QED 5-loop on the lattice

Ryuichiro Kitano (KEK)

Reference:
Kitano, 2411.11554 [hep-lat]
Kitano and Takura, PTEP 2023 (2023) 10, 103B02, 2210.05569 [hep-lat]
Kitano, Takaura and Hashimoto, JHEP 05 (2021) 199, 2103.10106 [hep-lat]

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QED 5-loop

The two groups have independently calculated the 5-loop coefficient of lepton g-2.

electron g-2: agreed!

$$a_e(\text{theory} : \alpha(\text{Rb})) = 1\,159\,652\,182.037\,(720)(11)(12) \times 10^{-12},$$

$$a_e(\text{theory} : \alpha(\text{Cs})) = 1\,159\,652\,181.606\,(229)(11)(12) \times 10^{-12},$$

$$a_e(\text{expt.}) = 1\,159\,652\,180.73\,(28) \times 10^{-12}.$$ [Harvard '08]

1 2 3 4 5-loop

Wonderful achievement of theorists!!

A little bit of discrepancy?

$$A_1^{(10)} = 6.737\,(159)$$ [Aoyama, Hayakawa, Kinoshita, Nio '19]

$$A_1^{(10)}[\text{Volkov}] = 5.891(61)$$ [Volkov '24]

→ seems to be resolved recently. [Muon g-2 Theory Initiative@KEK]

I tried to develop a numerical method to evaluate the perturbative coefficients in QED on the **lattice**.

Path integral

throw dice many times and take an average.

For example, two point functions of electrons:

$$\langle \psi(x)\bar{\psi}(0)\rangle = \int [dA] \det D \, D^{-1}_{(x,0)} e^{-S[A]}$$

Dice part

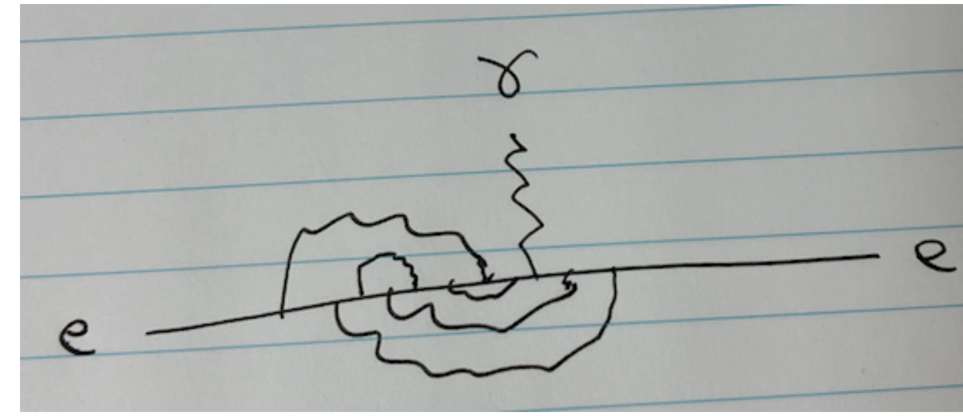
take average of this

Lattice people do this everyday.

No Feynman diagram needed.

Simple!

Even simpler



The most difficult and the most important part of the contributions are from diagrams with no lepton loops.

That's actually the easiest part for lattice.

$$\langle \psi(x)\bar{\psi}(0)\rangle = \int [dA]\, \det D\, D^{-1}_{(x,0)} e^{-S[A]}$$

Ignore lepton loops

$$\langle \psi(x)\bar{\psi}(0)\rangle = \int [dA]\, \det D\, D^{-1}_{(x,0)} e^{-S[A]}$$

In QED, this part is **free** theory!

throwing the dice part is trivially done!

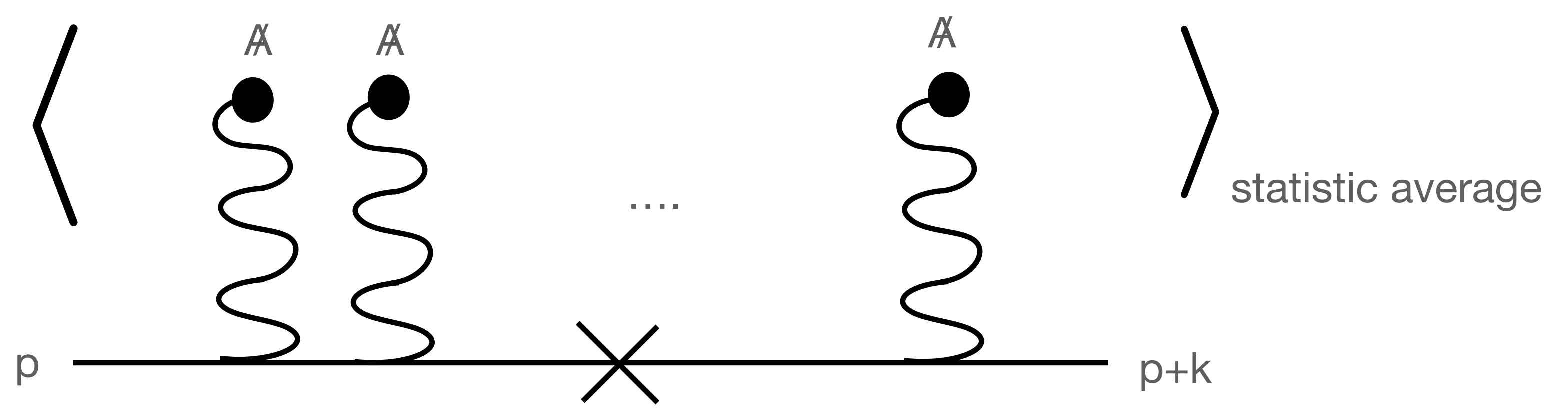
(Gaussian noise)

