Global Bayesian Fits Of Minimal Composite Higgs Models

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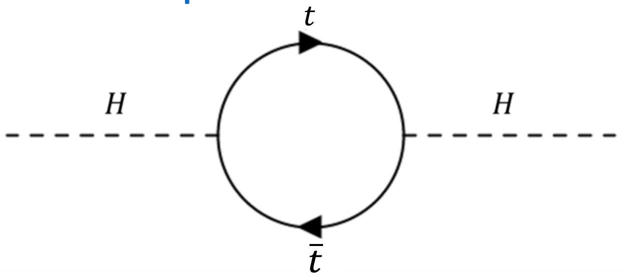
Naturalness of the Higgs mass

If the Standard Model is accurate up to some high energy scale Λ_{UV},

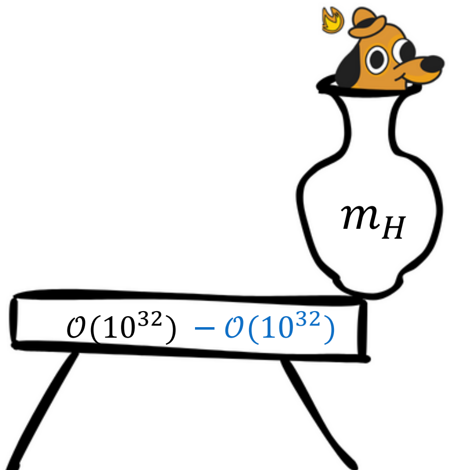
Higgs mass will receive loop contributions that scale with Λ_{UV}, mainly from t-quark:



$$m_H^2 = m_0^2 + \Delta m^2, \text{ where } \Delta m^2 = -\frac{3y_t^2}{8\pi^2}\Lambda_{UV}^2 + \mathcal{O}(m_t^2 \log\frac{\Lambda_{UV}}{m_t})$$

Fine-tuning

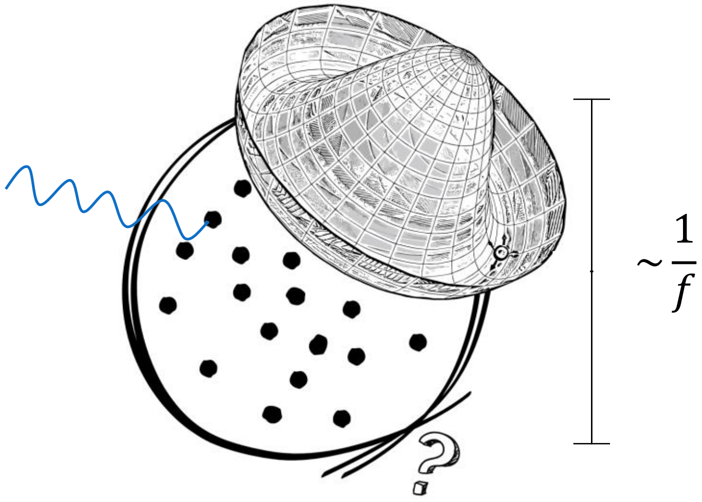
- If Λ_{UV} (e.g. Planck scale) $\gg m_H$, require very precise cancellation from m_0^2 to restore $m_H = 125$ GeV



(not to scale)

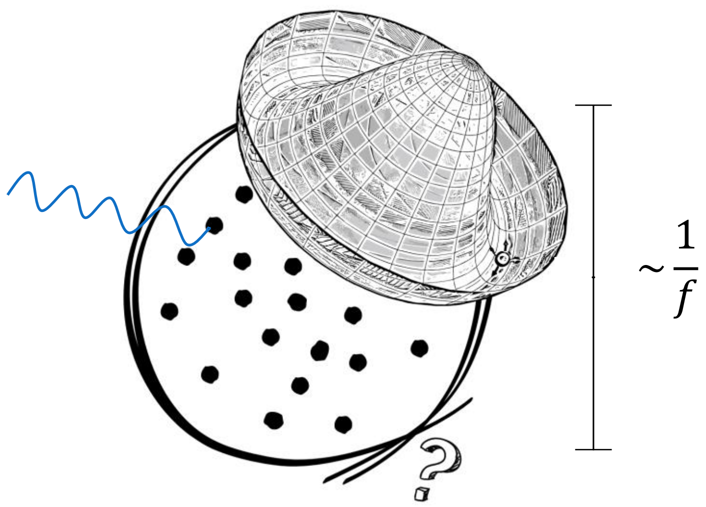
Why a composite Higgs?

- Consider some strongly interacting dynamics at scale, f $(\sim \mathrm{TeV})$
- Higgs appears as spin-0 bound state of said dynamics (finite size)



- Loop integrals are cut off at scale f

Composite Higgs Models (CHMs)



