

# The QCD-Axion-Nucleon Coupling at Finite Density

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in collaboration with Konstantin Springmann, Stefan Stelzl and Andreas Weiler

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FÜR PHYSIK



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& 3rd Gordon Godfrey Workshop on Astroparticle Physics



# Outline

- Axion Introduction
- Axion-nucleon couplings in Chiral Perturbation Theory
- Density dependence of the axion-nucleon couplings
- Model (in)dependent astrophysical axion bounds

# Why Axions?

## CP violation in the strong sector

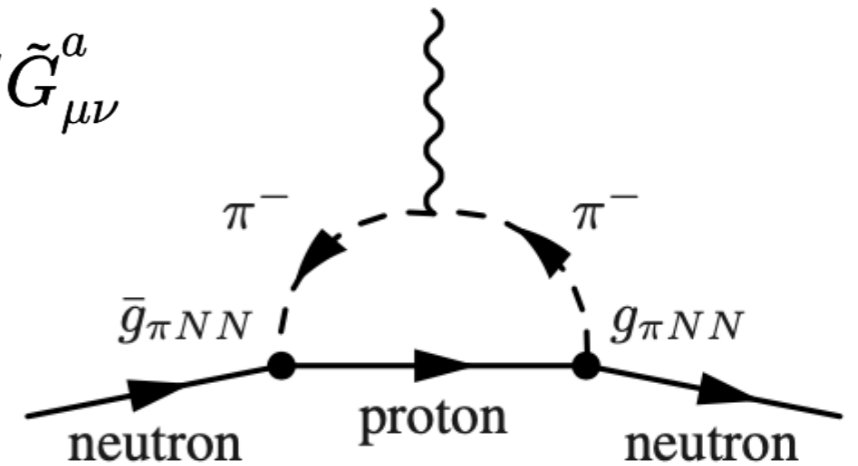
$$\mathcal{L}_{\text{QCD}} = \sum_q \bar{q} (i\not{D} - m_q e^{i\theta_q}) q - \frac{1}{4} G_a^{\mu\nu} G_{\mu\nu}^a - \theta \frac{\alpha_s}{8\pi} G_a^{\mu\nu} \tilde{G}_{\mu\nu}^a$$

## Predicts neutron EDM

$$\mathcal{L}_\chi \supset d_n \bar{n} \sigma^{\mu\nu} \gamma_5 n F_{\mu\nu}$$

$$d_n \approx \frac{e |\bar{\theta}| m_\pi^2}{m_n^3} \approx 10^{-16} |\bar{\theta}| e \text{ cm} \quad \longrightarrow \quad |\bar{\theta}| \lesssim 10^{-10}$$

Why so small???



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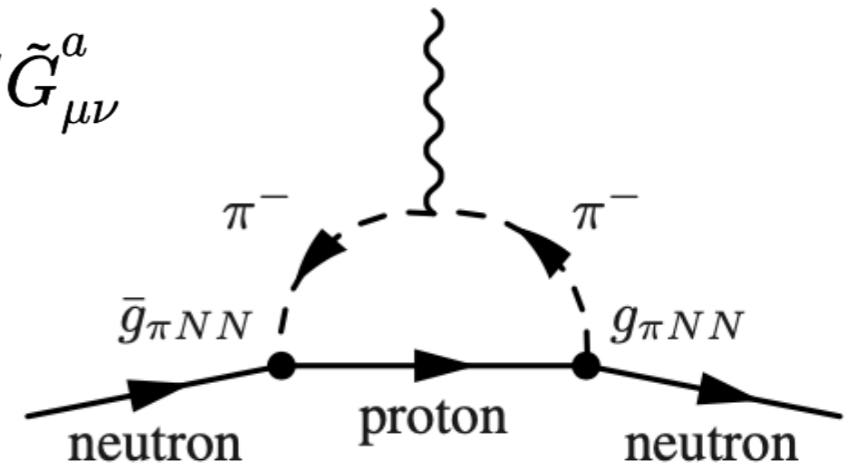
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## QCD Axion explains this by promoting $\theta$ to a dynamical field $a(x)$

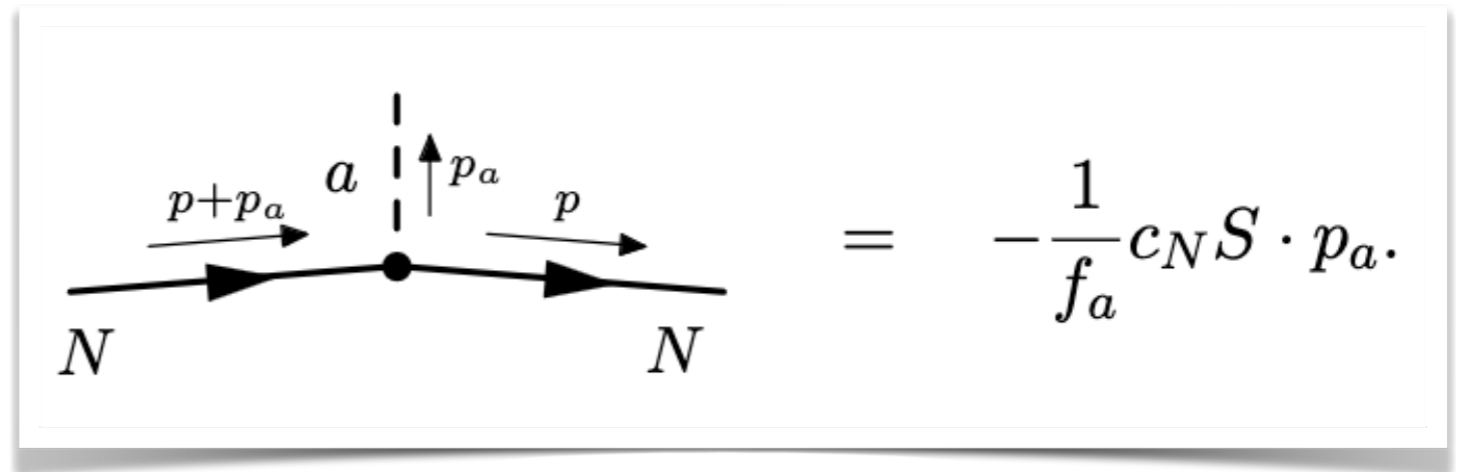
- Axion is very predictive
- Couplings to **nucleons**, photons, electrons, etc. determined by one scale  $f_a$

# Axion-Nucleon coupling

EFT valid for  $p \ll m_\pi$

$$\mathcal{L} \supset \frac{1}{f_a} \bar{N} c_N S \cdot \partial a N$$

$$N = (p, n)^T$$



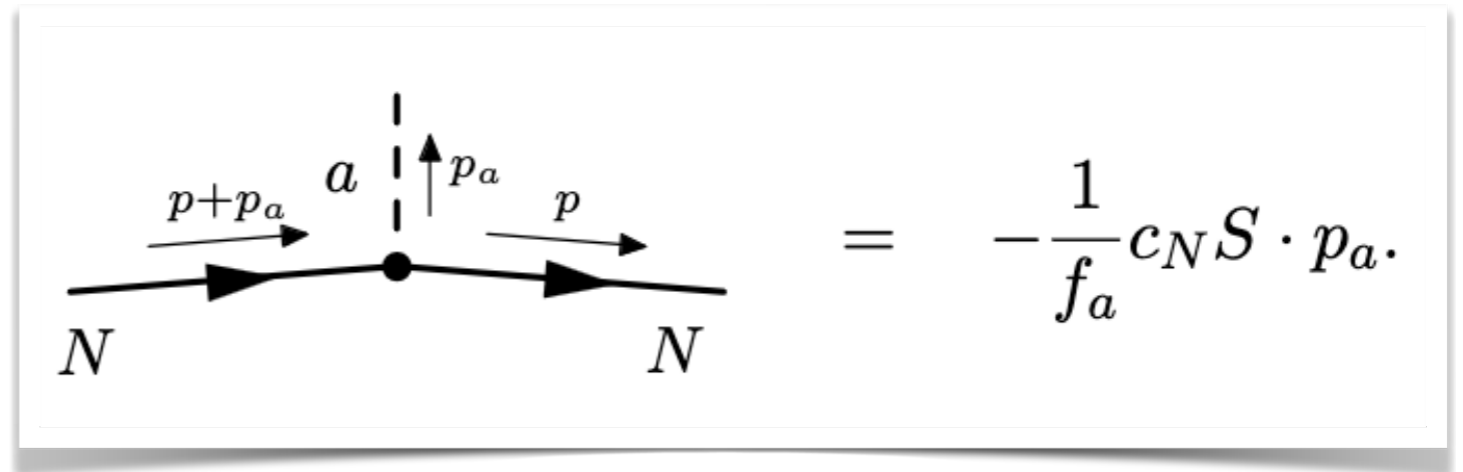
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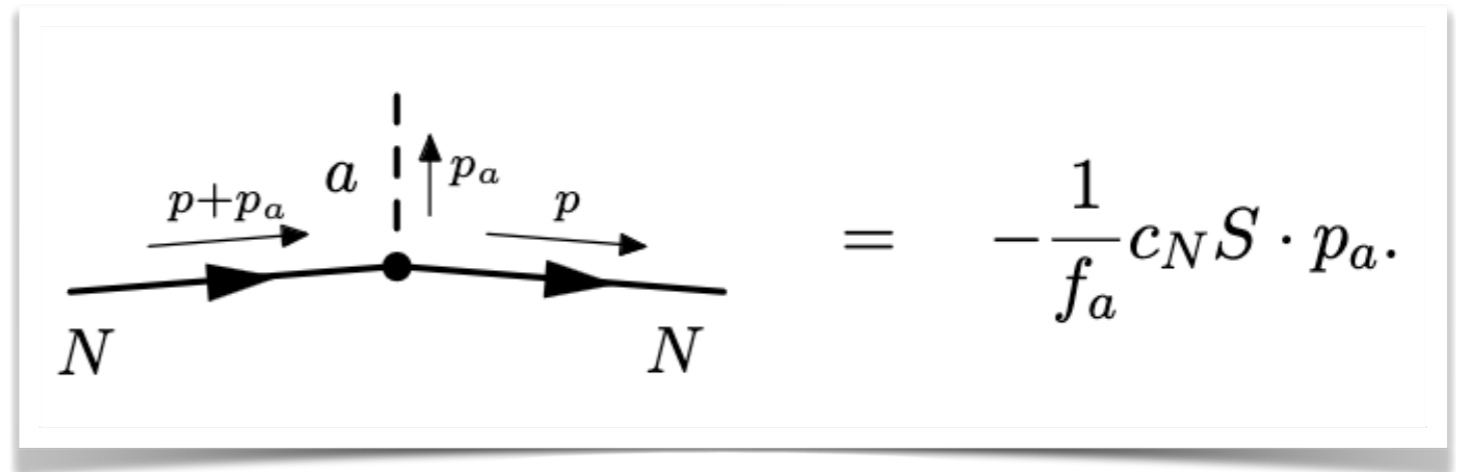
$$N_f = 2 \quad c_N = G_A c_{u-d} \tau^3 + G_0 c_{u+d} \mathbf{1}$$

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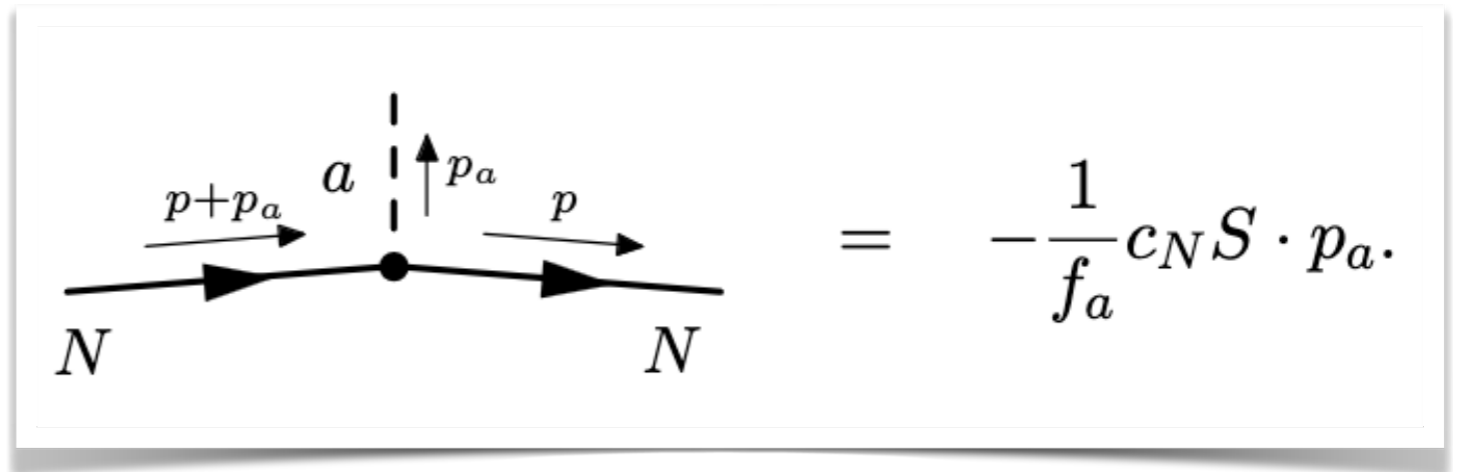
- KSVZ axion  $c_p^{\text{KSVZ}} = -0.47(3), \quad c_n^{\text{KSVZ}} = +0.02(3)$

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$$c_p^{\text{KSVZ}} = -0.47(3), \quad c_n^{\text{KSVZ}} = +0.02(3)$$

Compatible with zero due to accidental cancellation



## Is this EFT valid in astrophysical environments?



This Hubble Space Telescope image shows Supernova 1987A within the Large Magellanic Cloud

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**Not really...**

- Typical momenta  $k_F \simeq (3\pi^2 n_0)^{1/3} \simeq 260 \text{ MeV}$        $n_0 \simeq 0.16 \text{ fm}^{-3}$

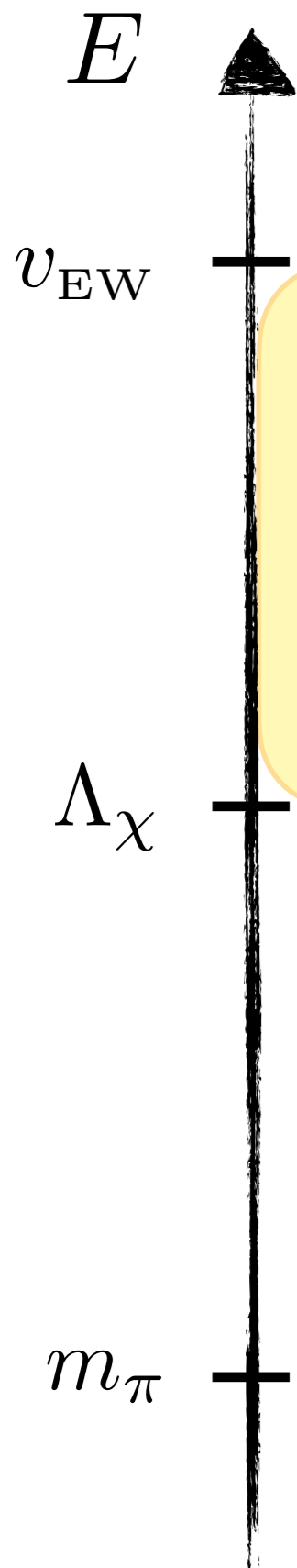
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**Need to construct EFT of pions and nucleons**

# Axion EFTs



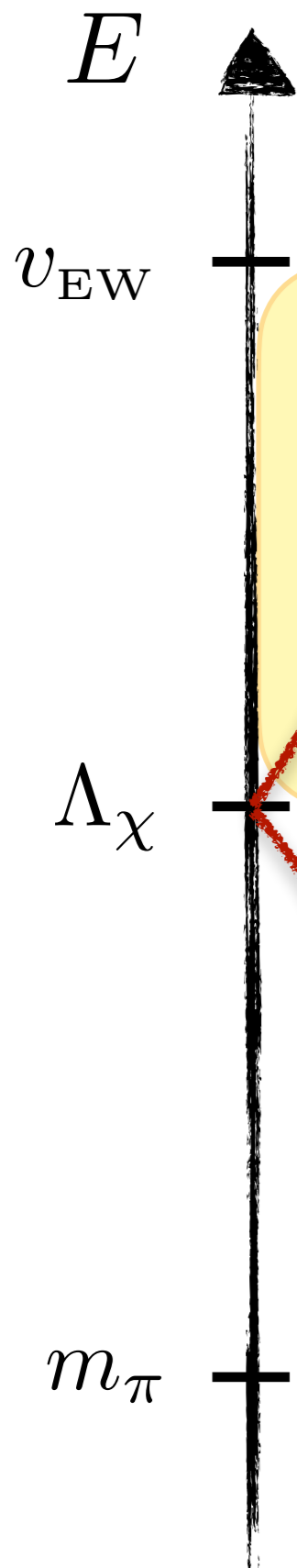
After chiral quark rotation

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{QCD},0} - (\bar{q}_L M_a q_R + \text{h.c.}) + \frac{1}{2} (\partial a)^2 + \frac{\partial_\mu a}{2f_a} J_{\text{PQ}}^\mu$$

$$M_a \equiv e^{\frac{ia(x)}{2f_a}} Q_a M_q e^{\frac{ia(x)}{2f_a}} Q_a \quad Q_a \equiv M_q^{-1} / \text{Tr}(M_q^{-1})$$

