Model-independent Extraction of Form Factors and $|V_{cb}|$ in $\overline{B} \to D \ell^- \overline{\nu}$ with hadronic tagging at BABAR

G. Eigen, OrcID: 0000000335295171

On behalf of the BABAR collaboration

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Outline

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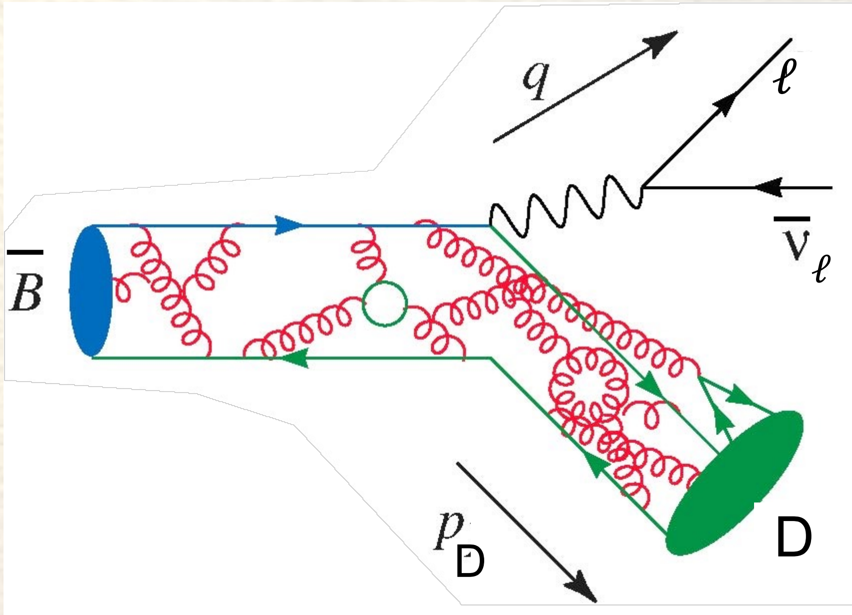
Introduction

- The decay $\bar{B}\to D\ell^-\bar{\nu}$ proceeds through a simple tree-level diagram and has been studied by many experiments



- The decay proceeds via the vector current only

$$\langle D|\bar{c}\gamma_\mu b|\bar{B}\rangle_V = f_+(q^2)\left((p_B+p_D)_\mu - \frac{(p_B+p_D)\cdot q}{q^2}q_\mu\right) + f_0(q^2)\frac{(p_B+p_D)\cdot q}{q^2}q_\mu$$

- In the limit of vanishing lepton masses $f_0(q^2)$ becomes zero

- The differential $\bar{B}\to D\ell^-\bar{\nu}$ decay rate is

$$\frac{d\Gamma}{dq^2 d\cos\theta_l} = \frac{G_F^2|V_{cb}|^2\eta_{EW}^2}{32\pi^3}k^3|f_+(q^2)|^2\sin^2\theta_l$$

with $k = m_D\sqrt{w^2-1}$ and $w = \frac{m_B^2+m_D^2-q^2}{2m_Bm_D}$

- So, $d\Gamma$ is a function of q^2 and $\cos\theta_\ell$ and depends on form factor $f_+(q^2)$ and CKM element $|V_{cb}|$

Introduction cont.

- Using the full data set, *BABAR* has performed a new study of $\bar{B}\to D\ell^-\bar{\nu}$ by analyzing the process $e^+e^- \to \Upsilon(4S) \to B_{tag}\bar{B}_{sig}$, where B_{tag} is reconstructed in *B* hadronic decays and $\bar{B}_{sig}$ represents the $\bar{B}\to D\ell^-\bar{\nu}$ signal mode

- Two different form factor parametrizations are employed, the model-independent Boyd-Grinstein-Lebed (BGL) expansion and the model-dependent Caprini-Lellouch-Neubert (CLN) expansion

  Nucl.Phys. **B461**, 493 (1996)
  Nucl.Phys. **B530**, 153 (1998)

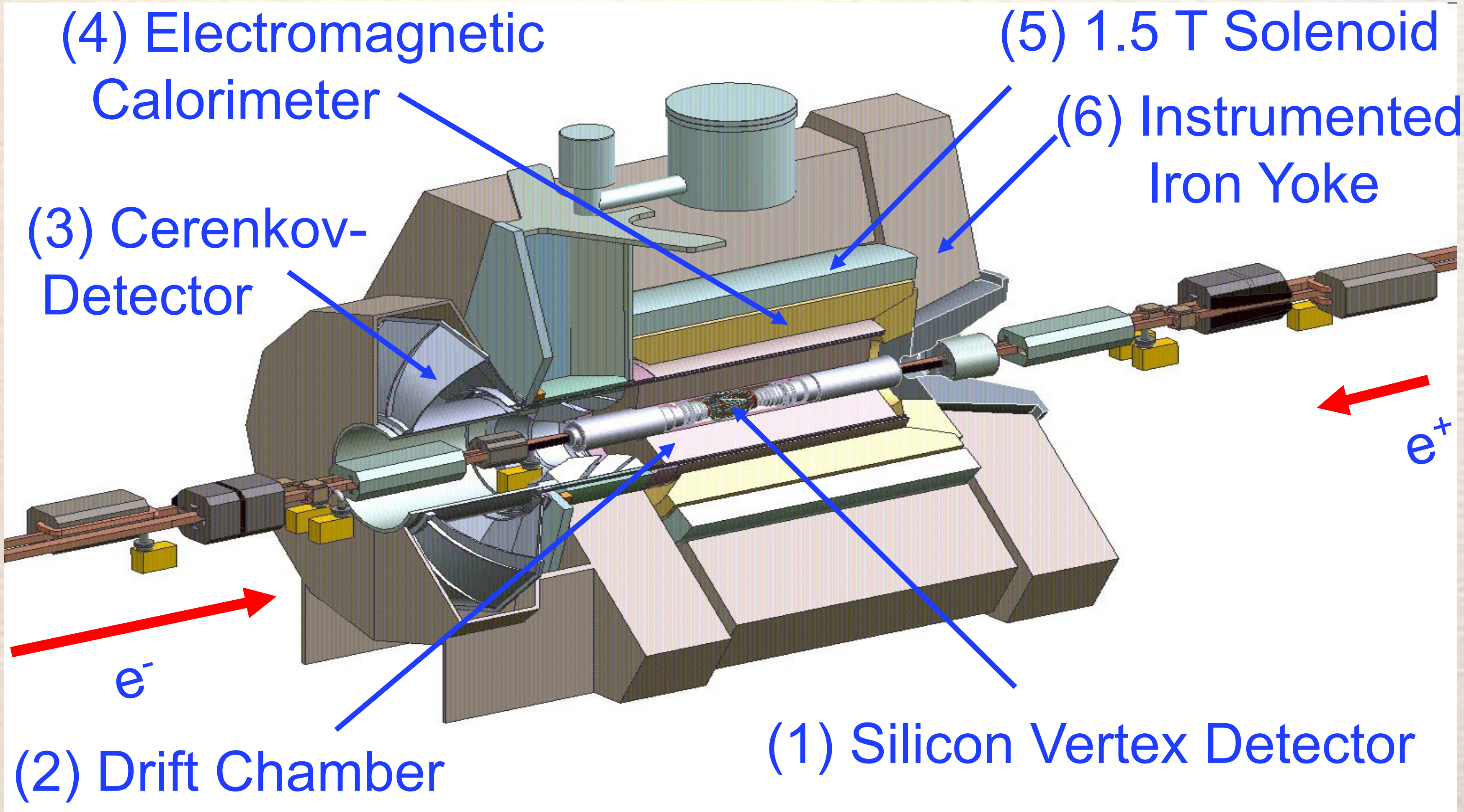
- BGL form factors $f_{0,+}$ are expressed as an expansion in variable $z(w) = \frac{\sqrt{w+1}-\sqrt{2}}{\sqrt{w+1}+\sqrt{2}}$ with free coefficients $a_{0,+}$ constrained by $\sum_{i=1}^{n} |a_i^{0,+}|^2 \leq 1$ normalized to Blaschke factors $P_{0,+}(z)$ that remove contributions of bound state $B_c^{(*)}$ poles and non-perturbative outer functions $\phi_{0,+}(z)$

