Dark matter candidate emerging from 3-form gauge theory

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Dark matter and dark energy

- Nature of dark matter (DM) and dark energy (DE) is currently unknown.

Dark matter (DM)	**Dark energy (DE)**
Properties	
• *Matter*: clumps under gravity • *Dark*: Does not emit/absorb light • *Cold*: Most non-relativistic today	• Causes accelerated expansion (negative pressure) • Doesn’t clump under gravity
Candidates	
• New particles (e.g. WIMPs, ALPs) • Primordial black holes • Modified gravity theories	• Cosmological constant(?) Λ • Quintessence

- This work proposes a model where a source of DM and DE can originate from **3-form gauge theory**.

3-form gauge theory

Electromagnetism	3-form gauge theory
• Photon A_μ with gauge redundancy: $$\delta A_\mu = \partial_\mu \theta$$ • Field strength tensor: $$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = \partial_{[\mu} A_{\nu]}$$ • 2 propagating d.o.f. (2 polarisation states)	• Tensor field $A_{\nu\rho\sigma}$ with gauge redundancy: $$\delta A_{\nu\rho\sigma} = \partial_{[\nu} \Omega_{\rho\sigma]} \propto \epsilon_{\mu\nu\rho\sigma} \partial^\mu \theta$$ • Field strength tensor: $$F_{\mu\nu\rho\sigma} = \partial_{[\mu} A_{\nu\rho\sigma]}$$ • $F_{\mu\nu\rho\sigma}$ is dual to a scalar F $$F_{\mu\nu\rho\sigma} = \epsilon_{\mu\nu\rho\sigma} F$$ • 0 propagating d.o.f. (no particles)

Dynamics of 3-forms in vacuum

$$\mathcal{L}_{gauge} = \sqrt{-g}\left[-\frac{1}{2\cdot 4!}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma}\right]$$

- Equations of motion: $\partial_\mu F = 0 \implies F = \text{constant} \equiv \lambda$

No new particles!

- Energy-momentum tensor:

$$T_{\mu\nu} = g_{\mu\nu}\cdot\frac{1}{2}\lambda^2 \implies \begin{cases} \rho = T_{00} = \frac{1}{2}\lambda^2 \\ p = T_{ii} = -\frac{1}{2}a^2\lambda^2 \end{cases}$$

- Negative pressure $\implies$ dark energy!

3-form in vacuum behaves like a cosmological constant!

Generating a mass for 3-forms

- Need to get DM candidate – apply **Anderson-Higgs mechanism** to 3-form.

- Analogous to photons propagating in plasma:
  - $+1$ d.o.f. due to collective plasma oscillations.
  - Photon now has 3 d.o.f. $\Longrightarrow$ **effective mass**.

- 3-form permeates Universe containing ordinary matter.

Anderson-Higgs mechanism on 3-form $\Longrightarrow$ $+1$ d.o.f. for 3-form $\Longrightarrow$ Effective mass arises (DM candidate)

Modelling the Universe

$$g_{\mu\nu} = \mathrm{diag}\left(1, -a(t)^2, -a(t)^2, -a(t)^2\right)$$

- *Cosmological principle* $\implies$ model ordinary matter as a **perfect cosmic fluid**.
  - Described by real scalar field $\phi(x)$ with shift symmetry $\delta\phi = c$.

- Construct Lagrangian (using first-order formalism):

$$\mathcal{L}_{fluid} = \sqrt{-g}\left[\nabla_\mu \phi V^\mu - \underbrace{\mu^4}_{\text{Energy scale of theory}} P(X)\right] \quad \text{where} \quad X = \frac{1}{2\mu^4} V_\mu V^\mu$$

Ordinary matter as a perfect fluid

$$\mathcal{L}_{fluid} = \sqrt{-g}\left[\nabla_\mu \phi V^\mu - \mu^4 P(X)\right] \quad \text{where} \quad X = \frac{1}{2\mu^4} V_\mu V^\mu$$

