Constraining millicharged dark matter with gravitational positivity bounds

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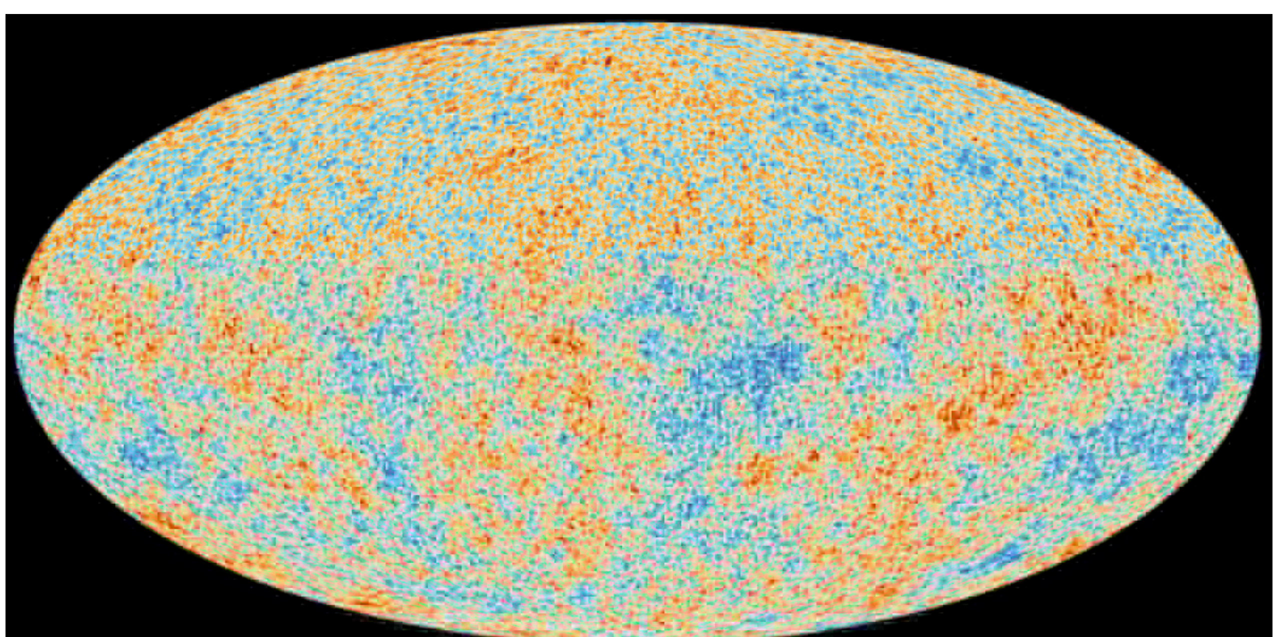
12/12 International Joint Workshop on the Standard Model and Beyond 2024 & 3rd Gordon Godfrey Workshop on Astroparticle Physics

Dark Matter

- Many evidences of dark matter

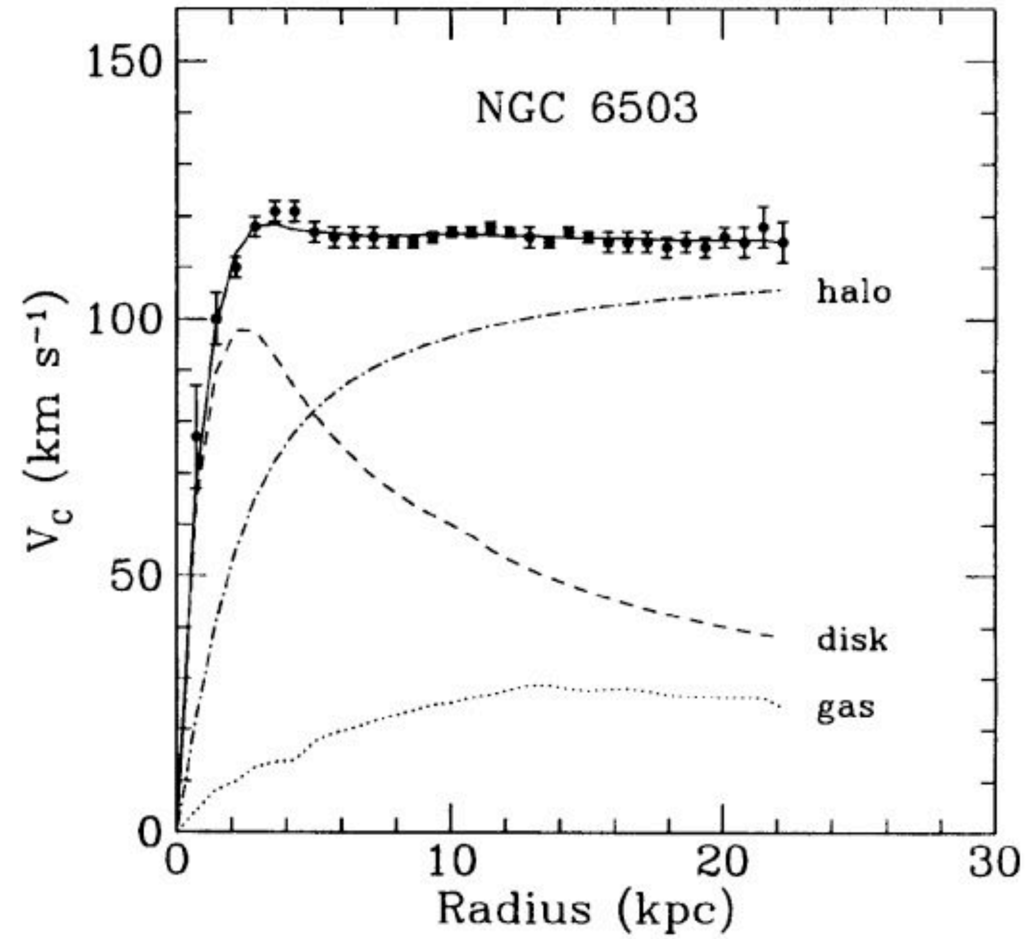
Bullet cluster

(Chandra X-Ray observatory)



Cosmic Microwave Background

(Planck Collaboration)

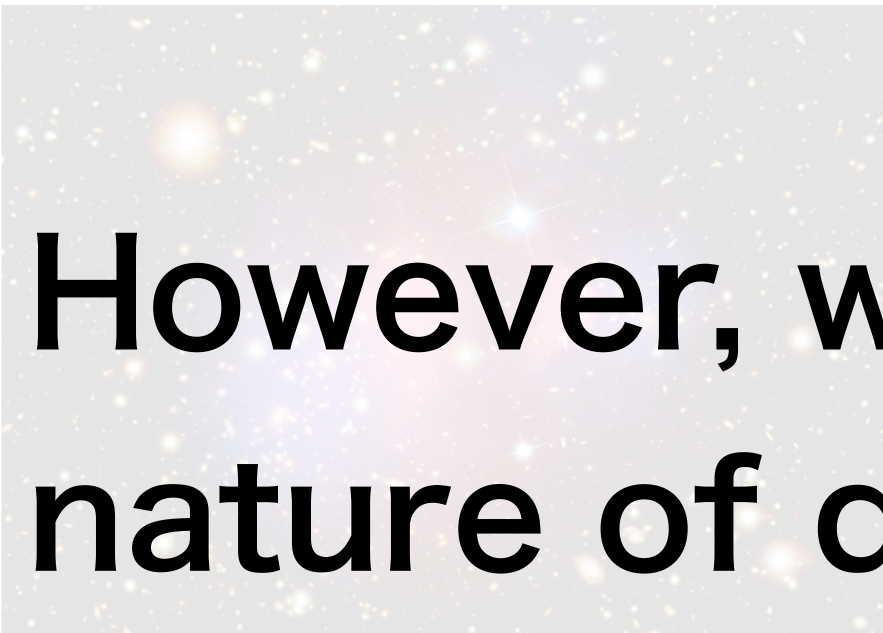


Galaxy rotation curve

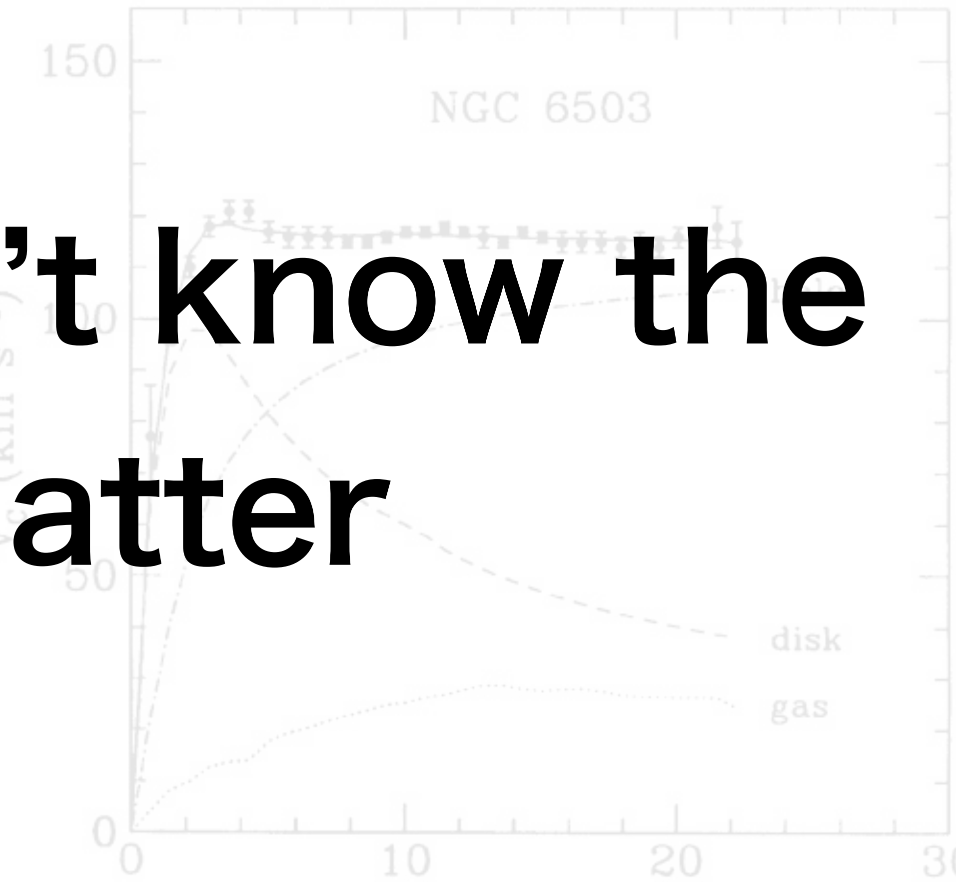
(NGC 6503)

Dark Matter

- Many evidences of dark matter



Bullet cluster
(Chandra X-Ray observatory)