- CLN form factors, based on QCD dispersion relations and HQET, have been used in most analyses and are expressed as

$$\mathcal{G}(w) = \mathcal{G}(1)\left(1 - 8\rho_D^2 z(w) + (51\rho_D^2 - 10)z(w)^2 - (252\rho_D^2 - 84)z(w)^3\right)$$

  where $\mathcal{G}(1)$ is the normalization and ρ_D is the slope

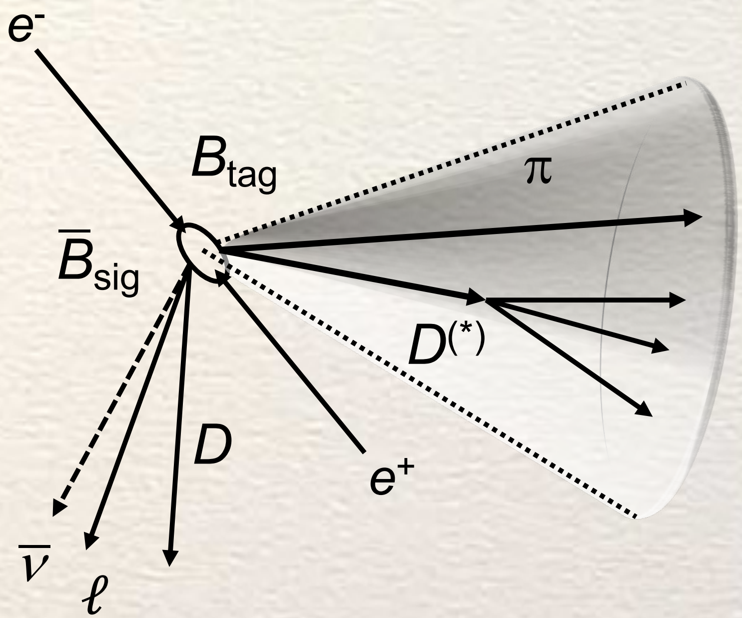
The BABAR Detector



Analysis Strategy

- Data sample consist of 471×10^6 $\Upsilon(4S)\to B\bar{B}$ events (426 fb^{-1}) NIM **A726**, 203 (2013)

- One B is tagged via a hadronic decay ($D^{(*)0}$, $D^{(*)+}$, $D_s^{(*)+}$, J/ψ) plus up to 5 charged charmless light mesons and 2 neutral mesons PRD **110**, 032018 (2024)



- The reconstruction relies on 2 variables

$$m_{ES} = \sqrt{\tfrac{1}{4}s - \left|\vec{p}^{\,*}_{tag}\right|^2}$$
$$\Delta E = E^*_{tag} - \tfrac{1}{2}\sqrt{s}$$

where $\vec{p}^{\,*}_{tag}$ and E^*_{tag} are 3-momentum and energy of B_{tag} in the CM frame and s is center-of-mass energy squared

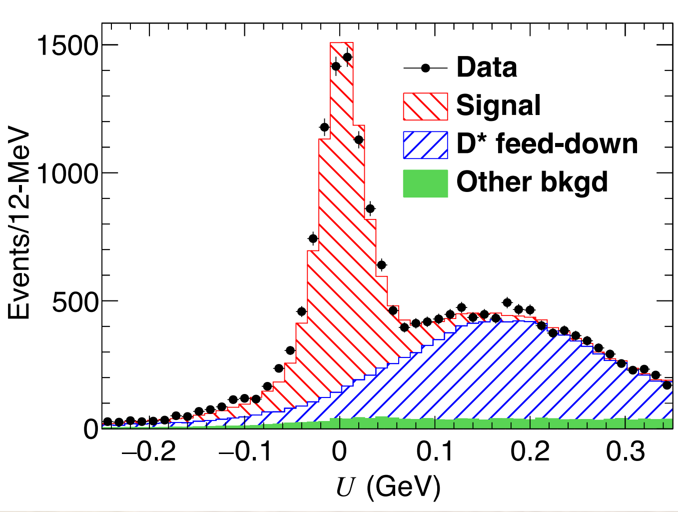
- Select events with m_{ES}>5.27 GeV/c^2 and $|\Delta E|$<72 MeV

- Select 10 modes on signal side: $D^0 \to K^-\pi^+$, $K^-\pi^+\pi^0$, $K^-\pi^+\pi^+\pi^-$, $D^+ \to K^-\pi^+\pi^+$, $K^-\pi^+\pi^-\pi^0$ plus an e^- with p_e>200 MeV/c or a μ^- with p_μ> 300 MeV/c

- Analysis is similar to that of old $\bar{B}\to D^*\ell^-\bar{\nu}$ PRL **123**, 091801 (2019)

Analysis Strategy cont.