- Energy-momentum tensor: $T_{\mu\nu} = 2V_\mu \partial_\nu \phi - V_\mu V_\nu - g_{\mu\nu}(V^\alpha \partial_\alpha \phi - \mu^4 P)$
  - 4-velocity of fluid: $V^\mu = \mu^2 \sqrt{2X} u^\mu$
  - Equations of motion: $P_X V^\mu = \nabla^\mu \phi$

- Rewrite as: $T_{\mu\nu} = \underbrace{2\mu^4 X P_X}_{\rho_f + p_f} u_\mu u_\nu - g_{\mu\nu} \underbrace{(2\mu^4 X P_X - \mu^4 P)}_{p_f}$

- Fix equation of state of cosmic fluid $w_f = p_f/\rho_f$ and solve for ρ_f in terms of w_f:

$$\boxed{\rho_f = \mu^4 P(X) \propto \mu^4 X^{(1+w_f)/2}}$$

Gauging the theory with a 3-form

$$\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2\cdot 4!}F_{\mu\nu\rho\sigma}F^{\mu\nu\rho\sigma} + \underbrace{\left(\partial_\mu\phi - \frac{\overbrace{g_A\mu}^{\text{Coupling strength of 3-form}}}{3!}\epsilon_{\mu\nu\rho\sigma}A^{\nu\rho\sigma}\right)}_{\text{Gauge coupling between 3-form and cosmic fluid}}V^\mu - \overbrace{\mu^4}^{\text{Energy scale of theory}}P(X)\right]$$

- Energy-momentum tensor:

$$T_{\mu\nu} = \underbrace{2\mu^4 XP_X}_{\rho+p}u_\mu u_\nu - g_{\mu\nu}\underbrace{\left(2\mu^4 XP_X - \mu^4 P - \frac{1}{2}F^2\right)}_{p}$$

- Read off the energy density and pressure:

$$\boxed{\rho = \mu^4 P + \frac{1}{2}F^2 \quad \text{and} \quad p = w_f\mu^4 P - \frac{1}{2}F^2}$$

Effective equation of state of universe

- Identify $\rho_{3-form} \equiv \frac{1}{2}F^2$ as the total energy density from the 3-form and its interaction with the cosmic fluid.

- Define new parameter $\kappa \equiv \frac{\rho_{3-form}}{\rho_f}$

- Effective equation of state is then:

$$\boxed{w_{eff} \equiv \frac{w_f - \kappa}{1+\kappa}}$$

Evolution of effective equation of state

- We know how w_{eff} behaves if we know how κ behaves.
- From here, assume working with background solutions, so fields are *only* time-dependent.

Equations of motion for 3-form + 1st Friedmann equation

$$\Downarrow$$

$$\boxed{\kappa' \rho_f + \kappa \rho_f' = \frac{g_A \mu^3 M_{Pl}}{a^4} \sqrt{\frac{\kappa}{1+\kappa}}}$$