Perturbative calculations on the lattice

[Di Renzo, Scorzato '00]

only one diagram to calculate:

generated gauge field configurations



Sink $t=t_{\mathrm{snk}}$ | D^{-1} | γ_μ Vertex | D^{-1} | Source $t=0$

diagonal in the position space.

$$\eta(t_{\mathrm{snk}}) \ \underset{x\leftarrow p}{\mathrm{FFT}} \ \boxed{\frac{1}{i p\!\!\!/ + m}} \ \underset{p\leftarrow x}{\mathrm{FFT}} \ \boxed{ieA\!\!\!/} \ \underset{x\leftarrow p}{\mathrm{FFT}} \ \cdots\cdots \gamma_\mu \cdots\cdots \ \boxed{\frac{1}{i p\!\!\!/ + m}} \ \underset{p\leftarrow x}{\mathrm{FFT}} \ \boxed{ieA\!\!\!/} \ \underset{x\leftarrow p}{\mathrm{FFT}} \ \boxed{\frac{1}{i p\!\!\!/ + m}} \ \underset{p\leftarrow x}{\mathrm{FFT}} \ \eta(0)$$

diagonal in the momentum space

Sequence of multiplying diagonal matrices and FFT.

Very effectively done on computers.

We simply store the values at **each order** in the perturbation.
Averaging process adds up all the diagrams at each order **automatically**.

Renormalization?

This is the three-point function I can calculate perturbatively. This is a **divergent** quantity.

$$G_\mu(t) = \left\langle \sum_{\mathbf{p}'} D^{-1}(t_{\text{sink}}, t; \mathbf{p}, \mathbf{p}')\gamma_\mu D^{-1}(t, t_{\text{src}}; \mathbf{p}' + \mathbf{k}, \mathbf{p} + \mathbf{k}) \right\rangle$$

But anyway, by separating t_{sink}, t, t_{src}, this quantity is dominated by the contributions from **on-shell fermion states**.

Electric and Magnetic projections:

$$G_E(t) = \text{tr}\left[\frac{1+\gamma_4}{2} G_4(t)\right], \quad G_M(t) = i\sum_{i,j,k} \epsilon_{ijk} \text{tr}\left[\frac{1+\gamma_4}{2}\gamma_5\gamma_i G_j(t)\right]\mathbf{k}_k,$$

Repeat the same calculations for

$$G_\mu^{\text{norm}}(t) = \sum_{\mathbf{p}'} \left\langle D^{-1}(t_{\text{sink}}, t; \mathbf{p}, \mathbf{p}') \right\rangle \gamma_\mu \left\langle D^{-1}(t, t_{\text{src}}; \mathbf{p}' + \mathbf{k}, \mathbf{p} + \mathbf{k}) \right\rangle$$

and normalize

$$F_E(t) = \frac{G_E(t)}{G_E^{\text{norm}}(t)}, \quad F_M(t) = \frac{G_M(t)}{G_M^{\text{norm}}(t)},$$

now **external legs are taken away**. We get form factors.

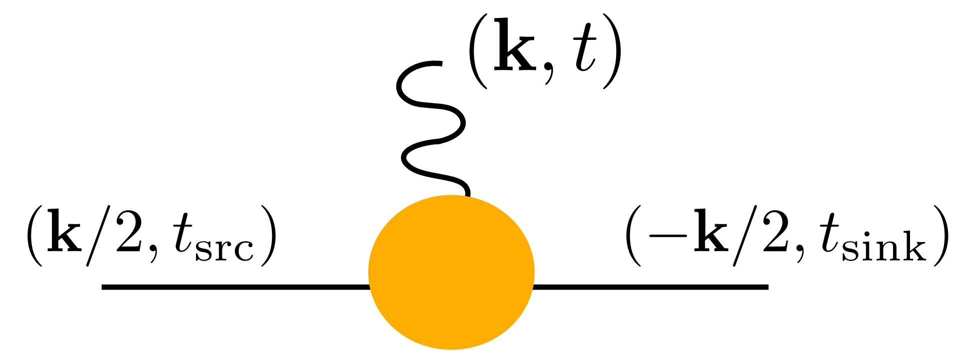
Finally, we get the g-factor $$\frac{g(t)}{2} = \frac{F_M(t)}{F_E(t)},$$ perturbatively.

All the divergence is gone, because this is a physical quantity!

Done!

Of course, the life is not so easy.

Limit, limit, limit…



We need to take the limits of

IR cutoff $m_\gamma \to 0$

continuum limit $m \to 0$
(fermion mass parameter)

infinite volume $L \to \infty$

(in the lattice unit, a=1)

while keeping

$$1/L \ll m_\gamma \ll m$$

The strategy is to keep $m_\gamma L \gg 1$,
and take the double limit, $m_\gamma/m \to 0$ and $m \to 0$.

We need a large volume!!

