Revisiting Metastable Cosmic String Breaking

Akifumi Chitose (ICRR, U. Tokyo)

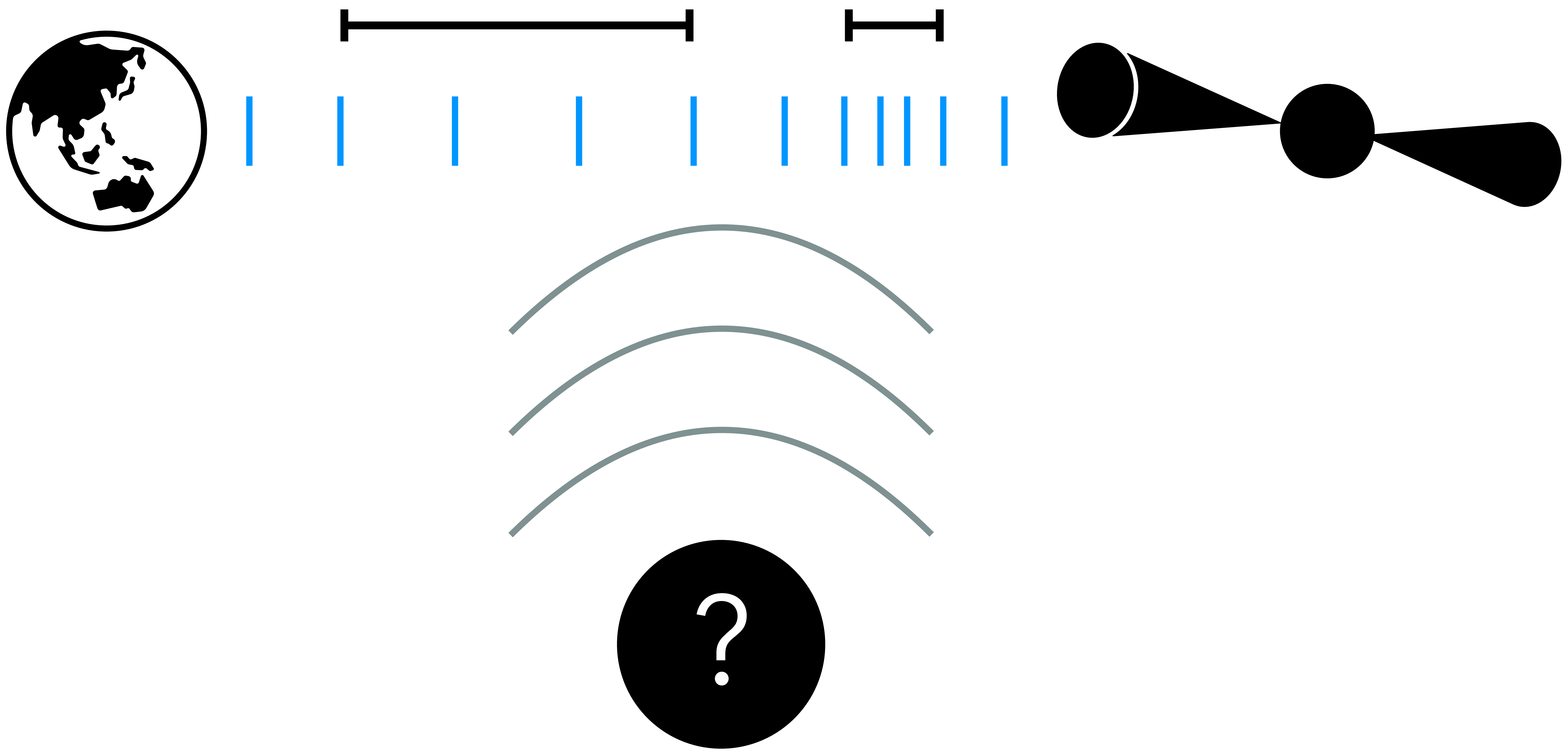
Based on:

JHEP 04 (2024) 068 [arXiv:2312.15662]

Akifumi Chitose, Masahiro Ibe, Yuhei Nakayama, Satoshi Shirai and Keiichi Watanabe

Stochastic Gravitational Wave Background

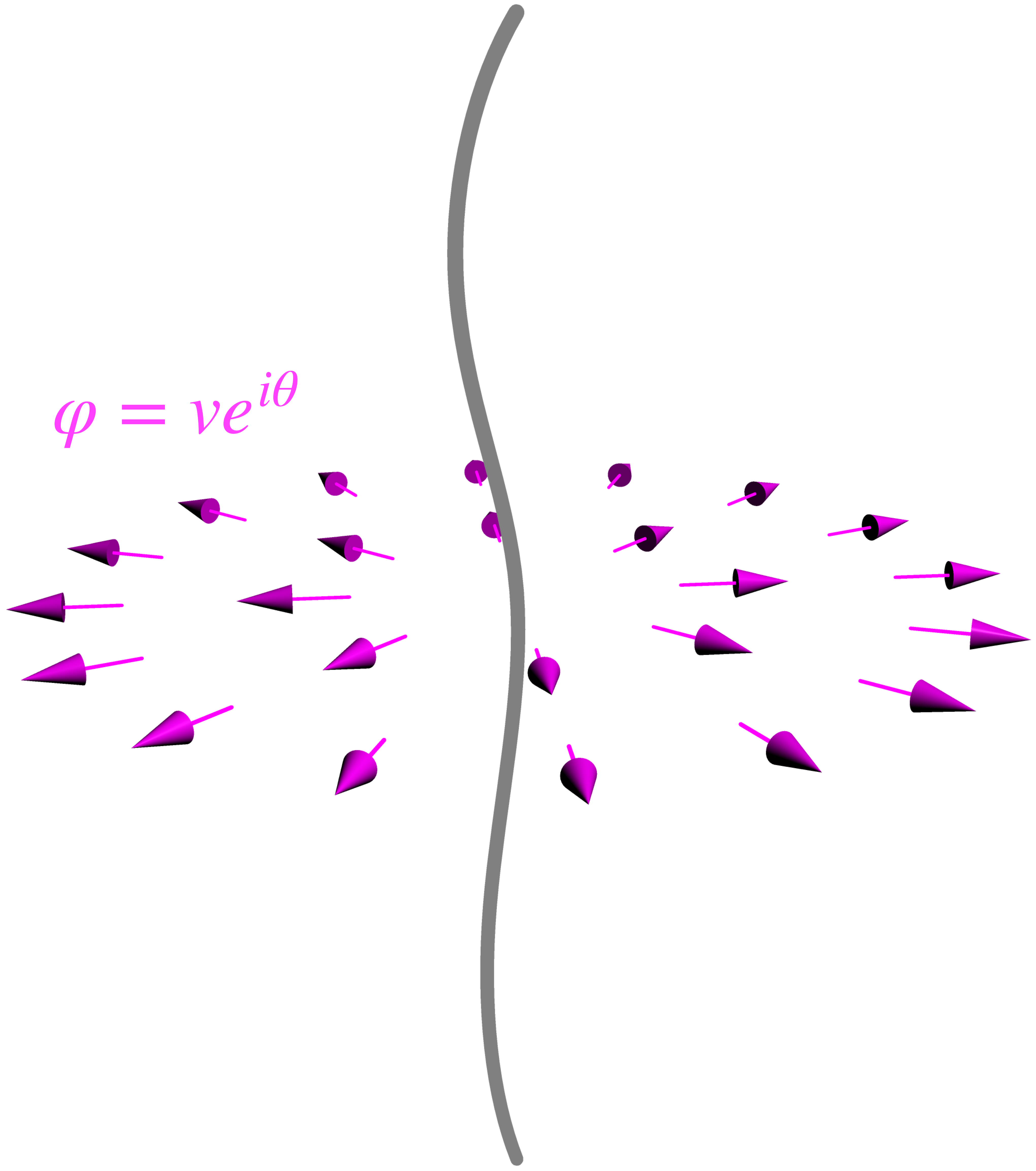
- Evidenced by PTA observations (NANOGrav, InPTA, EPTA, PPTA, CPTA)



Cosmic Strings

Probing BSM with GW

- Created in the Universe by spontaneous U(1) breaking
- Predicted by many BSM physics
  - GUT
  - Dark photon



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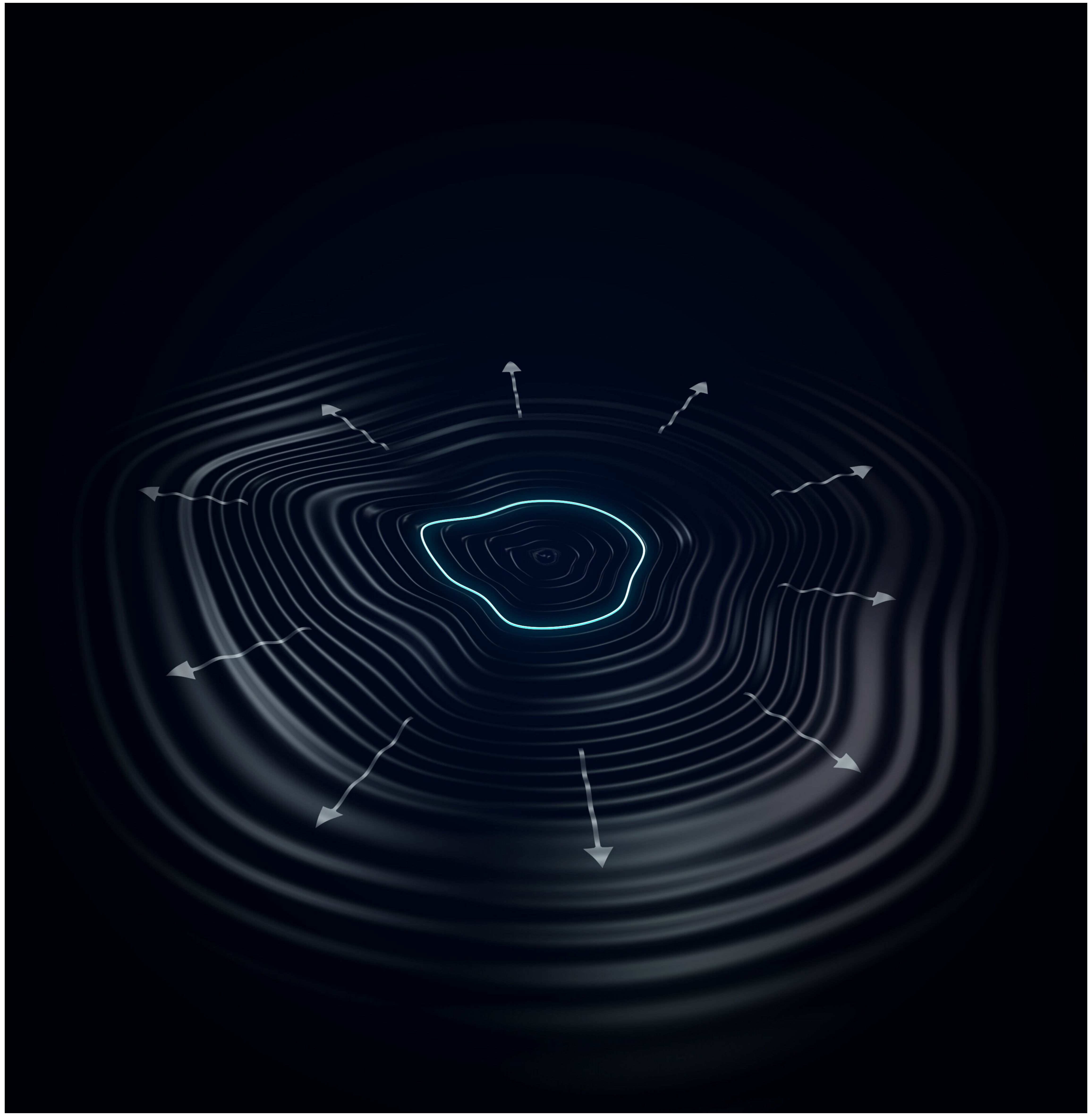
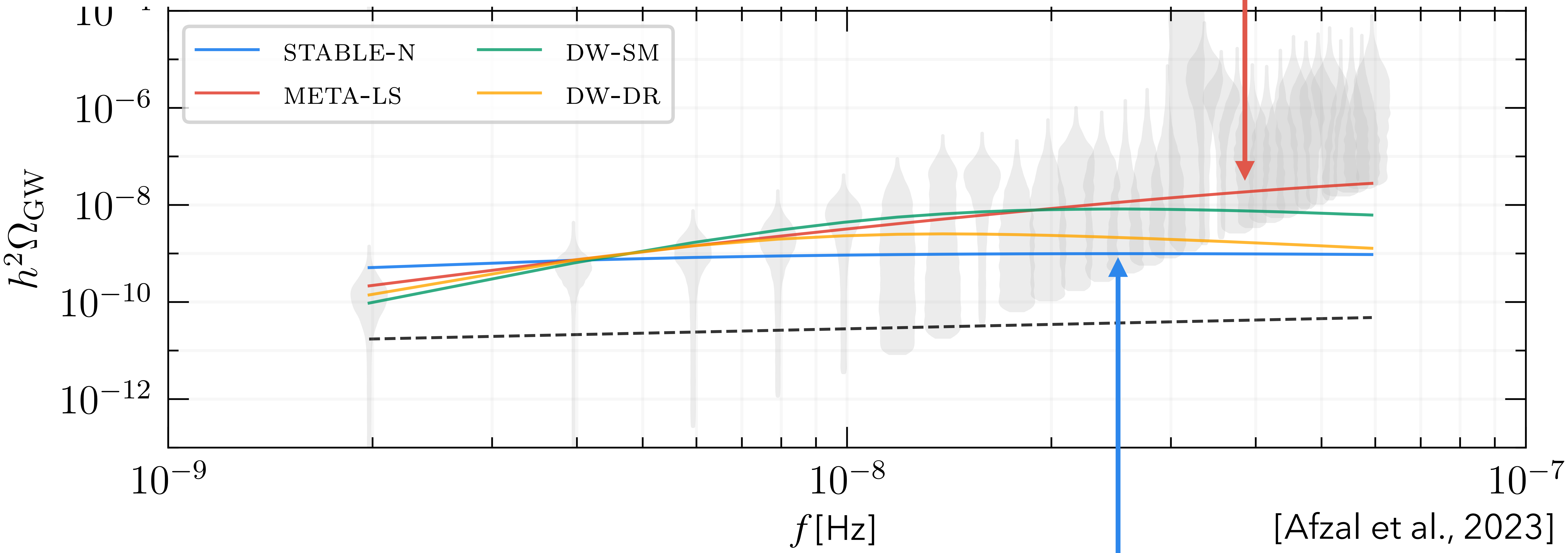


Credit: Daniel Dominguez from CERN's Education, Communications & Outreach (ECO) Department.

Metastable Cosmic Strings for PTA

Metastable cosmic strings can explain the NANOGrav signal

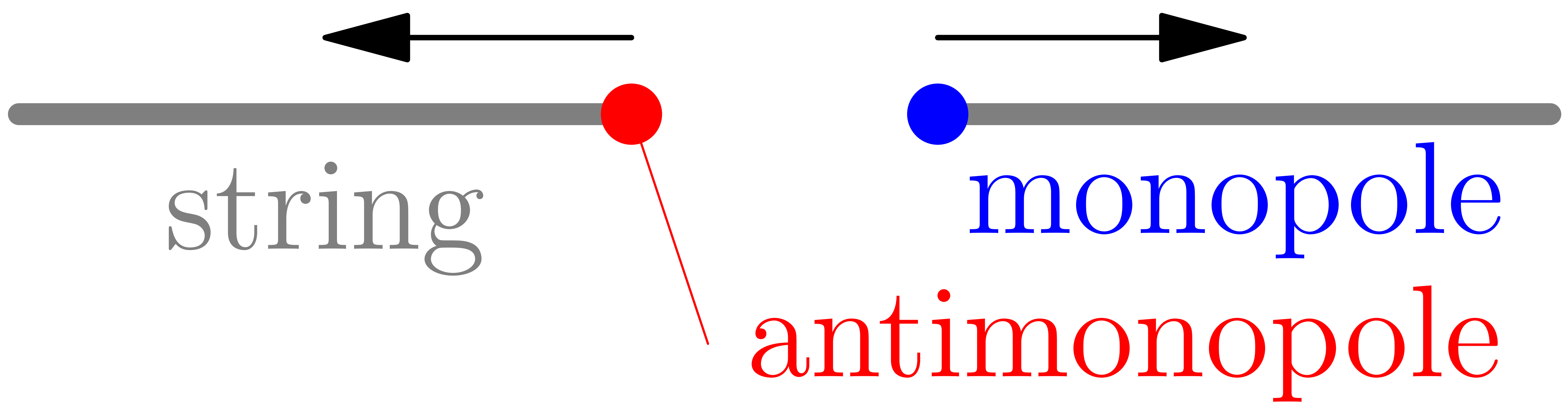


[Afzal et al., 2023]

Stable cosmic strings do not work

Metastable Cosmic Strings

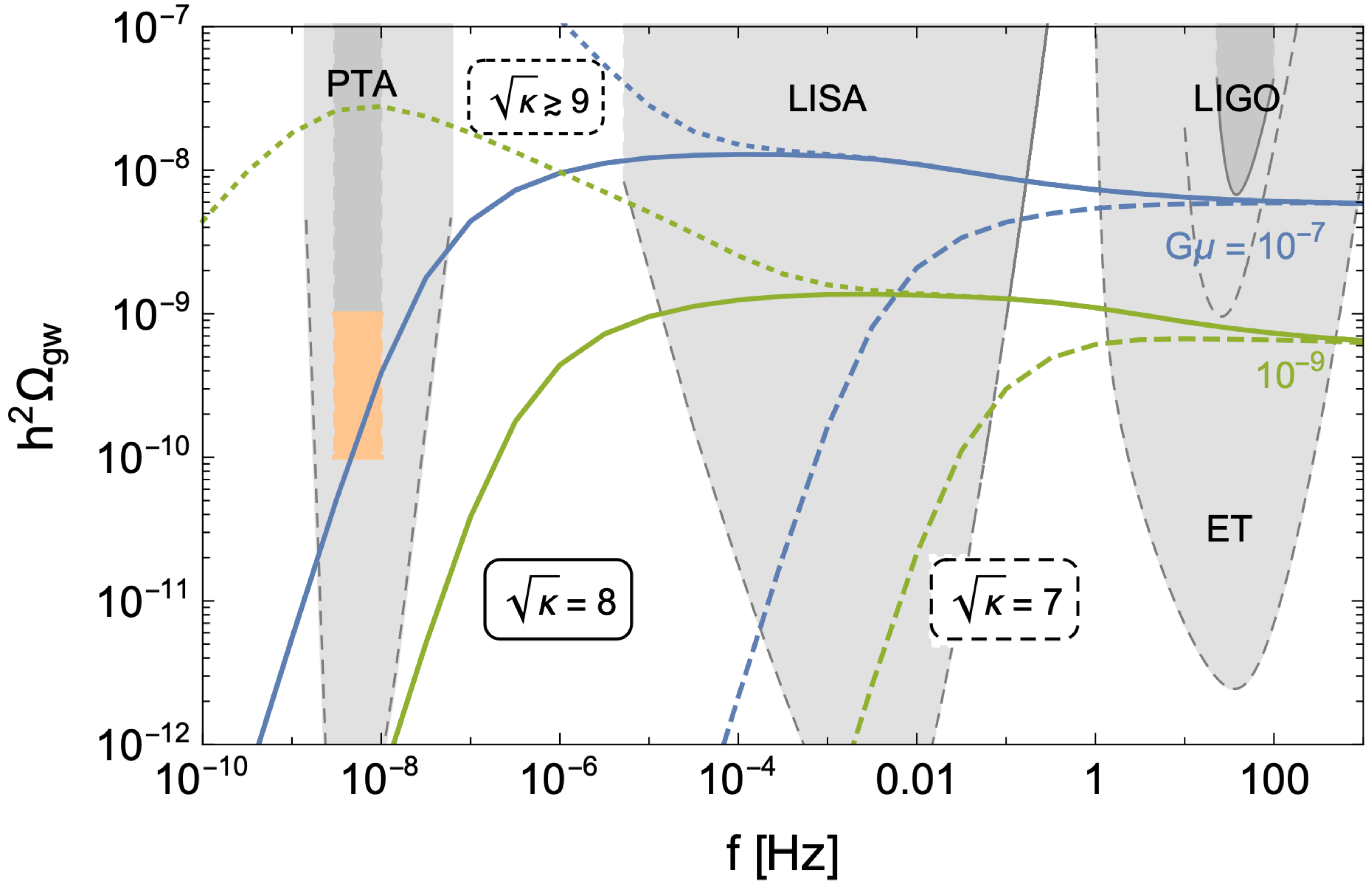
- Spontaneously cut by monopole-antimonopole pair creation
- Arise from e.g. $G \to \mathrm{U}(1) \to 1$ w/ G: simply connected



Metastable Cosmic Strings

GW spectrum depends on the decay rate

- $\Gamma \sim \exp[-\pi\kappa]$
- NANOGrav: $\sqrt{\kappa} \sim 8$
- κ must be calculated precisely