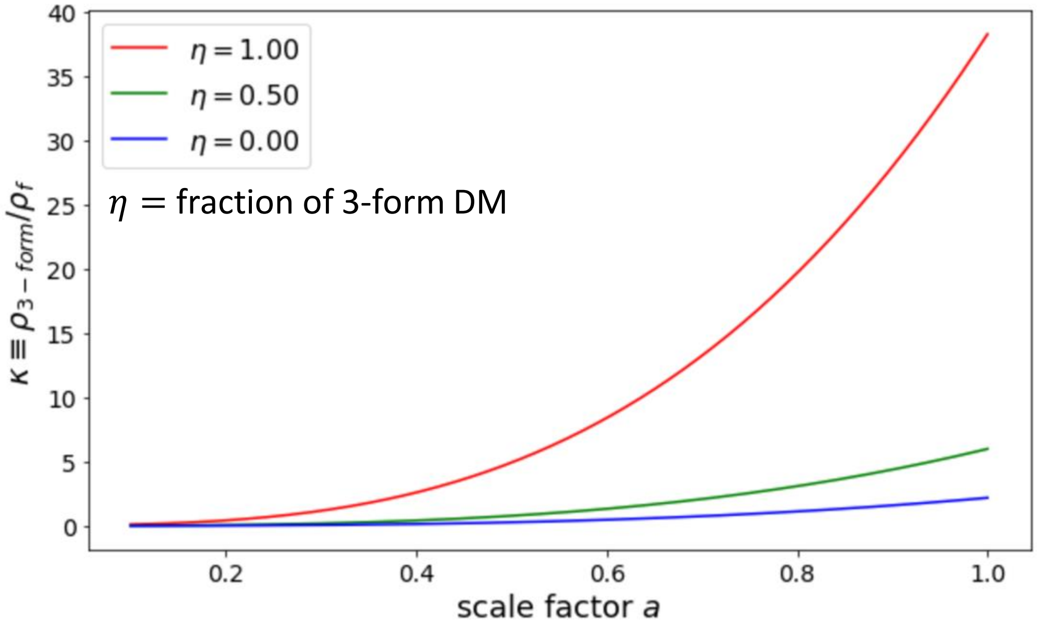
where primes denote derivatives w.r.t. $a(t)$