Galaxy rotation curve
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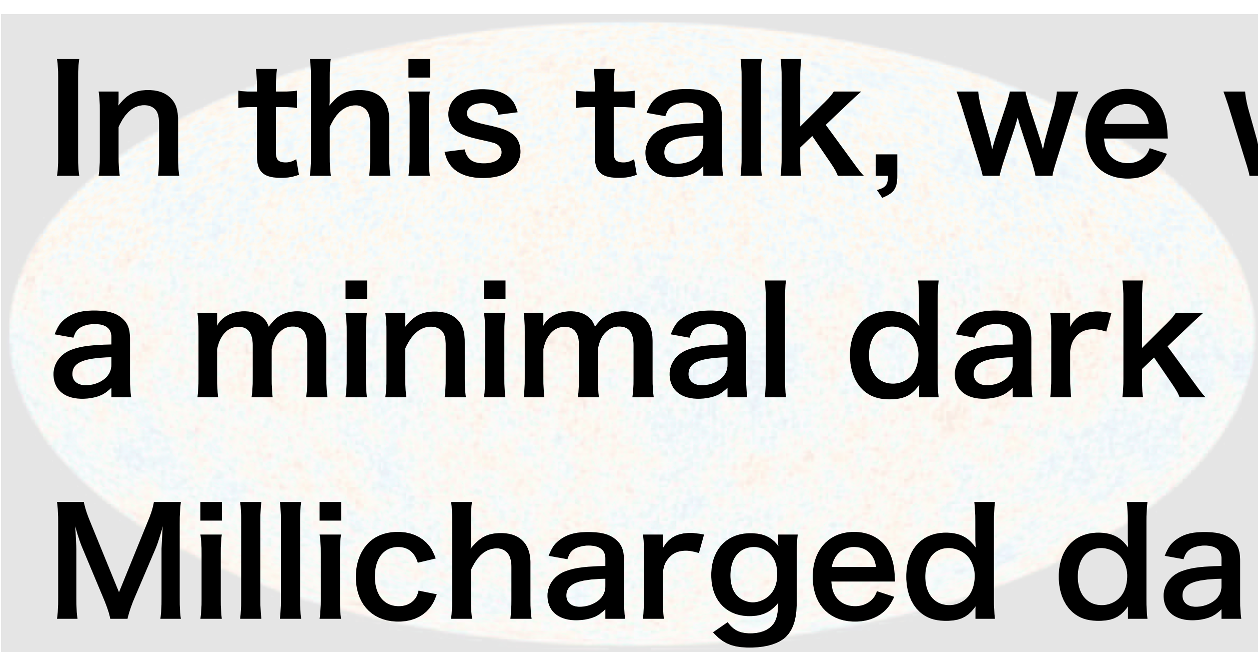


Cosmic Microwave Background
(Planck Collaboration)

However, we don’t know the nature of dark matter

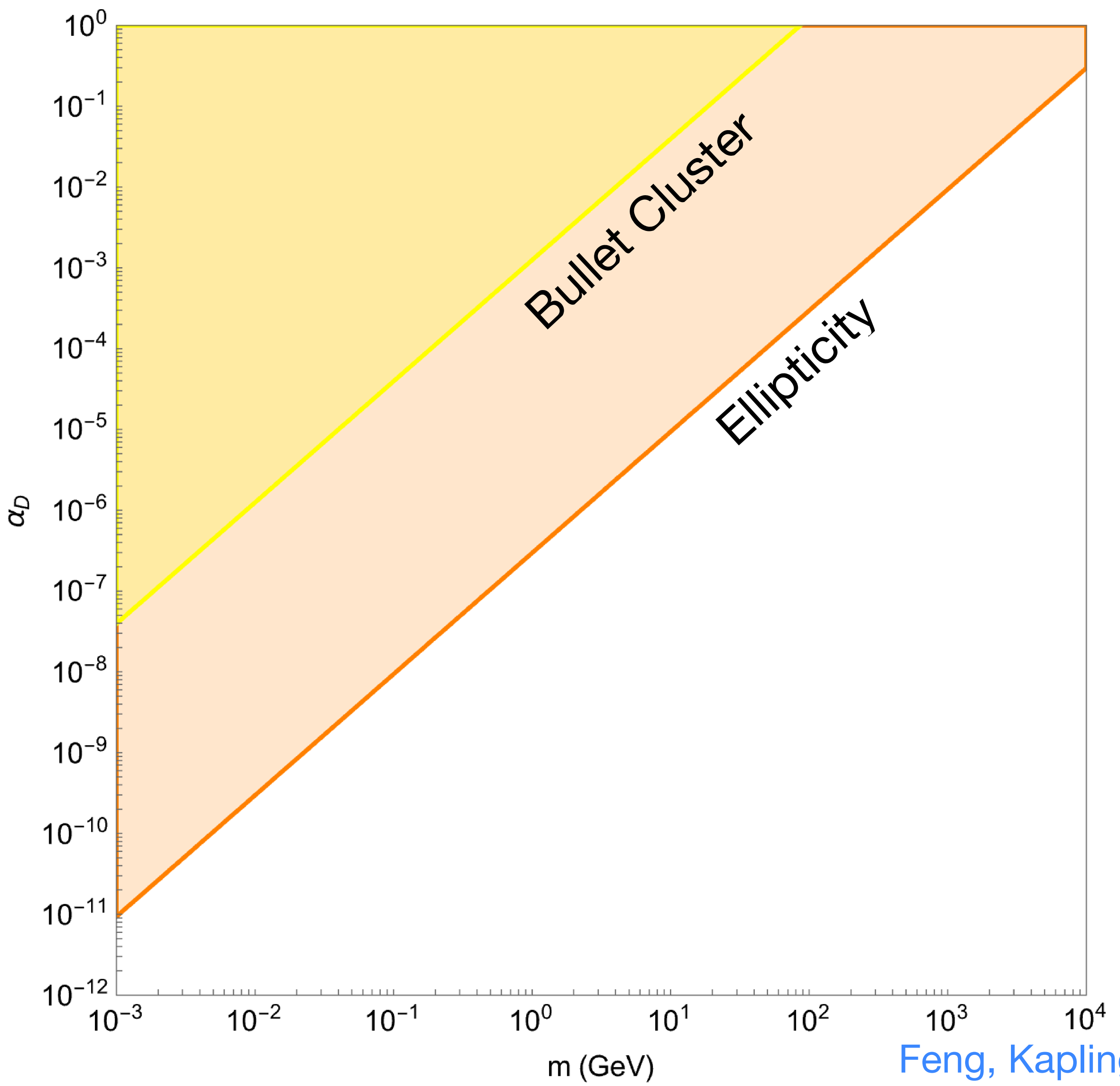
In this talk, we will consider a minimal dark sector model: Millicharged dark matter

Milli-charged dark matter

- Dark matter charged under the hidden $U(1)$

$$S_{\rm DS} = \int d^4x \sqrt{-g} \left[-\frac{1}{4} F_{b,\mu\nu} F_b^{\mu\nu} - \frac{\varepsilon}{2} F_{a,\mu\nu} F_b^{\mu\nu} + \bar{\chi}(i\gamma^\mu \nabla_\mu + e_D \gamma^\mu A_{b,\mu} - m)\chi \right]$$

- 3 parameters $\varepsilon, m, \alpha_D = e_D^2/4\pi$



Feng, Kaplinghat, Tu, Yu [2009]

Upper bound from observations

Milli-charged dark matter

- Dark matter charged under the hidden $U(1)$

$$S_{\rm DS} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{b,\mu\nu}F_b^{\mu\nu} - \frac{\varepsilon}{2}F_{a,\mu\nu}F_b^{\mu\nu} + \bar{\chi}(i\gamma^\mu\nabla_\mu + e_D\gamma^\mu A_{b,\mu} - m)\chi\right]$$

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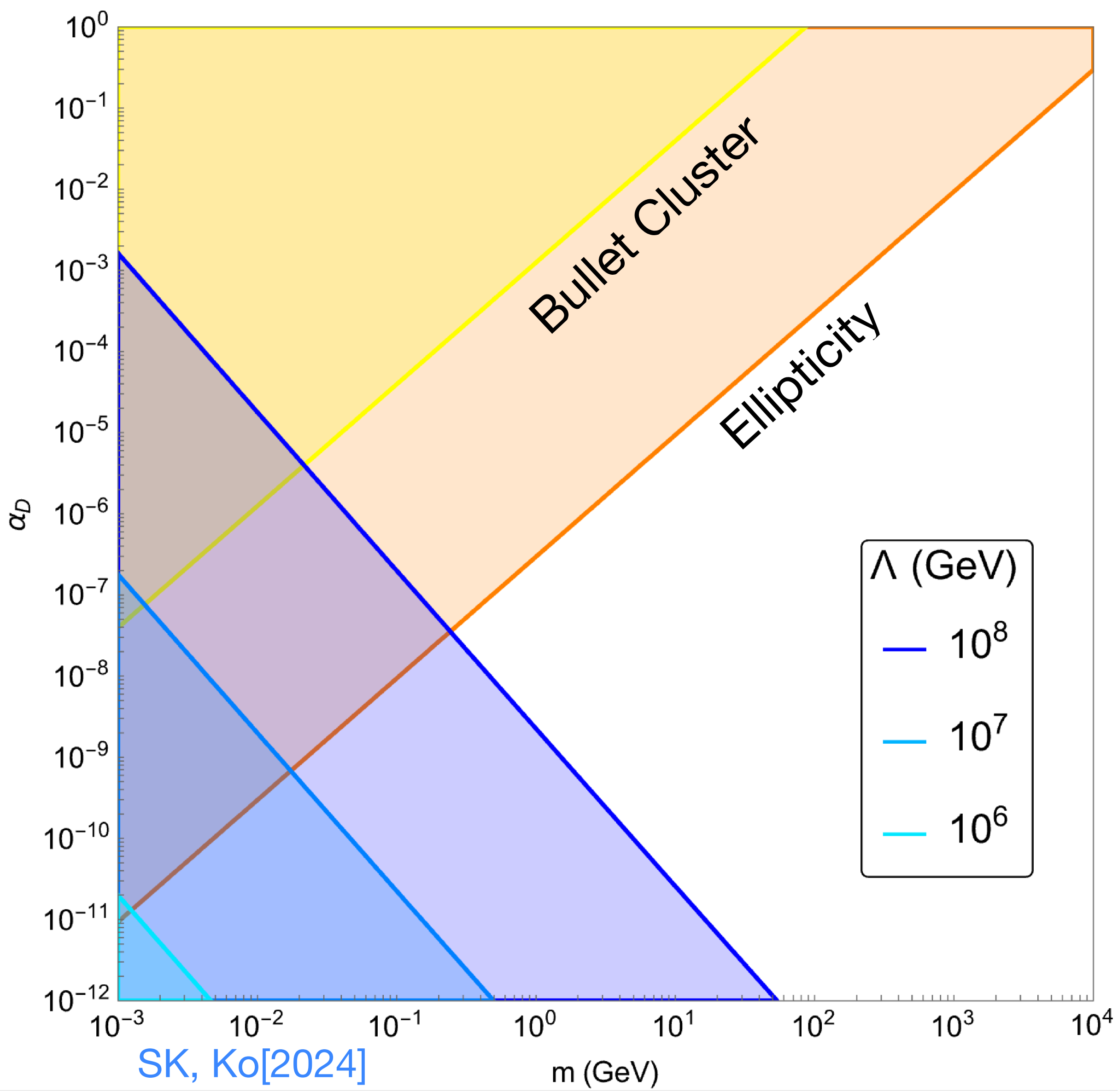

SK, Ko[2024]

Upper bound from observations

+

Lower bound from theoretical consistency

(Gravitational positivity bound)