- Determine missing momentum $p_{\bar{\nu}} \equiv p_{\text{miss}} = p_{e^+e^-} - p_{tag} - p_D - p_\ell$
- For a semileptonic decay with one missing neutrino this is fulfilled
- We use the discriminating variable $U = E^{**}_{\text{miss}} - \left|\vec{p}^{**}_{\text{miss}}\right|$
  (E^{**}_{miss} and $\vec{p}^{**}_{\text{miss}}$ are $\bar{\nu}$ energy and 3-momentum in $\bar{B}_{\text{sig}}$ rest frame)

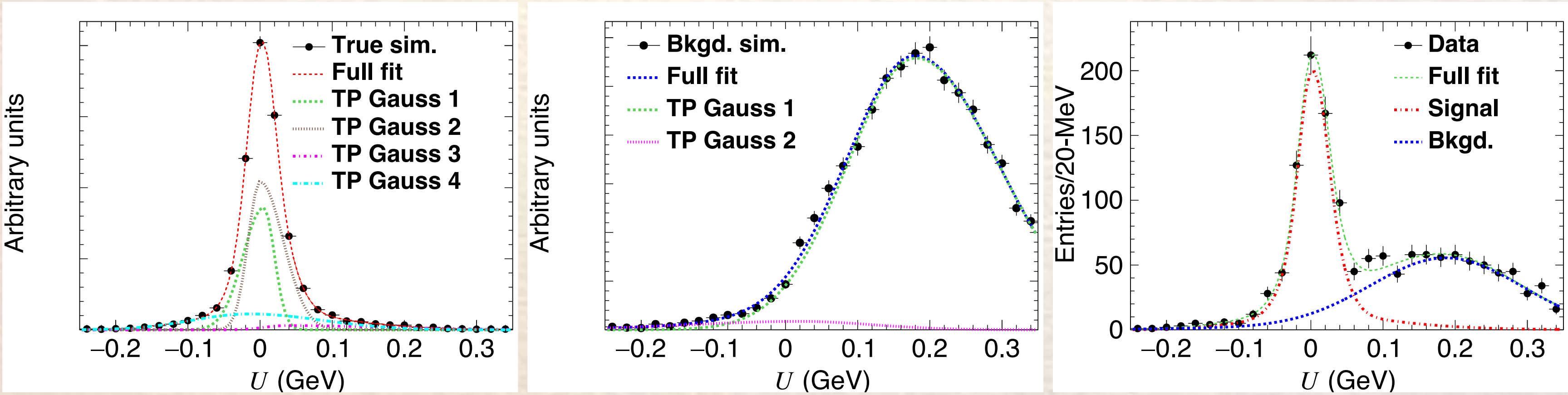


- We measure the extra energy in the calorimeter, require E_{Extra} ($\leq$ 80 MeV)
- We perform a kinematic fit of the entire event, constraining B_{tag}, B_{sig} and D mesons to their nominal masses, constrain B and D decay products to separate vertices
- In case of multiple candidates, we retain that with the lowest E_{Extra}
- A second kinematic fit with a U=0 constraint is done to improve the resolution in the variables q^2 and cos θ_ℓ (q is the momentum transfer to the $\ell^- \bar{\nu}$ system and θ_ℓ is the lepton helicity angle)

Signal-to-Background Separation

- We use a novel technique to separate signal from background since the background shape varies across phase space
- Primary background is from $\overline{B} \to D^* \ell^- \overline{\nu}$ with $D^* \to D\pi$ or $D^* \to D\gamma$

signal pdf | background pdf | Fit to data $K^-\pi^+e^-\overline{\nu}$ mode

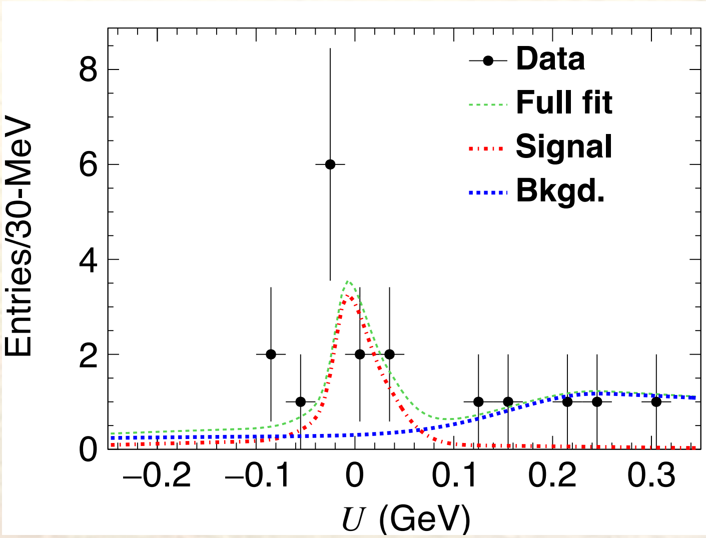


- Background from charmless B decays and $q\bar{q}$ continuum is small
- We define pdfs for signal (4 two-piece Gaussians) and background (2 two-piece Gaussians)
- We test the binned fit on the U distribution for the $K^-\pi^+e^-\overline{\nu}$ mode