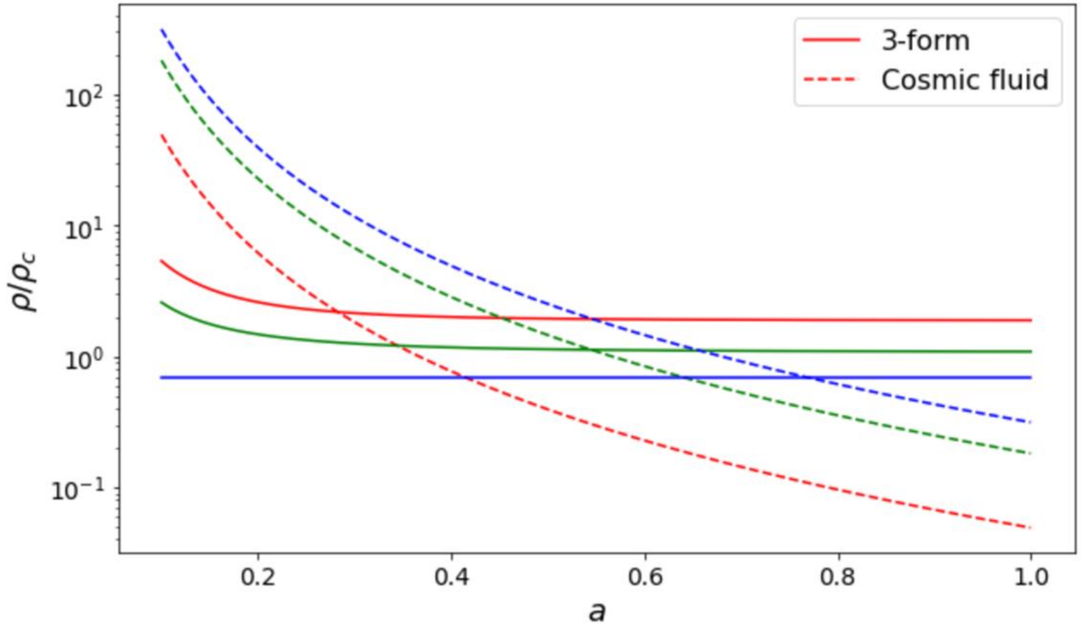
Solving for effective equation of state

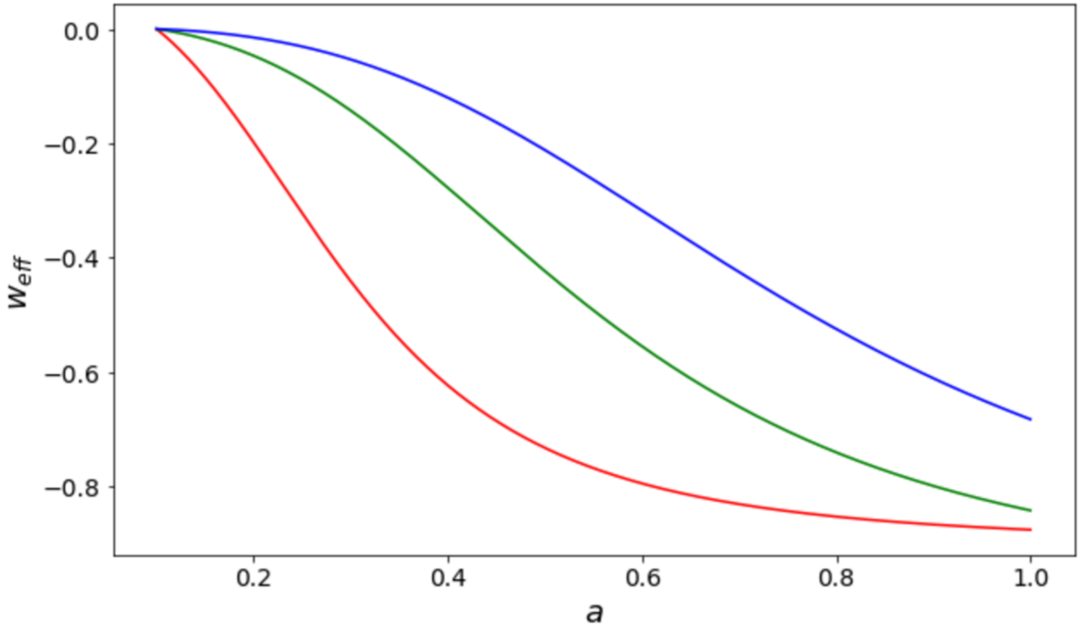
$$\boxed{\kappa'\rho_f + \kappa\rho_f' = \frac{g_A\mu^3 M_{Pl}}{a^4}\sqrt{\frac{\kappa}{1+\kappa}}}$$

To solve, we assume the following:

- Consider cosmology after radiation-matter equality.
- Cosmic fluid is a single perfect fluid with $w_f = 0$.
- All of the DE is explained by the 3-form.
- In the beginning of the matter-dominated era, just after radiation-matter equality, $0 < \kappa \ll 1$.


$\eta = 1.00$
$\eta = 0.50$
$\eta = 0.00$
η = fraction of 3-form DM
$\kappa \equiv \rho_{3-form}/\rho_f$
scale factor a
0
5
10
15
20
25
30
35
40
0.2
0.4
0.6
0.8
1.0


3-form
Cosmic fluid
ρ/ρ_c
a
10^{2}
10^{1}
10^{0}
10^{-1}
0.2
0.4
0.6
0.8
1.0


w_{eff}
a
0.0
−0.2
−0.4
−0.6
−0.8
0.2
0.4
0.6
0.8
1.0

Contributions to 3-form energy density

$$\boxed{\kappa'\rho_f + \kappa\rho_f' = \frac{g_A\mu^3 M_{Pl}}{a^4}\sqrt{\frac{\kappa}{1+\kappa}}}$$

- Can get approximate solutions to 3-form energy density today by assuming $\kappa \gg 1$.

$$\kappa(a) = ca^3 - \frac{g_A\mu^3 M_{Pl}}{3\left(\rho_{B,0} + (1-\eta)\rho_{DM,0}\right)}$$

$$\Downarrow$$

$$\rho_{3-form} = \frac{1}{2}F^2 = \frac{1}{2}\lambda^2 \overbrace{- \frac{g_A\mu^3 M_{Pl}}{3}}^{+\eta\rho_{DM,0}}\frac{1}{a^3}$$

Matter perturbations

- To understand clumping of matter by dark matter 3-form, we can study the perturbations of the 3-form gauge field.

$$A^{\nu\rho\sigma}(x,t) = \bar{A}^{\nu\rho\sigma}(t) + \delta A^{\nu\rho\sigma}(x,t)$$