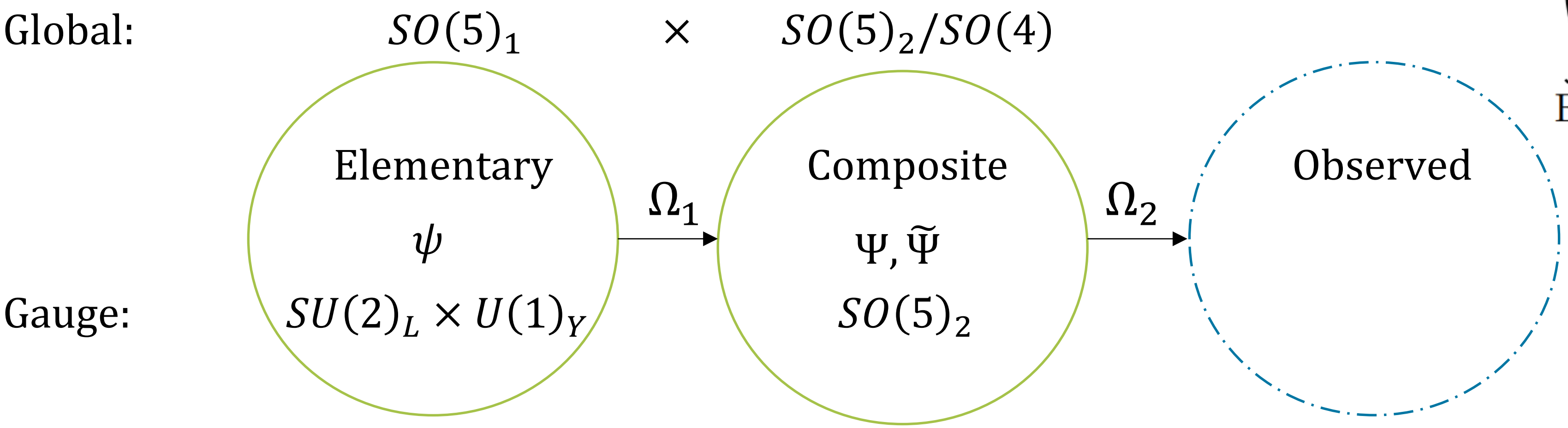
- Higgs is a **pseudo Nambu-Goldstone boson** (like pions in QCD)
- SM particles get masses by mixing with their partners in the composite sector
- TeV-scale composite resonances

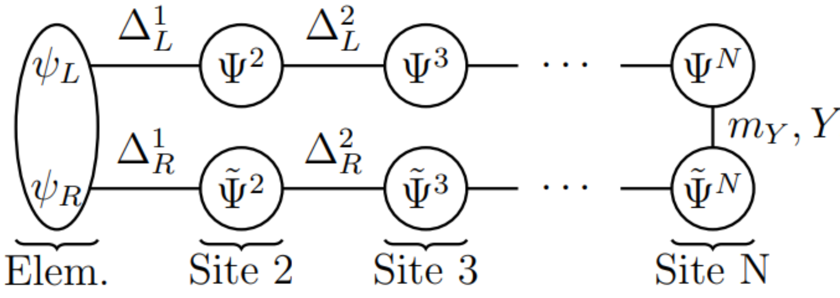
Minimal Composite Higgs Models (MCHMs)

- Encode global $SU(2)_L \times SU(2)_R \cong \mathrm{SO}(4)$

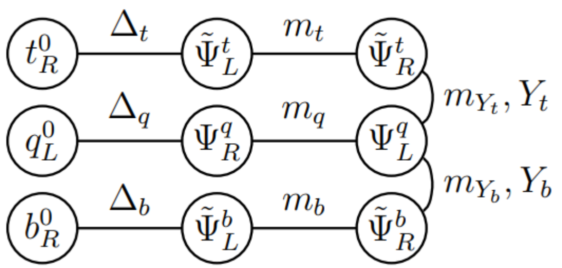
  - $\dim(SO(N)) = \frac{N(N-1)}{2}$

- $SO(5) \rightarrow SO(4)$ gives exactly 4 Higgs doublet fields





e.g.



- Various fermion representations ($\mathbf{1}, \mathbf{5}, \mathbf{10}, \mathbf{14}, \ldots$)

Minimal Composite Higgs Models (MCHMs)

$$\mathcal{L}_{\text{boson}} = -\frac{1}{4}\text{Tr}[G^0_{\mu\nu}G^{0\mu\nu}] - \frac{1}{4}\text{Tr}[W^0_{\mu\nu}W^{0\mu\nu}] - \frac{1}{4}B^0_{\mu\nu}B^{0\mu\nu} \quad \Big\} \text{ elementary}$$

$$- \frac{1}{4}\text{Tr}[\rho_{G\mu\nu}\rho_G^{\mu\nu}] - \frac{1}{4}\text{Tr}[\rho_{\mu\nu}\rho^{\mu\nu}] - \frac{1}{4}\rho_{X\mu\nu}\rho_X^{\mu\nu} \quad \Big\} \text{ composite}$$

$$+ \sum_{i=1,X,G} \frac{f_i^2}{4} Tr[(D_\mu\Omega_i)^\dagger(D^\mu\Omega_i)] + \frac{f_2^2}{2}(D_\mu\Omega_2\Phi_0)^\dagger(D^\mu\Omega_2\Phi_0) \quad \Big\} \text{ NGB}$$

$$SO(5)^0 \times SO(5)^1 : \Omega_1 \to g_0\Omega_1 g_1^{-1}, \qquad U(1)_X^0 \times U(1)_X^1 : \quad \Omega_X \to g_0\Omega_X g_1^{-1}$$

$$SO(5)^1 \times SO(4) \ : \Omega_2 \to g_1\Omega_2 h^{-1}, \qquad SU(3)_C^0 \times SU(3)_C^1 : \Omega_G \to g_0\Omega_G g_1^{-1}$$

$$\mathcal{L}_{\text{fermion}} = \bar{\psi}i\not{D}\psi \quad \Big\} \text{ elementary kinetic}$$

$$+ \sum_{k=2}^{N}\left(\bar{\Psi}^k(i\not{D} - m_L^k)\Psi^k + \bar{\tilde{\Psi}}^k(i\not{D} - m_R^k)\tilde{\Psi}^k\right) \quad \Big\} \text{ composite kinetic}$$

$$+ \sum_{k=1}^{N}\left(\Delta_L^k\bar{\Psi}_L^k\Omega_k\Psi_R^{k+1} + \Delta_R^k\bar{\tilde{\Psi}}_L^k\Omega_k\tilde{\Psi}_R^{k+1}\right) + \text{h.c.} \quad \Big\} \text{ link}$$

$$- m_Y\bar{\Psi}_L^N\tilde{\Psi}_R^N - Y\bar{\Psi}_L^N\Phi\Phi^\dagger\tilde{\Psi}_R^N + \text{h.c.} \quad \Big\} \text{ Yukawa-like.}$$

Minimal Composite Higgs Models (MCHMs)