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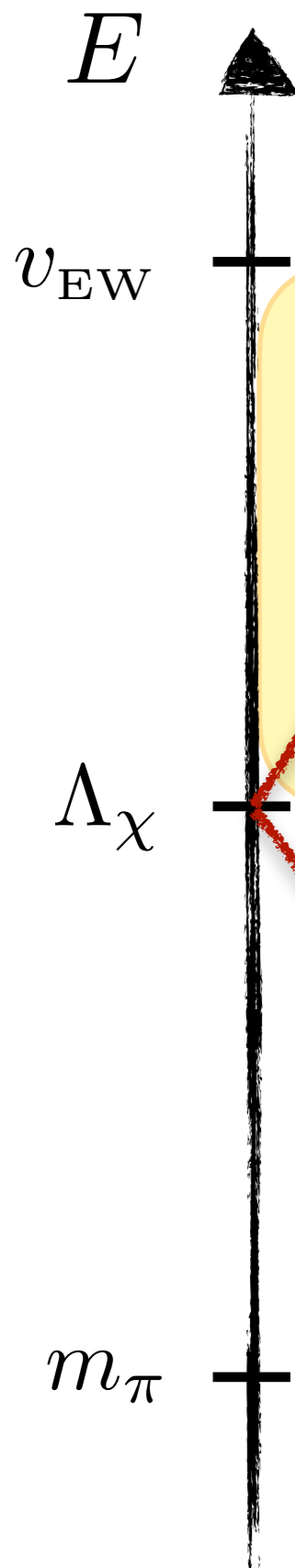


**QCD confines:**  $|\langle \bar{q}_L q_R \rangle| \equiv B f_\pi^2$

$$G_\chi = U(2)_L \times U(2)_R \rightarrow U(2)$$

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**Chiral Perturbation Theory:** spurion analysis

# Axion EFTs



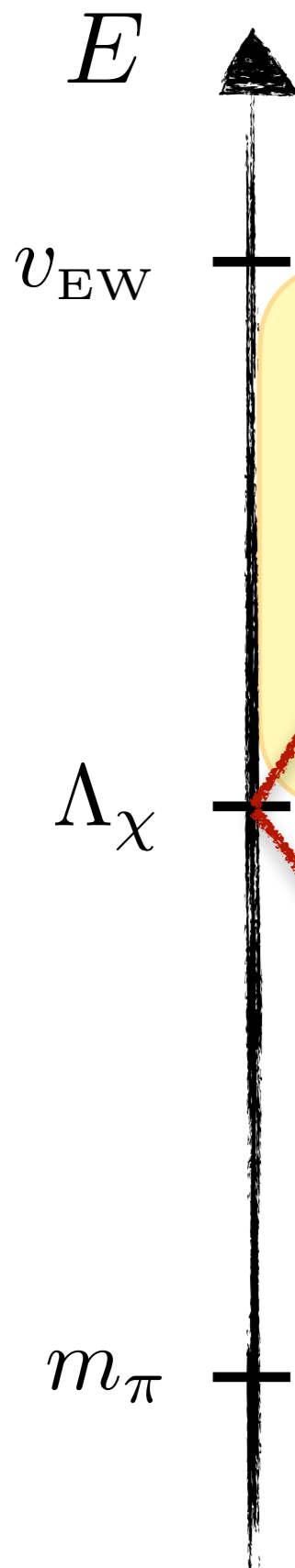
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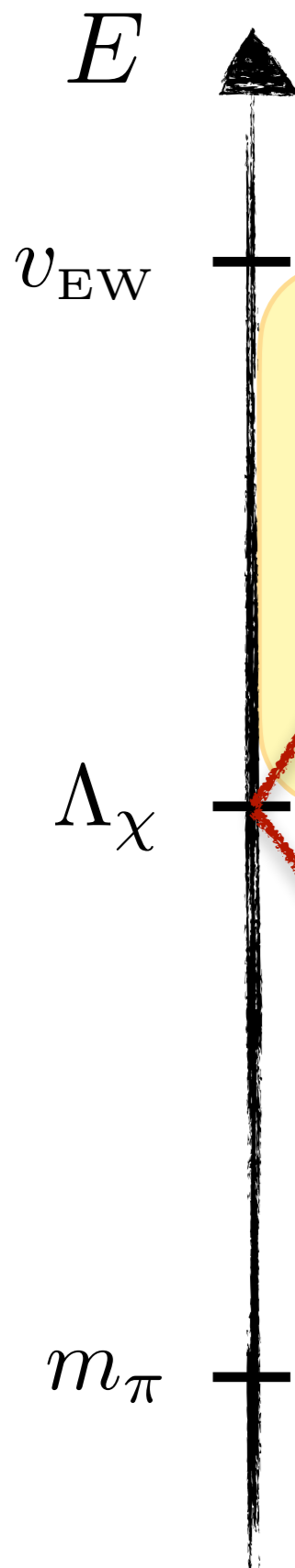
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1 GeV

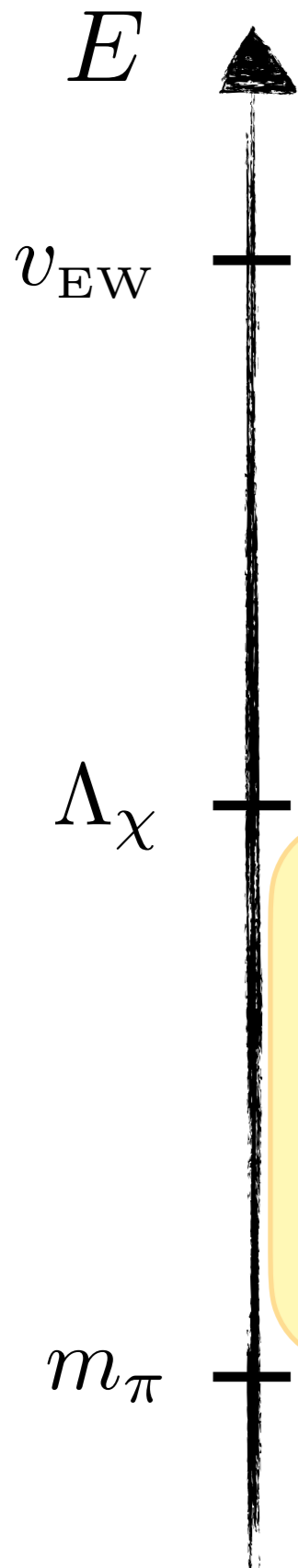
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Resonances, e.g.

$$\Lambda_\chi \sim (300 - 750) \text{ MeV}$$



# Axion EFTs



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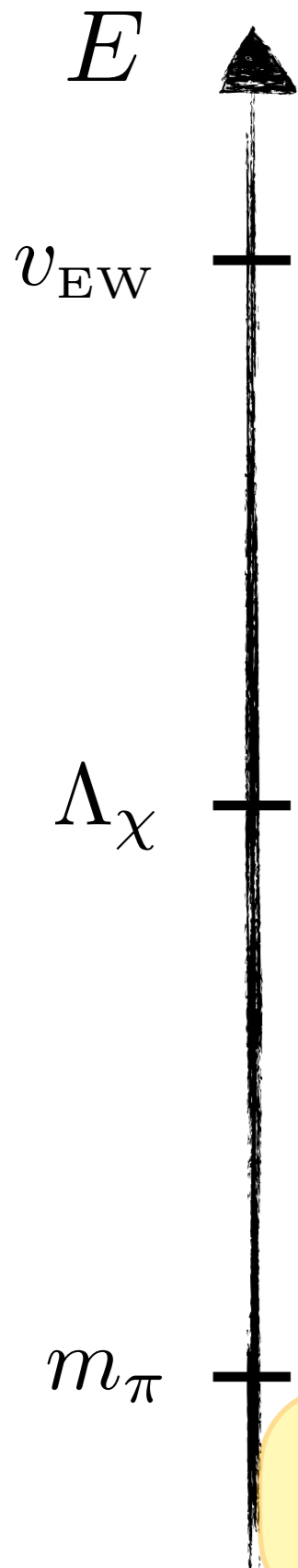
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**LO:**  $\hat{\mathcal{L}}_{\pi N}^{(1)} = \bar{N} (i v \cdot D + g_A S \cdot u + g_0 S \cdot \hat{u}) N$

$$\hat{u}_\mu = c_{u+d} \left( \frac{\partial_\mu a}{f_a} \right) + \dots \quad u_\mu = - \left( \frac{\partial_\mu \pi^a}{f_\pi} \right) \tau^a + c_{u-d} \left( \frac{\partial_\mu a}{f_a} \right) \tau_3$$

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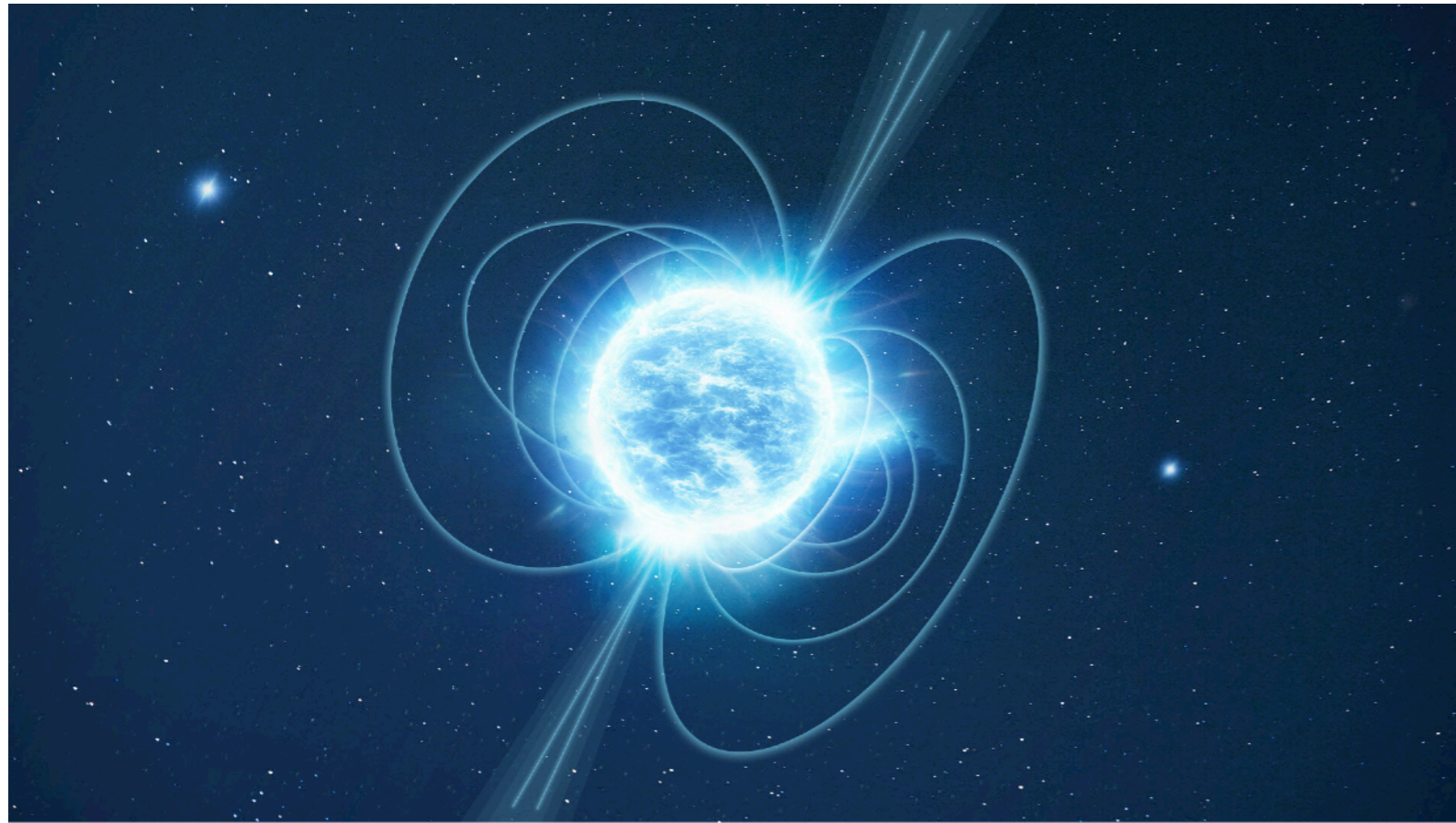
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**Integrate out pions:** theory of baryons and axion

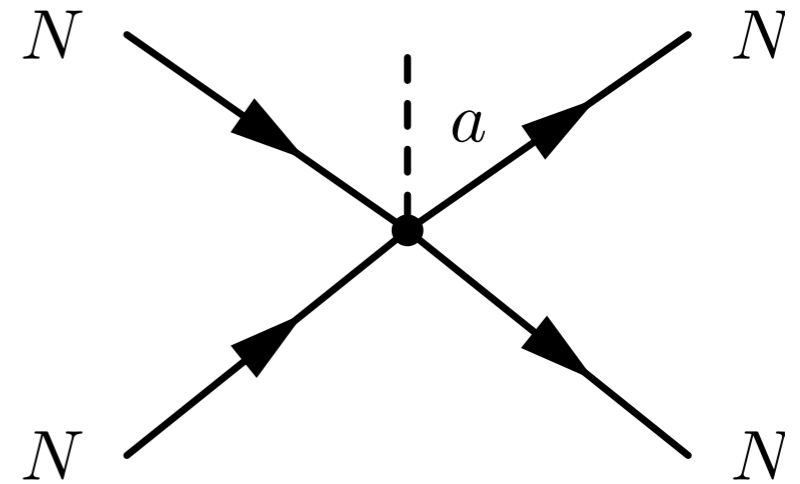
**How does a density background change these couplings?**



# Axion-Nuclon Coupling: Finite density

- Schematic example:

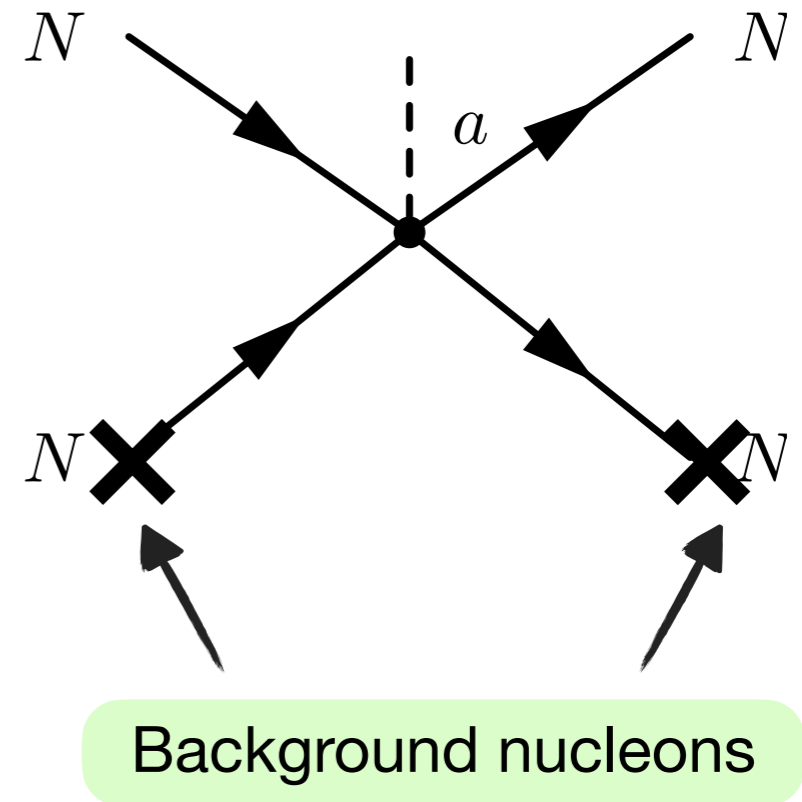
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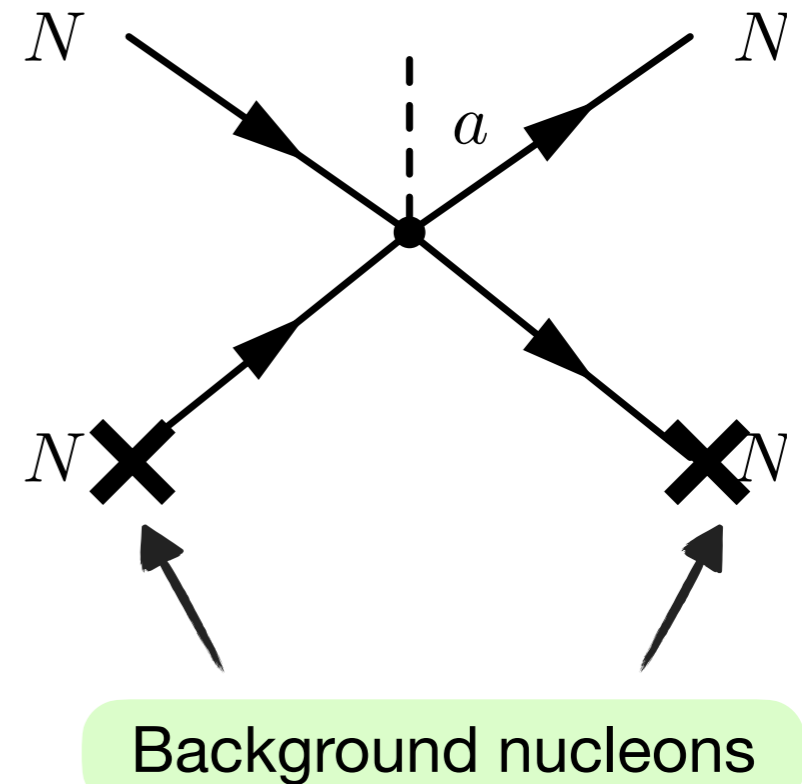
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Number density