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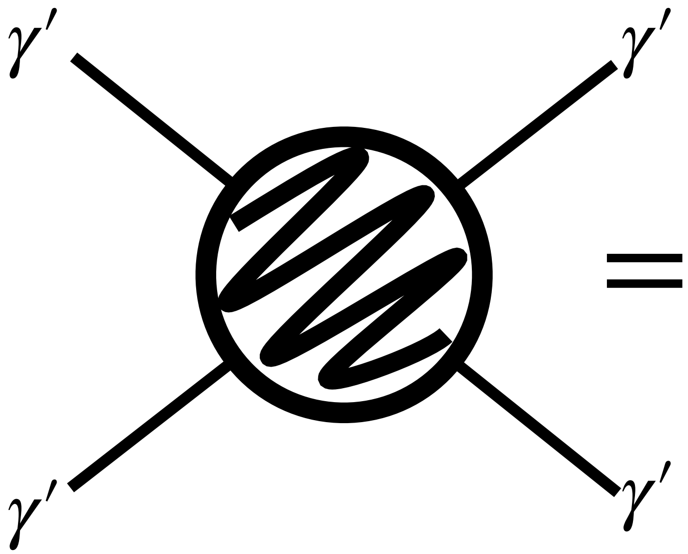
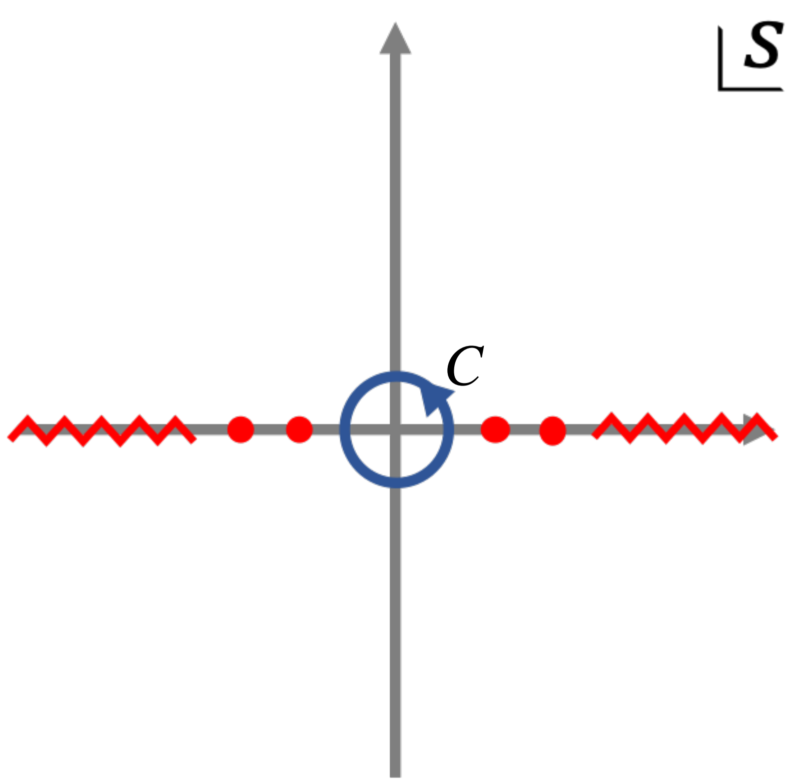
IR expansion of scattering amplitudes

- Expand the scattering amplitude in Mandelstam variables



$$= M(s, t \to 0) = \sum_p \frac{a_p s^p}{p!}$$

- Coefficients of s^p



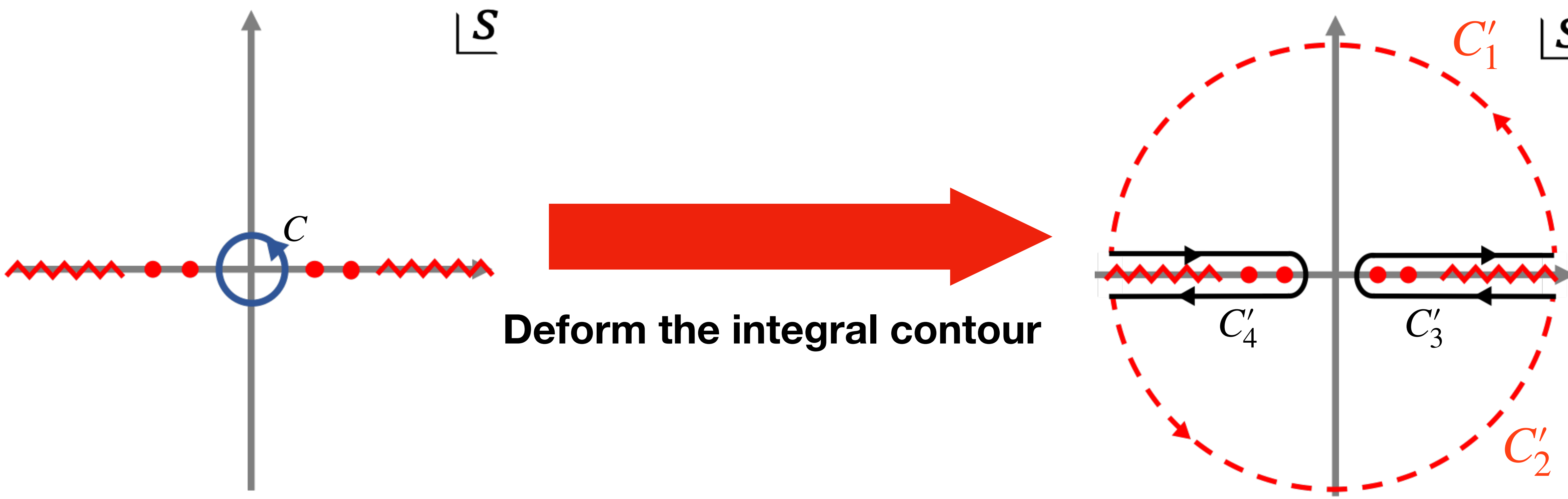
$$\frac{a_p}{p!} = \oint_C \frac{ds}{2\pi i} \frac{M(s,0)}{s^{p+1}}$$

Wilson-Coefficients

- Coefficients of s^p

$$\frac{a_p}{p!} = \oint_C \frac{ds}{2\pi i} \frac{M(s,0)}{s^{p+1}}$$

$$= \left(\int_{C_1'} + \int_{C_2'} + \int_{C_3'} + \int_{C_4'} \right) \frac{ds}{2\pi i} \frac{M(s,0)}{s^{p+1}}$$



Wilson-Coefficients

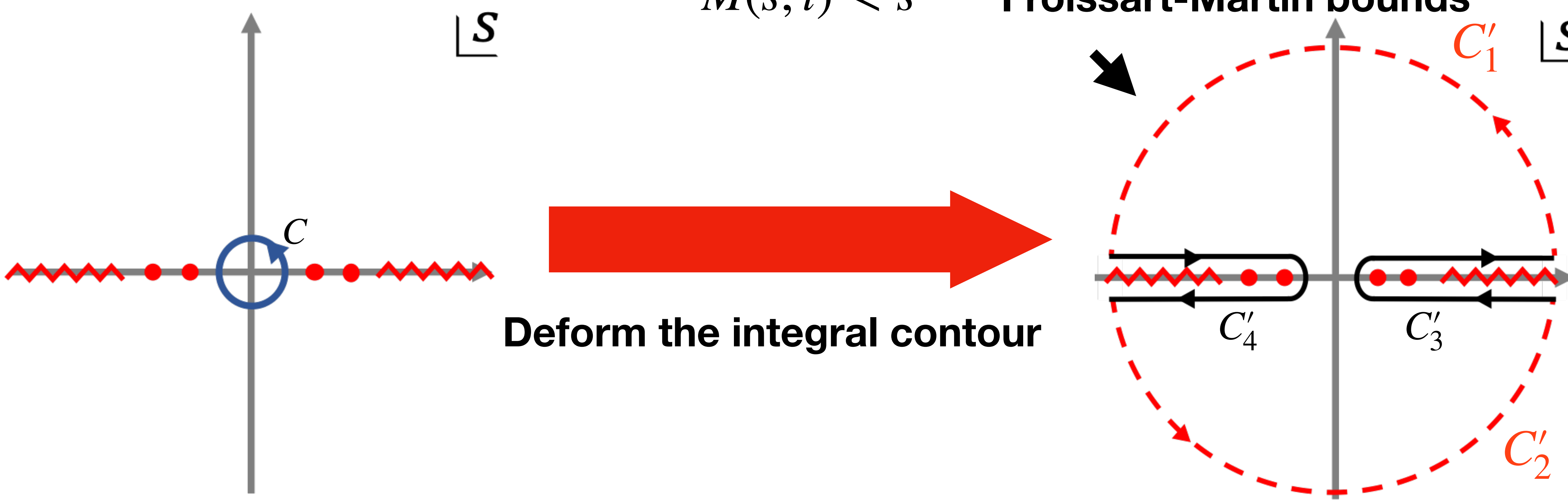
- Coefficients of s^p

$$\frac{a_p}{p!} = \oint_C \frac{ds}{2\pi i} \frac{M(s,0)}{s^{p+1}}$$

0 for $p \geq 2$

$$= \left(\int_{C_1'} + \int_{C_2'} + \int_{C_3'} + \int_{C_4'} \right) \frac{ds}{2\pi i} \frac{M(s,0)}{s^{p+1}}$$

$M(s,t) < s^2$



Wilson-Coefficients

- Coefficients of s^p

$$\frac{a_p}{p!} = \left(\int_{C_3'} + \int_{C_4'}\right)\frac{ds}{2\pi i}\frac{M(s,0)}{s^{p+1}}$$

$$= \frac{2}{\pi}\int_{m_{\rm th}^2}^{\infty} ds \frac{{\rm Im}M(s,0)}{s^{p+1}} > 0$$

For even p

Positivity bounds

- This is well-known as positivity bound

Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi[2006]

- Universal and elegant, but hard to give a phenomenologically useful constraints

- Let's improve positivity bounds
  1. Fully utilize information from EFT
  2. Incorporate gravity

Improved positivity bounds

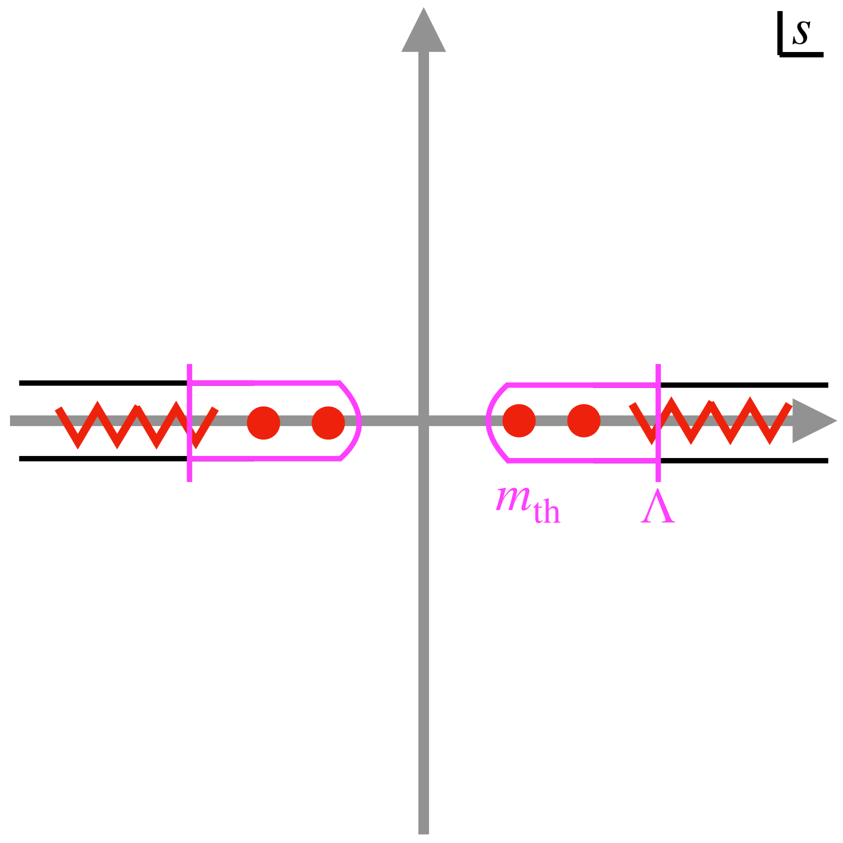
- Focus on s^2 coefficient

$$a_2 = \frac{\partial^2 M(s, t=0)}{\partial s^2} = \frac{4}{\pi}\int_{m_{\rm th}^2}^{\infty} ds \frac{{\rm Im}M(s,0)}{s^3}$$

- Introduce reference scale Λ which below our EFT is valid, then we can evaluate M

$$B(\Lambda) := a_2 - \frac{4}{\pi}\int_{m_{\rm th}^2}^{\Lambda^2} ds \frac{{\rm Im}M(s, t=0)}{s^3}$$

$$= \frac{4}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{{\rm Im}M(s, t=0)}{s^3} > 0$$