In this work

- **Quarks** are embedded in $(5-5-5)$; **Leptons** in $(14-10)$ or $(5-5)$

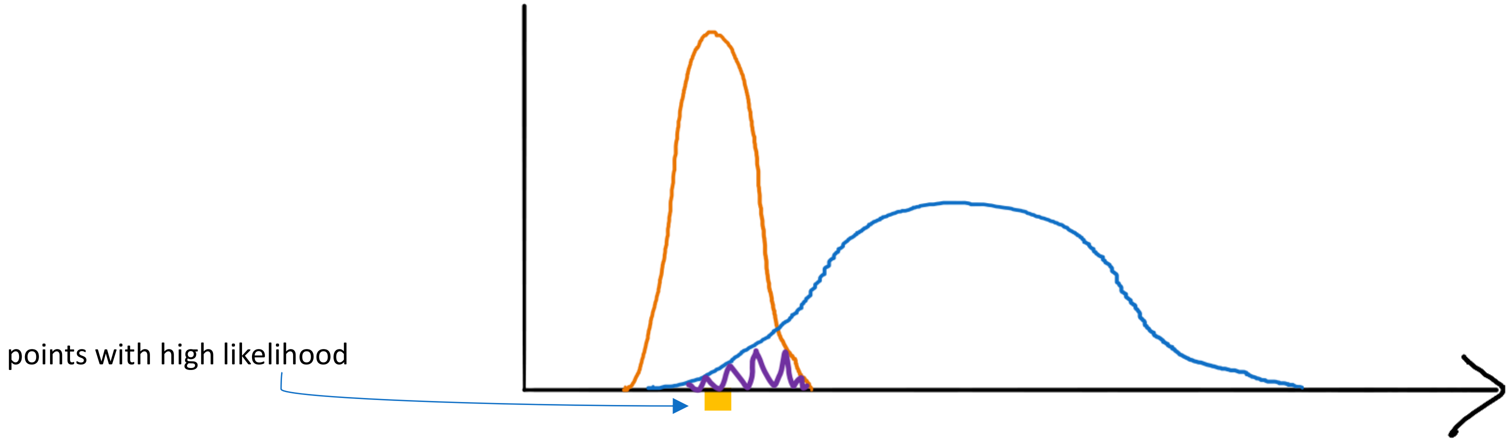
$(q_L - t_R - b_R)$ $(l_L - \tau_R)$

- Choice of representations based on work by:
  - M. Carena, L. Da Rold and E. Pont´on (JHEP 06 (2014) 159, [1402.2987])
    - Models that could fit collider constraints
  - J. Barnard and M. White (JHEP 09 (2017) 049, [1703.07653])
    - Explored leptonic-inclusive models based on fine-tuning, given by BG-measure
  - E. Carragher, W. Handley, D. Murnane, P. Stangl, W. Su, M. White, A.G. Williams (JHEP 21 (2021) 237, [2101.00428])
    - Convergent global fits with partially-composite 3rd generation quark

BSM Particle content

	Quark-like resonance					Lepton-like resonance		
Model	U	D	$Q_{4/3}$	$Q_{5/3}$	$Q_{8/3}$	N	L	E_2
$\text{LM4DCHM}^{5-5-5}_{5-5}$	8	8	2	2	0	2	6	2
$\text{LM4DCHM}^{5-5-5}_{14-10}$						10	12	2

Bayesian statistics to compare models



- Prior $\pi(p)$: Initial guess of some parameter (Distribution of points p)
- Likelihood $L(p)$: How well data fits a point
- Posterior $P(p)$: Updated guess based on the likelihood(p)

$$P(p) = \frac{L(p)\pi(p)}{Z}$$

- Kullback-Leibler (KL) Divergence:

$$D_{\mathrm{KL}} = \int dp P(p) \ln(P(p)/\pi(p))$$

- Evidence:

$$Z = \int dp\, L(p)\pi(p) \quad \rightarrow \ln(Z) = \langle \ln(L) \rangle_P - D_{\mathrm{KL}}$$

Fitting the models

Constraints

SM masses, EW precision observables, Z boson decay ratios, Higgs signal strengths, LHC fermion partner searches

Models

$$\mathrm{LM4DCHM}^{5-5-5}_{5-5} \qquad \mathrm{LM4DCHM}^{5-5-5}_{14-10}$$

- Lots of parameters to scan over
- E.g. fermion couplings:

$$L \subset -m_{\Psi}\bar{\Psi}\Psi - \tilde{m}_{\Psi}\bar{\tilde{\Psi}}\tilde{\Psi} + \Delta_L \psi_{\mathrm{L}} \Psi_{\mathrm{R}} + \Delta_R \psi_{\mathrm{R}} \tilde{\Psi}_{\mathrm{L}} + m_Y \bar{\Psi}_L \Psi_R + \cdots$$

- For now, only 3rd generation fermions couple to composite sector

Scan Parameters

LM4DCHM	**5 − 5**	**14 − 10**
Decay constants	$f,\ f_1,\ f_X,\ f_G$	$f,\ f_1,\ f_X,\ f_G$
Gauge couplings	$g_\rho,\ g_X,\ g_G$	$g_\rho,\ g_X,\ g_G$
Quark link couplings	$\Delta_{t_L},\ \Delta_{t_R},\ \Delta_{b_L},\ \Delta_{b_R}$	$\Delta_{t_L},\ \Delta_{t_R},\ \Delta_{b_L},\ \Delta_{b_R}$
Quark on-diagonal masses	$m_t,\ m_{\tilde{t}},\ m_b,\ m_{\tilde{b}}$	$m_t,\ m_{\tilde{t}},\ m_b,\ m_{\tilde{b}}$
Quark off-diagonal masses	$m_{Y_t},\ m_{Y_b}$	$m_{Y_t},\ m_{Y_b}$
Quark proto-Yukawa couplings	$Y_t,\ Y_b$	$Y_t,\ Y_b$
Lepton link couplings	$\Delta_{\tau_L},\ \Delta_{\tau_R}$	$\Delta_{\tau_L},\ \Delta_{\tau_R}$
Lepton on-diagonal masses	$m_\tau,\ m_{\tilde{\tau}}$	$m_\tau,\ m_{\tilde{\tau}}$
Lepton off-diagonal masses	m_{Y_τ}	
Lepton proto-Yukawa couplings	Y_τ	Y_τ
Dimensionality	25	24