- Gives contribution to coupling:  $\sim \frac{k_f^3}{(4\pi f_\pi)^2 \Lambda_\chi}$

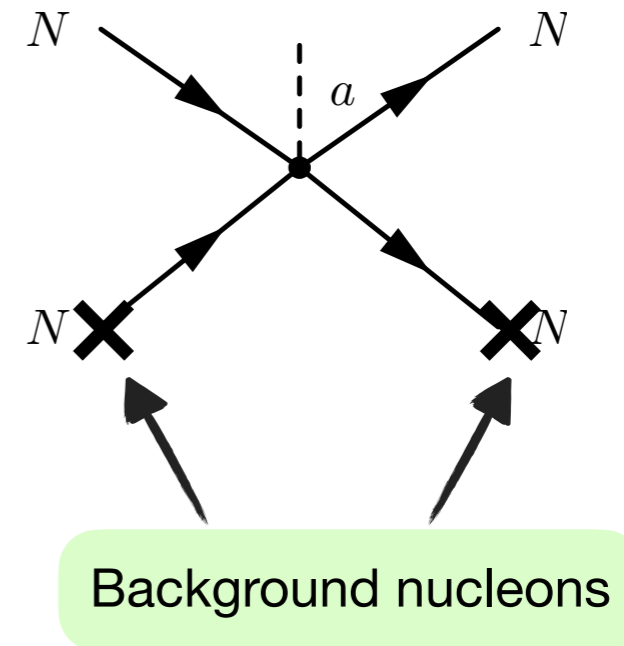
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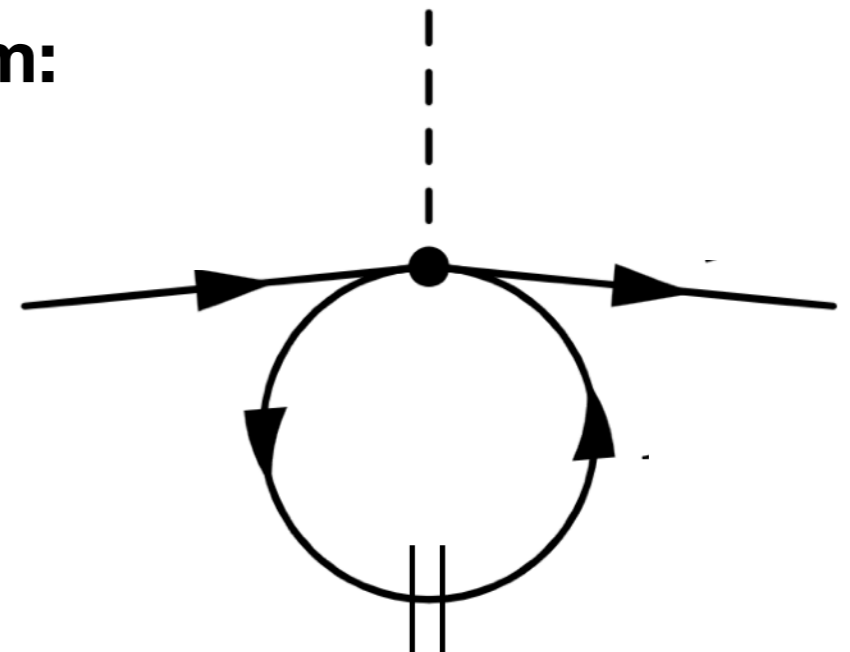
Number density



- **Systematically via QFT in Real-Time Formalism:**

Nucleon propagator at finite density

$$iG(k) = \frac{i}{k^0 + i\epsilon} - 2\pi\delta(k^0)\theta(k_f - |\vec{k}|)$$



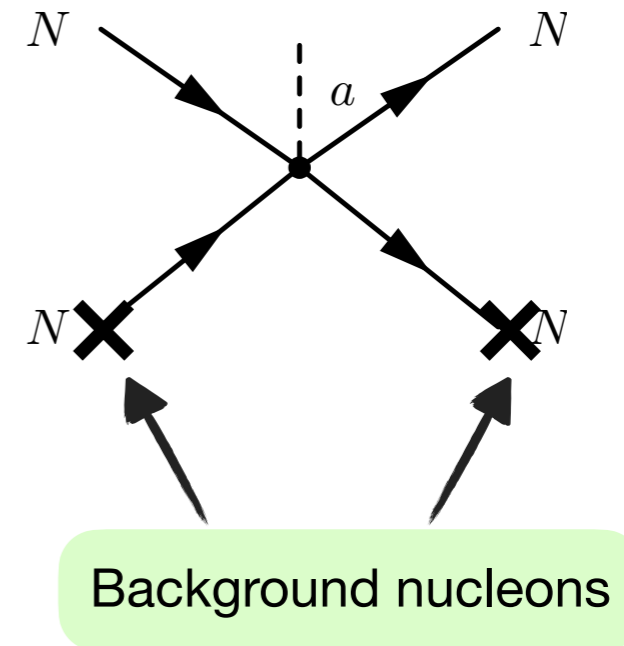
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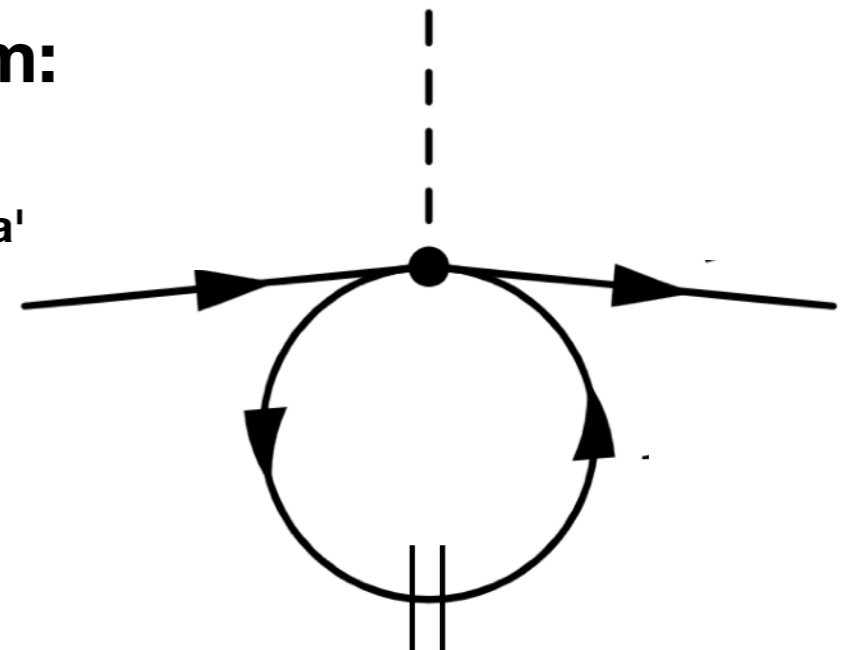
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Filled 'Fermi sea'

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NR fermion propagator

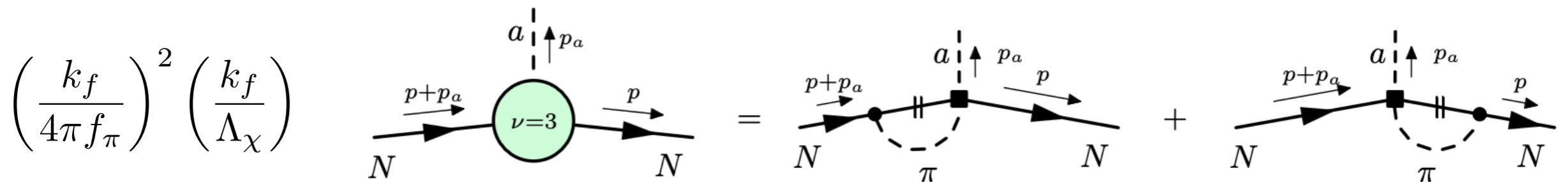
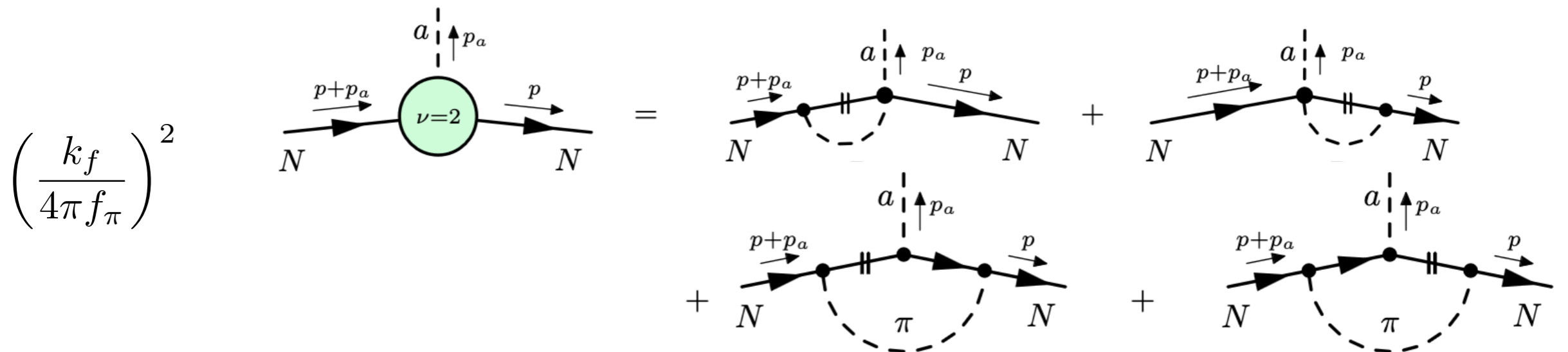




# Axion-Nuclon Coupling: Finite density

Get corrections systematically

$$\left( \frac{p}{4\pi f_\pi} \right)^\nu \rightarrow \left( \frac{k_f}{4\pi f_\pi} \right)^\nu$$



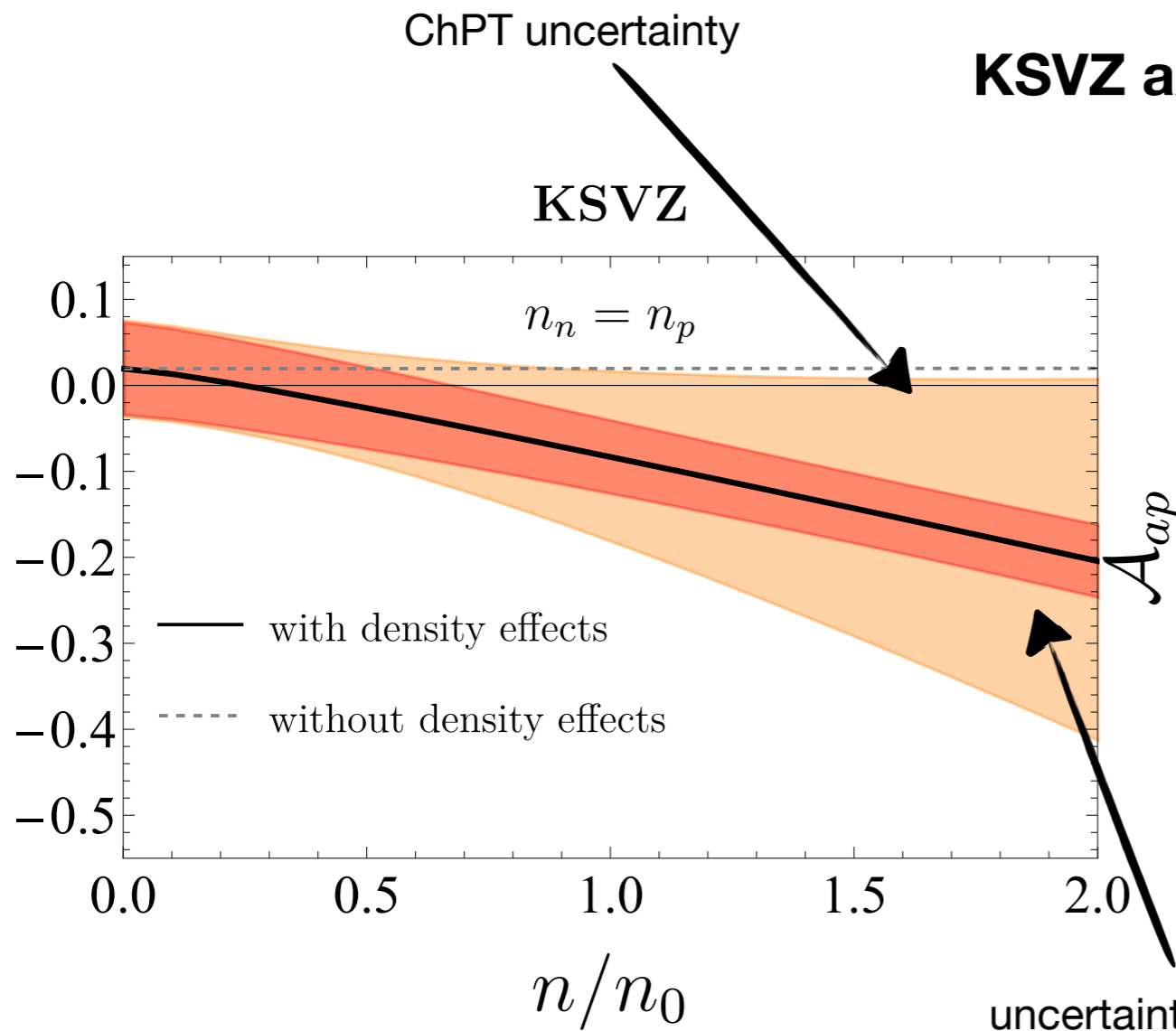
# Results

Simplifying assumption:

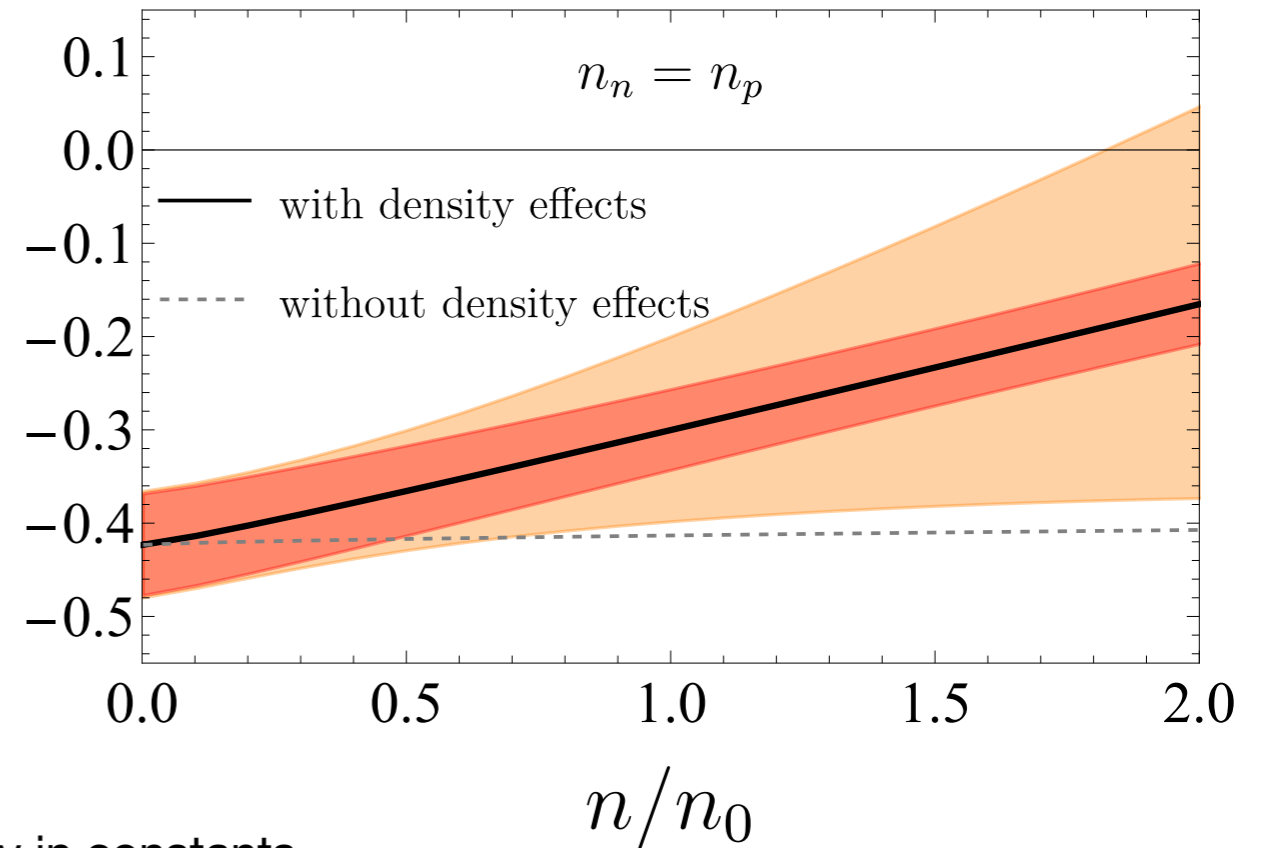
$$p \sim k_f$$

## KSVZ axion

### KSVZ



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At nuclear density  $\mathcal{A}_{ap}^{\text{KSVZ}}(n_0) = -0.299(43)(98)$

