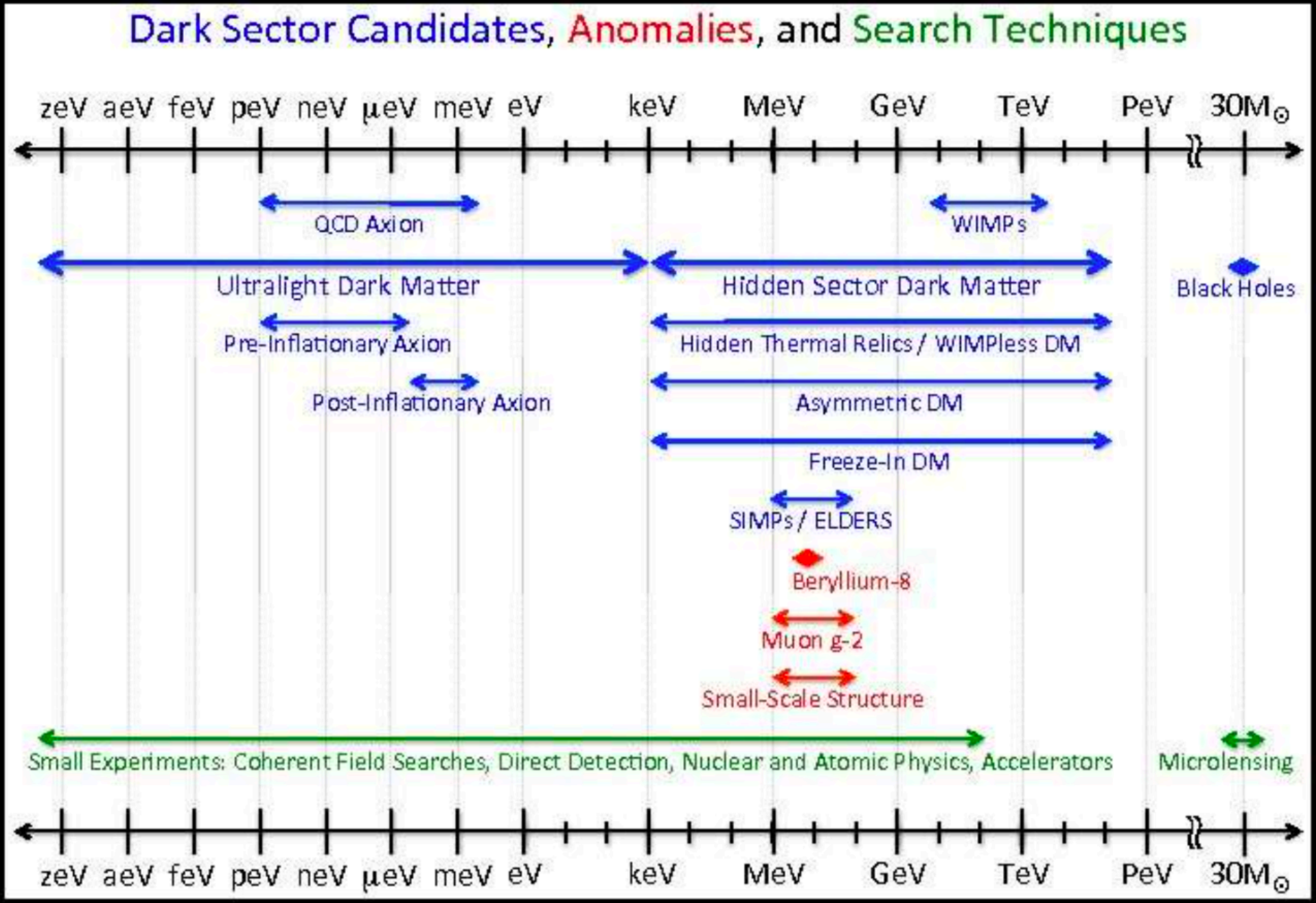
The International Joint Workshop on the Standard Model and Beyond 2024
& 3rd Gordon Godfrey Workshop on Astroparticle Physics
December 12, 2024

Ultralight Axion DM from Inflation-driven Quantum Phase Transition

KIAS
Seung J. Lee
in collaboration with
Ameen Ismail and Bingrong Yu: 2402.08716
+ work in progress (Ameen Ismail, Sarunas Verner, and Bingrong Yu)

