Neutrino Mass & Mass Ordering with Cosmic Gravitational Focusing

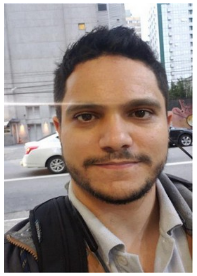
Shao-Feng Ge

葛韶锋

gesf@sjtu.edu.cn

SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

SFG & Liang Tan [arXiv:2409.11115]



Pedro Pasquini

Liang Tan

上海交通大學

SHANGHAI JIAO TONG UNIVERSITY

UNSW, Sydney

December 13, 2024

李政道研究所

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Minimal Neutrinos

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Georg G. Raffelt

Stars as Laboratories for Fundamental Physics

The Astrophysics of Neutrinos, Axions, and Other Weakly Interacting Particles

In the standard model, neutrinos have been assigned the most minimal properties compatible with experimental data: zero mass, zero charge, zero dipole moments, zero decay rate, zero almost everything.

Importance of Neutrino Masses

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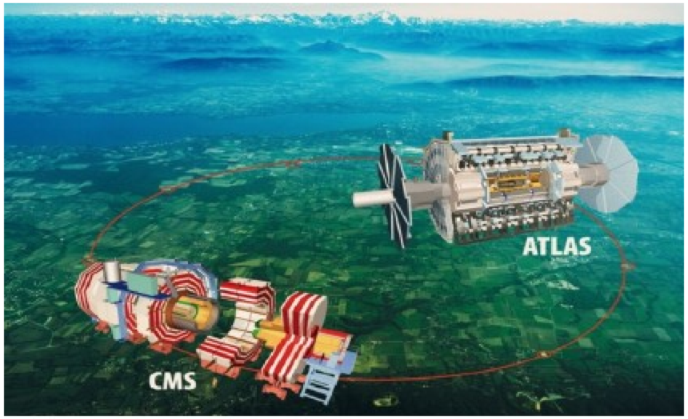
- **Higgs boson** ⇒ electroweak symmetry breaking & mass. ~ O(100)GeV
- **Chiral symmetry breaking** ⇒ majority of mass.
- The world seems not affected by the tiny neutrino mass?
  - Neutrino mass ⇒ Mixing
  - 3 Neutrino ⇒ possible **CP violation**
  - CP violation ⇒ **Leptogenesis**
  - ⇒ **Matter-Antimatter Asymmetry**
  - There is something left in the Universe.
  - **EW Baryogenesis** is not enough.

Daya Bay @ **March 8, 2012**



LHC @ **July 4, 2012**

Global Fit of Oscillation Parameters

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(for **NO**)	-1σ	Best Value	$+1\sigma$
$\Delta m_s^2 \equiv \Delta m_{21}^2 \, (10^{-5} \mathrm{eV}^2)$	7.30	7.50	7.72
$\left\|\Delta m_a^2 \equiv \Delta m_{31}^2\right\| \, (10^{-3} \mathrm{eV}^2)$	2.52	2.56	2.59
$\sin^2 \theta_s \; (\theta_s \equiv \theta_{12})$	0.302 (33.3°)	0.318 (34.3°)	0.334 (35.3°)
$\sin^2 \theta_a \; (\theta_a \equiv \theta_{23})$	0.544 (47.54°)	0.566 (48.79°)	0.582 (49.72°)
$\sin^2 \theta_r \; (\theta_r \equiv \theta_{13})$	0.02147 (8.43°)	0.02225 (8.58°)	0.02280 (8.69°)
δ_D	191°	216°	257°
δ_{Mi}	??	??	??

Salas, Forero, Gariazzo, Martinez-Mirave, Mena, Ternes, Tortola & Valle, [arXiv:2006.11237]

ν Mass Ordering

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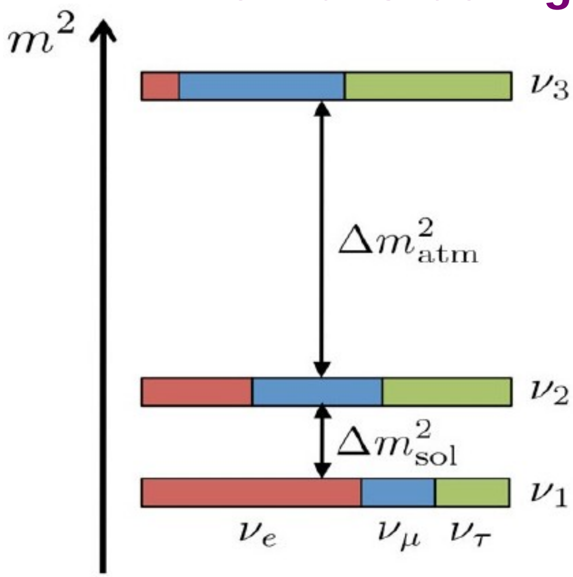
Normal Ordering (正序)



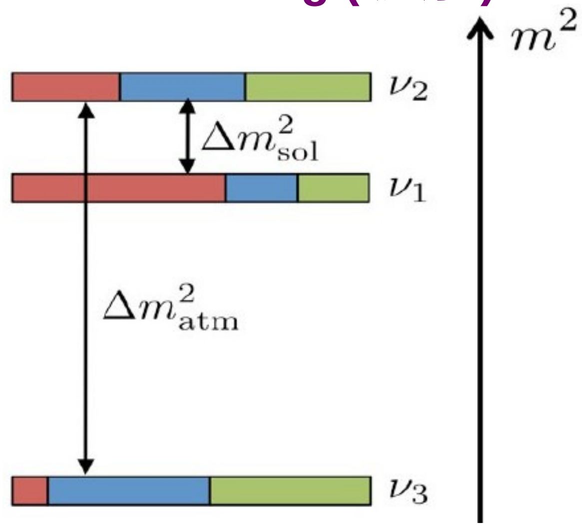
$$m_1 \lesssim m_2 < m_3$$

$$m_1$$

$$m_2 = \sqrt{m_1^2 + \Delta m_s^2}$$

$$m_3 = \sqrt{m_1^2 + \Delta m_a^2}$$

Inverted Ordering (反序)



$$m_3 < m_1 \lesssim m_2$$

$$m_1 = \sqrt{m_3^2 + \Delta m_a^2}$$

$$m_2 = \sqrt{m_3^2 + \Delta m_a^2 + \Delta m_s^2}$$

$$m_3$$

江门中微子实验 (JUNO)

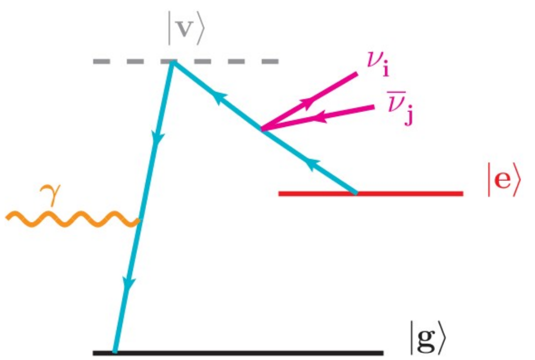
However, ν oscillation experiment cannot measure the absolute mass!

Neutrino Mass Measurements

ν Oscillation

Beta Decay

Radiative Emission of ν Pairs

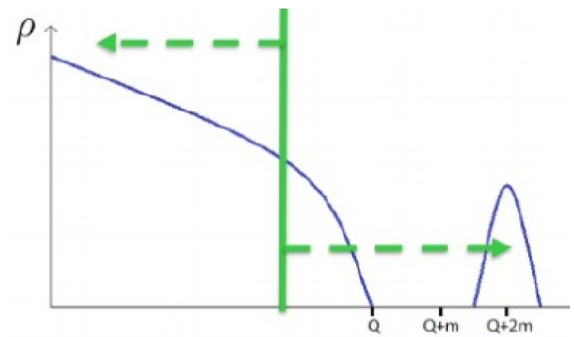


SFG, Pedro Pasquini, **JHEP 12 (2023) 083** [arXiv:2306.12953]
Phys.Lett.B 841 (2023) 137911 [arXiv:2206.11717]
Eur.Phys.J.C 82 (2022) 3, 208 [arXiv:2110.03510]

Supernova ν Time Delay

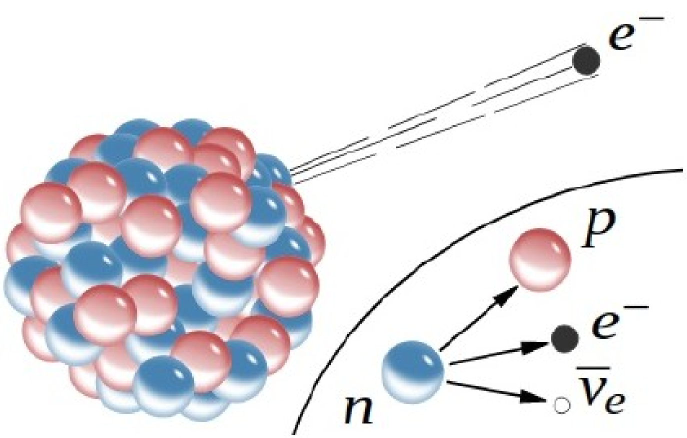
SFG, Chui-Fan Kong, Alexei Smirnov, **PRL 133 (2024) 121802** [arXiv:2404.17352]

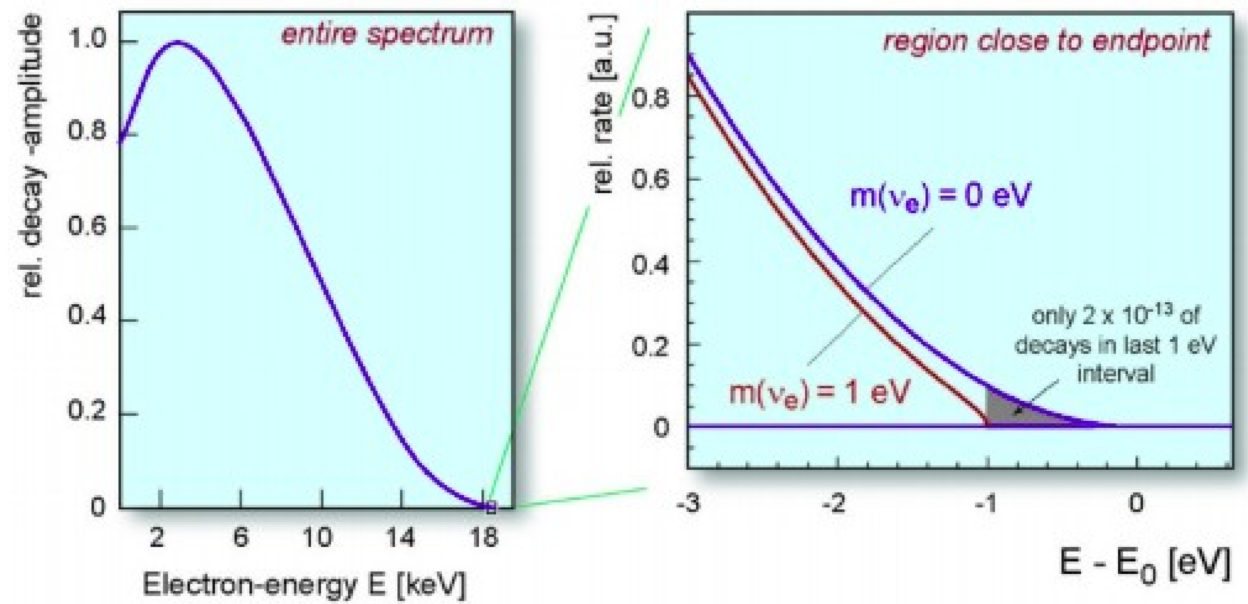
CνB Detection

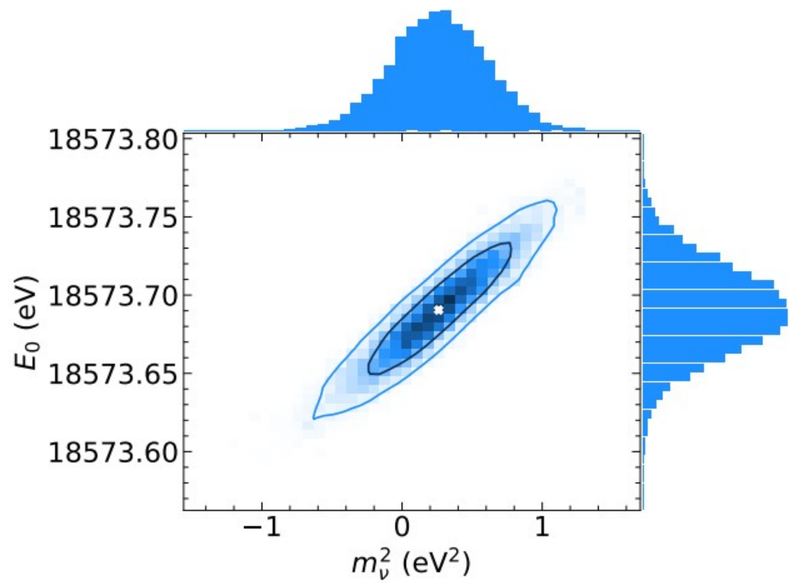


CMB & LSS

Neutrino Mass @ β Decay







Nature Phys. 18 (2022) 2, 160-166

CMB for ν Mass Measurement

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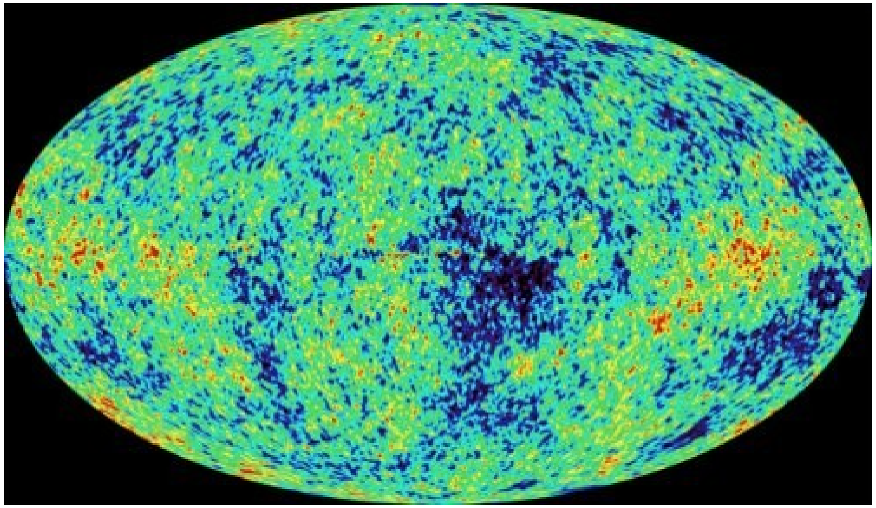
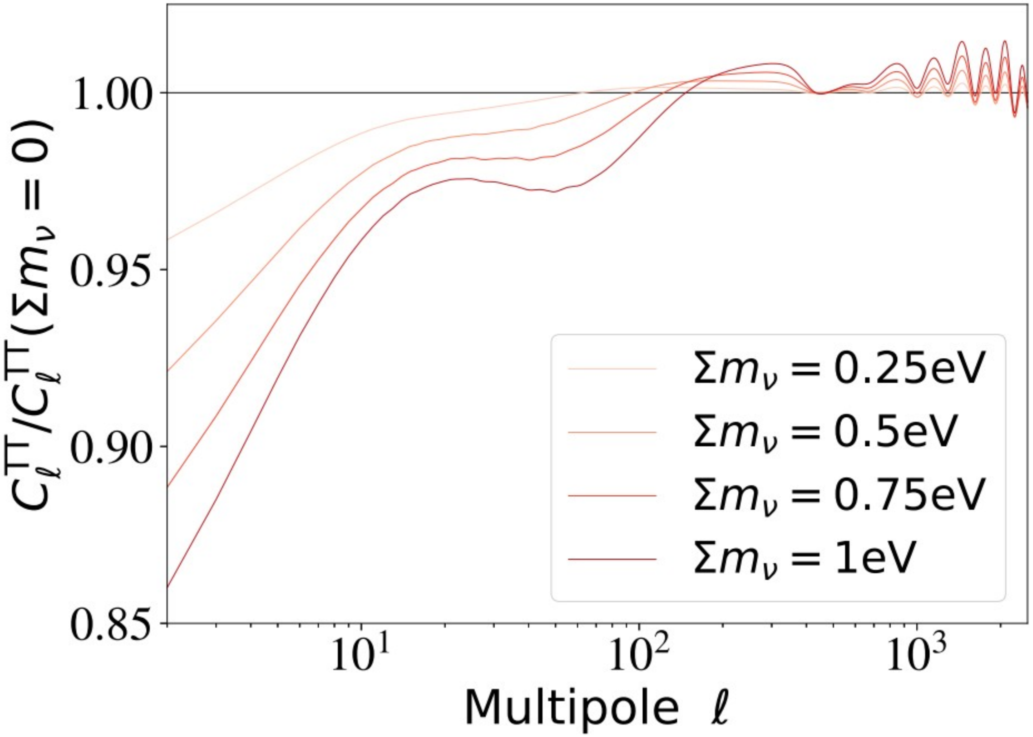
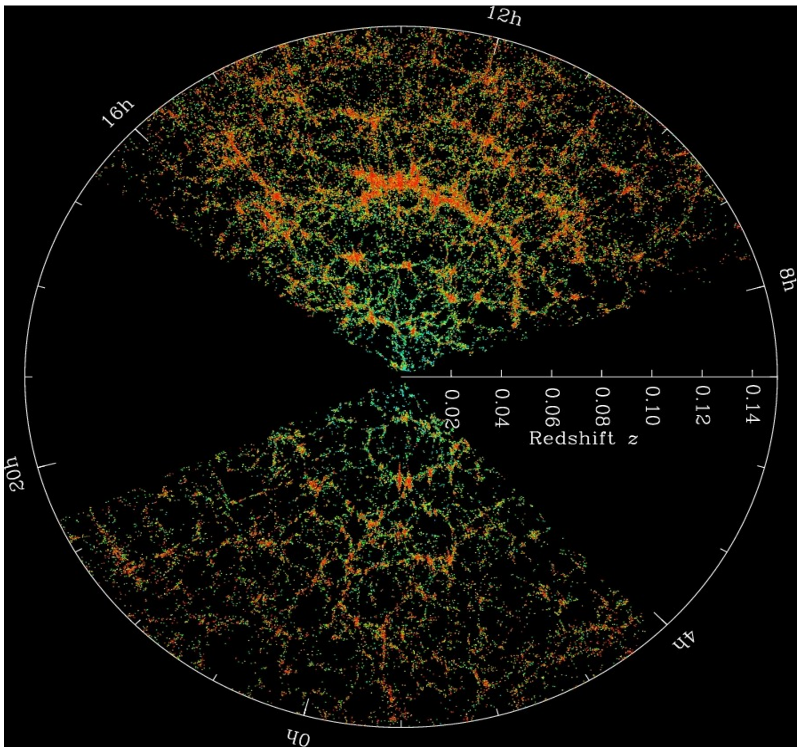
Massive neutrino decreasing CMB lensing power spectrum