$\mathcal{A}_{an}^{\text{KSVZ}}(n_0) = -0.083(43)(98)$

vs. vacuum  $\mathcal{A}_{ap}^{\text{KSVZ}} = -0.42(5)$

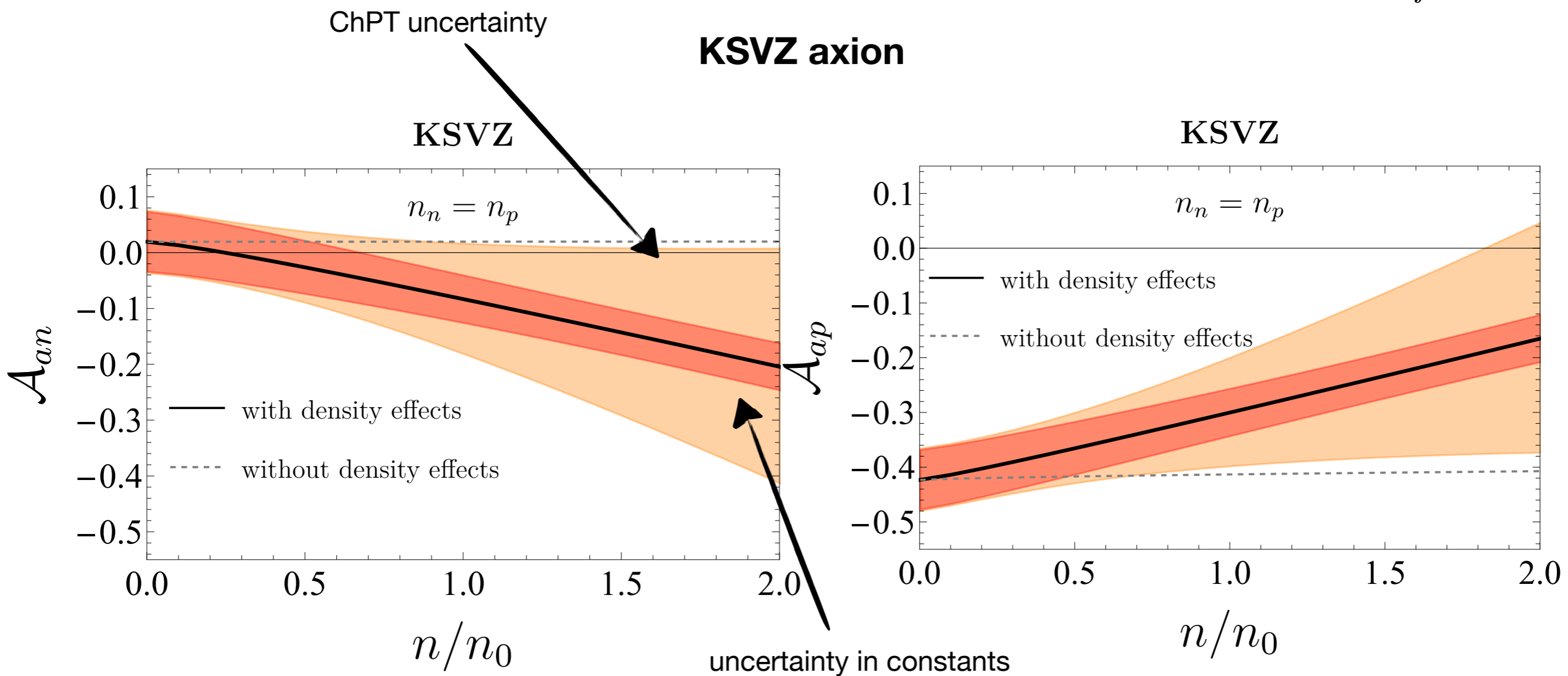
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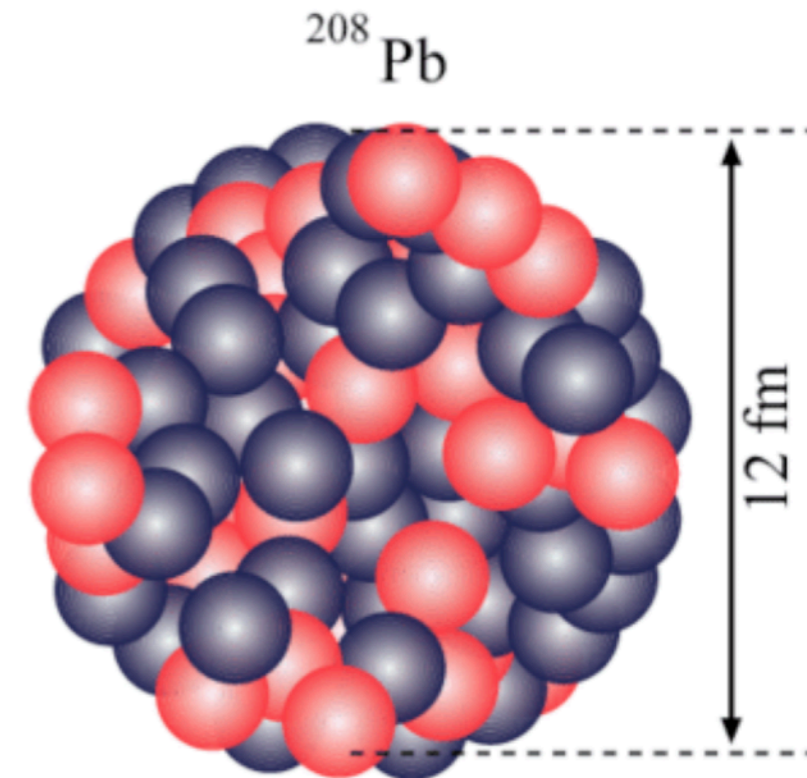
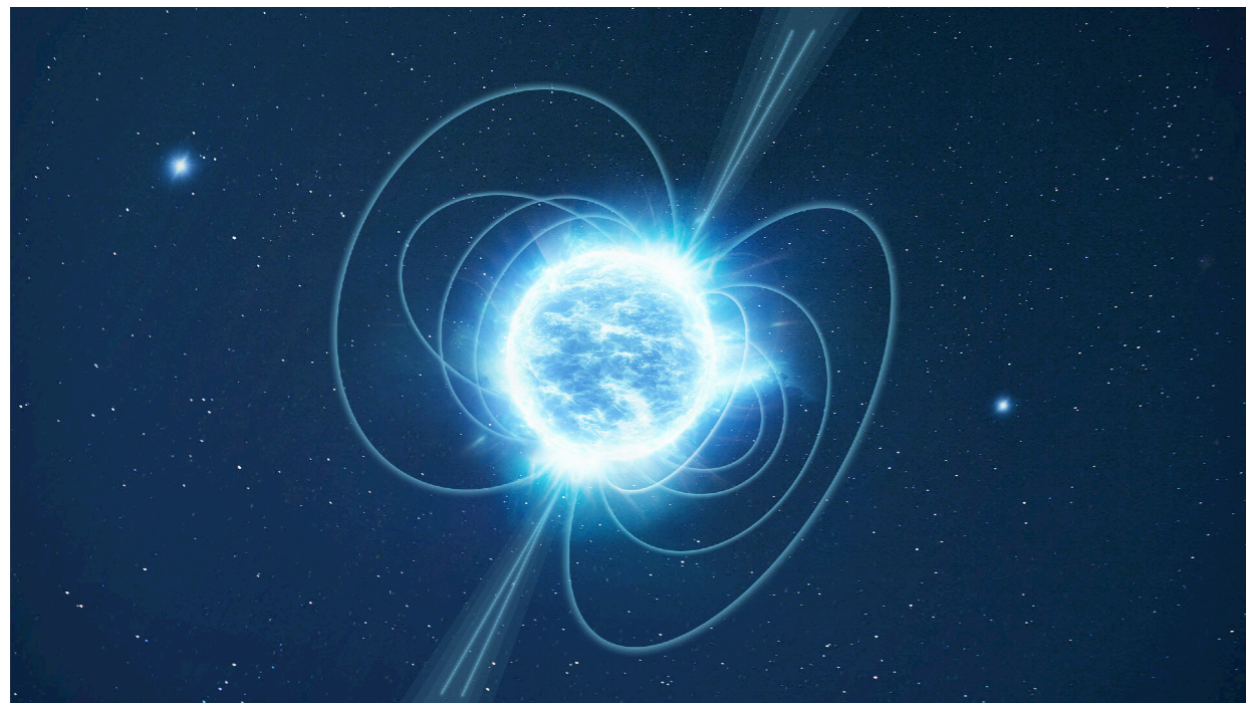
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**Accidental cancellation is lifted!**

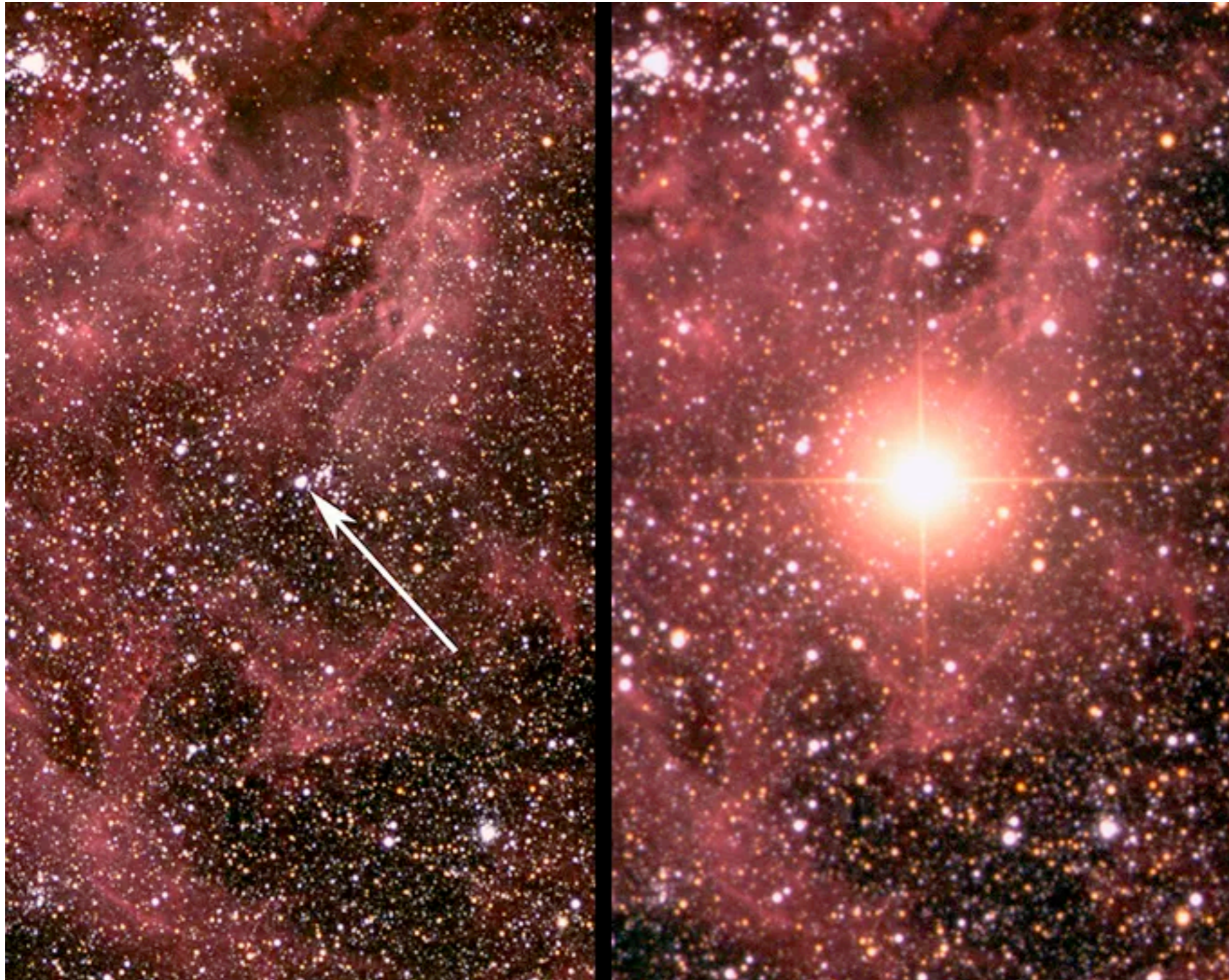


# Implications for phenomenology



# Bound from SN 1987A

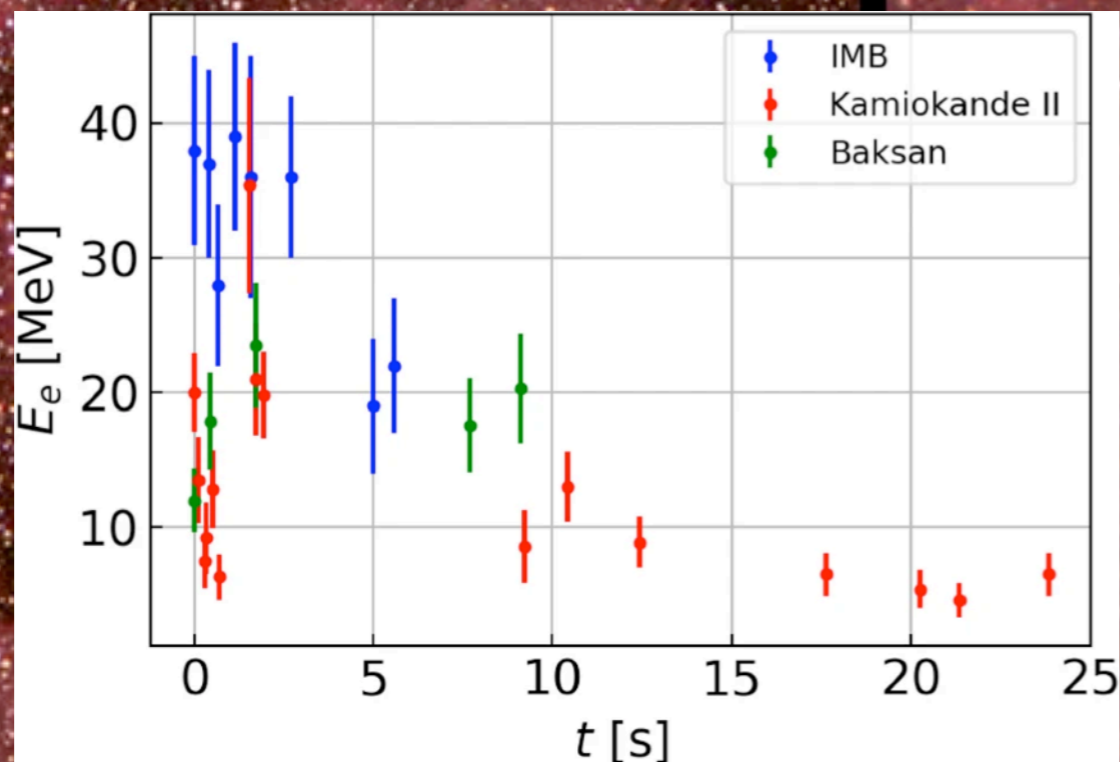
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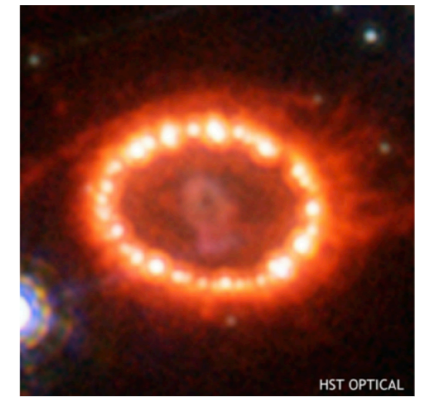
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~ 20 neutrinos within ~ 10 sec

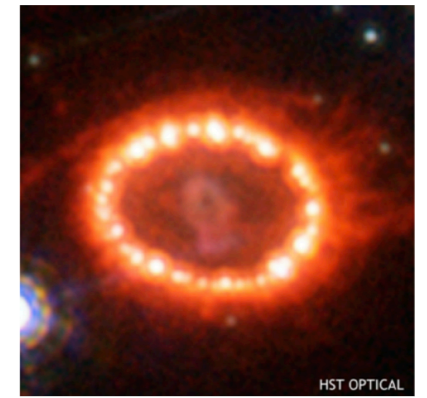


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*(today we would observe significantly more if a SN happens close to us)*
- By energy loss arguments additional new particles emitted by the SN would alter the signal duration (only neutrinos have a large enough mean free path to escape the SN)