Incorporating gravity into PB

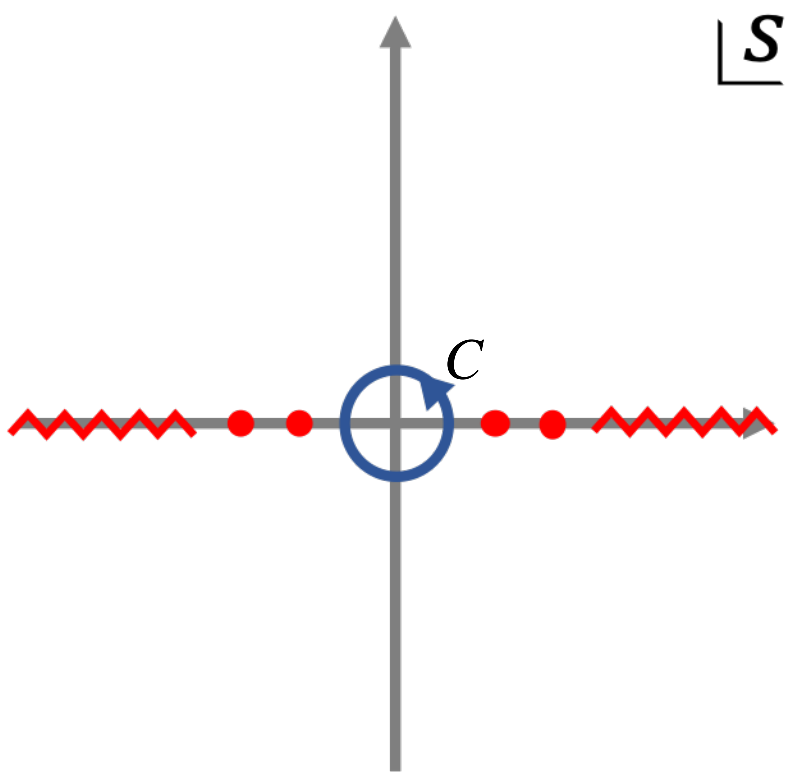
- Coefficients of s^p

$$\frac{a_p}{p!} = \oint_C \frac{ds}{2\pi i}\frac{M(s,0)}{s^{p+1}}$$

0 for $p \geq 2$

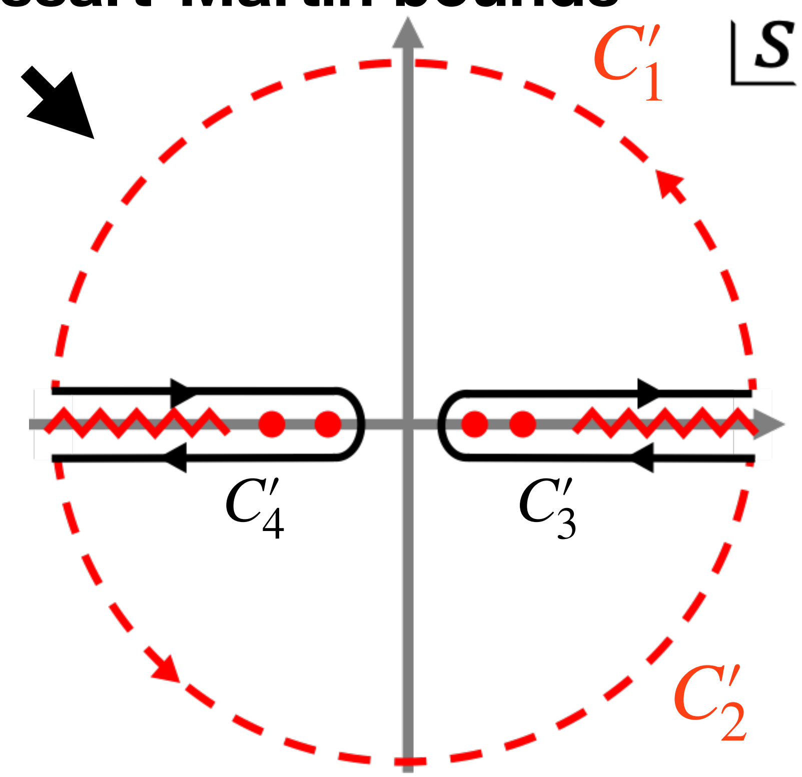
$$= \left(\int_{C_1'} + \int_{C_2'} + \int_{C_3'} + \int_{C_4'}\right)\frac{ds}{2\pi i}\frac{M(s,0)}{s^{p+1}}$$

$M(s,t) < s^2$



Deform the integral contour

Froissart-Martin bounds



Incorporating gravity into PB

- Coefficients of s^p

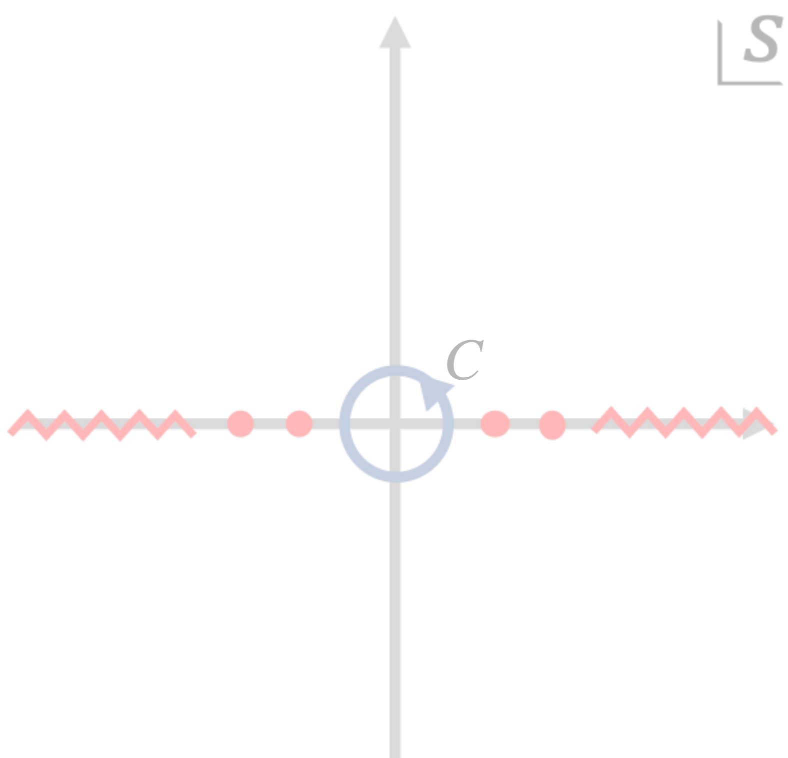
$$\frac{a_p}{p!} = \oint_C \frac{ds}{2\pi i} \frac{M(s,0)}{s^{p+1}}$$

0 for $p \geq 2$

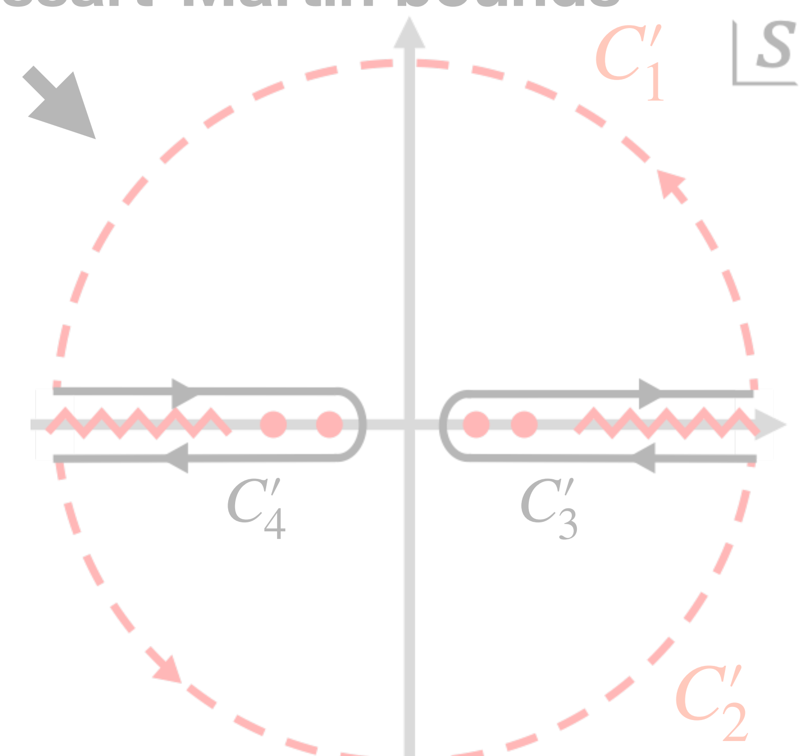
$$= \left(\int_{C'_1} + \int_{C'_2} + \int_{C'_3} + \int_{C'_4} \right) \frac{ds}{2\pi i} \frac{M(s,0)}{s^{p+1}}$$

$M(s,t) < s^2$

Froissart-Martin bounds

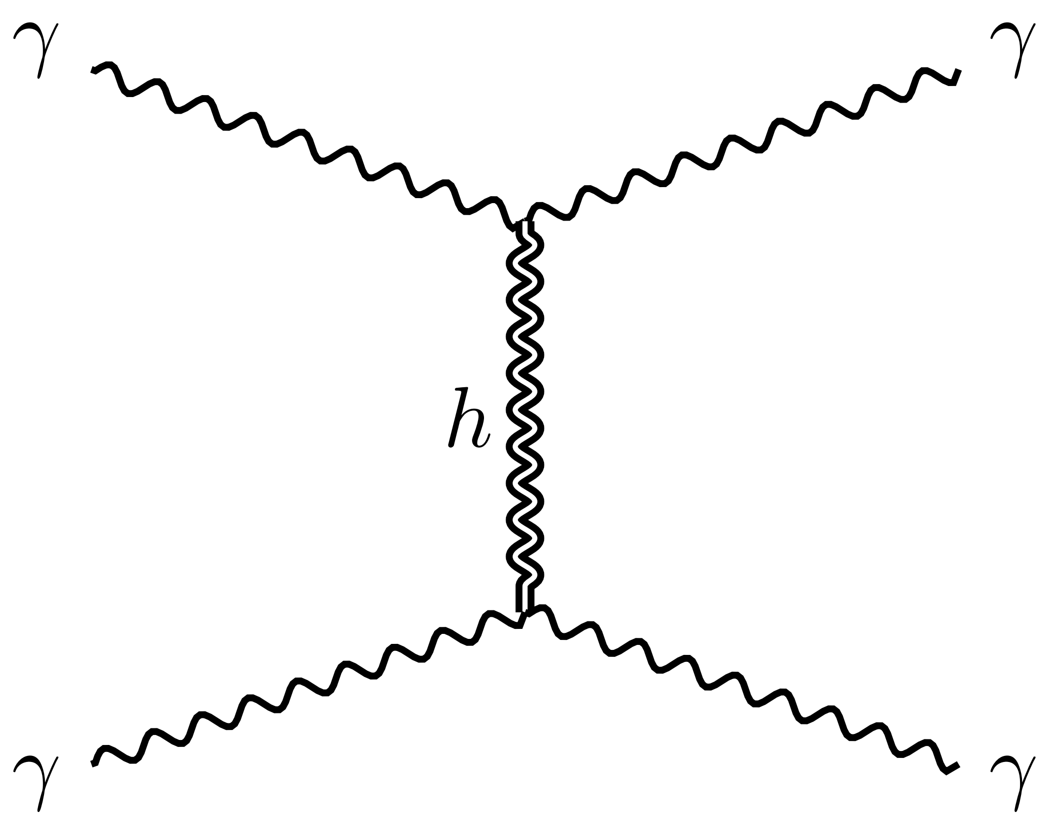


Deform the integral contour



Graviton t-channel pole

- t-channel pole of light-by-light scattering mediated by graviton breaks the Froissart-Martin bound w/ finite t



$$\propto -\frac{s^2}{t}$$

Regge behavior

- Additional assumption on high energy behavior

Tokuda, Aoki, and Hirano [2020]

$$\lim_{|s|\to\infty} \mathrm{Im}M(s,t) \approx f(t)\left(\frac{\alpha' s}{4}\right)^{2+j(t)}$$