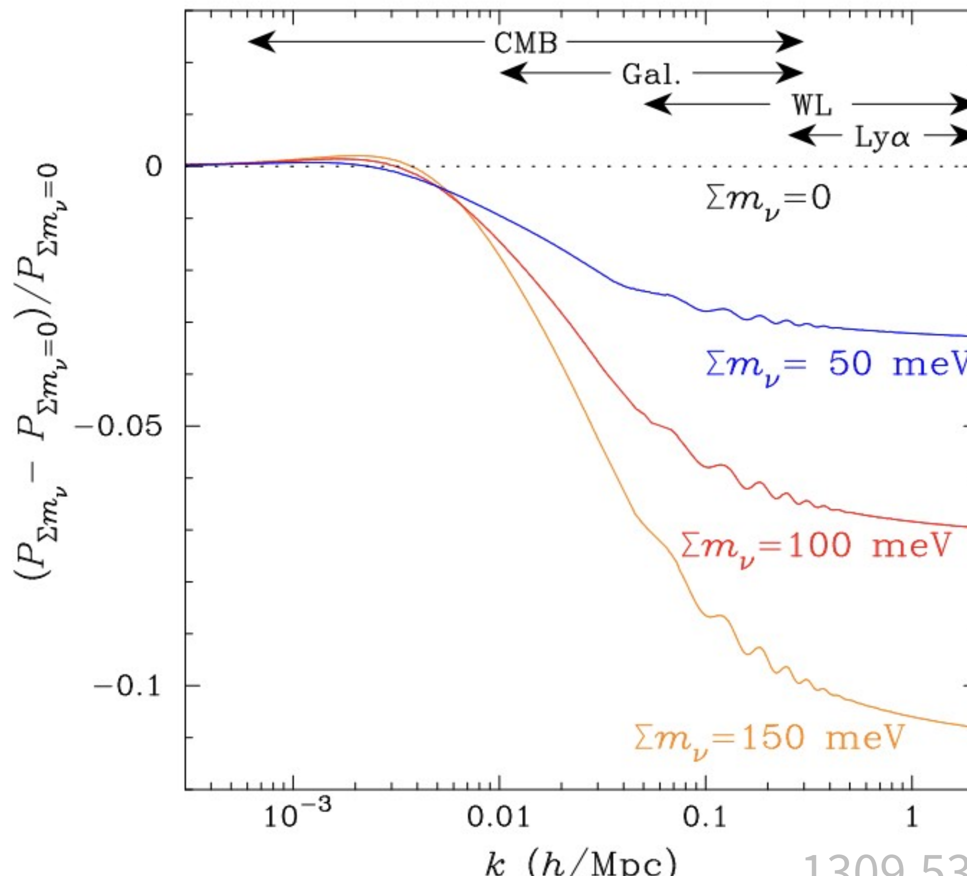




$$C_\ell^{\kappa\kappa} \propto (\Omega_m h^2)^2 A_s \left(1 - 0.02 \frac{f_\nu}{4 \times 10^{-3}}\right) \qquad f_\nu = \Omega_\nu / \Omega_m$$

Suppress matter power spectrum





1309.5383

$$P^{(\sum m_\nu)}(k \gg k_{\rm fs}, z) \approx \left(1 - 2f_\nu - \frac{6}{5} f_\nu \log \frac{1+z_\nu}{1+z}\right) P^{(\sum m_\nu = 0)}(k \gg k_{\rm fs}, z)$$

Cosmological Systematics

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model / dataset	Ω_m	H_0 [km s^{-1} Mpc^{-1}]	Σm_ν [eV]	N_{eff}
ΛCDM+$\sum m_\nu$				
DESI+CMB	0.3037 ± 0.0053	68.27 ± 0.42	< 0.072	—
ΛCDM+N_{eff}				
DESI+CMB	0.3058 ± 0.0060	68.3 ± 1.1	—	3.10 ± 0.17
wCDM+$\sum m_\nu$				
DESI+CMB	0.282 ± 0.013	$71.1^{+1.5}_{-1.8}$	< 0.123	—
DESI+CMB+Panth.	0.3081 ± 0.0067	67.81 ± 0.69	< 0.079	—
DESI+CMB+Union3	0.3090 ± 0.0082	67.72 ± 0.88	< 0.078	—
DESI+CMB+DESY5	0.3152 ± 0.0065	67.01 ± 0.64	< 0.073	—
wCDM+N_{eff}				
DESI+CMB	0.281 ± 0.013	$71.0^{+1.6}_{-1.8}$	—	2.97 ± 0.18
DESI+CMB+Panth.	0.3090 ± 0.0068	67.9 ± 1.1	—	3.07 ± 0.18
DESI+CMB+Union3	0.3097 ± 0.0084	67.8 ± 1.2	—	3.06 ± 0.18
DESI+CMB+DESY5	0.3163 ± 0.0067	67.2 ± 1.1	—	3.09 ± 0.18
w_0w_aCDM+$\sum m_\nu$				
DESI+CMB	$0.344^{+0.032}_{-0.026}$	$64.7^{+2.1}_{-3.2}$	< 0.195	
DESI+CMB+Panth.	0.3081 ± 0.0069	68.07 ± 0.72	< 0.155	—
DESI+CMB+Union3	0.3240 ± 0.0098	66.48 ± 0.94	< 0.185	—
DESI+CMB+DESY5	0.3165 ± 0.0069	67.22 ± 0.66	< 0.177	—

DESI [arXiv:2404.03002]

Results from DESI BAO + CMB

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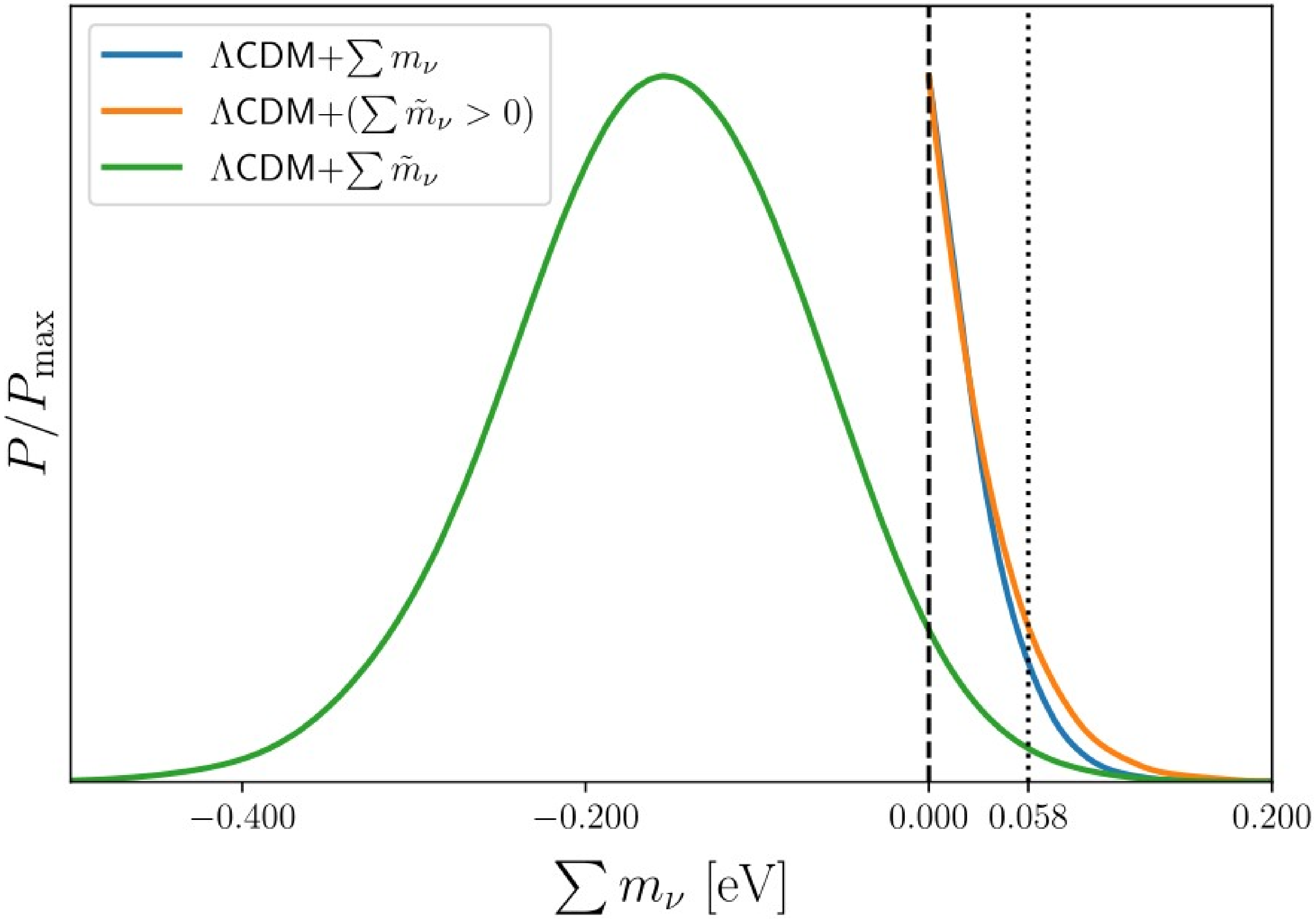

Craig, Green, Meyers & Rajendran [arXiv:2405.00836]

Cosmic ν Fluid (CνF) vs DM Halo

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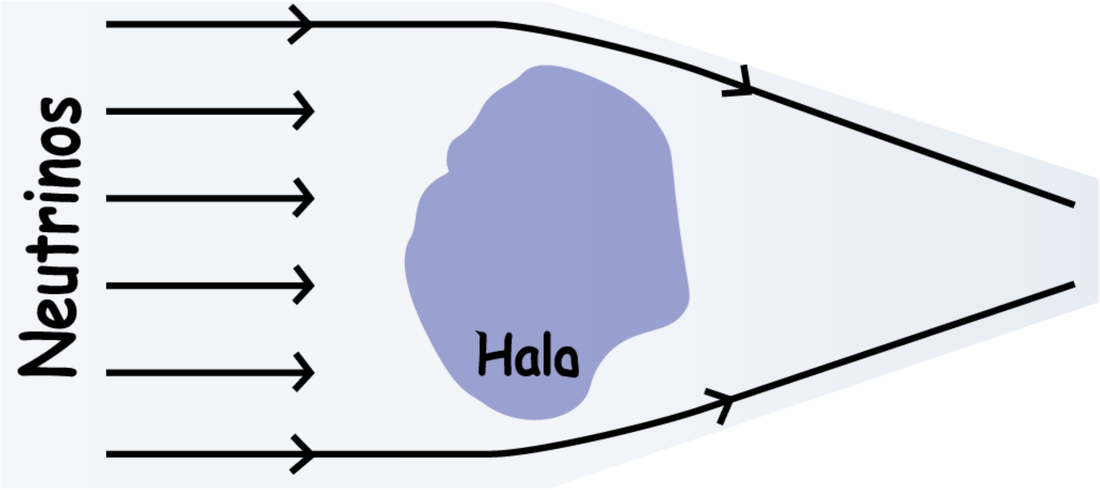
Gravitational attraction between DM halo & CνF



CνF

DM Halo

vs Stone in Water Flow

Gravitational Deflection

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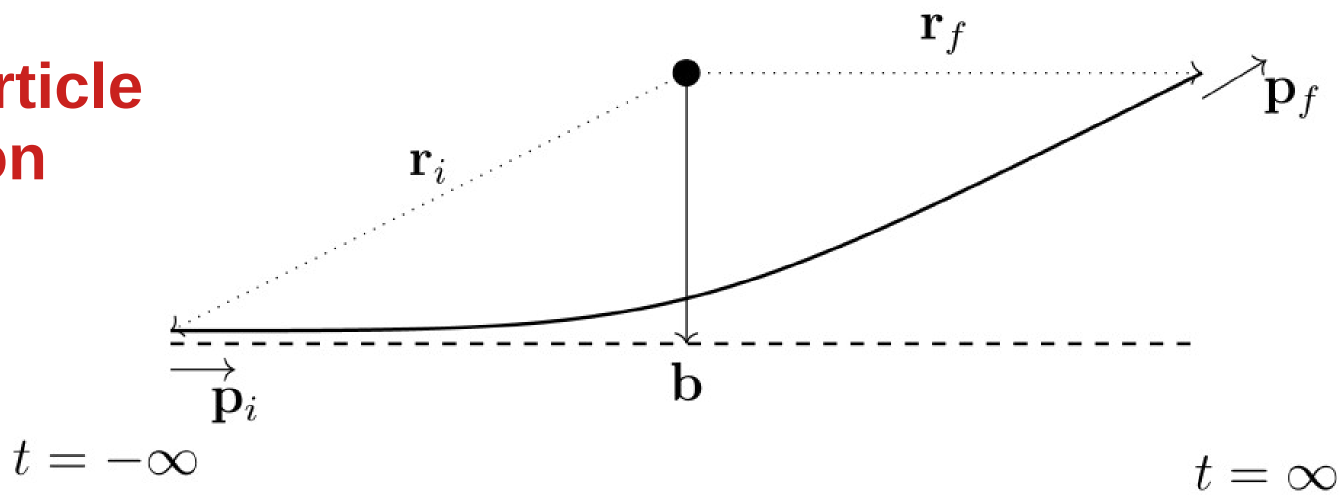
Single-particle description



$$ds^2 \equiv -\left(1 - \frac{2GM}{r}\right) dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1} dr^2 + r^2 d\Omega^2$$

$$\Delta\phi = 2|\boldsymbol{b}| \int_{r_{\min}}^{\infty} \frac{dr}{r^2} \left[1 + \frac{2b_{90}}{r} - \frac{|\boldsymbol{b}|^2}{r^2} + \frac{2GM|\boldsymbol{b}|^2}{r^3}\right]^{-1/2}$$

$$b_{90} \equiv GM m_\nu^2 / |\boldsymbol{p}_i|^2 \qquad r_{\min}^3 + 2b_{90} r_{\min}^2 - |\boldsymbol{b}|^2 r_{\min} + 2GM|\boldsymbol{b}|^2 = 0$$

SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

Dynamical Friction

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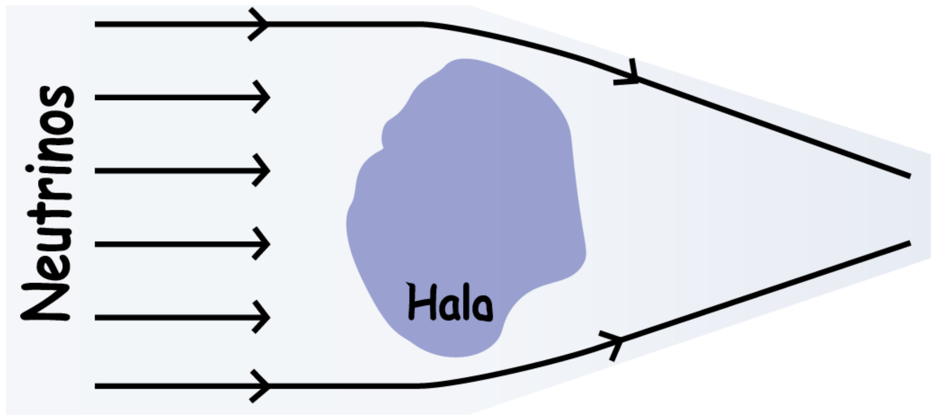
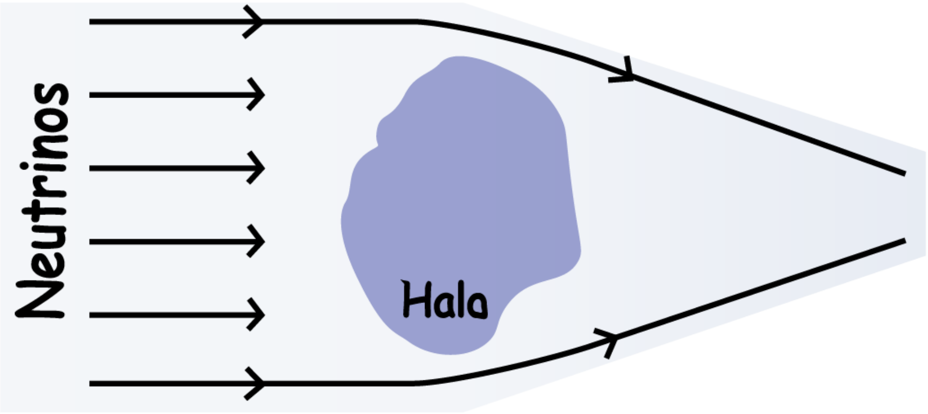
Single-particle description



$$\Delta\phi \approx \pi + 2\frac{GM}{|\boldsymbol{b}|}\left(\frac{m_\nu^2}{|\boldsymbol{p}_i|^2} + 2\right) \qquad f_\nu(\boldsymbol{p}_i, \boldsymbol{v}) \approx \frac{2}{e^{|\boldsymbol{p}_i - E_{\boldsymbol{p}_i}\boldsymbol{v}_{\nu c}|/T} + 1}$$

$$\Delta\boldsymbol{p}^{\|} \equiv \boldsymbol{p}_f^{\|} - \boldsymbol{p}_i = (-\cos\Delta\phi - 1)\,\boldsymbol{p}_i \approx -\frac{2G^2M^2}{|\boldsymbol{b}|^2}\left(\frac{m_\nu^2}{|\boldsymbol{p}_i|^2} + 2\right)^2 \boldsymbol{p}_i$$

Gravitational Focusing



Boltzmann description

$$f_\nu(\boldsymbol{x},\boldsymbol{p}) \equiv \overline{f}_\nu(\boldsymbol{p}) + \delta f_\nu(\boldsymbol{x},\boldsymbol{p})$$

$$\left\{\partial_t + \frac{\boldsymbol{p}\cdot\nabla_{\boldsymbol{x}}}{aE_{\boldsymbol{p}}} - \left[(H+\dot{\Phi})\boldsymbol{p} + \frac{E_{\boldsymbol{p}}}{a}\nabla_{\boldsymbol{x}}\Psi - \frac{|\boldsymbol{p}|^2\nabla_{\boldsymbol{x}}\Phi - \boldsymbol{p}(\boldsymbol{p}\cdot\nabla_{\boldsymbol{x}}\Phi)}{aE_{\boldsymbol{p}}}\right]\cdot\nabla_{\boldsymbol{p}}\right\} f_\nu(\boldsymbol{x},\boldsymbol{p}) = 0$$

$$\delta\widetilde{f}_\nu(\boldsymbol{k},\boldsymbol{p}) = \widetilde{\Psi}(\boldsymbol{k})\left(\frac{m_\nu^2+2\boldsymbol{p}^2}{\boldsymbol{p}\cdot\boldsymbol{k}}\boldsymbol{k} - \boldsymbol{p}\right)\cdot\nabla_{\boldsymbol{p}}\overline{f}_\nu(\boldsymbol{p})$$

$$\mathrm{Im}[\delta\widetilde{\rho}_\nu] = -\frac{(\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}})\widetilde{\Psi}}{4\pi}\int d|\boldsymbol{p}'|\left(m_\nu^4 + 3m_\nu^2|\boldsymbol{p}'|^2 + 2|\boldsymbol{p}'|^4\right)\frac{d\overline{f}_\nu}{d|\boldsymbol{p}'|}\Theta(|\boldsymbol{p}'| - E_{\boldsymbol{p}'}|\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}}|)$$

Density Dipole

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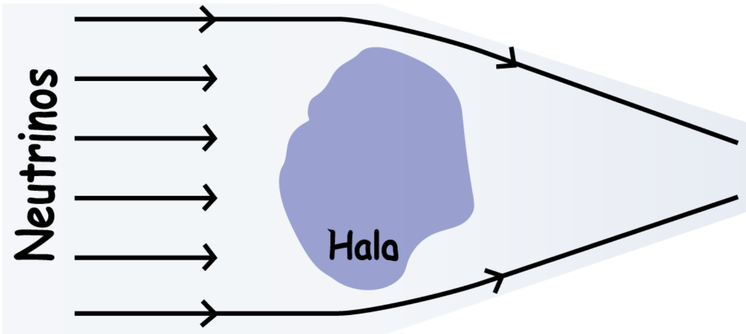
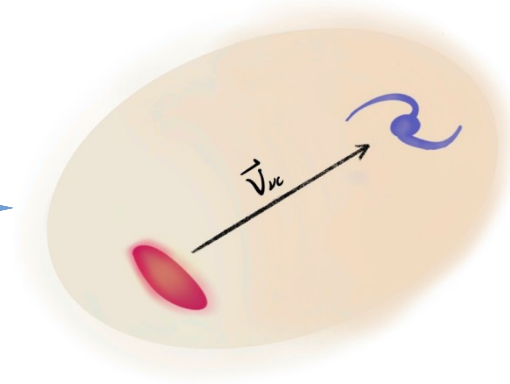




$$\delta_m = \delta_{m0} + \sum_{i=1}^{3} \frac{\delta\rho_{\nu_i}}{\rho_m}$$

Density enhancement downwind!

$$\delta\rho_\nu(-\mathbf{x}) = -\delta\rho_\nu(\mathbf{x})$$

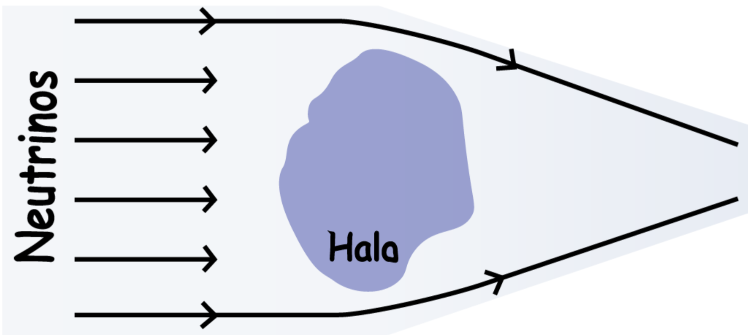
$$\widetilde{\delta}_m \equiv \widetilde{\delta}_{m0}(1 + i\widetilde{\phi})$$

$$[\widetilde{A}(\mathbf{k})]^* = \int d\mathbf{x} e^{i\mathbf{k}\cdot\mathbf{x}} A(\mathbf{x}) = \int d\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} A(-\mathbf{x})$$

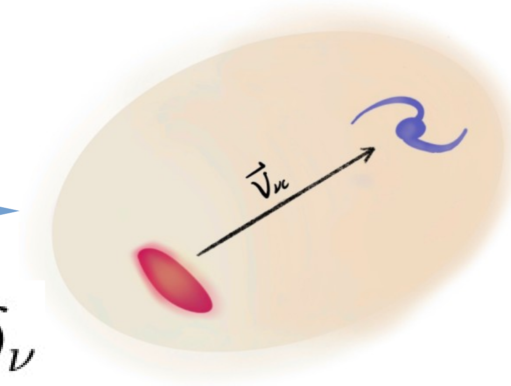
$$= -\int d\mathbf{x} e^{-i\mathbf{k}\cdot\mathbf{x}} A(\mathbf{x}) = -\widetilde{A}(\mathbf{k})$$

SFG, Pedro Pasquini, Liang Tan [arXiv:2312.16972]

Galaxy Correlation & Bias



$$\delta_m = \delta_{m0} + \sum_{i=1}^{3} \frac{\delta \rho_{\nu_i}}{\rho_m}$$



$$\delta_{g\alpha} = b_c^\alpha F_c \delta_c + b_\nu^\alpha F_\nu \delta_\nu$$

$$\delta_{g\alpha,\mathrm{RSD}}(\boldsymbol{x}) \equiv \delta_{g\alpha}(\boldsymbol{x}) - \frac{\partial}{\partial x}\left(\frac{\boldsymbol{u}_m \cdot \hat{\boldsymbol{x}}}{aH}\right)$$

$$\mathrm{Im}[\widetilde{\delta}_{g\alpha,\mathrm{RSD}} \widetilde{\delta}^*_{g\beta,\mathrm{RSD}}] = -i \Delta b \left[\mu_{\boldsymbol{k}}^2 \frac{\dot{\widetilde{\phi}}}{H} + (f \mu_{\boldsymbol{k}}^2 + 1)\widetilde{\phi}\right] \widetilde{\delta}^2_{m0}$$

Galaxies with different bias

$$\Delta b \equiv b_c^\alpha - b_c^\beta$$

Halo Mass Function & HOD for BGS

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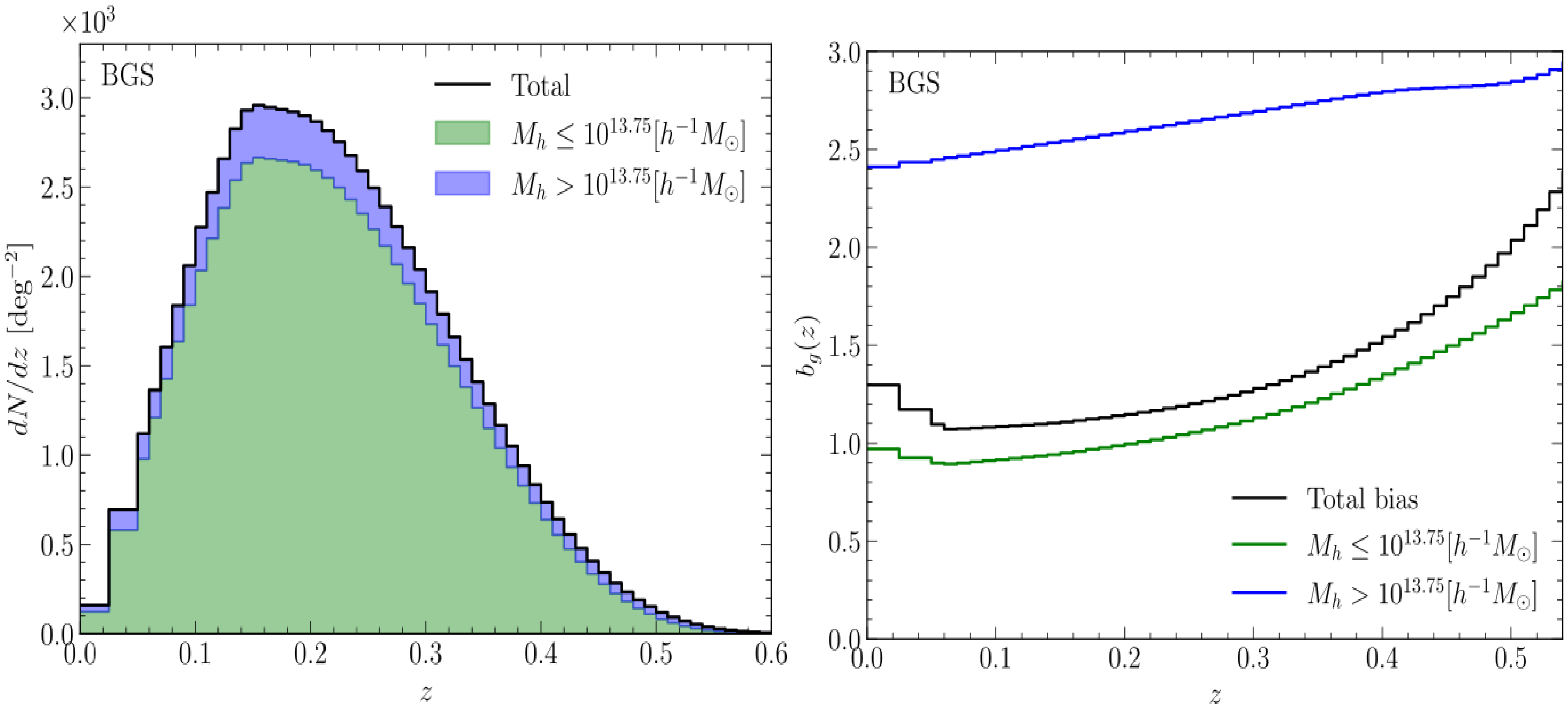
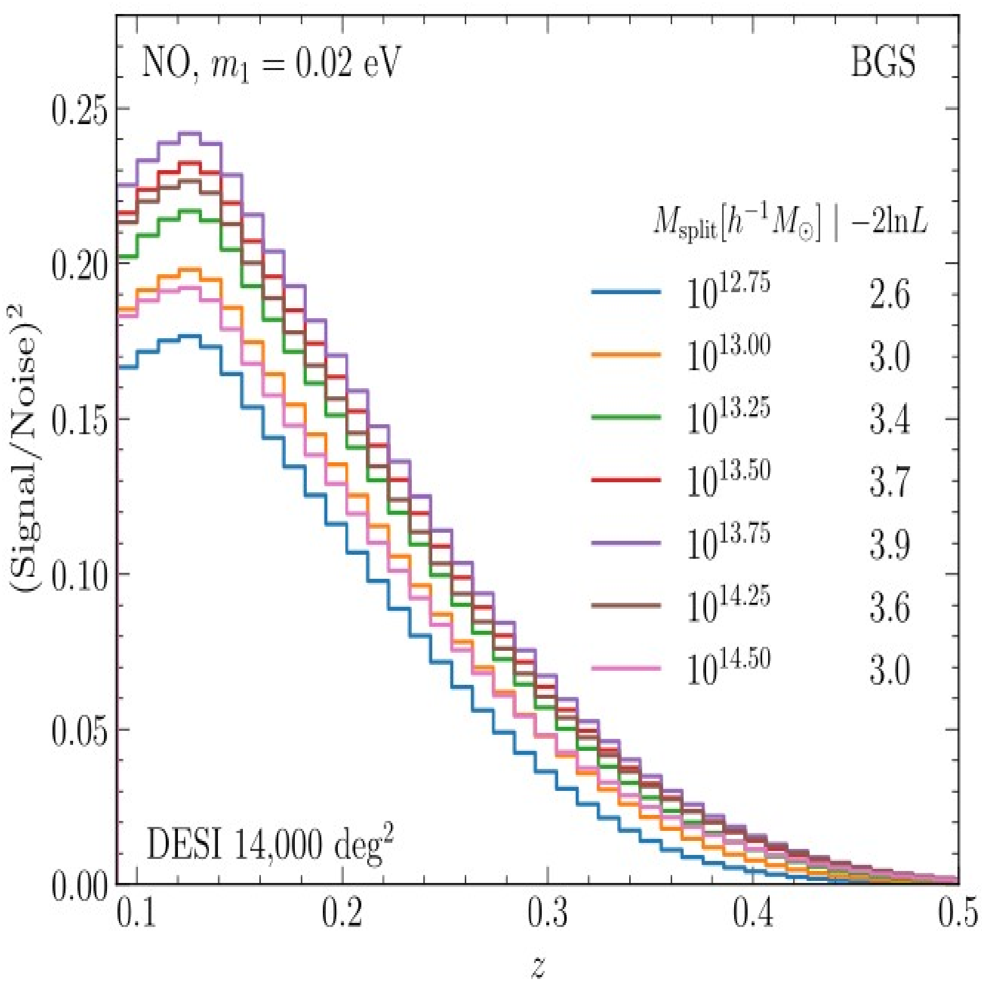
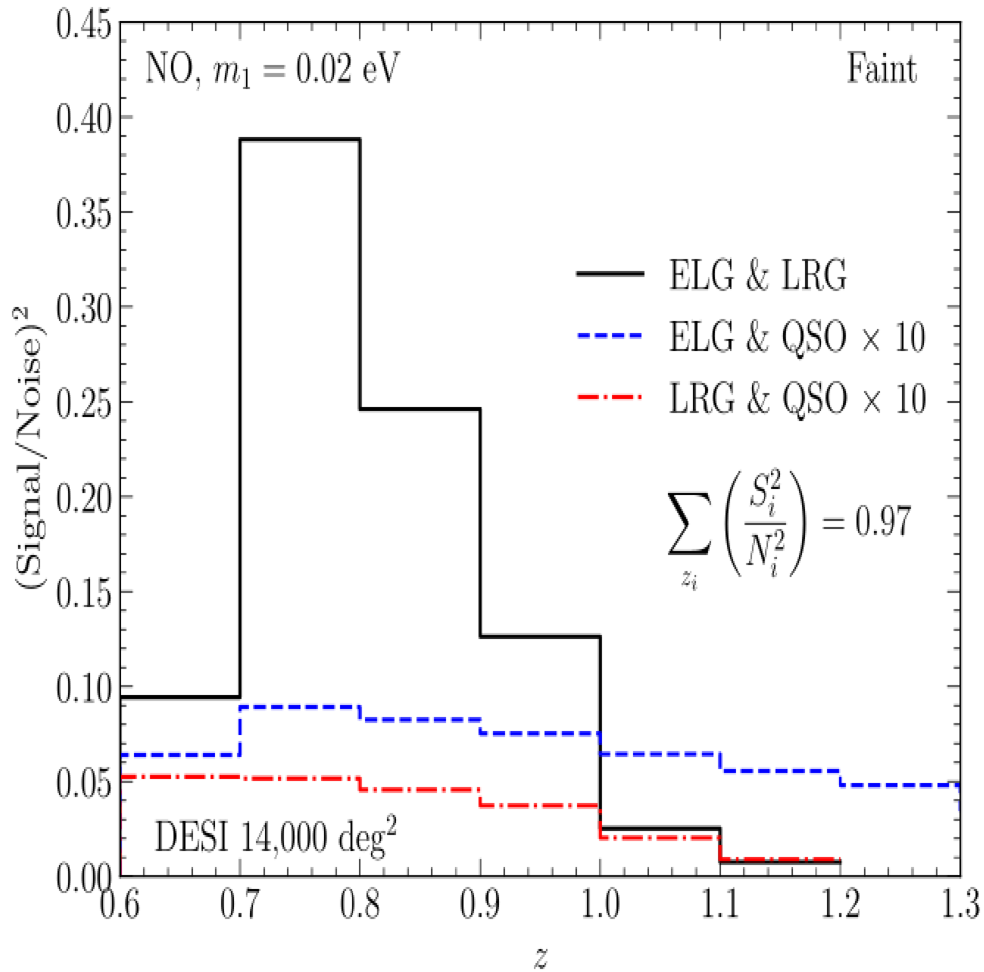