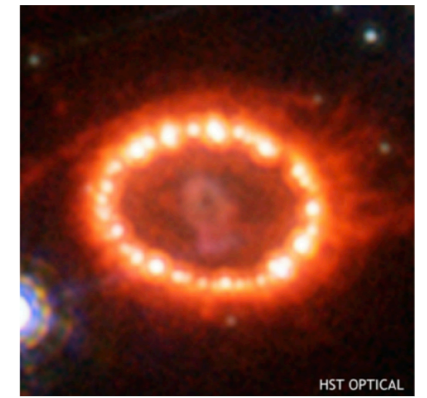
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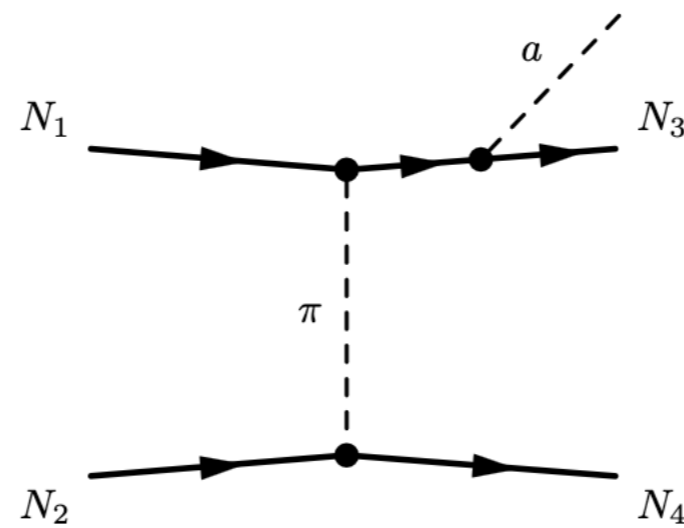


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- For a QCD axion this constrains  $m_a$  and  $f_a$

# Implications for supernova bound

## Calculation of axion emissivity of supernova

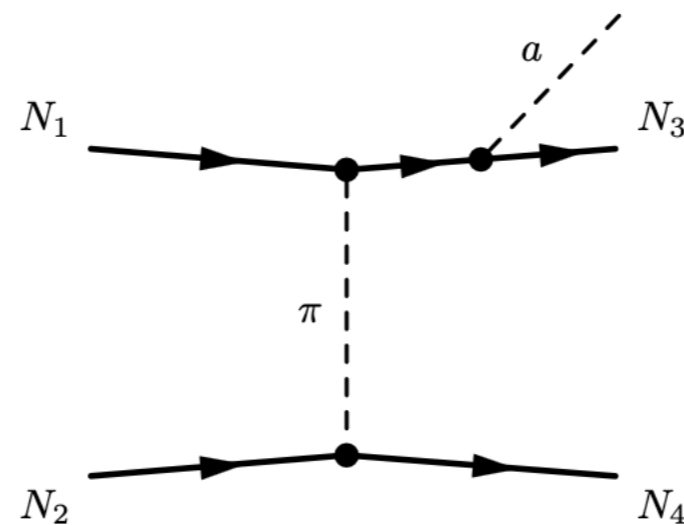
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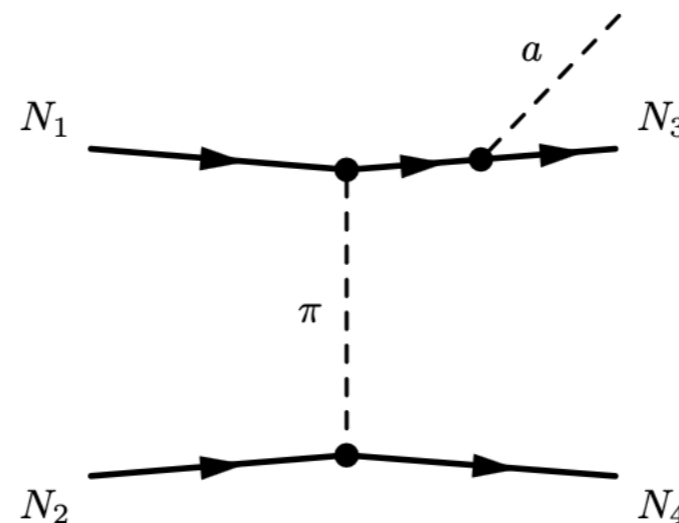


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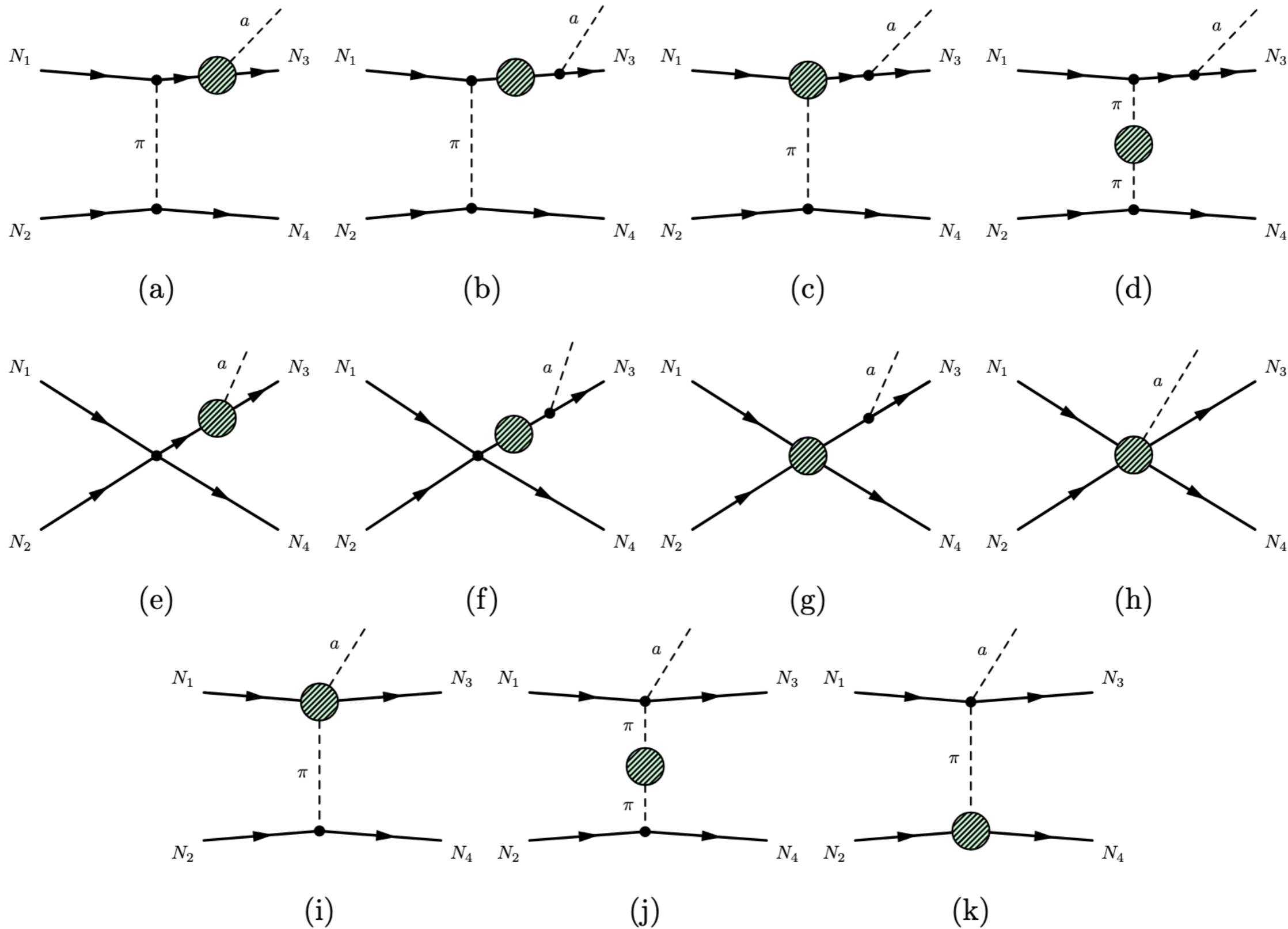


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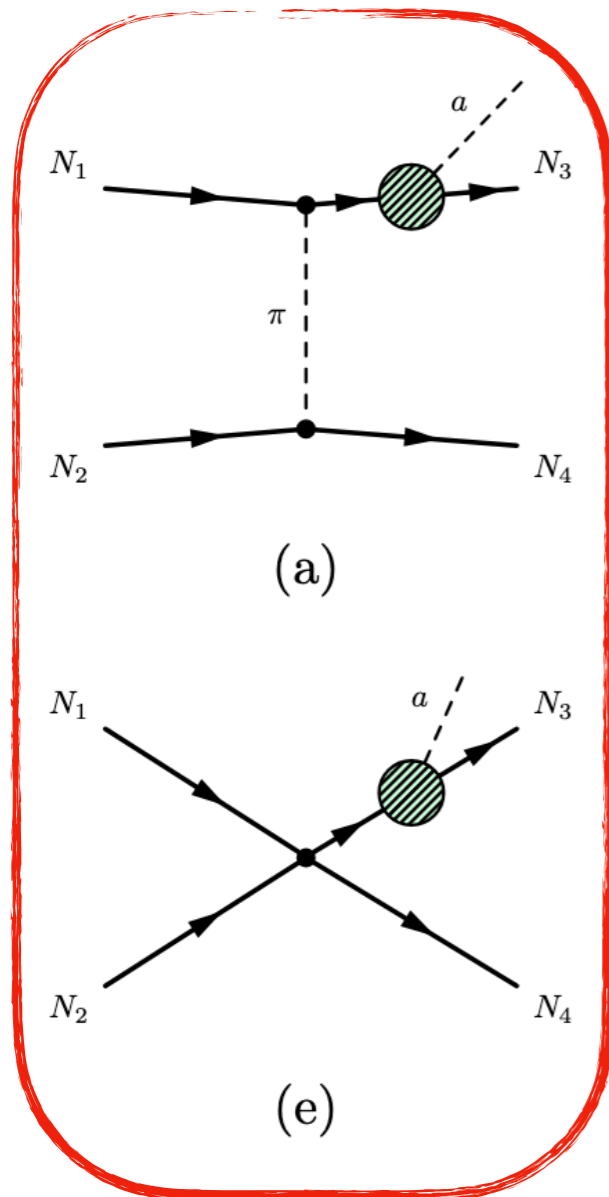
- Multiply rate by fudge factors:  $\Gamma_a = \Gamma_a^{\text{tree}} \gamma_f \gamma_p \gamma_h$  Chang, Essig, McDermott ('18)

# Supernova bound revisited

## Relevant diagrams up to NLO



# Supernova bound revisited



Outlined for the first time

Springmann, MS, Stelzl, Weiler ('24)

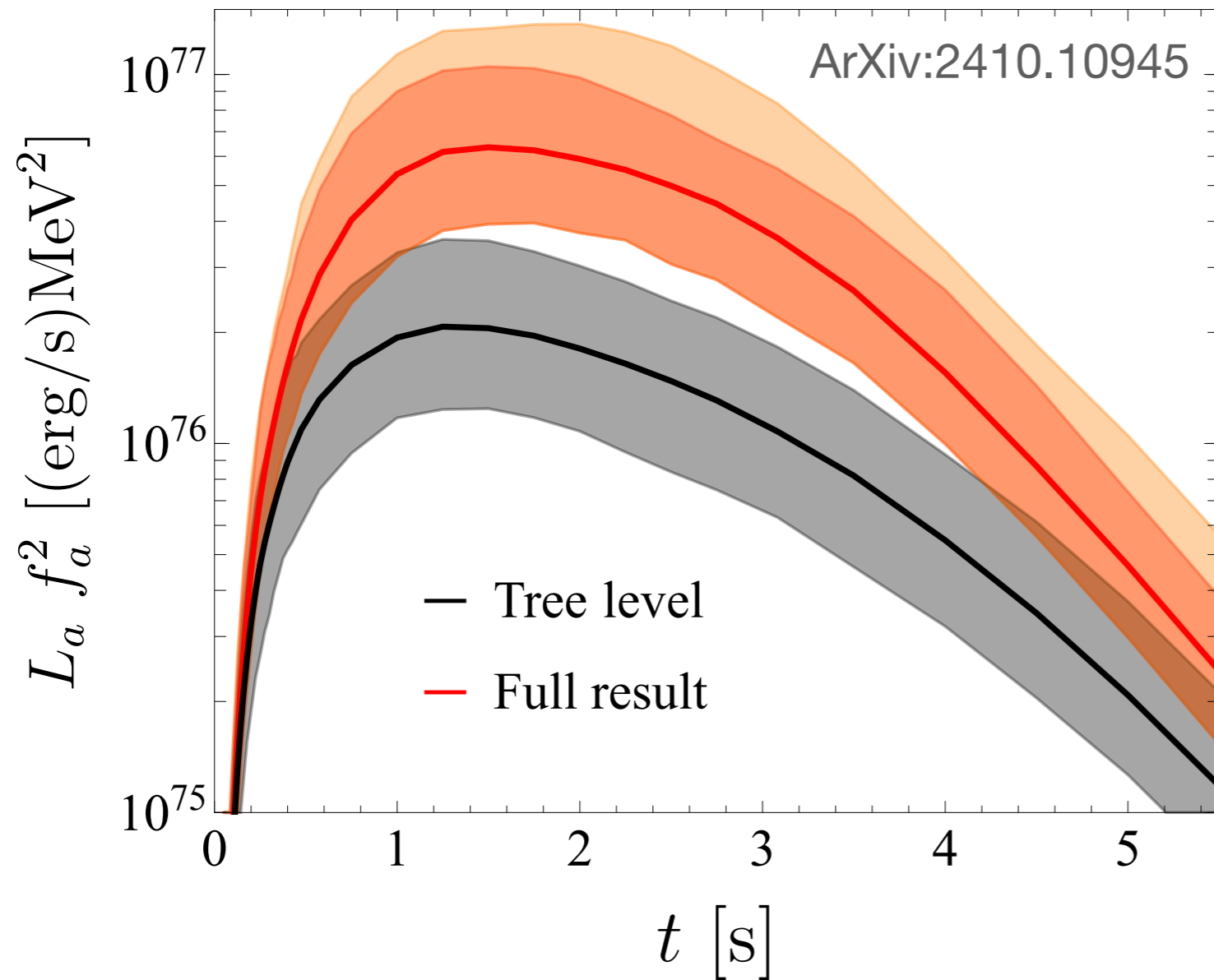
← Modified couplings

Focus on these for now

Fully systematic evaluation should take into account all diagrams up to given order

# Implications for supernova bound

example: KSVZ axion



$$L_a = \int dr 4\pi r^2 \dot{\epsilon}_a(r)$$

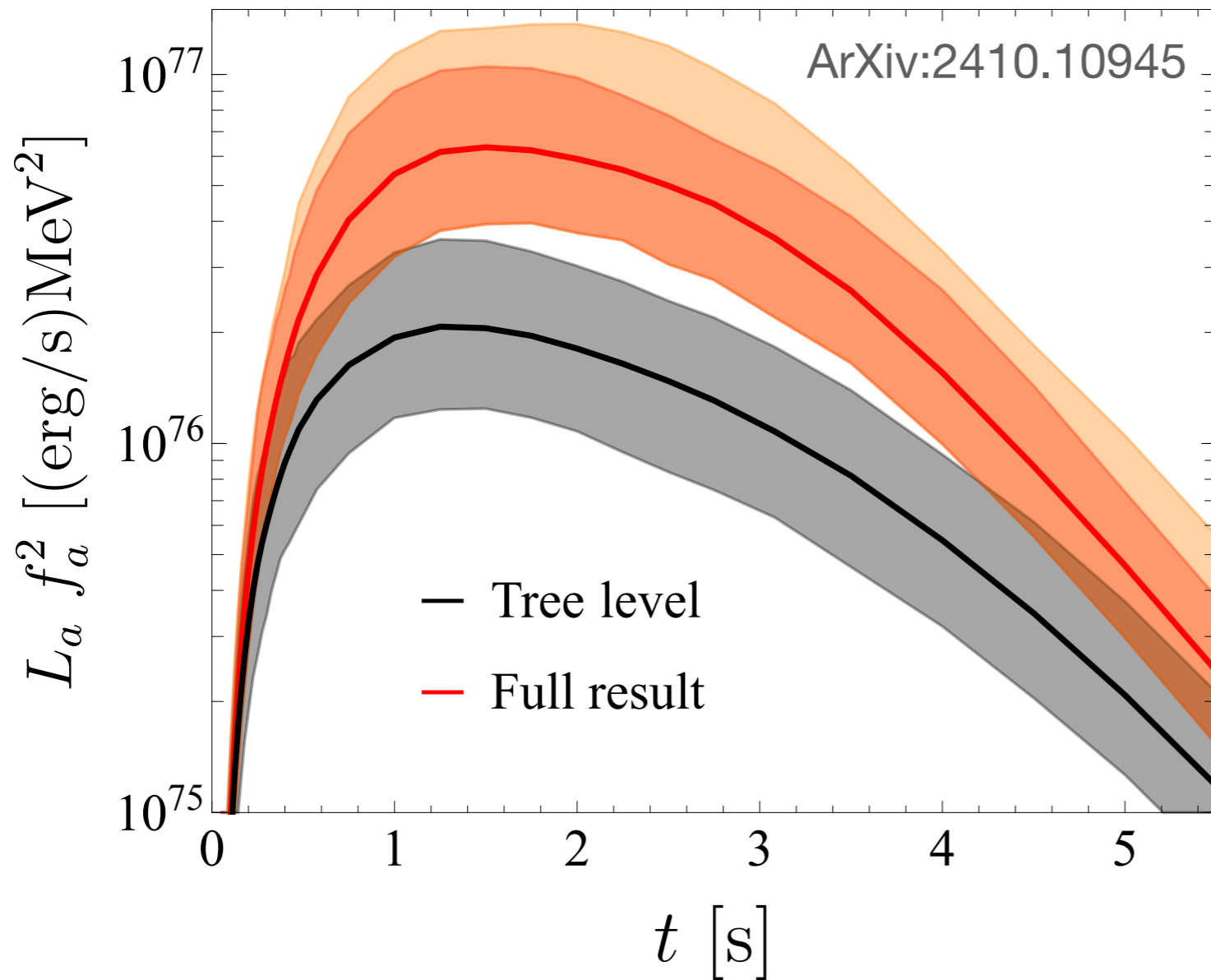
Supernova profile from <https://www.mpa.mpa-garching.mpg.de/ccsnarchive/>