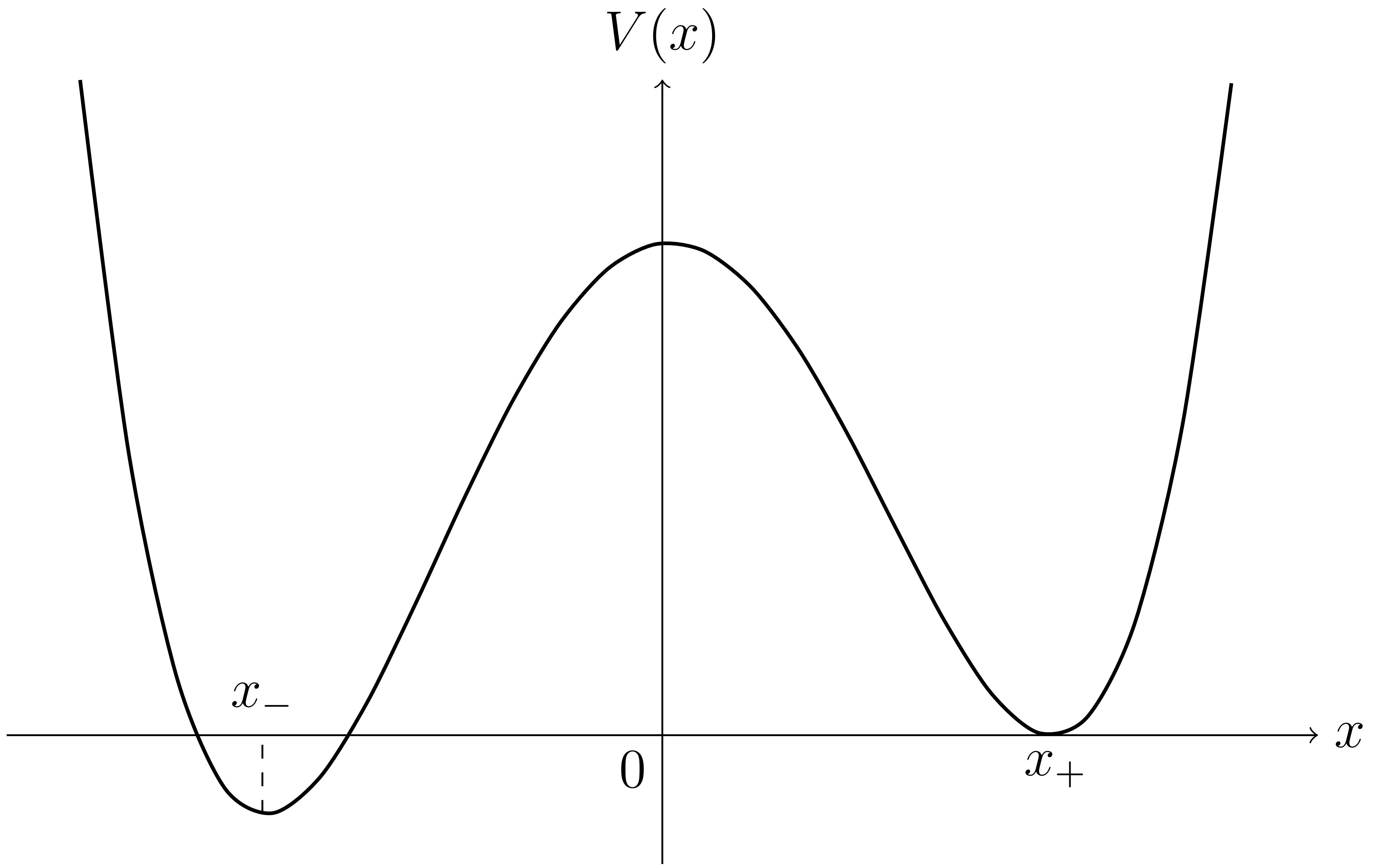


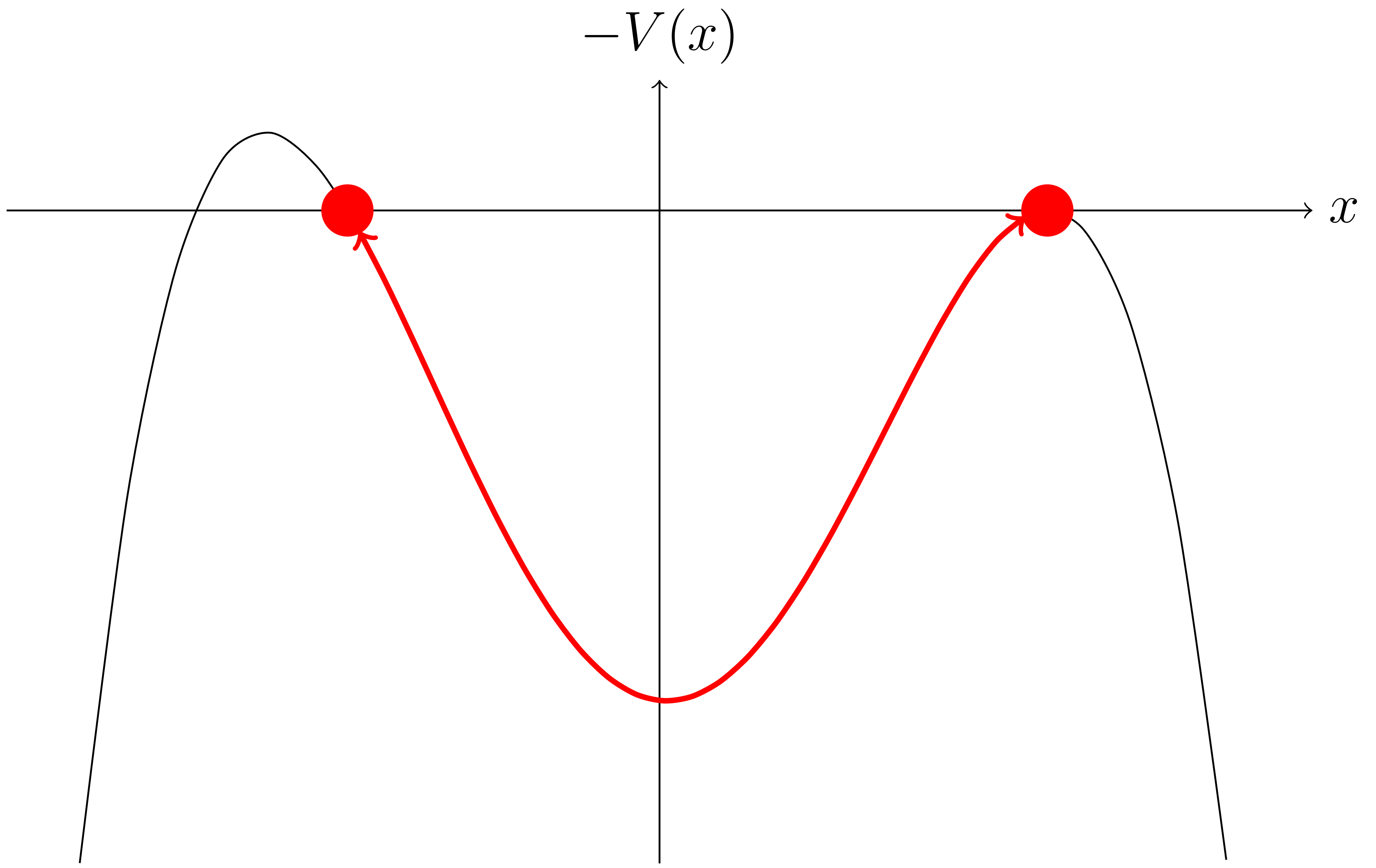
[Buchmüller et al., 2023]

String breaking rate

Tunneling and bounce see e.g. [Coleman, 1985]

- Procedure:
  - Go to imaginary time
    - $\approx$ invert the potential
  - Find the bounce solution
    - Action: S_B
  - Decay rate: $\Gamma \sim \exp[-S_B]$

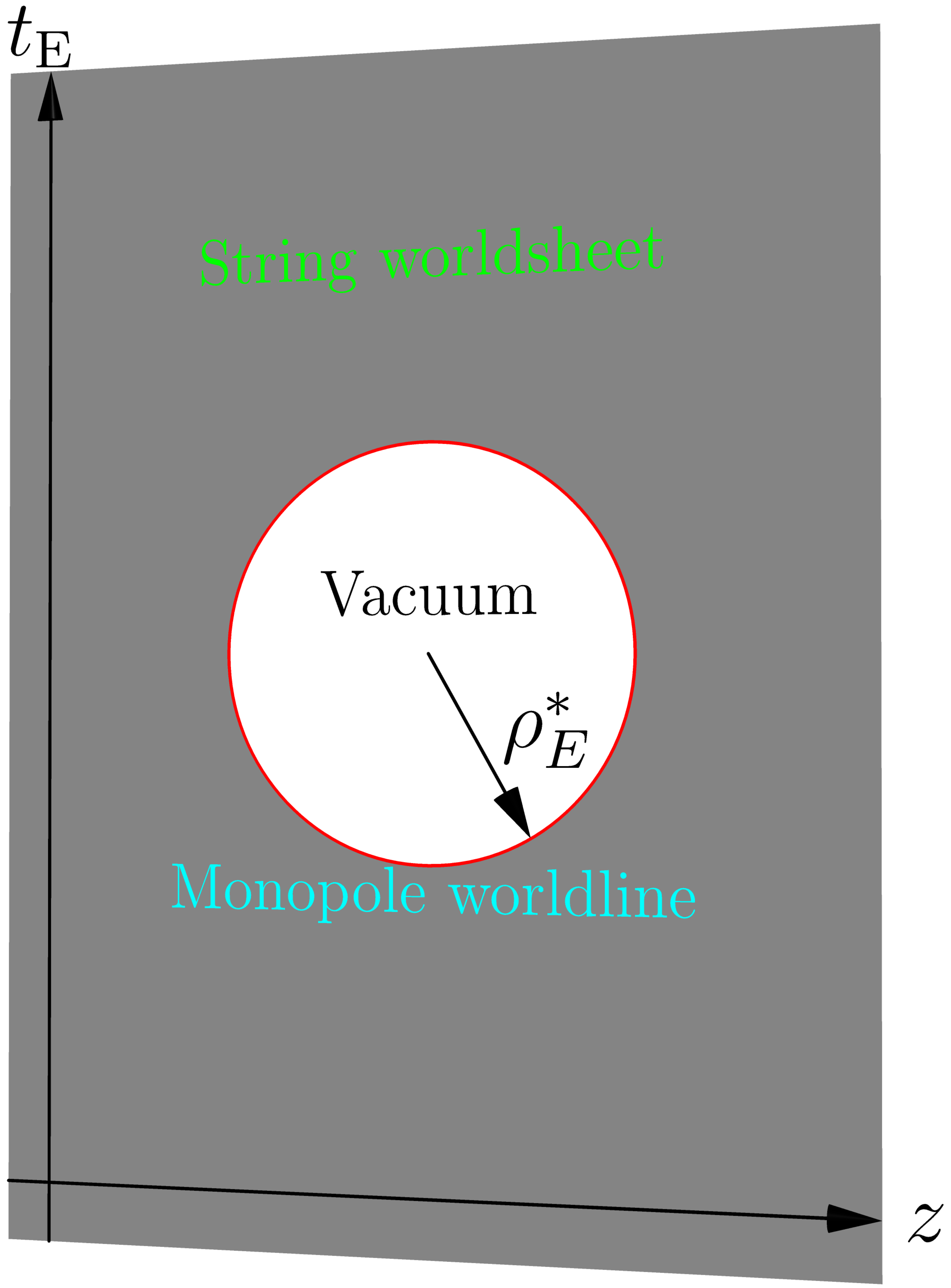




String breaking rate

Preskill-Vilenkin approximation [Preskill & Vilenkin, 1992]

- Neglect monopole size and string width
- $S_E = 2\pi\rho_E^* M_{\text{mono}} - \pi\rho_E^{*2} T_{\text{str}}$
  - $\rho_E^* = M_{\text{mono}} / T_{\text{str}}$
  - $\pi\kappa = S_B = \pi M_{\text{mono}}^2 / T_{\text{str}}$

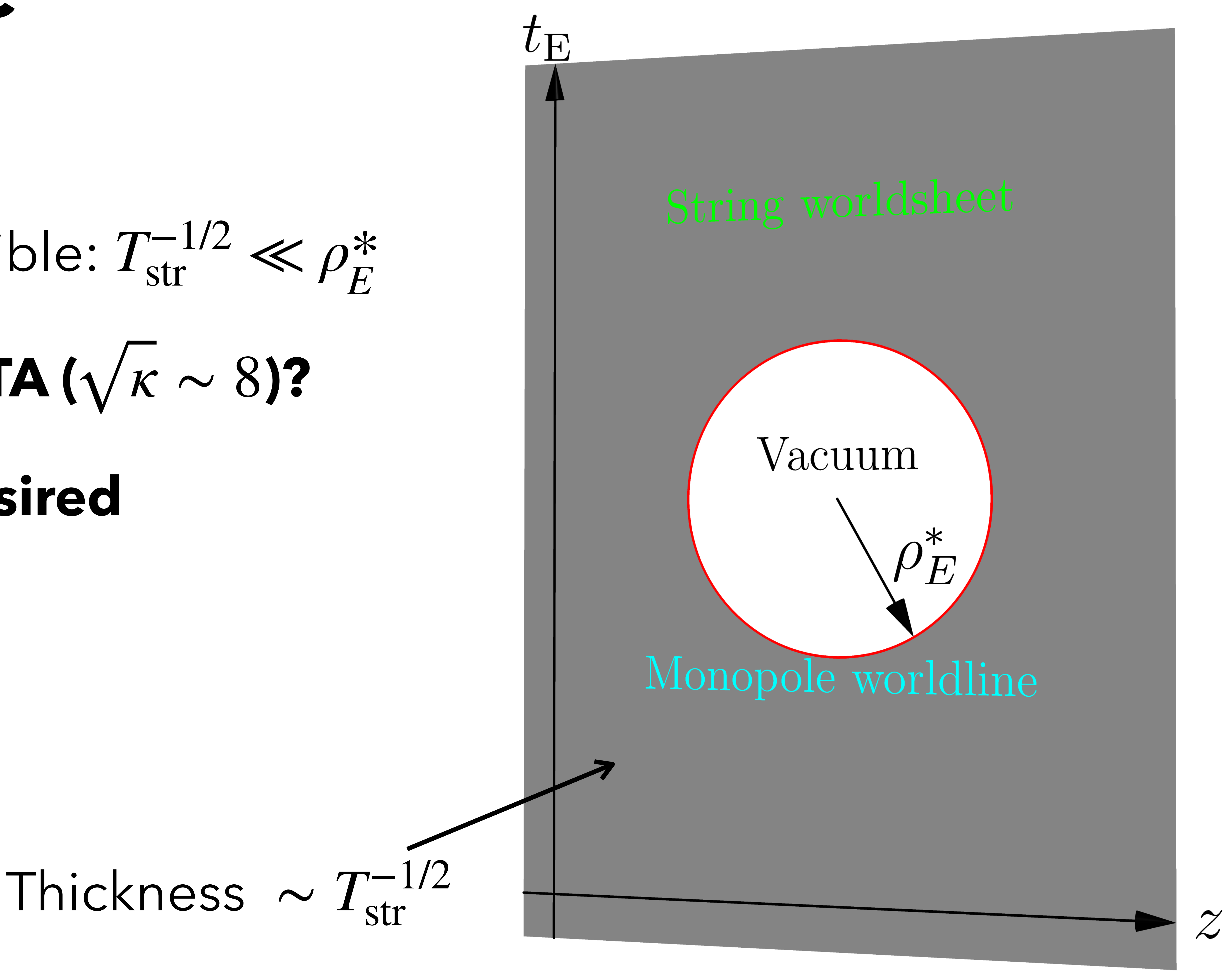


String breaking rate

Is Preskill-Vilenkin valid here?

- String width should be negligible: $T_{\rm str}^{-1/2} \ll \rho_E^*$
  - $\sqrt{\kappa} \gg 1 \cdots$ **Is this OK for PTA ($\sqrt{\kappa} \sim 8$)?**

→Alternative evaluation desired



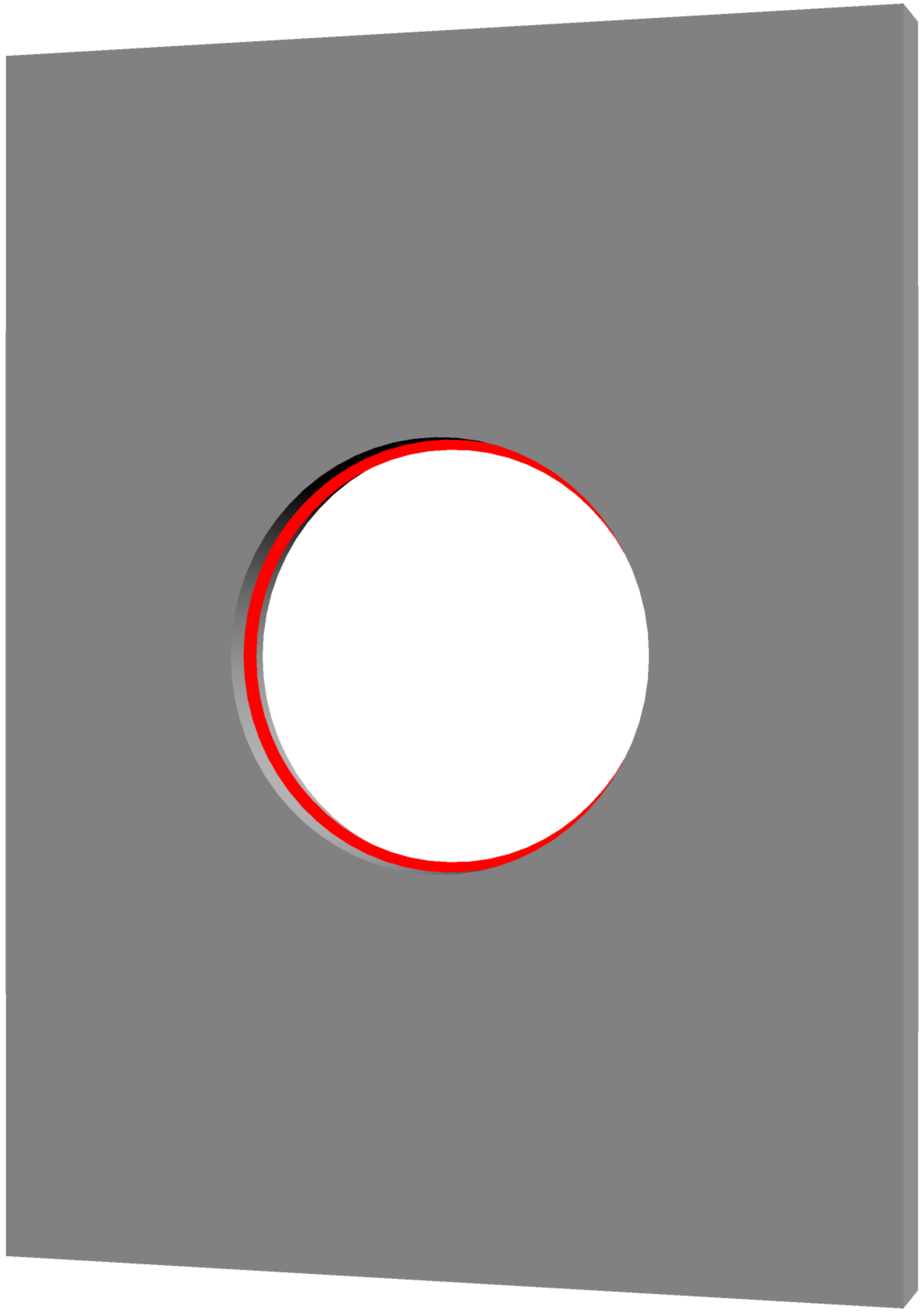
Re-evaluation of Bounce Action

Strategy

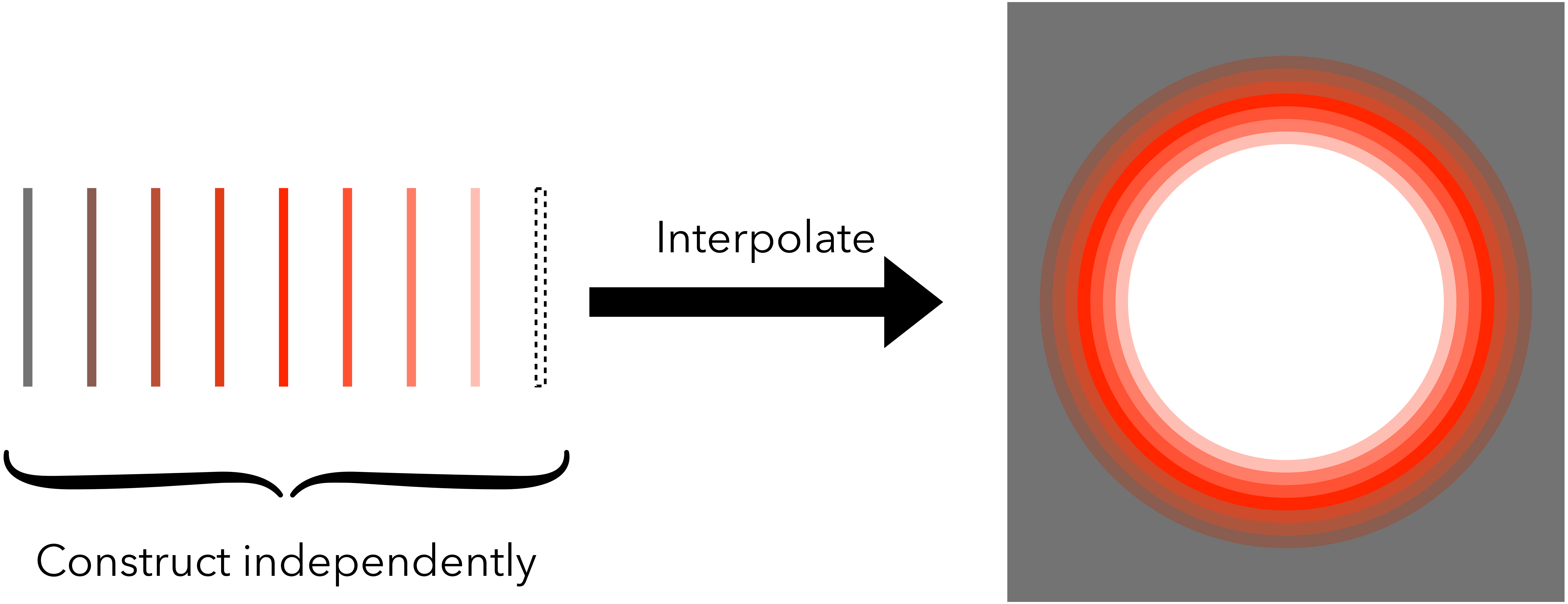
How to evaluate the bounce action?

- Solve 4D Euclidean field equation?
  - Stiff equation
  - Bounce: saddle point of S_E
    → nontrivial algorithm needed

→ Alternative strategy



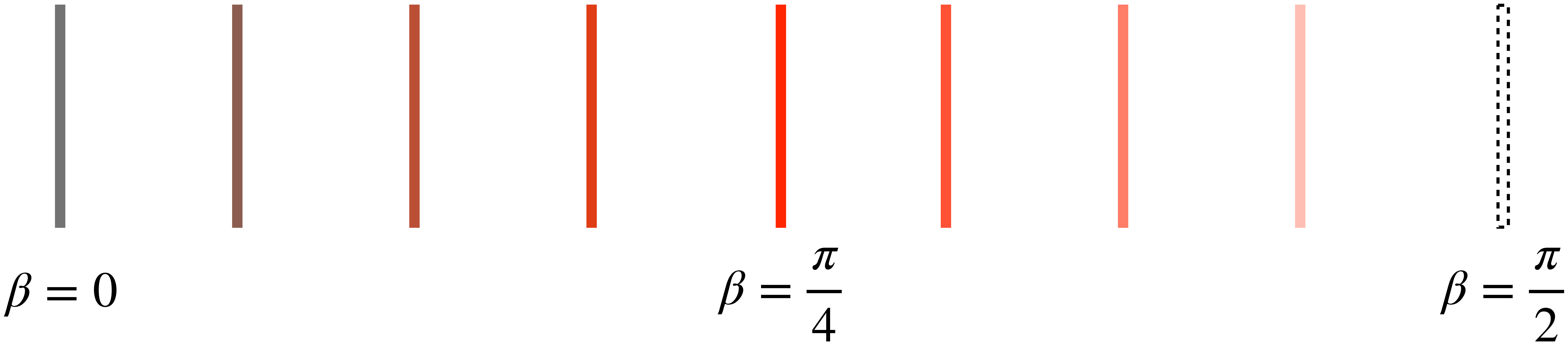
Strategy
Conceptual sketch



Strategy

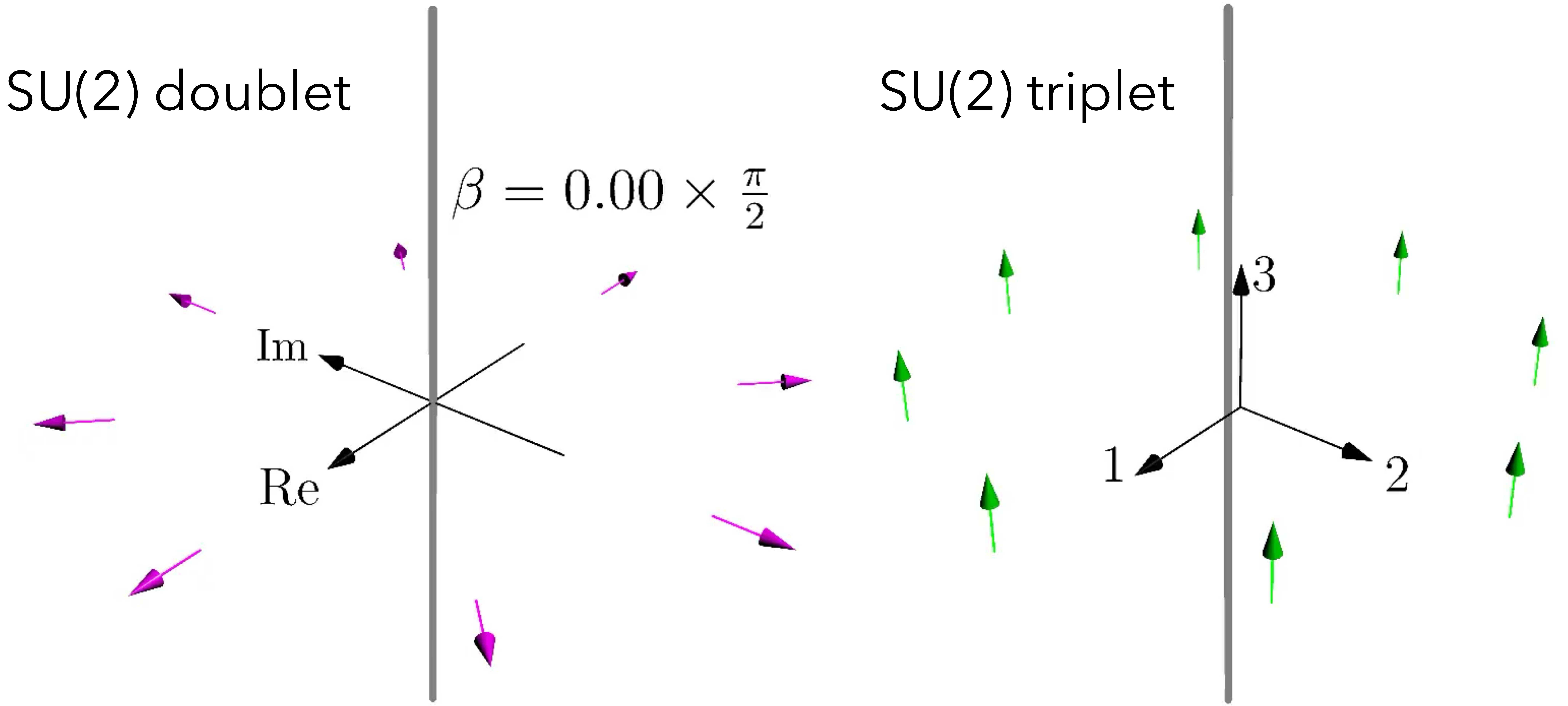
Step 1: Build "excited strings" with an Ansatz