$$n_g(z) \equiv \int d\ln M_h \frac{dn(z)}{d\ln M_h} \langle N(M_h) \rangle \qquad b_g(z) \equiv \frac{1}{n_g} \int d\ln M_h \frac{dn(z)}{d\ln M_h} \langle N(M_h) \rangle b_h(M_h, z)$$

SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

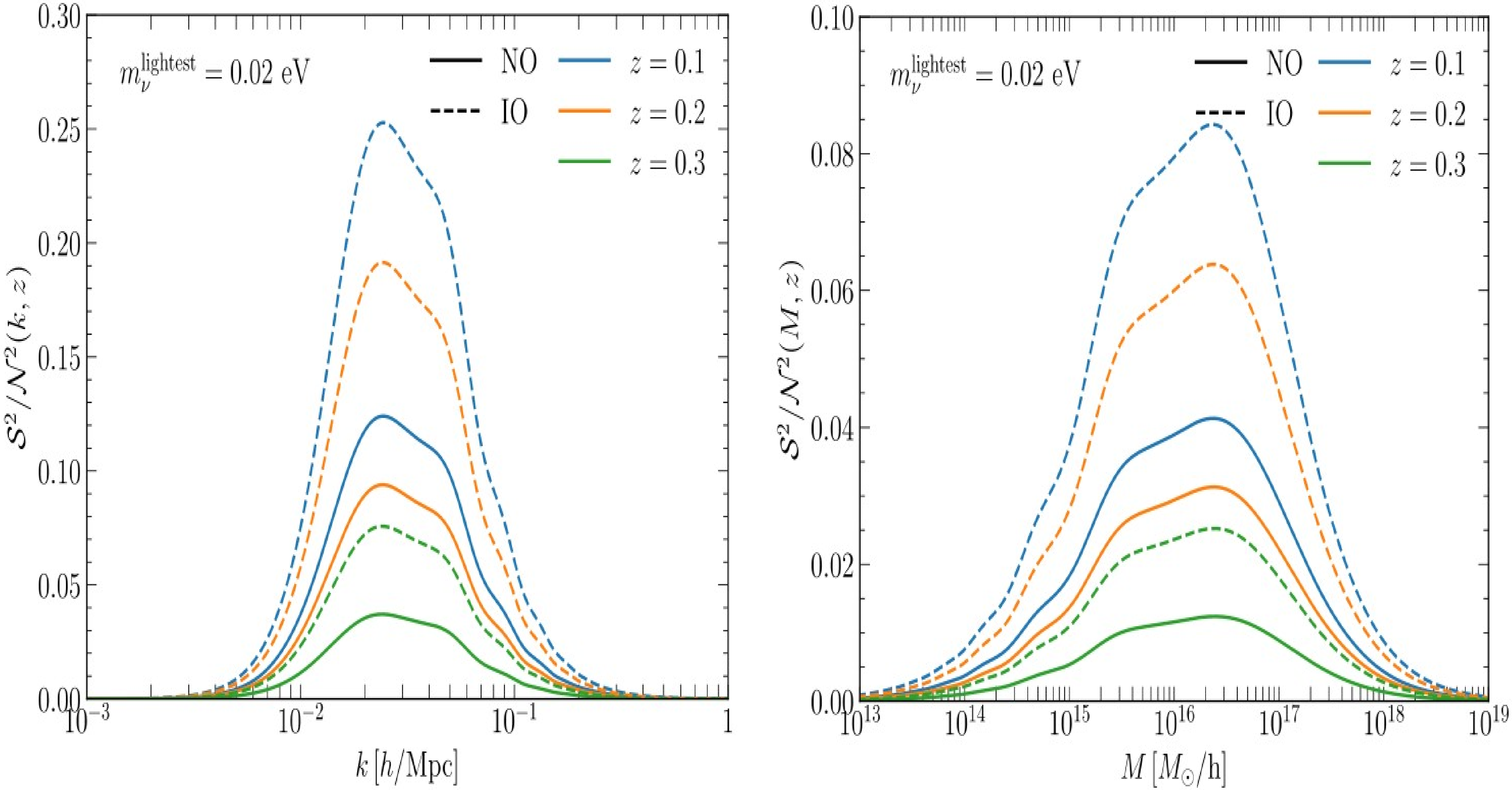
SNR @ DESI





SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

Redshift Distribution



SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

Sensitivities on ν Masses

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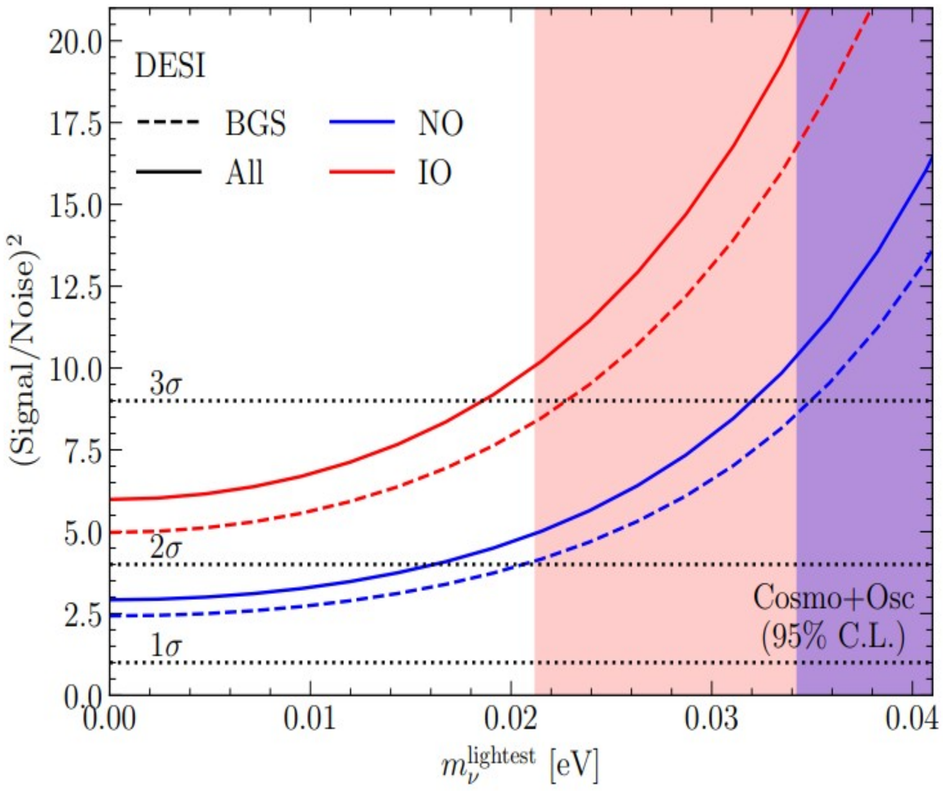
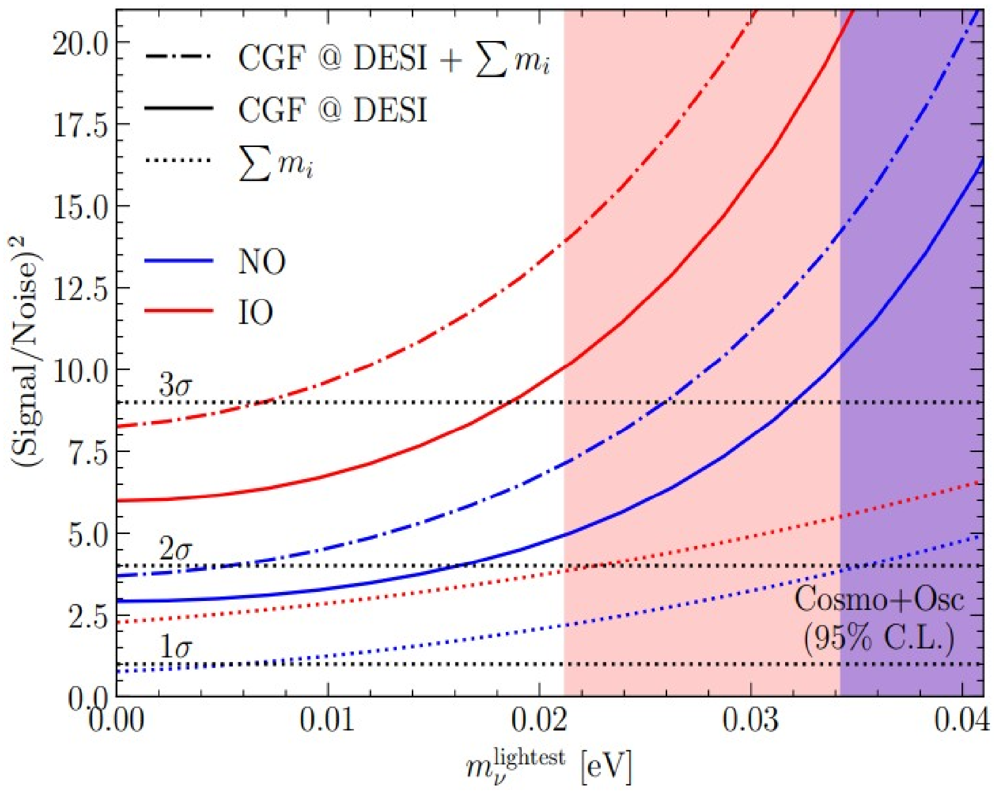
$$\mathrm{Im}\left[\tilde{\delta}_{g\alpha}\left(\tilde{\delta}_{g\beta}\right)^*\right] \propto \left(\mathbf{v}_{\nu c}\cdot\hat{\mathbf{k}}\right)\left(f_0 m_\nu^4 + f_1 m_\nu^2 T^2 + f_2 T^4\right)$$





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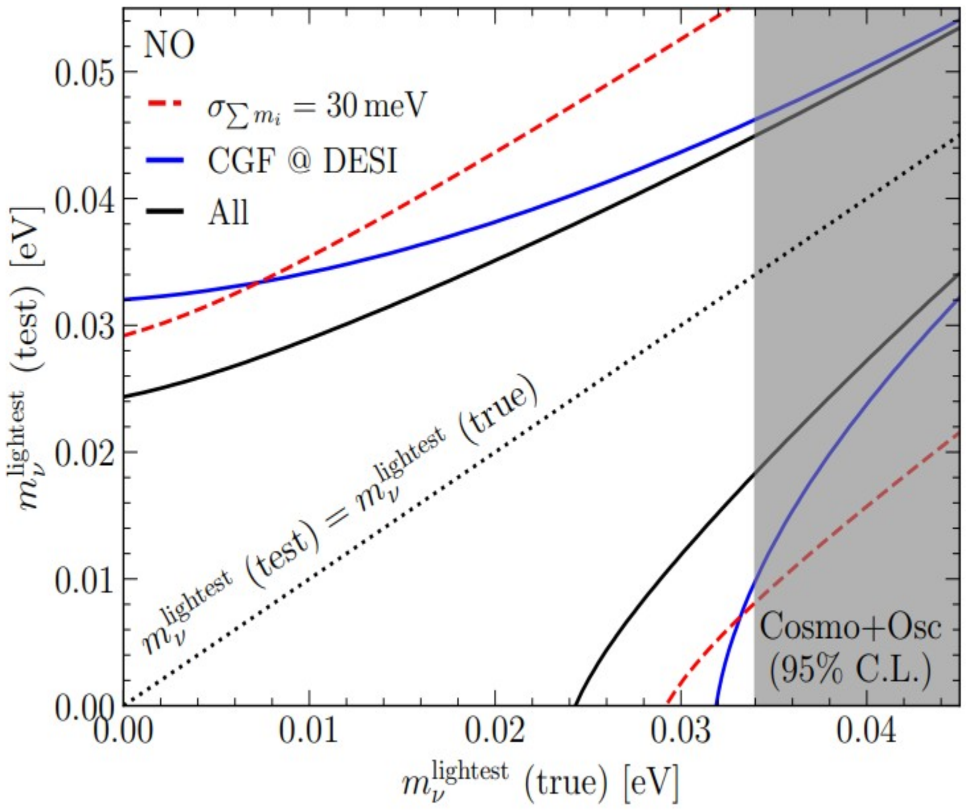
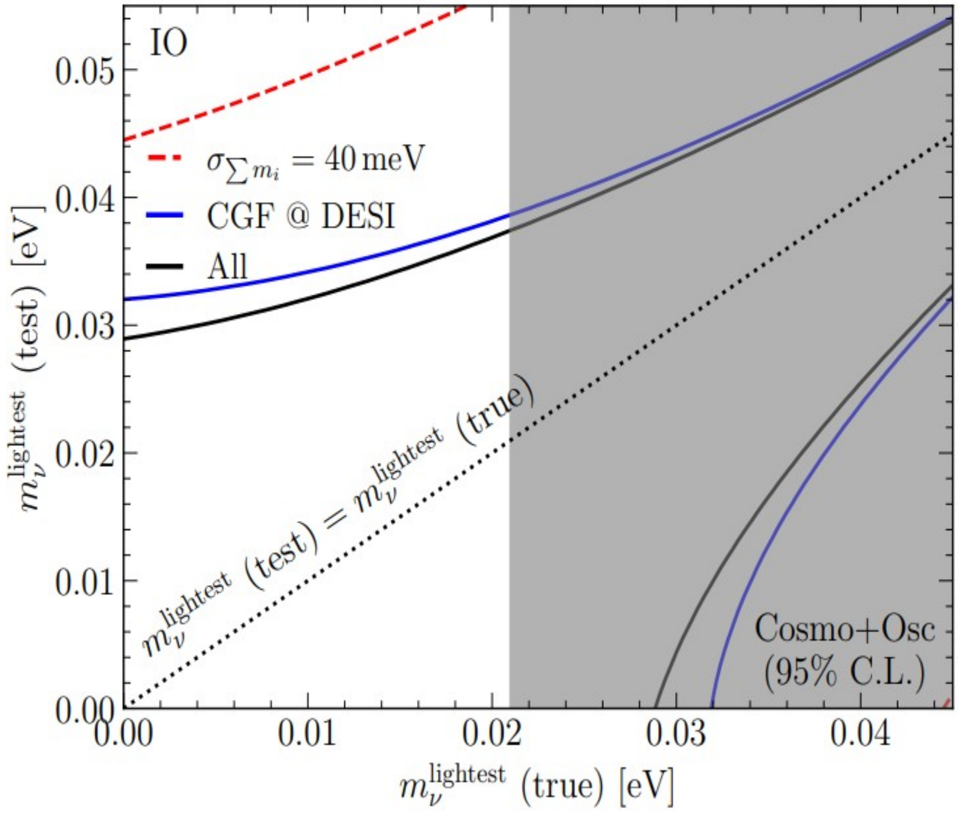
Sensitivities on ν Masses

$$\mathrm{Im}\left[\tilde{\delta}_{g\alpha}\left(\tilde{\delta}_{g\beta}\right)^*\right] \propto \left(\mathbf{v}_{\nu c}\cdot\hat{\mathbf{k}}\right)\left(f_0 m_\nu^4 + f_1 m_\nu^2 T^2 + f_2 T^4\right)$$