~80 orders of magnitude



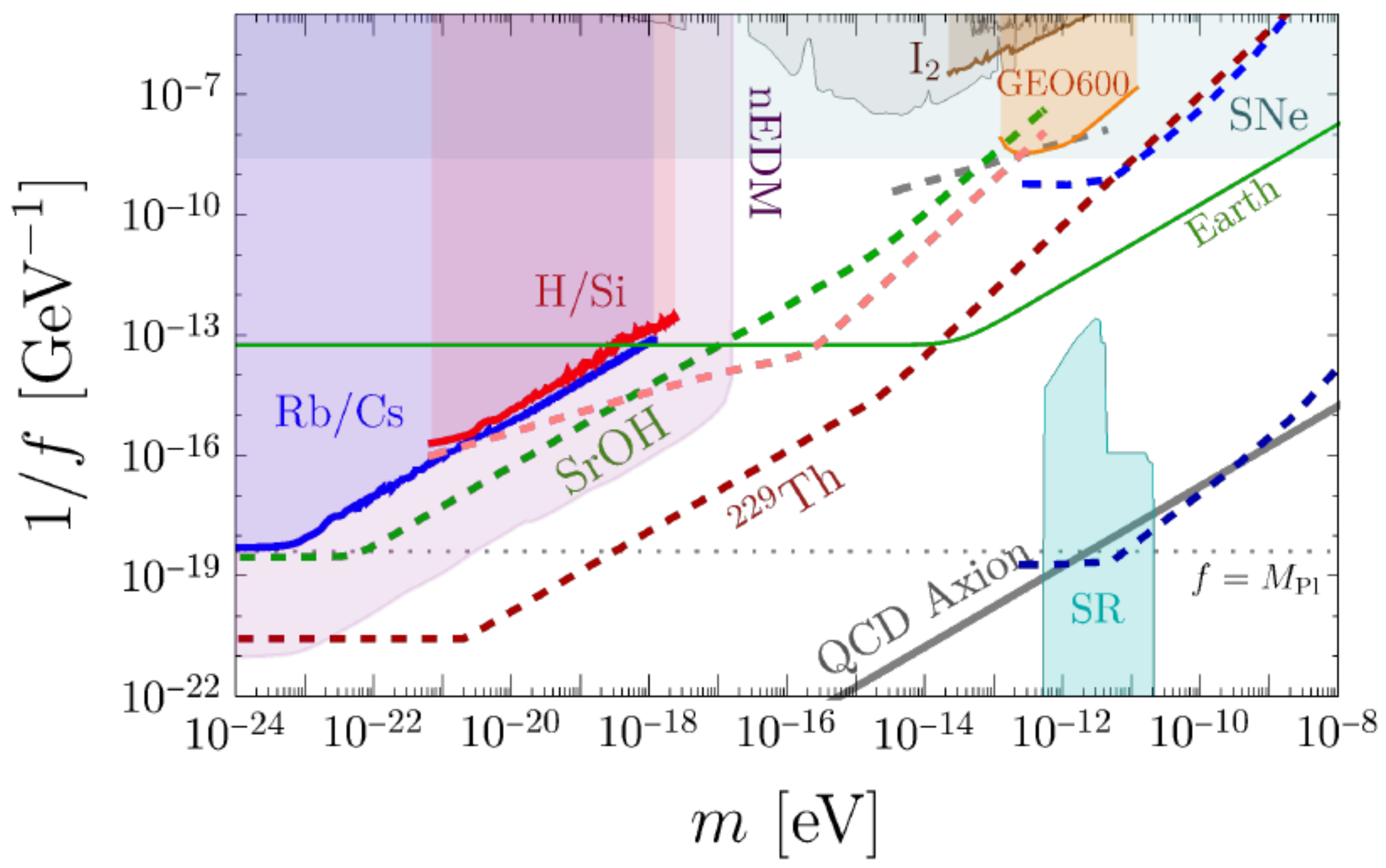
Beyond WIMP, so many new ways to probe possible DM, But mostly for (ultra)light DM

- Cosmological/astrophysical Probes (indirect, CMB, star cooling, LSST,PTA,gravitational wave, lensing,…)
- Table Top experiments (nuclear or electron scatteribg/absorption) for direct detection
- Cavity experiments for axion like particles, Beam Dump Experiments, Quantum Sensing (atomic physics)
- At colliders (including facilities for LLP such as FASER II, SHiP,…)

