BY OLEG POPOV

NON-HOLOMORPHIC MODULAR $\mathscr{A}_4$ SYMMETRIC SCOTOGENIC MODEL

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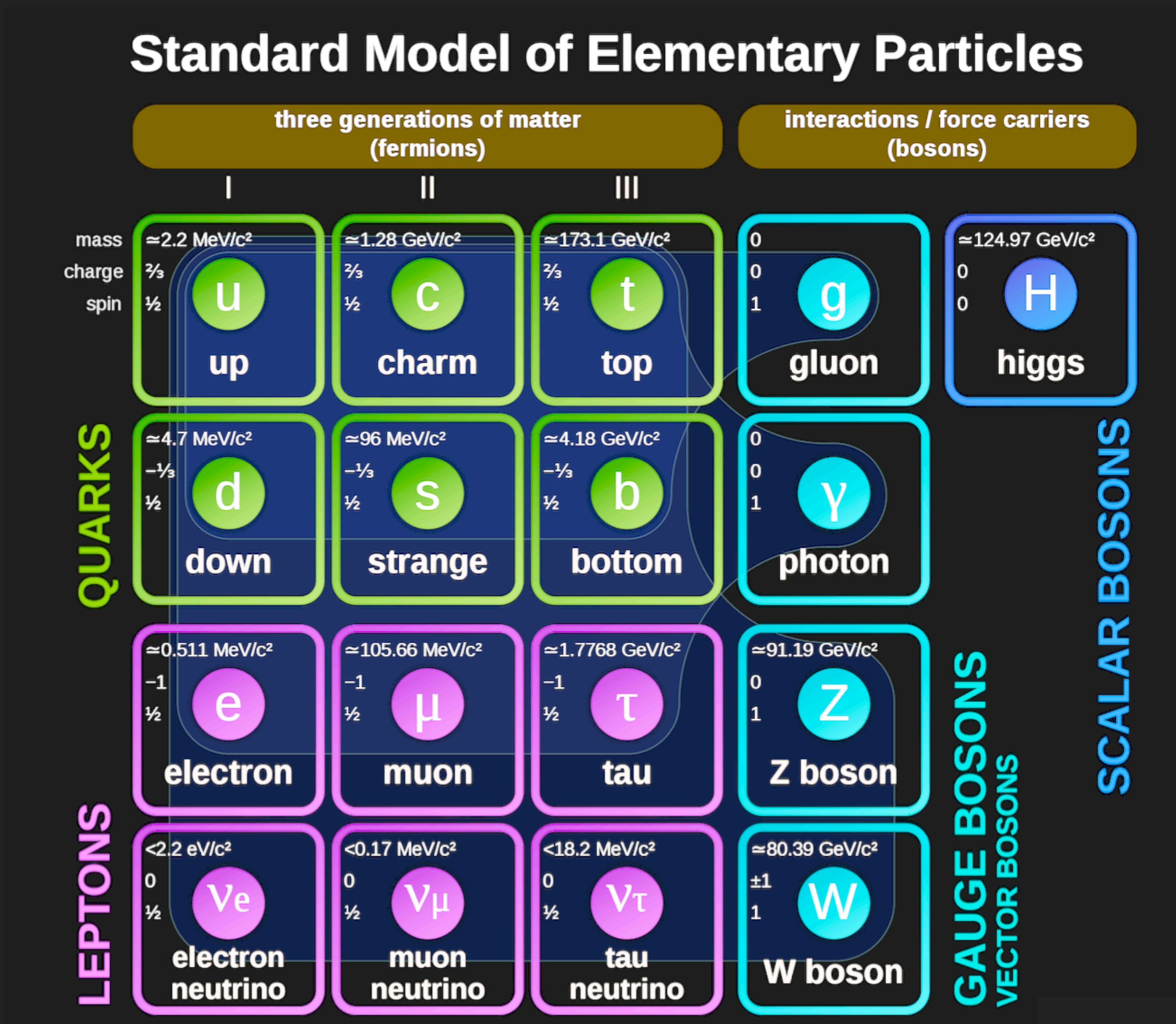
In collaboration with Hiroshi Okada and Takaaki Nomura

OVERVIEW

MOTIVATION



- Neutrino Mass Origin?
- Source of Neutrino mixing? PMNS matrix
- Dark Matter?

$$\mathbb{G}_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$$

$$\mathcal{L}^Y = \bar{u} Y_u Q H + \bar{Q} Y_d d H + \text{h.c.}$$

$$\langle h^0 \rangle = v \rightarrow \quad m_f = Y_f v$$

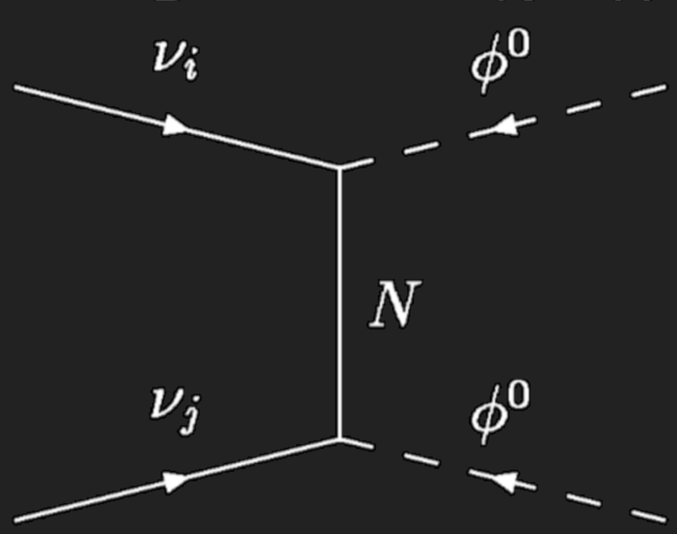
ORIGIN OF NEUTRINO MASSES

Weinberg Operator 1979: $\frac{LHLH}{\Lambda} = \frac{(l^- H^+ - \nu H^0)(l^- H^+ - \nu H^0)}{\Lambda}$