Scanning

- Bayesian view of these parameters
- Log-spaced priors
- Likelihood is taken to be Gaussian in observables
- Posteriors will show us what parameter volume likely fits our constraints

Computation

Nested sampling (`PolyChord`)

W.J. Handley, M.P. Hobson, A.N. Lasenby [1506.00171] github.com/PolyChord

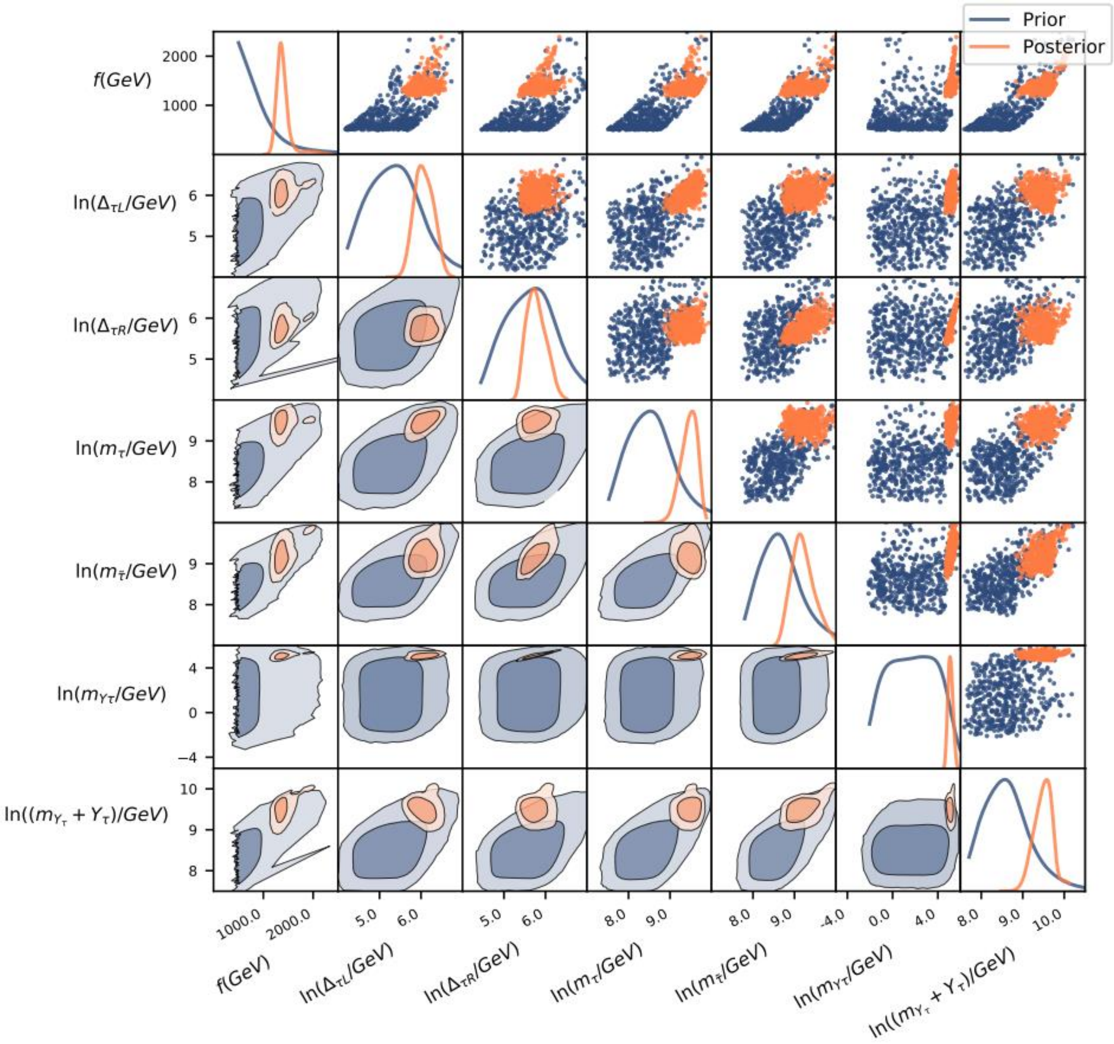
- Sensitive to multi-modal distributions
- Scales well with high dimensionality