α': Reggeization scale

- Define s^2 coefficient subtracting gravitational t pole

$$\tilde{B}(\Lambda) := \lim_{t\to -0}\left[B(\Lambda) + \frac{2}{M_{\rm pl}^2 t}\right]_{s=0}$$

$$= \lim_{t\to -0}\left[\frac{4}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}M(s,t)}{s^3} + \frac{2}{M_{\rm pl}^2 t}\right]_{s=0} \gtrsim \frac{\sigma}{M_s^2 M_{\rm pl}^2}$$

$\sigma = \pm\, \mathcal{O}(1)$ which depends on the details of UV completion of gravity

Regge behavior

- Additional assumption on high energy behavior

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Gravitational positivity bounds

Gravitational positivity bounds

- Gravitational positivity bounds as a consistency condition with quantum gravity

$$\tilde{B}(\Lambda) := = \lim_{t\to -0}\left[\frac{4}{\pi}\int_{\Lambda^2}^{\infty} ds \frac{\mathrm{Im}M(s,t)}{s^3} + \frac{2}{M_{\rm pl}^2 t}\right]_{s=0} \gtrsim 0$$

- Phenomenological applications to
  - PB in QED w/ gravitational diagrams Alberte, de Rham, Jaitly, Tolley[2020]
  - Gravitational PB in the Standard model Aoki,Loc, Noumi, Tokuda [2021]
    in scalar potential Noumi, Tokuda [2021]
    in massive dark photon Noumi, Sato, Tokuda [2022]
    Aoki, Noumi, Saito, Sato, Shirai, Tokuda, Yamazaki [2023]
- We will discuss the constraints from gravitational positivity bounds in millicharged dark matter model

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Millicharged dark matter

- Action

$$S_{\rm DS} = \int d^4x\sqrt{-g}\left[-\frac{1}{4}F_{b,\mu\nu}F_b^{\mu\nu} - \frac{\varepsilon}{2}F_{a,\mu\nu}F_b^{\mu\nu} + \bar{\chi}(i\gamma^\mu\nabla_\mu + e_D\gamma^\mu A_{b,\mu} - m)\chi\right]$$

- 3 parameters $\varepsilon, m, \alpha_D = e_D^2/4\pi$

- After diagonalization

$$S_{\rm DS} \approx \int d^4x\sqrt{-g}\left[-\frac{1}{4}F'_{\mu\nu}F'^{\mu\nu} + \bar{\chi}\left(i\gamma^\mu\nabla_\mu + e_D\gamma^\mu A'_\mu - \frac{\varepsilon}{2}e_D\gamma^\mu A_\mu - m\right)\chi + \mathcal{O}(\epsilon^2)\right]$$

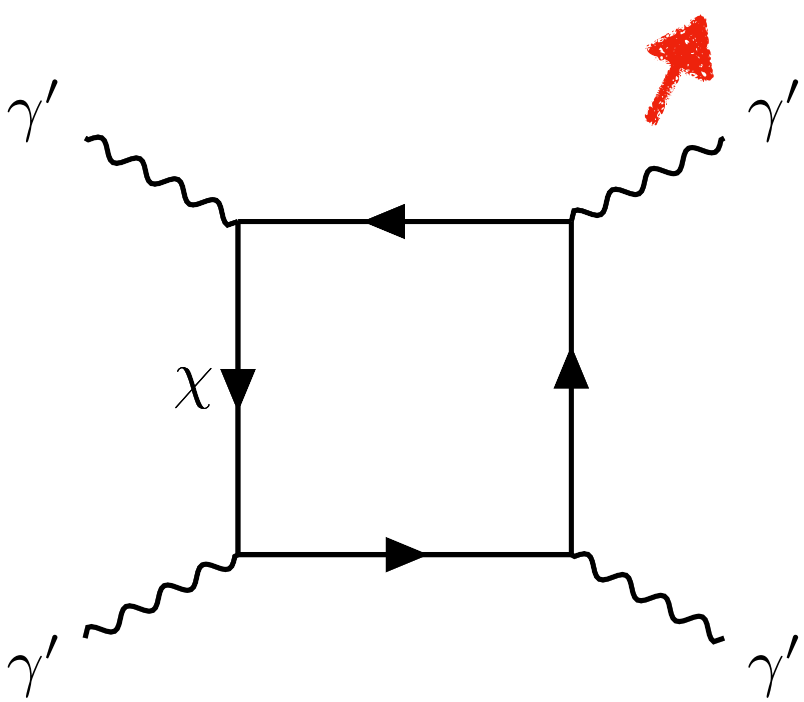
- Dark matter couples to SM photon by $A_b \to A' - \varepsilon A$

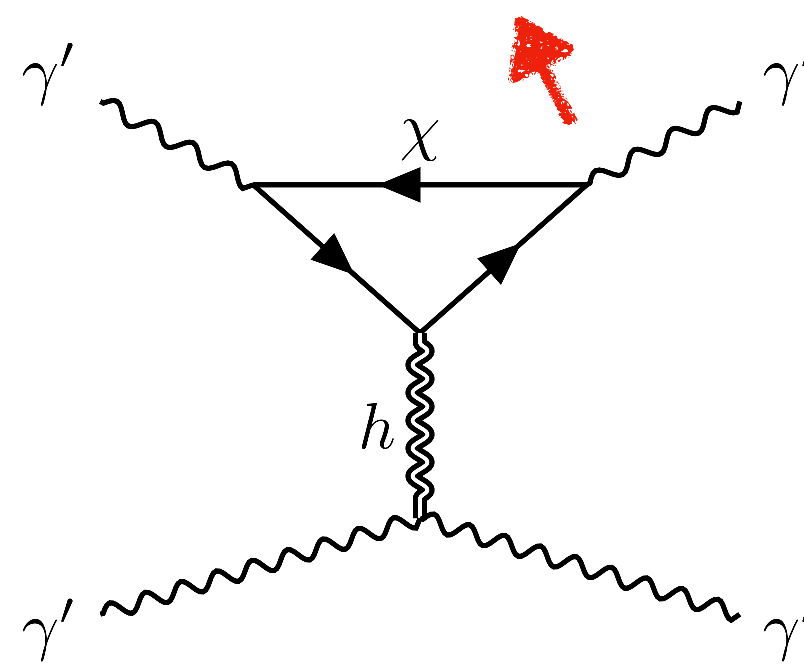
- Let's investigate the positivity constraints on $\gamma'\gamma' \to \gamma'\gamma'$ and $\gamma\gamma' \to \gamma\gamma'$ scattering amplitudes

$\gamma'\gamma' \to \gamma'\gamma'$ Scattering

- $\gamma'\gamma' \to \gamma'\gamma'$ cases

$$B_{\text{total}}(\Lambda) = B_{\text{non-grav}}(\Lambda) + B_{\text{grav}}(\Lambda)$$





$$\frac{16\alpha_D^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right)$$

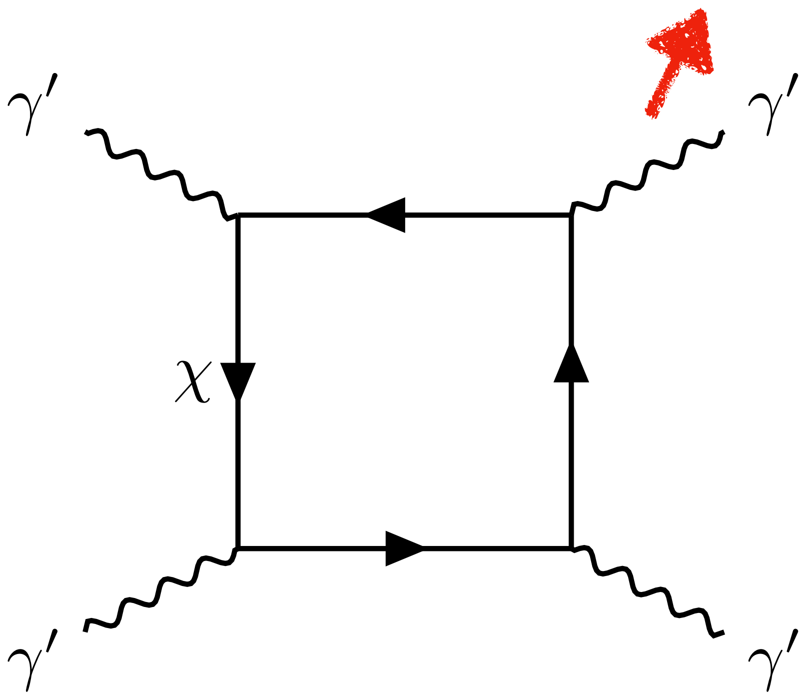
$$-\frac{11\alpha_D}{90\pi m^2 M_{\text{pl}}^2}$$

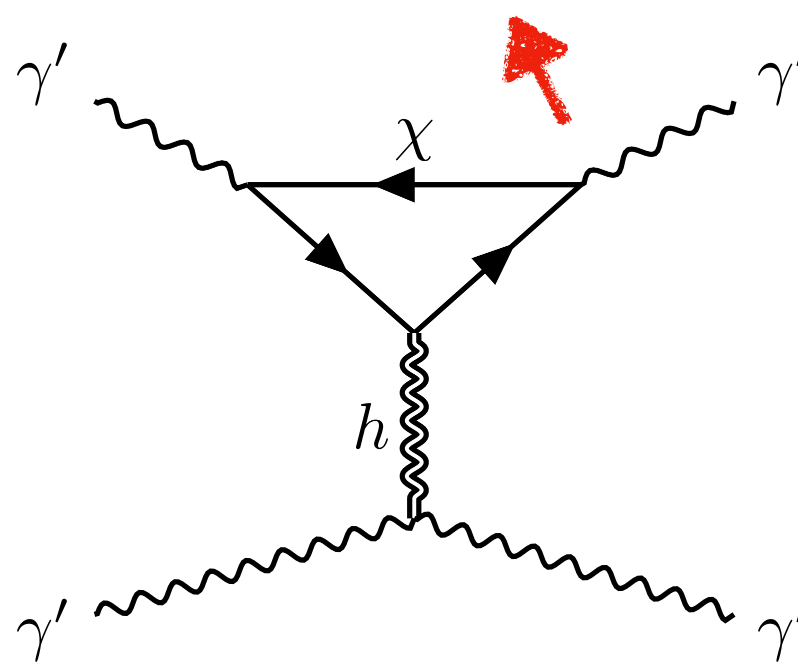
where $\alpha'_D = \frac{e_D^2}{4\pi}$

$\gamma'\gamma' \to \gamma'\gamma'$ Scattering

Point 1: Gravitational contribution can be negative so that we can obtain stronger bound

$$B_{\rm total}(\Lambda) = B_{\rm non-grav}(\Lambda) + B_{\rm grav}(\Lambda)$$





$$\frac{16\alpha_D^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m}-\frac{1}{4}\right) \qquad -\frac{11\alpha_D}{90\pi m^2 M_{\rm pl}^2}$$

where $\alpha'_D = \frac{e_D^2}{4\pi}$

$$\alpha_D\left(\ln\frac{\Lambda}{m}-\frac{1}{4}\right) > \frac{11}{1440\pi}\left(\frac{\Lambda}{m}\right)^2\left(\frac{\Lambda}{M_{\rm pl}}\right)^2 .$$