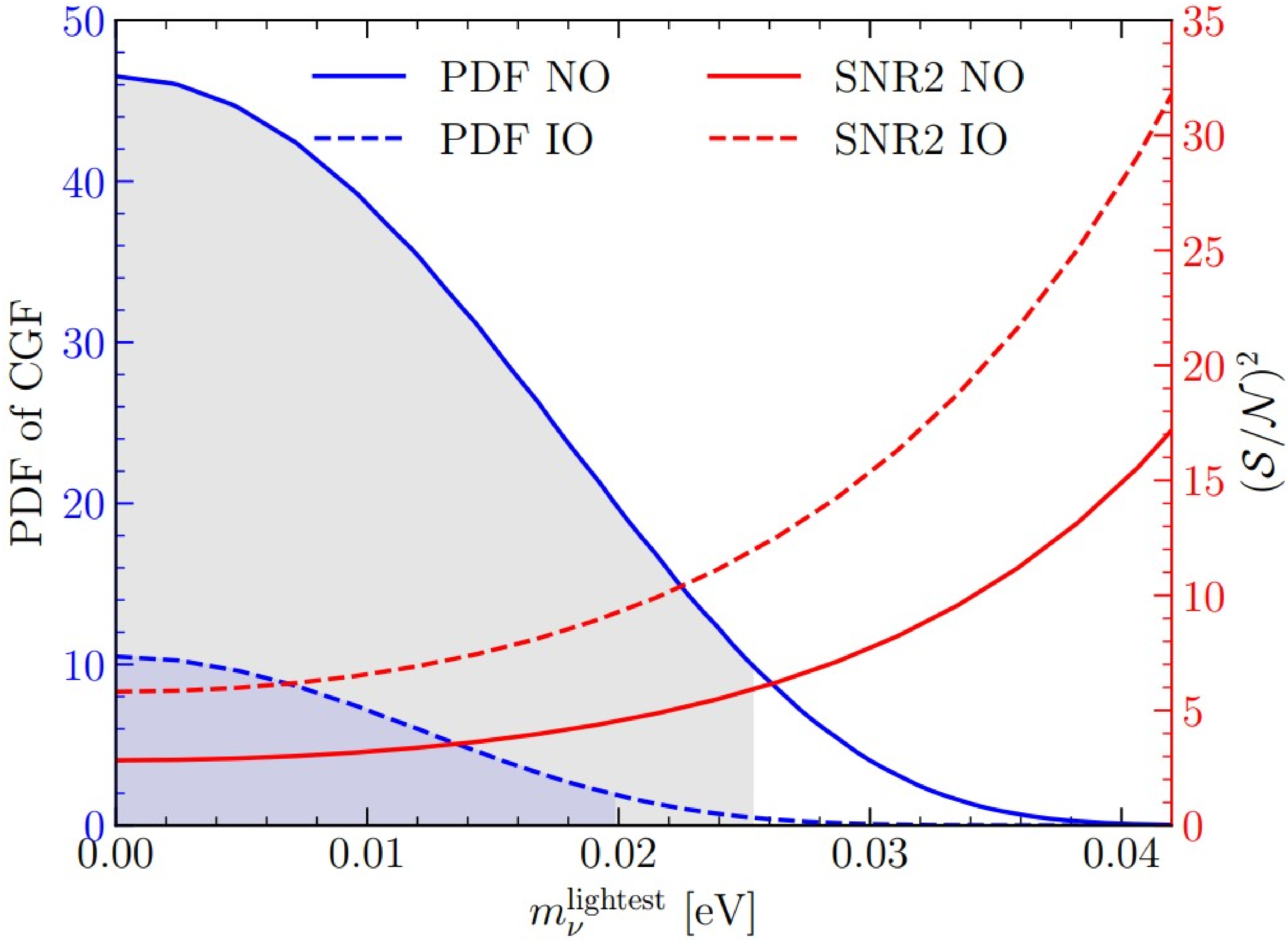




DESI, Euclid, Subaru PFS, **CSST**

SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]

Neutrino Mass Orderings



SFG & Liang Tan [arXiv:2409.11115]

Sensitivities on ν Mass Ordering

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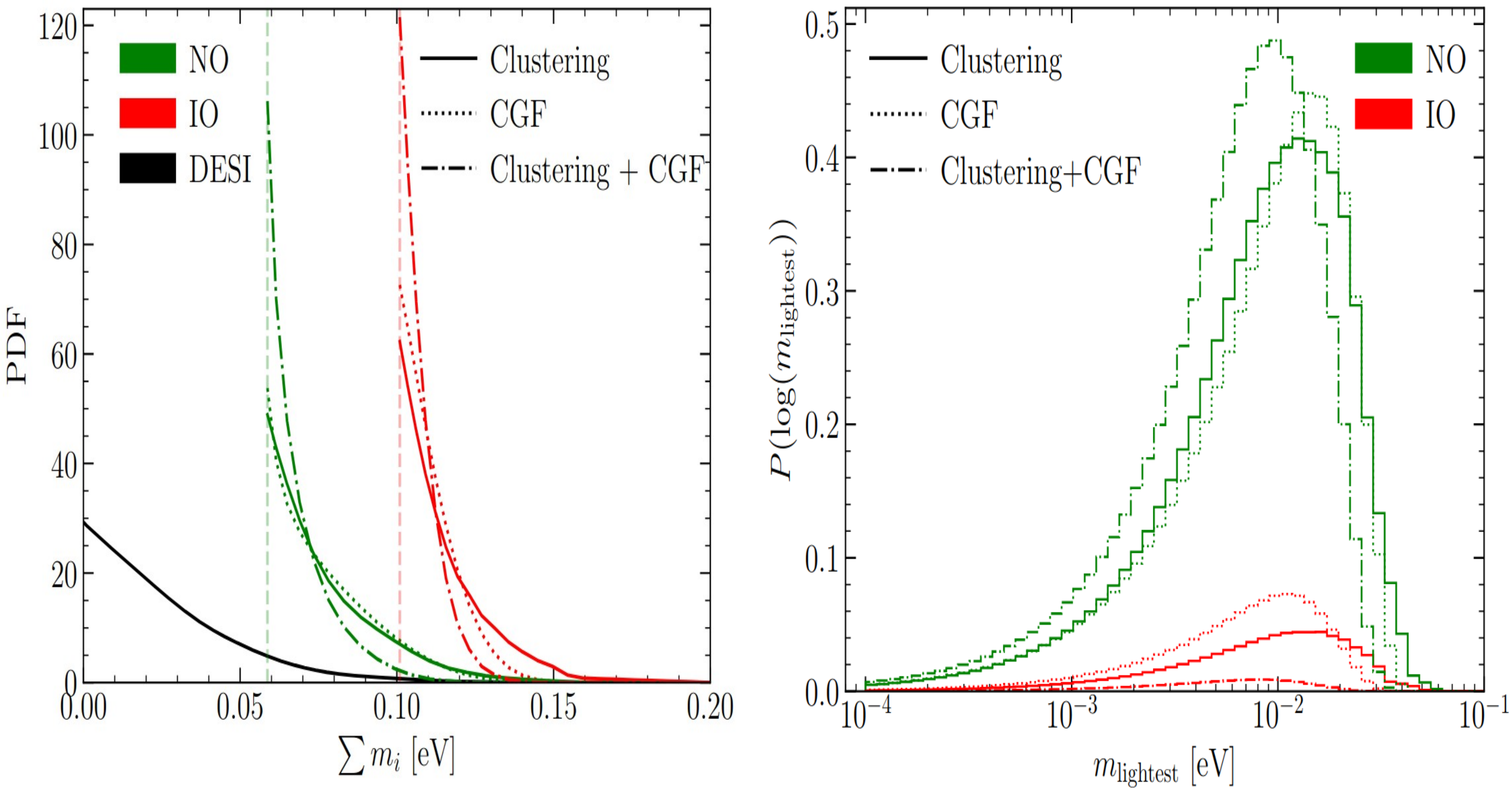


89.9% vs 10.1% → **98.2% vs 1.8%**

SFG & Liang Tan [arXiv:2409.11115]

Summary

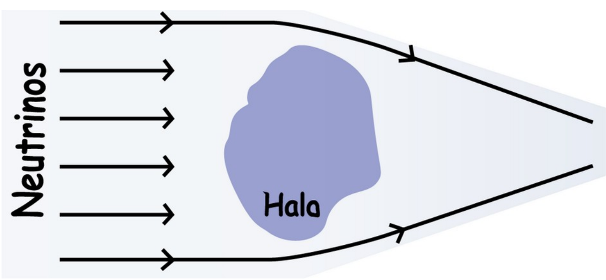
1) Overview of Neutrino Mass

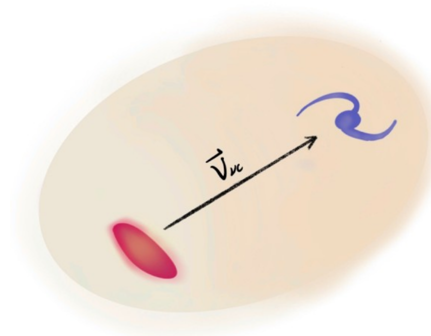
2) 3rd Cosmological Way

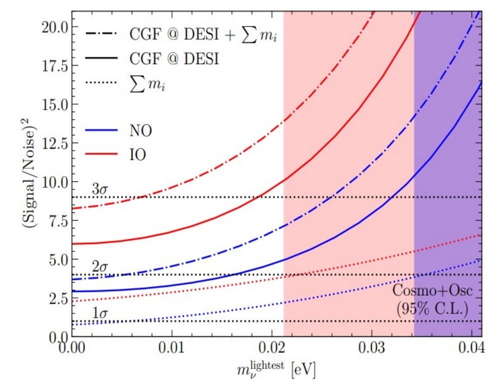
Cosmic Gravitational Focusing

Dipole Structure in Galaxy Correlation Function

Neutrino Mass Measurement @ DESI







3) Mass Ordering

4) Summary

Thank You

Imaginary Density & Average

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$$\mathrm{Im}[\delta\rho_\nu] = \frac{\widetilde{\Psi}}{4\pi}\sum_i (\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})\left[m_i^4\left(1-\frac{m_i|\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}}|}{2T_\nu}\right)+\pi^2 m_i^2 T_\nu^2+\frac{14}{15}\pi^4 T_\nu^4\right]$$

$$\langle(\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}})^2\rangle = \frac{1}{3}\int\frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2\Delta_\zeta^2(\boldsymbol{k}')\left|\frac{T_{\theta_{\nu_i c}}(\boldsymbol{k}',z)}{|\boldsymbol{k}'|}\right|^2,$$

$$\langle(\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})[\partial_z(\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})]\rangle = \frac{1}{3}\int\frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2\Delta_\zeta^2(\boldsymbol{k}')\left[\frac{T_{\theta_{\nu_i c}}}{|\boldsymbol{k}'|}\frac{\partial_z T_{\theta_{\nu_i c}}}{|\boldsymbol{k}'|}\right],$$

$$\langle[\partial_z(\boldsymbol{v}_{\nu_i c}\cdot\hat{\boldsymbol{k}})]^2\rangle = \frac{1}{3}\int\frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2\Delta_\zeta^2(\boldsymbol{k}')\left[\frac{\partial_z T_{\theta_{\nu_i c}}}{|\boldsymbol{k}'|}\right]^2,$$

Decoherence & Non-Linear

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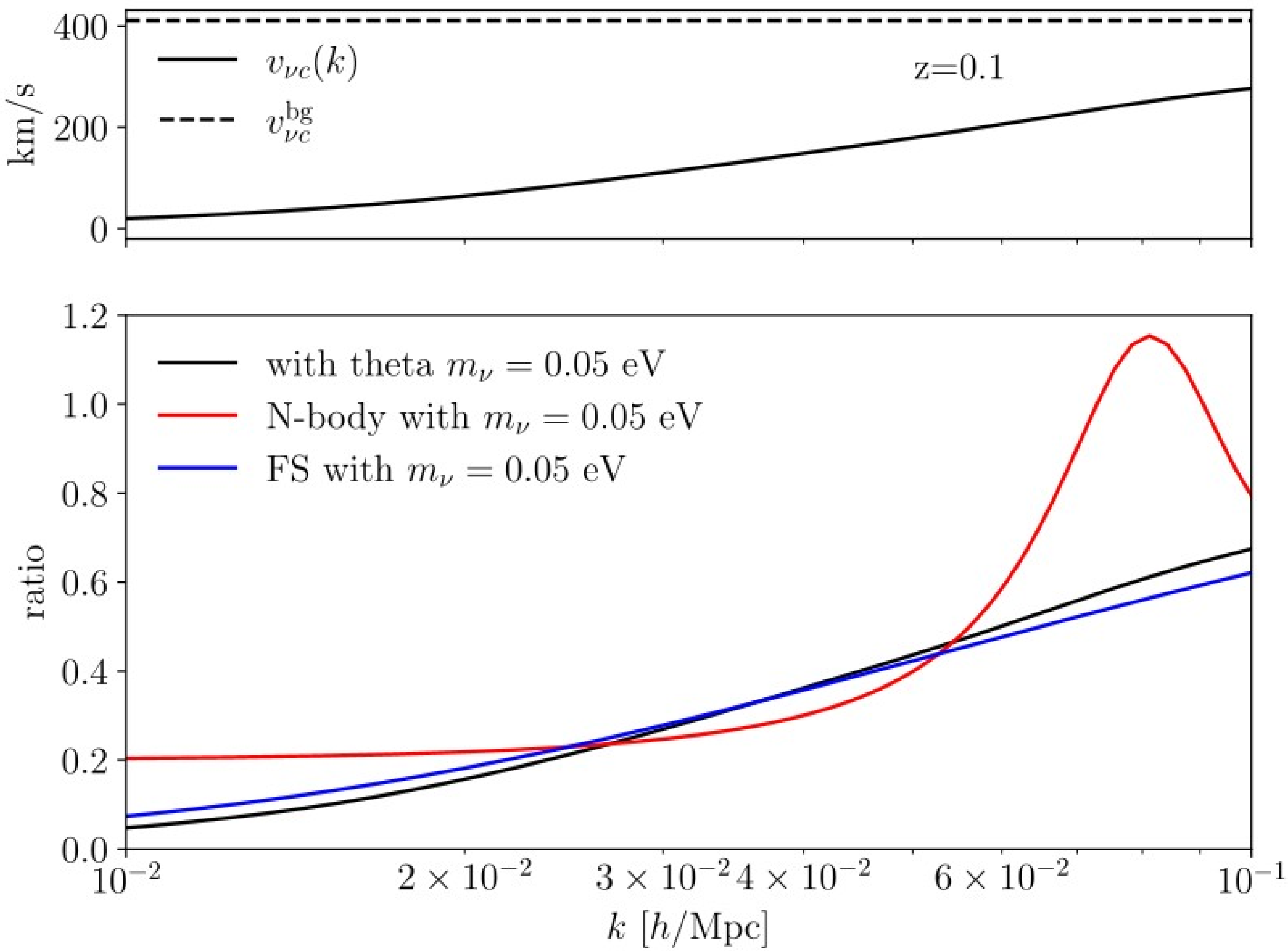
$$\langle(\boldsymbol{v}_{\nu c}\cdot\hat{\boldsymbol{k}})^2\rangle = \frac{1}{3}\int \frac{d|\boldsymbol{k}'|}{|\boldsymbol{k}'|}\Theta(|\boldsymbol{k}|-|\boldsymbol{k}'|)\left|\widetilde{W}(|\boldsymbol{k}'|R)\right|^2 \Delta_\zeta^2(\boldsymbol{k}')\left|\frac{T_{\theta_{\nu_i c}}(\boldsymbol{k}',z)}{|\boldsymbol{k}'|}\right|^2$$