**SN bound on axion  
mass O(few) stronger**

**Adds theory uncertainty!**

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Tree level:

$$f_a \gtrsim 6.1_{-1.4}^{+1.7} \times 10^8 \text{ GeV}, \quad m_a \lesssim 9.8_{-2.2}^{+3.0} \text{ meV}.$$

Vertex corrections:

$$f_a \gtrsim 1.0_{-0.2}^{+0.5} \times 10^9 \text{ GeV}, \quad m_a \lesssim 5.9_{-2.0}^{+1.8} \text{ meV}.$$



# Model dependence - QCD axion-nucleon coupling

Requiring that the axion solve the strong CP problem, i.e.

$$\mathcal{L}_{\text{QCD}} \supset \frac{a}{f_a} \frac{g^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

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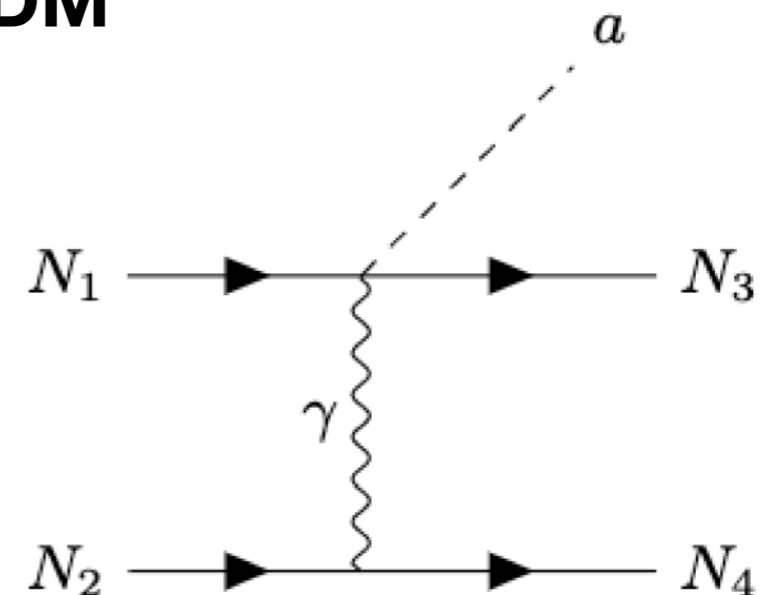
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**Bound on axion from EDM operator in SN:**

Lucente, Mastrototaro, Carenza, DiLuzio, Giannotti, Mirizzi



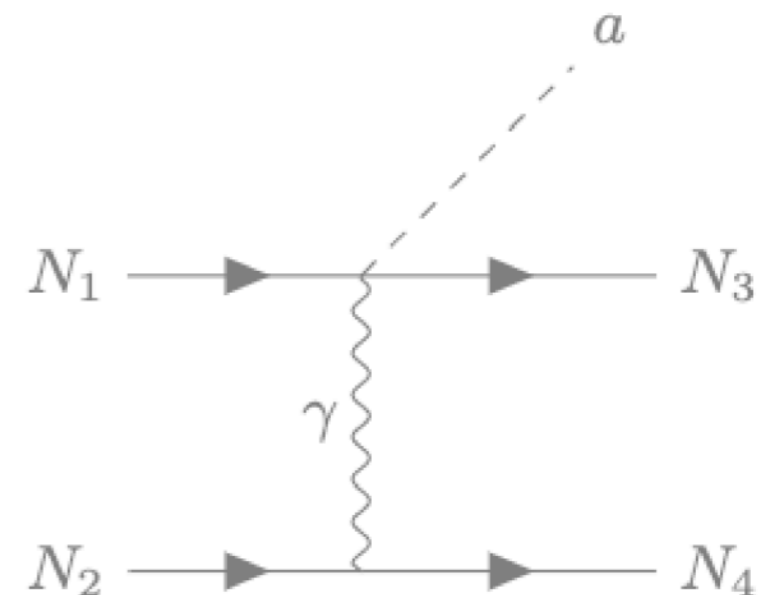
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**We found that this is not the dominant process**

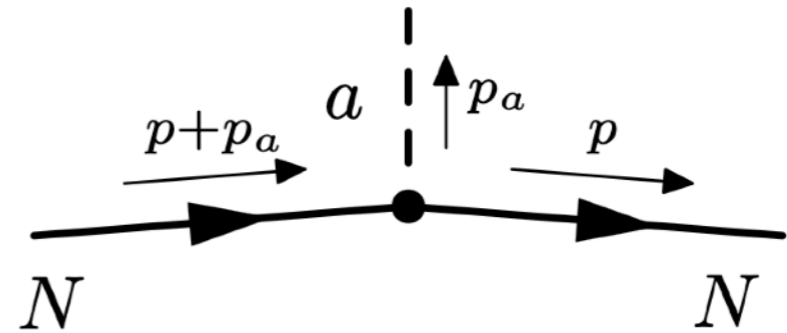


# Model dependence - QCD axion-nucleon coupling

LO:

$$c_p = g_0 c_{u+d} + g_A c_{u-d}$$

$$c_n = g_0 c_{u+d} - g_A c_{u-d}$$



$$c_{u\pm d} = (c_u \pm c_d)/2$$

$$c_q \equiv c_q^0 - [Q_a]_q$$

$$Q_a = \frac{\text{Diag}[1, z]}{1+z}, \quad z \equiv \frac{m_u}{m_d}$$

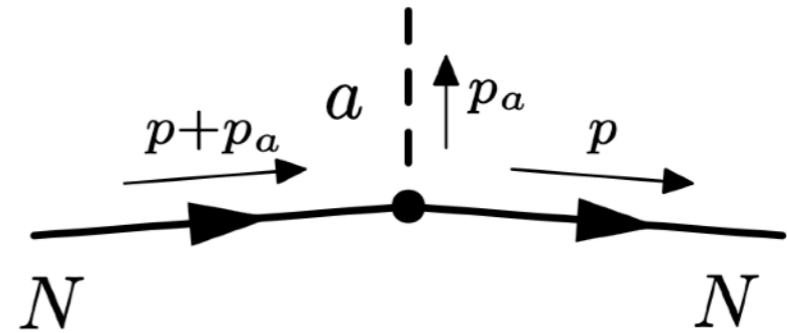
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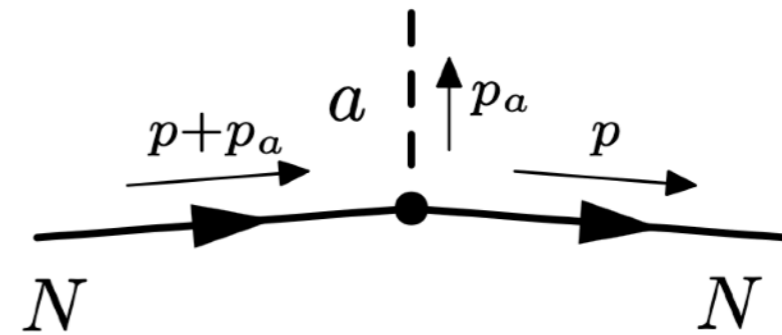
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DiLuzio, Mescia, Nardi, Panci, Ziegler ('17)

Badziak, Harigaya ('23)

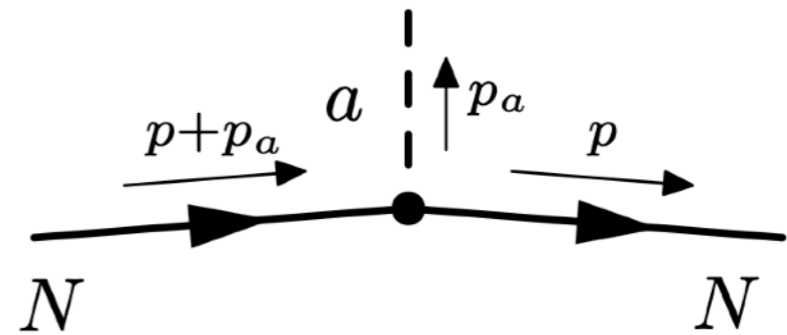
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**What is left at NLO?**



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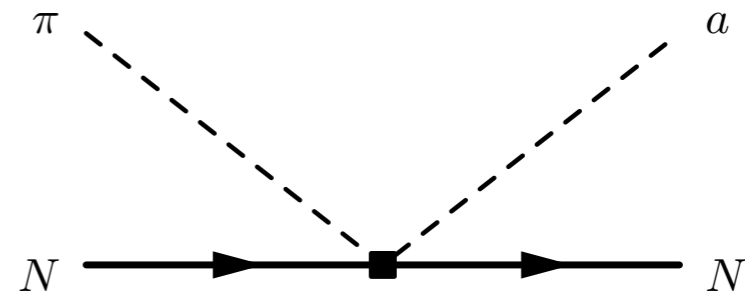
**Which interactions does this give rise to?**

# Model dependence - QCD axion-nucleon coupling

Isospin breaking term survives independent of the type of axion model

$$\mathcal{L}_{\pi N}^{(2)} \supset \hat{c}_5 \bar{N} \tilde{\chi}_+ N$$

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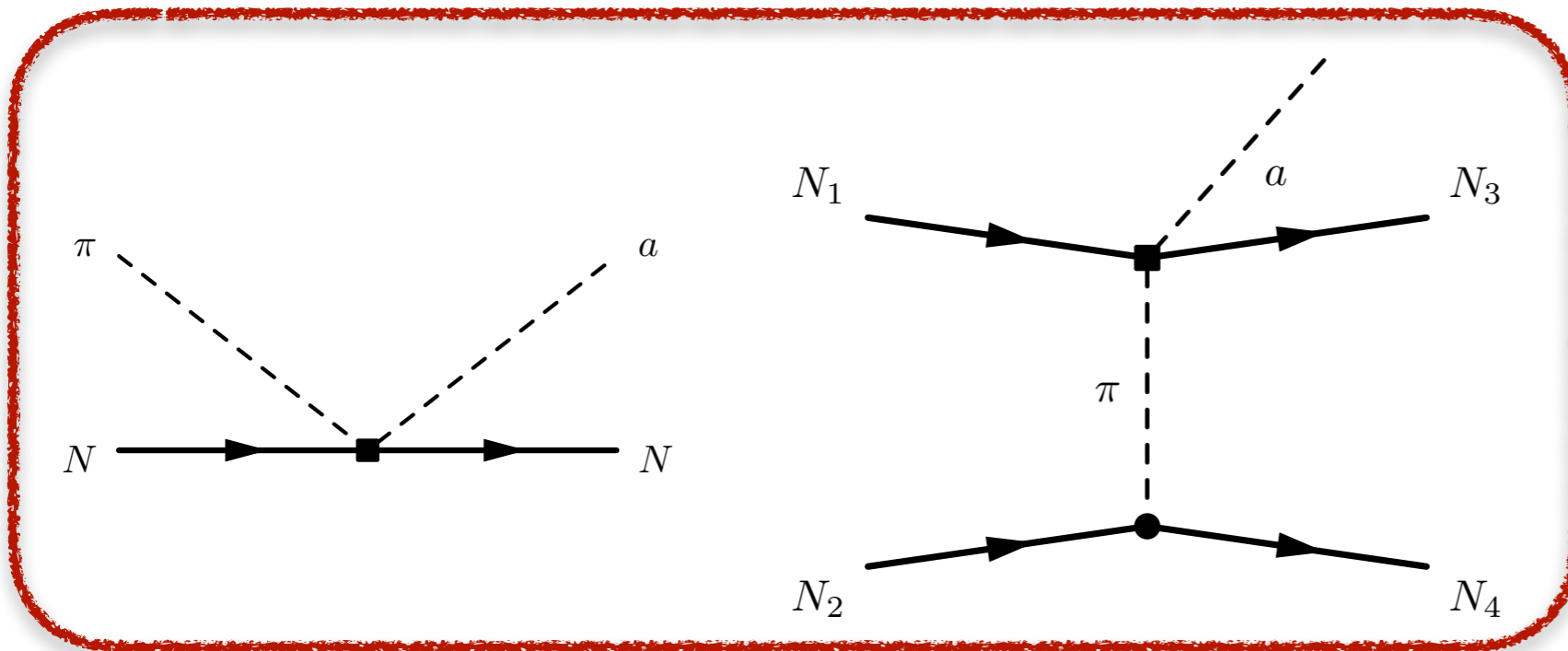
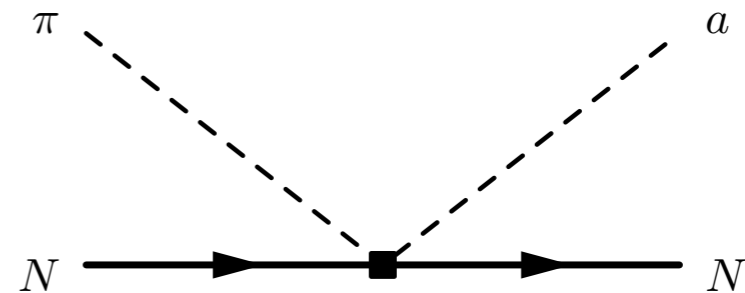


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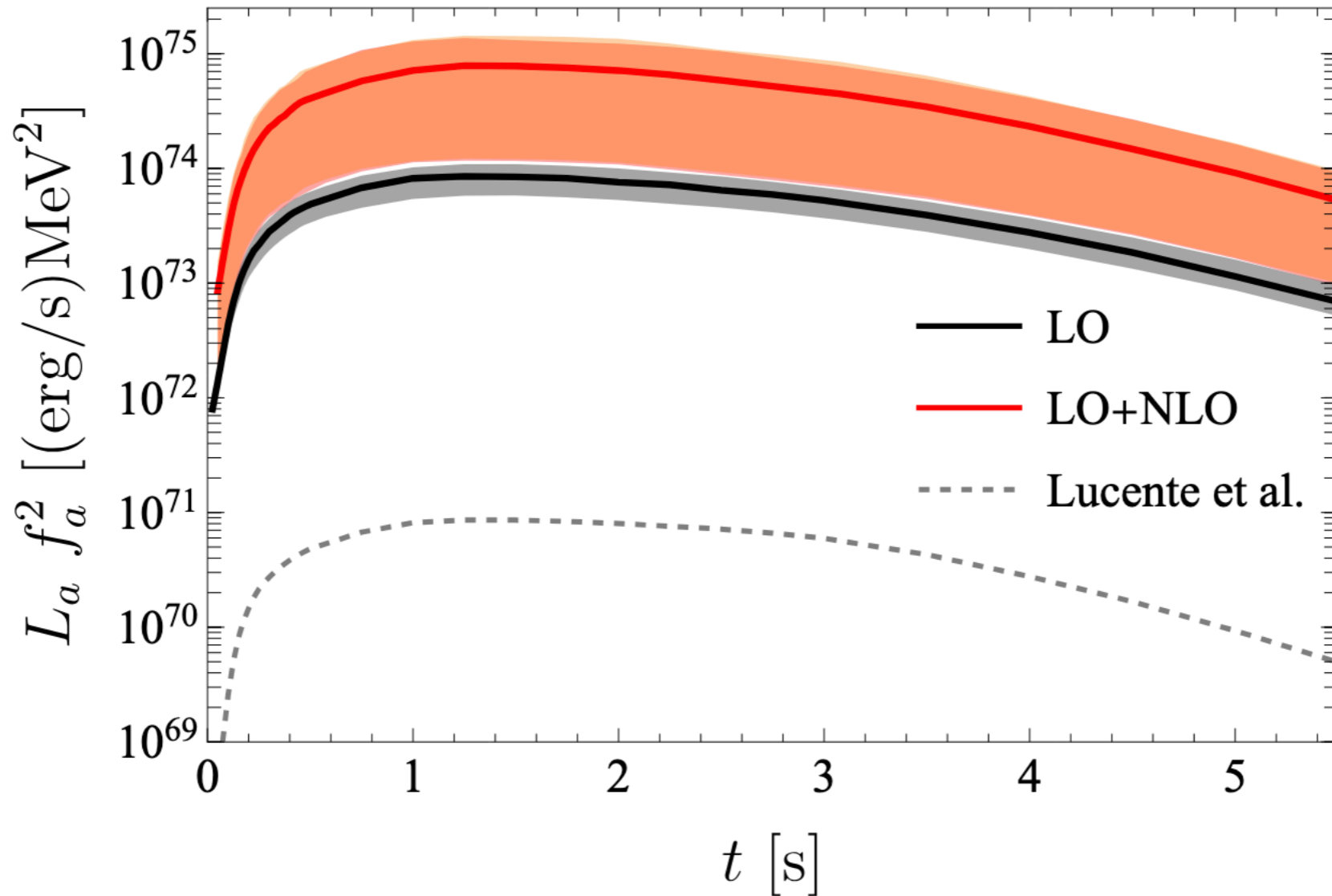
**Dominant  
production channels**

These diagrams dominate for the model independent SN bound

# Astrophobic axions

- Loose the loop-suppression compared to EDM operator

$$L_a^{\text{tree}, \hat{c}_5} \simeq (4\pi)^4 L_a^{\text{EDM}} \simeq 10^4 L_a^{\text{EDM}}$$