Add $N_R \sim (1,1,0)$ under $\mathbb{G}_{SM}$

$$\mathcal{L}_{new} = \bar{N} Y_D L H + m_N N_R N_R + \text{h.c.}$$

Seesaw-I



$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}$$

$v \ll m_N \sim 10^{11}\text{GeV} \rightarrow m_\nu \ll v$ with $Y_D \sim 1$

or $Y_D \ll 1 \rightarrow m_\nu \ll v$ with $m_N \sim O(10^{2-3} GeV)$

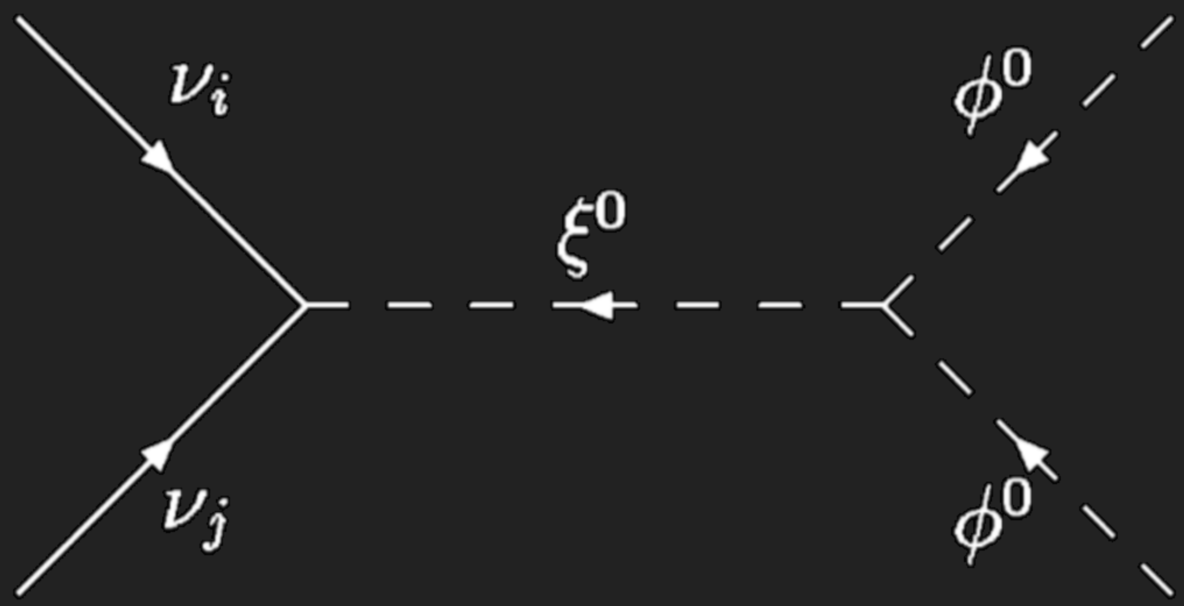
Seesaw−I

ORIGIN OF NEUTRINO MASSES

Seesaw-II

Add[1980] $\xi = (\xi^{++}, \xi^{+}, \xi^{0}) \sim (1, 3, 1)$ under $\mathbb{G}_{SM}$

$$\mathcal{L}_{new} = YL\xi L - \mu H\xi H + \text{h.c.} \rightarrow m_\nu = Y \langle \xi^0 \rangle = -2\frac{Y\mu v^2}{M_\xi}$$

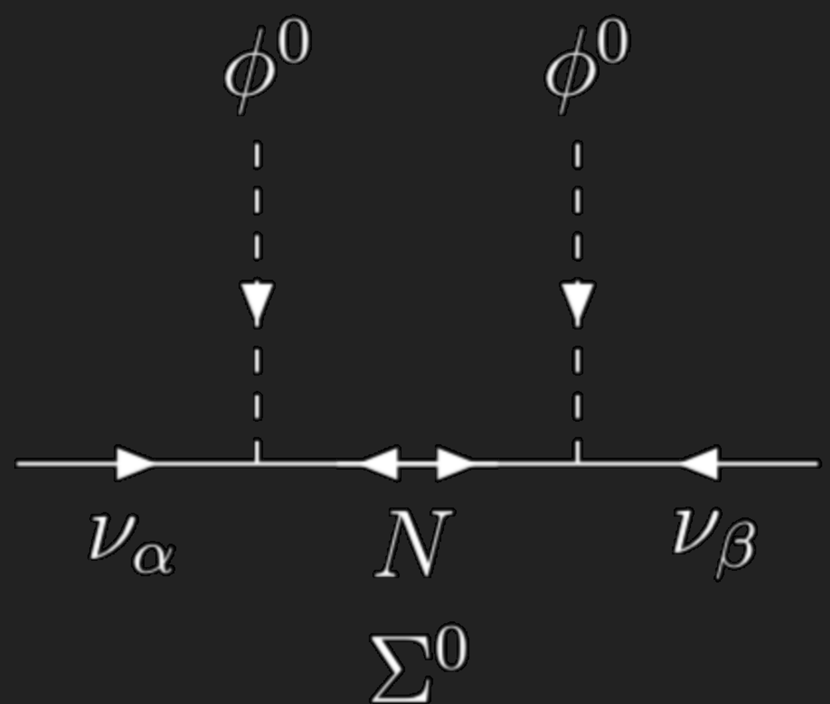


ORIGIN OF NEUTRINO MASSES

SEESAW-III

Add $\Sigma_R \sim (1,3,0)$ under $\mathbb{G}_{SM}$

$$\mathcal{L}_{new} = \bar{\Sigma} Y_D L H + m_N \Sigma_R \Sigma_R + \text{h.c.}$$



$$\begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix} \Rightarrow m_\nu \simeq \frac{-m_D^2}{m_N}$$

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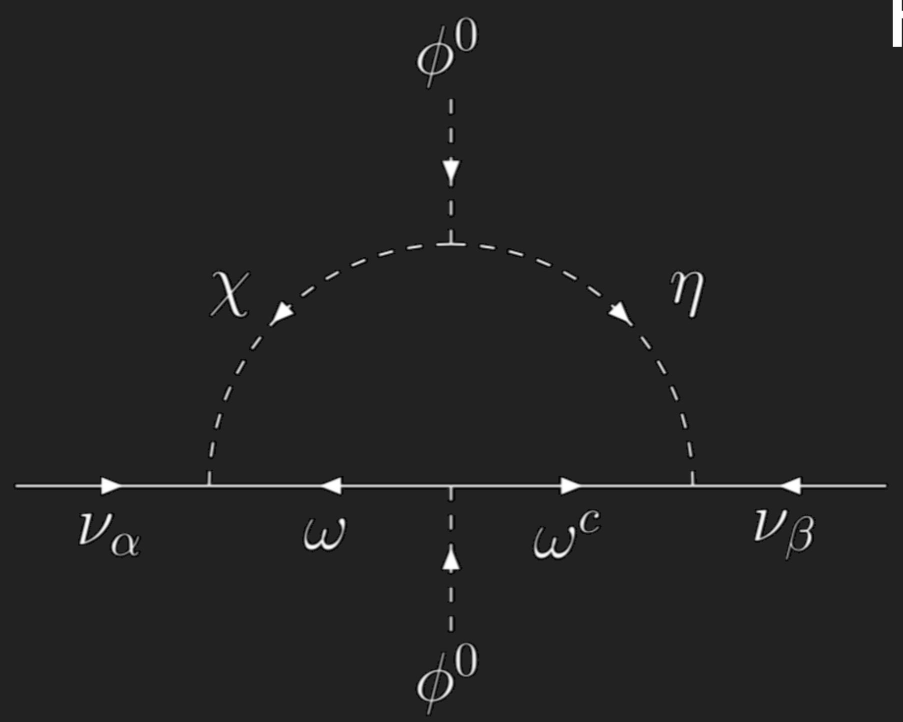
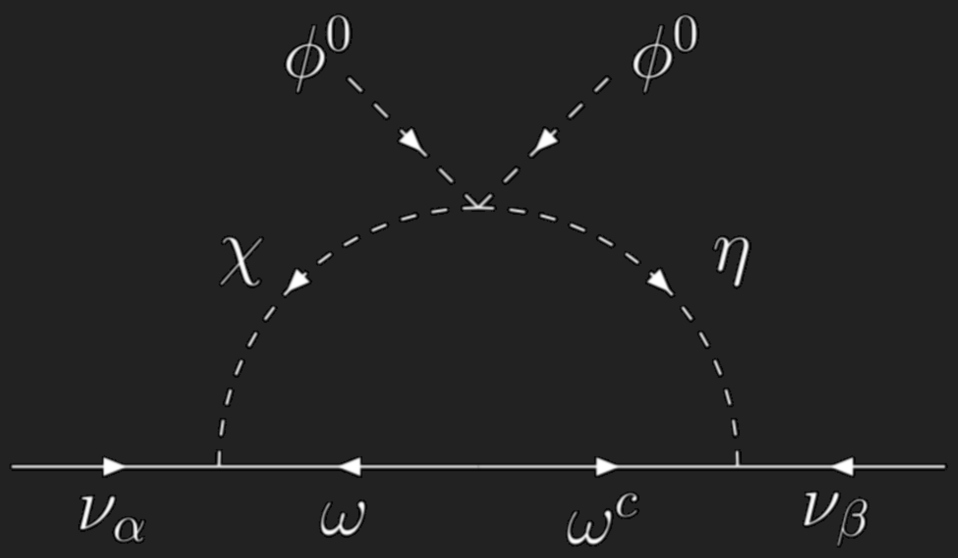
ORIGIN OF NEUTRINO MASSES

