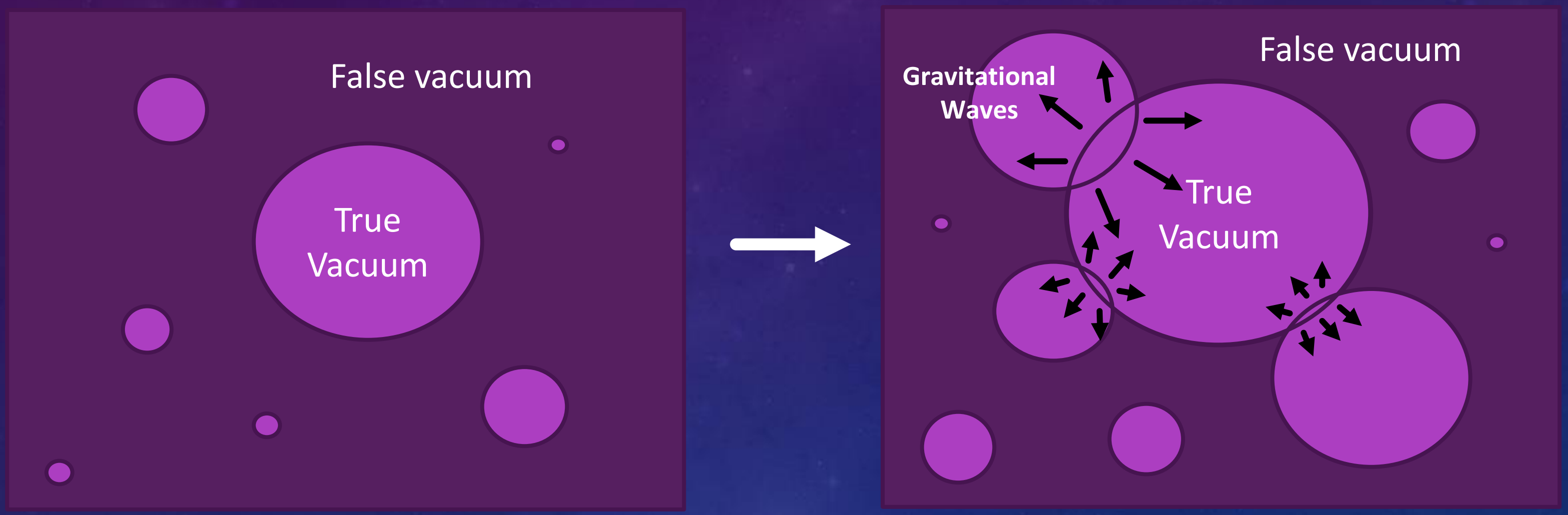
$$\theta_{min} \approx 0.67$$

Critical temperature:

$$T_c = 27 \text{ MeV}$$

FIRST ORDER PHASE TRANSITION

- Metastable vacuum means the electroweak phase and chiral phase transitions are now first-order



Semiclassical approximation for tunnelling (in the high-T limit) gives $S(T)$ → Calculate the transition probability $\Gamma(T)$ → Gives the percolation temperature $T_p \approx 26$ MeV

IMPLICATIONS OF THE MODEL

SPEED OF BUBBLE WALLS AND ENERGY BUDGET

- Where does the released vacuum energy go?

$$\frac{F_{net}}{A} = \frac{F_{driving}}{A} - \frac{F_{friction}(v_w)}{A}$$

- If $\frac{F_{net}}{A} > 0$ for all v_w then we get a runaway bubble, and the energy mostly goes to the walls
- Otherwise, we get a terminal velocity v_w and the energy is mostly deposited inside the bubble

$$\partial^2 \rho + \frac{\partial\, V_{eff}(\rho, T)}{\partial \rho} - \frac{\partial_\mu \rho\, u^\mu\, f(\rho)}{\sqrt{1 + \left(\partial_\mu \rho\, u^\mu\, g(\rho)\right)^2}} = 0$$

- Gives a terminal velocity $v_w = 0.99986$ (thanks to scale invariance and 0 cosmological constant)