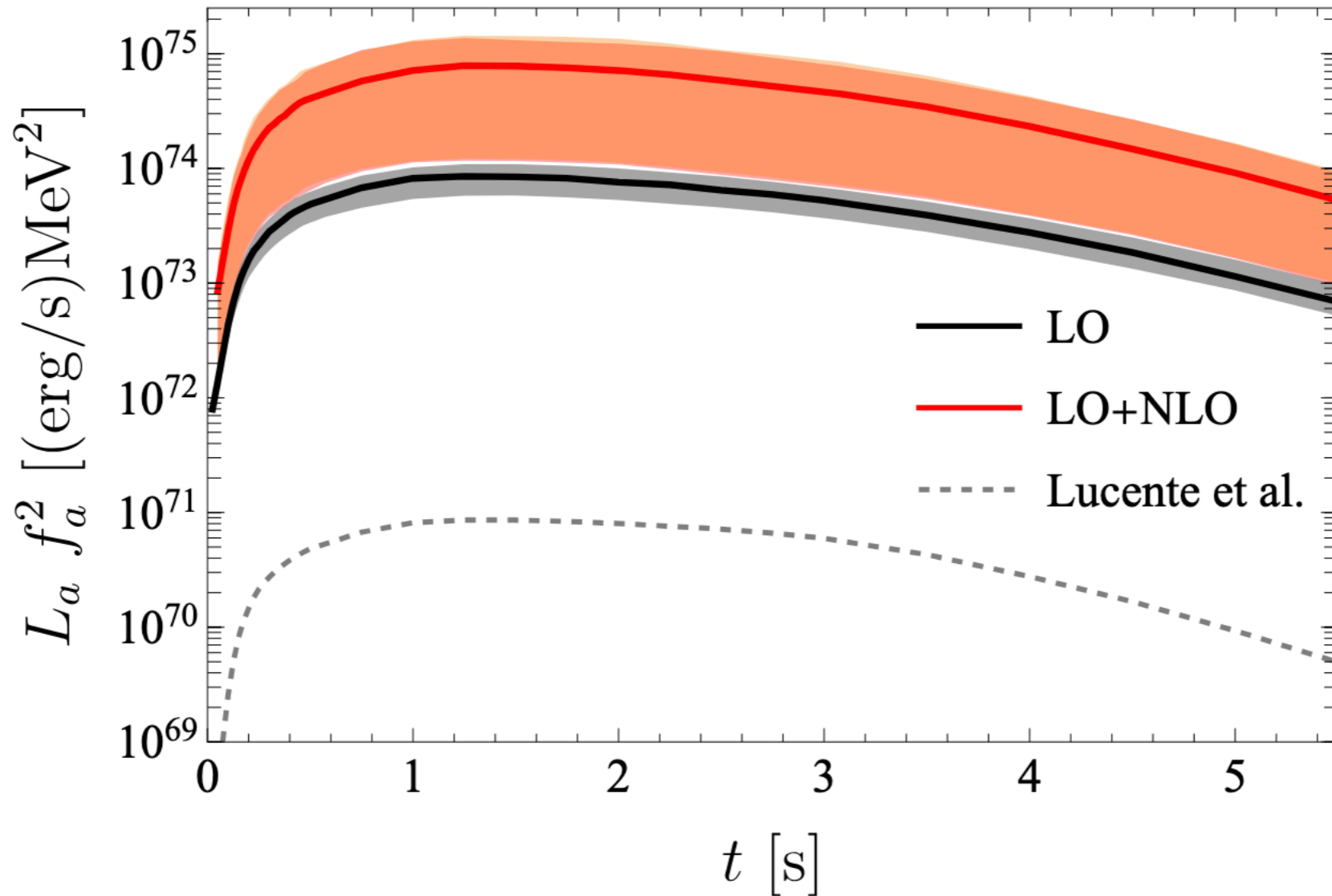
ArXiv:2410.19902



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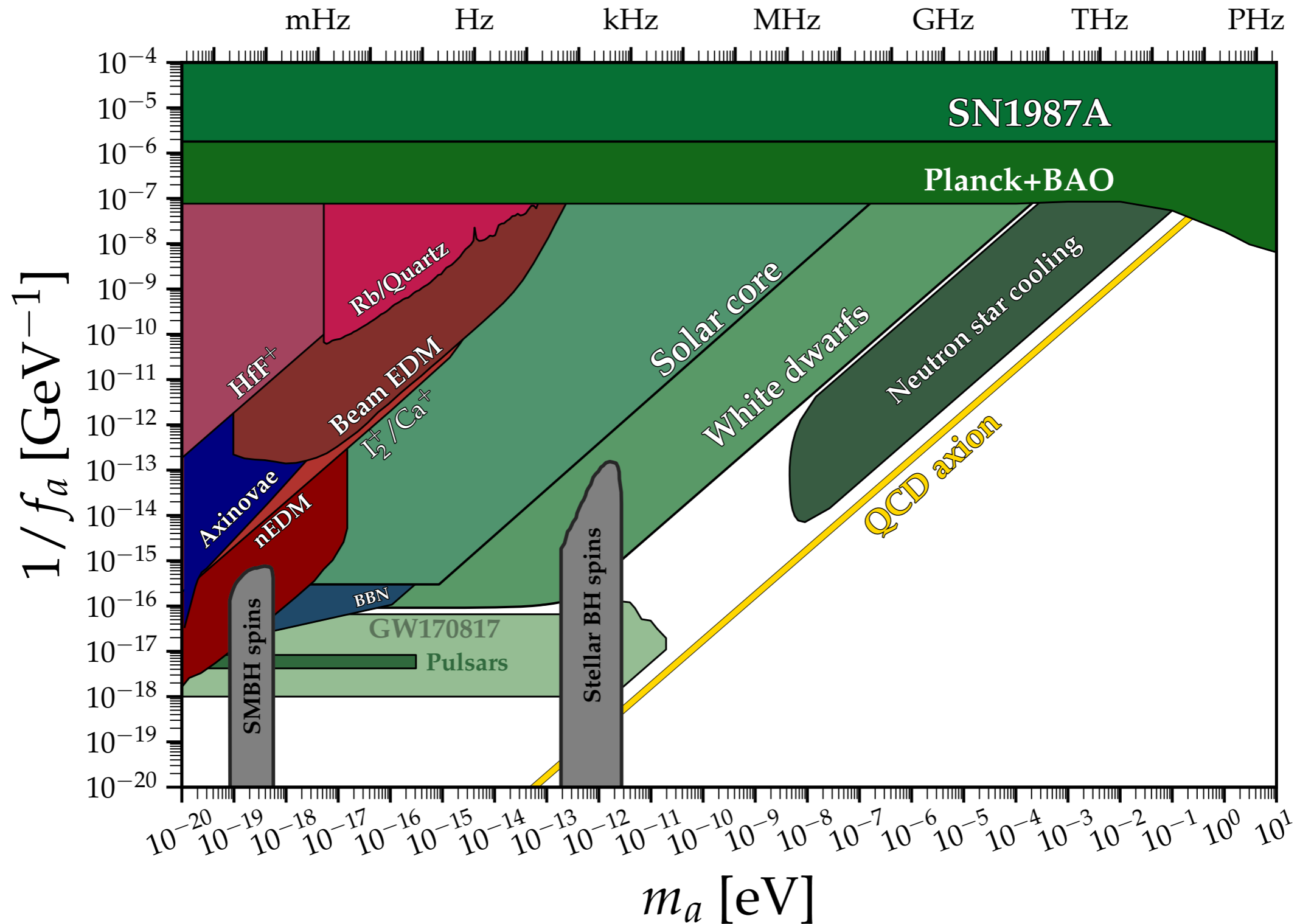


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**Strong universal bound on QCD axions:**

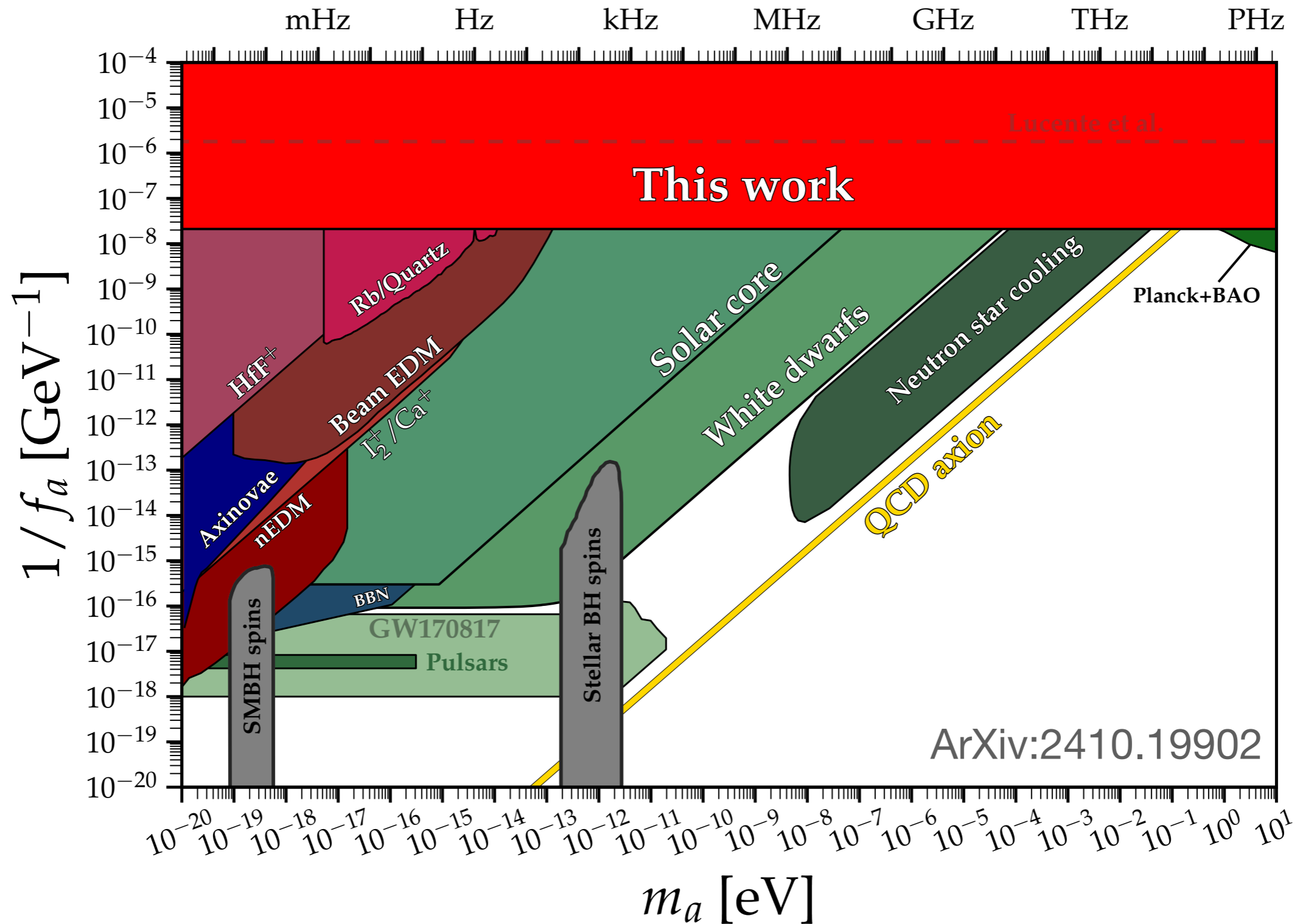
$$f_a > 1.1_{-0.6}^{+0.4} \times 10^8 \text{ GeV}, \quad (68\% \text{ C.L.})$$

# Astrophobic axions



Exclusion Plot from <https://github.com/cajohare>

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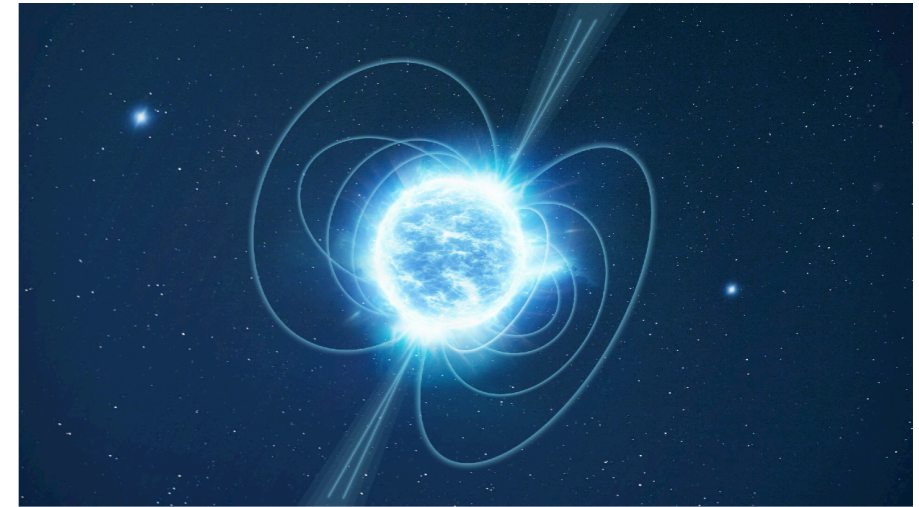


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# Conclusions

- **QCD axion couplings are density dependent!**
- **Systematic calculation of axion couplings within ChPT**
- **Significant changes of supernova bound**
- **Large uncertainty at high densities**

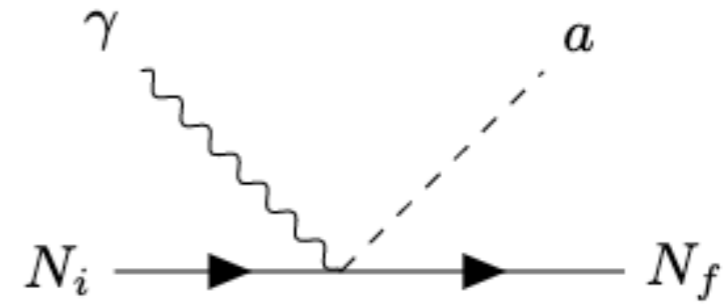




**Backup slides**

# Comparison with current literature

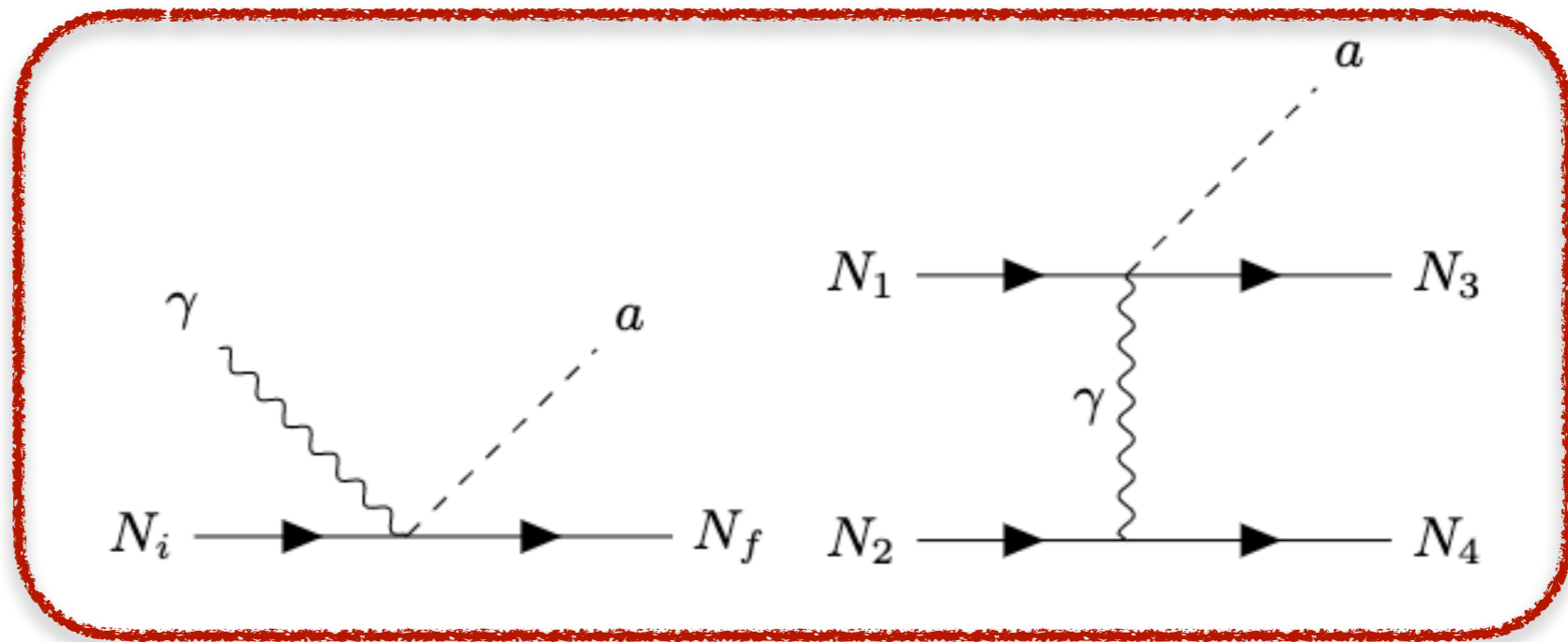
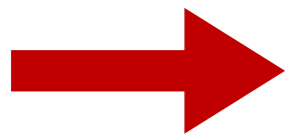
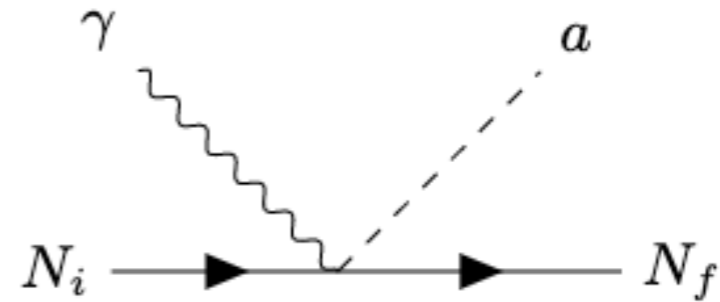
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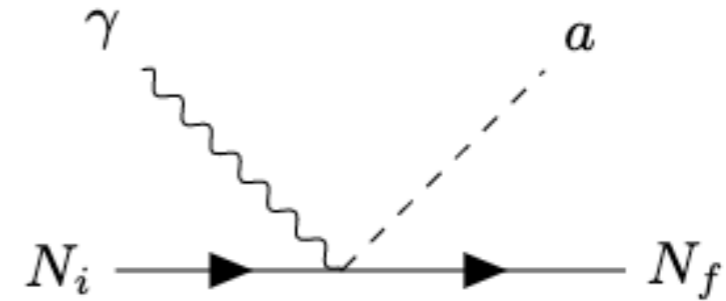
see Lucente et al. '22