《 Physics Letters A 》

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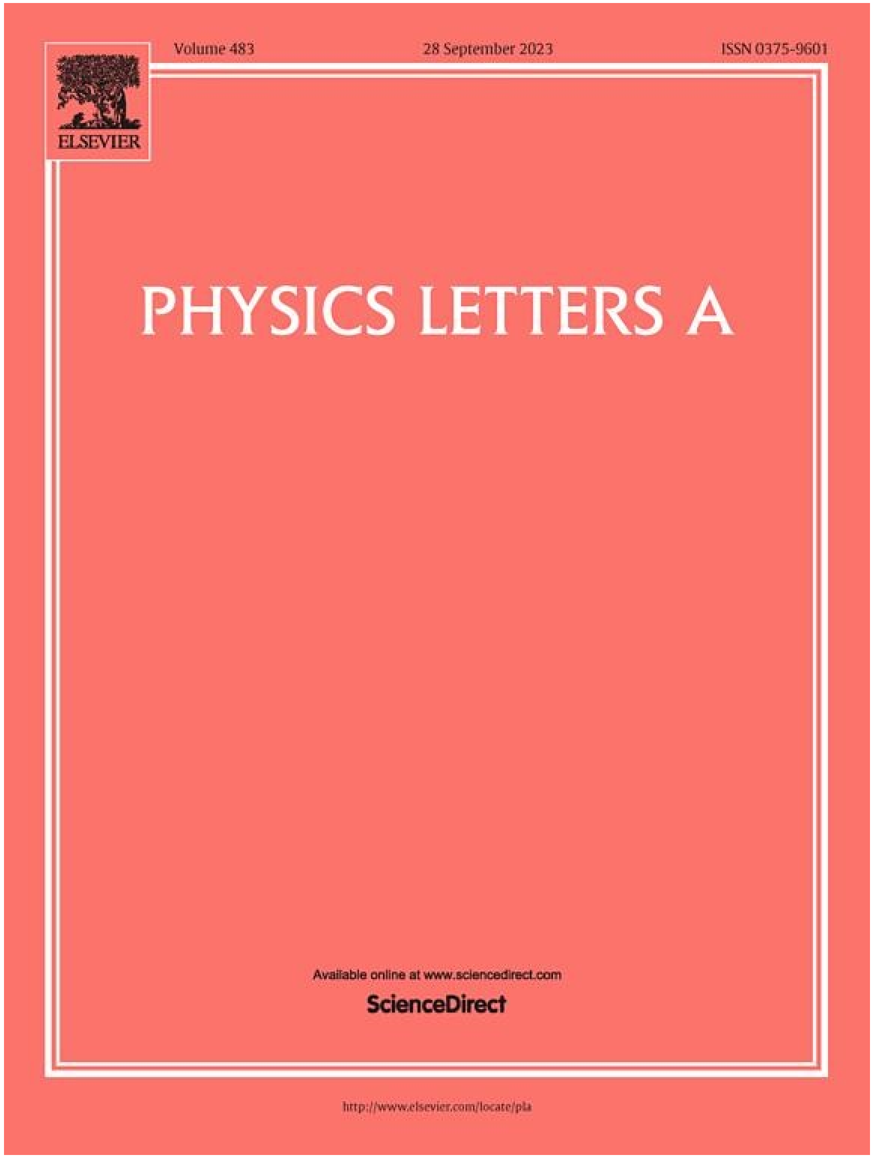

Aims & Scope

- Nonlinear science,
- Statistical physics,
- Mathematical and computational physics,
- AMO and physics of complex systems,
- Plasma and fluid physics,
- Optical physics,
- General and cross-disciplinary physics,
- Biological physics and nanoscience,
- Astrophysics, Particle physics and Cosmology.

为什么存在物质世界?

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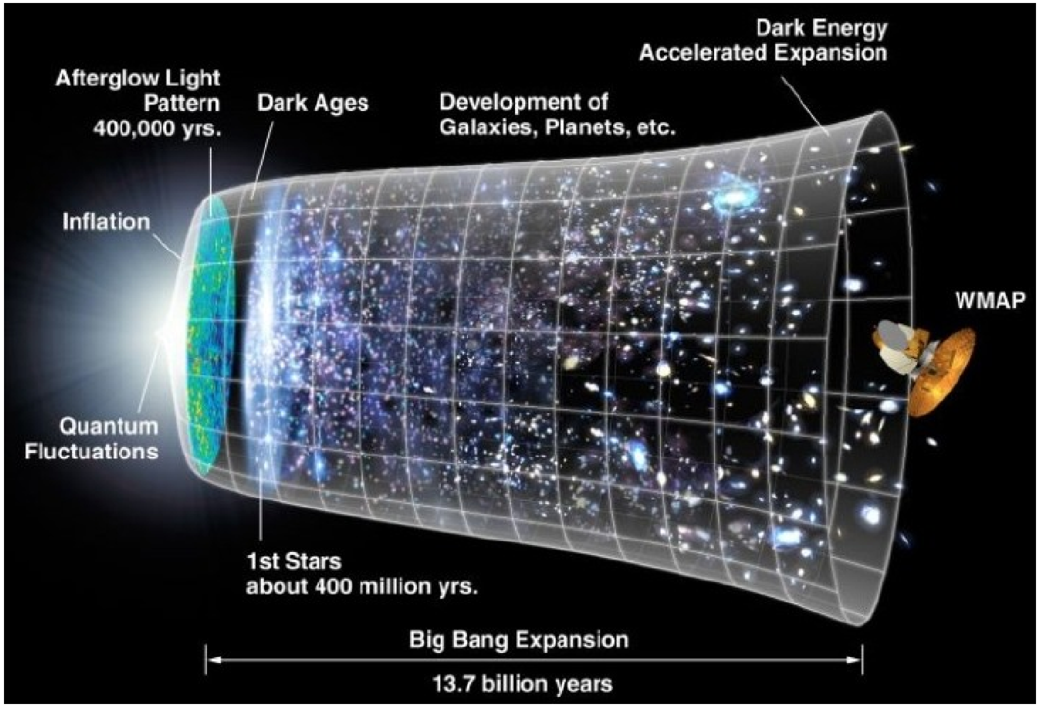
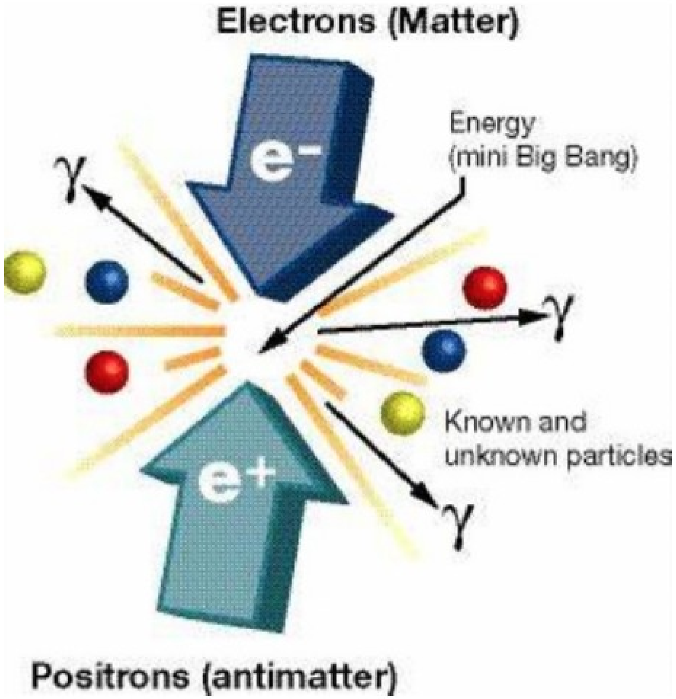
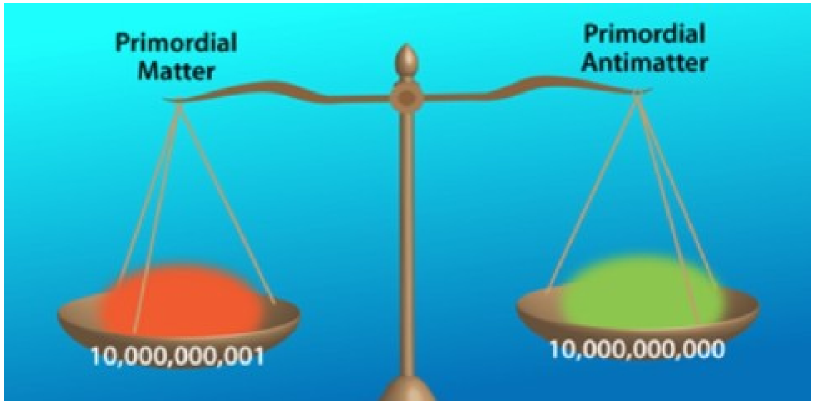




- Primordial asymmetry diluted by **Inflation (暴涨理论)**

$$a(t) = e^{Ht}$$

- Starting from symmetric phase
- Why still some asymmetry?
- **Something is wrong!**

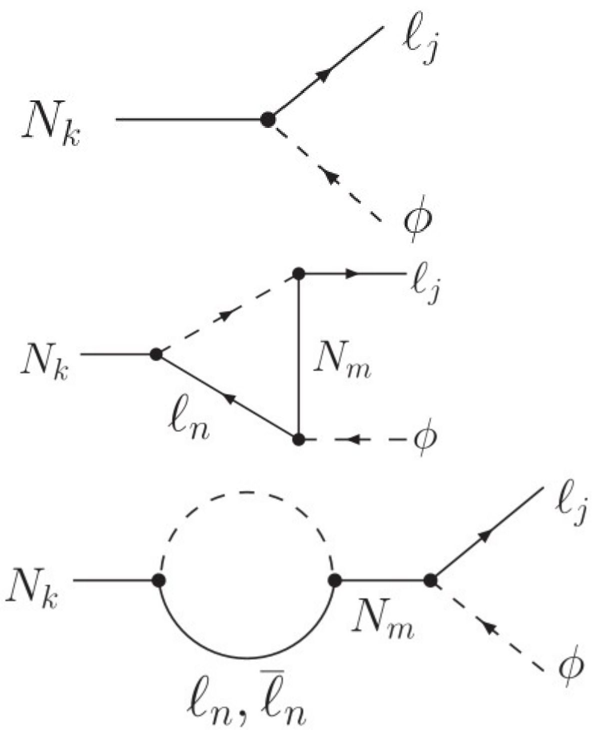


Leptogenesis (轻子创生机制)

宇宙早期有大量右手中微子，随着宇宙温度下降，衰变到轻的粒子

$$N_k \to \begin{cases} \ell_j + \overline{\phi} \\ \overline{\ell}_j + \phi \end{cases}$$

正反物质不对称



- **Interference between tree & loop diagrams**

$$\Gamma = \Gamma_{\text{tree}} + \Gamma_{\text{loop}}(+\delta_D, +\delta_M)$$
$$\overline{\Gamma} = \Gamma_{\text{tree}} + \Gamma_{\text{loop}}(-\delta_D, -\delta_M)$$

The matter-antimatter asymmetry needs Dirac/Majorana CP phases.

Yanagida

Fukugita

Three Neutrino Mixing

Tsung-Dao Lee Institute

- **Three Neutrino Mixing**

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \mathbf{U} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \qquad \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} = \mathbf{U}^\dagger \begin{pmatrix} A & B_1 & B_2 \\ & C_1 & D \\ & & C_2 \end{pmatrix} \mathbf{U}$$

- **PMNS Matrix**

$$U_{\rm PMNS} = \mathcal{P} \begin{pmatrix} c_s c_r & s_s c_r & s_r e^{-i\delta_D} \\ -s_s c_a - c_s s_a s_r e^{i\delta_D} & +c_s c_a - s_s s_a s_r e^{i\delta_D} & s_a c_r \\ +s_s s_a - c_s c_a s_r e^{i\delta_D} & -c_s s_a - s_s c_a s_r e^{i\delta_D} & c_a c_r \end{pmatrix} \mathcal{Q}$$

with $\mathcal{P} \equiv \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ & $\mathcal{Q} \equiv \text{diag}(e^{i\delta_{M1}}, 1, e^{i\delta_{M3}})$
[$(s, a, r) \equiv (12, 23, 13)$ for (**solar, atmospheric, reactor**) angles]

如果中微子没有质量，就没有 CP 相角 δ_D！

Seesaw Mechanism (跷跷板机制)

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- **Heavy neutrinos (*N*)**

$$\bar{\nu} \mathrm{M_D} \mathcal{N} + h.c. + \overline{\mathcal{N}} \mathrm{M_N} \mathcal{N} = \begin{pmatrix} \bar{\nu} & \overline{\mathcal{N}} \end{pmatrix} \begin{pmatrix} 0 & \mathrm{M_D} \\ \mathrm{M_D^T} & \mathrm{M_N} \end{pmatrix} \begin{pmatrix} \nu \\ \mathcal{N} \end{pmatrix}$$

required by **Grand Unification Theory (大统一理论)**

- **Seeaw Mechanism**

The diagonalization of the full mass matrix

$$\begin{pmatrix} 0 & \mathrm{M_D} \\ \mathrm{M_D^T} & \mathrm{M_N} \end{pmatrix} \Rightarrow M_\nu = -\mathrm{M_D} \frac{1}{\mathrm{M_N}} \mathrm{M_D^T}$$

$\mathrm{M_D} \sim O(100)$ GeV, $\mathrm{M_N} \sim O(10^{15})$ GeV $\rightarrow \mathrm{M_\nu} \sim O(0.01)$ eV

柳田勉、何小刚：
小朋友，我们来玩跷跷板吧

Light neutrino mass $\mathrm{M_\nu}$ is suppressed by the heavy ones

What is Mass? (质量是什么 ?)

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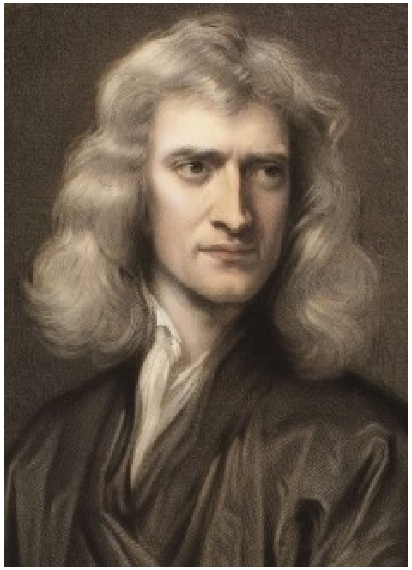
- Mass is a long-standing issue. **Long history**



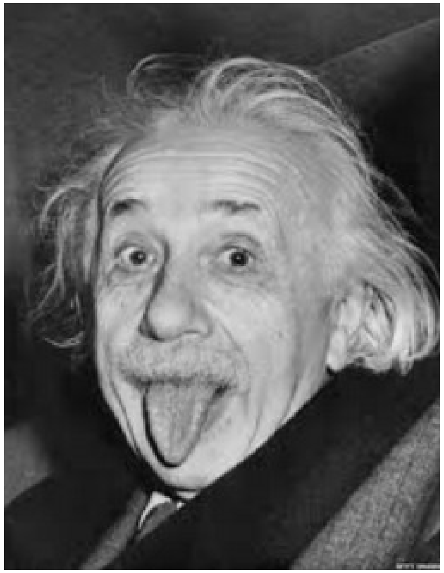
牛顿：如有雷同，纯属巧合

Inertial Mass

vs

Gravitational Mass

爱因斯坦：小样，你以为披个马甲我就不认识你了吗?