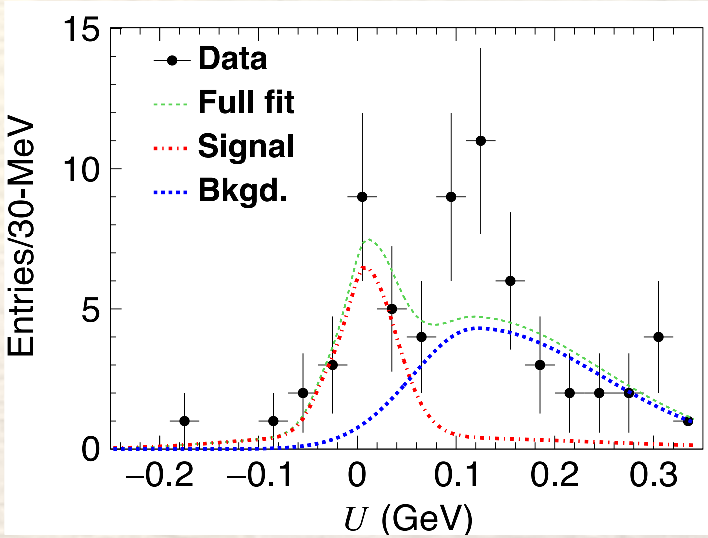
Background Varies across Phase Space

- We show that this method works in different regions of cos θ_ℓ and q^2

$|\cos\theta_\ell + 0.85| < 0.05$

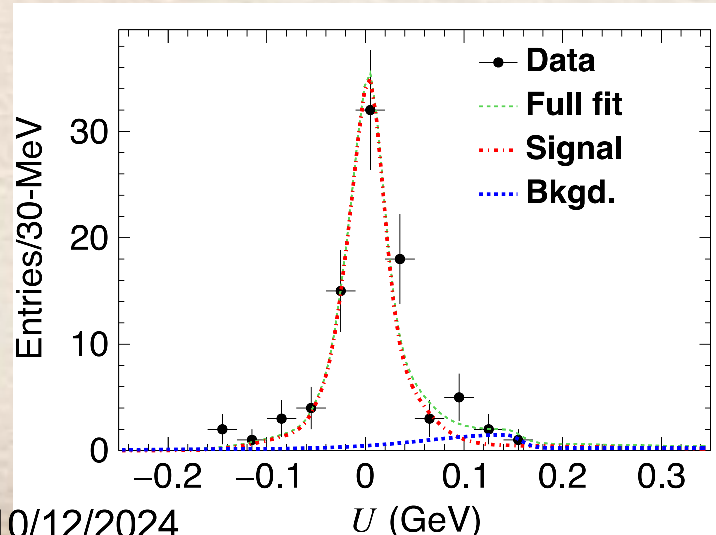


$|\cos\theta_\ell - 0.85| < 0.05$

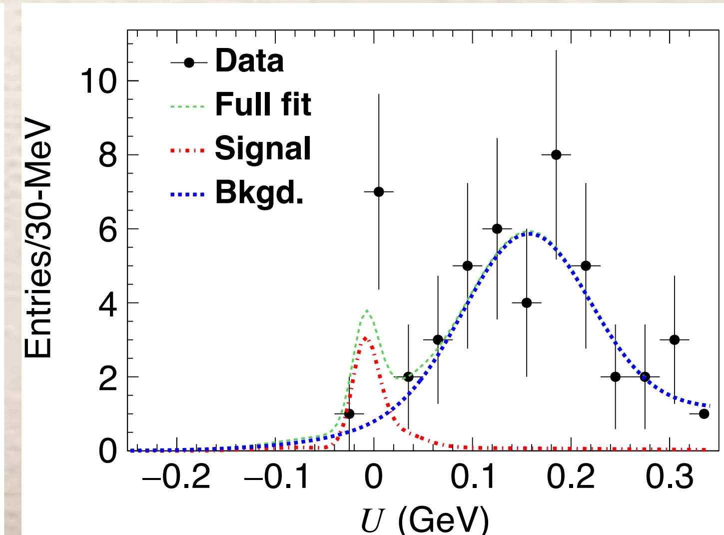


- Binned fits to data in $K^-\pi^+\pi^+e^-\bar{\nu}$ mode
- Fits describe data well

$|q^2 - 0.75| < 0.25$ GeV^2/c^2



$|q^2 - 9.75| < 0.25$ GeV^2/c^2



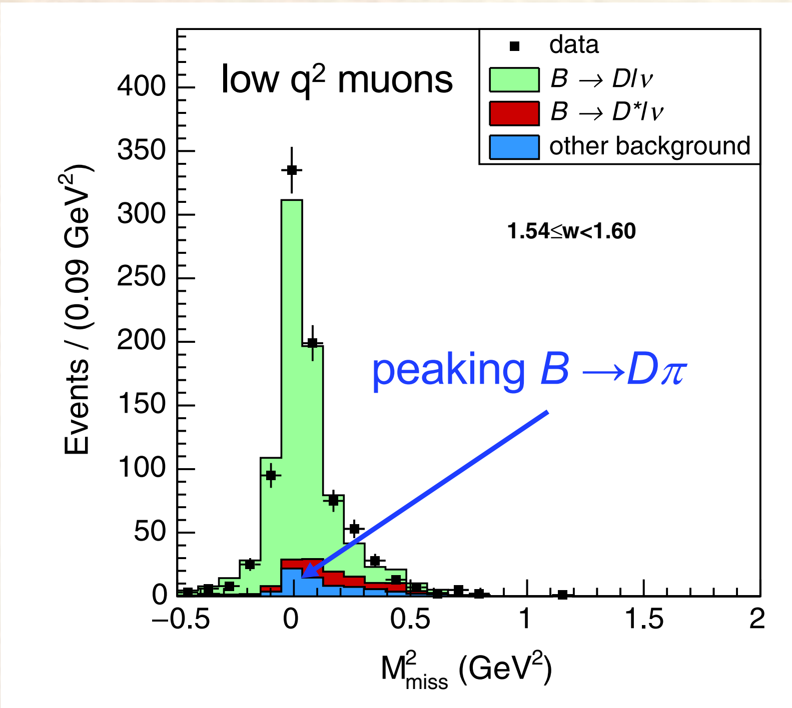
- Binned fits to data in $K^-\pi^+\pi^-\pi^+e^-\bar{\nu}$ mode
- Fits describe data well
- Distributions illustrate different background shapes

Remove Peaking Background at low q^2

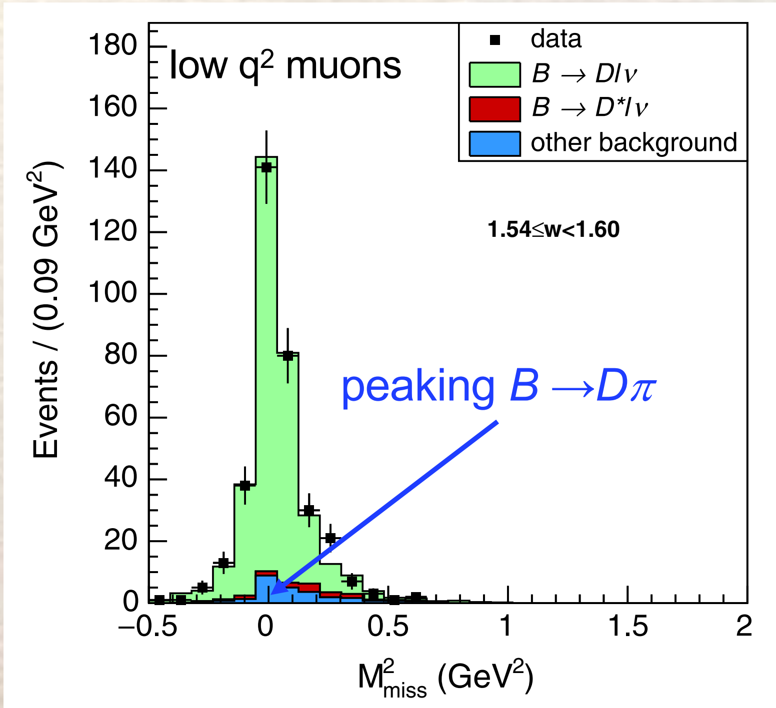
- For low q^2, the squared missing-mass distribution shows a small peaking background from $\overline{B} \rightarrow D\pi$, particularly in muon modes
- Probably caused by $\mu \leftrightarrow \pi$ misidentification in the muon channel
- We remove this peaking background by requiring $q^2 > 0.5\ \mathrm{GeV}^2/c^2$

PRD **93**, 032006 (2016)