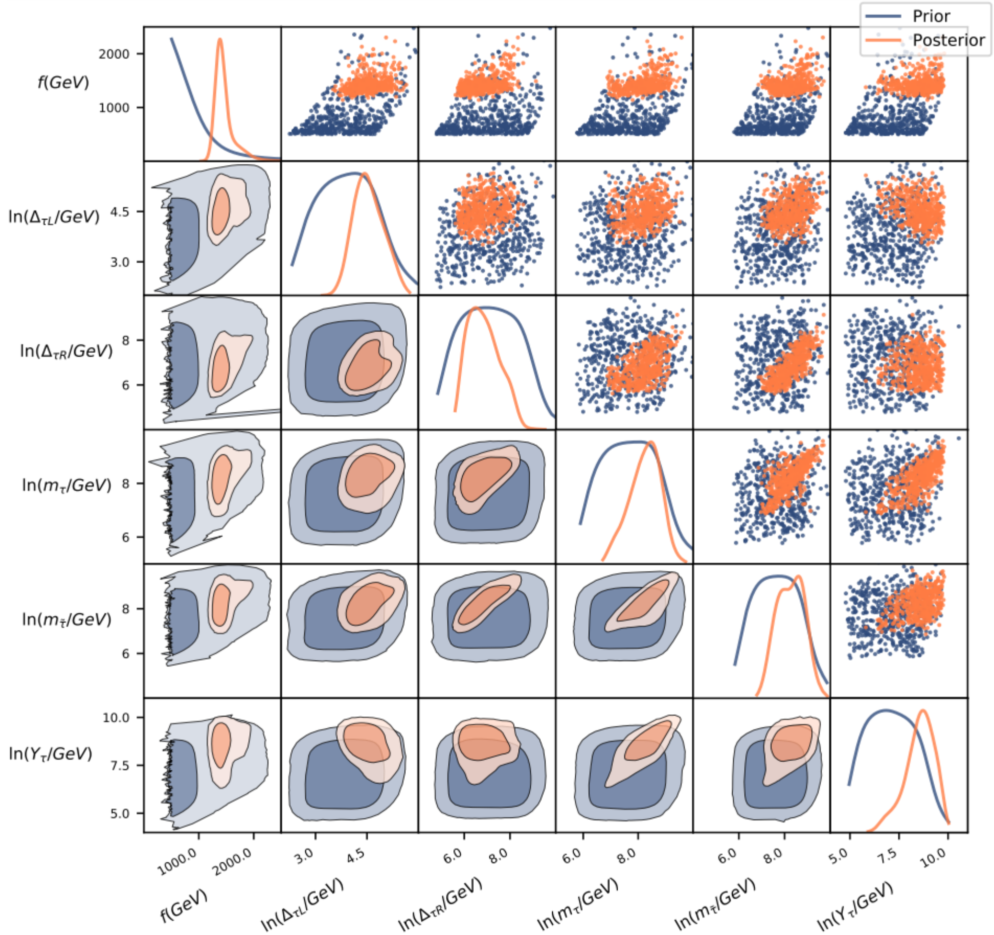
Results

$\mathrm{LM4DCHM}^{5-5-5}_{5-5}$ (5 − 5)

$\mathrm{LM4DCHM}^{5-5-5}_{14-10}$ (14 − 10)

Prior
Posterior





Bayesian view of the models

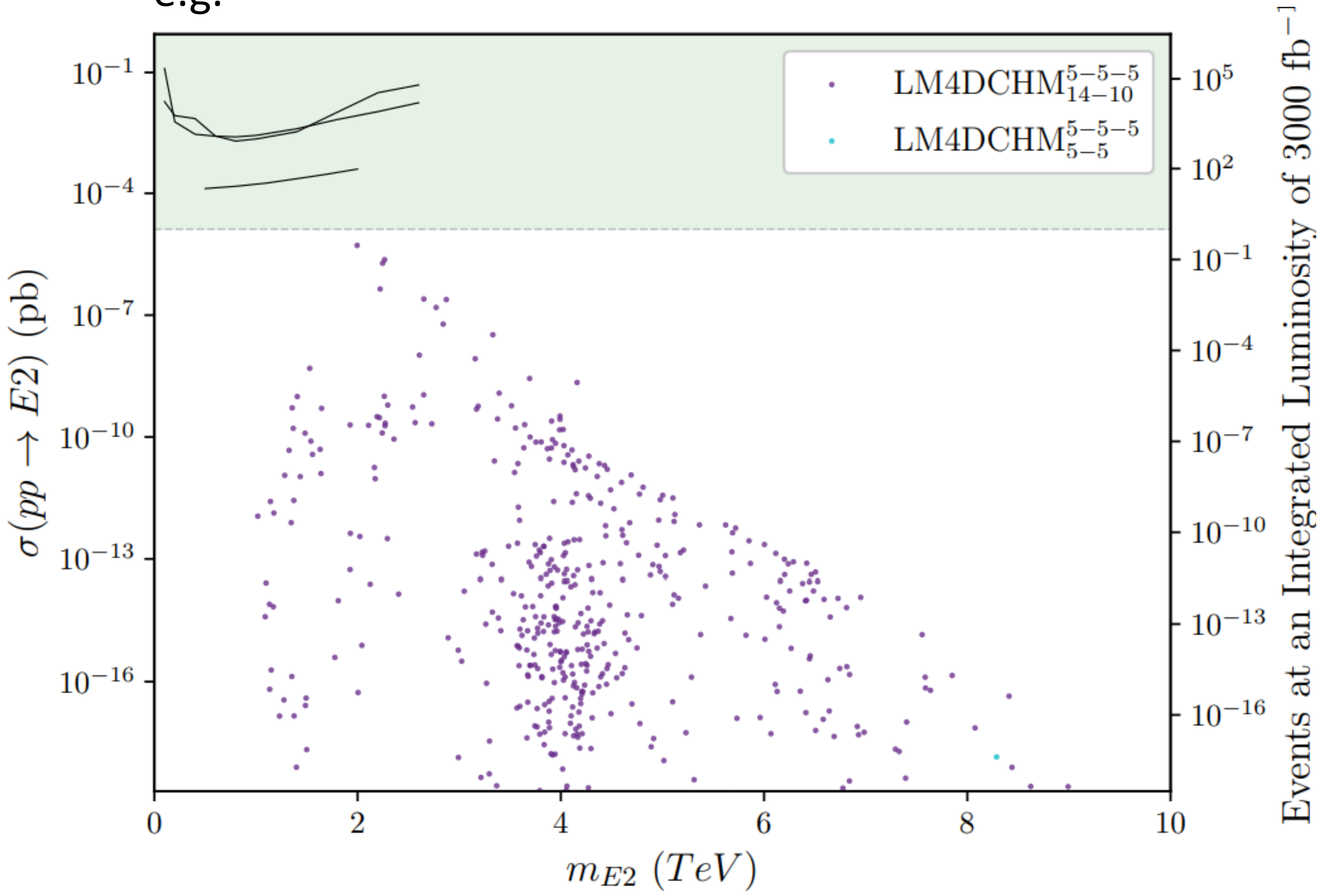
Model	$\ln(\mathcal{Z})$	$\langle \ln(\mathcal{L}) \rangle_P$	D_{KL}
$\mathrm{LM4DCHM}^{5-5-5}_{5-5}$	-45.60 ± 0.06	-17.27	28.33
$\mathrm{LM4DCHM}^{5-5-5}_{14-10}$	-36.30 ± 0.05	-14.63	21.67

