Lessons from the Seiberg-Witten Axion

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Outline

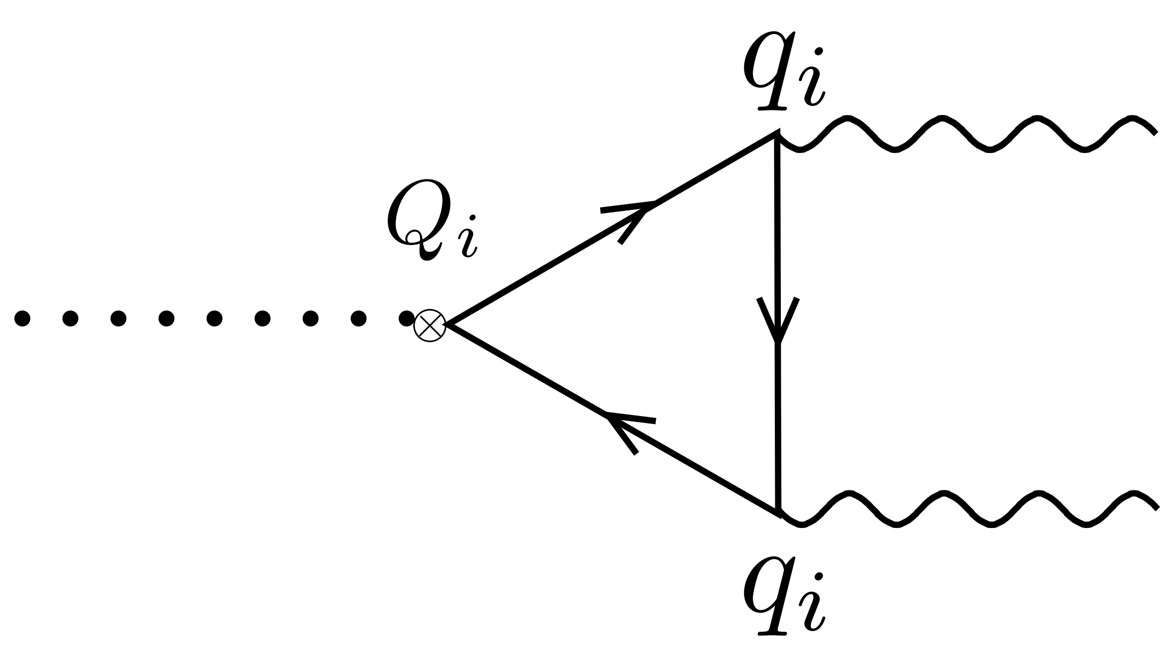
- The photon axion coupling and quantization - puzzles when theory has monopoles/duality
- The SW axion
- EM duality and the Seiberg-Witten solution
- Couplings of the SW axion - electric and magnetic frames
- Duality invariance of the axion decay rate
- Explicit instanton calculation
- Lessons learned

The axion-photon coupling

- Determined by the anomaly $$\partial_\mu j^\mu_A = \frac{q^2}{8\pi^2} F\tilde{F}$$

$$\mathcal{L}_{a,EM} = -\frac{a}{Nf}\sum_i Q_i \left(\frac{q_i^2}{16\pi^2} F^{\mu\nu}\widetilde{F}_{\mu\nu}\right)$$

- Q_i the PQ charge, q_i the electric charge of fermion



- Important for axion detection experiments…

Axion coupling quantization

- PQ axion a Goldstone boson, compact internal direction, expect $a \to a + 2\pi v$ discrete shift symmetry to be exact. But we have a linear coupling from anomaly…

$$\mathcal{L}_{a,EM} = -\frac{a}{Nf}\sum_i Q_i \left(\frac{q_i^2}{16\pi^2} F^{\mu\nu}\widetilde{F}_{\mu\nu}\right)$$

- Generally expect to be quantized!

$$Ch_2 = \frac{q^2}{16\pi^2}\int d^4x\, F^{\mu\nu}\,\widetilde{F}_{\mu\nu}$$

- Due to Atiyah-Singer index theorem:

$$n_L - n_R = \frac{1}{2}\int d^4x\,\partial_\mu j_A^\mu(x) = \int d^4x \frac{q^2}{16\pi^2} F\tilde{F} = q^2 Ch_2$$

- Assuming all PQ charges integer - axion coupling quantized

Anomalies and monopoles

- In the presence of monopoles the anomaly is modified

- Monopole charge due to Dirac quantization where g is half-integer…

$$\frac{4\pi g}{e^2}$$

- Monopoles can also run in the triangle diagram, anomaly:

$$\partial_\mu j^\mu_A(x) = 2\sum_i \left(\mathcal{A}_i F'^{\mu\nu}\, \widetilde{F}'_{\mu\nu} + \mathcal{B}_i F'^{\mu\nu}\, F'_{\mu\nu}\right)$$

$$\mathcal{A}_i = \frac{q_i^2}{16\pi^2} - \frac{g_i^2}{e^4} \quad , \quad \mathcal{B}_i = \frac{q_i g_i}{4\pi e^2} \, ,$$

- Note that $dF' \neq 0$ and we set $\theta = 0$

Puzzle #1

- What is the axion-photon coupling for the case of magnetic monopoles?

- Anomalies suggest $\mathcal{L}_{a,EM} = -\frac{a}{Nf}\sum_i Q_i \left(\mathcal{A}_i F'^{\mu\nu}\widetilde{F}'_{\mu\nu} + \mathcal{B}_i F'^{\mu\nu}F'_{\mu\nu}\right)$

$$\mathcal{A}_i = \frac{q_i^2}{16\pi^2} - \frac{g_i^2}{e^4} \quad , \quad \mathcal{B}_i = \frac{q_i g_i}{4\pi e^2}$$

- Does not seem to be quantized. Also can be much larger than the usual coupling by factor of $1/\alpha^2 \sim 10^4$

- Indeed Sokolov and Ringwald claimed this to be the size (though somewhat more complicated form) of the coupling

Electric-magnetic duality and axions

- The free Maxwell equations exhibit electric-magnetic duality

$$\mathbf{E} \to \mathbf{B},\ \mathbf{B} \to -\mathbf{E}$$

- Can extend this to full SL(2,Z) symmetry if we also introduce the **θ** angle

$$\mathcal{L}_{\rm free} = -\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu} - \frac{\theta}{16\pi^2} F^{\mu\nu} \widetilde{F}_{\mu\nu}$$

- Most useful to introduce ``holomorphic coupling" **τ**:

$$\tau \equiv \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$$

Electric-magnetic duality and axions

- Under SL(2,Z) duality transformation: $$\tau' = \frac{a\tau + b}{c\tau + d}$$

- a,b,c,d integers with ad-bc=1

- If you have charges $\begin{pmatrix} g' \\ q' \end{pmatrix} = \mathcal{M}^T \begin{pmatrix} g \\ q \end{pmatrix}$ $\quad \mathcal{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

- Physical charges $(4\pi g/e, qe)$

- Usual duality called $S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ exchanges electric and magnetic fields AND charges, inverts coupling $\tau' = -\frac{1}{\tau}$

- NOT a traditional symmetry - different descriptions of same physics

Electric-magnetic duality and axions

- 2π shift of θ angle usual symmetry, $T = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$
- Field transform as $\left(F'^{\mu\nu} + i\,\widetilde{F}'^{\mu\nu}\right) = \frac{1}{c\tau^* + d}\left(F^{\mu\nu} + i\,\widetilde{F}^{\mu\nu}\right)$
- Maxwell equations covariant under SL(2,Z):

$$\frac{\mathrm{Im}\,(\tau)}{4\pi}\,\partial_\nu\left(F^{\mu\nu} + i\,\widetilde{F}^{\mu\nu}\right) = J^\mu + \tau K^\mu$$

- What happens when we have an axion? Sikivie axion electrodynamics

$$\nabla\cdot\vec{E} = \frac{e^2N}{3\pi^2 v}\vec{B}\cdot\nabla a, \quad \nabla\times\vec{B} - \frac{\partial\vec{E}}{\partial t} = \frac{e^2N}{3\pi^2 v}\left[\vec{E}\times\nabla a - \vec{B}\frac{\partial a}{\partial t}\right], \quad \Box a = \frac{e^2N}{3\pi^2 v}\vec{E}\cdot\vec{B} - m_a{}^2 a.$$

Puzzle #2: what are the duality invariant Maxwell-axion equations?