$B^- \longrightarrow D^0 \ell^- \overline{\nu}$

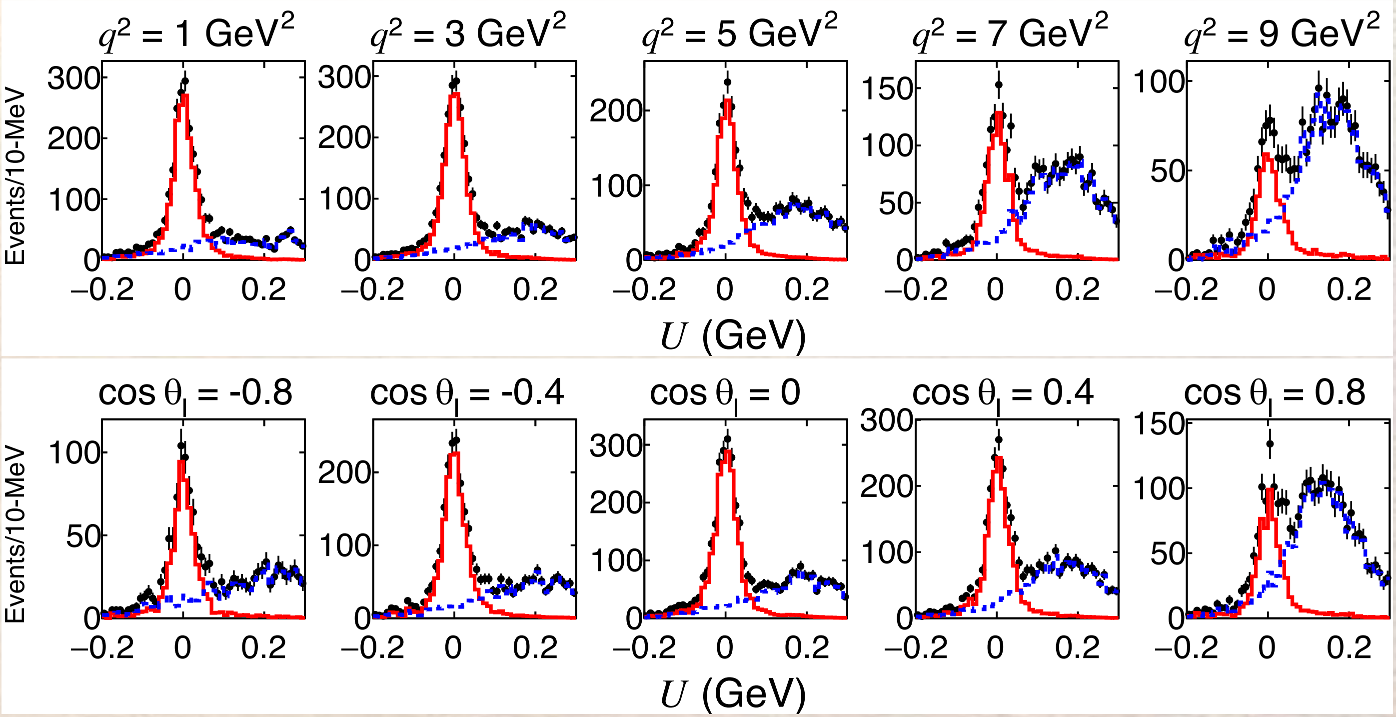


$\overline{B}^0 \longrightarrow D^+ \ell^- \overline{\nu}$



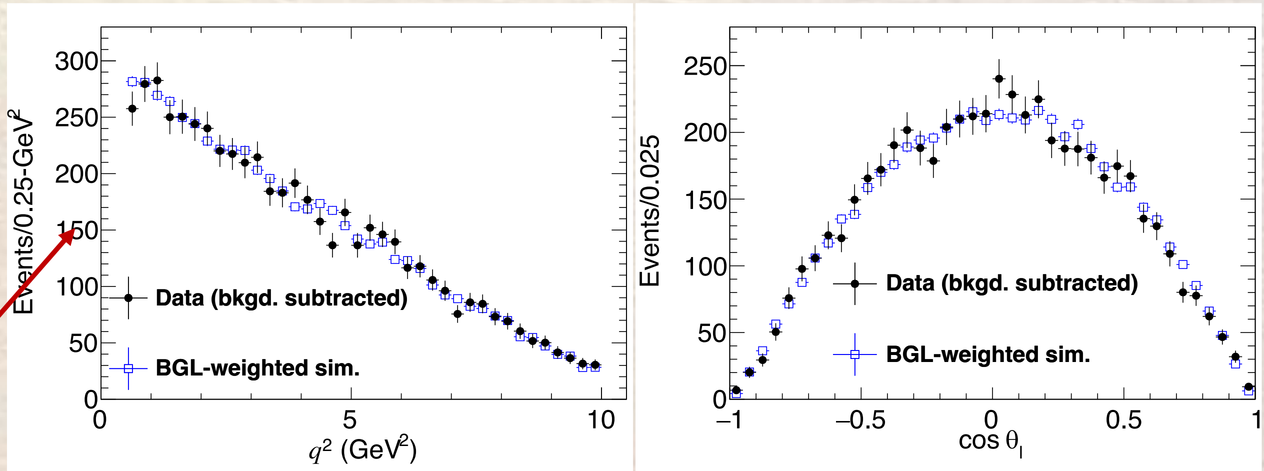
Extraction of Signal Weight Factors

- We perform continuous U-variable fits in q^2 and cos θ_ℓ regions, selecting 50 events at a time that are closest to a selected event to determine signal and background components from which we determine signal weights for each event
- Signal weight $\mathcal{Q}_i = \frac{\mathcal{S}_i(U_i)}{\mathcal{S}_i(U_i)+\mathcal{B}_i(U_i)}$ and background weight $1-\mathcal{Q}_i = \frac{\mathcal{B}_i(U_i)}{\mathcal{S}_i(U_i)+\mathcal{B}_i(U_i)}$
- We observe 16,701 events in all ten modes
- To illustrate how well this procedure works, we show the U variable distributions for different q^2 and cos θ_ℓ regions, summing the $\mathcal{Q}_i$ values of all 10 modes
- Red points result from signal weights $\mathcal{Q}_i$ and blue points from background weights (1-$\mathcal{Q}_i$)



Unbinned Angular Fits

- We require $|U|$<50 MeV, $0.5 \leq q^2 \leq 10$ GeV$^2/c^2$ & $|\cos\theta_\ell|$<0.97 for the final sample
- We perform ML fits in the q^2-$\cos\theta_\ell$ plane using only signal weights Q_i
- We add two external constraints
  - To set normalization of the form factors, the $w\rightarrow1$ region calculations from lattice QCD are added as Gaussian constraints (6 $f_{0,+}(w)$ MILC data points) PRD **92**, 034506 (2015)
  - To access $|V_{cb}|$ the absolute q^2–differential decay rate data from Belle are also incorporated as Gaussian constraints (40 $d\Gamma/dw$ data points) PRD **93**, 032006 (2016)
- The total likelihood function is $\mathcal{L}(\vec{x})_{|tot} = -2\ln\mathcal{L}(\vec{x})_{|BABAR} + \chi^2(\vec{x})_{|Belle} + \chi^2(\vec{x})_{|FNAL/MILC}$
- We perform fits both with the BGL (N=2,3) and CLN forms
- 1d projections of the nominal fit in comparison with simulation using the BGL form