- Both scans are convergent!
- Bayesian evidence of $14-10$ is $\times\, 10^4$ better than $5-5$

Using best fit points (points that satisfy all constraints at 3σ), we can also look at pheno:

Direct detection (best fit points)

e.g.



m_{E2} :mass of lightest exotic particle with electric charge=2

- $\mathrm{LM4DCHM}^{5-5-5}_{14-10}$
- $\mathrm{LM4DCHM}^{5-5-5}_{5-5}$

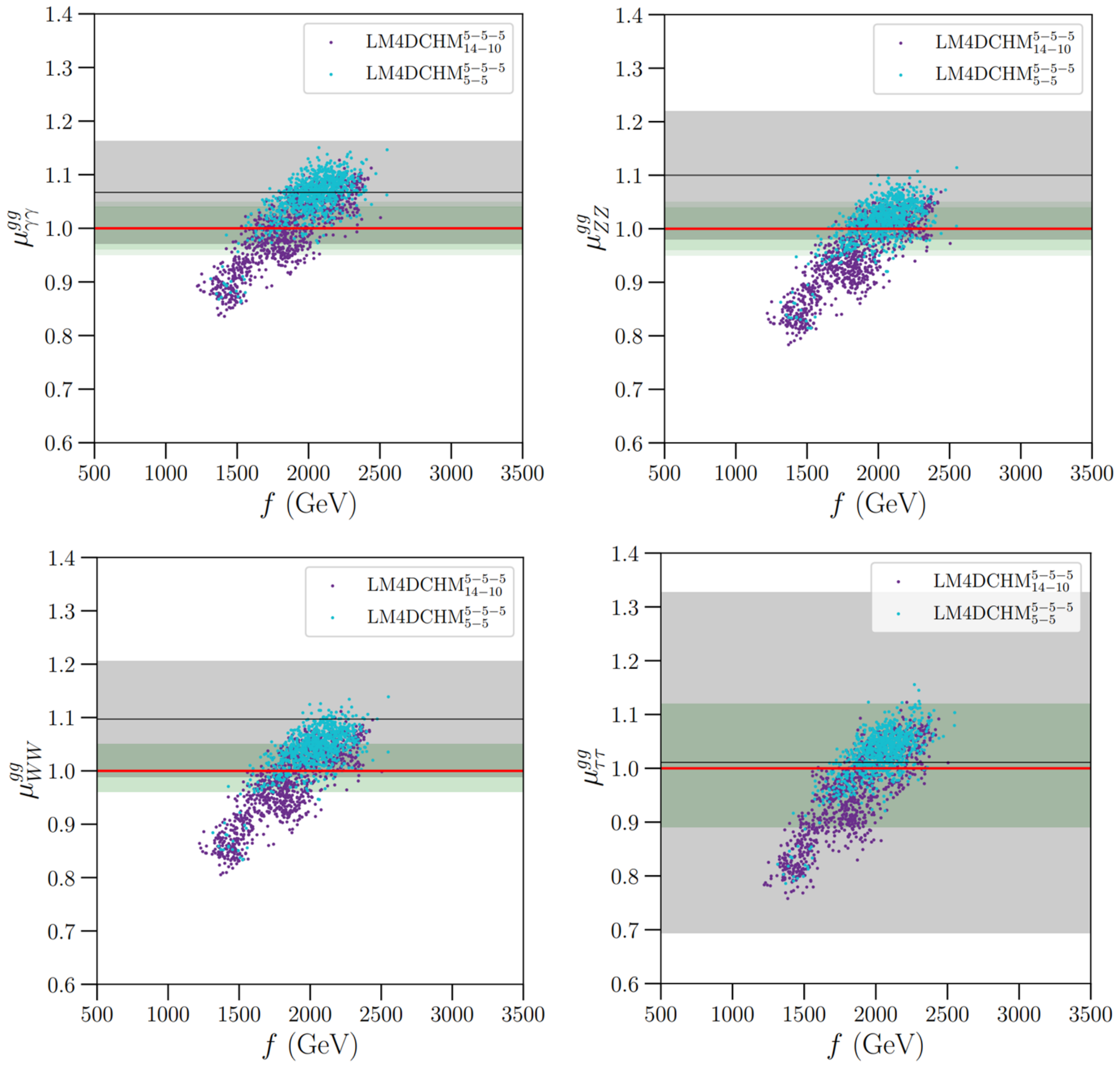
: Producable range at HL−LHC

— : 13 TeV search bounds

- Each model has heavy quark and lepton partners
- Look for them at LHC?

Not really…

Higgs signal strengths (best fit points)



- Another avenue is Higgs signal strengths (gluon fusion)
- Sensitive to modifications of Higgs-SM couplings, as well as loop contributions from composite resonances

$$\mu^{gg}_{jj} = \frac{[\sigma(gg \to H)BR(H \to jj)]_{measured}}{[\sigma(gg \to H)BR(H \to jj)]_{SM}}$$

- LM4DCHM$^{5-5-5}_{14-10}$
- LM4DCHM$^{5-5-5}_{5-5}$

: Current measurements

: Projected precision at HL−LHC

: SM

Takeaway

- Scans of lepton-inclusive MCHMs are convergent
- Both models satisfy all imposed experimental constraints
- But leptons embedded in $(14-10)$ is a better Bayesian fit than $(5-5)$

- SM partners are generally too heavy to be seen, even at HL-LHC
- Best shot is indirect tests via Higgs signal strengths

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Back up slide(s?)