- Sikivie axion electrodynamics

$$\nabla\cdot\vec{E}=\frac{e^2N}{3\pi^2v}\vec{B}\cdot\nabla a,\quad \nabla\times\vec{B}-\frac{\partial\vec{E}}{\partial t}=\frac{e^2N}{3\pi^2v}\left[\vec{E}\times\nabla a-\vec{B}\frac{\partial a}{\partial t}\right],\quad \Box a=\frac{e^2N}{3\pi^2v}\vec{E}\cdot\vec{B}-m_a{}^2a.$$

- With added assumption $\nabla\cdot\vec{B}=0,\quad \nabla\times\vec{E}+\dfrac{\partial\vec{B}}{\partial t}=0$

- However, this is clearly NOT incorporating electric-magnetic duality

- In covariant form: $\partial_\mu F^{\mu\nu}=g\tilde{F}^{\sigma\nu}\partial_\sigma a,\quad \partial_\mu\tilde{F}^{\mu\nu}=0$

- Alternative form that seems to be duality invariant:

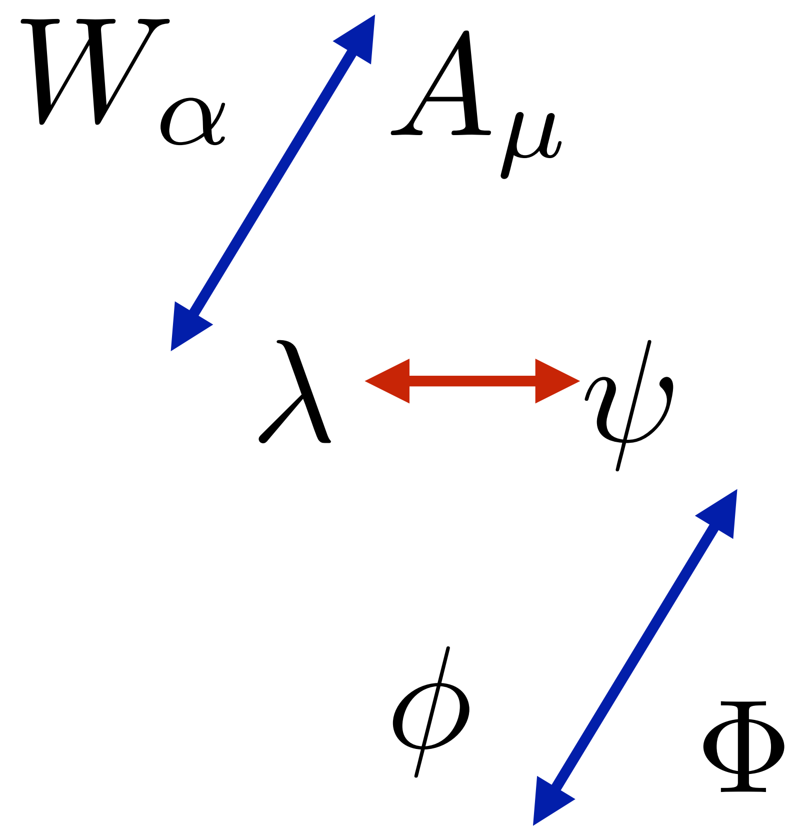
$$\partial_\mu F^{\mu\nu}=g\tilde{F}^{\sigma\nu}\partial_\sigma a,\quad \partial_\mu\tilde{F}^{\mu\nu}=-gF^{\sigma\nu}\partial_\sigma a$$

- What's going on here?

The Seiberg-Witten axion

- These questions can be addressed within a calculable UV complete toy-example: The original N=2 supersymmetric SU(2) Seiberg-Witten theory

- Matter content: N=2 vector superfield - in N=1 language a vector superfield + a chiral superfield in the adjoint, no superpotential



- Global symmetries: SU(2)$_R$ $\lambda \leftrightarrow \psi$ U(1)$_R$ Φ has R-charge 2, but anomalous

The Seiberg-Witten axion

- The theory has a ``moduli space" of vacua - essentially **Φ** has no potential, adjoint scalar gets a VEV breaking $SU(2) \rightarrow U(1)$ giving rise to a Coulomb branch (ordinary QED-like theory, except it is N=2 supersymmetric)

- The Coulomb branch parametrized by the gauge invariant $u \equiv \frac{1}{2}\mathrm{tr}(\phi^2)$

- SU(2) everywhere broken, for all values of u. Large $u \gg \Lambda^2$: SU(2) broken before it becomes strongly coupled - perturbative regime. The $W^{\pm}$ become massive, along with the charginos via the super-Higgs mechanism

- For $u \ll \Lambda^2$: strongly coupled regime, non-perturbative effects important

The Seiberg-Witten axion

- Since **Φ** has R-charge 2, the R-symmetry will be spontaneously broken everywhere. There will be a PQ-axion in the spectrum (the R-axion)

- R-symmetry spontaneously broken and anomalous - a nice toy example to study for axion physics

- Important: as we will see the dynamics of the SW solution will imply the presence of magnetic monopoles/dyons that may become light at certain points in the moduli space

- Monopoles/dyons do carry $U(1)_R$ charge as well - can view this as monopole getting mass from PQ breaking

- Perfect example to study effect of monopoles on axion coupling!

The Seiberg-Witten solution

- Effective theory N=2 SUSY U(1) theory, can be written in terms of chiral superfield A + vector superfield V (A, V) OR the dual variable (A_D, V_D)

- They are not independent - there is a complicated non-linear relation between A and A_D:

$$A_D = \frac{\partial \mathcal{F}(A)}{\partial A} \quad , \quad A = -\frac{\partial \mathcal{F}_D(A_D)}{\partial A_D} \, .$$