$\gamma'\gamma' \to \gamma'\gamma'$ Scattering

- Point 1: Gravitational contribution is negative so that we can obtain stronger bound

$$B_{\rm total}(\Lambda) = B_{\rm non-grav}(\Lambda) + B_{\rm grav}(\Lambda)$$

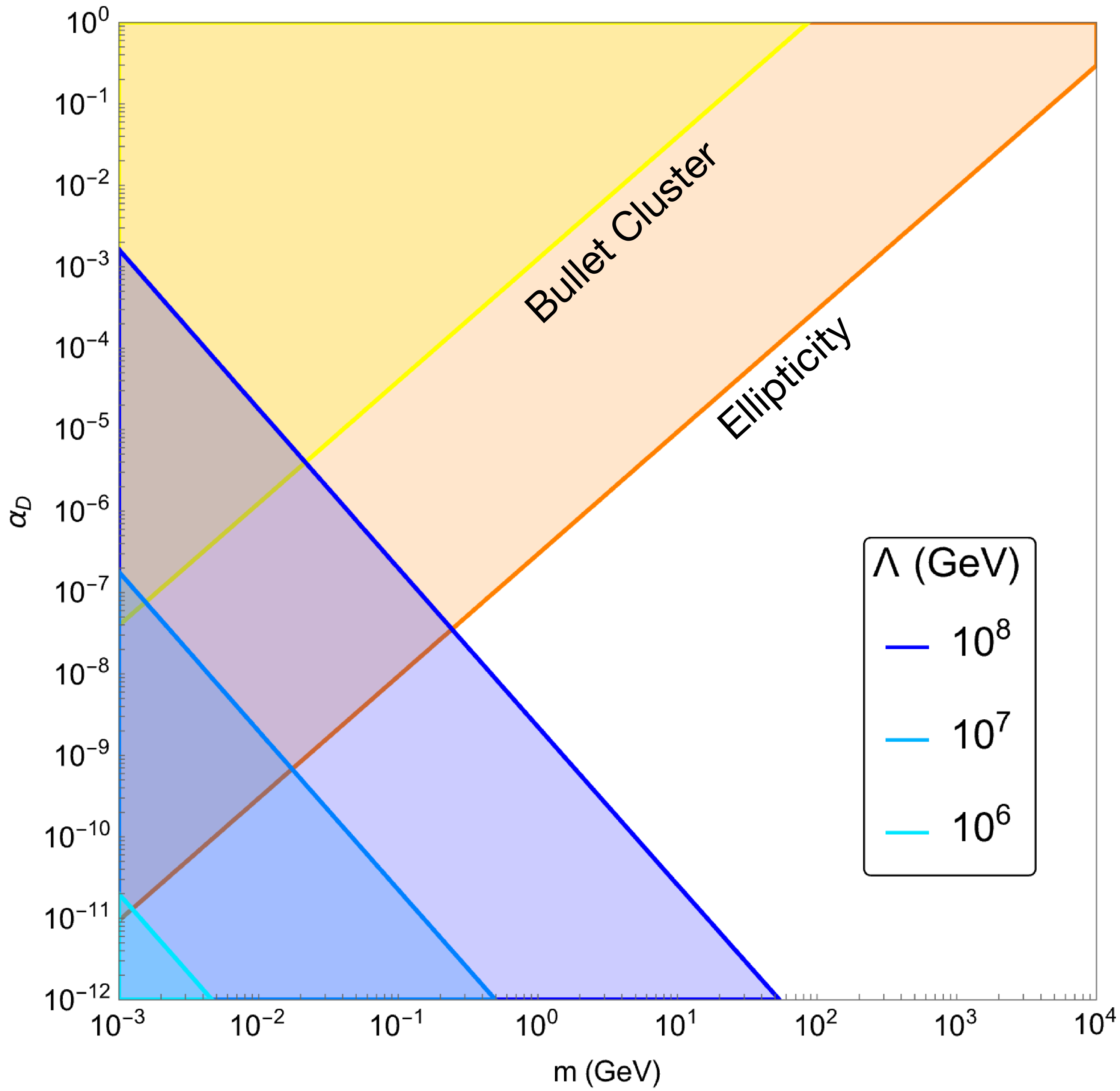
$$\frac{16\alpha_D^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right) \qquad -\frac{11\alpha_D}{90\pi m^2 M_{\rm pl}^2} \qquad \text{where } \alpha_D' = \frac{e_D^2}{4\pi}$$

Point 2: Constraints on renormalizable couplings

$$\alpha_D\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right) > \frac{11}{1440\pi}\left(\frac{\Lambda}{m}\right)^2\left(\frac{\Lambda}{M_{\rm pl}}\right)^2.$$

Constraint from $\gamma'\gamma' \to \gamma'\gamma'$ scattering

$$\alpha_D \left(\ln \frac{\Lambda}{m} - \frac{1}{4} \right) > \frac{11}{1440\pi} \left(\frac{\Lambda}{m} \right)^2 \left(\frac{\Lambda}{M_{\rm pl}} \right)^2 .$$

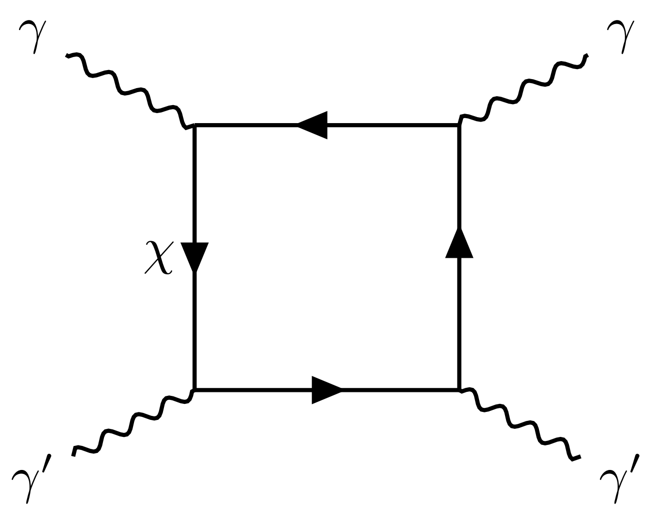


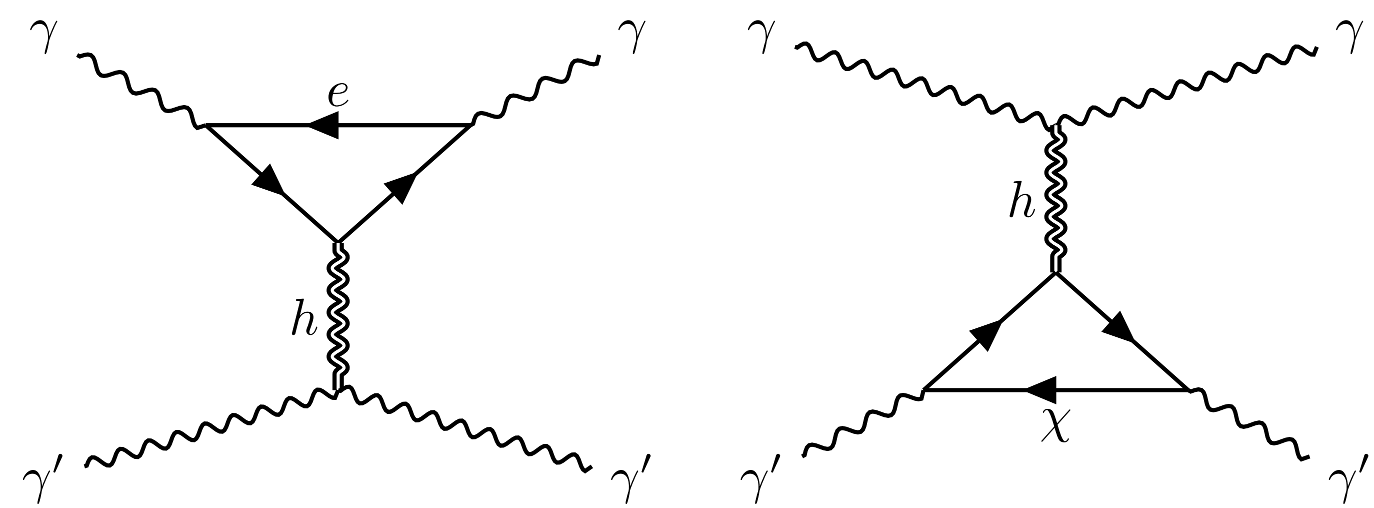
When $m \sim 10^{-3}$, and $\Lambda \gtrsim 10^6$
the whole parameter range
is constrained by combining
observation and positivity bound

$\gamma\gamma' \to \gamma\gamma'$ scattering

- $\gamma\gamma' \to \gamma\gamma'$ cases

$$B_{\text{total}}(\Lambda) = B_{\text{non-grav}}(\Lambda) + B_{\text{grav}}(\Lambda)$$





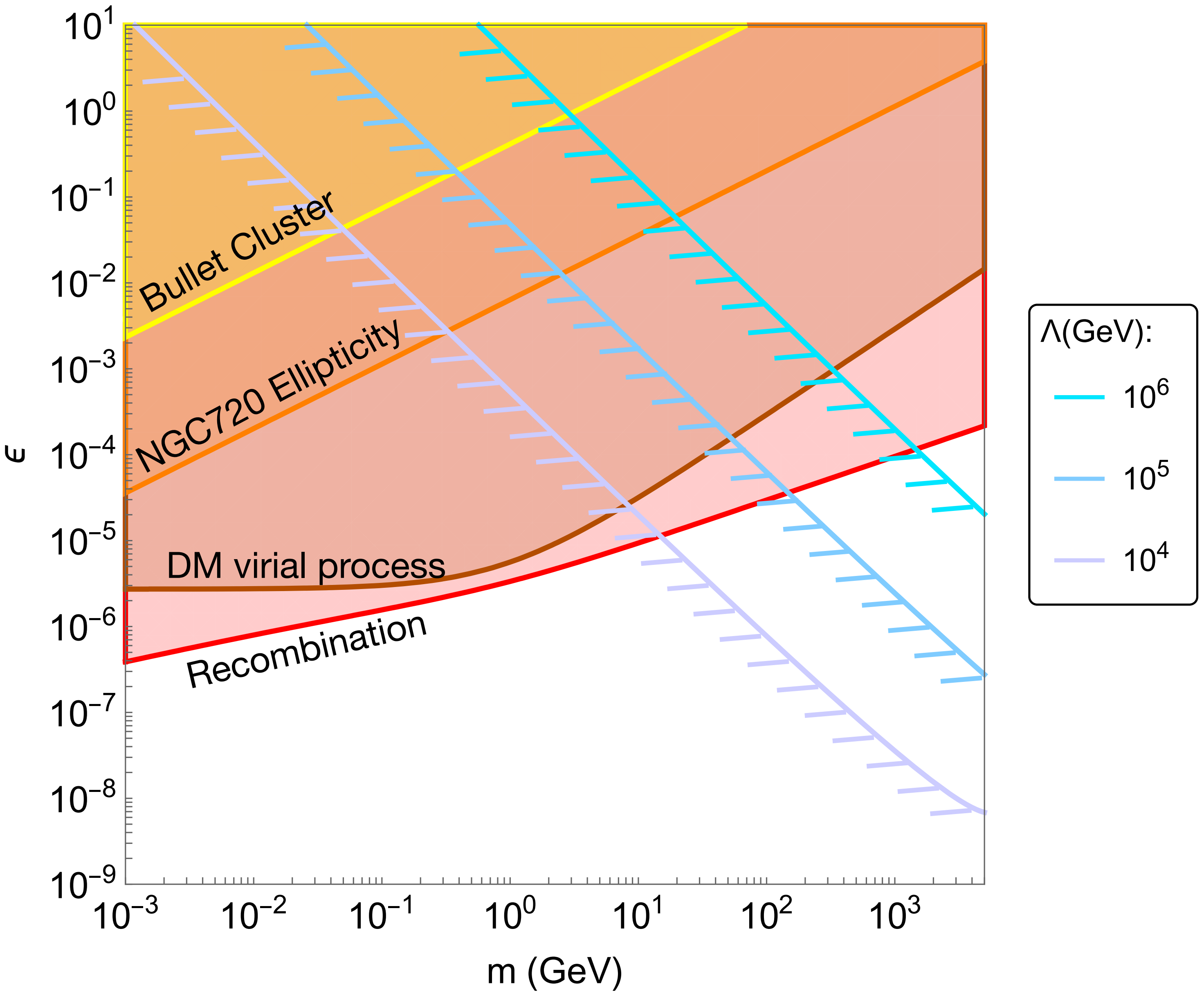
$$\frac{16\varepsilon^2\alpha_D^2}{\Lambda^4}\left(\ln\frac{\Lambda}{m} - \frac{1}{4}\right)$$

$$-\frac{11\alpha}{180\pi m_e^2 M_{\text{pl}}^2} - \frac{11\alpha_D}{180\pi m^2 M_{\text{pl}}^2}$$