Higgs vev

$$\mathcal{L}^{\text{eff}}_{\text{comp. fermions}} = \sum_{\psi=t,b,\nu,\tau} [\bar{\psi}^0_L \not{p}(1+\Pi_{\psi L}(p^2))\psi^0_L + \bar{\psi}^0_R \not{p}(1+\Pi_{\psi R}(p^2))\psi^0_R + \bar{\psi}^0_L M_\psi(p^2)\psi^0_R + h.c.]$$

$$\gamma_{\text{gauge}} = -\frac{9m_\rho^4\left(m_a^2 - m_\rho^2\right)t_\theta}{64\pi^2\left(m_a^2 - (1+t_\theta)\,m_\rho^2\right)} \ln\left[\frac{m_a^2}{(1+t_\theta)\,m_\rho^2}\right]$$

$$V^{\text{eff}}_{\text{fermion}}(h) = -2\sum_{\psi=t,b,\tau,\nu} N_\psi \int \frac{dp_E^2}{16\pi^2} \ln\left[(1+\Pi_{\psi L}(-p_E^2))(1+\Pi_{\psi R}(-p_E^2)) - \frac{|M_\psi(-p_E^2)|^2}{p_E^2}\right]$$

$$V(h) := \gamma^2 s_h^2 + \beta^4 s_h^4 \qquad s_{\langle h\rangle} = \frac{\gamma}{2\beta} \qquad m_H := \sqrt{8\beta(1-s^2_{\langle h\rangle})}\frac{s_{\langle h\rangle}}{f}$$

masses of the lightest composite gauge bosons m_ρ and m_a

$$m_\rho^2 := \frac{1}{2}g_\rho^2 f_1^2, \qquad m_a^2 := \frac{1}{2}g_\rho^2(f_1^2 + f_2^2).$$

Constraints

- SM masses: $\mathrm{m_t}, \mathrm{m_b}, \mathrm{m_\tau}, \mathrm{m_H}$
- Oblique parameters: S and T

$$S = \frac{1}{4\,\alpha_{\mathrm{em}}}\left(1 - \frac{m_W^2}{m_Z^2} - \frac{g_{Zee}^R}{2(g_{Zee}^R - g_{Zee}^L)}\right)$$

$$\mathrm{T} = \frac{1}{\alpha_{\mathrm{em}}}\left(\frac{\Pi_{WW}^T(p^2 = 0)}{m_W^2} - \frac{\Pi_{ZZ}^T(p^2 = 0)}{m_Z^2}\right)$$

- Z decay ratios: $R_i = \frac{\Gamma(Z\to i\bar{\imath})}{\Gamma(Z\to q\bar{q})}$, for $i = b, e, \mu, \tau$
- Higgs signal strengths: $\mu_{jj}^{gg} = \frac{[\sigma(gg\to H)BR(H\to jj)]_{measured}}{[\sigma(gg\to H)BR(H\to jj)]_{SM}}$
- Collider searches*

  (lower mass bounds for fermion resonances)

Lepton resonance	Lower Mass Bound
N	90.3 GeV
$E2$	370 GeV
L_1	300 GeV
L_2	790 GeV
L_3	225 GeV

Symmetries

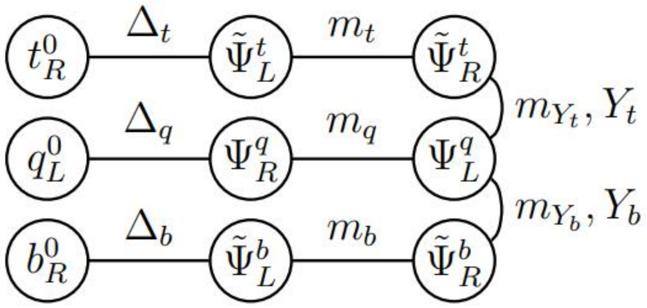
$$
\begin{aligned}
&SO(5)^0 \times SO(5)^1 : \Omega_1 \to g_0 \Omega_1 g_1^{-1}, \qquad && U(1)_X^0 \times U(1)_X^1 : \ \Omega_X \to g_0 \Omega_X g_1^{-1}, \\
&SO(5)^1 \times SO(4) \ \ : \Omega_2 \to g_1 \Omega_2 h^{-1}, \qquad && SU(3)_C^0 \times SU(3)_C^1 : \Omega_G \to g_0 \Omega_G g_1^{-1},
\end{aligned}
$$

where g_a denotes transformations from Site a, and $h \in SO(4)$. The decay constants f_i in the NGB terms correspond to the scales of these symmetry breakings. Most NGBs are unphysical and can be gauged away, with the sole exception of the Higgs field, which is parameterised in the product $\Omega := \Omega_1 \Omega_2$. Because of this, it has an associated symmetry breaking scale f given by

$$\frac{1}{f^2} = \frac{1}{f_1^2} + \frac{1}{f_2^2}.$$

This is related to the Higgs vev v by

$$f \equiv \frac{v}{s_{\langle h \rangle}} = \frac{246}{s_{\langle h \rangle}} \text{ GeV},$$