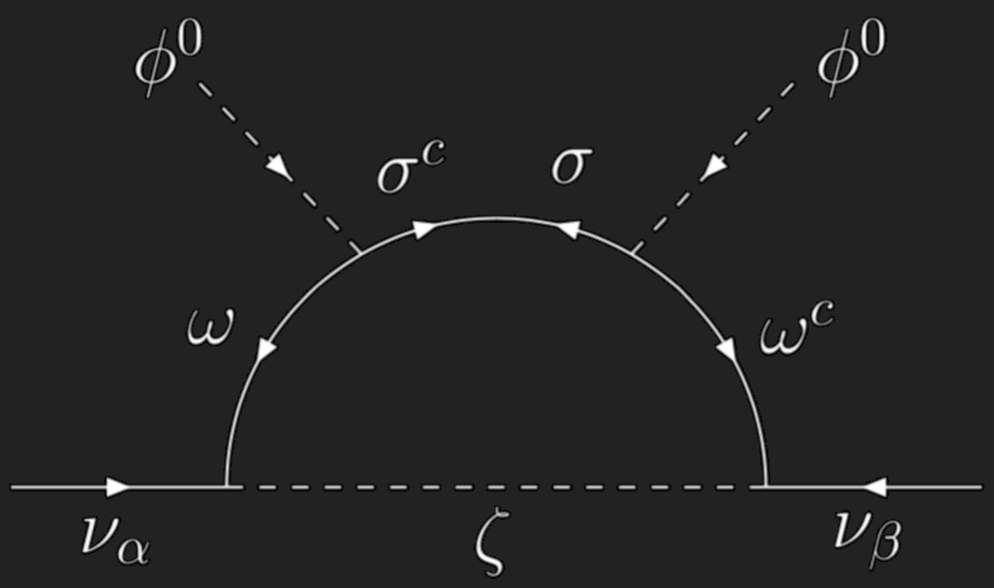
Radiative Majorana Neutrino Masses



Zee[1986]

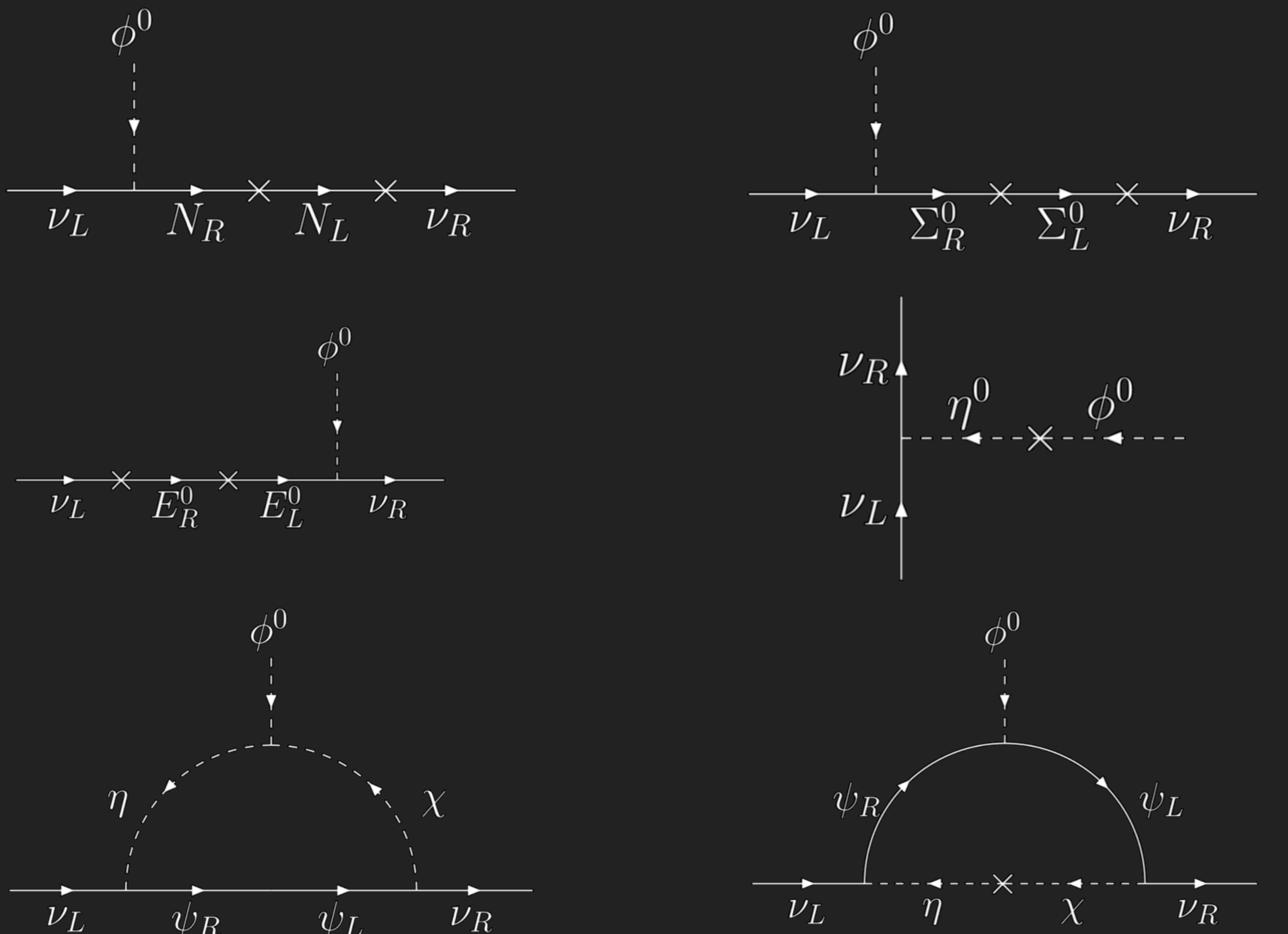


Ma[2006]