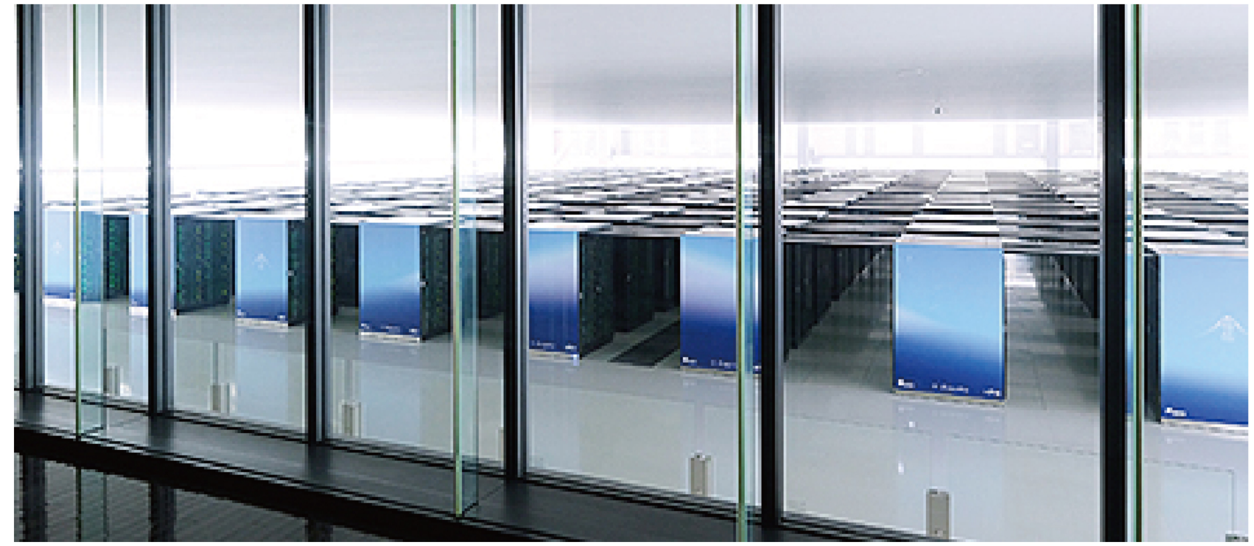
For example, if we want $m_\gamma L \sim 4$, $m_\gamma/m \sim 0.1$, and $m^2 \sim 0.1$, we need $L \sim 100$!

We need a supercomputer.

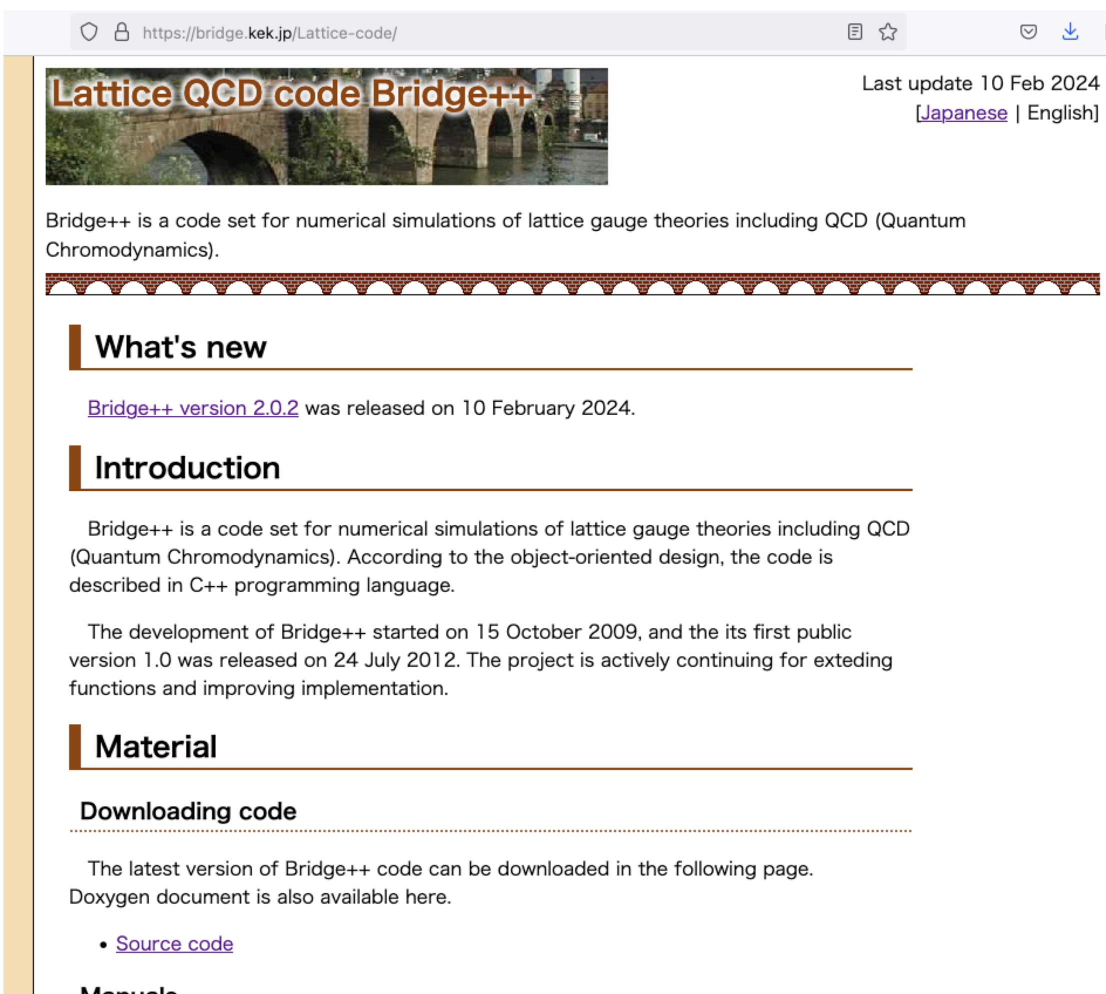
Supercomputer and code:

We had a good one in the next building. (-2024)

FUGAKU is also open for researchers.



Matsufuru-san in the next building has been developing a user friendly open lattice codes:
(Thanks, Matsufuru san!)


https://bridge.kek.jp/Lattice-code/

Lattice QCD code Bridge++

Last update 10 Feb 2024
[Japanese | English]

Bridge++ is a code set for numerical simulations of lattice gauge theories including QCD (Quantum Chromodynamics).

What's new

Bridge++ version 2.0.2 was released on 10 February 2024.

Introduction

Bridge++ is a code set for numerical simulations of lattice gauge theories including QCD (Quantum Chromodynamics). According to the object-oriented design, the code is described in C++ programming language.