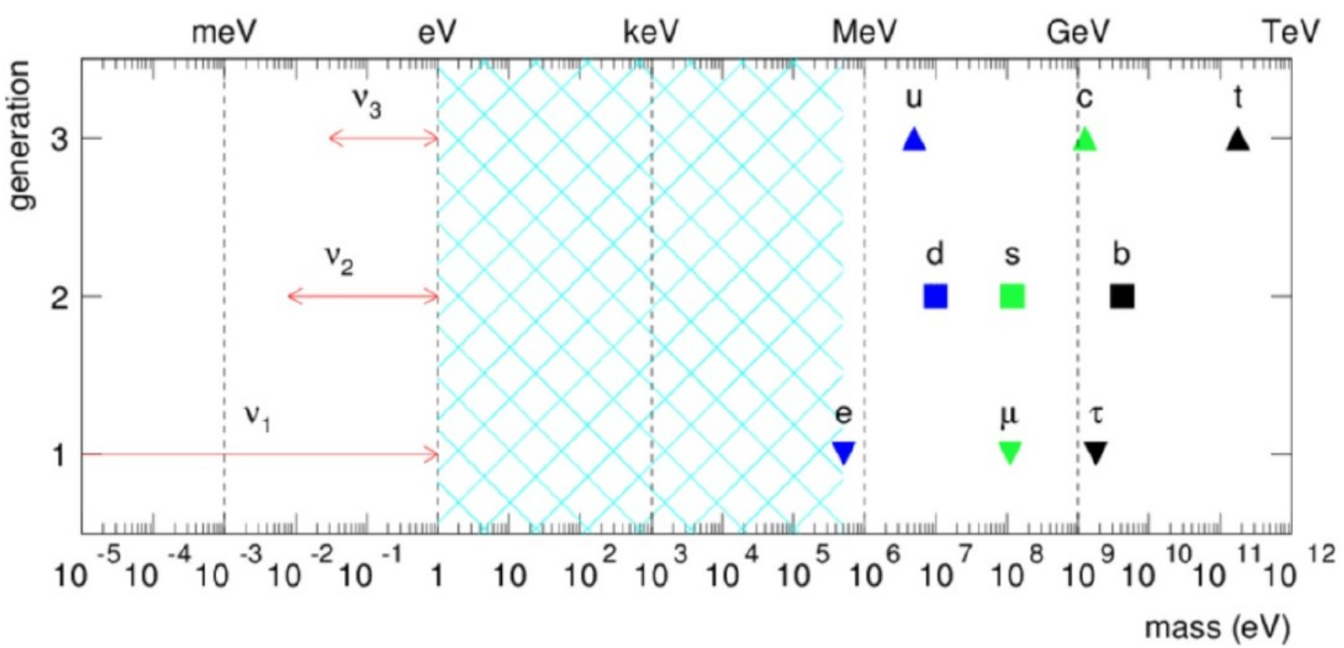


- Higgs Mechanism (希格斯机制)

$$y\overline{\psi}\phi\psi \quad \Rightarrow \quad \overline{\psi}M\psi = \overline{\psi}y\langle\phi\rangle\psi$$

希格斯：问题不在马甲，关键在于帽子!

Tiny Neutrino Mass



希格斯：大家都喜欢戴墨西哥草帽

$$m_t = 173\,\text{GeV} = 1.73 \times 10^{11}\,\text{eV}$$
$$m_e = 0.51\,\text{MeV} = 5.1 \times 10^{5}\,\text{eV}$$
$$m_\nu \lesssim 10^{-2}\,\text{eV}$$

How to explain the huge difference?

中微子：希格斯你这帽子太大了，我就一小孩，戴着不合适

Neutrino Mass Measurements

李政道研究所 Tsung-Dao Lee Institute

ν Oscillation @ JUNO

SFG, Kaoru Hagiwara, Naotoshi Okamura, Yoshitaro Takaesu, **JHEP 05 (2013) 131** [arXiv:1210.8141]

Beta Decay

SFG, Manfred Lindner, **Phys.Rev.D 95 (2017) 3, 033003** [arXiv:1608.01618]
SFG & Werner Rodejohann, **Phys.Rev.D 96 (2017) 5, 055019** [arXiv:1707.07904]
SFG & Jing-Yu Zhu, **Chin.Phys.C 44 (2020) 8, 083103** [arXiv:1910.02666]

Radiative Emission of ν Pairs

SFG, Pedro Pasquini, **JHEP 12 (2023) 083** [arXiv:2306.12953]
Phys.Lett.B 841 (2023) 137911 [arXiv:2206.11717]
Eur.Phys.J.C 82 (2022) 3, 208 [arXiv:2110.03510]

Supernova ν Time Delay

SFG, Chui-Fan Kong, Alexei Smirnov, **PRL 133 (2024) 121802** [arXiv:2404.17352]

CνB Detection

CMB & LSS

SFG, Pedro Pasquini, Liang Tan, **JCAP 05 (2024) 108** [arXiv:2312.16972]
SFG & Liang Tan [arXiv:2409.11115]

Neutrino Mass & Mixing

Tsung-Dao Lee Institute

- Neutrino Kinetic & Mass Terms

$$\mathcal{L}_\nu = \overline{\nu}_\alpha \left(i \not{\partial} \delta_{\alpha\beta} + M_{\alpha\beta} \right) \nu_\beta$$

- Correlated Mass Matrix

$$\overline{\nu}_\alpha M_{\alpha\beta} \nu_\beta = -\frac{1}{2} \begin{pmatrix} \nu_e^T & \nu_\mu^T & \nu_\tau^T \end{pmatrix} \mathcal{C} \begin{pmatrix} A & B_1 & B_2 \\ & C_1 & D \\ & & C_2 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} + \cdots$$

- Mass Diagonalization

$$\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = V^\dagger \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \qquad \begin{pmatrix} m_1 & & \\ & m_2 & \\ & & m_3 \end{pmatrix} = V^T \begin{pmatrix} A & B_1 & B_2 \\ & C_1 & D \\ & & C_2 \end{pmatrix} V$$

Mass Eigenstate — **Flavor Eigenstate**

Mass eigenstate is truly physical!

Neutrino Oscillation & Mass

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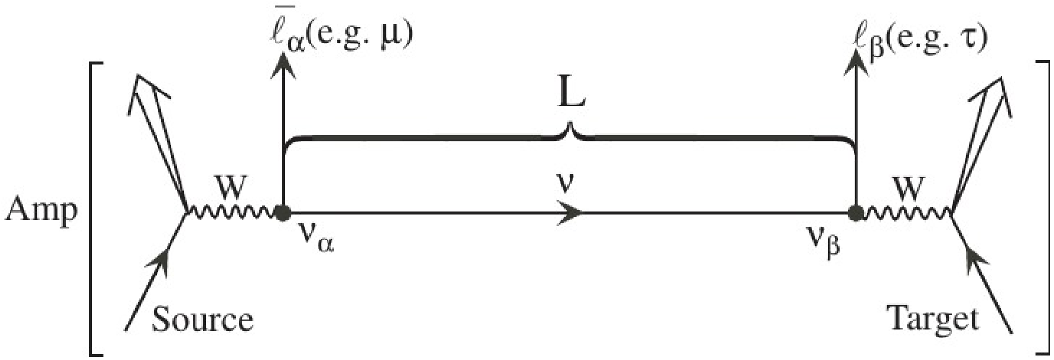
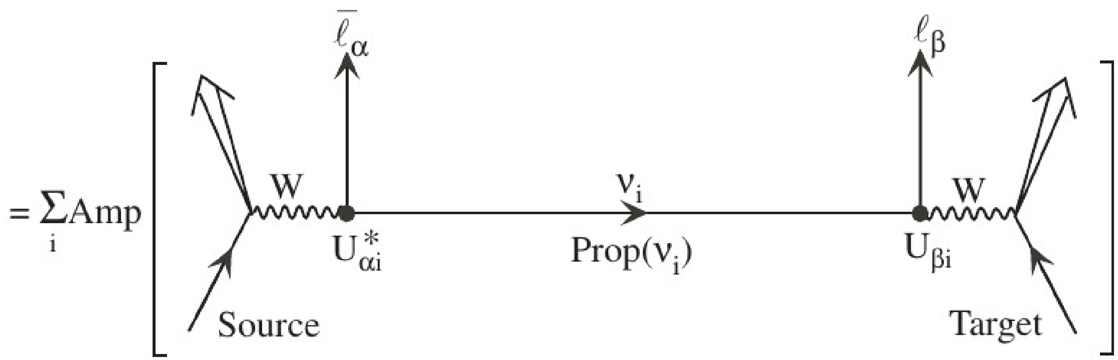




$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i$$

$$\rightarrow \boxed{\sum_i U_{\alpha i} e^{i(E_i t - \vec{P}_i \cdot \vec{x})} \nu_i}$$

$$= \boxed{\sum_i U_{\alpha i} P_i U^\dagger_{\beta i} \nu_\beta}$$

$$\equiv \sum_\beta A_{\alpha\beta} \nu_\beta$$

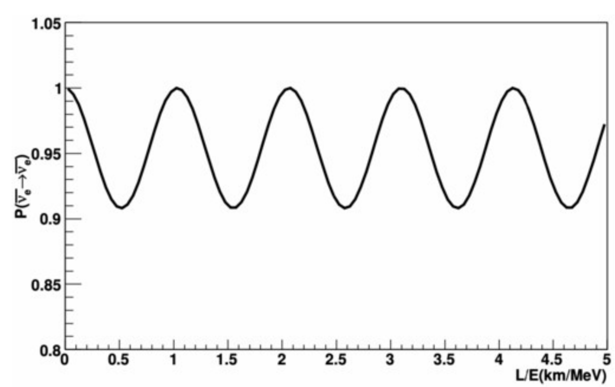
B. Kayser, [hep-ph/0506165]

$$P_{\alpha\beta}|_{\alpha\neq\beta} \equiv |A_{\alpha\beta}|^2 = \sin^2 2\theta \sin^2\left(\delta m^2 \frac{L}{4E}\right)$$

Mass

1st New Physics

Quantum Interference over Macroscopic Distance!



Three Neutrino Oscillation

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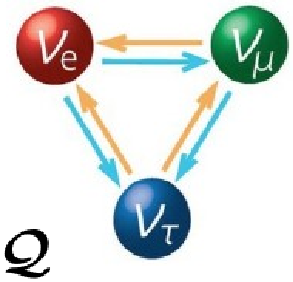

- **PMNS Matrix**

$$U_{\rm PMNS} = \mathcal{P} \begin{pmatrix} c_s c_r & s_s c_r & s_r e^{-i\delta_{\rm D}} \\ -s_s c_a - c_s s_a s_r e^{i\delta_{\rm D}} & +c_s c_a - s_s s_a s_r e^{i\delta_{\rm D}} & s_a c_r \\ +s_s s_a - c_s c_a s_r e^{i\delta_{\rm D}} & -c_s s_a - s_s c_a s_r e^{i\delta_{\rm D}} & c_a c_r \end{pmatrix} \mathcal{Q}$$

with $\mathcal{P} \equiv {\rm diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ & $\mathcal{Q} \equiv {\rm diag}(e^{i\delta_{\rm M1}}, 1, e^{i\delta_{\rm M3}})$
[$(s, a, r) \equiv (12, 23, 13)$ for (**solar, atmospheric, reactor**) angles]

$$\begin{aligned} P(\nu_\alpha \to \nu_\beta) &= |{\rm Amp}(\nu_\alpha \to \nu_\beta)|^2 \\ &= \delta_{\alpha\beta} - 4 \sum_{i>j} \Re(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin^2(\Delta m^2_{ij} \frac{L}{4E}) \\ &\quad + 2 \sum_{i>j} \Im(U^*_{\alpha i} U_{\beta i} U_{\alpha j} U^*_{\beta j}) \sin(\Delta m^2_{ij} \frac{L}{2E}) \end{aligned}$$

$$\Delta m^2_{ij} \equiv m^2_i - m^2_j \qquad \Delta m^2_{ij} \frac{L}{4E} = 1.27\, \Delta m^2_{ij}({\rm eV}^2) \frac{L\,({\rm km})}{E\,({\rm GeV})}$$

DM Gravitational Focusing

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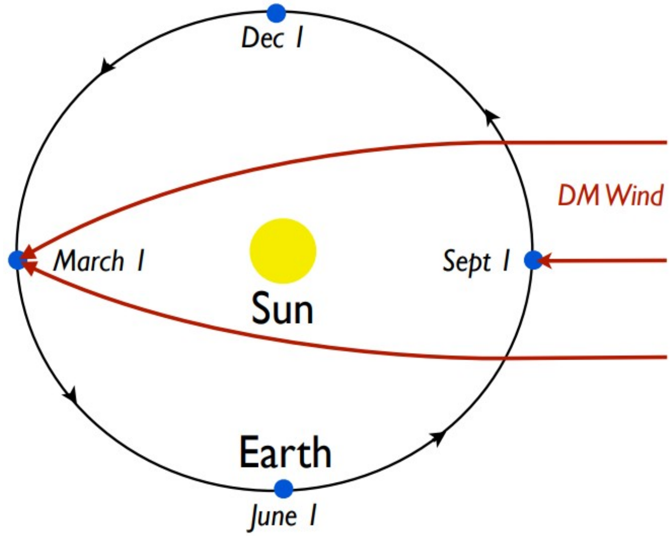
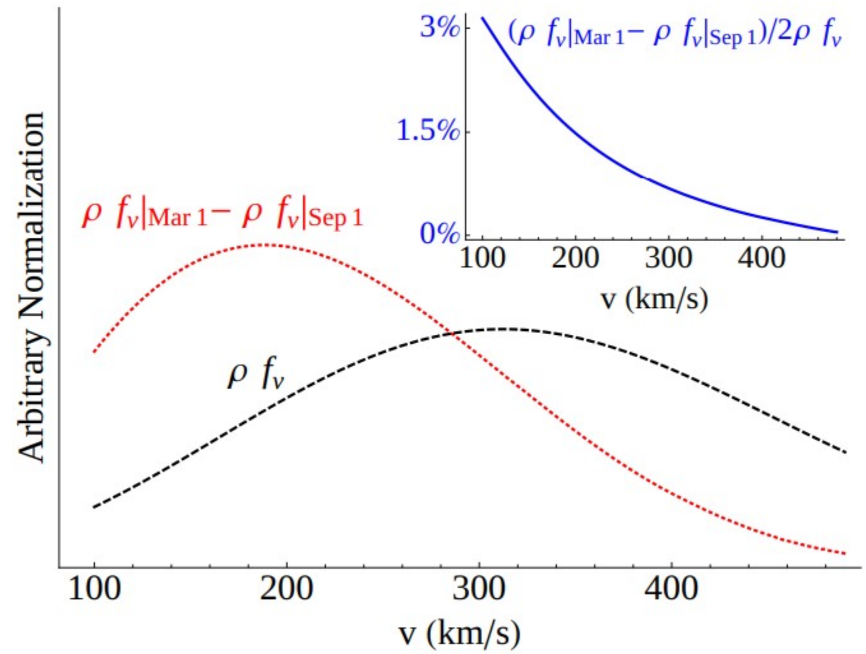
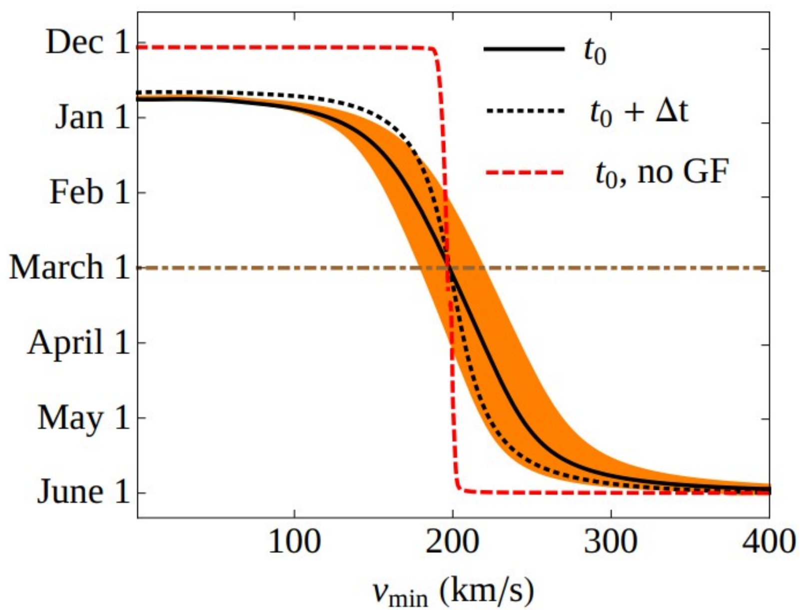