- $\mathcal{F}(A)$ is the prepotential - exactly calculable in SW

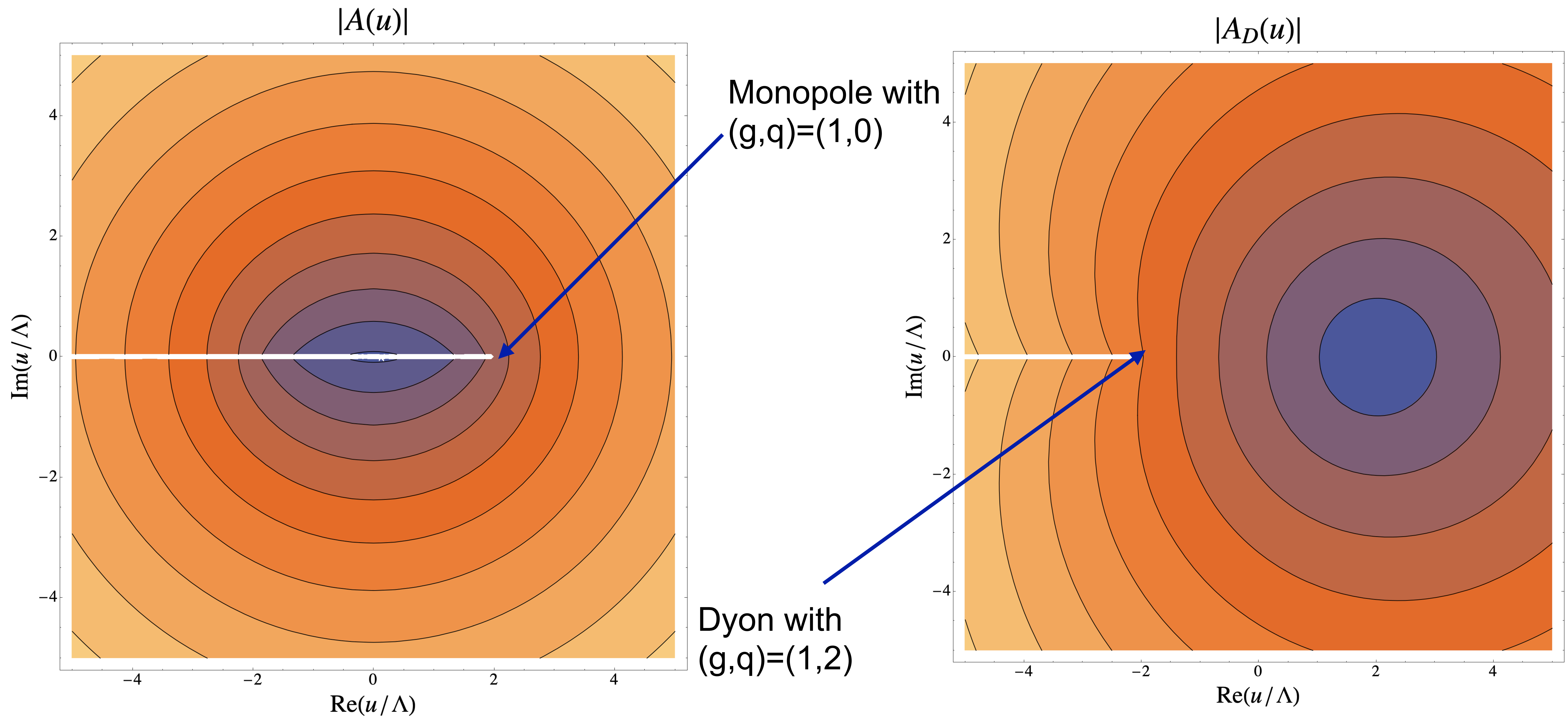
- VEVs also calculable:

$$A^v(u) = \sqrt{u + 2\Lambda^2}\, {}_2F_1\left(-\frac{1}{2}, \frac{1}{2}, 1; \frac{4\Lambda^2}{u + 2\Lambda^2}\right)$$

$$A^v_D(u) = i\frac{u - 2\Lambda^2}{2\Lambda}\, {}_2F_1\left(\frac{1}{2}, \frac{1}{2}, 2; \frac{2\Lambda^2 - u}{4\Lambda^2}\right) ,$$

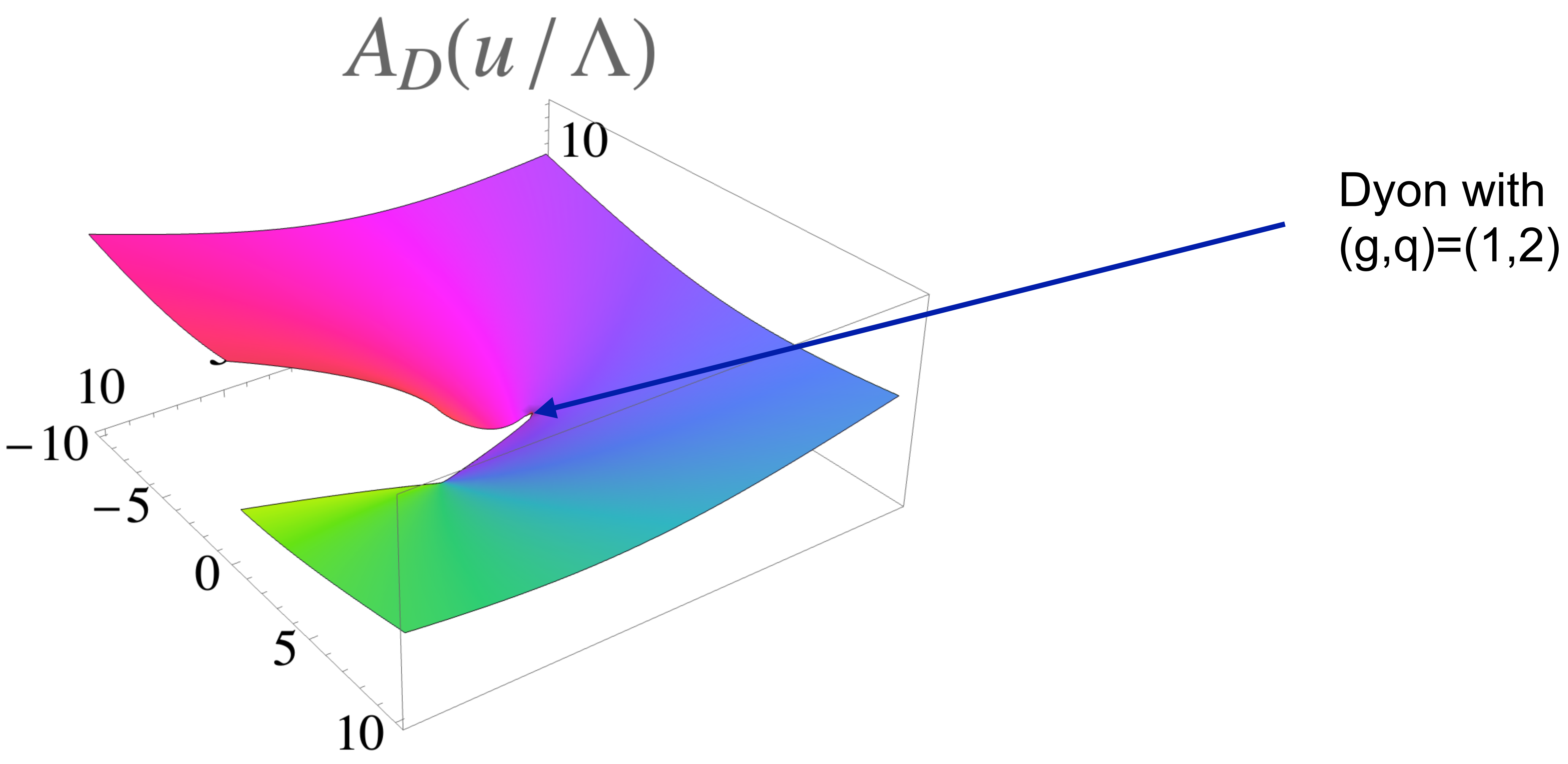
The Seiberg-Witten solution

- A has branching point at $u = \pm 2\Lambda^2$ - massless monopole/dyon
- A_D has branching point at $u = -2\Lambda^2$ - massless dyon



The Seiberg-Witten solution

- A better visualization of the branch cut in A_D:



The Strong Coupling region of SW

- Once $u \sim \Lambda^2$ the IR EFT in terms of A is strongly coupled. Better to use dual variables in the vicinity of singularity $u = 2\Lambda^2$

- The S-dual coupling $\tau_D = -1/\tau$

$$\tau_D = \partial^2 \mathcal{F}_D(A_D)/\partial A_D^2 \qquad\qquad \tau_D \equiv \frac{\theta_D}{2\pi} + \frac{4\pi i}{e_D^2}$$

- e_D runs logarithmically to 0 near $u=2\Lambda^2$ where the (g,q)=(1,0) monopole becomes massless

$$W = A_D M \widetilde{M}$$

- A_D=0 at $u=2\Lambda^2$ leading to vanishing monopole mass, the BPS formula

$$m^{BPS}_{(g,q)} = |A^v(u)\, q + A^v_D(u)\, g|\,,$$

The Strong Coupling region of SW

- The effective theory here better described as

$$\mathcal{L}_{IR}^{D} = \frac{1}{8\pi i}\int d^4\theta \frac{\partial \mathcal{F}_D}{\partial A_D}\overline{A}_D +$$