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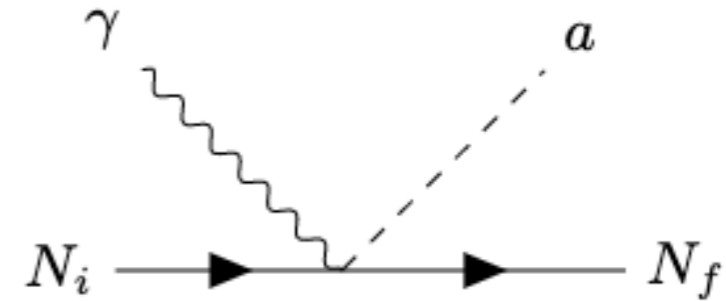
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Crewther, Vecchia, Veneziano, Witten ('79)

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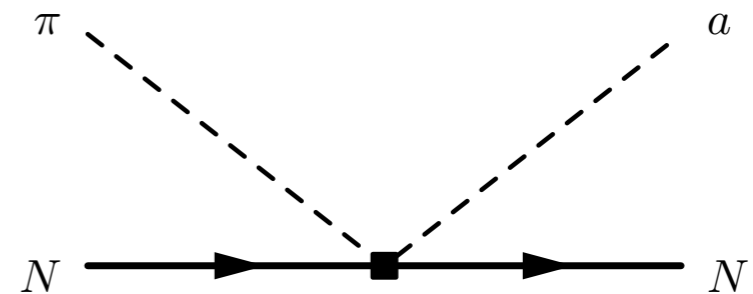
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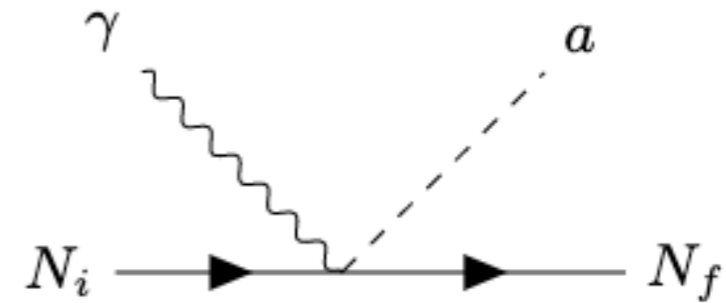
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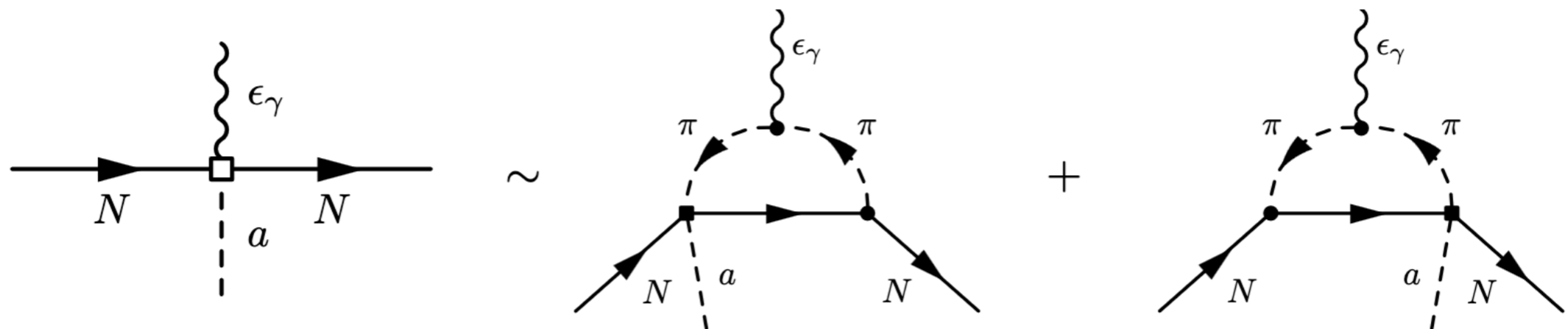
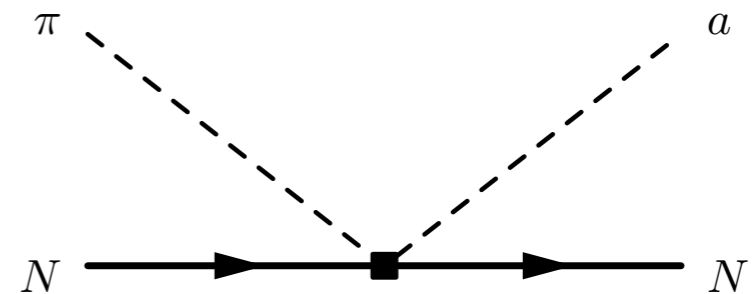
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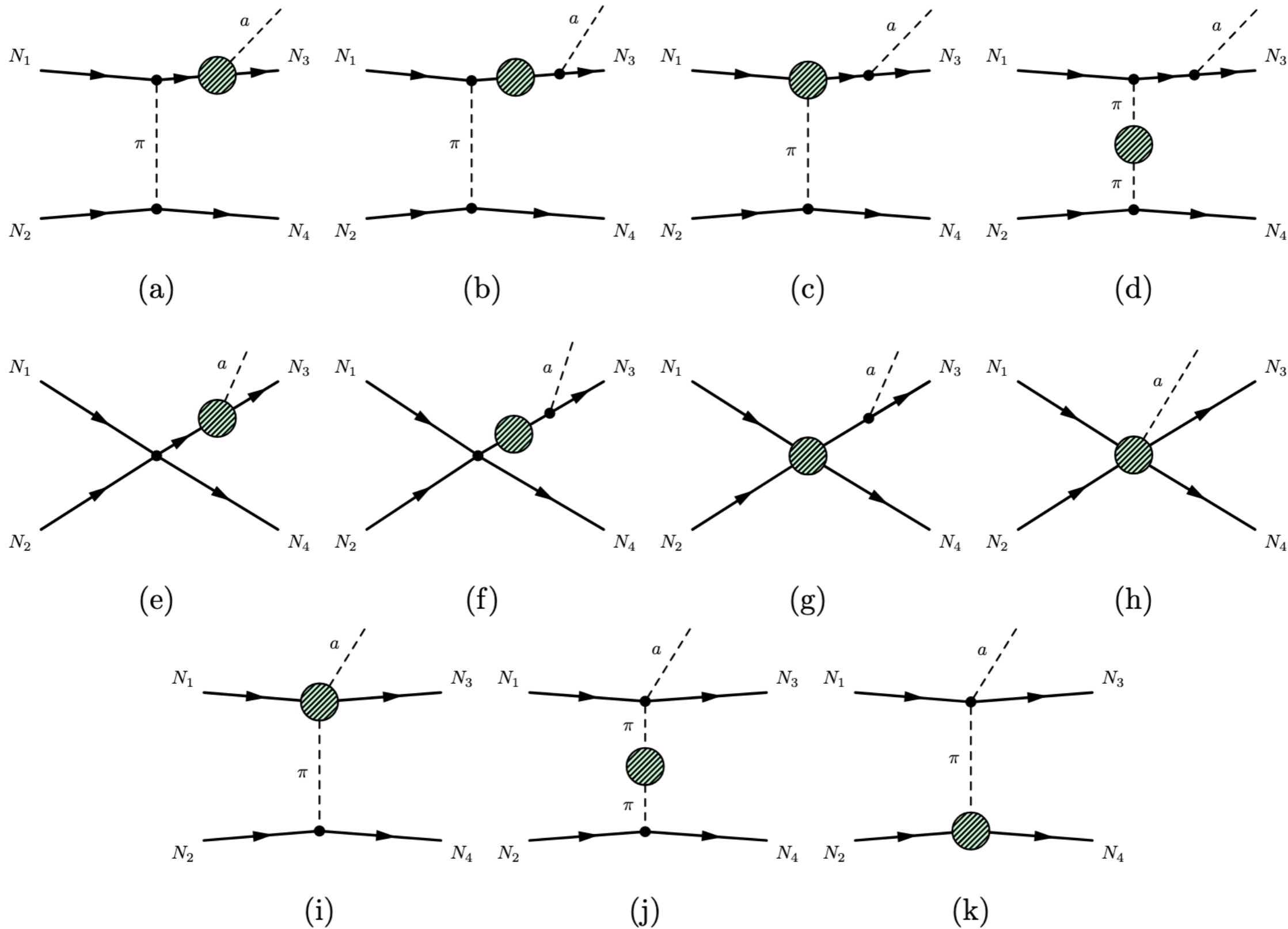
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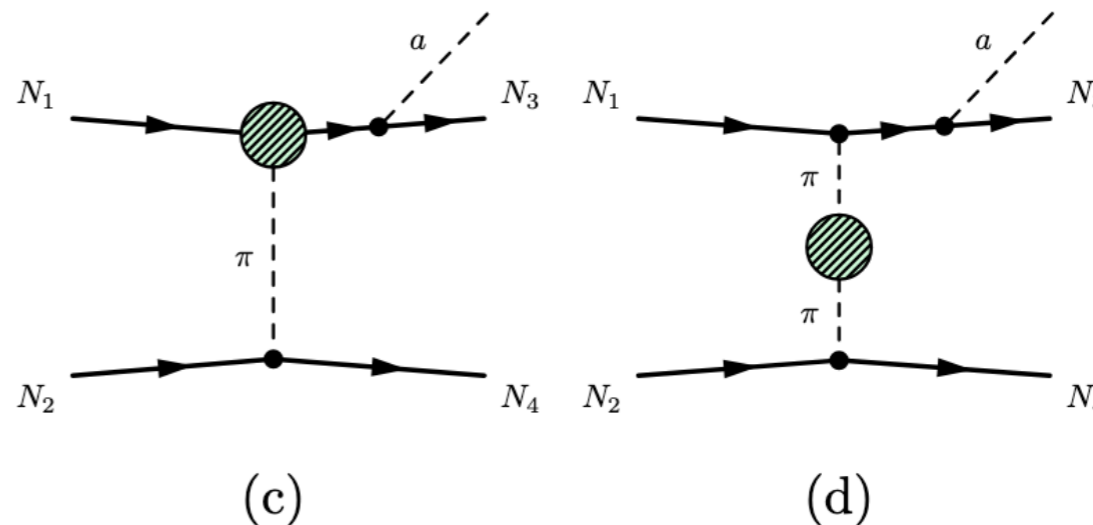
# Supernova bound revisited

## Relevant diagrams up to NLO



# Supernova bound revisited

Neglected



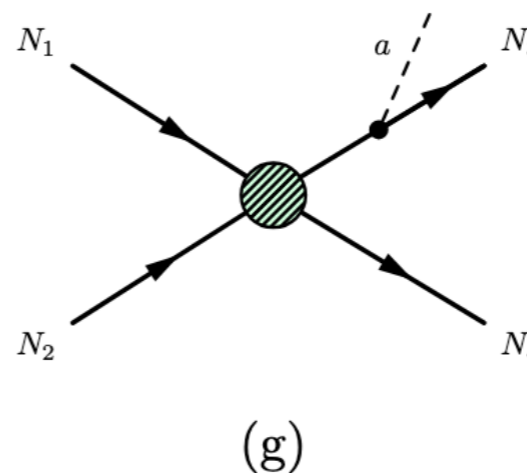
Modification of nuclear interaction:

- Fudge factor  $\gamma_p$

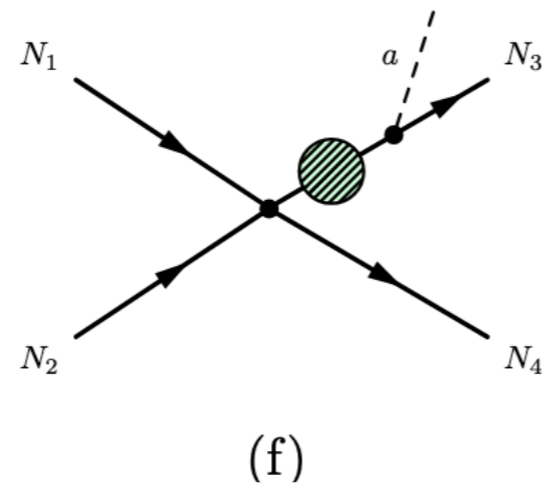
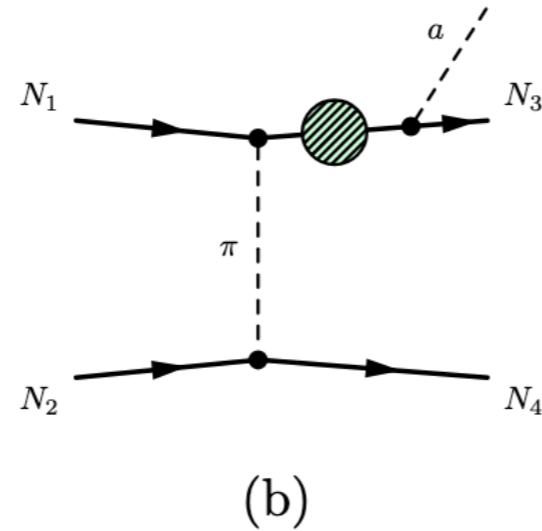
Chang, Essig, McDermott ('18)

- Phenomenologically modelled

Carenza, Fischer, Giannotti, Guo, Martinez-Pinedo, Mirizzi ('19)



# Supernova bound revisited



Modelled as nucleon re-scatterings

- Fudge factor  $\gamma h$

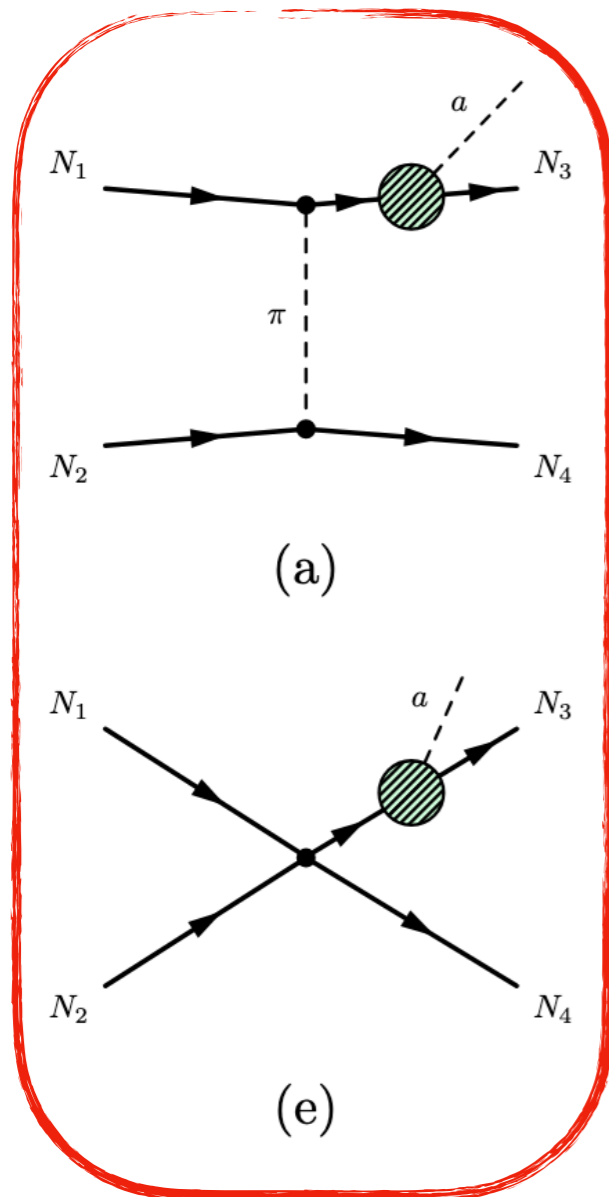
Raffelt, Seckel ('88)

Chang, Essig, McDermott ('18)

- Phenomenologically

Carenza, Fischer, Giannotti, Guo, Martinez-Pinedo, Mirizzi ('19)

# Supernova bound revisited



Outlined for the first time

Springmann, MS, Stelzl, Weiler ('24)

← Modified couplings

Focus on these for now

Fully systematic evaluation should take into account all diagrams up to given order