Wimp mass range

$$\varepsilon^2 \alpha_D^2 \left(\ln \frac{\Lambda}{m} - \frac{1}{4} \right) > \frac{11\alpha}{2880\pi} \left(\frac{\Lambda}{M_{\rm Pl}} \right)^2 \left(\frac{\Lambda}{M_{\rm e}} \right)^2 \left[1 + \frac{\alpha_D}{\alpha} \left(\frac{m_e}{m} \right)^2 \right]$$

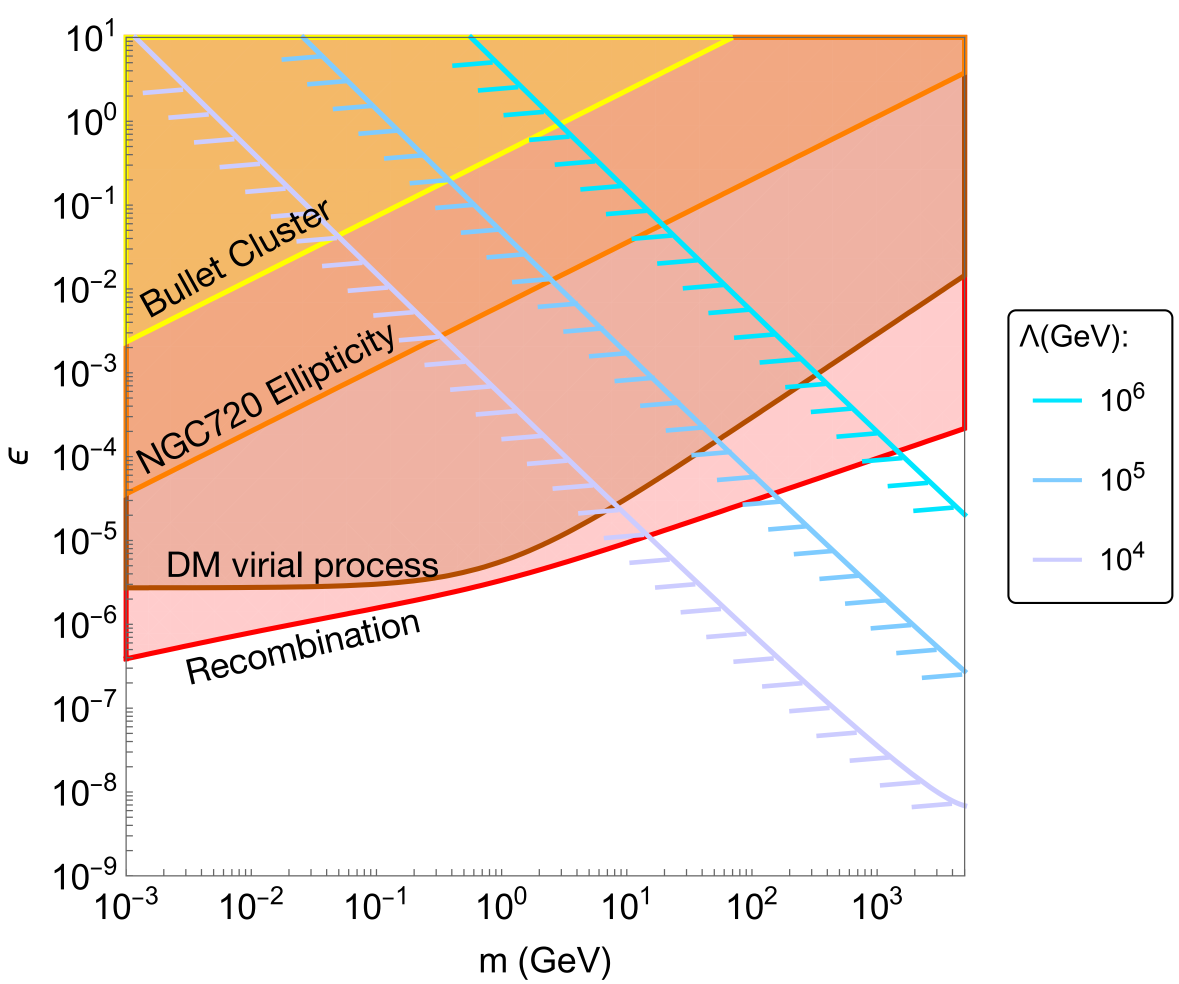
$\alpha \sim \alpha_{\rm bound}$



Interpretation of the result

- Case 1: When we get stronger constraints from the observation (the shaded regions go down)
  or we could not find any new physics even in high energy (Blue lines go up) or both

$$\varepsilon^2\left[\alpha^2\left(\ln\frac{\Lambda}{m_e}-\frac{1}{4}+2\left(\frac{\Lambda^2}{m_W^2}\right)^2\right)+\alpha'^2\left(\ln\frac{\Lambda}{m}-\frac{1}{4}\right)\right]>\frac{11\alpha}{720\pi}\left(\frac{\Lambda}{M_{\rm pl}}\right)^2\left(\frac{\Lambda}{M_{\rm e}}\right)^2\left[1+\frac{\alpha'}{\alpha}\left(\frac{m_e}{m}\right)^2\right]$$



- We can rule out the this model by combining observations and theoretical constraints

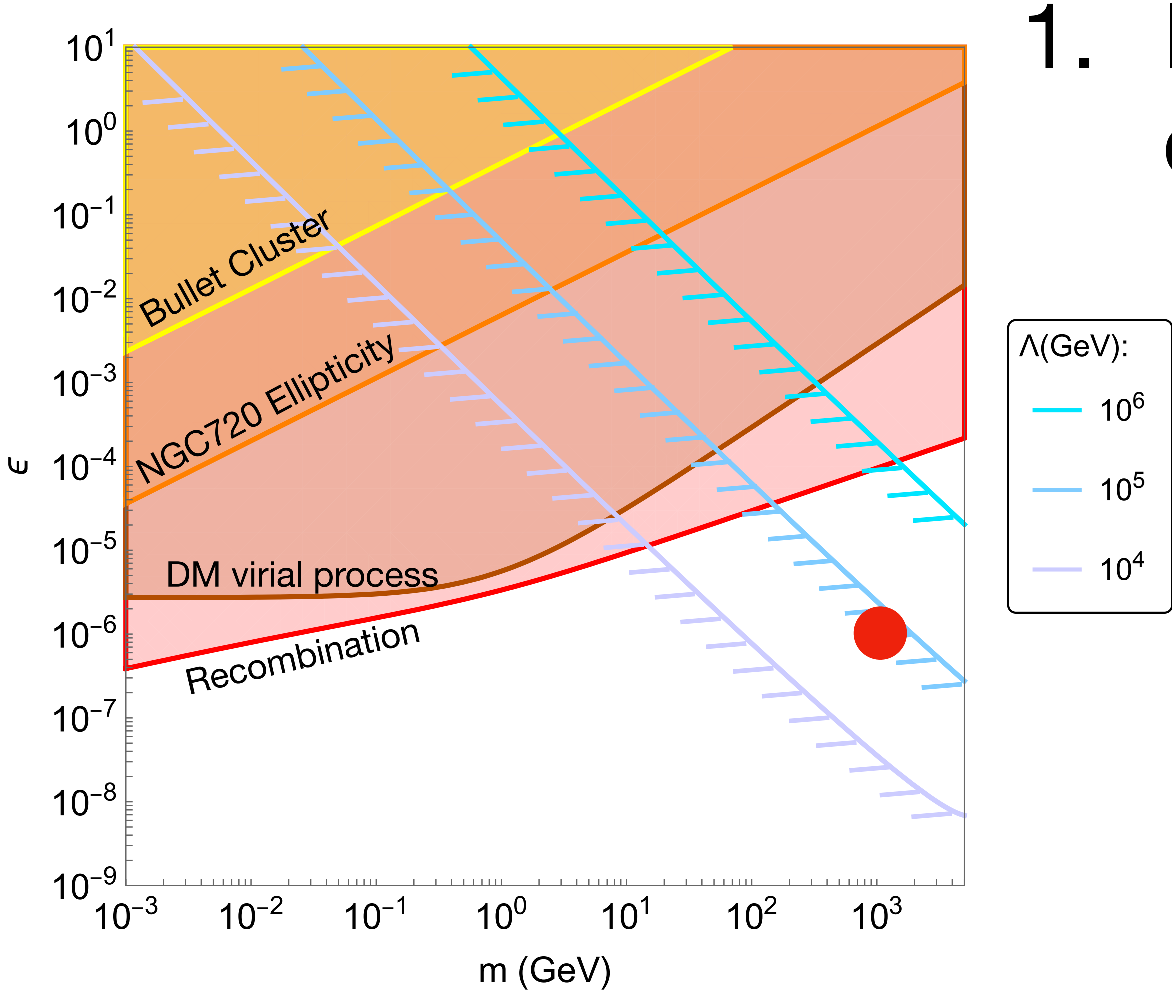
Interpretation of the result

- Case 2: We could find the dark matter from the observation and pin down on the parameter space

$$\varepsilon^2\left[\alpha^2\left(\ln\frac{\Lambda}{m_e}-\frac{1}{4}+2\left(\frac{\Lambda^2}{m_W^2}\right)^2\right)+\alpha'^2\left(\ln\frac{\Lambda}{m}-\frac{1}{4}\right)\right]>\frac{11\alpha}{720\pi}\left(\frac{\Lambda}{M_{\rm pl}}\right)^2\left(\frac{\Lambda}{M_{\rm e}}\right)^2\left[1+\frac{\alpha'}{\alpha}\left(\frac{m_e}{m}\right)^2\right]$$