- The actual expressions of the prepotential:

$$\mathcal{F}(A) = \frac{1}{2\pi i}\left\{-4A^2\left(\ln\frac{2A}{\Lambda} - \frac{3}{2}\right) + A^2\sum_{k=1}^{\infty} d_k \left(\frac{\Lambda}{A}\right)^{4k}\right\}$$

$$\mathcal{F}_D(A_D) = \frac{1}{4\pi i}\left\{A_D^2\left(\ln\frac{-iA_D}{32\Lambda} - \frac{3}{2}\right) - 16iA_D + A_D^2\sum_{k=1}^{\infty} d_k^D \left(\frac{A_D}{\Lambda}\right)^{k}\right\}$$

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Perturbative part

$$\mathcal{F}(A) = \frac{1}{2\pi i}\left\{-4A^2\left(\ln\frac{2A}{\Lambda} - \frac{3}{2}\right) + A^2\sum_{k=1}^{\infty} d_k \left(\frac{\Lambda}{A}\right)^{4k}\right\}$$

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$$\mathcal{L}^D_{IR} = \frac{1}{8\pi i}\int d^4\theta \frac{\partial \mathcal{F}_D}{\partial A_D}\overline{A}_D +$$

Instanton corrections
= BPS states in loops

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The Non-perturbative contributions

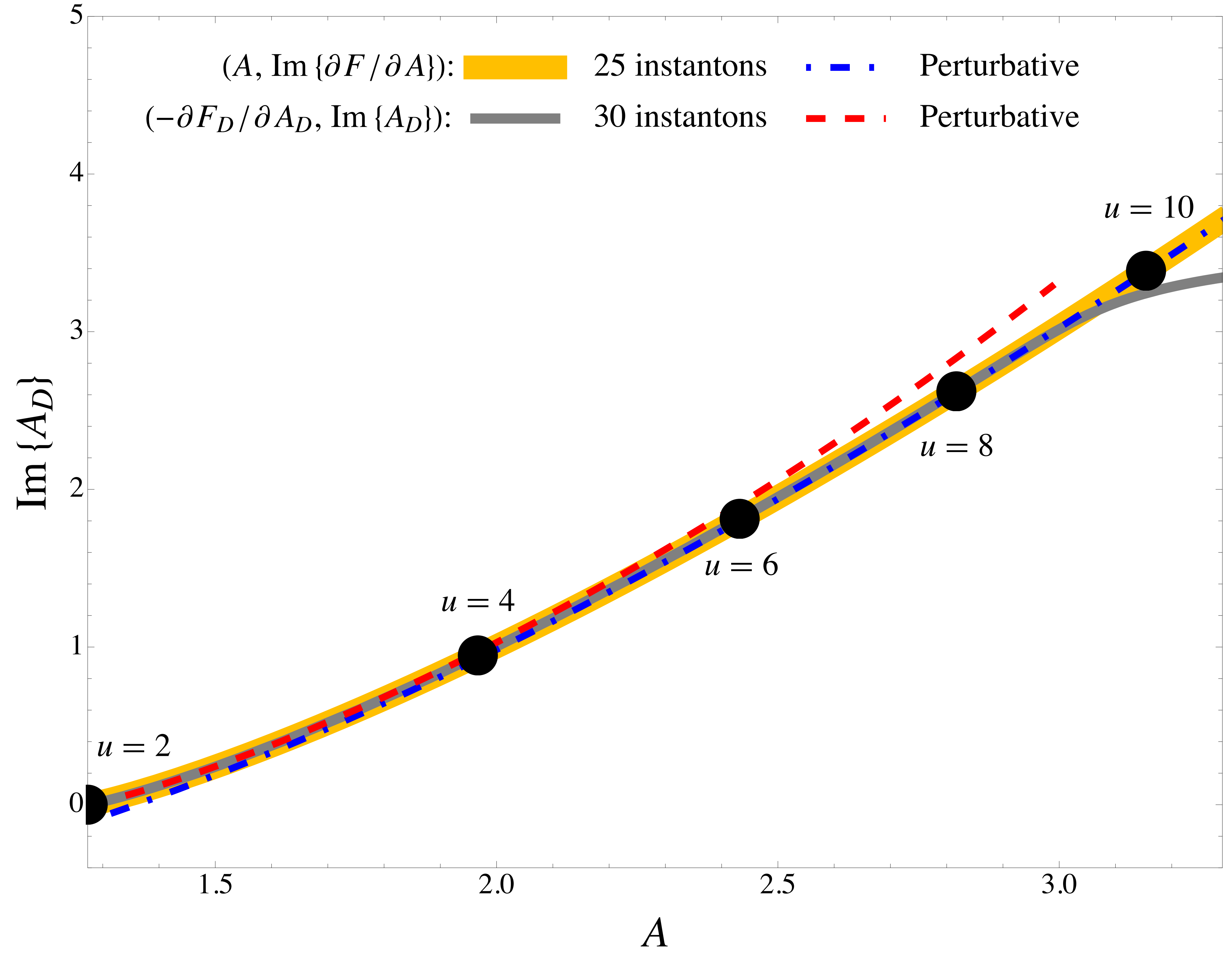
- d_k and d_k^D are the k-instanton coefficients, that are calculable.

$$d_1 = \frac{1}{2},\ d_2 = \frac{5}{64},\ d_3 = \frac{3}{64},\ d_4 = \frac{1469}{32768}$$

$$d_1^D = \frac{i}{16},\ d_2^D = -\frac{5}{1024},\ d_3^D = -\frac{11i}{16384},\ d_4^D = \frac{63}{524288}.$$

- Using up to 30 instantons

- Very good agreement between two expansions. Dual instanton breaks down for large u



The SW Axion - Electric frame

- In the effective U(1) theory only field N=2 vector SF, whose scalar component is A: it's the <A> that breaks the PQ symmetry, so the axion will be

$$A(x) = A^v(u)\, e^{i\frac{a(x)}{f(u)}}$$

- Just the phase of the A field - as any GB. Note $f = \sqrt{2}|A^v|/e$

- The axion dependence of τ will fix the photon-axion coupling

- We find:

$$-\frac{e^2}{16\pi^2 f} F_{\mu\nu}\widetilde{F}^{\mu\nu} \quad \left\{ N_a a - \sum_{k=1}^{\infty} \left[b_k \sin\left(\frac{4ka}{f}\right) + c_k \cos\left(\frac{4ka}{f}\right) \right] \right\}$$

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Usual coupling from anomaly

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Non-perturbative contributions due to monopole etc

The SW Axion - Electric frame

- The coefficients are related to the instanton factors

$$b_k - ic_k = (4k-1)(4k-2)\left(\frac{\Lambda}{A^v}\right)^{4k} d_k$$

- Explicitly known

- Note we also have imaginary parts - will get both $F_{\mu\nu}\widetilde{F}^{\mu\nu}$ and $F_{\mu\nu}F^{\mu\nu}$ couplings! The full form of term in the action:

$$\left(-\frac{1}{2e_p^2} + \frac{\operatorname{Im} G(\alpha)}{16\pi^2}\right) F_{\mu\nu}F^{\mu\nu} + \frac{8\alpha + \operatorname{Re} G(\alpha)}{16\pi^2} F_{\mu\nu}\widetilde{F}^{\mu\nu}$$

$$G(\alpha) \equiv \sum_{k=1}^{\infty} (\tilde{b}_k - i\tilde{c}_k) \left|\frac{\Lambda}{A^v}\right|^{4k} [\sin(4k\alpha) + i\cos(4k\alpha)] \qquad \alpha \equiv \frac{a}{f} - \frac{\theta_p}{8} \qquad \tilde{b}_k - i\tilde{c}_k = (4k-1)(4k-2)d_k$$