- Unwind the whole string gradually
  - $\beta = 0$: ordinary string, $\beta = \pi/2$: vacuum
  - Field configuration: given by Ansatz [Shifman & Yung, 2002]



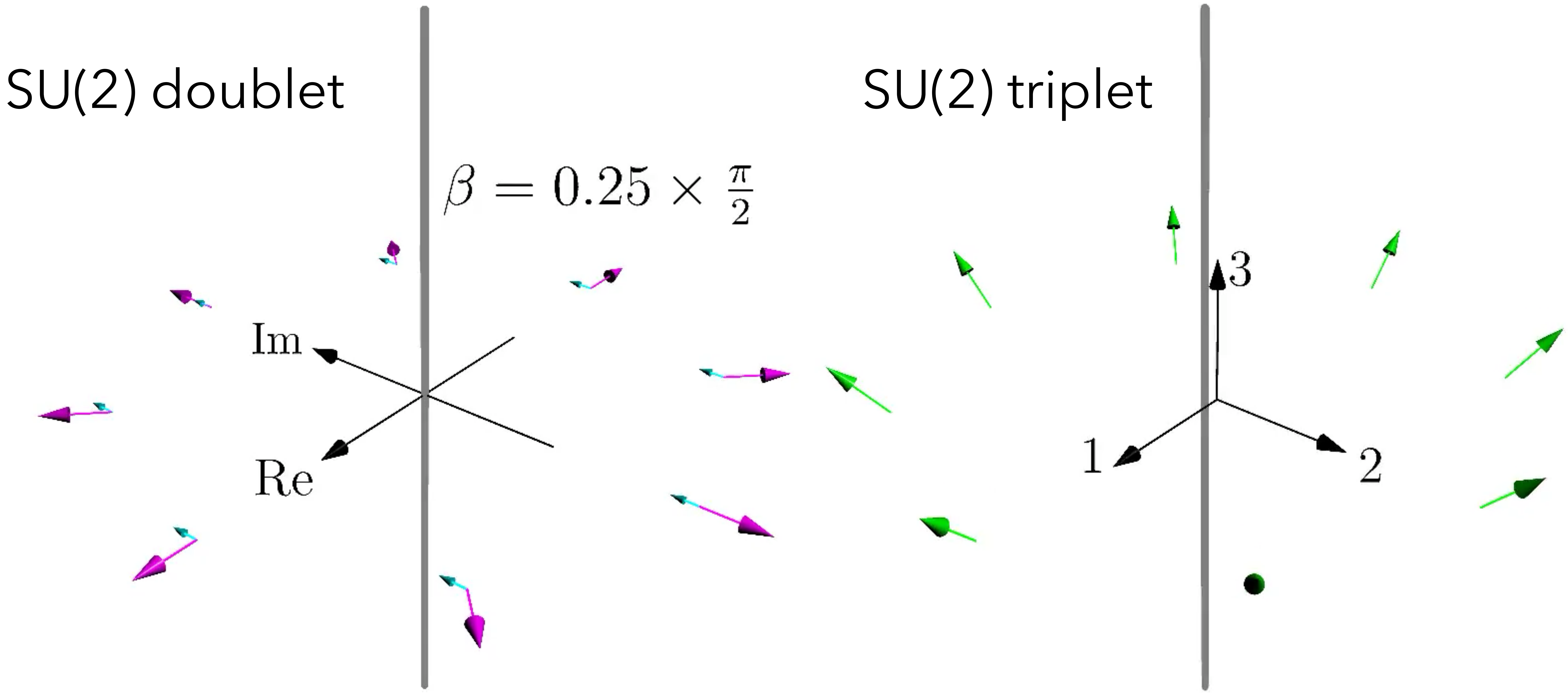
Strategy

Unwinding the string



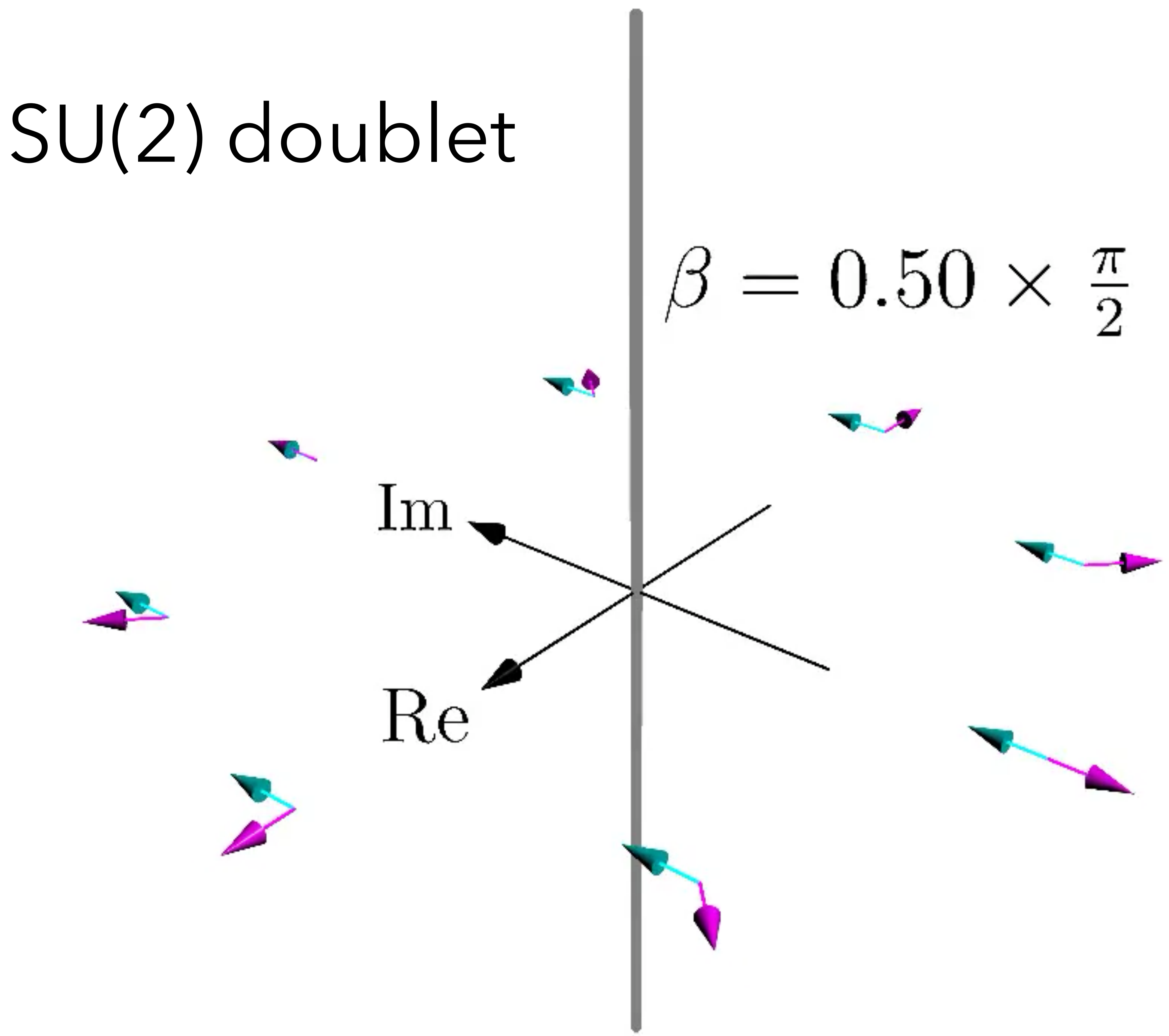
Strategy

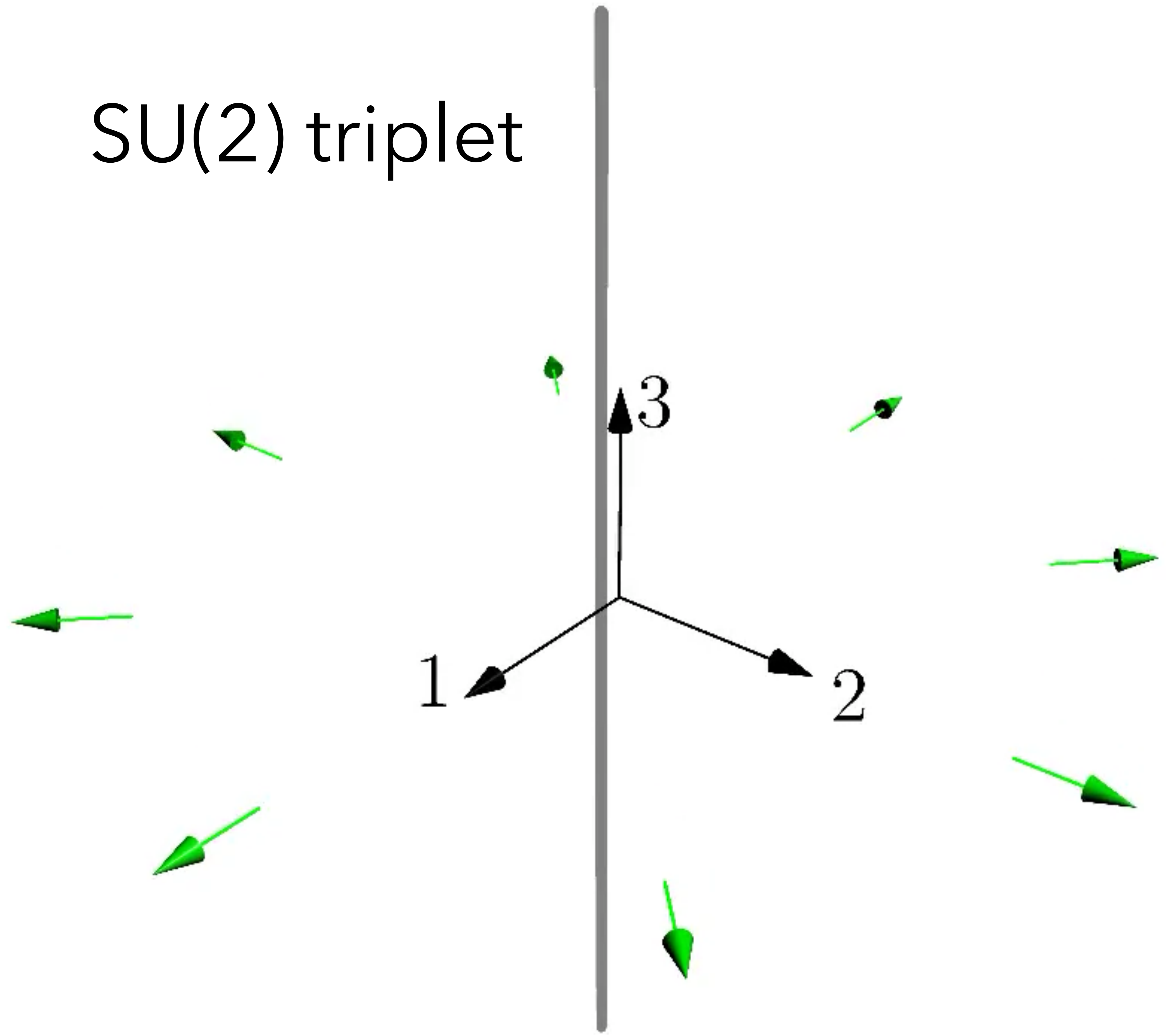
Unwinding the string



Strategy

Unwinding the string

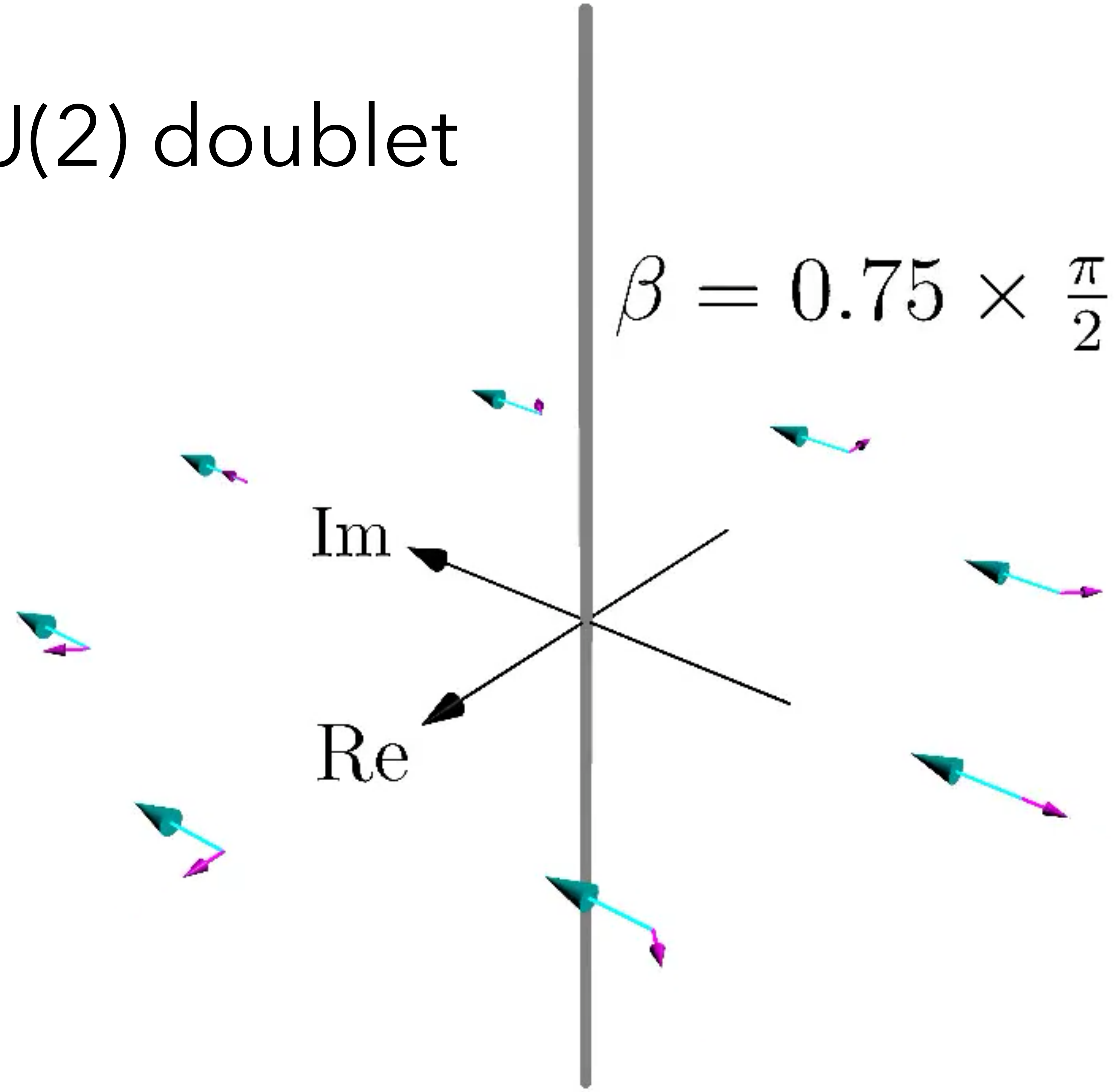




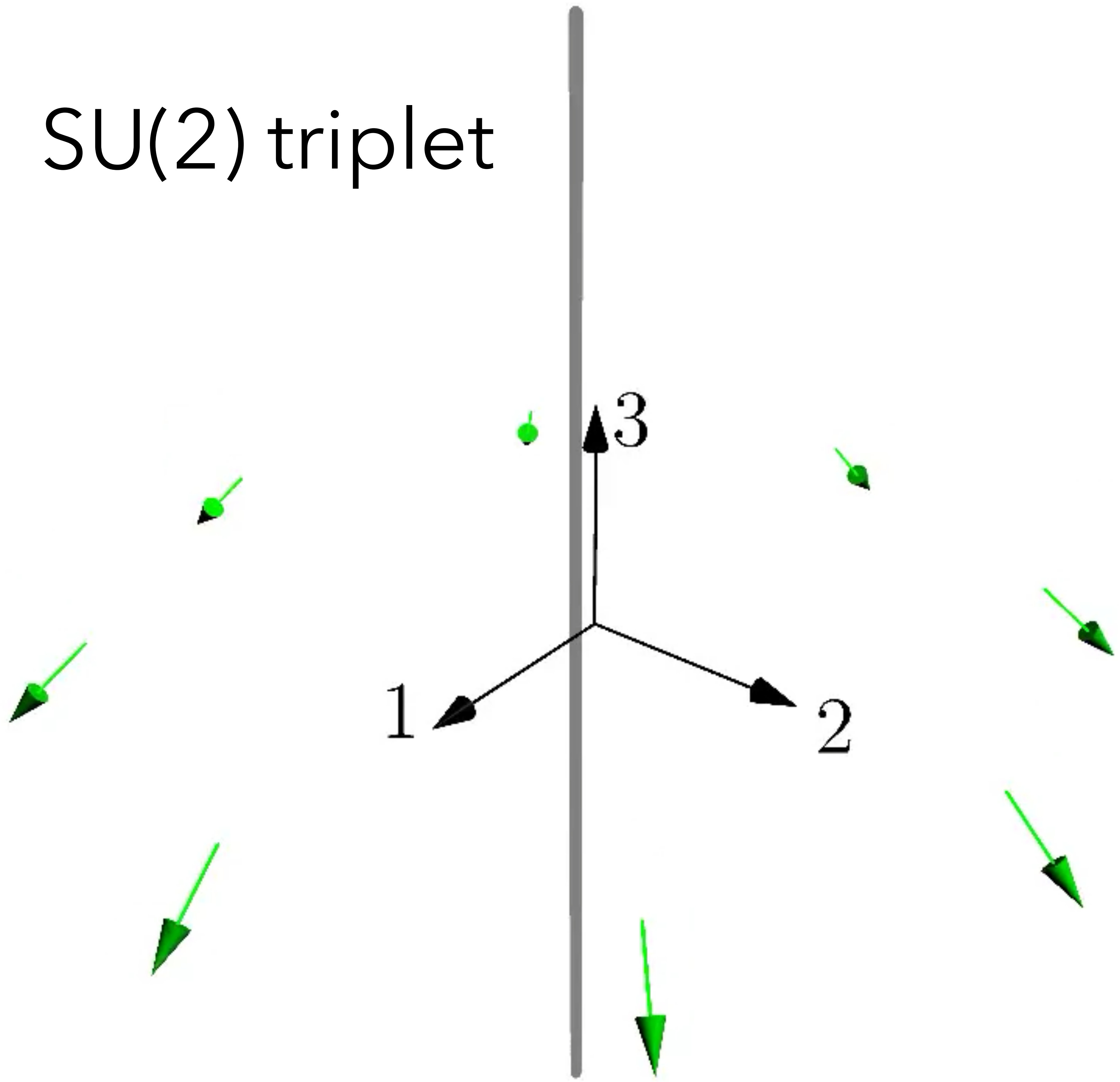
Strategy

Unwinding the string

SU(2) doublet



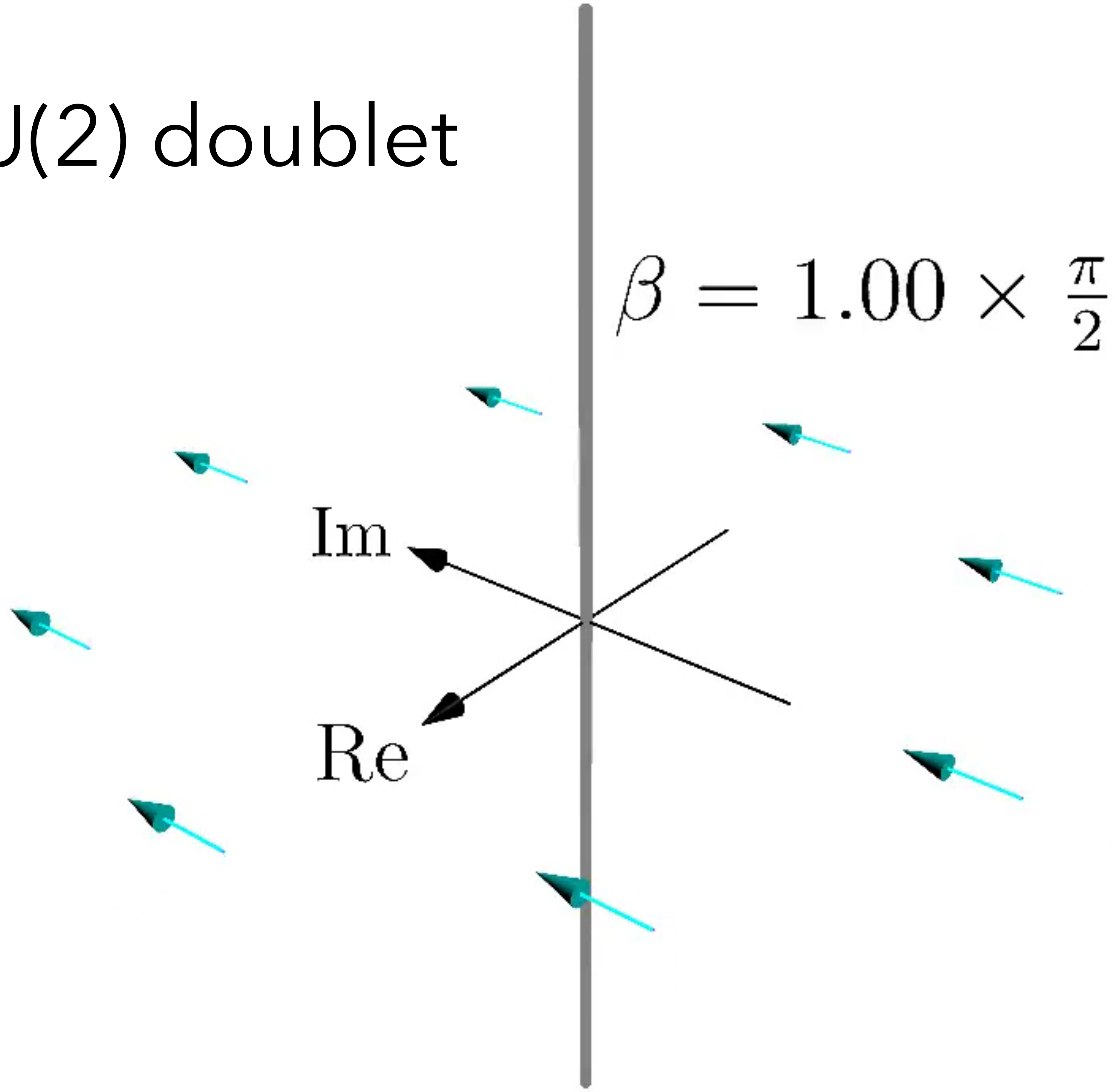
SU(2) triplet



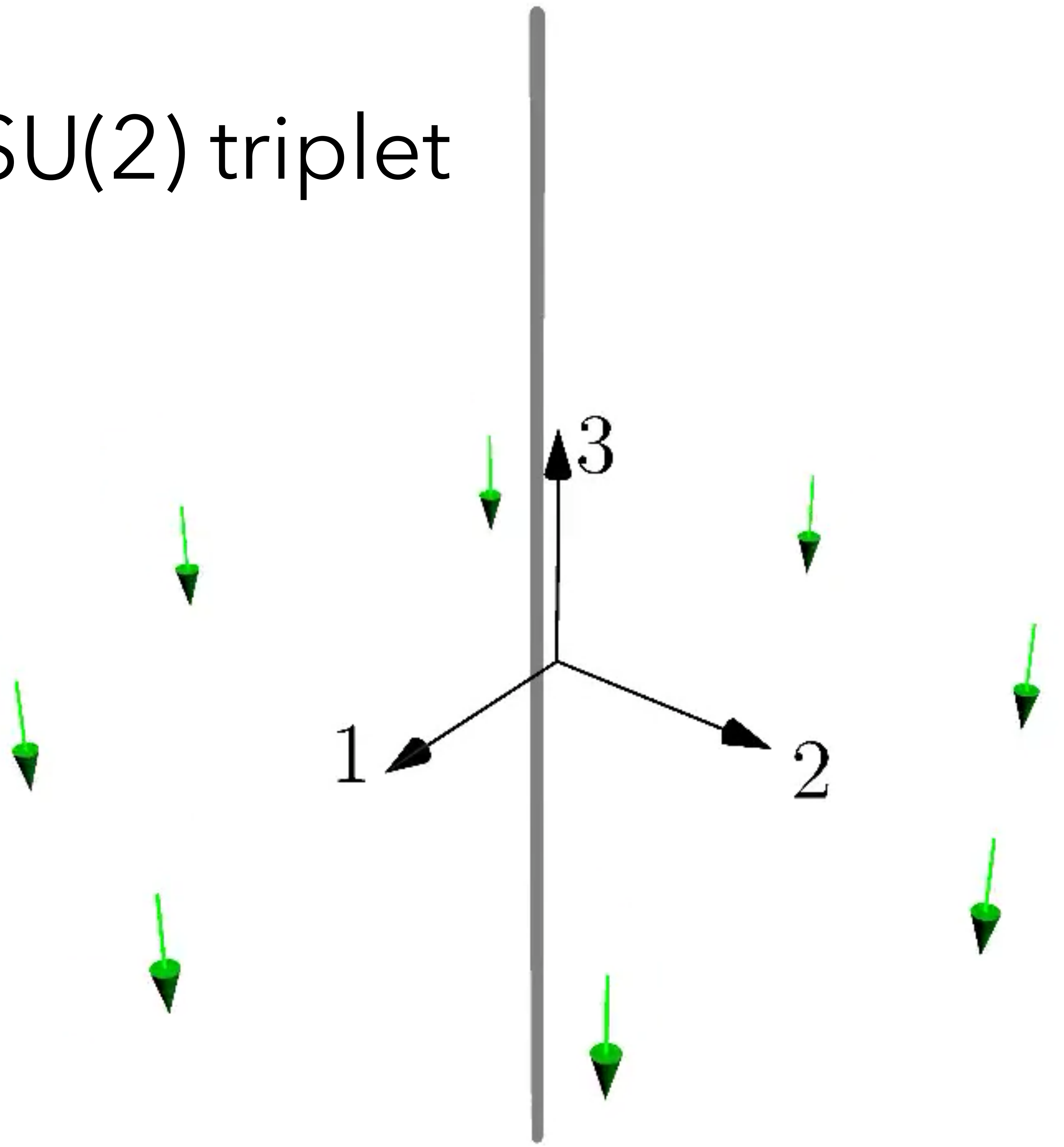
Strategy

Unwinding the string

SU(2) doublet

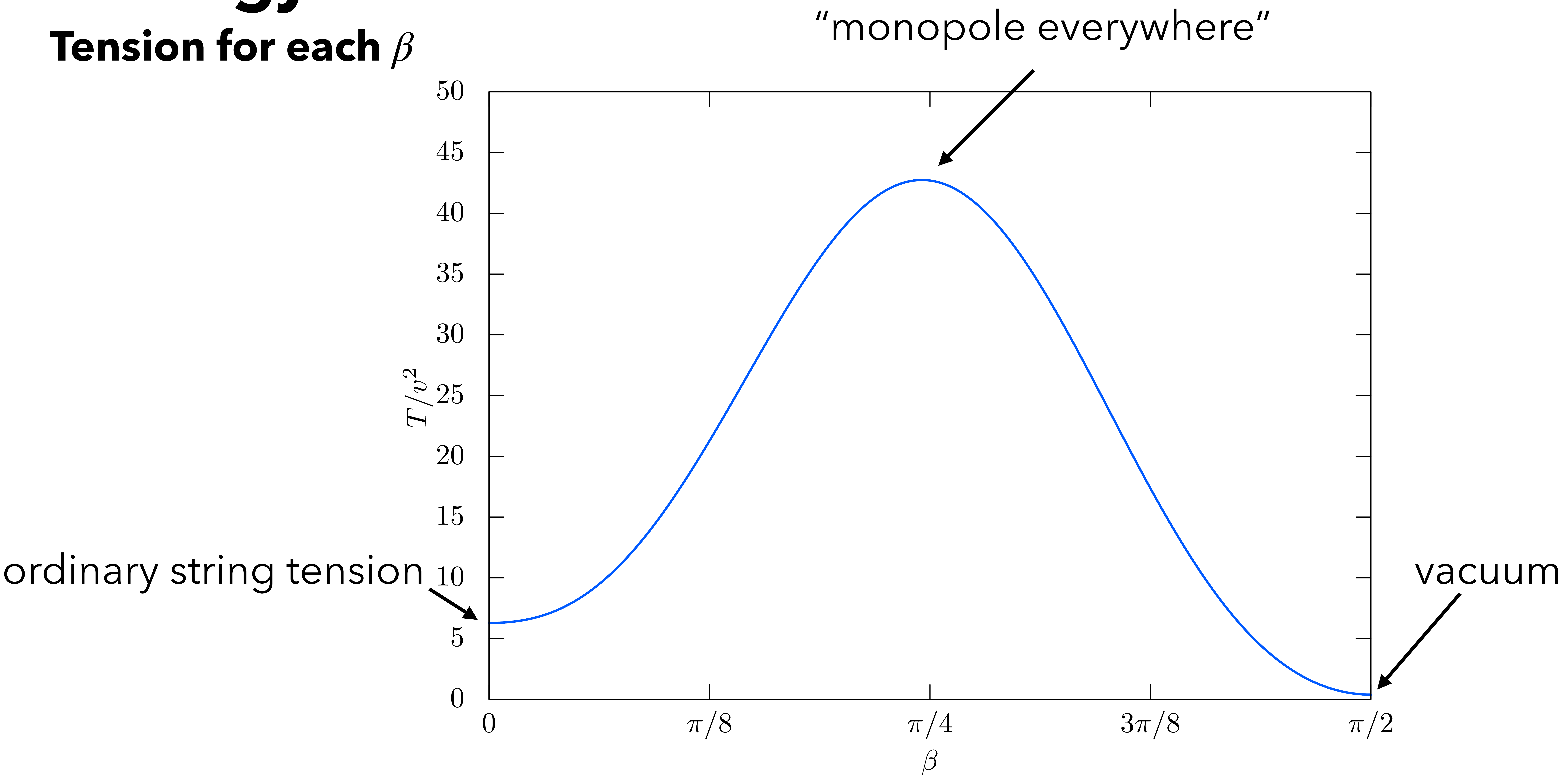


SU(2) triplet



Strategy

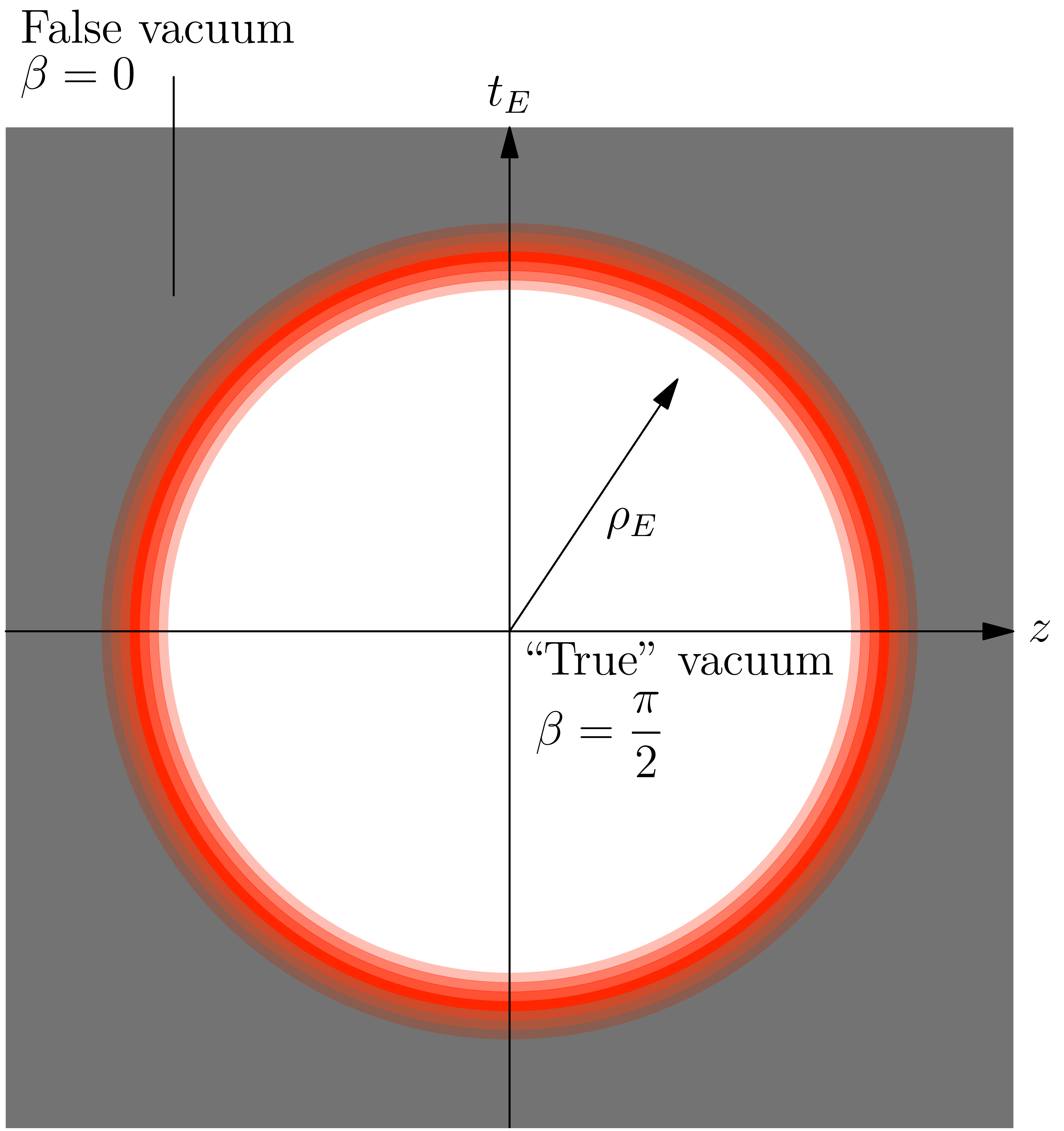
Tension for each β



Strategy

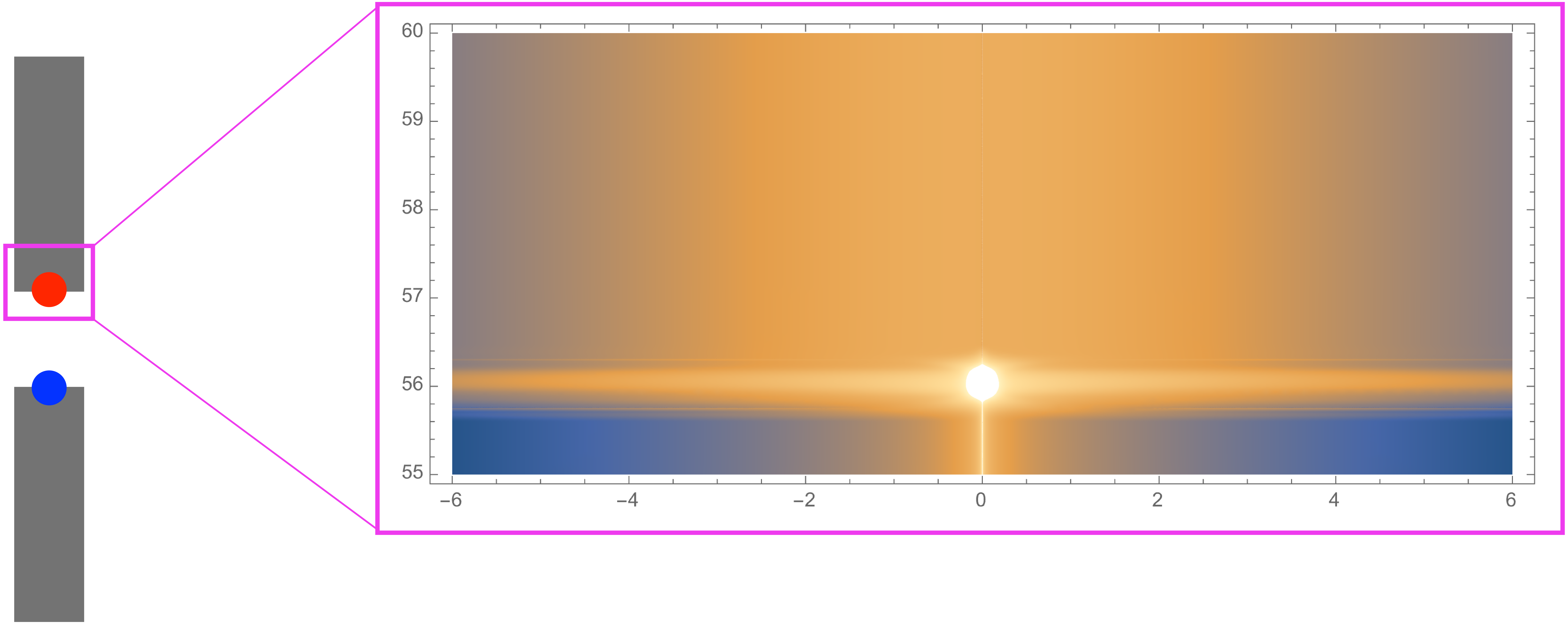
Step 2: Promote β to a field on the string

- Vary β on the worldsheet
- Effective 1D theory for $\beta(t_E, z) = \beta(\rho_E)$
  - Solve EoM → bounce action
- Upper bound on true S_B



Results

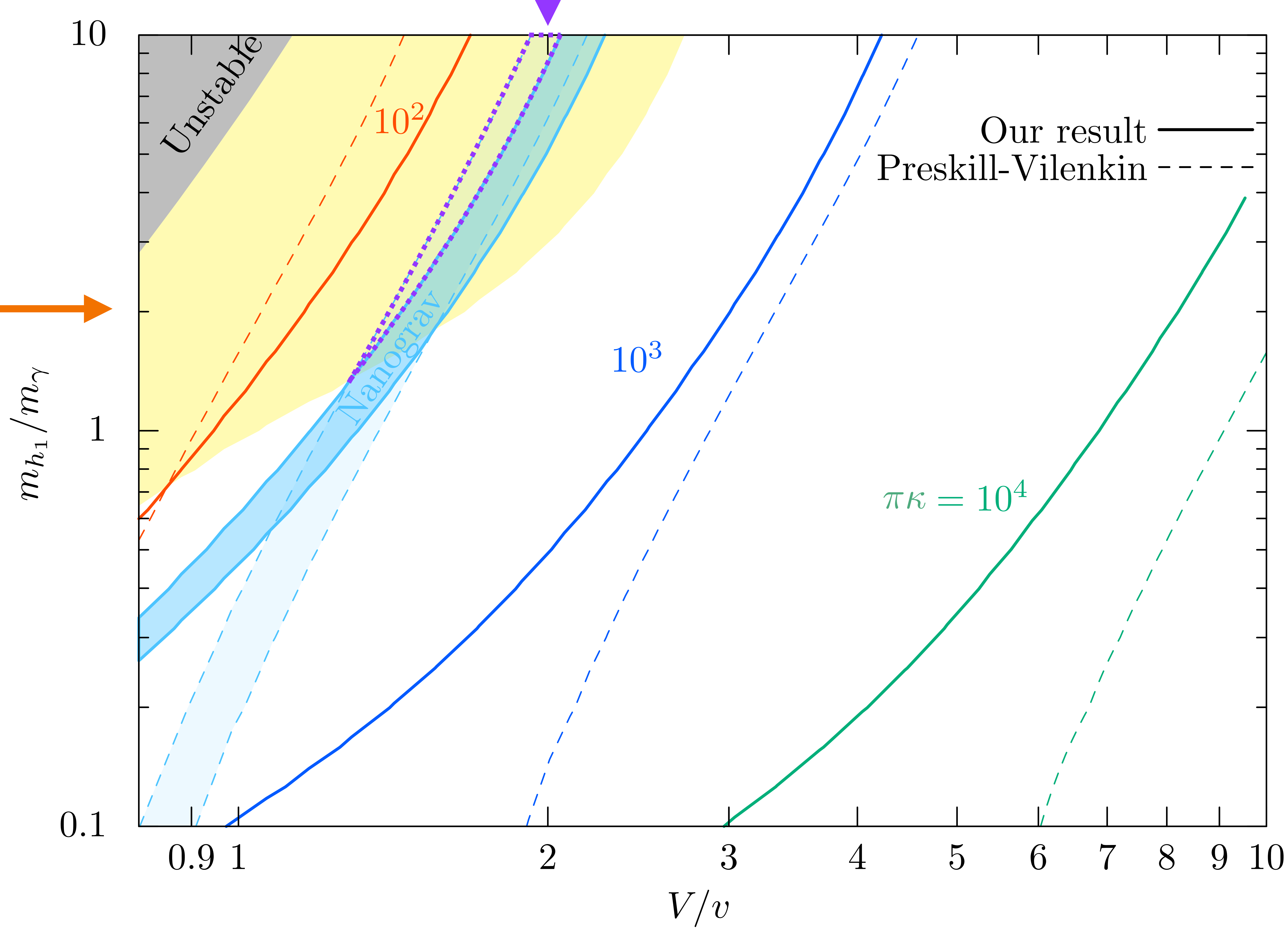
String being cut



Results

Interpretation of NANOGrav results modified

Preskill-Vilenkin < our bound



Conclusions & Outlook

- An upper bound on the bounce action for string breaking was calculated
  - free of the conventional assumption
- The Preskill-Vilenkin approximation can be inappropriate for the PTA data
- Next steps:
  - Optimal bounce action?
  - More realistic setup?