The development of Bridge++ started on 15 October 2009, and the its first public version 1.0 was released on 24 July 2012. The project is actively continuing for exteding functions and improving implementation.

Material

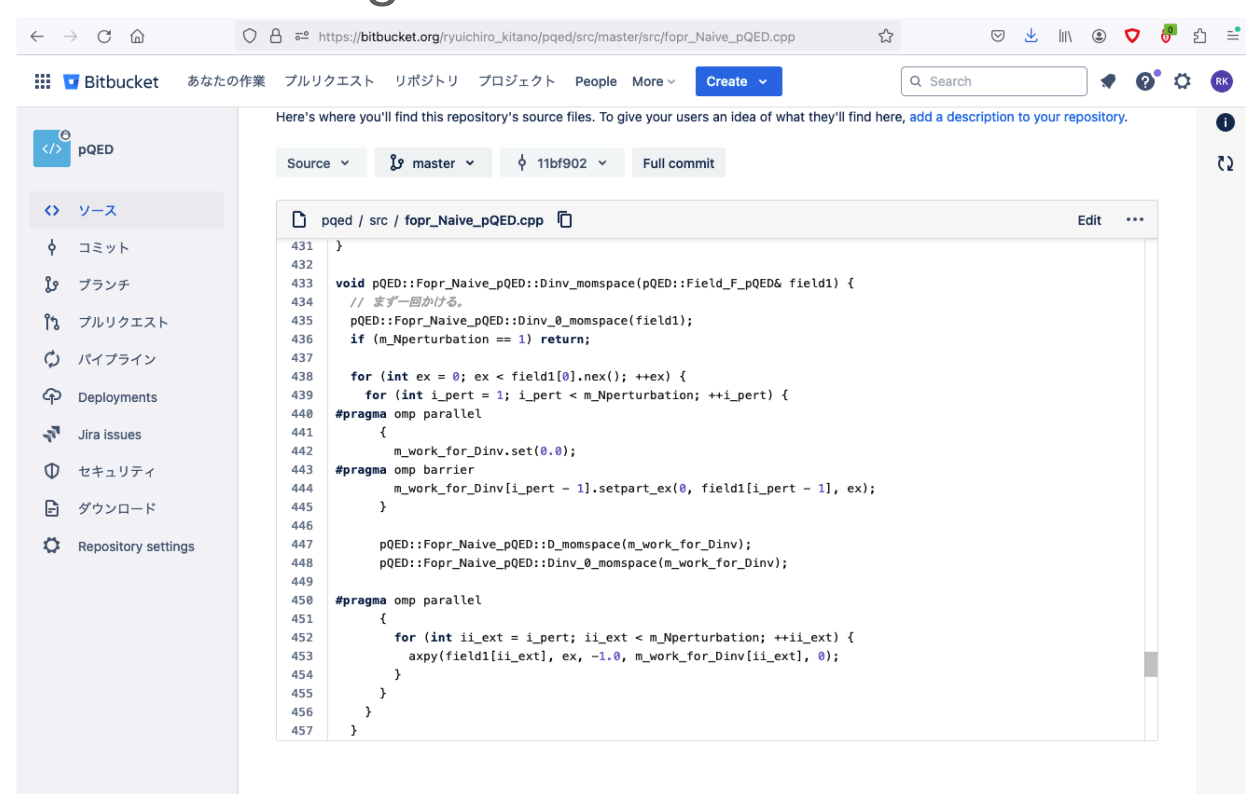
Downloading code

The latest version of Bridge++ code can be downloaded in the following page. Doxygen document is also available here.

- Source code

Manuals

It is a good summer homework!


https://bitbucket.org/ryuichiro_kitano/pqed/src/master/src/fopr_Naive_pQED.cpp

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pQED

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Here's where you'll find this repository's source files. To give your users an idea of what they'll find here, add a description to your repository.