The SW Axion - Electric frame

- Expanding to linear order in the axion get coupling:

$$-\frac{a}{16\pi}\mathrm{Im}\left\{\left[i\frac{e^3}{\sqrt{2}}\frac{\partial\tau}{\partial A}\right]_{a=0}(F^{\mu\nu}+i\widetilde{F}^{\mu\nu})^2\right\}$$

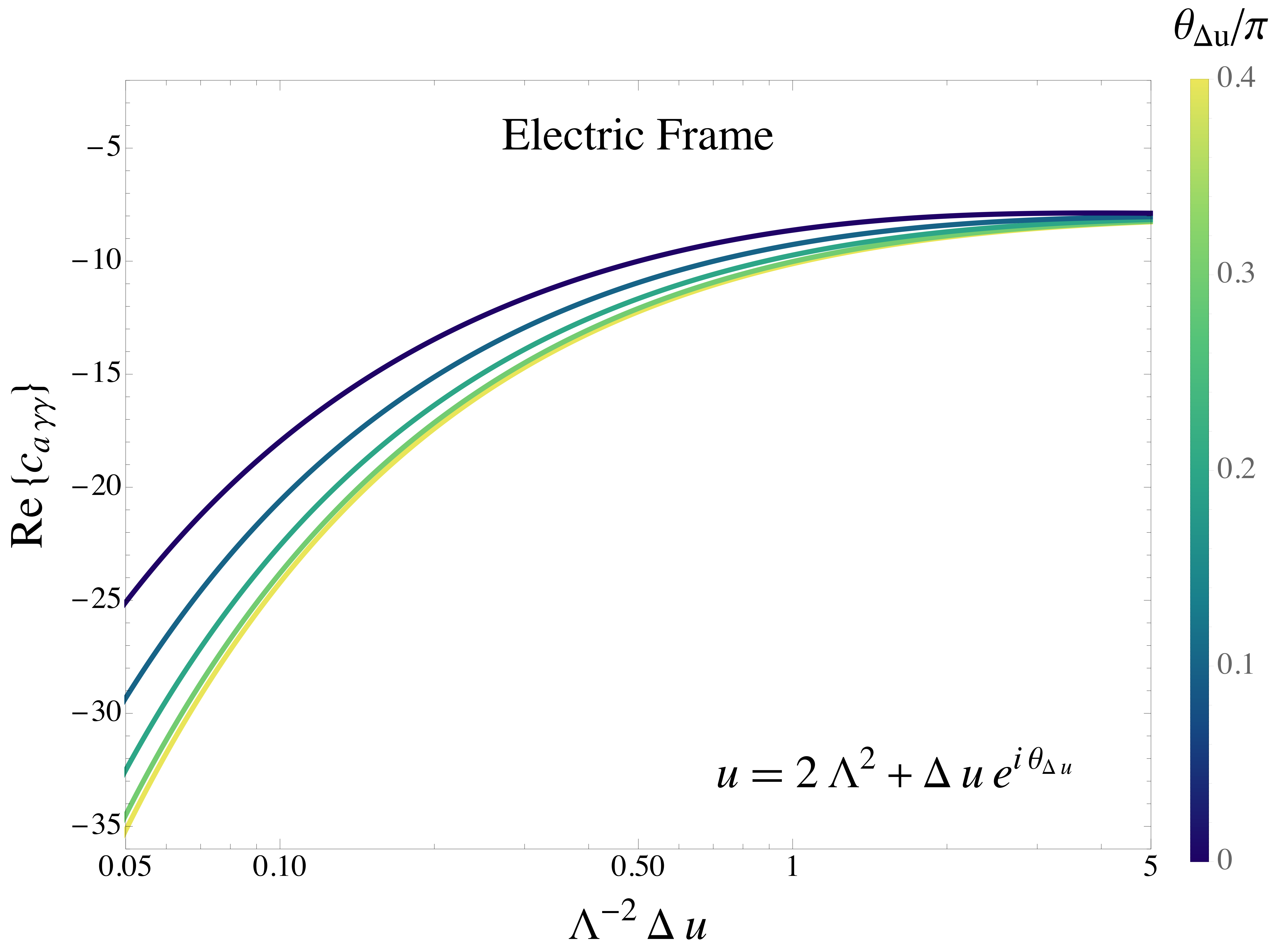
- Defining the coupling coefficient: $$\frac{g_{a\gamma\gamma}}{4}=\frac{e^2}{16\pi^2 f}c_{a\gamma\gamma}=\frac{i}{\sqrt{2}}\frac{e^3}{8\pi}\frac{\partial\tau}{\partial A}$$

$$c_{a\gamma\gamma}=-8-\sum_{k=1}^{\infty}4k(\tilde{b}_k-i\tilde{c}_k)\left(\frac{\Lambda}{A^v}\right)^{4k}$$

- -8 perturbative anomaly from gauginos

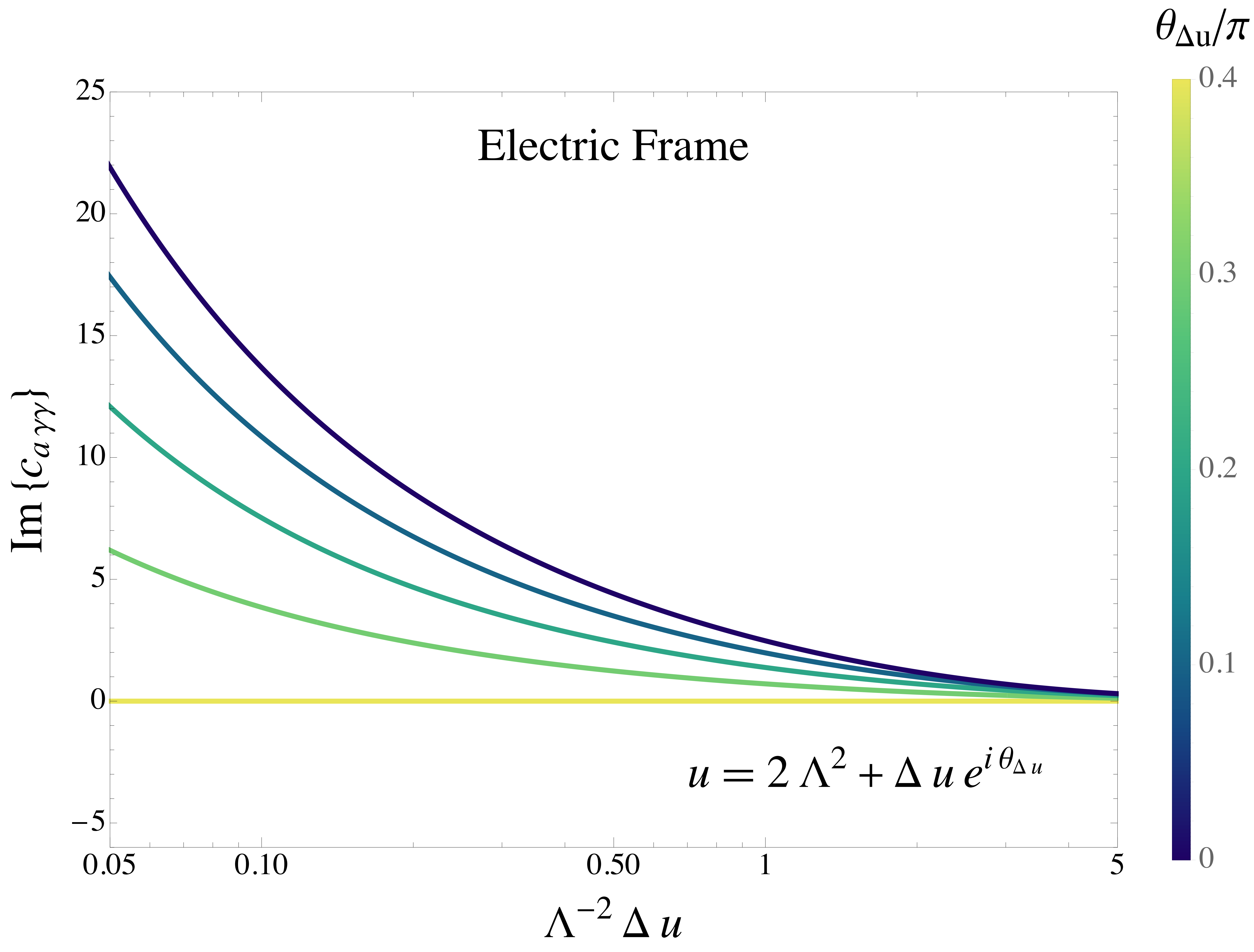
- Instantons: contributions of monopole/BPS states