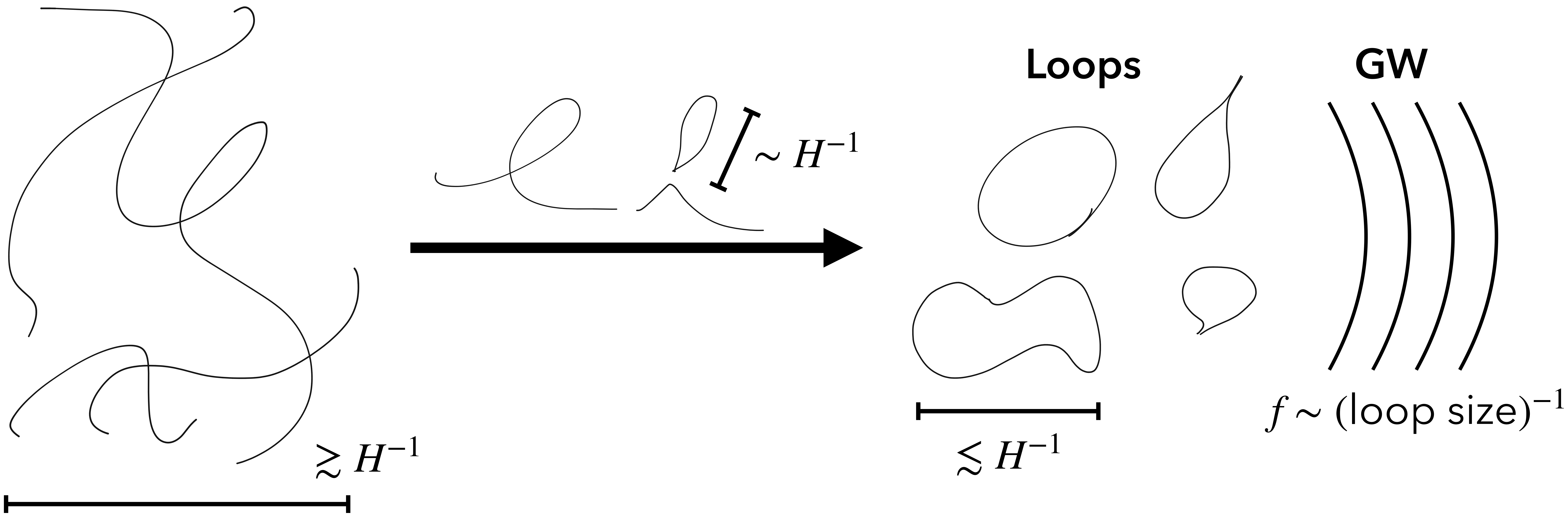
Thank you!

Backup

Cosmic Strings

Gravitational waves from loops

Network of long strings



Setup

A toy model for SU(2) → U(1) → 1

- SU(2) gauge theory
- Higgses:
  - ϕ: SU(2) triplet
    - $\langle\phi\rangle = V$: SU(2) → U(1), monopoles formed
  - h: SU(2) doublet
    - $\langle h\rangle = v$: U(1) → 1, strings formed

Setup

SU(2) gauge theory w/ adjoint Higgs & fundamental Higgs

- $\mathscr{L} = -\frac{1}{4g^2}F^2 - |Dh|^2 - \left(D\overrightarrow{\phi}\right)^2 - V_{\text{Higgs}}(h,\phi)$
  - h: SU(2) fundamental, ϕ: SU(2) adjoint
- $V_{\text{Higgs}}(h,\phi) = \lambda\left(|h|^2 - v^2\right)^2 + \tilde{\lambda}\left(\overrightarrow{\phi}^2 - V^2\right)^2 + \gamma\left|\left(\phi^a\frac{\tau^a}{2} - \frac{V}{2}\right)h\right|^2$
  - Assumptions: $\lambda,\ \tilde{\lambda},\ \gamma > 0$, $V > v$

Setup

Symmetry breaking pattern

- $V_{\text{Higgs}}(h,\phi) = \lambda\left(|h|^2 - v^2\right)^2 + \tilde{\lambda}\left(\vec{\phi}^2 - V^2\right)^2 + \gamma\left|\left(\phi^a\frac{\tau^a}{2} - \frac{V}{2}\right)h\right|^2$
- SU(2) → U(1) by $\phi^a = V\delta_3^a$
  - U(1) generator: $\tau^3/2$
- U(1) → 1 by $h_i = v\delta_i^1$

$\phi^a = V\delta_3^a$

$\gamma V^2 |h_2|^2$

Setup

Cosmic Strings and Monopoles

- 1st SSB: SU(2) → U(1) by $\phi = V\delta_3^a$
  - Monopoles formed by ϕ
- 2nd SSB: U(1) → 1 by $h_1 = ve^{i\times 0}$
  - Cosmic strings formed by h_1
  - But SU(2) is simply connected → only metastable