Fraser, Ma, OP[2014]

ORIGIN OF NEUTRINO MASSES Dirac Neutrino Masses



E. Ma and OP, [PLB(2017)]

MODULAR SYMMETRY

$$\tau \longrightarrow \gamma\tau = \frac{a\tau + b}{c\tau + d} \qquad S^2 = \mathcal{I}\,, \qquad (ST)^3 = \mathcal{I}\,.$$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \qquad S : \tau \longrightarrow -\frac{1}{\tau}\,, \qquad T : \tau \longrightarrow \tau + 1\,.$$

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{Z})\,, \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} (\mathrm{mod} N) \right\} \qquad \Gamma_N \equiv \bar{\Gamma}/\bar{\Gamma}(N)$$

$$f_i(\gamma\tau) = (c\tau + d)^k \rho_{ij}(\gamma) f_j(\tau)\,, \quad \gamma \in \Gamma(N) \qquad \Gamma_3 \simeq \mathscr{A}_4$$

$$y_1(\tau) = 1 + 12q + 36q^2 + 12q^3 + \dots$$

[arXiv:1706.08749]

$$y_2(\tau) = -6q^{1/3}(1 + 7q + 8q^2 + \dots) \qquad q \equiv e^{i2\pi\tau}$$

$$y_3(\tau) = -18q^{2/3}(1 + 2q + 5q^2 + \dots)$$

$$Y_3^{(4)} = (y_1^2 - y_2y_3, y_3^2 - y_1y_2, y_2^2 - y_1y_3) \qquad Y_1^{(6)} = y_1^3 + y_2^3 + y_3^3 - 3y_1y_2y_3$$

$$Y_1^{(4)} = y_1^2 + 2y_2y_3 \qquad Y_{3a}^{(6)} = (y_1^3 + 2y_1y_2y_3, y_1^2y_2 + 2y_2^2y_3, y_1^2y_3 + 2y_3^2y_2)$$

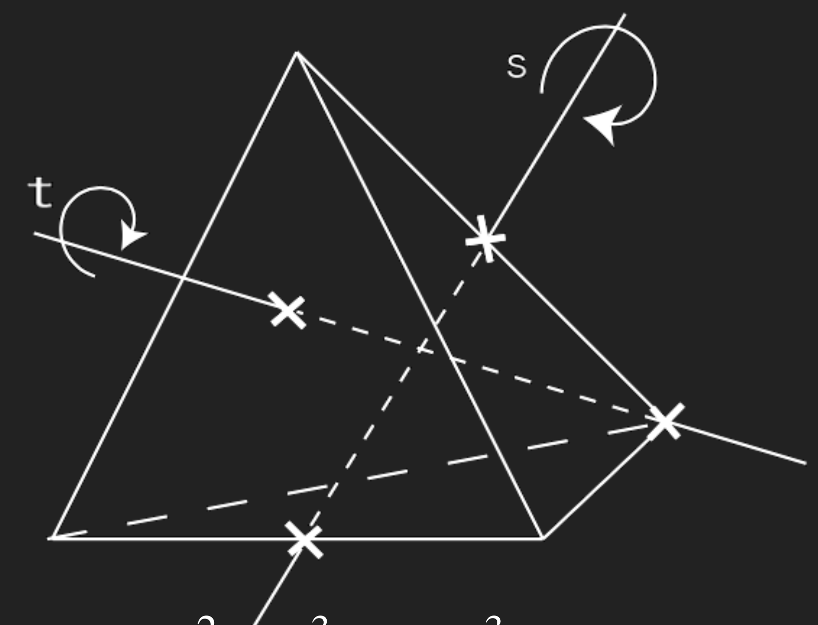
$$Y_{1'}^{(4)} = y_3^2 + 2y_1y_2 \qquad Y_{3b}^{(6)} = (y_3^3 + 2y_1y_2y_3, y_3^2y_1 + 2y_1^2y_2, y_3^2y_2 + 2y_2^2y_1)$$

$$Y_{1''}^{(4)} = y_2^2 + 2y_1y_3 \qquad Y_{3c}^{(6)} = (y_2^3 + 2y_1y_2y_3, y_2^2y_3 + 2y_3^2y_1, y_2^2y_1 + 2y_1^2y_3)$$

NEUTRINO MASSES VIA $\mathscr{A}_4$ SYMMETRY

$$\begin{array}{lll} C_1: & \{e\}, & h=1, \\ C_3: & \{s, tst^2, t^2st\}, & h=2, \\ C_4: & \{t, ts, st, sts\}, & h=3, \\ C_{4'}: & \{t^2, st^2, t^2s, tst\}, & h=3. \end{array}$$

	h	χ_1	$\chi_{1'}$	$\chi_{1''}$	χ_3
C_1	1	1	1	1	3
C_3	2	1	1	1	-1
C_4	3	1	ω	ω^2	0
$C_{4'}$	3	1	ω^2	ω	0



$$s^2 = t^3 = (st)^3 = e$$

$$\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}.$$

$$\underline{1} = a_1 a_2 + b_1 b_2 + c_1 c_2,$$

$$\underline{1}' = a_1 a_2 + \omega^2 b_1 b_2 + \omega c_1 c_2,$$

$$\underline{1}'' = a_1 a_2 + \omega b_1 b_2 + \omega^2 c_1 c_2.$$

$$(\mathbf{A})_\mathbf{3} \times (\mathbf{B})_\mathbf{3} = (\mathbf{A}\cdot\mathbf{B})_\mathbf{1} + (\mathbf{A}\cdot\Sigma\cdot\mathbf{B})_{\mathbf{1}'} + (\mathbf{A}\cdot\Sigma^*\cdot\mathbf{B})_{\mathbf{1}''} + \begin{pmatrix} \{A_y B_z\} \\ \{A_z B_x\} \\ \{A_x B_y\} \end{pmatrix}_\mathbf{3} + \begin{pmatrix} [A_y B_z] \\ [A_z B_x] \\ [A_x B_y] \end{pmatrix}_\mathbf{3}.$$

[arXiv:1003.3552]