The Axion Coupling - Electric frame



- For large u recover perturbative answer from gauginos
- For $u=2\Lambda^2$ ($\Delta u=0$) diverges - due to massless monopole
- Divergence due to contribution of light monopole to anomaly as expected initially

The Axion Coupling - Electric frame



- Coupling to F^2: only if theta angle non-zero (can rotate away)
- For large u always vanishes
- For $u=2\Lambda^2$ ($\Delta u=0$) diverges - due to massless monopole

The SW Axion - Magnetic frame

- We can go to the magnetic duality frame, and define axion there!

$$A_D(x) = iA_D^v\, e^{i\frac{a_D(x)}{f_D}}$$

- This axion non-linearly related to original axion (and also the massless radial modes) - NOT the same field.

- Can calculate its couplings similarly to that of the electric axion:

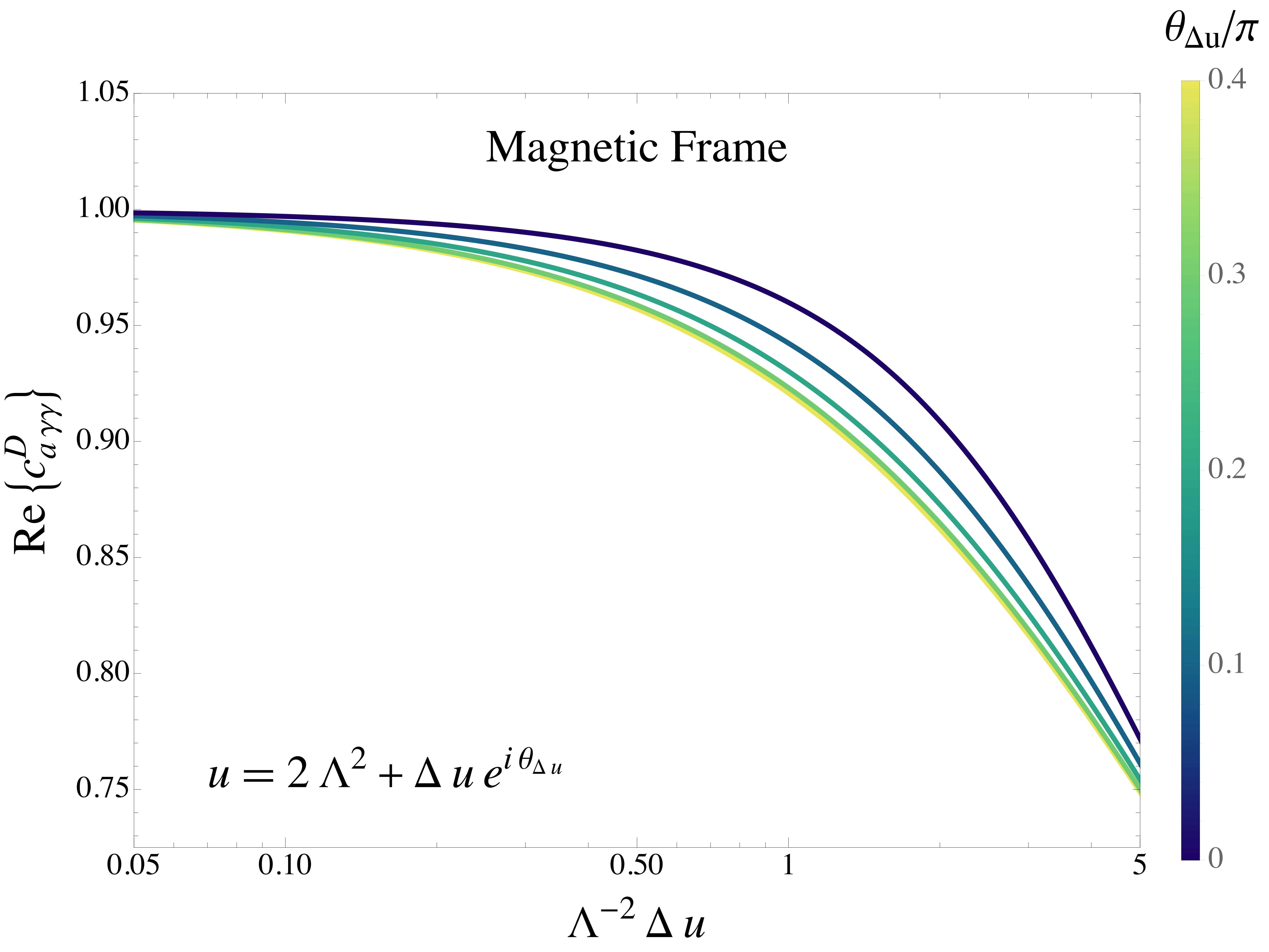
$$-\frac{a_D}{16\pi}\mathrm{Im}\left\{\left[-\frac{e_D^3}{\sqrt{2}}\frac{\partial\tau_D}{\partial A_D}\right]_{a_D=0}(F_D^{\mu\nu}+i\widetilde{F}_D^{\mu\nu})^2\right\}$$

$$\frac{g_{a\gamma\gamma}^D}{4} = \frac{e_D^2}{16\pi^2 f_D}c_{a\gamma\gamma}^D = -\frac{1}{\sqrt{2}}\frac{e_D^3}{8\pi}\frac{\partial\tau_D}{\partial A_D}$$

$$c_{a\gamma\gamma}^D = 1+\sum_{k=1}^{\infty}\frac{k}{2}\left(\tilde{b}_k^D - i\tilde{c}_k^D\right)\left(\frac{A_D^v}{\Lambda}\right)^k$$