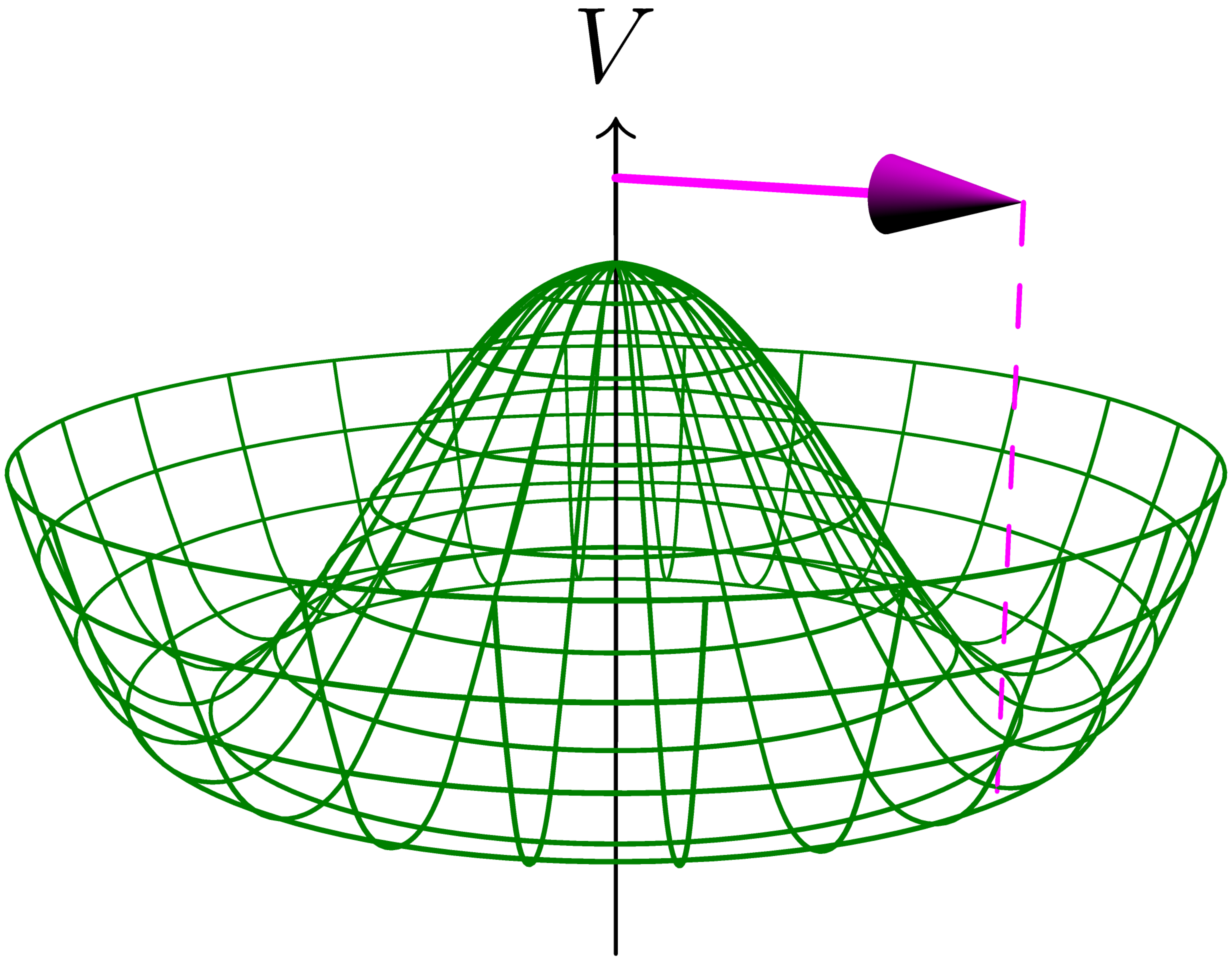
$\sqrt{\kappa_{PV}} \propto V/v$

→ interested in $V/v = \mathcal{O}(1)$

Cosmic Strings

from U(1) breaking

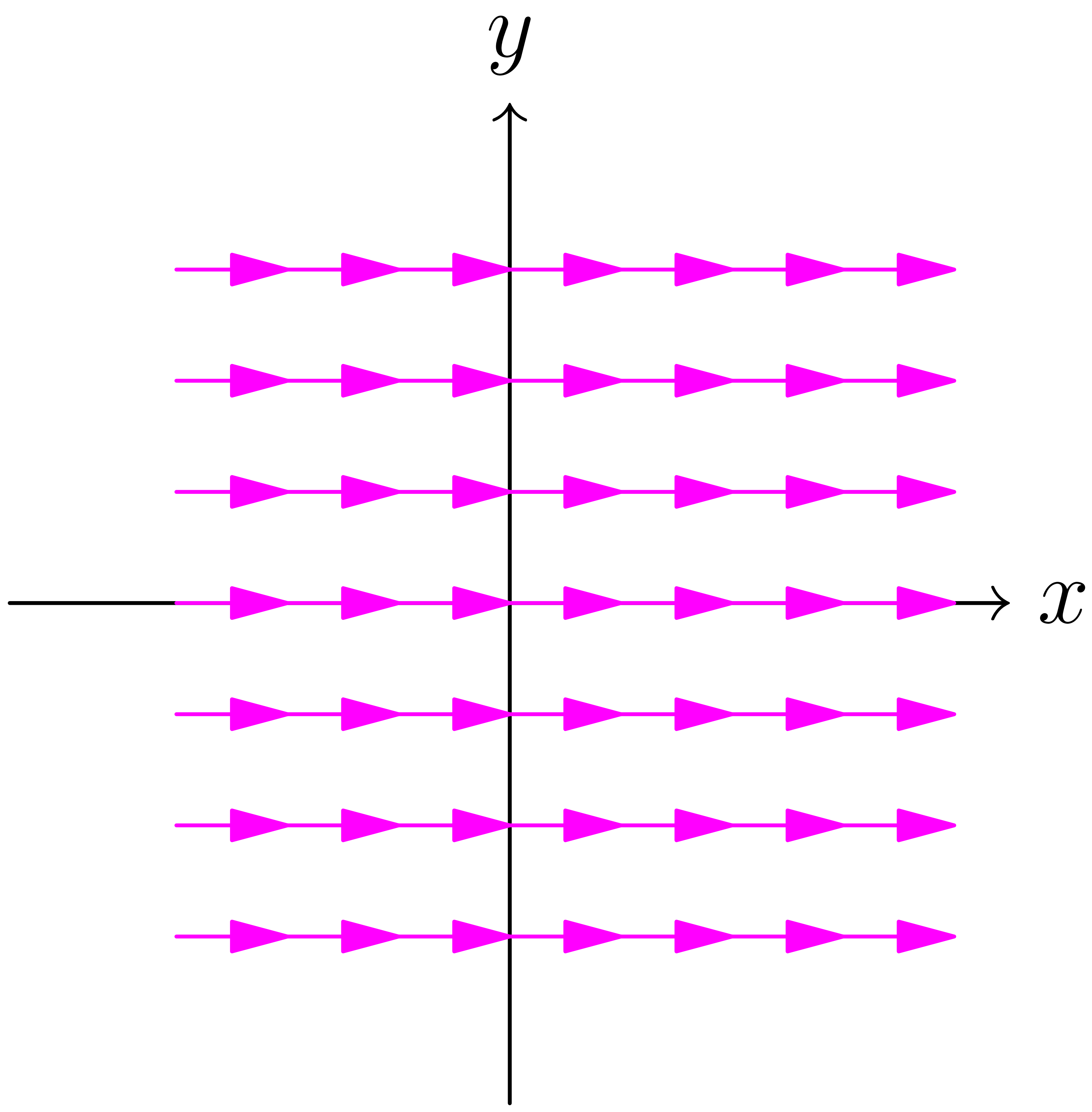
- Simplest setup: abelian Higgs
- $V(\phi) = \lambda \left(\phi^\dagger \phi - v^2\right)^2$
- U(1): $\phi \to e^{i\alpha}\phi$
  - broken by $\langle \phi \rangle = v$



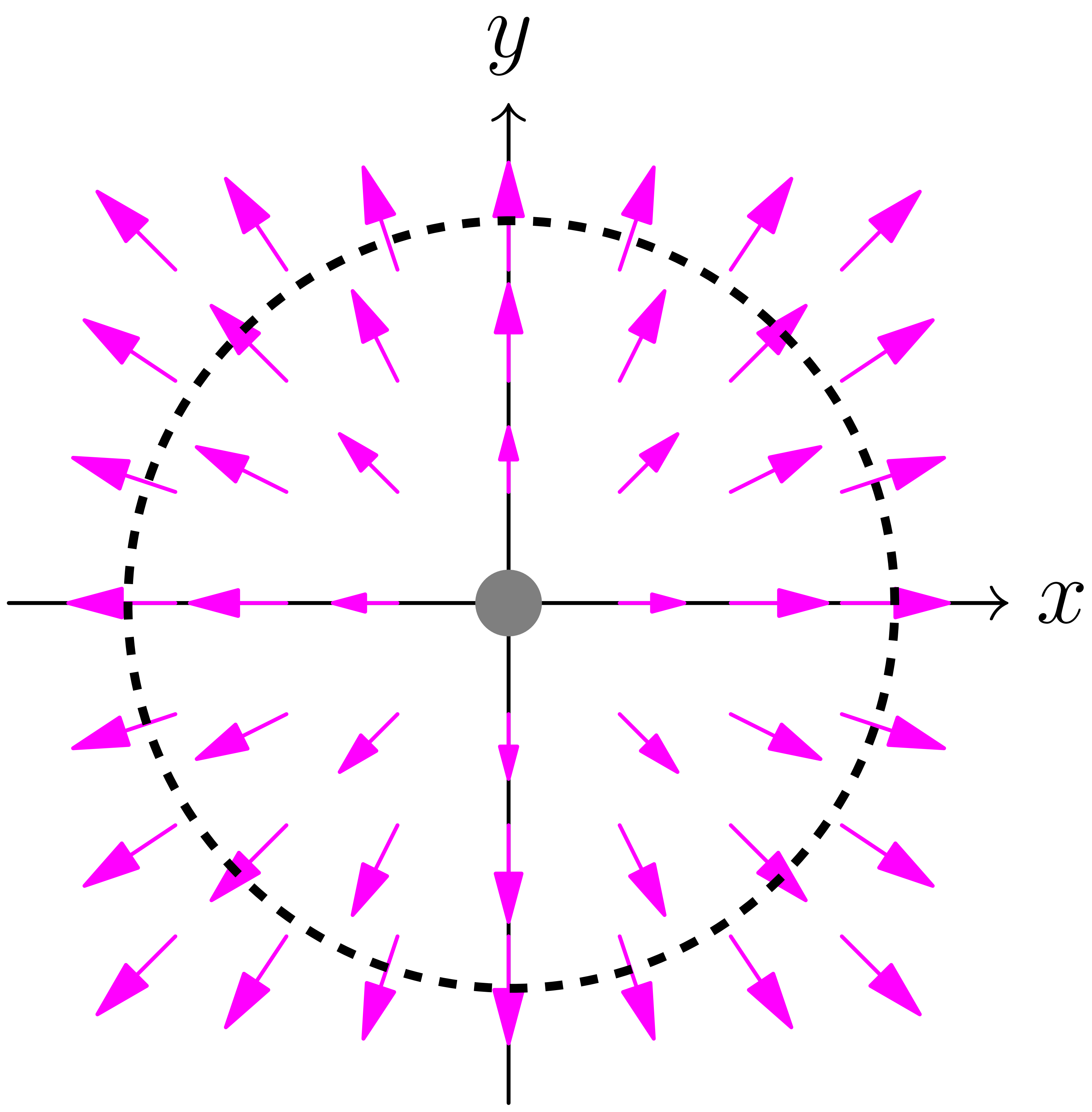
Cosmic Strings

from U(1) breaking (ctd.)

Vacuum

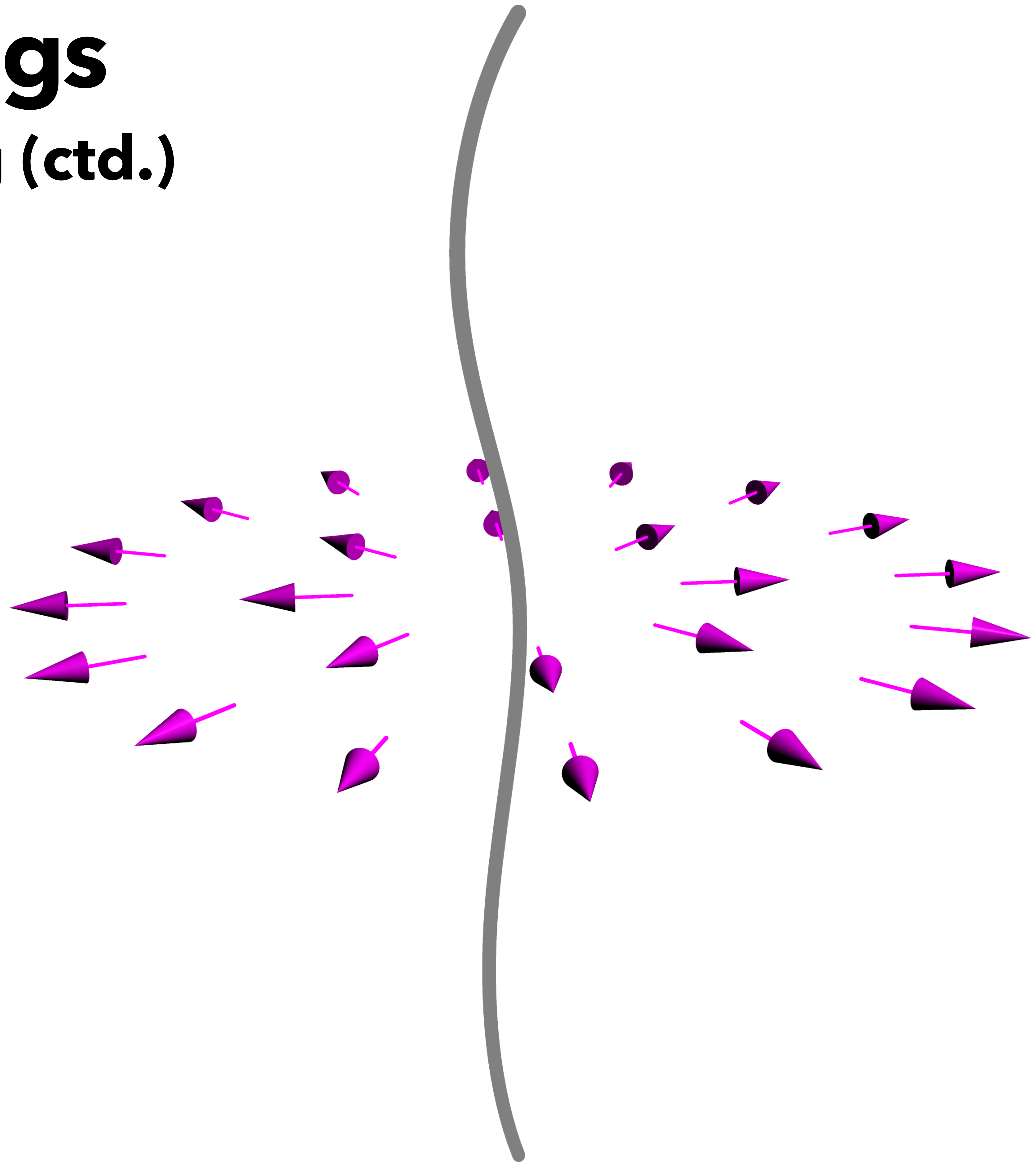


Wound about z axis



Cosmic Strings

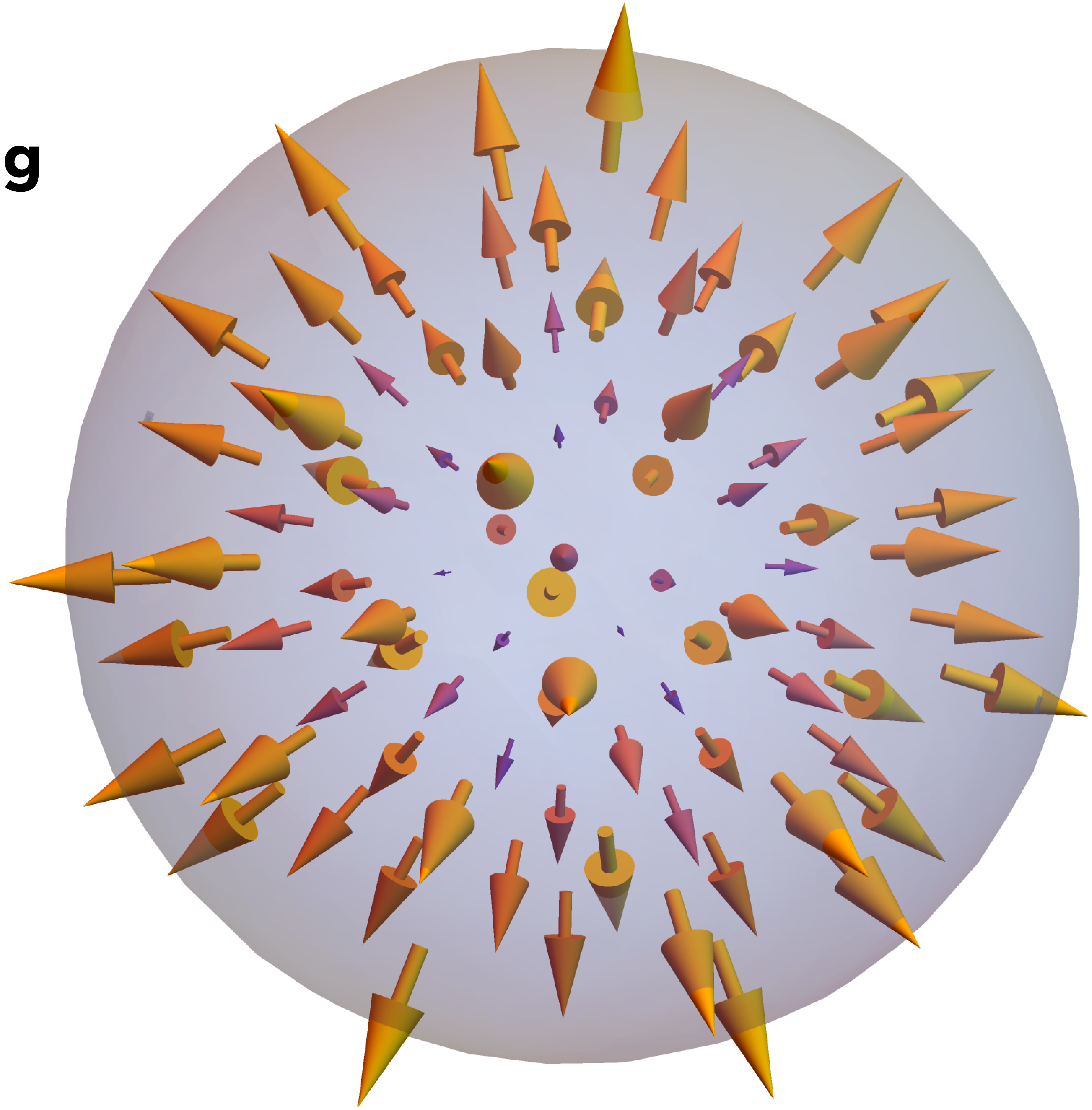
from U(1) breaking (ctd.)



Monopoles

0 dimensional cousin of cosmic string

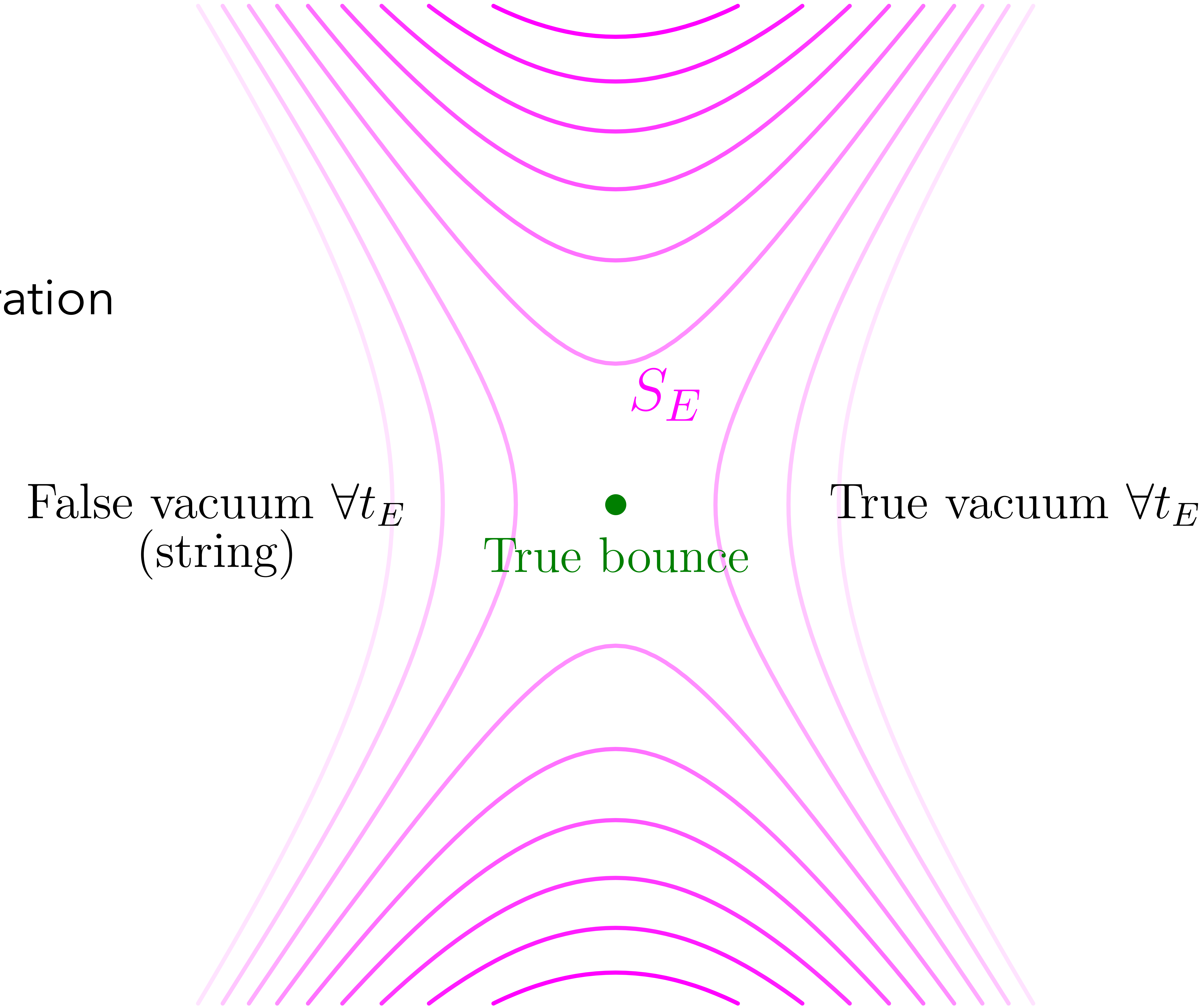
- Arise from winding on 2D sphere
- Behave like point-like particles*