NEUTRINO MASSES VIA $\mathscr{A}_4$ SYMMETRY

$$\begin{aligned} &(\mathscr{A}_4, L) \\ (\nu_i, l_i)_L &\sim (\underline{3}, 1), \\ l_{1R} &\sim (\underline{1}, 1), \\ l_{2R} &\sim (\underline{1}', 1), \\ l_{3R} &\sim (\underline{1}'', 1), \\ N_{iR} &\sim (\underline{3}, 0), \\ \Phi_i = (\phi_i^+, \phi_i^0) &\sim (\underline{3}, 0), \\ \eta = (\eta^+, \eta^0) &\sim (\underline{1}, -1). \end{aligned}$$

$$\frac{1}{2} M N_{iR}^2 + f \bar{N}_{iR}(\nu_{iL}\eta^0 - l_{iL}\eta^+) + h_{ijk}\overline{(\nu_i, l_i)}_L l_{jR}\Phi_k + \text{H.c.},$$

$$U_L^\dagger \mathcal{M}_l U_R = \begin{bmatrix} \sqrt{3}h_1 v & 0 & 0 \\ 0 & \sqrt{3}h_2 v & 0 \\ 0 & 0 & \sqrt{3}h_3 v \end{bmatrix} = \begin{bmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{bmatrix},$$

$$U_L = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{bmatrix}, \quad U_R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E. Ma, G. Rajasekaran, PRD(2001)
K.S. Babu, E. Ma, J.W.F. Valle, PLB(2002)

* bimaximal and tribimaximal
* Max $\sin(\theta_{23})$; $\sin(\theta_{13}) = 0$;
* Degenerate neutrino masses
* Soft breaking → small mass splittings, $\sin(\theta_{13}) \neq 0$

$$\mathcal{M}_\nu = \frac{f^2 u^2}{M} U_L^T U_L = \frac{f^2 u^2}{M} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

A MODULAR $\mathscr{A}_4$ SYMMETRIC SCOTOGENIC MODEL

	Fermions			Bosons	
	$(\bar{L}_{L_e}, \bar{L}_{L_\mu}, \bar{L}_{L_\tau})$	$(e_{R_e}, e_{R_\mu}, e_{R_\tau})$	N_R	H	η^*
$SU(2)_L$	**2**	**1**	**1**	**2**	**2**
$U(1)_Y$	$\frac{1}{2}$	-1	0	$\frac{1}{2}$	$-\frac{1}{2}$
A_4	$1, 1', 1''$	$1, 1'', 1'$	3	1	1
$-k$	0	0	-1	0	-3

	Couplings		
	$Y^{(6)}_{\mathbf{1}}$	$Y^{(2)}_{\mathbf{3}}$	$Y^{(4)}_{\mathbf{3}}$
A_4	**1**	**3**	**3**
$-k$	6	2	4

$$-\mathcal{L}_{Lepton} = \sum_{\ell=e,\mu,\tau} y_\ell \bar{L}_{L_\ell} H e_{R_\ell}$$
$$+ \alpha_\nu \bar{L}_{L_e} (Y^{(4)}_{\mathbf{3}} \otimes N_R)_{\mathbf{1}} \tilde{\eta} + \beta_\nu \bar{L}_{L_\mu} (Y^{(4)}_{\mathbf{3}} \otimes N_R)_{\mathbf{1}''} \tilde{\eta}$$
$$+ \gamma_\nu \bar{L}_{L_\tau} (Y^{(4)}_{\mathbf{3}} \otimes N_R)_{\mathbf{1}'} \tilde{\eta}$$
$$+ M_1 (Y^{(2)}_{\mathbf{3}} \otimes \bar{N}^C_R \otimes N_R) + \text{h.c.},$$

$$Y^{(6)}_{\mathbf{1}} = y_1^3 + y_2^3 + y_3^3 - 3 y_1 y_2 y_3, \quad \rightarrow Y^{(6)}_{\mathbf{1}} \left(H^\dagger \eta\right)^2$$

T. Nomura, H. Okada, OP, PLB(2020)

MASS MATRICES

$$M_N = \frac{M_1}{3}\begin{bmatrix} 2y_1 & -y_3 & -y_2 \\ -y_3 & 2y_2 & -y_1 \\ -y_2 & -y_{3,1} & 2y_3 \end{bmatrix}$$

$$Y_{\mathbf{3}}^{(2)} = (y_1, y_2, y_3)$$

$$y_\eta = \begin{bmatrix} \alpha_\nu & 0 & 0 \\ 0 & \beta_\nu & 0 \\ 0 & 0 & \gamma_\nu \end{bmatrix}\begin{bmatrix} y'_1 & y'_3 & y'_2 \\ y'_3 & y'_2 & y'_1 \\ y'_2 & y'_1 & y'_3 \end{bmatrix}$$

$$Y_{\mathbf{3}}^{(4)} \equiv (y'_1, y'_2, y'_3) = \begin{bmatrix} y_1^2 - y_2 y_3 \\ y_3^2 - y_1 y_2 \\ y_2^2 - y_1 y_3 \end{bmatrix}$$

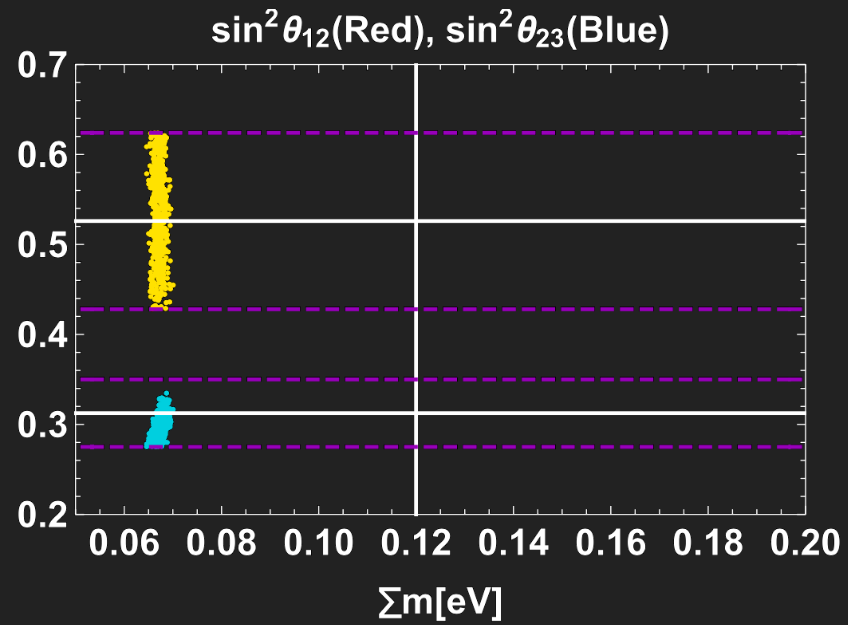
$$m_{\nu_{ij}} \approx \sum_{\alpha=1-3} \frac{Y_{\eta_{i\alpha}} D_{N\alpha} Y^T_{\eta_{\alpha j}}}{(4\pi)^2} \times \left(\frac{m_R^2}{m_R^2 - D_{N\alpha}^2} \ln\left[\frac{m_R^2}{D_{N\alpha}^2}\right] - \frac{m_I^2}{m_I^2 - D_{N\alpha}^2} \ln\left[\frac{m_I^2}{D_{N\alpha}^2}\right]\right)$$