- Helmholtz decomposition:

$$A^{\nu\rho\sigma}(x,t) = \epsilon^{\mu\nu\rho\sigma}\left(\frac{1}{\mu}\nabla_\mu\Phi(x,t) + \xi_\mu(x,t)\right)$$

- Can gauge away ξ_μ - left with $A^{\nu\rho\sigma} = \frac{1}{\mu}\epsilon^{\mu\nu\rho\sigma}\nabla_\mu(\bar{\Phi}(t) + \delta\Phi(x,t))$

Matter perturbations

- Assume we are in the matter-dominated era.
- Studied equations of motion up to first order in perturbations.

$$\nabla_\mu V^\mu = 0$$
$$P_X V_\mu = \nabla_\mu(\phi - g_A \Phi)$$
$$\nabla_\mu \Box \Phi = -16 g_A \mu^2 V_\mu$$

$$\Downarrow$$

$$\boxed{\left(\Box - m_\Phi^2(a)\right)\delta\Phi = m_\Phi^2(a)\,\delta_{M,f}\left(\bar{\Phi} + \frac{\bar{\phi}}{g_A}\right) + \text{non-linear terms}}$$

Matter perturbations

$$\left(\Box - m_\Phi^2\right)\delta\Phi = m_\Phi^2 \delta_{M,f} \left(\bar{\Phi} + \frac{\bar{\phi}}{g_A}\right) + \text{non-linear terms}$$

$$\boxed{m_\Phi^2 \approx \frac{g_A^2 \mu^6}{\rho_{M,f,0}} \frac{1}{a^3}}$$

- Describes a classical scalar field with source term.
  - Source $\propto \delta_{M,f}$ (density contrast of matter in cosmic fluid)
  - Effective mass $\propto a^{-3}$

Conclusion

- Introduced novel theory of DM involving a 3-form gauge field, which simultaneously provides an origin of DE.

- By considering energy density of background fields, one can show that the 3-form provides a matter component emerging from interactions of the cosmic fluid.
  - However, 3-form DM drives universe to a DE-dominated era at earlier times.

- To make a viable DM candidate, need to study 3-form perturbations to verify large scale structure formation.

BACKUP SLIDES

Levi-Civita tensor

Normalisation:

- $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma} = -4!$
- $\epsilon_{0123} = \sqrt{-g}$
- $\epsilon^{0123} = 1/\sqrt{-g}$

Alternatively, can write in terms of Levi-Civita *symbol* $\tilde{\epsilon}_{\mu\nu\rho\sigma}$:

- $\epsilon_{\mu\nu\rho\sigma} = \sqrt{-g}\tilde{\epsilon}_{\mu\nu\rho\sigma}$
- $\epsilon^{\mu\nu\rho\sigma} = (1/\sqrt{-g})\tilde{\epsilon}^{\mu\nu\rho\sigma}$

Energy-momentum tensors

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\partial \mathcal{L}}{\partial g^{\mu\nu}} = 2\frac{\partial(\mathcal{L}/\sqrt{-g})}{\partial g^{\mu\nu}} - g_{\mu\nu}(\mathcal{L}/\sqrt{-g})$$

$$T_{\mu\nu}^{fluid} = (\rho + p)u_\mu u_\nu - g_{\mu\nu}p$$

Cosmic fluid through first-order formalism

$$\mathcal{L}_{fluid} = \sqrt{-g}\left[\partial_\mu \phi V^\mu - \mu^4 P(X)\right] \quad \text{where} \quad X = \frac{1}{2\mu^4} V_\mu V^\mu$$

Equations of motion:

- $\partial_\mu\left(\sqrt{-g}V^\mu\right) = 0$
- $V^\mu = \frac{1}{P_X}\partial^\mu \phi$