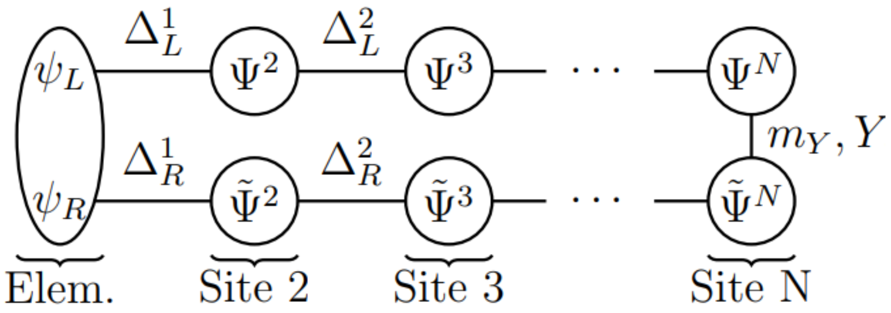


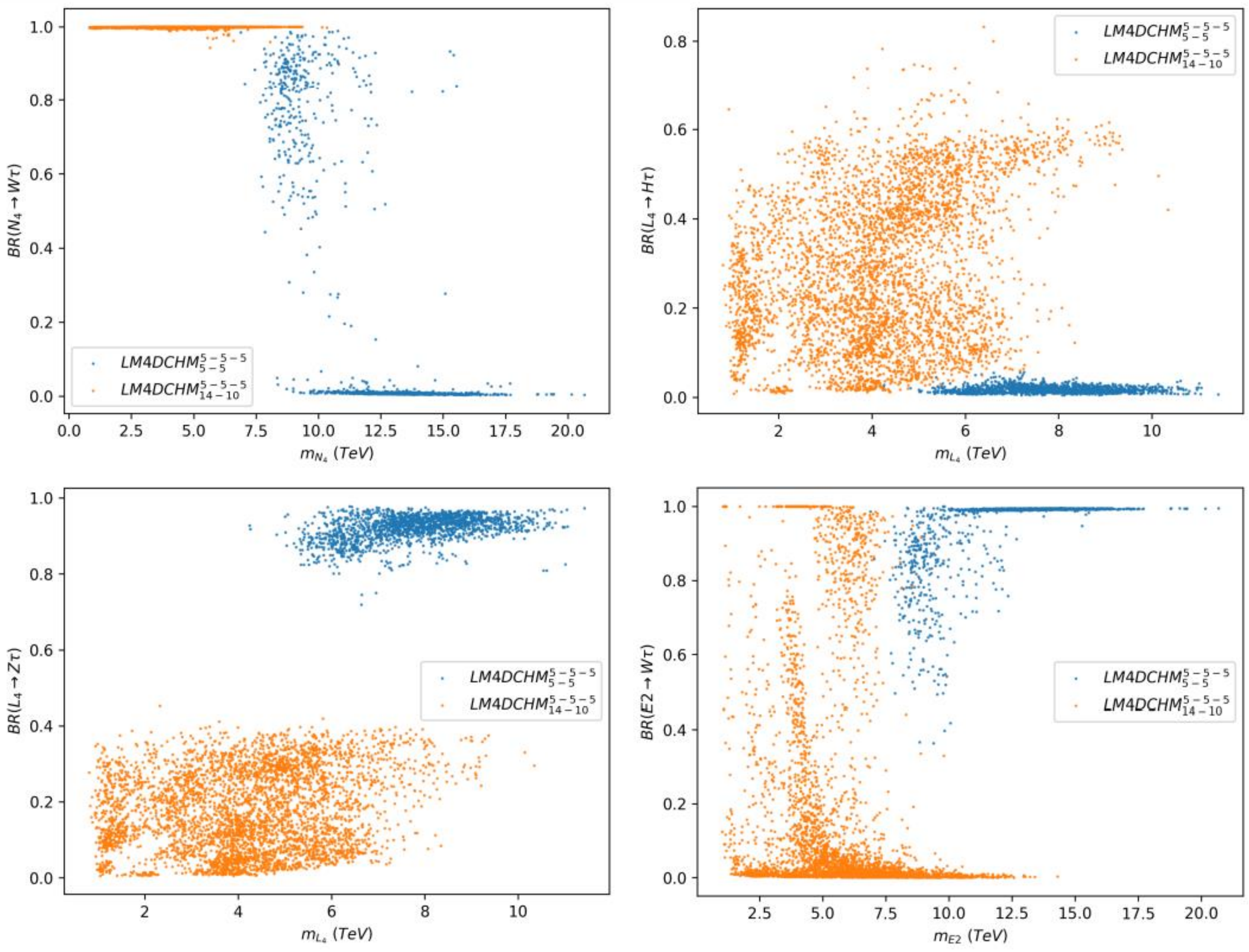
Figure 4.1: Structure of couplings between elementary and composite fermions.

Reweighted stats

Model	$\ln(\mathcal{Z})$	$\langle\ln(\mathcal{L})\rangle_P$	$\max\ln(\mathcal{L})$	D_{KL}
$\text{LM4DCHM}^{5-5-5}_{5-5}$	-65.06	-16.75	-10.79	48.31
$\text{LM4DCHM}^{5-5-5}_{14-10}$	-50.34	-15.37	-9.13	34.97

Table 5: Statistics from the combined Bayesian scans of each model, with the samples re-weighted as if all parameters had been given uniform priors with the same bounds as in Table 2.

Branching ratios



Naturalness of the Higgs mass

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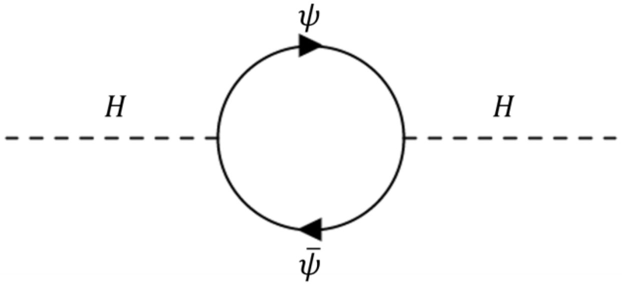
Higgs mass with receive loop contributions that scale with Λ_{UV}:

$$m_H^2 = m_0^2 + \Delta m^2$$



$$= iN_C \left(-i\frac{\lambda_\psi}{\sqrt{2}}\right)^2 \int \frac{d^4k}{(2\pi)^4} \frac{\mathrm{Tr}[(p+k+m_\psi)(k+m_\psi)]}{((p+k)^2 - m_\psi^2 - i\epsilon)(k^2 - m_\psi^2 - i\epsilon)}$$