Strategy

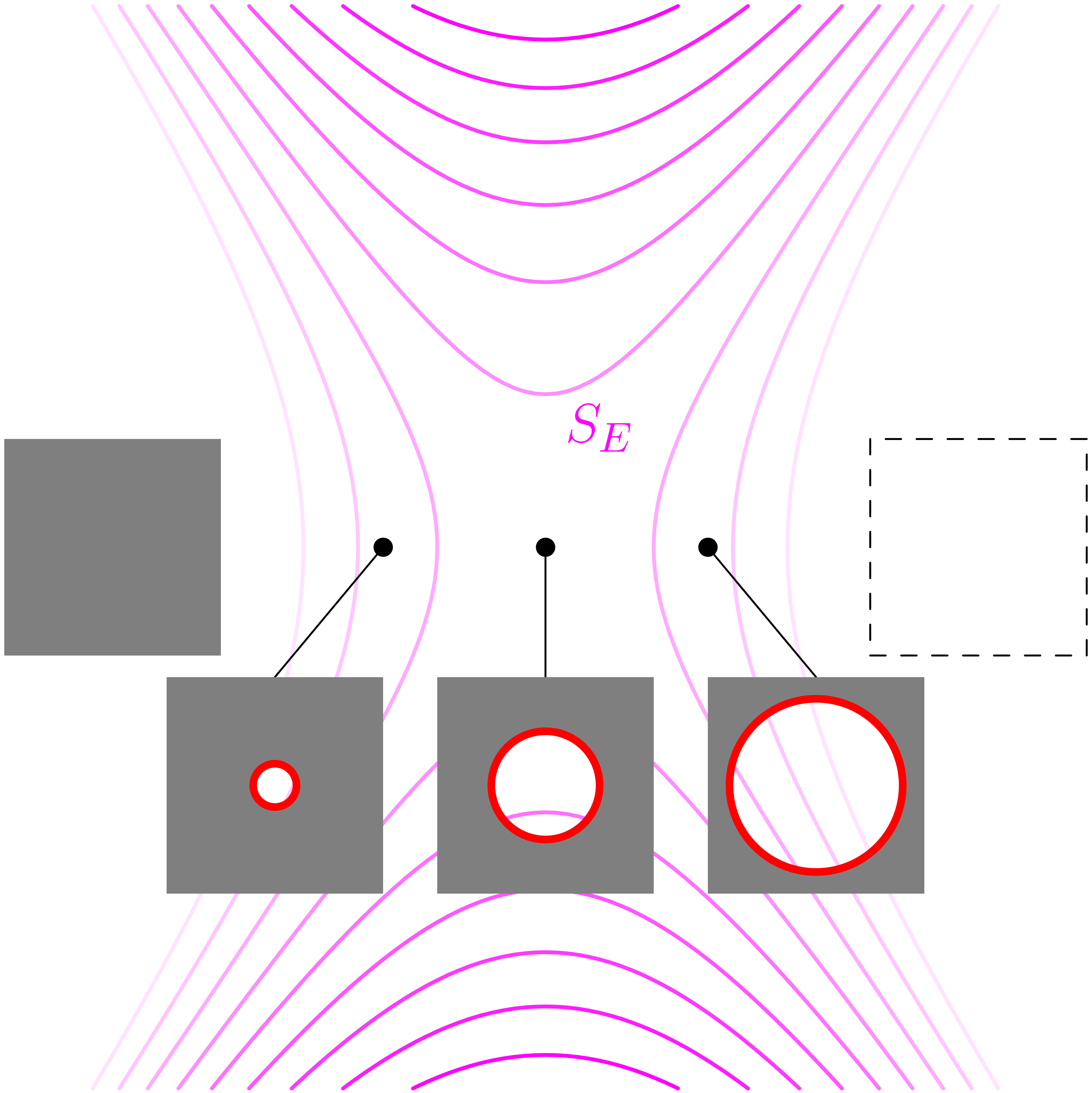
Why it is an upper bound

Each point: 4D field configuration



Strategy

Why it is an upper bound

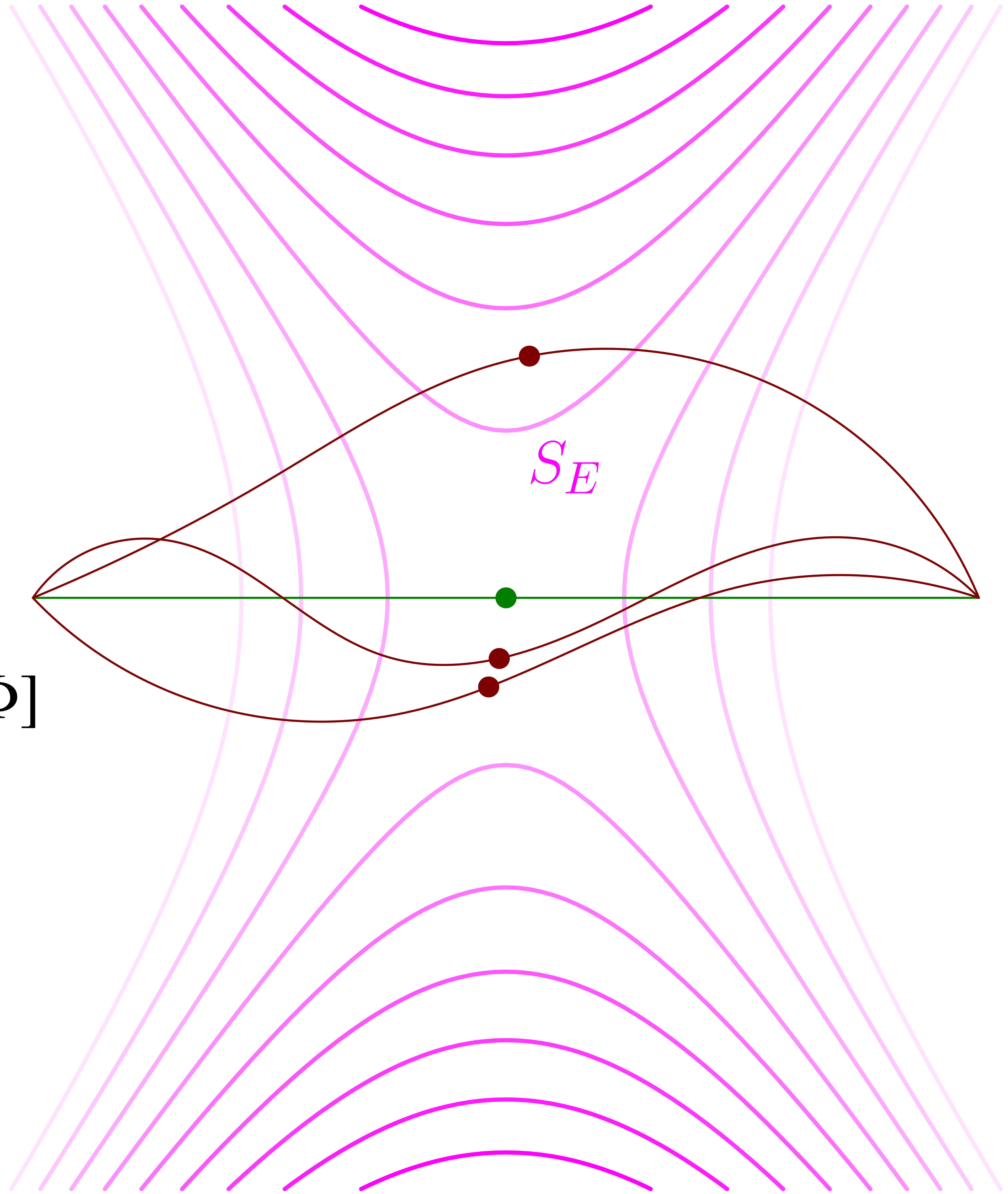


Strategy

Why it is an upper bound

True (optimal) bounce action:

$$S_E[\bullet] = \min_{\text{path joining the two sides}} \max_{\Phi \in \text{path}} S_E[\Phi]$$



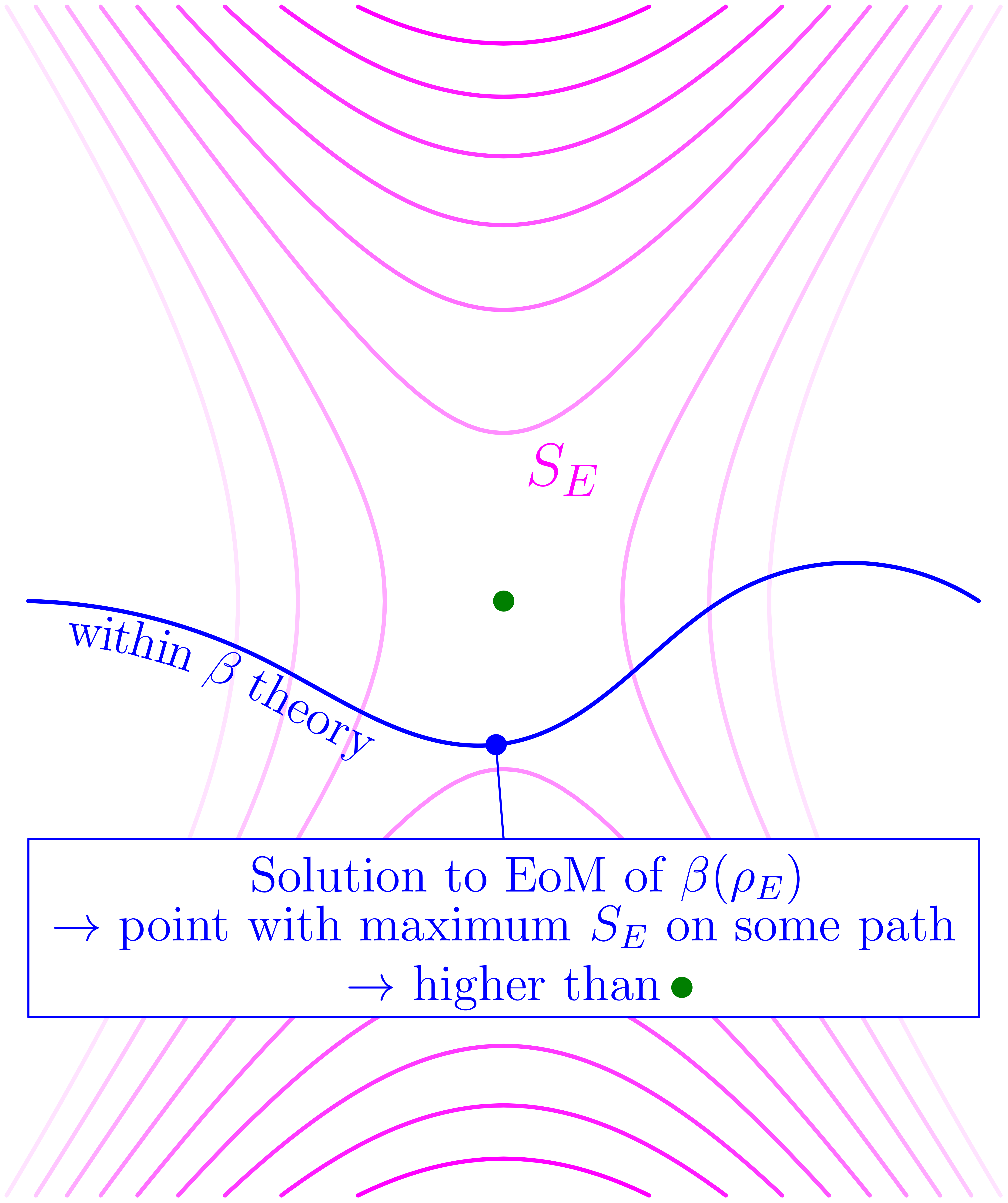
Strategy

Why it is an upper bound

∃path that

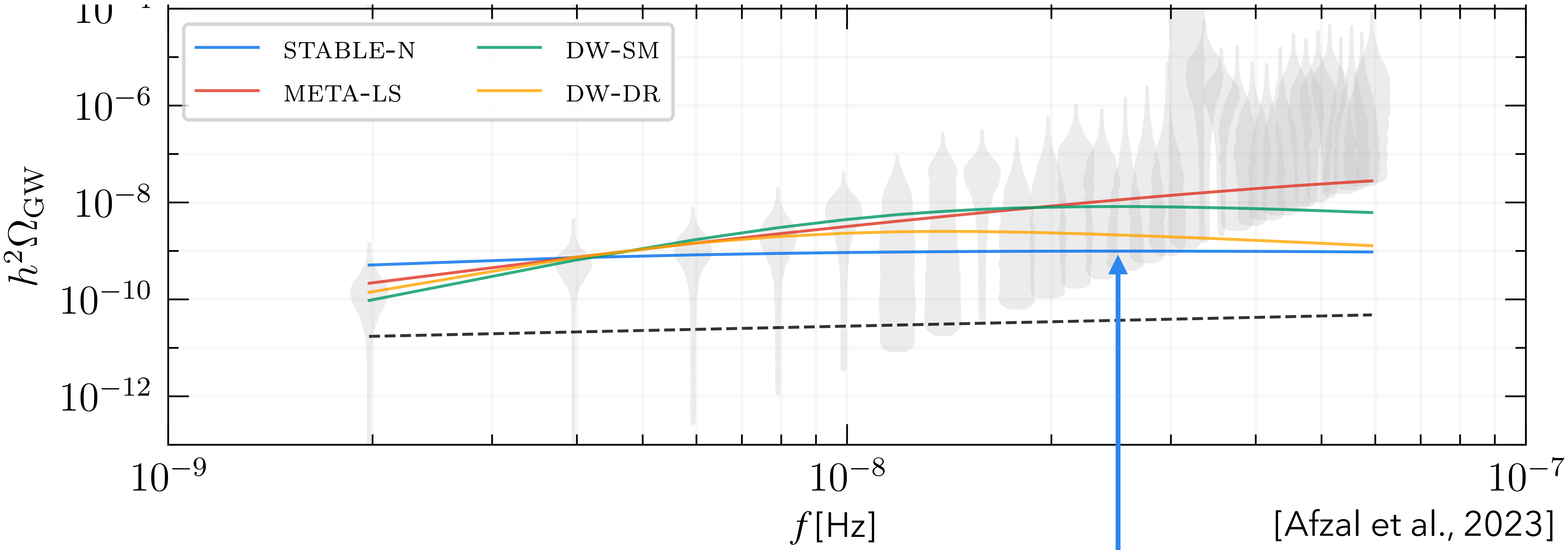
- joins the two vacua
- stays within the effective β theory
- has maximum S_E at ●

$\rightarrow S_E[\bullet] \geq S_E[\bullet]$



Cosmic Strings for PTA

Failure of stable cosmic strings

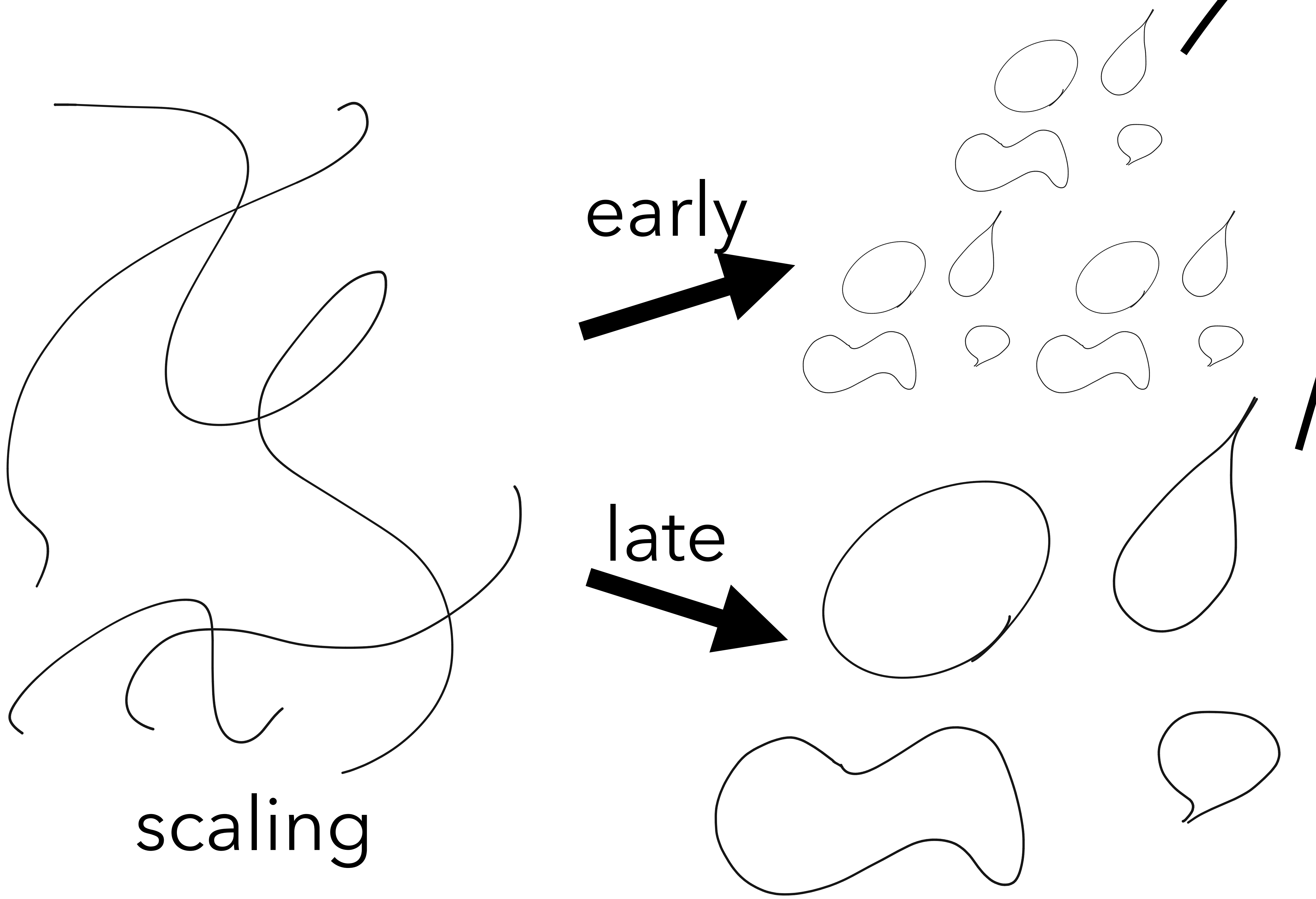


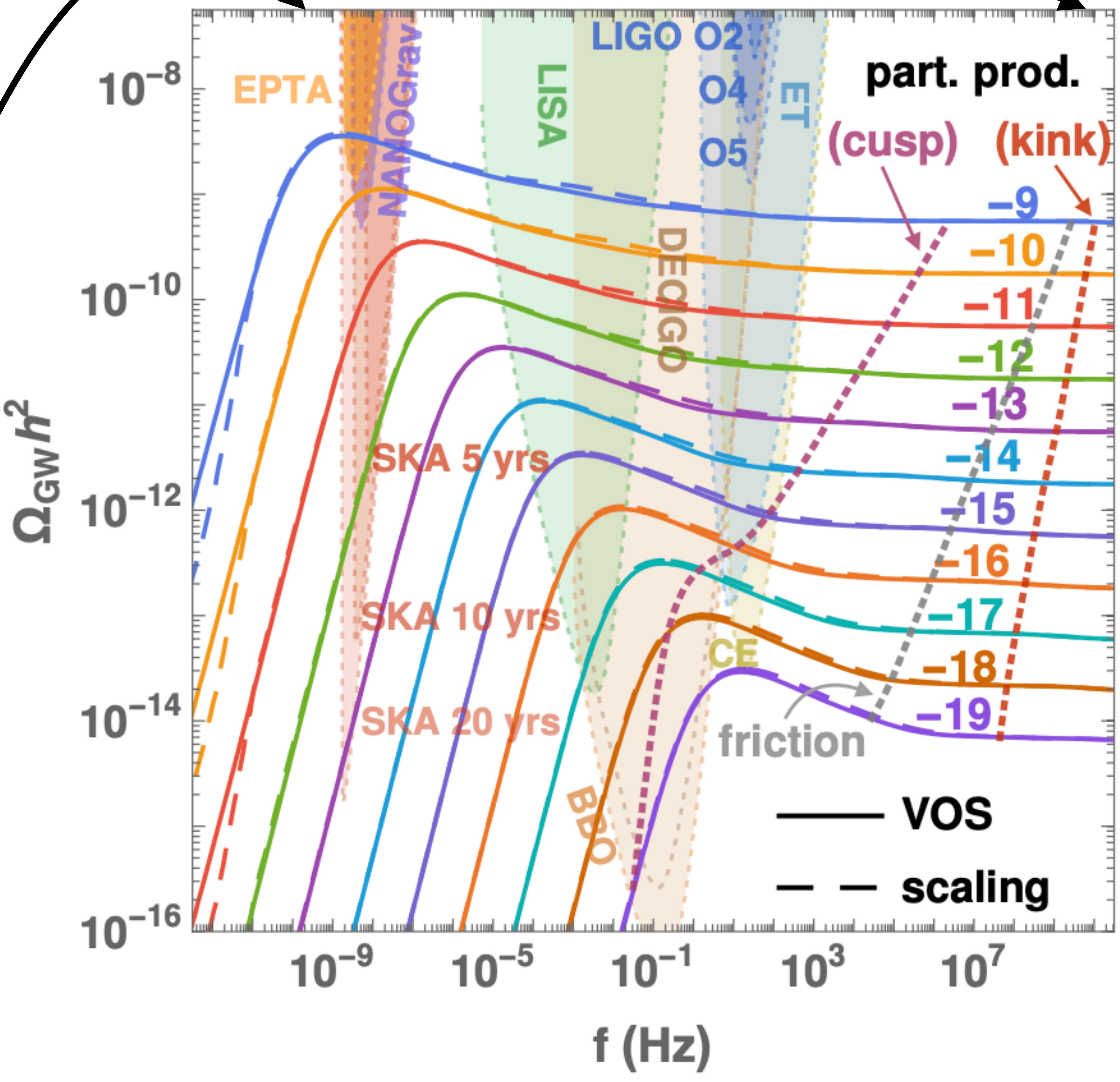
[Afzal et al., 2023]

Blue tilt not reproduced

Cosmic Strings for PTA

Stable strings vs. PTA



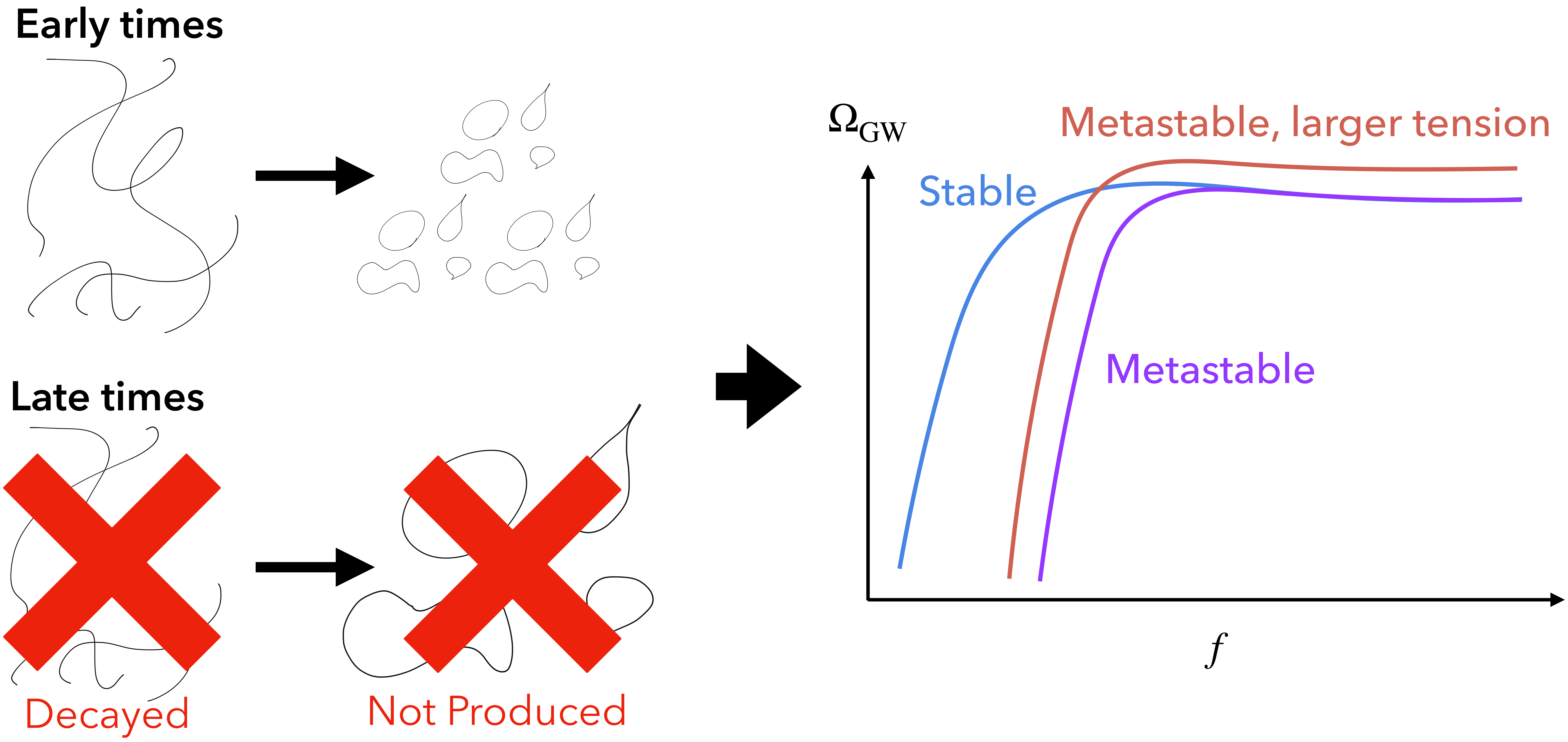


[Gouttenoire et al., 2019]

Only parameter: tension

→ cannot fit height & tilt

Metastable Cosmic Strings



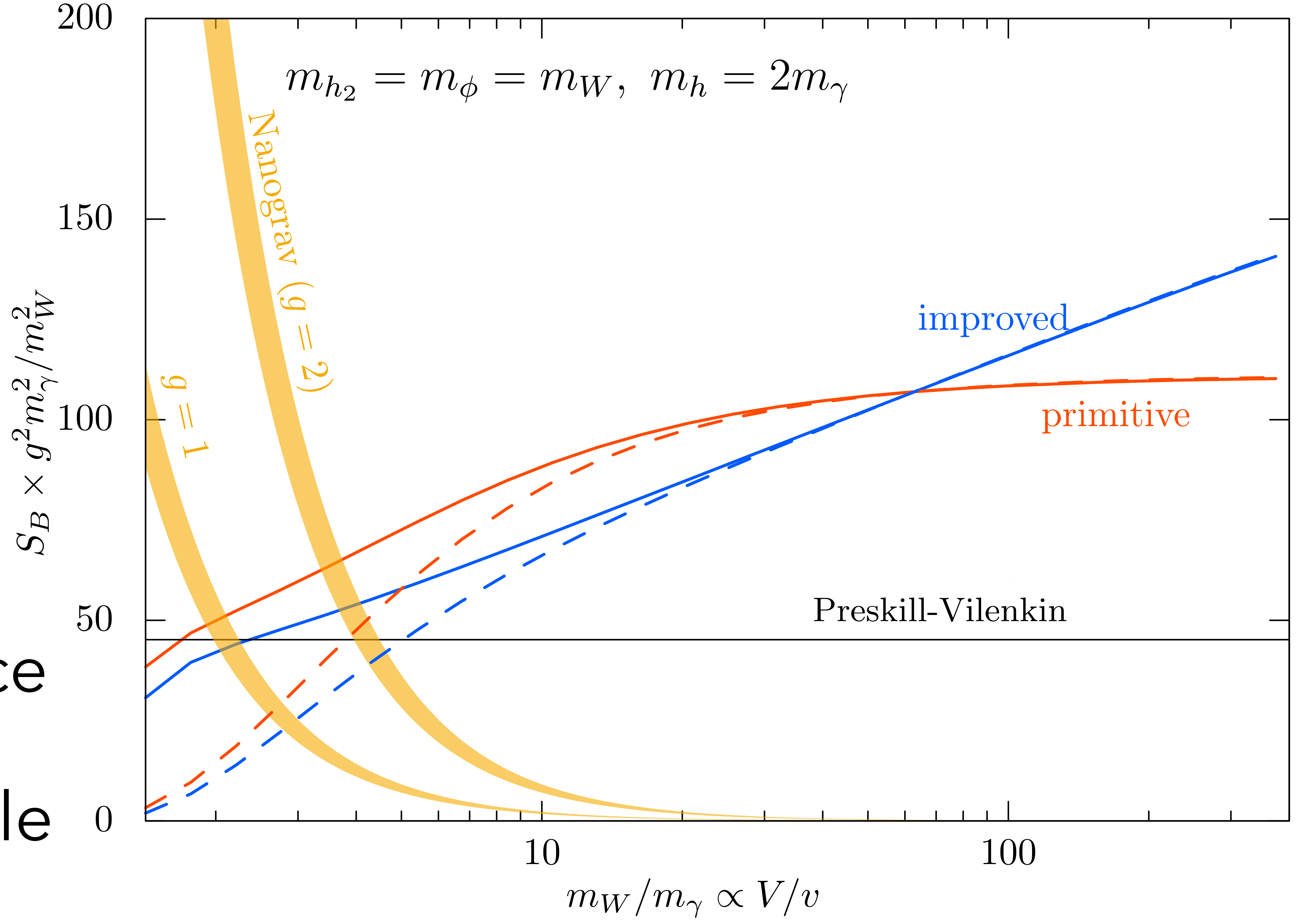
When does thin-wall break down?