4-velocity of fluid: $V^\mu = \mu^2\sqrt{2X}u^\mu$

$$T_{\mu\nu} = 2V_\mu \partial_\nu \phi - V_\mu V_\nu - g_{\mu\nu}(V^\alpha \partial_\alpha \phi - \mu^4 P) = 2\mu^4 X P_X u_\mu u_\nu - g_{\mu\nu}(2\mu^4 X P_X - \mu^4 P)$$

$$\rho(w) = \mu^4 P = \alpha \mu^4 X^{(1+w)/2} \quad \text{where} \quad w = p/\rho$$

3-form gauge theory in vacua

Gauge invariance:

- $\delta A_{\nu\rho\sigma} = \frac{1}{g_A \mu} \epsilon_{\mu\nu\rho\sigma} \nabla^{\mu} \theta$
- $\delta F_{\mu\nu\rho\sigma} = \frac{1}{g_A \mu} \nabla_{[\mu} \epsilon_{\nu\rho\sigma]\alpha} \nabla^{\alpha} \theta$
- $\delta\left(F_{\mu\nu\rho\sigma} F^{\mu\nu\rho\sigma}\right) = 0 \Longrightarrow \nabla_{\mu} \nabla^{\mu} \theta = 0$
- $\delta F \propto \nabla_{\mu} \nabla^{\mu} \theta = 0$

Dual 1-form:

- $B_{\mu} = \frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} A^{\nu\rho\sigma}$ where $\delta B_{\mu} = \frac{1}{g_A \mu} \nabla_{\mu} \theta$
- $F = -\frac{1}{4} \nabla_{\mu} B^{\mu}$

3-form gauge theory in vacua

Equations of motion:

$$\partial_\mu(\sqrt{-g}F^{\mu\nu\rho\sigma}) = 0 \implies \partial_\mu\left(\sqrt{-g}(\frac{1}{\sqrt{-g}}\tilde{\epsilon}^{\mu\nu\rho\sigma}F)\right) = 0 \implies \tilde{\epsilon}^{\mu\nu\rho\sigma}\partial_\mu F = 0 \implies \partial_\mu F = 0$$

Lagrangian requires a boundary term $+\frac{1}{4!}\partial_\mu\left(\sqrt{-g}F^{\mu\nu\rho\sigma}A_{\nu\rho\sigma}\right)$ to ensure:

1. The variation of the fields vanishes at the boundary.
2. The energy-momentum tensor derived from the *on-shell* Lagrangian reproduces the correct sign.

3-form gauge theory + cosmic fluid

Gauge invariance:

- $\delta A_{\nu\rho\sigma} = \frac{1}{g_A \mu} \epsilon_{\mu\nu\rho\sigma} \nabla^{\mu} \theta$
- $\delta B_{\mu} = \frac{1}{g_A \mu} \nabla_{\mu} \theta$
- $\delta F_{\mu\nu\rho\sigma} = \frac{1}{g_A \mu} \nabla_{[\mu} \epsilon_{\nu\rho\sigma]\alpha} \nabla^{\alpha} \theta$
- $\delta F = \nabla_{\mu} \nabla^{\mu} \theta = 0$
- $\delta \phi = \theta$

Equations of motion:

- $\partial_{\mu}\left(\sqrt{-g}\, V^{\mu}\right) = 0$
- $V^{\mu} = \frac{1}{P_X}\left(\partial^{\mu} \phi - g_A \mu B^{\mu}\right)$
- $\partial_{\mu}\left(\sqrt{-g}\, F^{\mu\nu\rho\sigma}\right) = 4 g_A \mu \sqrt{-g}\, \epsilon^{\mu\nu\rho\sigma} V_{\mu} \Longrightarrow \partial_{\mu} F = 4 g_A \mu V_{\mu}$

3-form gauge theory + cosmic fluid

Energy-momentum tensor:

$$T_{\mu\nu} = 2\mu^4 X P_X u_\mu u_\nu - g_{\mu\nu}\left(2\mu^4 X P_X - \mu^4 P - \frac{1}{2}F^2\right)$$

Energy density:

$$\rho = \mu^4 P + \frac{1}{2}F^2 = \rho_\Lambda X^{(1+w)/2}$$