- We have to modify the theory below $\Lambda = 10^5$ GeV

1. Introduce new fields which have a different high energy behavior

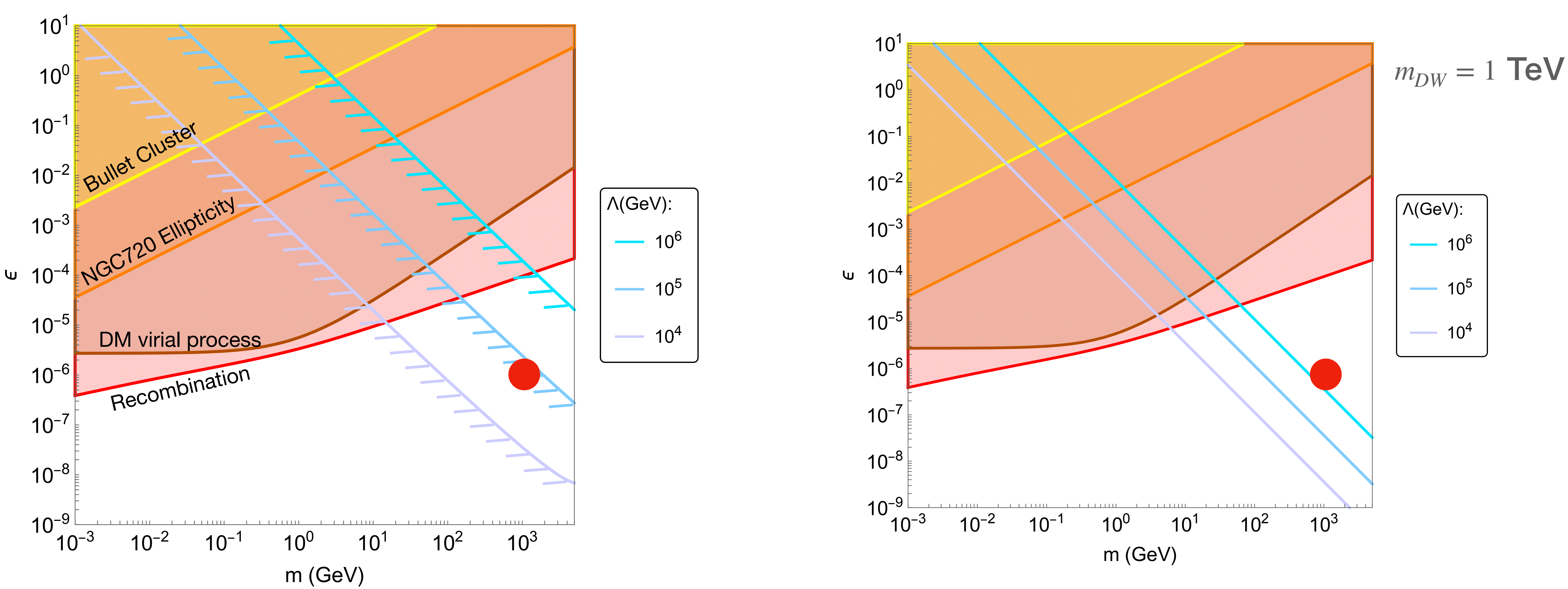


Interpretation of the result

- Case 2: We could find the dark matter from the observation and pin down on the parameter space
- We have to modify the theory below $\Lambda = 10^5$ GeV
    1. Introduce new fields which have a different high energy behavior

$$\varepsilon^2\left[\alpha^2\left(\ln\frac{\Lambda}{m_e}-\frac{1}{4}+2\left(\frac{\Lambda^2}{m_{DW}^2}\right)^2\right)+\alpha'^2\left(\ln\frac{\Lambda}{m}-\frac{1}{4}\right)\right]>\frac{11\alpha}{720\pi}\left(\frac{\Lambda}{M_{\rm pl}}\right)^2\left(\frac{\Lambda}{M_{\rm e}}\right)^2\left[1+\frac{\alpha'}{\alpha}\left(\frac{m_e}{m}\right)^2\right]$$

Without dark W boson **With dark W boson**



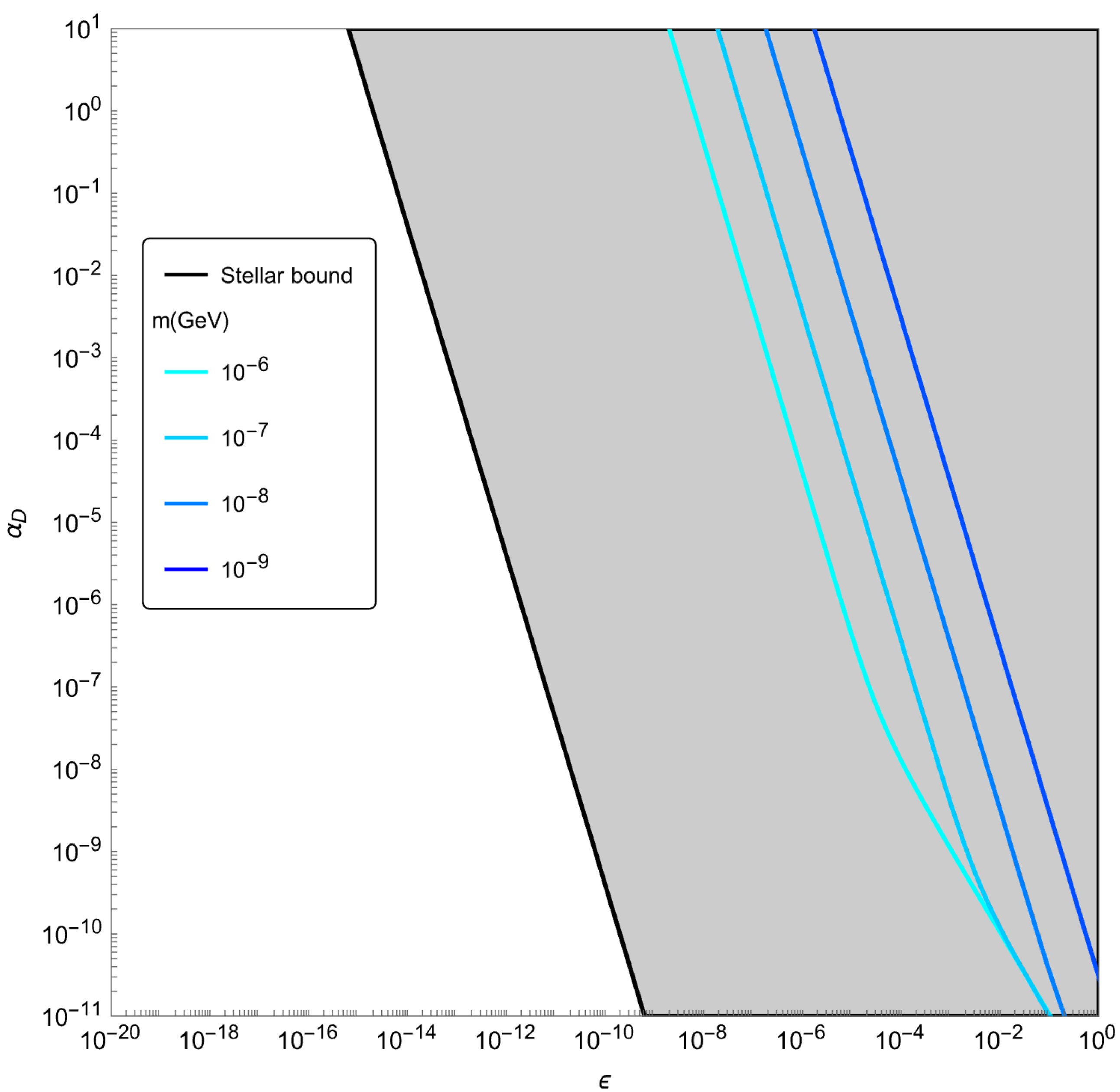
Interpretation of the result

- Case 2: We could find the dark matter from the observation and pin down on the parameter space
- We have to modify the theory below $\Lambda = 10^5$ GeV
  1. Introduce new fields which have a different high energy behavior
  2. Introduce enormous number of particles to modify the bounds (e.g. Kaluza-Klein modes)

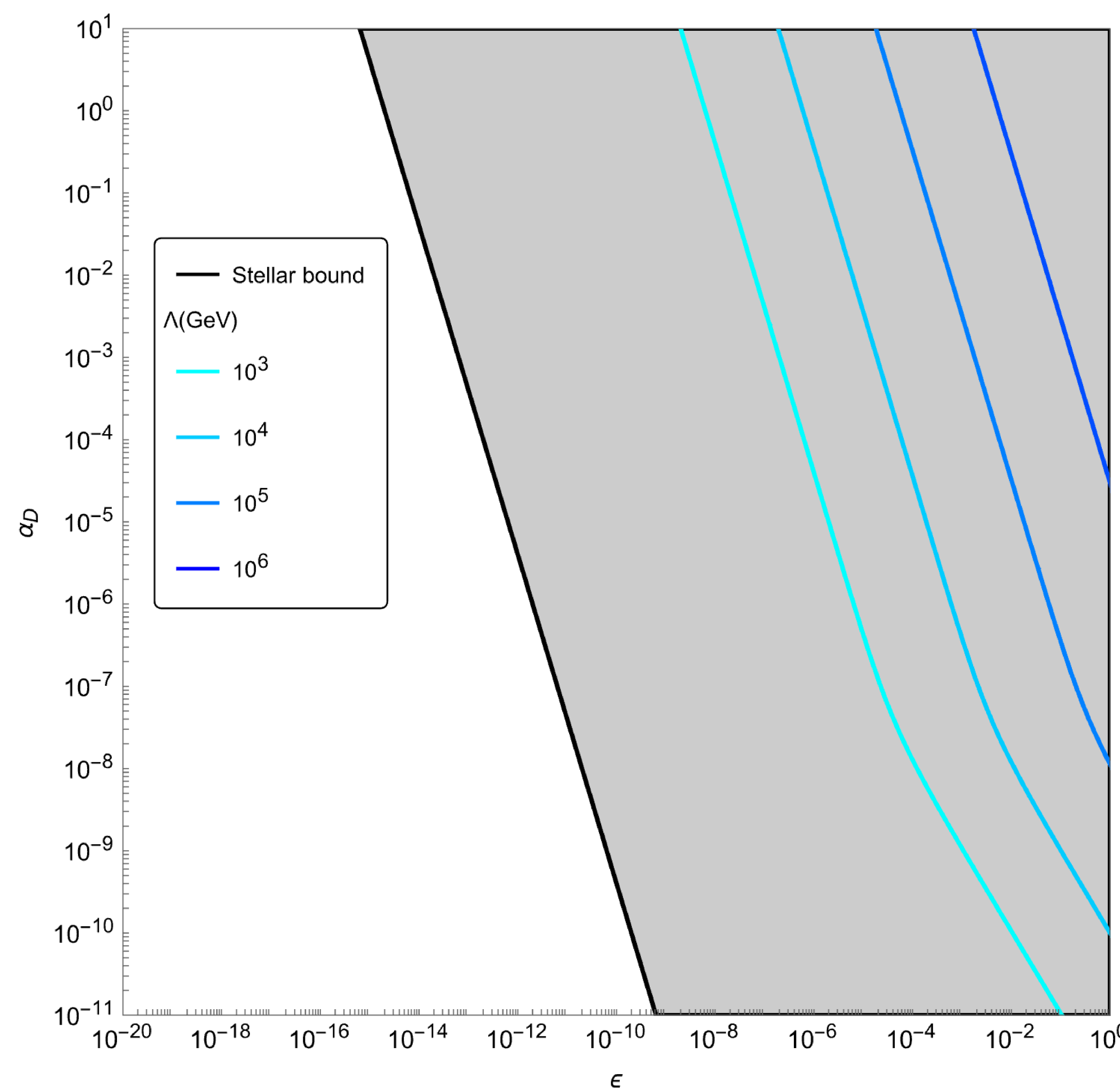
Light dark matter $m < T_\odot \sim 1\text{keV}$

$$\varepsilon^2 \alpha_D^2 \left(\ln \frac{\Lambda}{m} - \frac{1}{4} \right) > \frac{11\alpha}{2880\pi} \left(\frac{\Lambda}{M_{\rm Pl}} \right)^2 \left(\frac{\Lambda}{M_{\rm e}} \right)^2 \left[1 + \frac{\alpha_D}{\alpha} \left(\frac{m_e}{m} \right)^2 \right]$$

When we fix $\Lambda = 10^3$ GeV



When we fix $m = 10^{-6}$ GeV



Conclusion

- Positivity bounds are interesting as UV-IR consistency conditions
- Phenomenologically, we may get interesting bounds by incorporating gravity into positivity bounds
- Constraints on milli-charged dark matter from gravitational positivity bounds