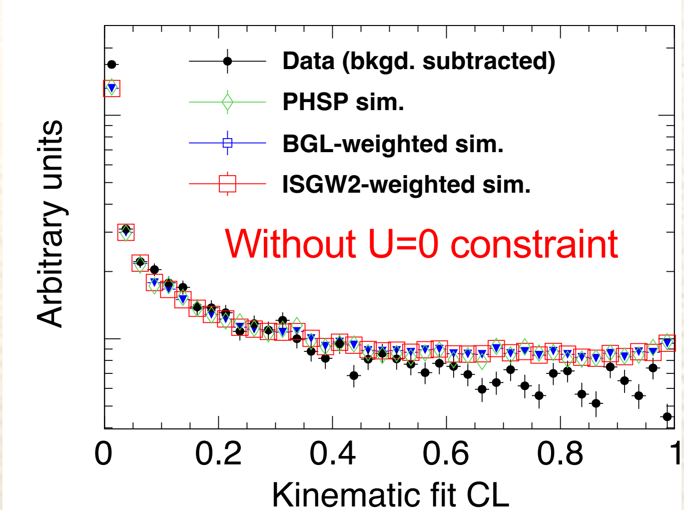


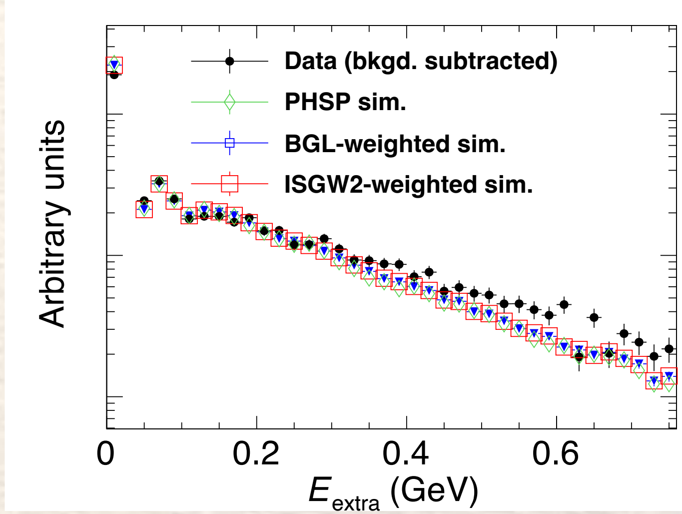
- The $\cos\theta_\ell$ distribution exhibits the $\sin^2\theta_\ell$ dependence expected in the SM ➡ this indicates that the $\bar{\nu}$ reconstruction works well

Cross Checks

- Besides the nominal fit, we perform 3 other fits with different background subtraction to study systematic uncertainties

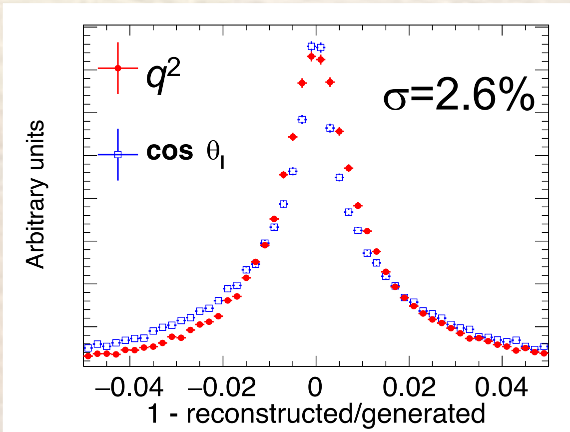
- We perform cross checks between background-subtracted data and efficiency-corrected simulations with BGL weighting and ISGW2 weighting for the confidence level of the fit and the E_{Extra} distribution

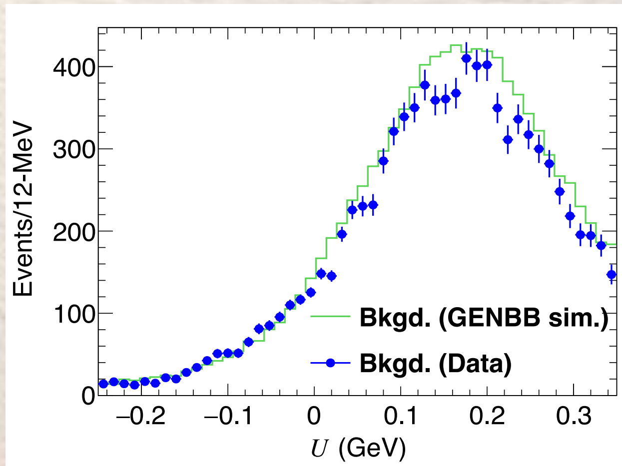




PRD **52**, 2783 (1995)

- The relative resolution of the deviation of the reconstructed-to-generated values for the q^2 and cos θ_ℓ distributions peak at 1, σ=2.6% →





- Comparison of (1-$\mathcal{Q}$) weighted data and background simulation

Systematic Errors

- Since background-subtracted data and simulations roughly agree, we assign no systematic error
- Take resolution (2.6%) in ratio of background-subtracted data and simulation as systematic error
- To evaluate systematic error associated with reconstruction we repeat unbinned fit employing kinematic variables without the kinematic fit and take difference of result wrt standard unbinned fit as systematic error
- To evaluate systematic error associated with background subtraction we perform 3 additional background subtractions and perform fits; the largest deviation wrt to the result of the nominal fit is taken as systematic error
- Variations in the background and signal line shapes are accounted for
  - for background line shapes we vary all 7 parameters in the pdf by 5% and redo fits; deviations from nominal fit are taken as systematic error
  - for signal line shapes we vary all parameters of the central two-piece Gaussians and weights of the two tail Gaussians by 5% and redo fits; deviations from the nominal fit are taken as systematic error

Fit Results

PRD **110**, 032018 (2024)

- Fit parameters for *BABAR*+*Belle*+*FNAL/MILC* data and BGL with *N*=2 expansion

a_0^{f+}	a_1^{f+}	a_2^{f+}	a_1^{f0}	a_2^{f0}
0.0126±0.0001	-0.096±0.003	0.352±0.052	-0.059±0.003	0.155±0.049