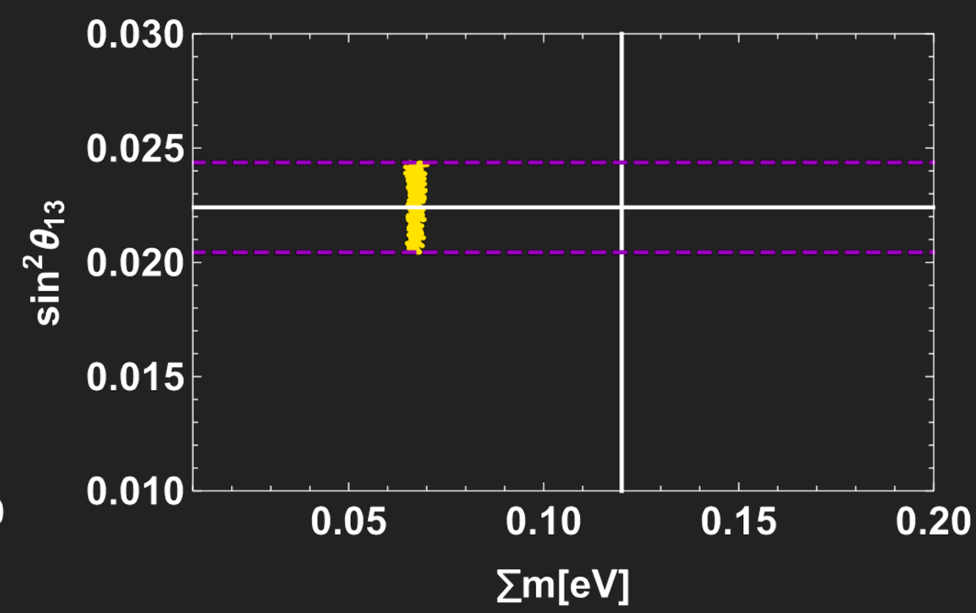
$$y_1(\tau) = \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right)$$

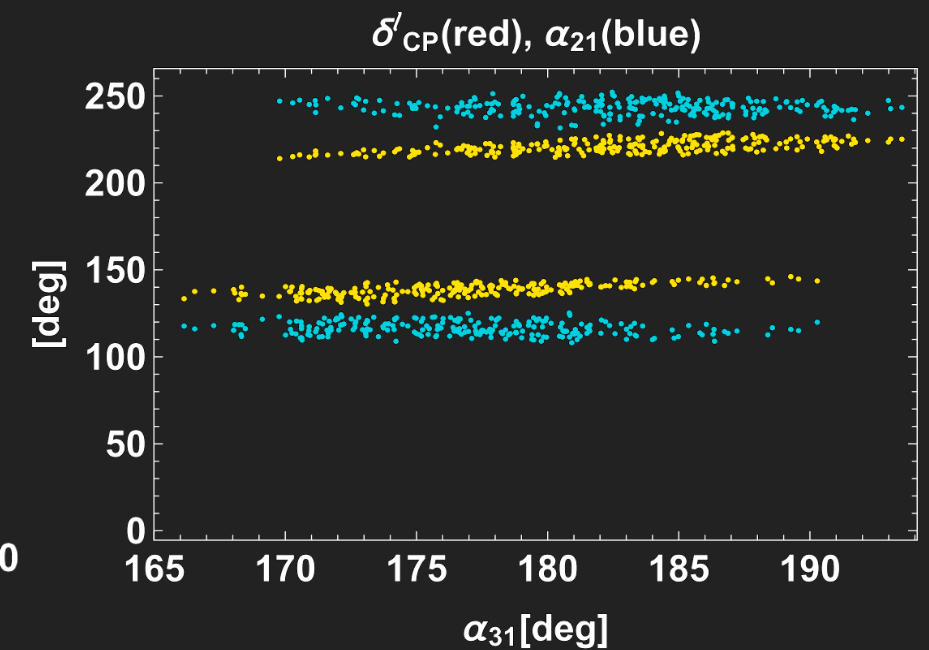
$$y_2(\tau) = \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2 \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right)$$

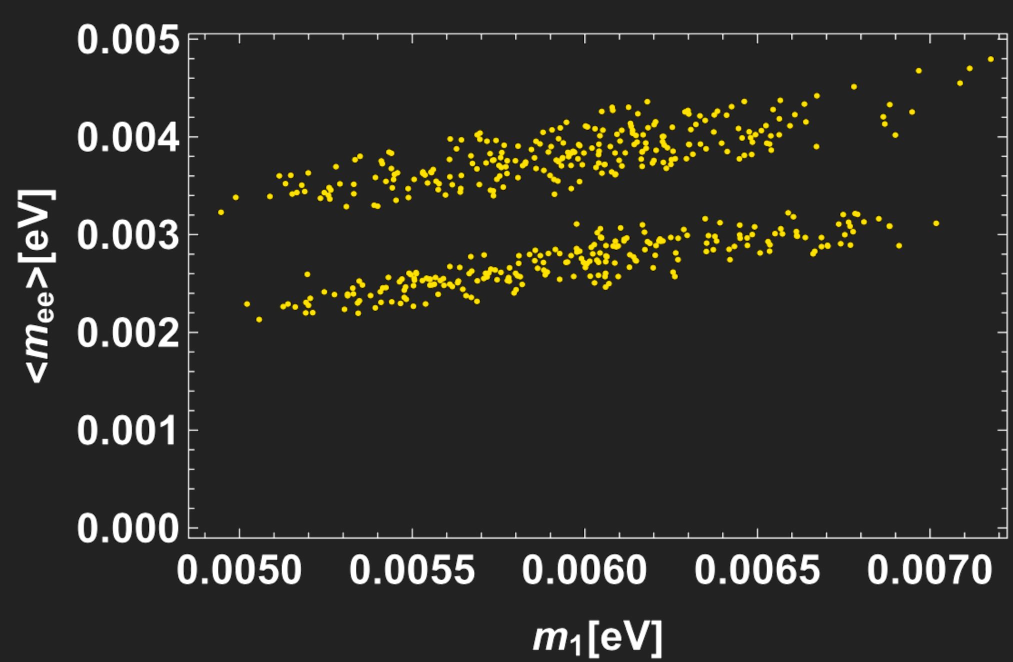
$$y_3(\tau) = \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2 \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right)$$