Source master 11bf902 Full commit

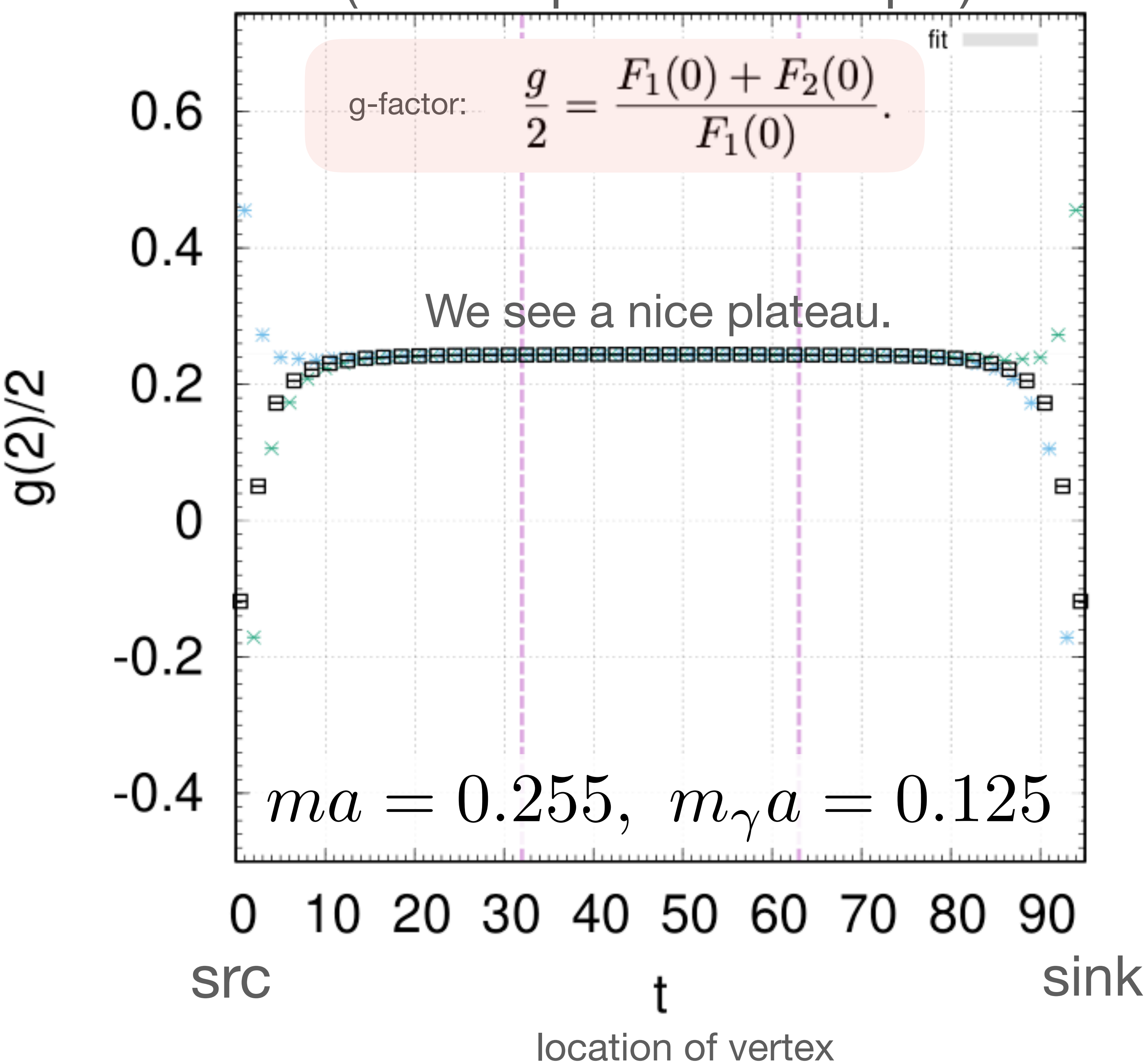
pqed / src / fopr_Naive_pQED.cpp Edit

```
}

void pQED::Fopr_Naive_pQED::Dinv_momspace(pQED::Field_F_pQED& field1) {
  // まず一回かける。
  pQED::Fopr_Naive_pQED::Dinv_0_momspace(field1);
  if (m_Nperturbation == 1) return;

  for (int ex = 0; ex < field1[0].nex(); ++ex) {
    for (int i_pert = 1; i_pert < m_Nperturbation; ++i_pert) {
#pragma omp parallel
      {
        m_work_for_Dinv.set(0.0);
#pragma omp barrier
        m_work_for_Dinv[i_pert - 1].setpart_ex(0, field1[i_pert - 1], ex);
      }

      pQED::Fopr_Naive_pQED::D_momspace(m_work_for_Dinv);
      pQED::Fopr_Naive_pQED::Dinv_0_momspace(m_work_for_Dinv);

#pragma omp parallel
      {
        for (int ii_ext = i_pert; ii_ext < m_Nperturbation; ++ii_ext) {
          axpy(field1[ii_ext], ex, -1.0, m_work_for_Dinv[ii_ext], 0);
        }
      }
    }
  }
```

64^3x128 lattice results:

O(200,000) configurations.
This is a result of FUGAKU 3days.

(one-loop order example)



plateau ~ on-shell

$$m_\gamma L = 8$$

finite volume effect under control.

I'm actually using a trick to make T-direction larger by averaging periodic and anti-periodic boundary conditions. No worry about backward propagation.

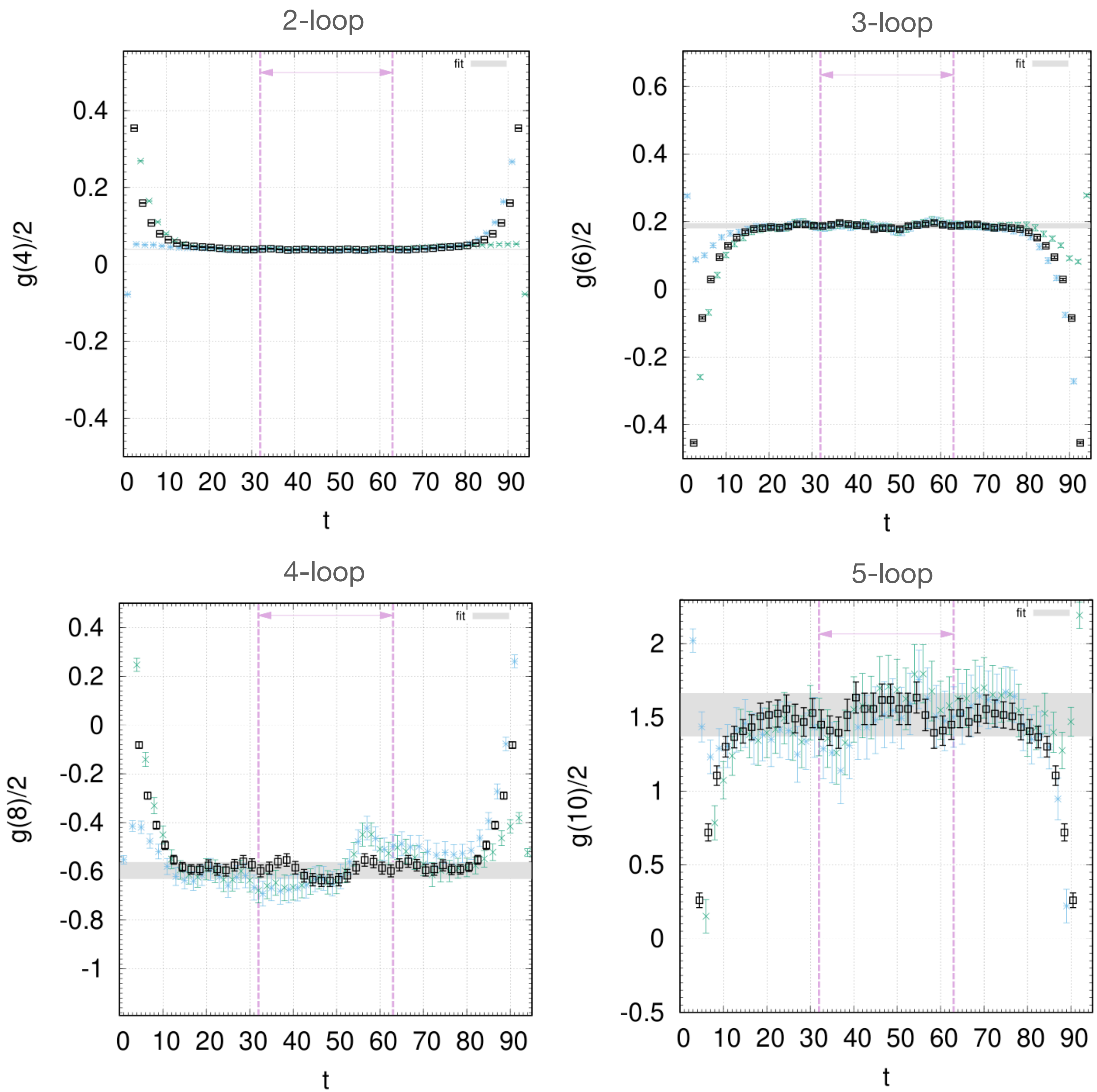
64³x128 lattice results:

O(200,000) configurations.

This is a result of FUGAKU 3days.

$$ma = 0.255,\ m_\gamma a = 0.125$$

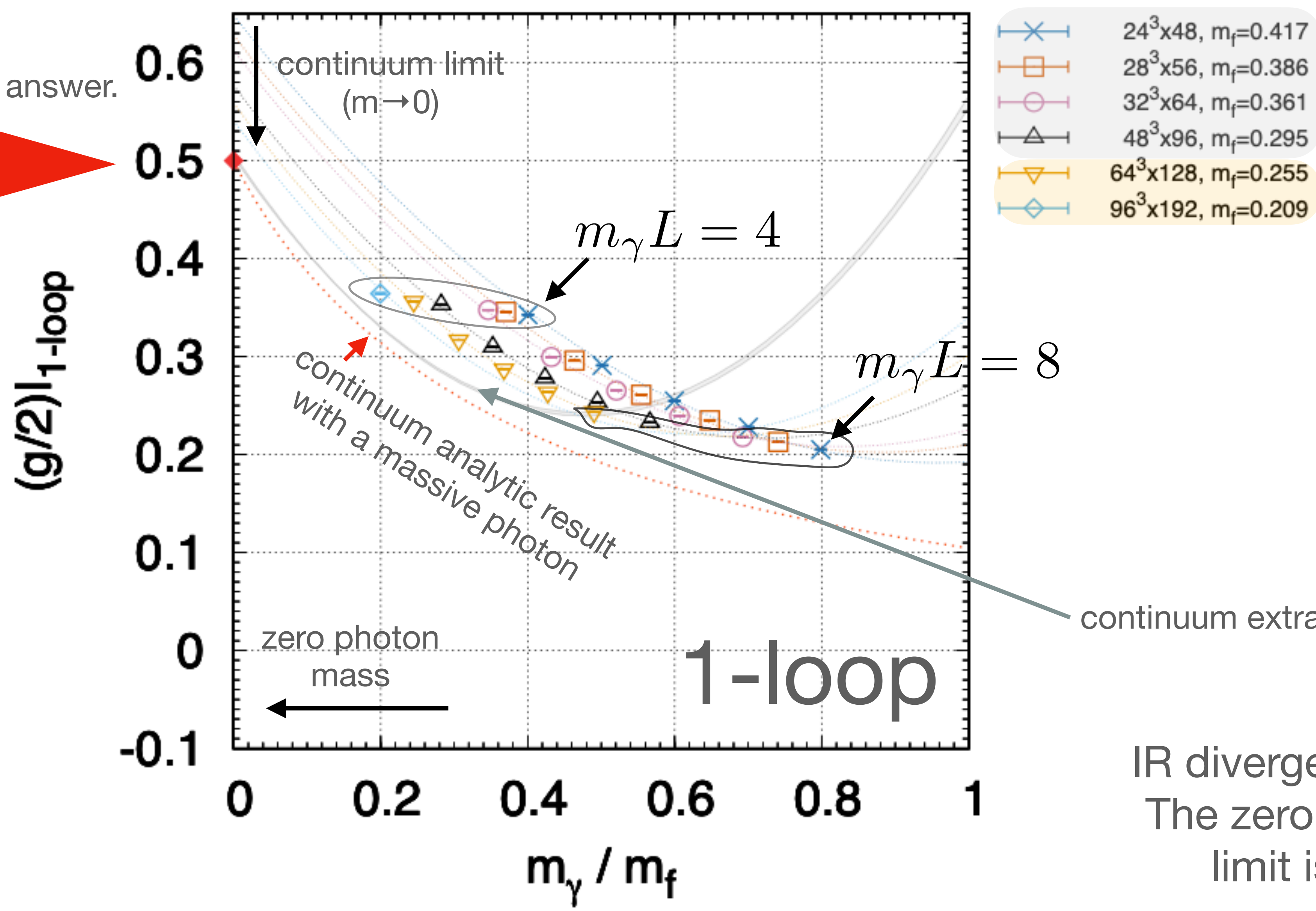
But the photon mass is still big.



Limits:

fitting with quadratic functions.

This is the answer.



NEC SX-Aurora TSUBASA at KEK.