IMPLICATIONS OF THE MODEL

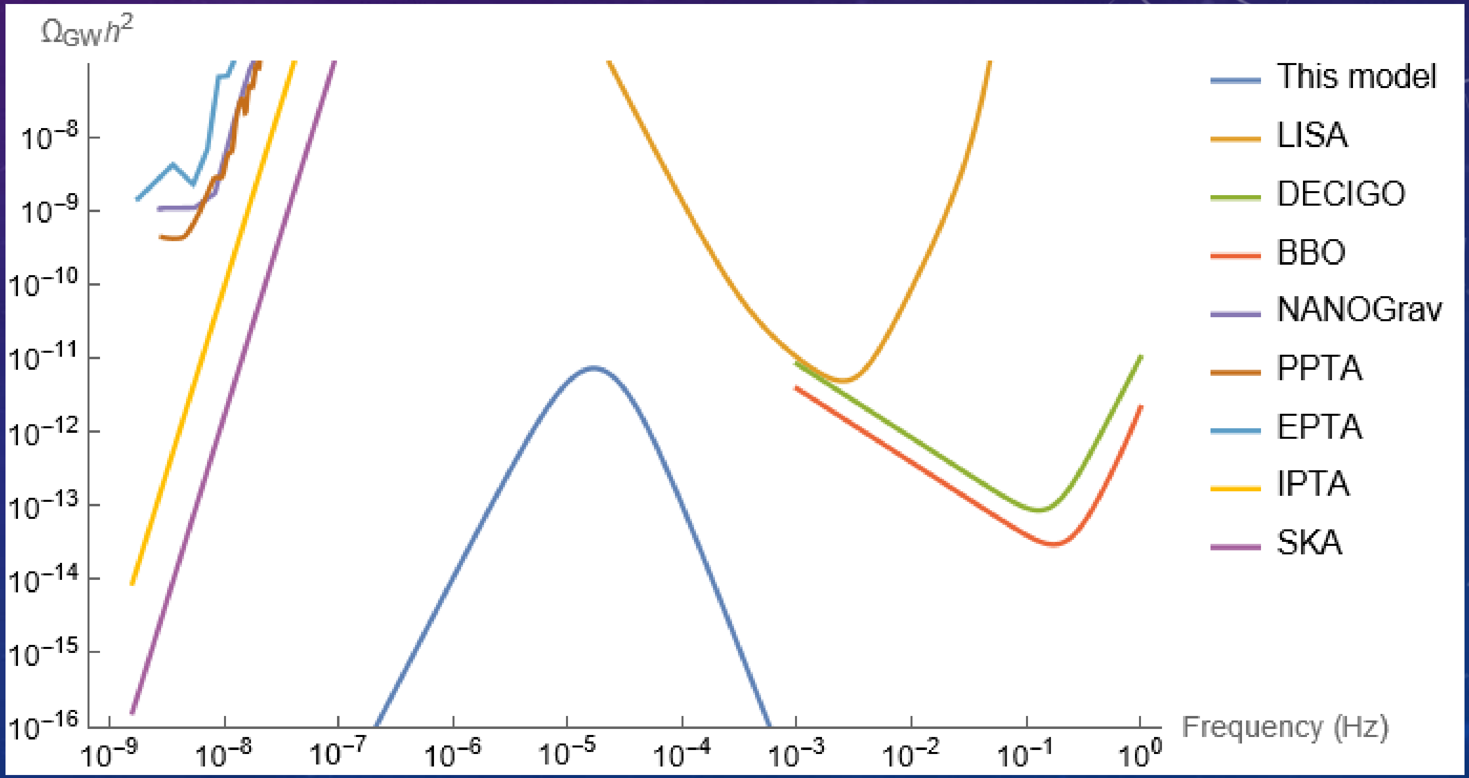
GRAVITATIONAL WAVES

$$\Omega_{GW}h^2 = 2.65 \times 10^{-6}\,\frac{H}{\beta}\left(\frac{k_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3} v_w \left(\frac{f}{f_{sw}}\right)^3\left(\frac{7}{4+3\left(\frac{f}{f_{sw}}\right)^2}\right)^{7/2}$$

$$f_{sw} = 1.9 \times 10^{-5} Hz \frac{1}{v_w}\frac{\beta}{H}\left(\frac{T}{100GeV}\right)\left(\frac{g_*}{100}\right)^{1/6}$$

- 3 sources of gravitational waves:
  - Collision of bubble walls
  - Sounds waves (dominant here)
  - Magnetohydrodynamic turbulence

- Ratio of latent heat to radiation energy $\alpha = \frac{\epsilon_{vac}}{\epsilon_{rad}} \approx 0.09$
- Ratio of transition rate to Hubble scale $\frac{\beta}{H(T_p)} \approx 4000$
- Fraction of latent heat that goes into bulk motion of the plasma $\kappa_v \approx 0.1$



IMPLICATIONS OF THE MODEL

PRIMORDIAL BLACK HOLES FROM LATE-TIME NUCLEATION

- Radiation energy density scales as $a(t)^{-4}$ while vacuum energy density doesn't scale
- So later nucleating patches become over-dense compared to earlier patches
- If the over-density $\delta(t_{late}) \equiv \frac{\epsilon_{rad}^{late}(t_{late}) - \epsilon_{rad}}{\epsilon_{rad}}$ reaches a critical value $\delta_c \approx 0.5$, it collapses to a primordial black hole

$$\max(T_{late}) = 12.5 \text{ MeV}$$

$$P(T_{late}) = \exp\left(\frac{-4\pi}{3} \frac{T_{late}}{H(T_{late})} \int_{T_{late}}^{T_c} \frac{dT}{H(T)} \left(\frac{S(T)}{2\pi}\right)^{\frac{3}{2}} e^{-S(T)}\right) = \exp(-4 \times 10^{36}) \longrightarrow \frac{\Omega_{PBH}}{\Omega_m} \approx 0$$

IMPLICATIONS OF THE MODEL

PRIMORDIAL BLACK HOLES FROM INFLATION OVER-DENSITIES

- The critical over-density for collapse δ_c depends on the equation of state of the fluid
- Near a first order phase transition the equation of state drops, $\omega_{min} \approx \frac{g_*^-}{4g_*^+ - g_*^-}$
- Mass given by $M_{PBH} \approx \frac{4\pi}{3}(c_s H^{-1})^3 \epsilon_{rad}$
- For the confinement transition:

$$T \approx 85 \text{ MeV}, \quad g_*^+ \approx 106.75, \quad g_*^- \approx 62.75, \quad \omega_{min} \approx \frac{1}{6}, \quad M_{PBH} \approx 1\text{M}_\odot$$

- For the chiral/electroweak transition:

$$T \approx 25 \text{ MeV}, \quad g_*^+ \approx 62.75, \quad g_*^- \approx 10.75, \quad \omega_{min} \approx \frac{1}{22}, \quad M_{PBH} \approx 10\text{M}_\odot$$

CONCLUSION

- The scale invariant extension solves the Hierarchy problem
- It has only a small effect on the Standard Model today, adding just one more light, weakly interacting particle but leaving the rest intact
- But it has a significant impact on cosmological history which may one day be detectable in the gravitational wave background or in the presence of primordial black holes