- Density enhancement



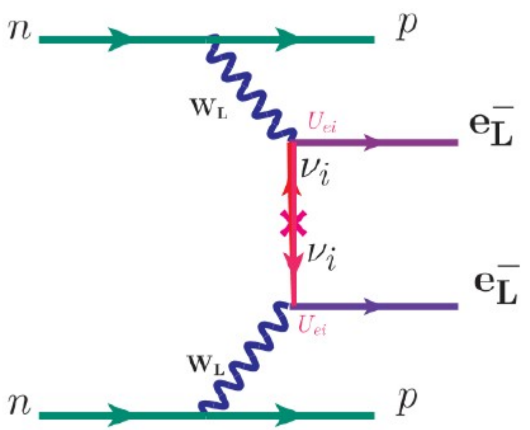
- Modulation phase shift

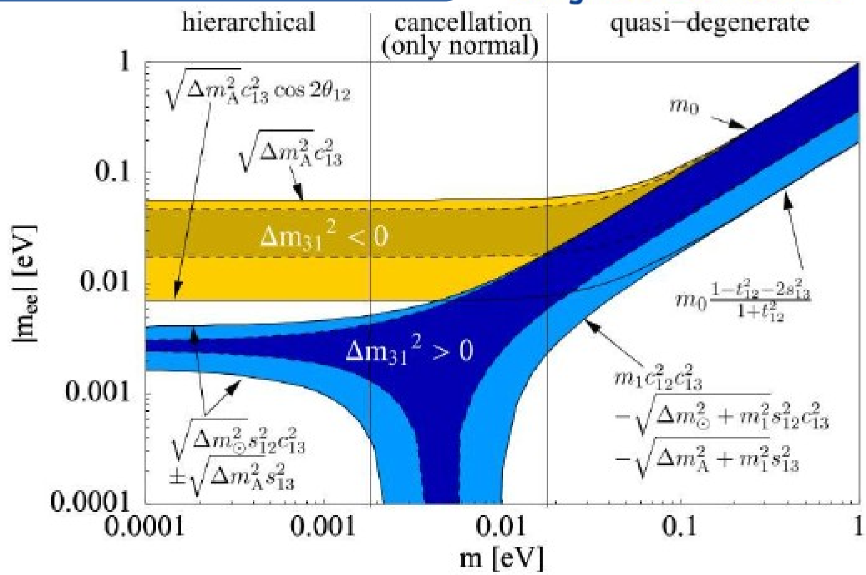


Lee, Lisanti, Peter, Safdi, Phys. Rev. Lett. 112, 011301 (2014) [arXiv:1308.1953]

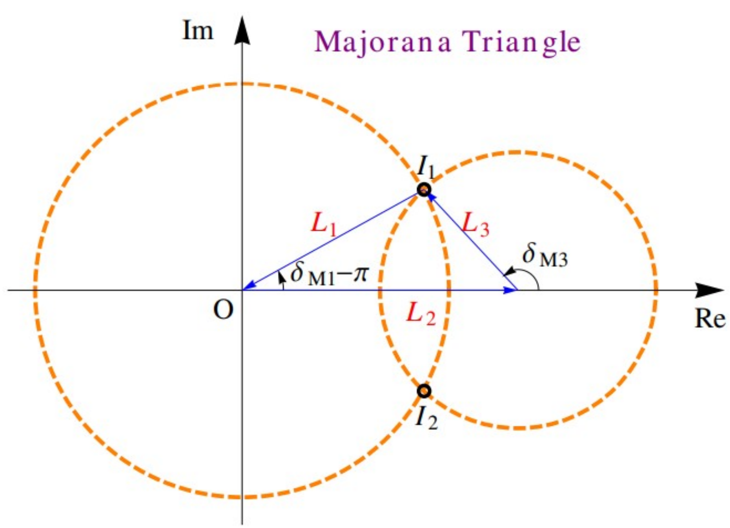
Bozorgnia & Schwetz, JCAP 08 (2014) 013 [arXiv:1405.2340]

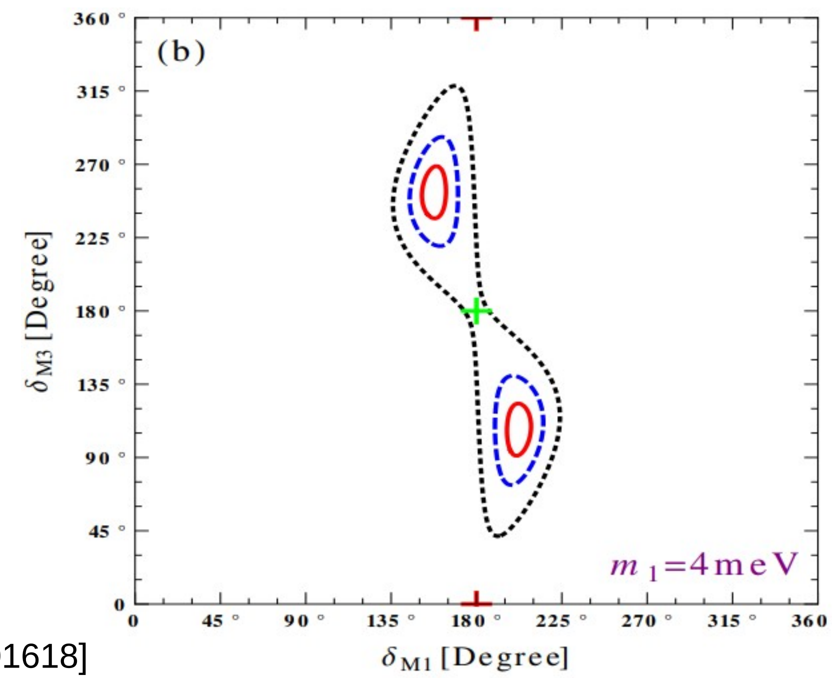
Neutrinoless Double Beta Decay





$$\langle m \rangle_{ee} \equiv \left| \sum_i m_i U_{ei}^2 \right| = \left| c_s^2 c_r^2 m_1 e^{i\delta_{M1}} + s_s^2 c_r^2 m_2 + s_r^2 m_3 e^{i\delta_{M3}} \right|$$





SFG & Manfred Lindner, PRD 95 (2017) No.3, 033003 [arXiv:1608.01618]

Radiative Emission of ν Pairs

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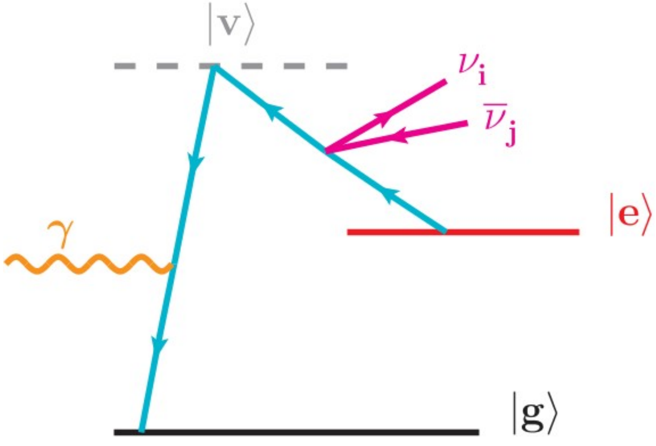
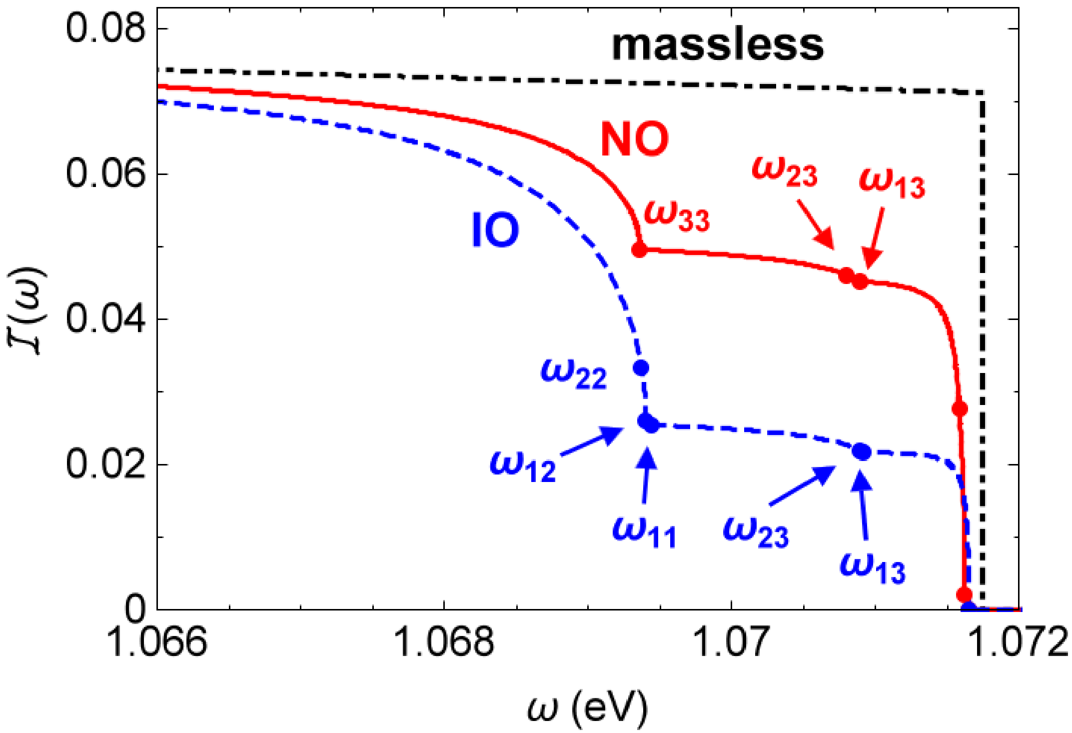


$$|e\rangle \to |v\rangle + \nu\bar{\nu}$$

$$\mathrm{E1} \times \mathrm{M1}$$

M. Yoshimura, Phys. Rev. D 75, 113007 (2007)

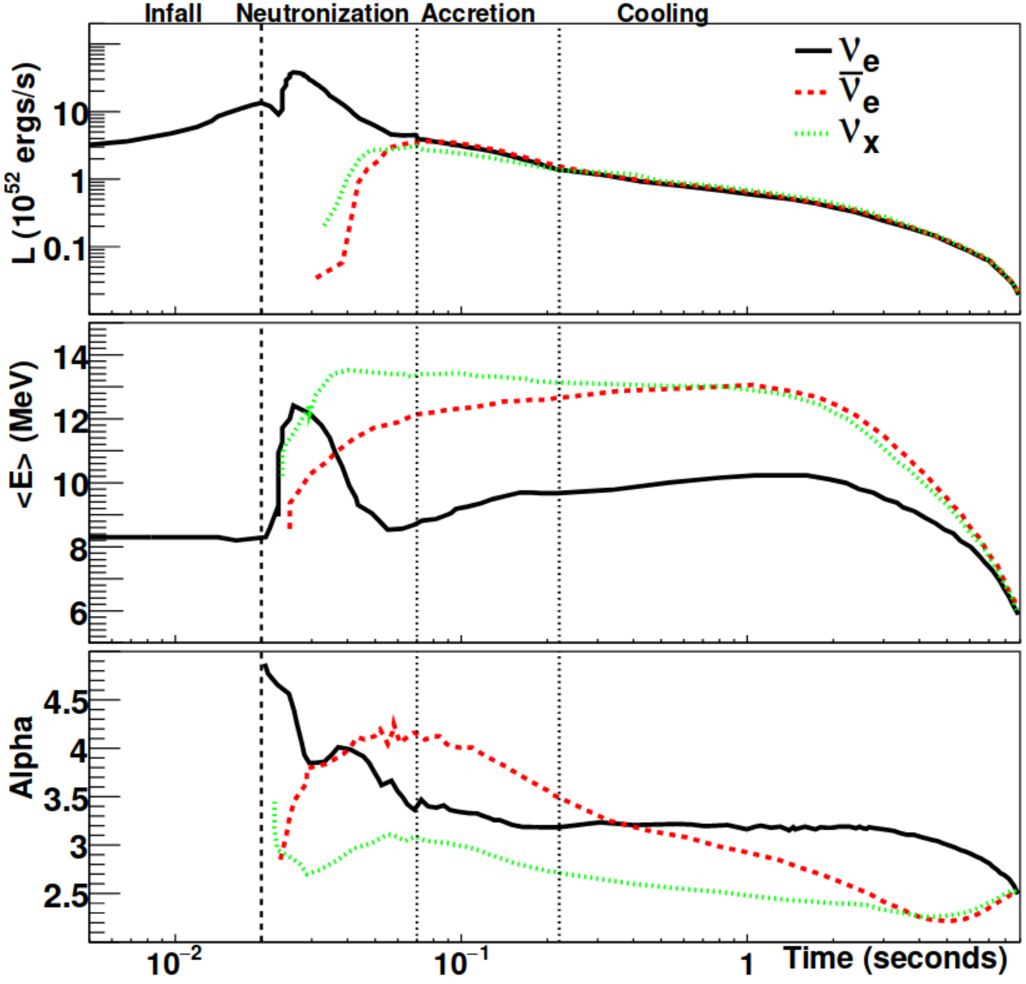
Jue Zhang & Shun Zhou, Phys.Rev.D 93 (2016) 11, 113020

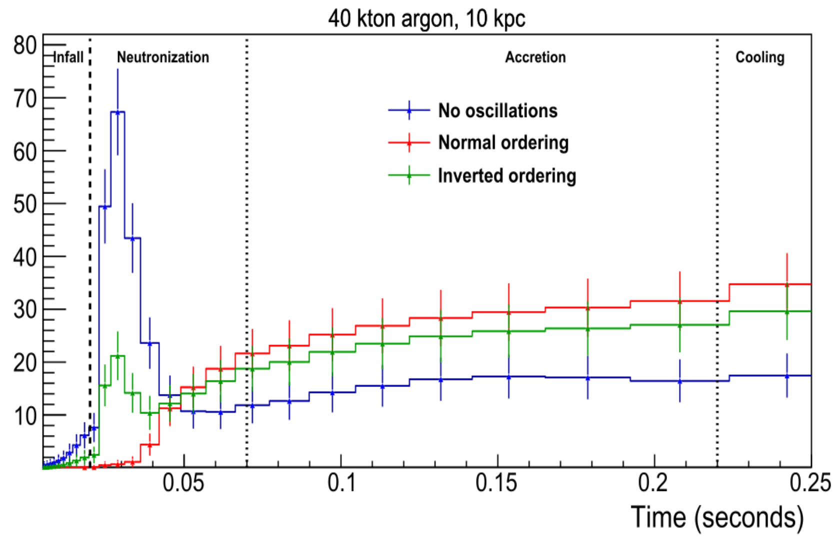


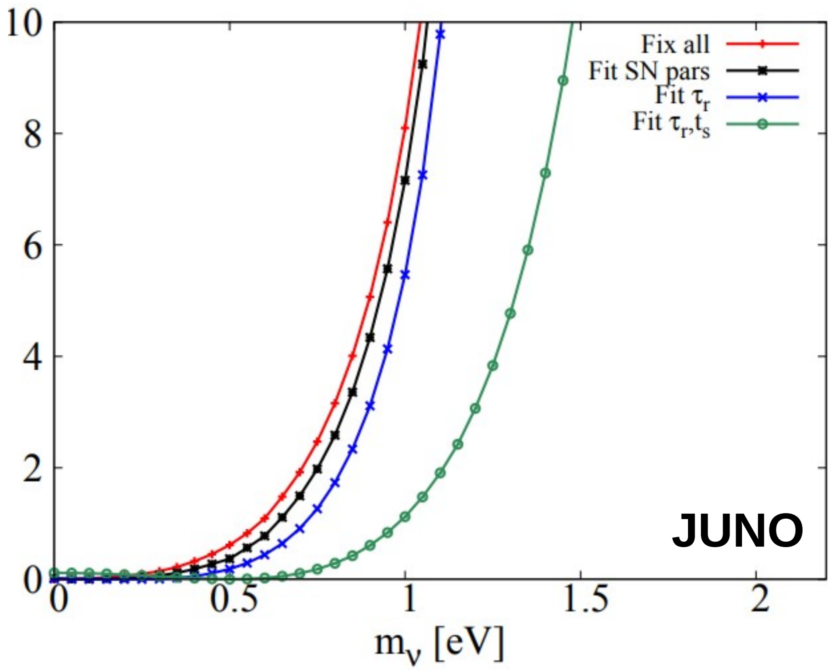
$$\omega_{ij}^{\max} \equiv \frac{E_e - E_g}{2} - \frac{1}{2}\frac{(m_i + m_j)^2}{(E_e - E_g)}$$

SFG & Pasquini
Eur.Phys.J.C 82 (2022) 3, 208;
Phys.Lett.B 841 (2023) 137911;
JHEP 12 (2023) 083

Time Delay in Supernova ν's







DUNE, 2008.06647

SFG, Chui-Fan Kong, Alexei Smirnov,
PRL 133 (2024) 121802 [arXiv:2404.17352]

Lu, Cao, Li & Zhou [JCAP 05 (2015) 044]