FUGAKU

continuum limit
(m→0)

continuum extrapolation

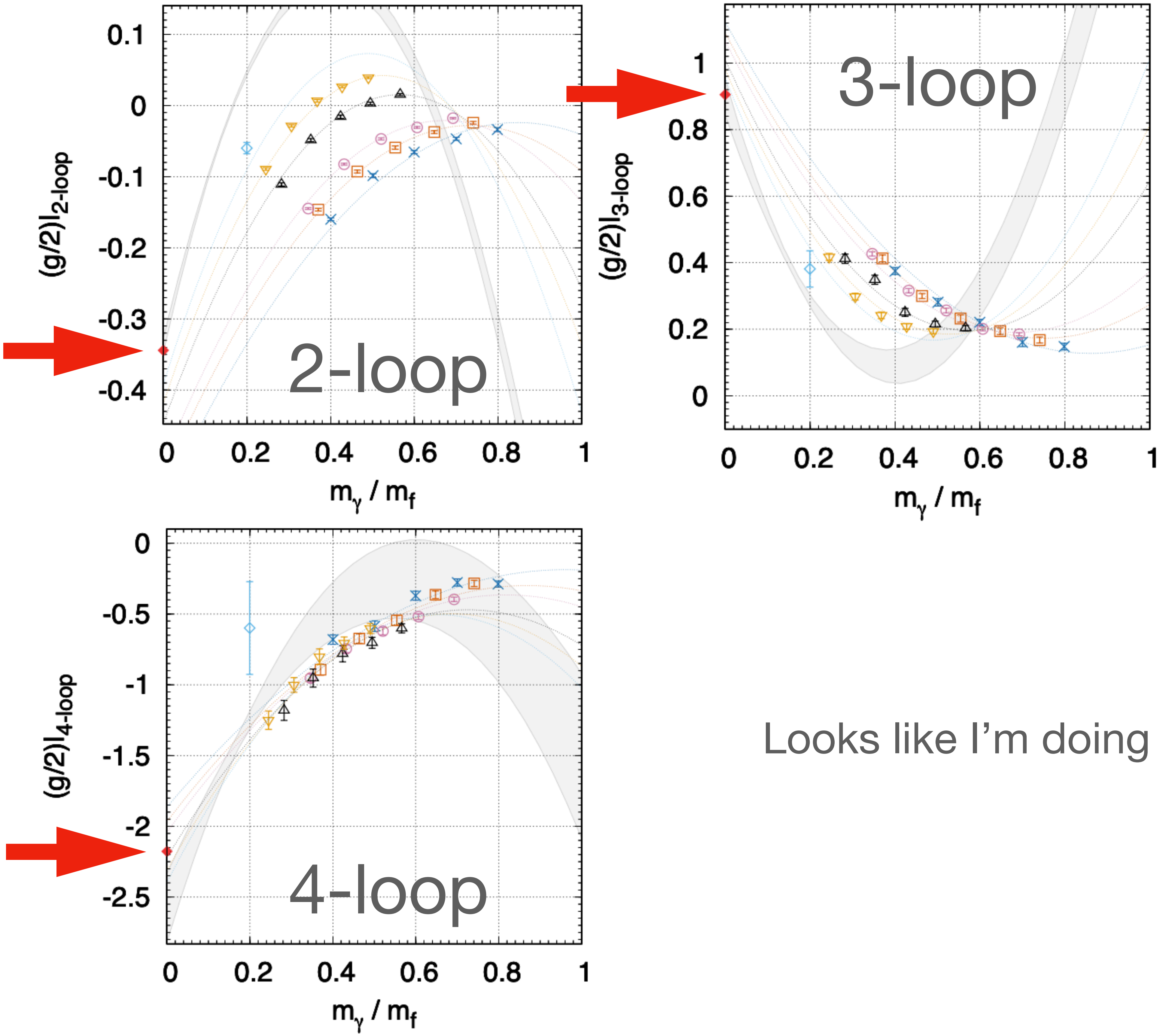
IR divergence is severe.
The zero photon mass
limit is a bit far..

Looks like we could reproduce α/π.

systematic error (including fitting, finite volume etc.) is a percent level. (hopefully)

higher loops:

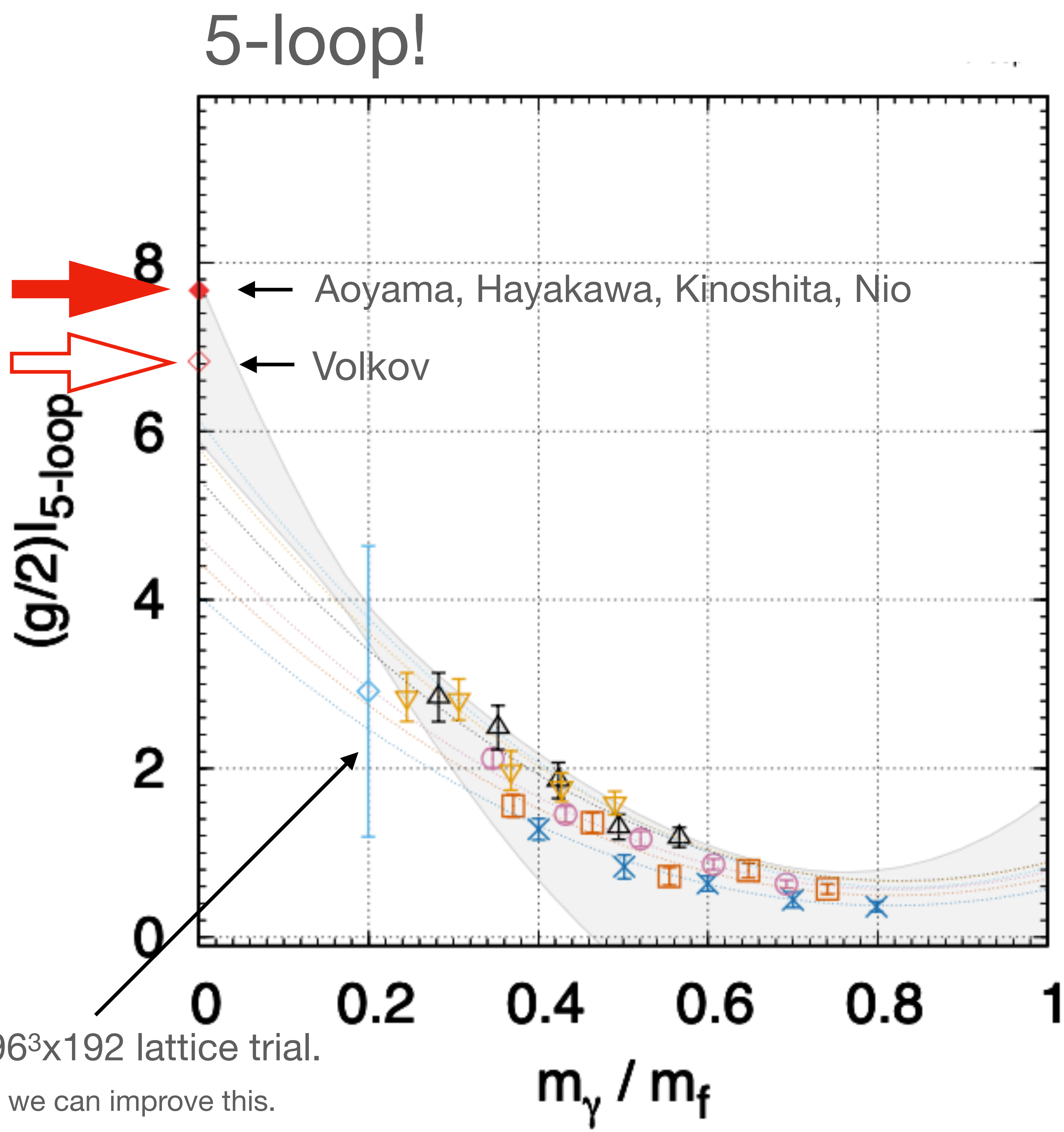
fitting with quadratic functions.



Looks like I'm doing all right.

5-loop results:

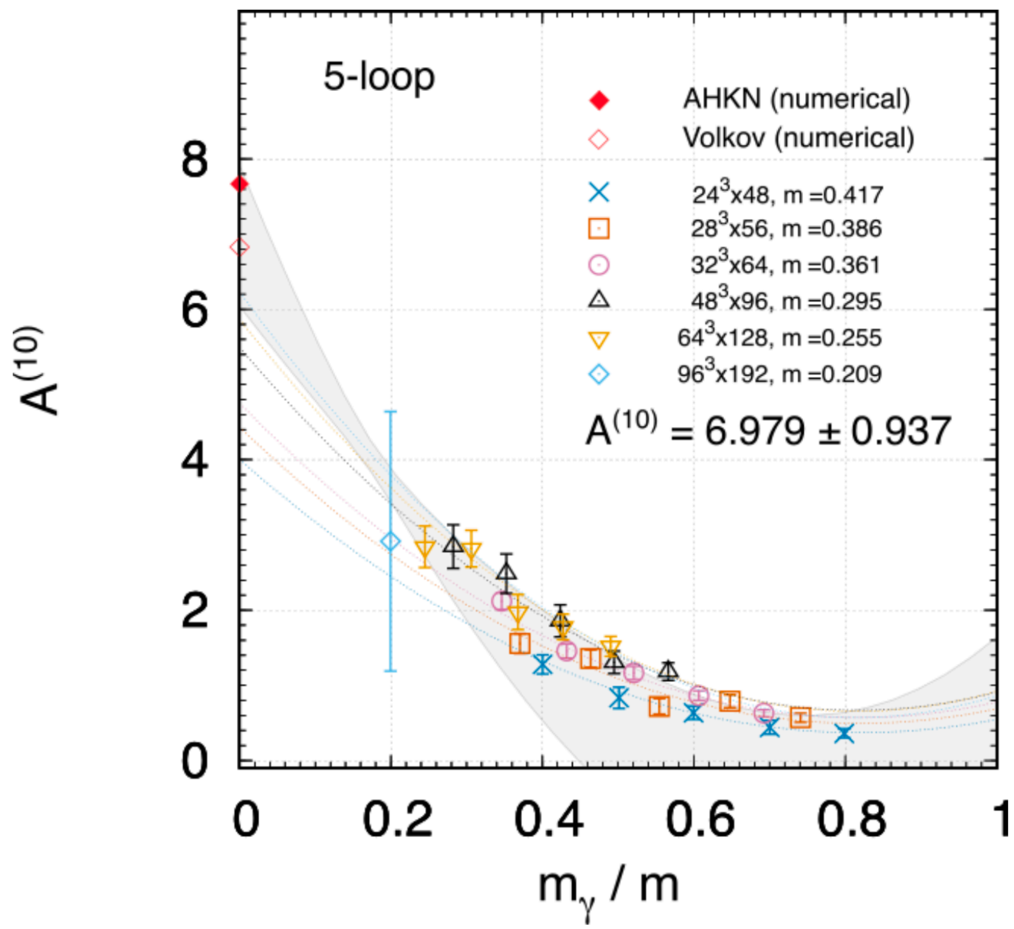
fitting with quadratic functions.



I guess I could give an independent confirmation.

This is 96^3x192 lattice trial.

Maybe we can improve this.



$$A^{(10)}(\text{no lepton loop}) = 7.0 \pm 0.9$$

to be compared with

7.668 ± 0.159 (AHKN)

6.828 ± 0.060 (Volkov)

I'm friendly with anybody.

Summary

I tried.

I couldn't quite reach the precision of the Feynman diagram method, but at least this gives a totally independent calculation/confirmation.