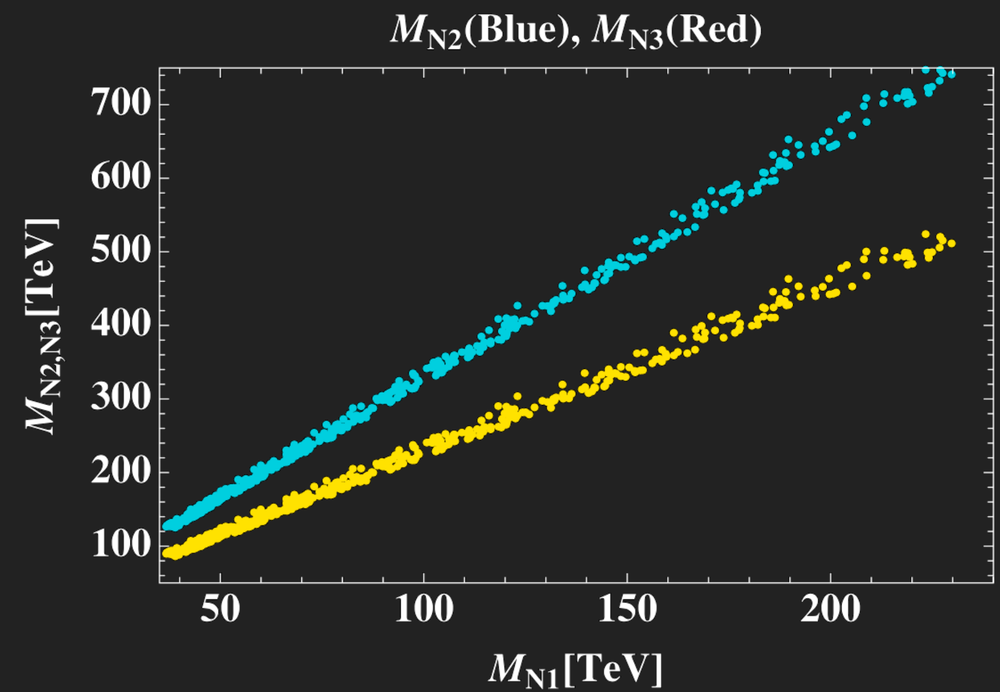
RESULT PLOTS











NON-HOLOMORPHIC MODULAR $\mathscr{A}_4$ SYMMETRIC SCOTOGENIC MODEL

Fields	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	$\mathcal{A}_4$	k
$\overline{L_L}$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	$\mathbf{3}$	-1
e_R	$\mathbf{1}$	$\mathbf{1}$	-1	$\mathbf{3}$	1
N_R	$\mathbf{1}$	$\mathbf{1}$	0	$\mathbf{3}$	0
H	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	$\mathbf{1}$	0
η	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	$\mathbf{1}$	1

T. Nomura, H. Okada, OP, PLB(2025)

MODEL LAGRANGIAN

$$
\begin{aligned}
\mathcal{L}_{Y_\ell} = {} & M_0[\overline{N_R^c}N_R]_{\mathbf{1}} + M_3 Y_{\mathbf{3}}^{(0)}[\overline{N_R^c}N_R]_{\mathbf{3}_s} \\
& + y_\ell[\overline{L_L}e_R]_{\mathbf{1}}H + y_{\ell_3}Y_{\mathbf{3}}^{(0)}[\overline{L_L}e_R]_{\mathbf{3}_s}H \\
& + y'_{\ell_3}Y_{\mathbf{3}}^{(0)}[\overline{L_L}e_R]_{\mathbf{3}_a}H + y_D[\overline{L_L}N_R]_{\mathbf{1}}\eta \\
& + y_{D_3}Y_{\mathbf{3}}^{(0)}[\overline{L_L}N_R]_{\mathbf{3}_s}\eta + y'_{D_3}Y_{\mathbf{3}}^{(0)}[\overline{L_L}N_R]_{\mathbf{3}_a}\eta \\
& + \text{h.c.},
\end{aligned}
$$

$$
\begin{aligned}
V = {} & \mu_\eta^2\eta^\dagger\eta + \tilde{\lambda}Y_{\mathbf{1}}^{(-2)}(H\eta)^2 + \lambda_{H\eta}\left(H^\dagger H\right)\left(\eta^\dagger\eta\right) \\
& + \lambda'_{H\eta}|H\eta|^2 + \text{other trivial terms},
\end{aligned}
$$

MASS MATRICES

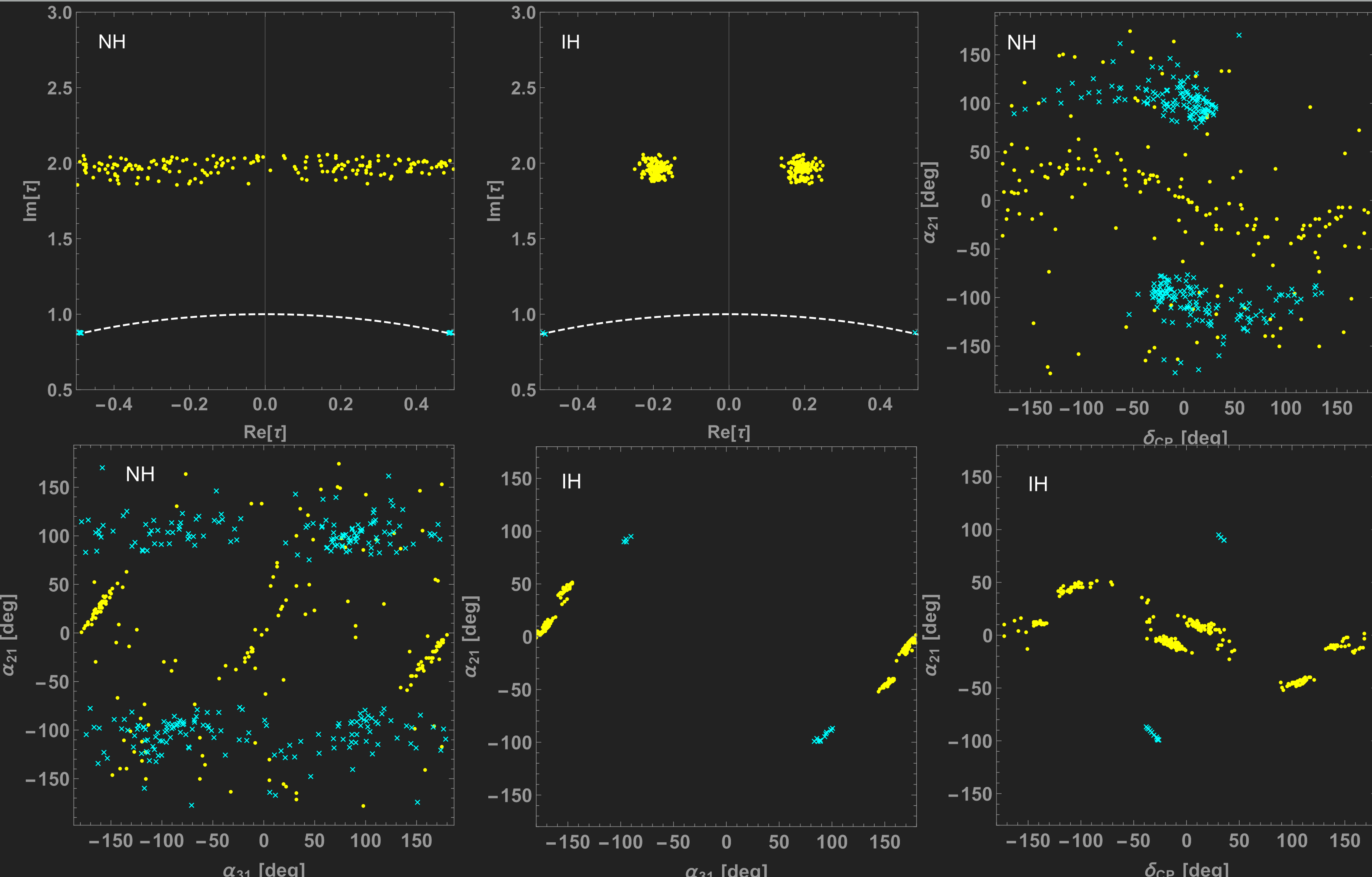
$$M_N = M_3 \begin{bmatrix} \tilde{M}_0 + 2y_1 & -y_3 & -y_2 \\ -y_3 & 2y_2 & \tilde{M}_0 - y_1 \\ -y_2 & \tilde{M}_0 - y_1 & 2y_3 \end{bmatrix}$$