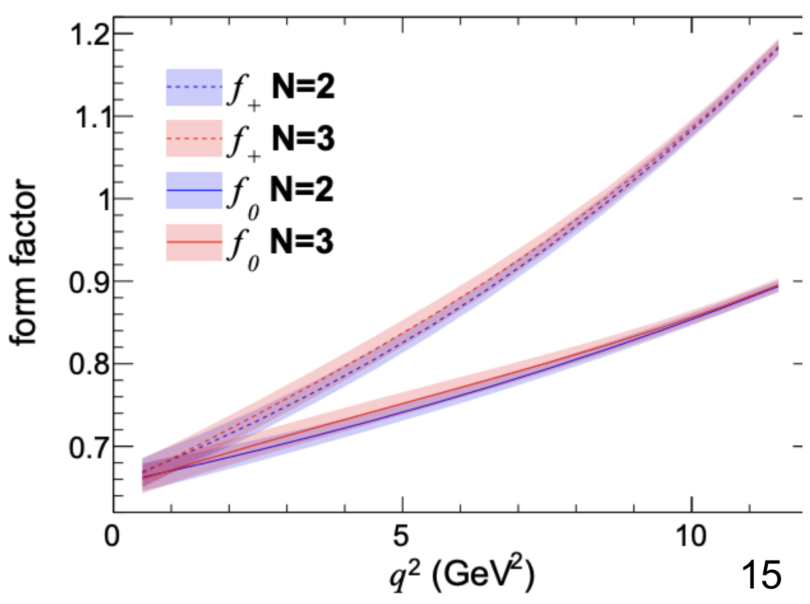
- Fit parameters for *BABAR*+*Belle*+*FNAL/MILC* data and BGL with *N*=3 expansion

a_0^{f+}	a_1^{f+}	a_2^{f+}	a_3^{f+}	a_1^{f0}	a_2^{f0}	a_3^{f0}
0.0126± 0.0001	-0.098± 0.004	0.626± 0.241	-3.939± 3.194	-0.061± 0.003	0.435± 0.205	-3.977± 2.840

- Fit parameters for *BABAR*+*Belle*+*FNAL/MILC* data and CLN

$\mathcal{G}(1)$	ρ^2_D
1.056±0.008	1.155±0.023

- Compare N=2 and N=3 BGL form factors
  - Both agree well though the N=2 results have higher precision
  - the 1σ error includes both statistical and systematic uncertainties



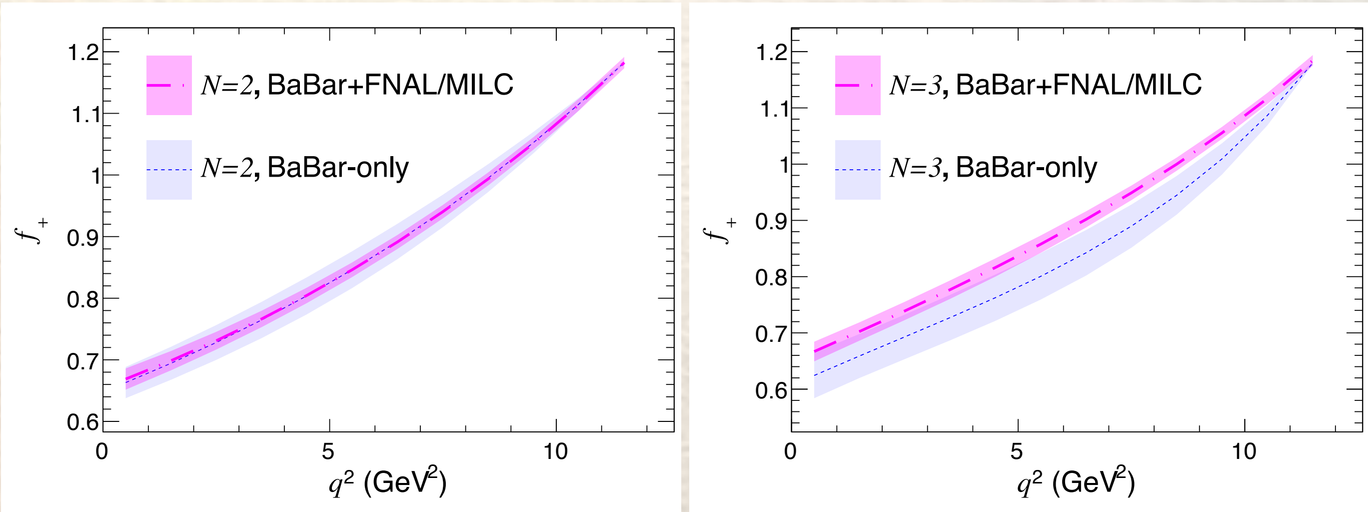
Form Factor Results

- Now let us look at the f^+ (N=2 and N=3) results for *BABAR* data only compared to *BABAR+FNAL/MILC* data

- For N=2, both results are in excellent agreement at the 1σ level

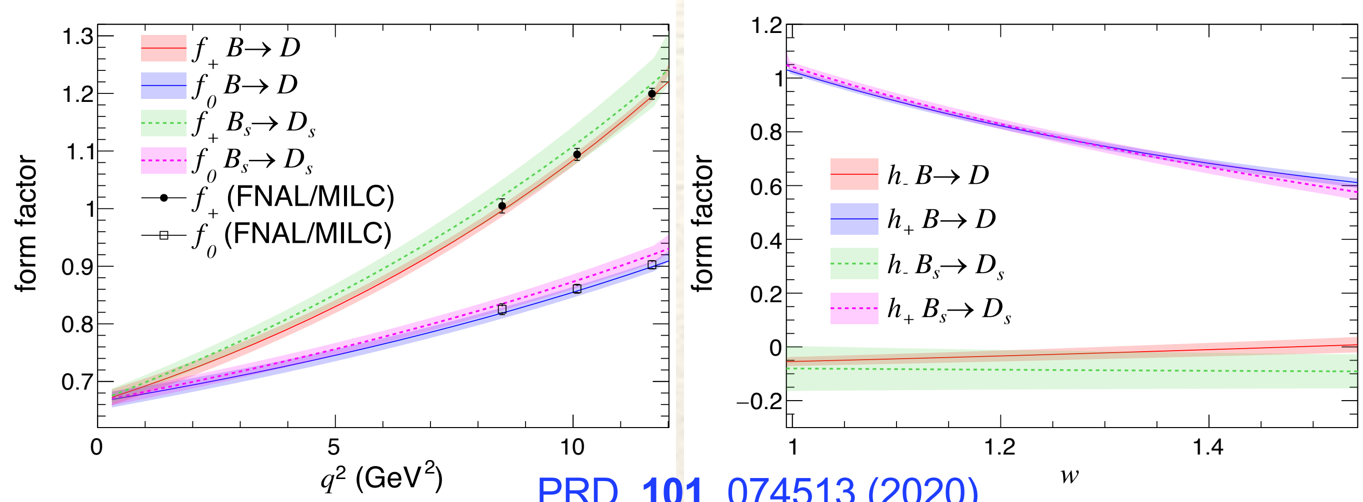
- For N=3, both results are consistent though the *BABAR* only result is systematically lower → at the 1σ level it disagrees with the fit to *BABAR+ FNAL/MILC* data