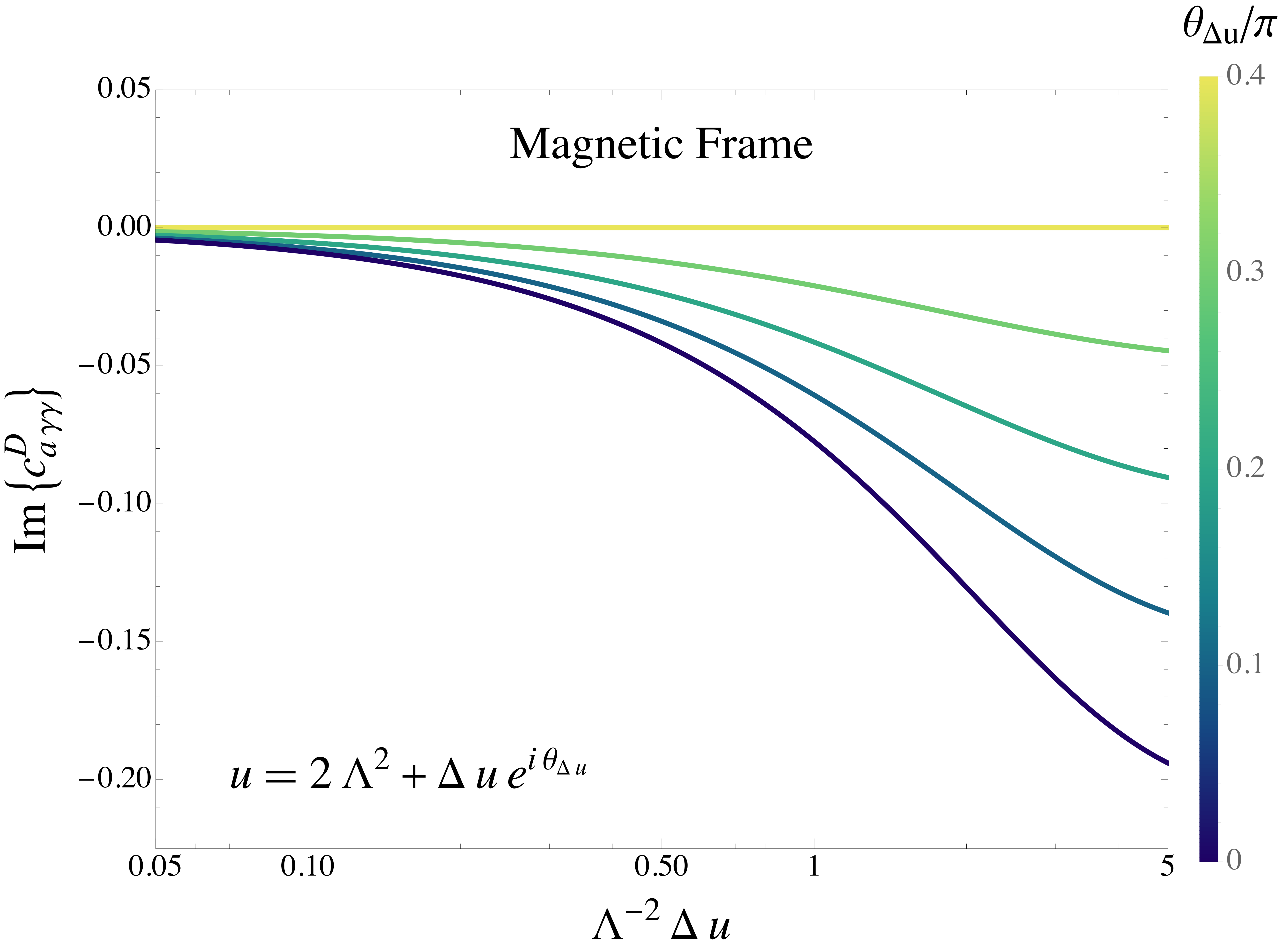
- 1: perturbative contribution of magnetic monopole
- Instantons here represent contributions of gauginos and other BPS states in this frame

The Axion Coupling - Magnetic frame



- For $u=2\Lambda^2$ ($\Delta u=0$) recover perturbative result due to monopole
- For large u diverges
- Divergence due to contribution of light gaugino to anomaly which in this frame becomes strongly coupled

The Axion Coupling - Magnetic frame



- Coupling to F^2: only if theta angle non-zero (can rotate away)
- For $u=2\Lambda^2$ ($\Delta u=0$) always vanishes
- For large u diverges - the effect of strongly coupled gauginos

Duality invariance of a→γγ rate?

- We have seen couplings in two frames seem to be very different. However duality merely gives different descriptions to SAME physics. So physical quantities better be duality invariant.

- a→γγ rate is physical observable. They have to agree in the two frames!

- Check this. If θ≠0 we have both $aF\widetilde{F}$ and aFF couplings.

$$\Gamma^{\rm el}_{a\to\gamma\gamma} \propto \sum_{h,h'} |\mathcal{M}_{hh'}|^2 \propto 2\left(|\mathcal{M}_{+-}|^2 + |\mathcal{M}_{++}|^2\right) \propto \frac{e^4}{f^2}\left|c_{a\gamma\gamma}\right|^2 \propto \left|g_{a\gamma\gamma}\right|^2$$

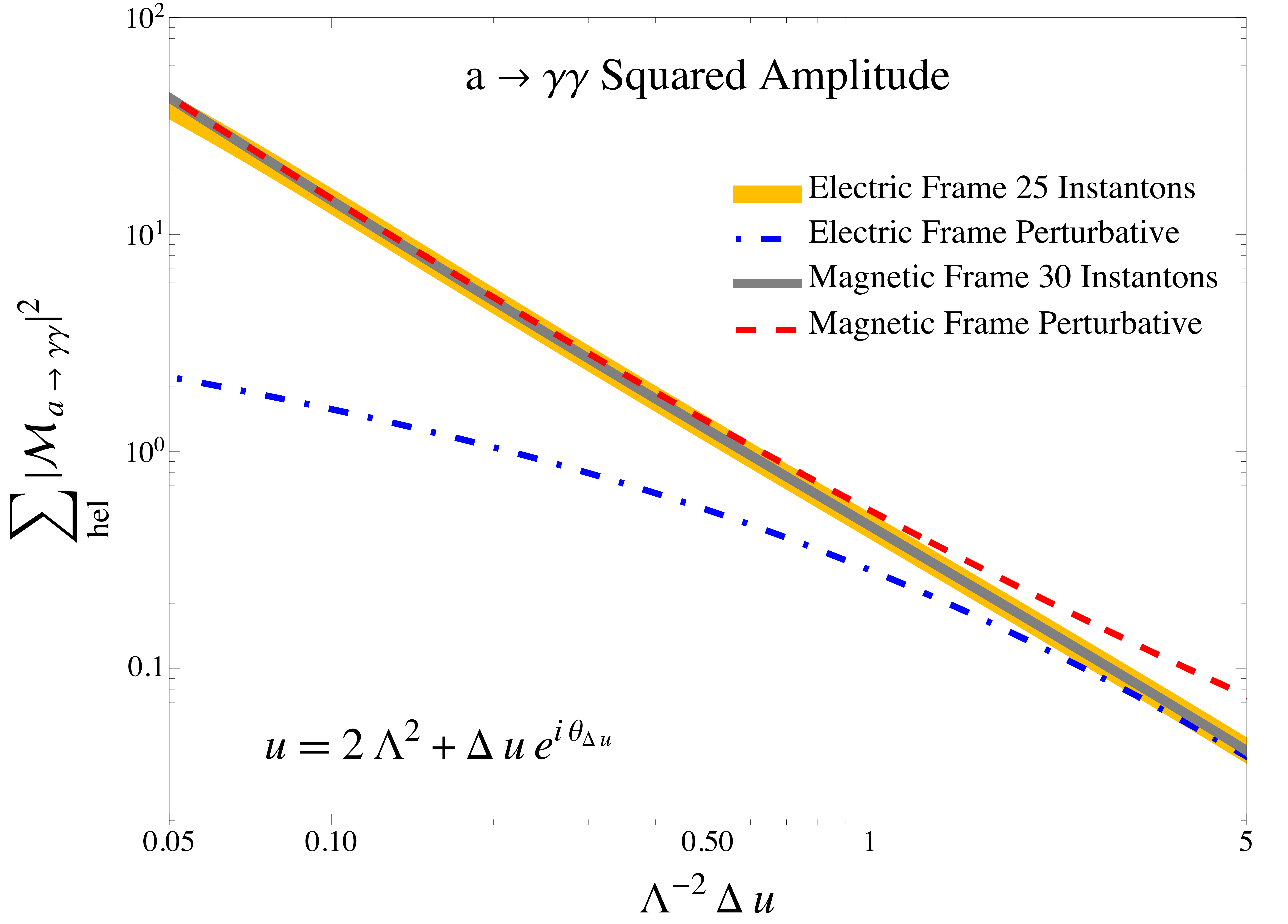
Duality invariance of a→γγ rate?