- Introduce "β-thin-wall approximation"
- Thin-wall approximation to the 1D effective theory of $\beta(\rho_E)$
  - Valid only for $V \gg v$
- Preskill-Vilienkin approximation: similar but different
  - β-thin-wall: Ansatz → effective 1D theory → thin-wall
  - Preskill-Vilenkin: assume thin-wall in the 4D theory

Results

Hint for stronger results?

- solid: bounce, dashed: β-thin-wall
- For large hierarchy:
  - Primitive: Preskill-Vilenkin × $\mathcal{O}(1)$
- For small hierarchy:
  - β-thin-wall deviates from the bounce
    - Preskill-Vilenkin: also questionable



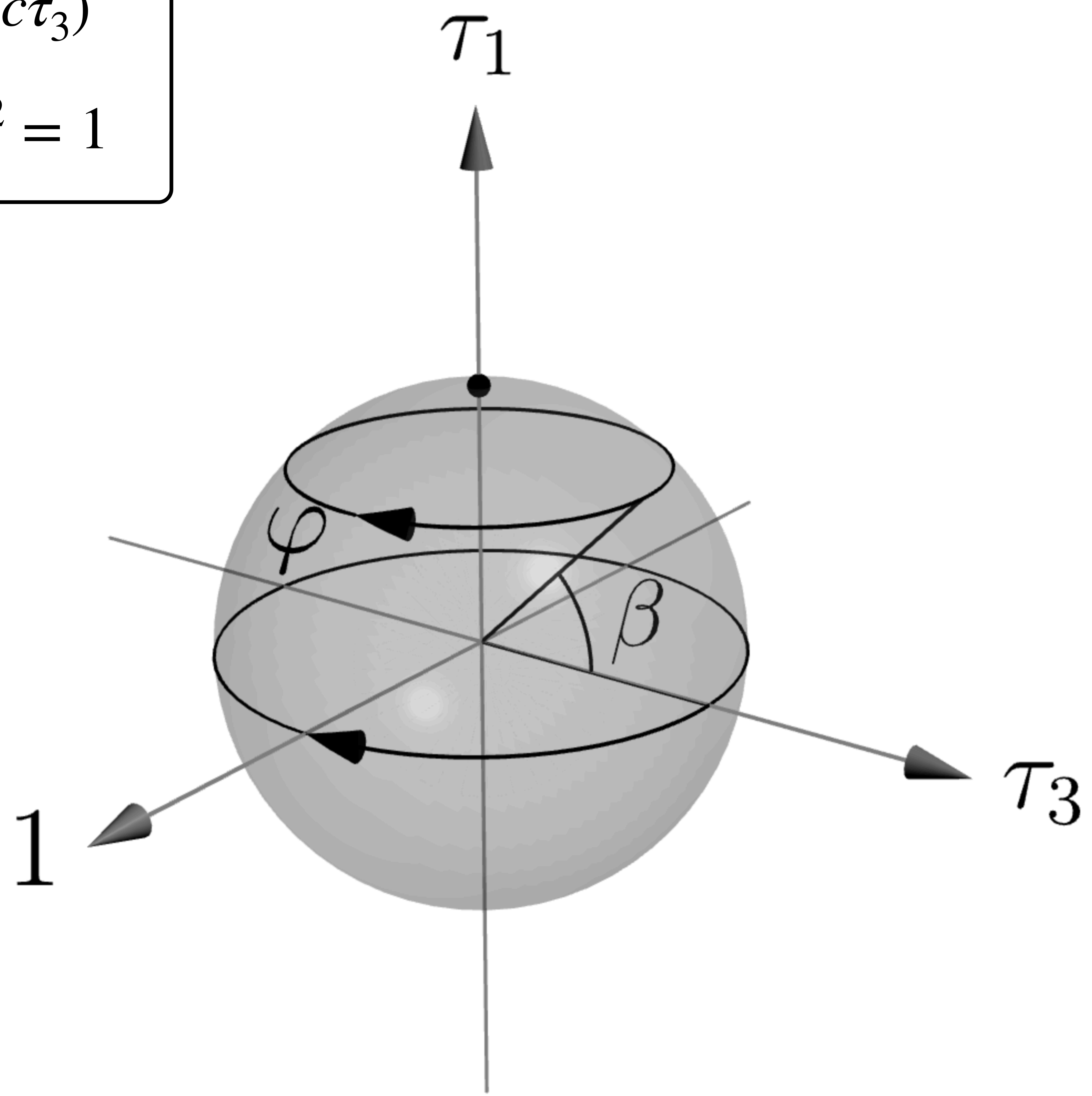
Strategy

Unwinding the string

$$a + i(b\tau_1 + c\tau_3)$$

$$a^2 + b^2 + c^2 = 1$$

- $U = e^{-i\tau_3\varphi} \cos\beta + i\tau_1 \sin\beta \in S^2 \subset \mathrm{SU}(2)$
  - $h = U(v\ 0)^\top,\ \phi = U(\tau_3/2)U^\dagger$
  - controls the U(1) winding
    - $h_1 = e^{-i\varphi} v$ for $\beta = 0$
  - $U = i\tau_1 = \mathrm{const}.$ for $\beta = \pi/2$
    - completely unwound

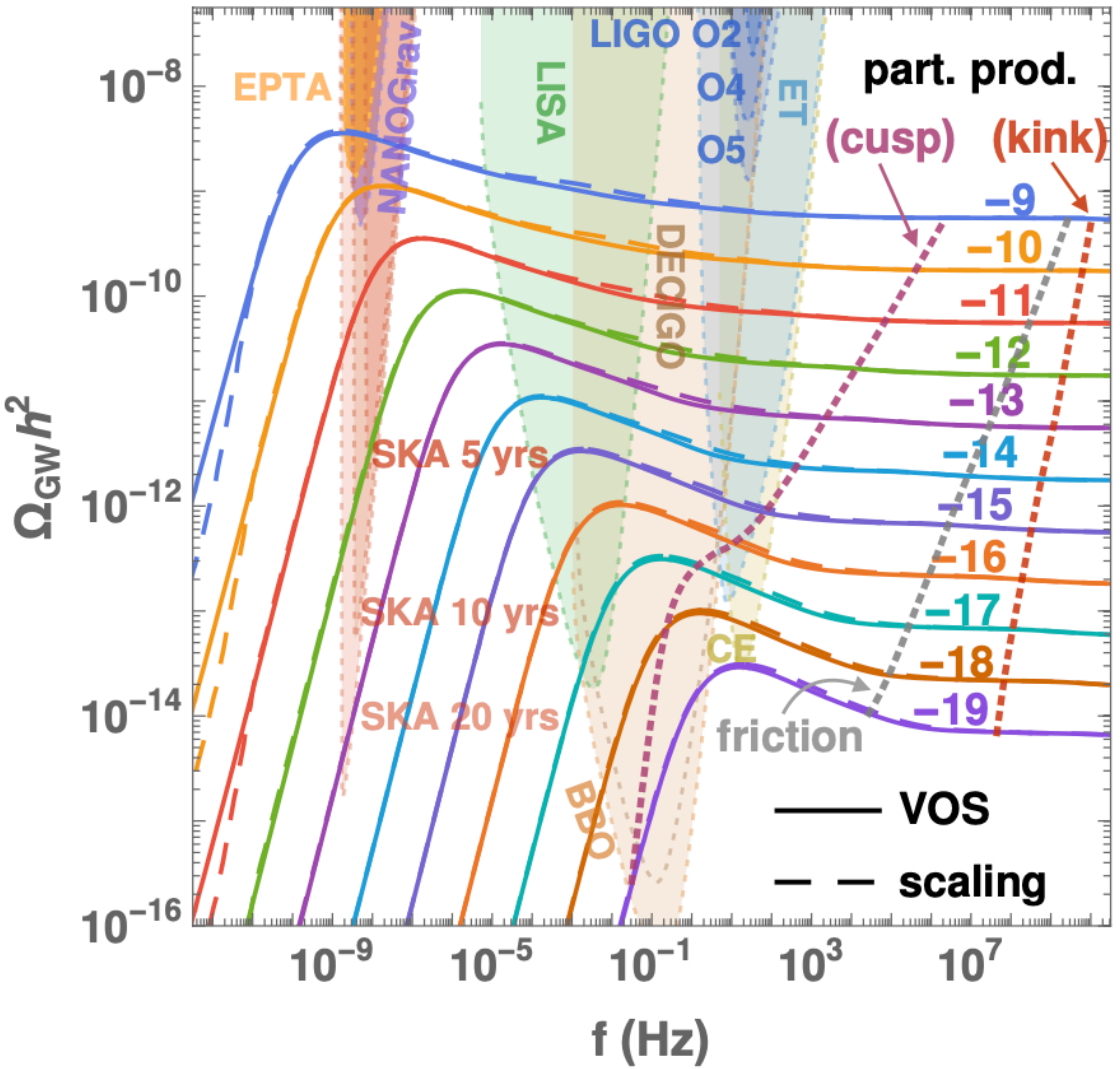


On metastability

Stable strings vs. PTA

- Nanograv's spectrum: blue tilted
- GW spectrum from stable cosmic strings →
  - The amplitude and the low-frequency cutoff correlate
  - → Mismatch with Nanograv

late emission ← early emission

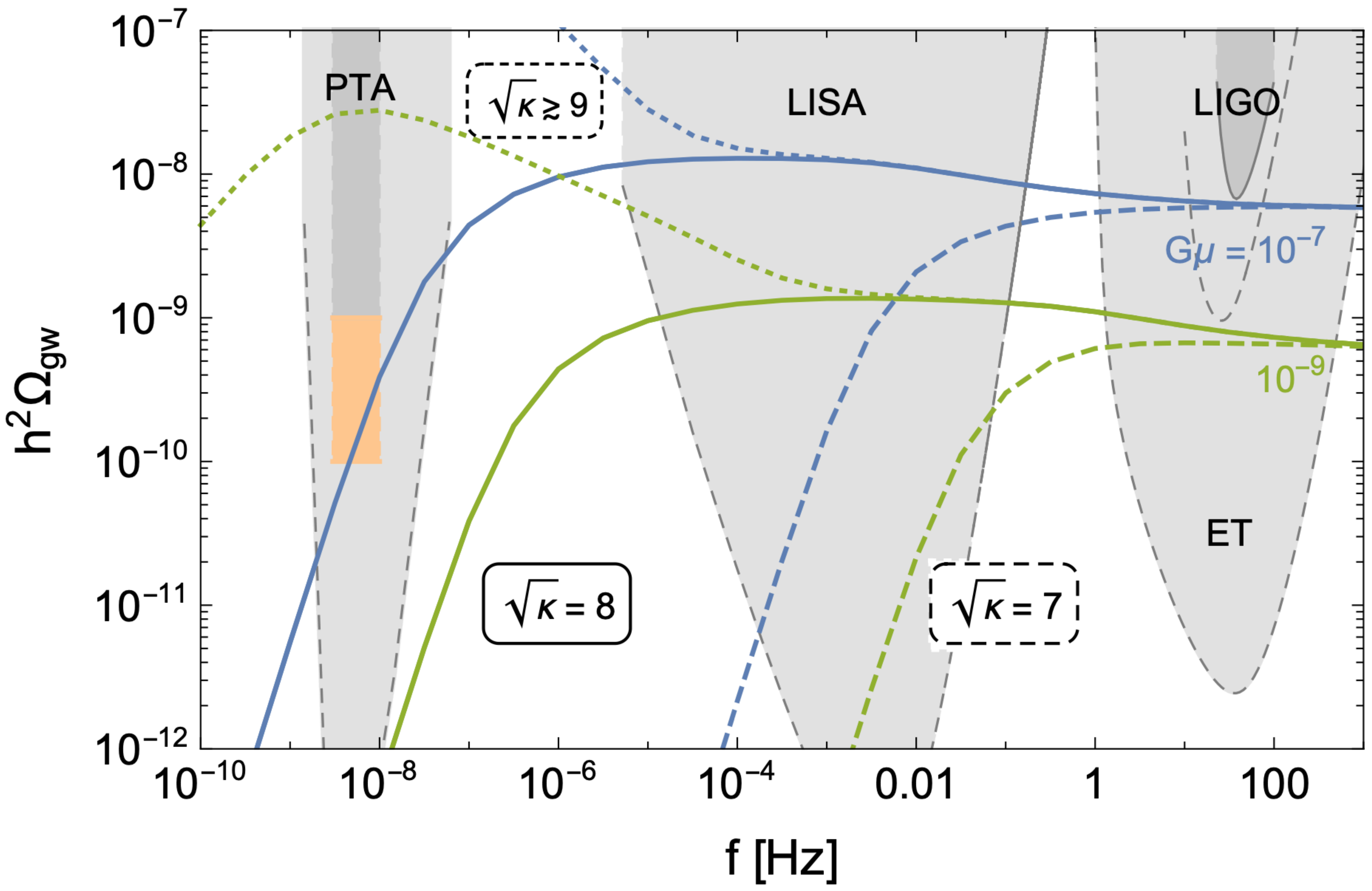


[Gouttenoire et al., 2019]

On metastability

Metastable strings vs. PTA

- Finite lifetime moves the cutoff to the right
  - → better fit with the PTA data



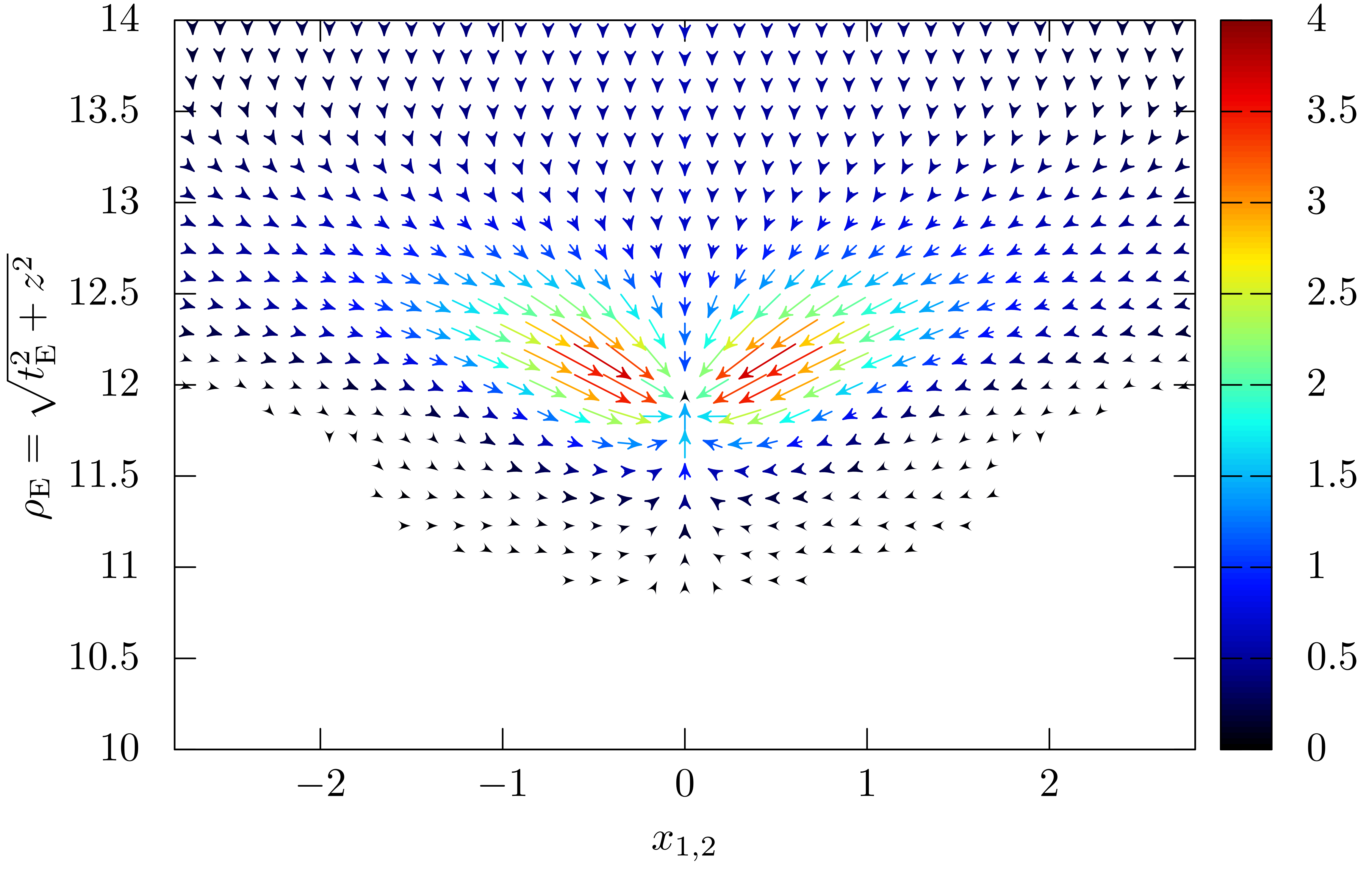
[Buchmüller et al., 2023]