- Including the lattice points reduces the total error



Form Factor Results

- The $B \rightarrow D$ form factors have improved precision and show good agreement with the new, full q^2 $B_s \rightarrow D_s$ calculation of the HPQCD Collaboration assuming flavor SU(3) symmetry



PRD **101**, 074513 (2020)

- Some slight tension exists for h_- in the HQET basis at maximum recoil point, $q^2 \rightarrow 0$, but otherwise the SU(3) flavor symmetry seems to hold ➔ SU(3) flavor symmetry breaking cannot be large

$|V_{cb}|$ Measurements

- The CKM parameter $|V_{cb}|$ is extracted either from exclusive $\bar{B}\to D\ell^-\bar{\nu}$ & $\bar{B}\to D^*\ell^-\bar{\nu}$ decay rates or from the inclusive $b\to c\ell^-\bar{\nu}$ decay rate

- There is a ~3σ tension between $|V_{cb}|_{D^*\ell\nu} = 0.0398 \pm 0.0006$ & $|V_{cb}|_{inc} = 0.0422 \pm 0.0005$ that is not understood yet

- We extract $|V_{cb}|$ by $|V_{cb}| = \sqrt{\frac{\mathfrak{B}}{\Gamma' \tau_B}}$, where B are semileptonic branching fractions taken from HFLAV, τ_B are the B lifetimes (τ_{B^+}=1.519±0.004 ps and τ_{B^0}=1.638±0.004 ps) and Γ' is the decay rate obtained from the fit

- Using our Γ' fit result (BGL with N=2), we obtain for HFLAV data

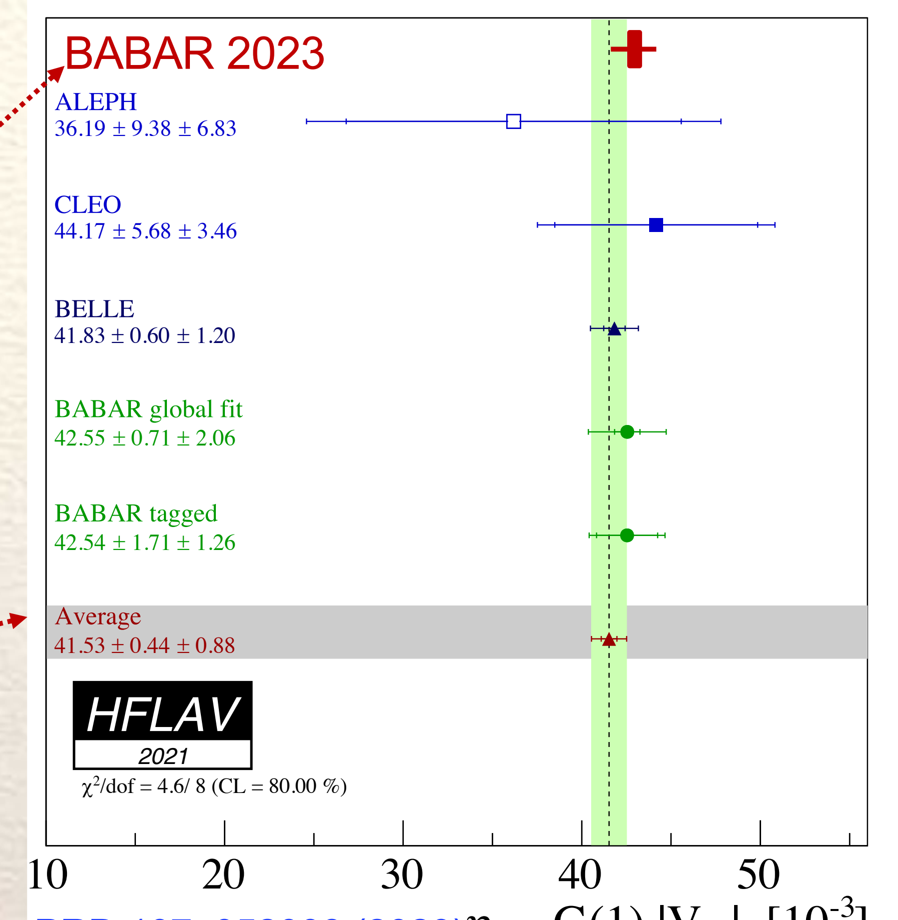
- All measurements agree within the errors

Data	$\|V_{cb}\|$
BABAR B^0	0.04036±0.00017±0.00010±0.00167
BABAR B^+	0.03898±0.00015±0.00009±0.00130
Belle B^0	0.04201±0.00018±0.00010±0.00106
Belle B^+	0.04160±0.00017±0.00010±0.00107

$|V_{cb}|$ Results from 2d Fit

- Nominal 2d fits to BABAR+Belle16+FNAL/MILC data yield
  PRD **93**, 032006 (2016)
  - $|V_{cb}|_{BGL} = 0.0411 \pm 0.0012$
  - $|V_{cb}|_{CLN} = 0.0409 \pm 0.0011$
- Compute $|V_{cb}|\mathcal{G}(1)\eta_{EW}$ with $\mathcal{G}(1)=1.0530\pm0.0083$, $\eta_{EW}=1.0066\pm0.0050$
  - $\eta_{EW}\mathcal{G}(1)|V_{cb}| = 0.0436 \pm 0.00129$ (1.3 σ higher)
- Compared to the world average
  - $\eta_{EW}\mathcal{G}(1)|V_{cb}|_{WA} = 0.04153 \pm 0.00098$
- Good agreement with the $|V_{cb}|$ from inclusive analysis

  $|V_{cb}|_{inc} = 0.0422 \pm 0.0005$
- Some tension with $|V_{cb}|$ from $\bar{B}\to D^*\ell^-\bar{\nu}$



PRD 107, 052008 (2023)

Conclusions and Outlook

- We performed the first 2-dimensional unbinned angular analysis in the q^2- cos θ_ℓ plane for the $\overline{B} \to D\ell^- \overline{\nu}$ process
- We used a novel event-wise signal-to-background separation
- The lepton helicity follows a $\sin^2 \theta_\ell$ distribution as expected in the SM; this is shown for the first time confirming that the ν reconstruction works well
- For the BGL form we measure $|V_{cb}|=0.0411\pm0.0012$, which is closer to the value measured in inclusive $b \to c\ell^- \overline{\nu}$ decays
- The $B\to D$ form factors are found to be consistent with the $B_s \to D_s$ form factors predicted by lattice calculations and expected by flavor SU(3) relations
- A similar analysis on $\overline{B} \to D^* \ell^- \overline{\nu}$ is in progress to measure BGL and CLN form factors (V, A_1, A_2 & A_3) and determine $|V_{cb}|$

Thank you for your attention