- Similarly in the magnetic frame $\Gamma^{\mathrm{mag}}_{a\to\gamma\gamma} \propto \left|g^{D}_{a\gamma\gamma}\right|^2$
- Claim: $\left|g_{a\gamma\gamma}\right|^2$ is duality invariant!
- Using $\tau = -\tau_D^{-1}$ and the SW relations for $\partial A/\partial A_D$ we find

$$e^3 \frac{\partial \tau}{\partial A} = \left(-\frac{\overline{\tau}_D}{|\tau_D|}\right)^3 e_D^3 \frac{\partial \tau_D}{\partial A_D}$$

- Up to a phase couplings equal, so $\Gamma^{\mathrm{el}}_{a\to\gamma\gamma} = \Gamma^{\mathrm{mag}}_{a\to\gamma\gamma}$
- Also evaluated numerically:

Duality invariance of a→γγ rate



Duality invariance of $a \to \gamma\gamma$ rate

- We see that physical observables are duality invariant (as they must be)

- KEY in this result: the axion transforms non-linearly under duality! a and a_D are different! They are uniquely defined by requiring they be Goldstone bosons in a given frame.

- If we defined a_D to be the axion in the ELECTRIC frame as well - then it is not GB of linearly realized $U(1)_R$ symmetry, but still a consistent definition of the theory.

- Mass of chargino $M = |A|$, but we are using a_D as variable rather than a. Mass will depend on a_D - as if it also had a magnetic charge - ``dual Witten effect”. But it just means these types of fluctuations in the electric frame are experimentally constrained, while a is not…

Origin of the periodic terms

- We saw form of axion coupling

$$-\frac{e^2}{16\pi^2 f} F_{\mu\nu}\widetilde{F}^{\mu\nu} \quad \left\{ N_a a - \sum_{k=1}^{\infty} \left[b_k \sin\left(\frac{4ka}{f}\right) + c_k \cos\left(\frac{4ka}{f}\right)\right]\right\}$$

- Where do the periodic terms come from? One interpretation contribution of monopoles/other BPS states

- Other interpretation: instanton effects

- Instantons break $U(1)_R$ explicitly. Can identify contribution to correlator

- For large u - weak coupling, adjoint VEV provides IR regulator - finite calculable contribution

A sample instanton calculation

- Contribution to $\langle\bar\psi\bar\psi\bar\lambda\bar\lambda\rangle$
- Interpreted as contribution to

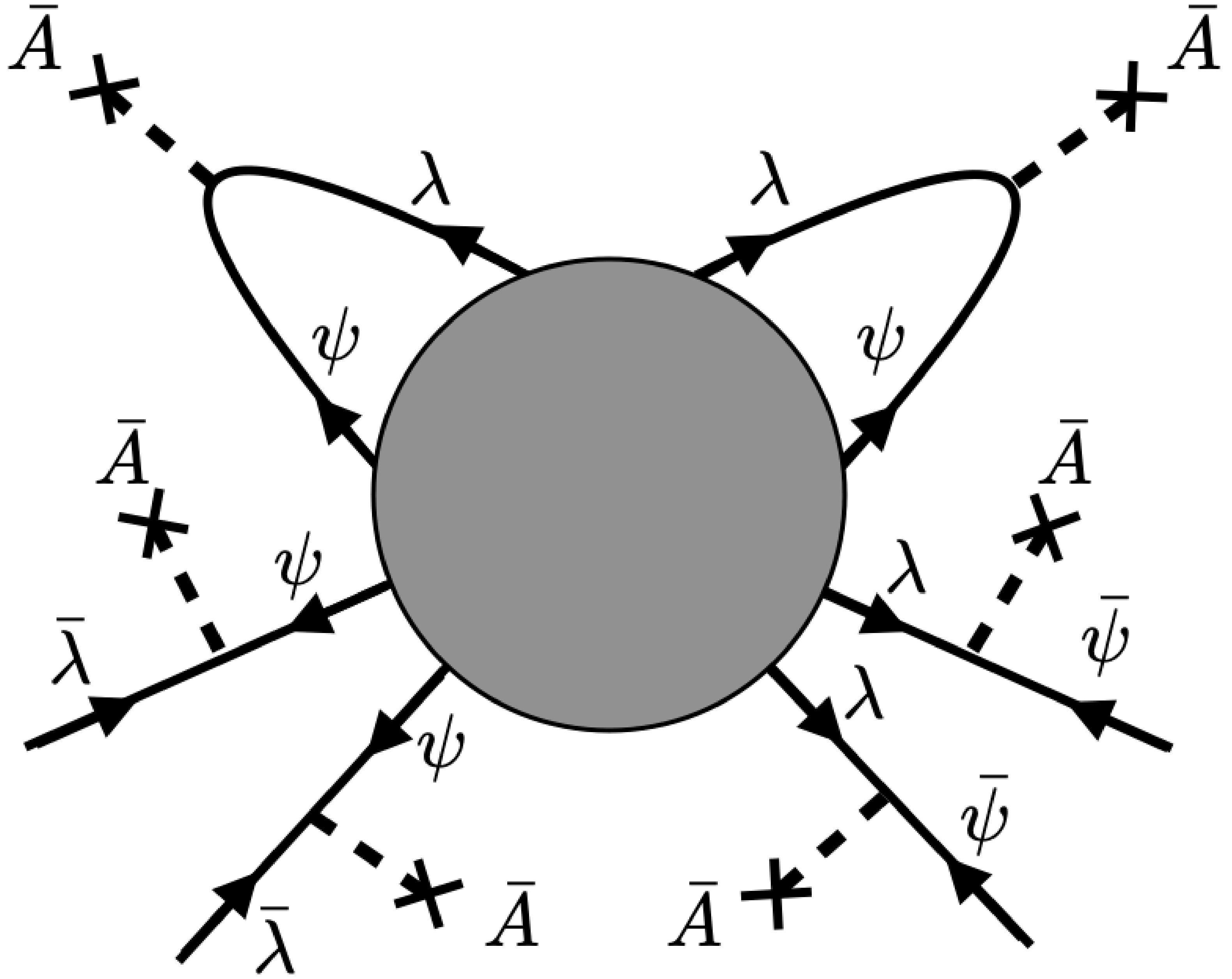
$$\mathrm{Im}\{(\partial^4\mathcal{F}/\partial A^4)\lambda^2\psi^2\}$$

- Instanton NDA gives the estimate

$$\langle\bar\psi(x_1)\bar\psi(x_2)\bar\lambda(x_3)\bar\lambda(x_4)\rangle$$

$$\propto \int d^4x_0 \int \frac{d\rho}{\rho^5}(\Lambda\rho)^4 \bar{A}^6 \rho^{12}\, e^{-4\pi^2 A\bar{A}\rho^2} \prod_{i=1}^{4} S_F(x_i - x_0)$$

$$\propto \frac{\Lambda^4}{A^6}\int d^4x_0 \prod_{i=1}^{4} S_F(x_i - x_0)\,,$$



- Holomorphic as SUSY required

- Match the coefficient $A^2(\Lambda/A)^4$

$d_1=1/2$

Lessons learned

- Axion couplings not always quantized, additional terms could sometimes be large

- Additional terms due to monopoles/BPS states - can also be interpreted as instantons. Will give periodic terms in the axion

- Seiberg-Witten gives concrete calculable toy model with light monopoles

- Key point: axion itself transforms under electric-magnetic duality, explains previous issues with axion vs. duality

- What is the right duality transformation of Maxwell-axion Sikivie equations? Guess is it should be secretly duality invariant if transformation of axion properly taken into account…