Magnetic fields

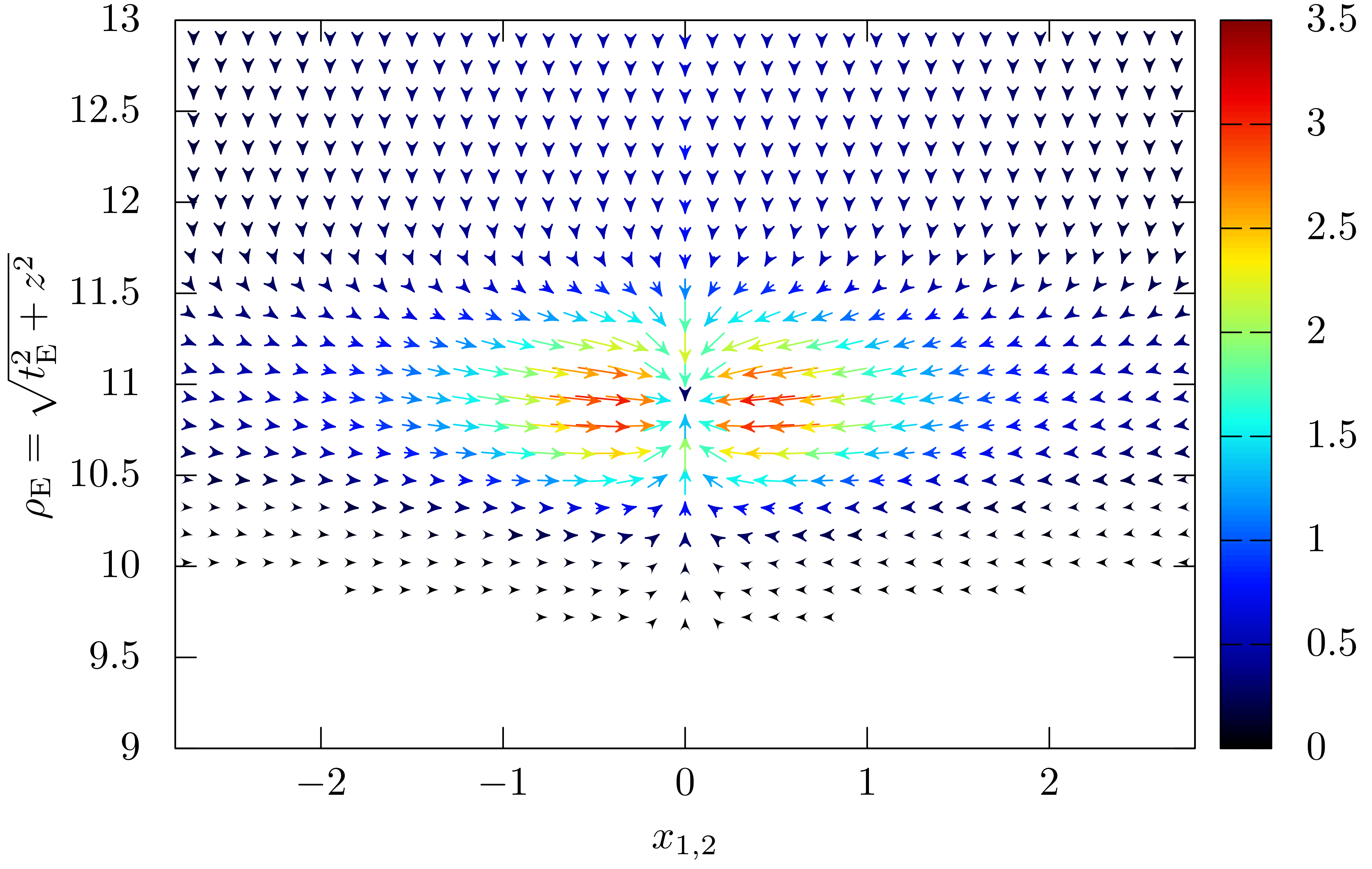
Cross section of the breaking string

$$B_i = \frac{1}{2}\epsilon^{ijk}\frac{\phi^a}{V}F^a_{jk}$$

Primitive

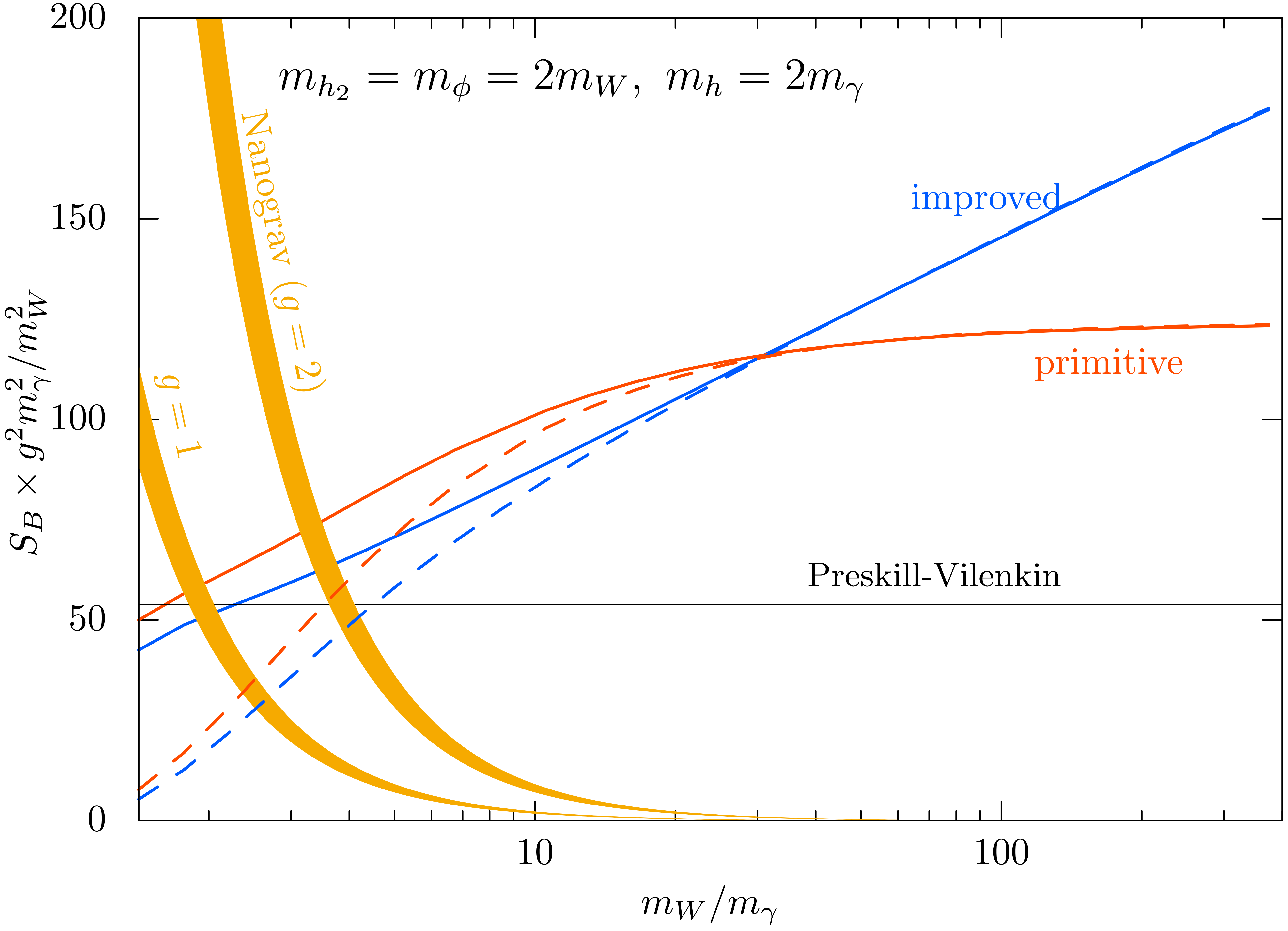


Improved



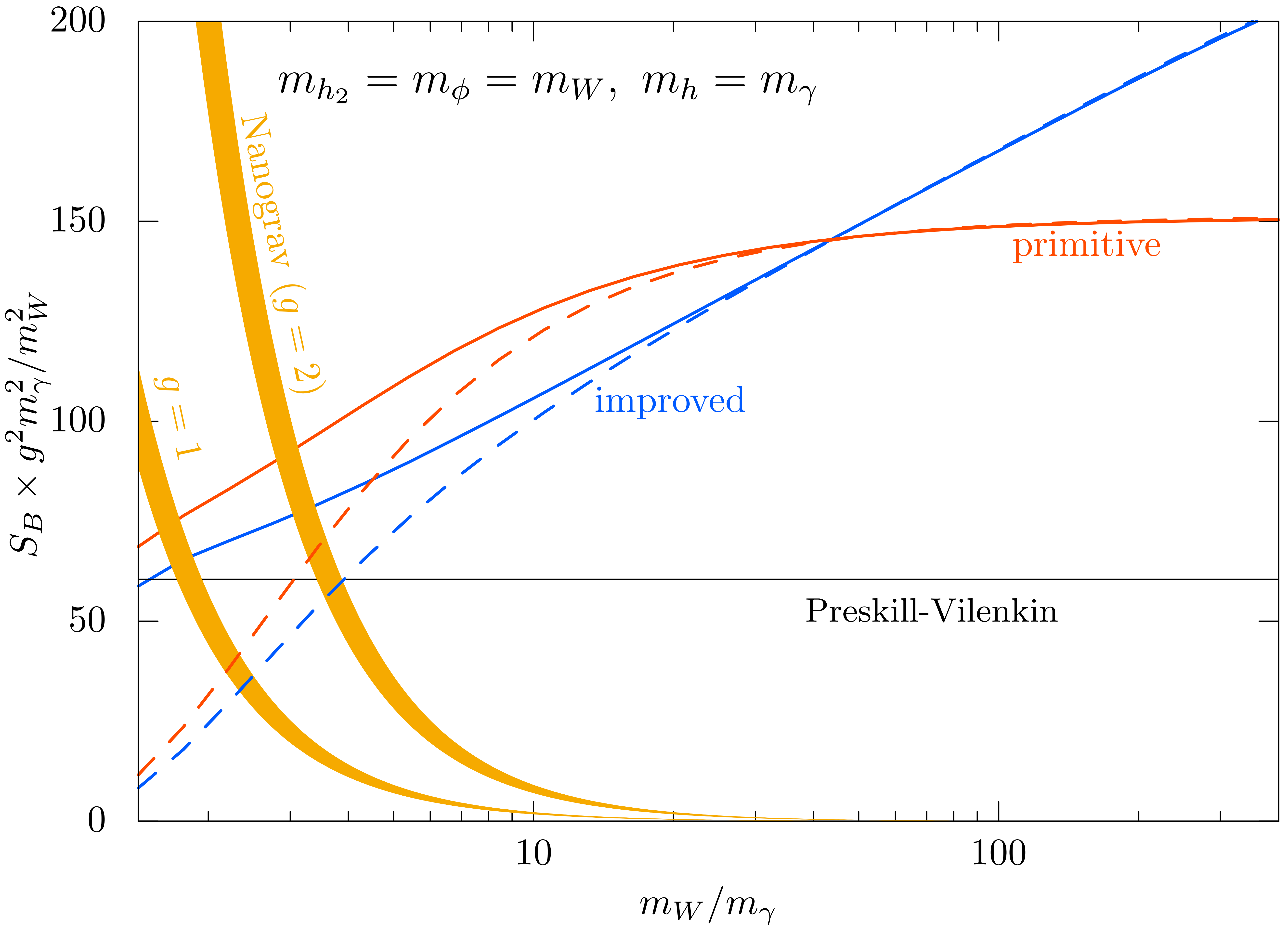
Other parameters

Light W



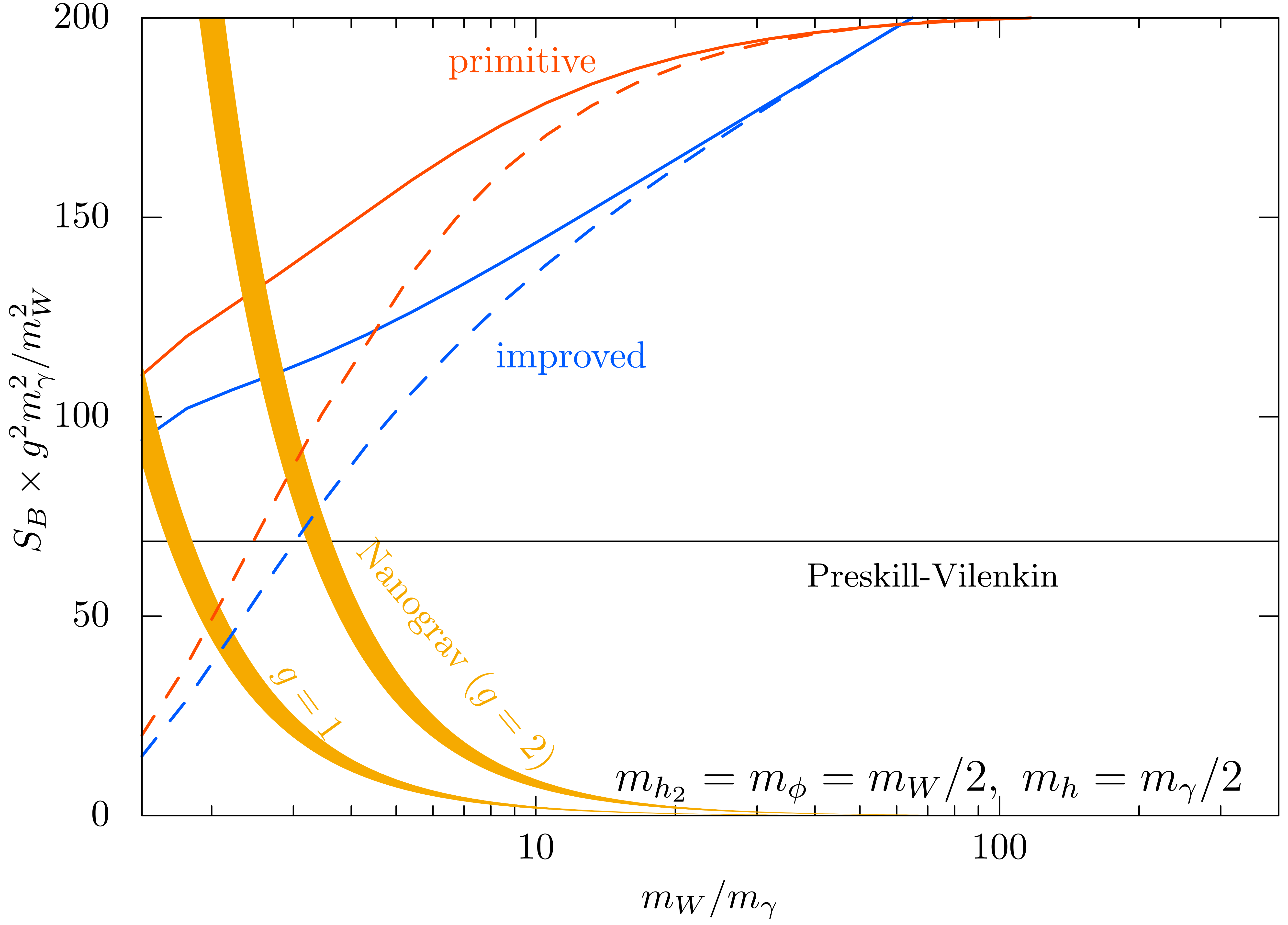
Other parameters

SUSY-like



Other parameters

Heavy W

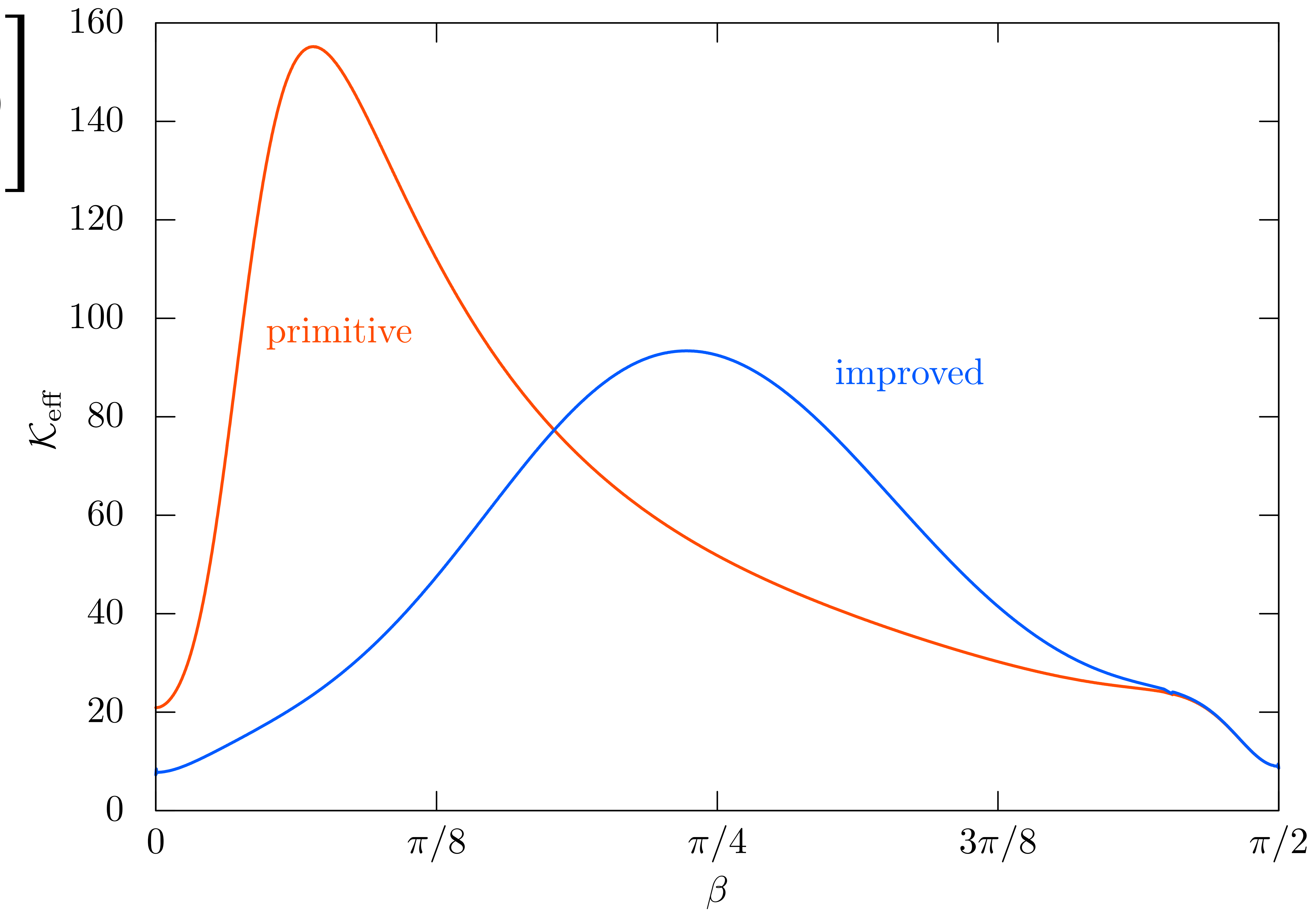


β-thin-wall approximation

- $$S_B = 2\pi \int_0^\infty \rho_E \mathrm{d}\rho_E \left[\frac{1}{2}\mathscr{K}_{\mathrm{eff}}(\beta)\beta'^2 + T(\beta) - T(0)\right]$$
  $$\approx -\pi\rho_E^{*2}\left[T(0) - T\left(\frac{\pi}{2}\right)\right] + 2\pi\rho_E^* \int_{\mathrm{wall}} \mathrm{d}\rho_E \left[\frac{1}{2}\mathscr{K}_{\mathrm{eff}}(\beta)\beta'^2 + T(\beta) - T(0)\right]$$
  $$= -\pi\rho_E^{*2}\left[T(0) - T\left(\frac{\pi}{2}\right)\right] + 2\pi\rho_E^* m_{\mathrm{eff}}$$
  - $m_{\mathrm{eff}} := \int_0^{\frac{\pi}{2}} \mathrm{d}\beta \sqrt{2\mathscr{K}_{\mathrm{eff}}(\beta)(T(\beta) - T(0))}$
- Maximum: $S_B = \pi \dfrac{m_{\mathrm{eff}}^2}{T(0) - T(\pi/2)}$

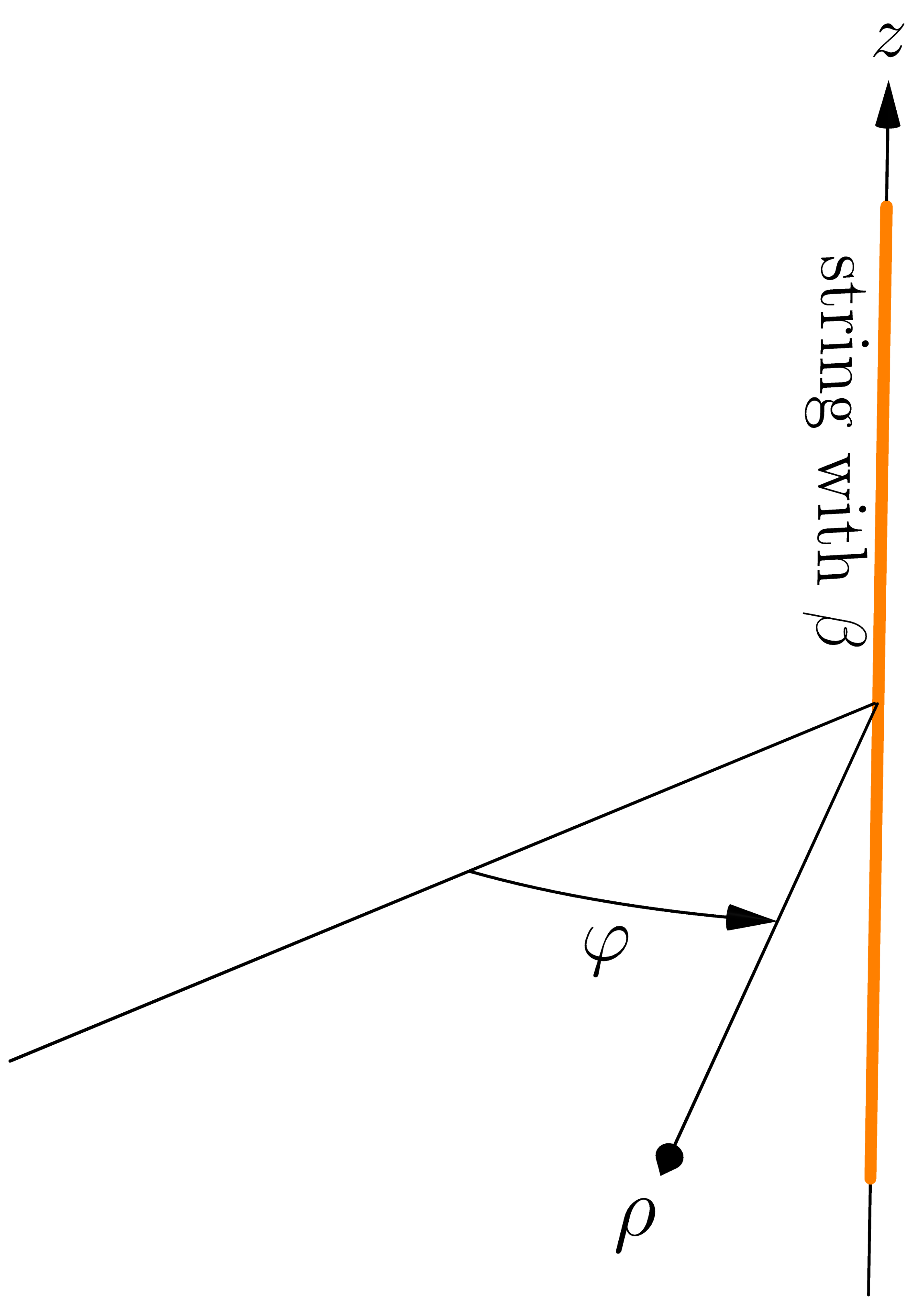
Kinetic term

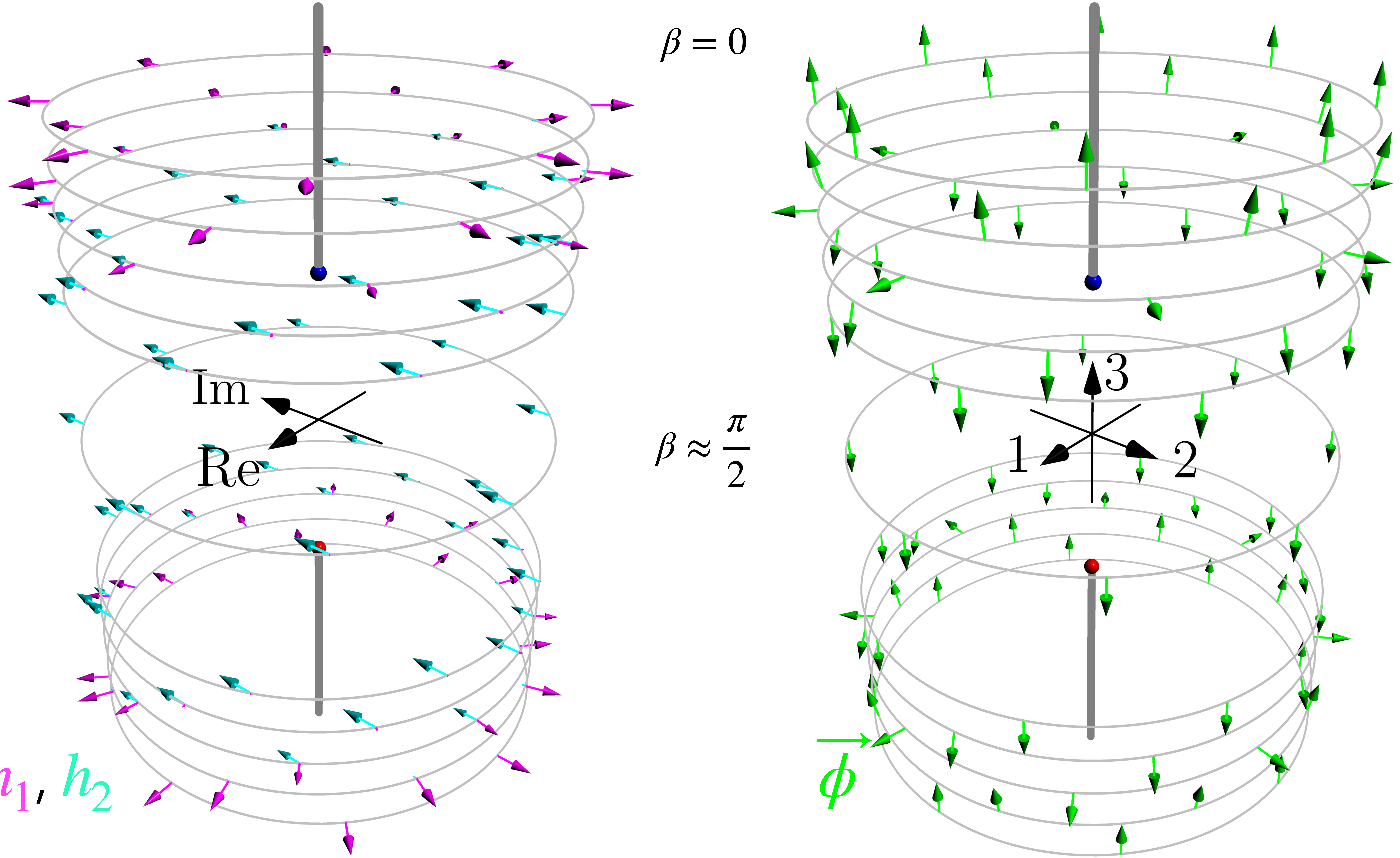
- $S_E = 2\pi \int_0^\infty \rho_E \mathrm{d}\rho_E \left[\frac{1}{2} \mathscr{K}_{\mathrm{eff}}(\beta)\beta'^2 + T(\beta) \right]$



Primitive Ansatz [Shifman & Yung, 2002]

- $h(x) = U \begin{pmatrix} \xi_\beta(\rho) \\ 0 \end{pmatrix}$
- $A_\theta(x) = iU\partial_\varphi U^{-1}[1 - f_\beta(\rho)]$, other components: 0
- $\phi(x) = VU\frac{\tau_3}{2}U^{-1} + \varphi_\beta(\rho)\left[\frac{\tau_1}{2}\sin\beta - \frac{\tau_2}{2}\cos\beta\right]$
- $U = e^{-i\tau_3\varphi}\cos\beta + i\tau_1\sin\beta$
- $\xi_\beta(0) = 0,\ \xi_\beta(\infty) = v, f_\beta(0) = 1, f_\beta(\infty) = 0,\ \varphi_\beta(0) = V\sin 2\beta,\ \varphi_\beta(\infty) = 0$





$\beta = 0$
$\beta \approx \frac{\pi}{2}$
Im
Re
3
1
2
h_1, h_2
ϕ

Setup

Couplings vs. Masses

- Scale hierarchy: $\sqrt{\kappa_{PV}} = M_M/\sqrt{T_{\rm str}} \sim V/v \propto m_W/m_\gamma$
  - Gauge field : $m_W = gV$, $m_\gamma = \frac{1}{\sqrt{2}}gv$
  - (Scalars : $m_\phi = \sqrt{8\tilde{\lambda}}V$, $m_{h_1} = 2\sqrt{\lambda}v$, $m_{h_2} = \sqrt{\gamma}V$)
- Euclidean action in terms of the masses: $S_E = \frac{1}{g^2}[g \text{ independent}]$

Couplings vs. Masses (detailed)

- Gauge field : $m_W = gV$, $m_\gamma = \frac{1}{\sqrt{2}} gv$
  - Scale hierarchy: $V/v \propto m_W/m_\gamma$
- Scalar triplet : $m_\phi = \sqrt{8\tilde{\lambda}}V$
- Scalar doublet: $m_{h_1} = 2\sqrt{\lambda}v$, $m_{h_2} = \sqrt{\gamma}V$
- Euclidean action:

$$g^2\mathscr{H} = \frac{1}{4}F^2 + \left|D\hat{h}\right|^2 + \frac{1}{2}\left(D\hat{\phi}\right)^2 + \frac{m_\phi^2}{8m_W^2}\left(\hat{\phi}^2 - m_W^2\right)^2 + \frac{m_{h_1}^2}{4m_\gamma^2}\left(|\hat{h}|^2 - 2m_\gamma^2\right)^2 + \frac{m_{h_2}^2}{m_W^2}\left|\left(\hat{\phi} - \frac{m_W}{2}\right)\hat{h}\right|^2$$