$$y_\eta = \begin{bmatrix} y_D + 2y_{D_3}y_1 & -(y_{D_3} - y'_{D_3})y_3 & -(y_{D_3} + y'_{D_3})y_2 \\ -(y_{D_3} + y'_{D_3})y_3 & 2y_{D_3}y_2 & y_D - (y_{D_3} - y'_{D_3})y_1 \\ -(y_{D_3} - y'_{D_3})y_2 & y_D - (y_{D_3} + y'_{D_3})y_1 & 2y_{D_3}y_3 \end{bmatrix} \qquad (\overline{L_L} y_\eta N_R)\eta$$

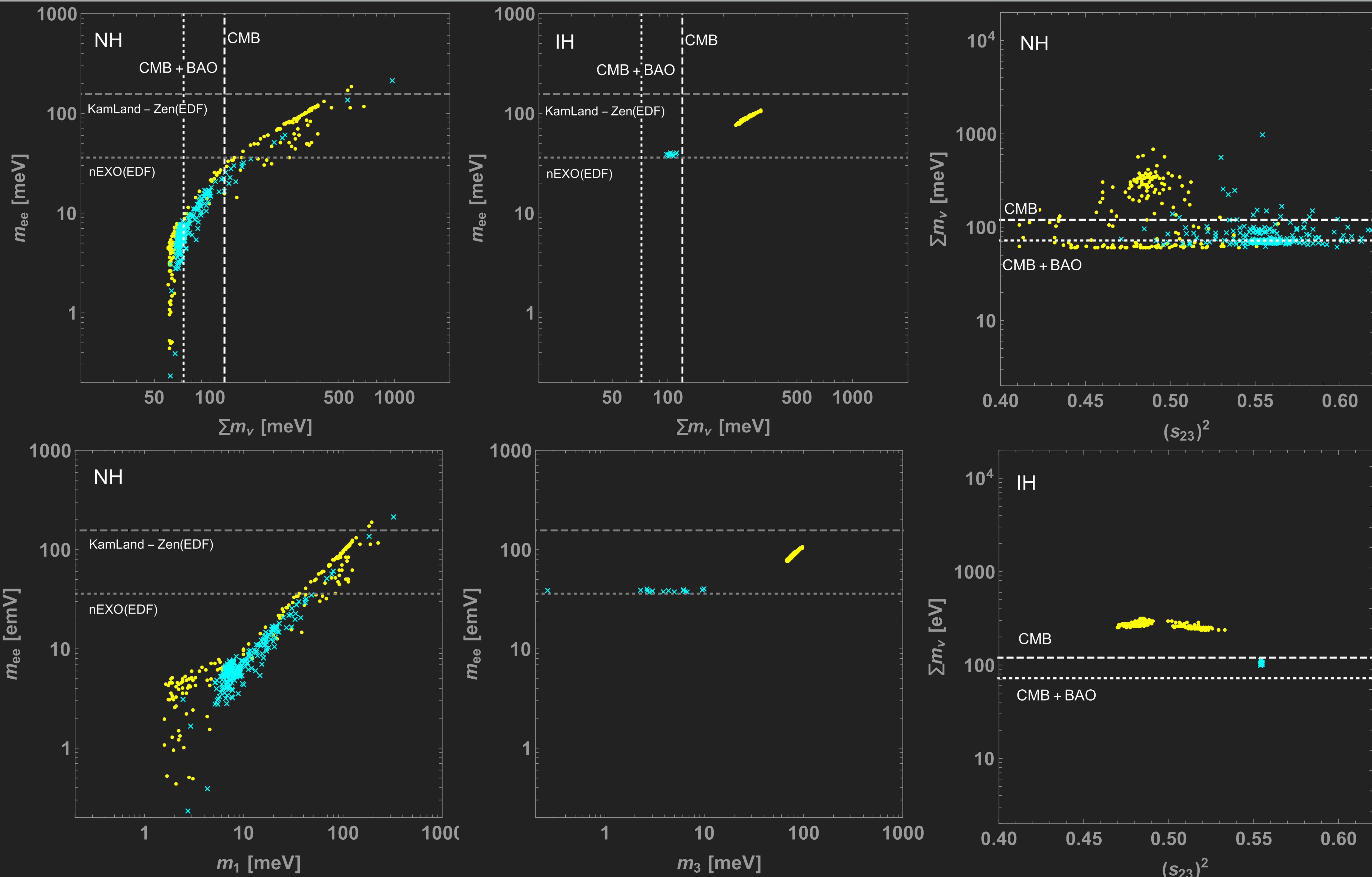
$$\frac{m_\ell}{v} = \begin{bmatrix} y_\ell + 2y_{\ell_3}y_1 & -(y_{\ell_3} - y'_{\ell_3})y_3 & -(y_{\ell_3} + y'_{\ell_3})y_2 \\ -(y_{\ell_3} + y'_{\ell_3})y_3 & 2y_{\ell_3}y_2 & y_\ell - (y_{\ell_3} - y'_{\ell_3})y_1 \\ -(y_{\ell_3} - y'_{\ell_3})y_2 & y_\ell - (y_{\ell_3} + y'_{\ell_3})y_1 & 2y_{\ell_3}y_3 \end{bmatrix}$$

$$m_{\nu_{ij}} \approx \sum_{\alpha=1-3} \frac{Y_{\eta_{i\alpha}} D_{N\alpha} Y^T_{\eta_{\alpha j}}}{(4\pi)^2} \times \left(\frac{m_R^2}{m_R^2 - D_{N\alpha}^2} \ln\left[\frac{m_R^2}{D_{N\alpha}^2}\right] - \frac{m_I^2}{m_I^2 - D_{N\alpha}^2} \ln\left[\frac{m_I^2}{D_{N\alpha}^2}\right] \right)$$

RESULT PLOTS



RESULT PLOTS



SUMMARY

- Naturally small neutrino masses (radiative model)
- Minimal field content $(\mathscr{A}_4)$
- Candidate for dark matter (scotogenic = darkness)
- Prediction for PMNS matrix/neutrino mixing (modular)
- Correlation between Majorana and Dirac phases
- Model is testable by current and future neutrino experiments (KATRIN, $0\nu\beta\beta$) and cosmological observations (DESI2024)