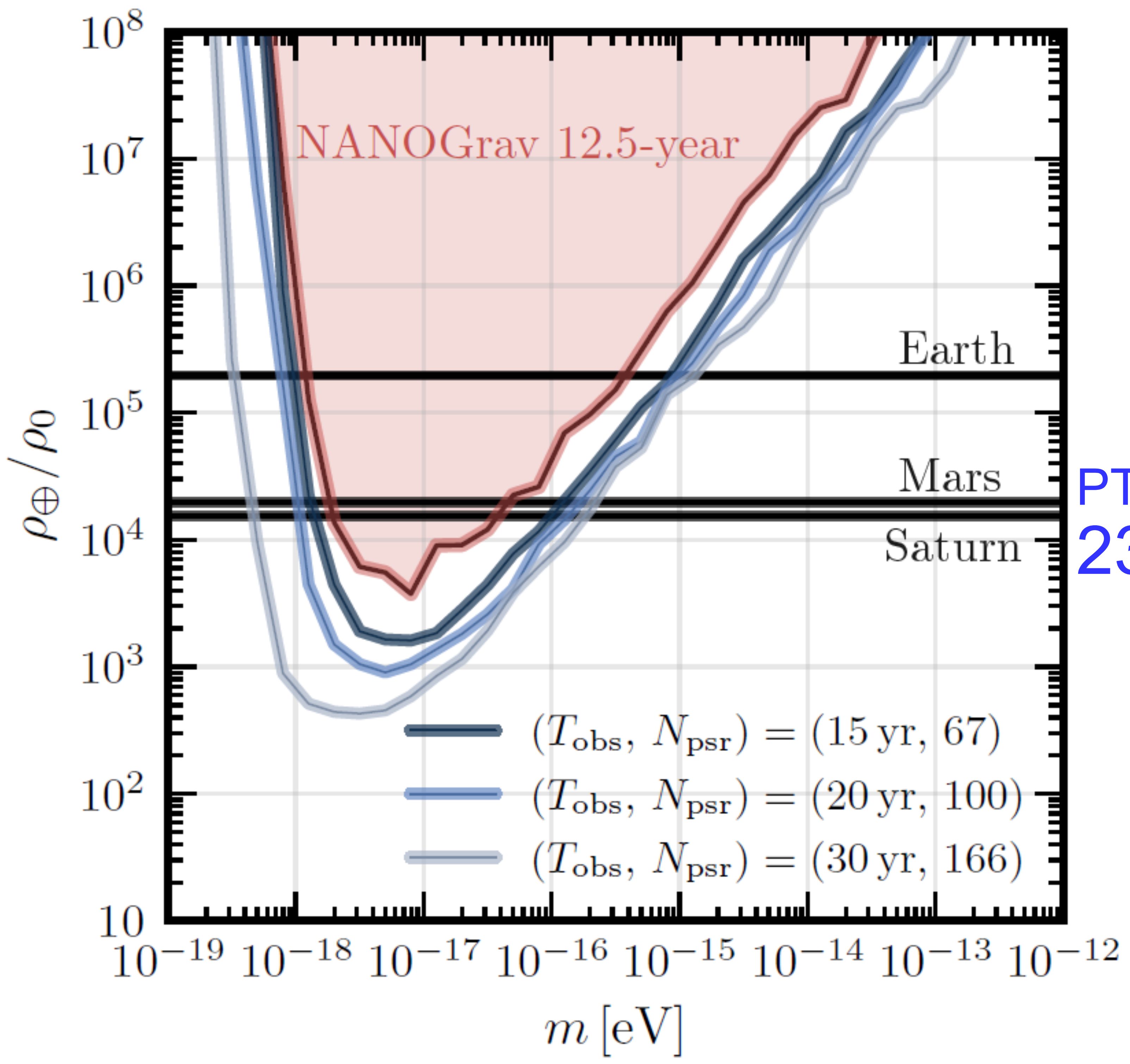
Ultralight DM (ULDM)

- Ultralight (wave) DM: $10^{-22}\,\mathrm{eV} < m < \mathrm{eV}$

Future atomic-/astro-physics experiments: $m < 10^{-10}\mathrm{eV}$



Quantum sensor, 2205.12988



PTA, 2312.12225

Axion from Misalignment Mechanism

- Axion-like particle (ALP): well-motivated ultralight DM
  (protected by shift symmetry)

$$\ddot{\eta} + 3H\dot{\eta} + m_\eta^2 \eta^2 = 0$$

$$V(\eta) = \Lambda_\eta^4 \left[1 - \cos\left(\frac{\eta}{f_\eta}\right)\right] \Rightarrow m_\eta = \Lambda_\eta^2 / f_\eta$$

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✦ Misalignment mechanism: axion starts to oscillate when $H \sim m_\eta$, and behaves as matter after then, $\rho_\eta \sim a^{-3}$

$$\left(\frac{\Omega_\eta h^2}{0.12}\right)_{\text{ALP, misalignment}} \sim \left(\frac{m_\eta}{10^{-10}\ \text{eV}}\right)^{1/2} \left(\frac{f_\eta}{10^{14}\ \text{GeV}}\right)^2$$

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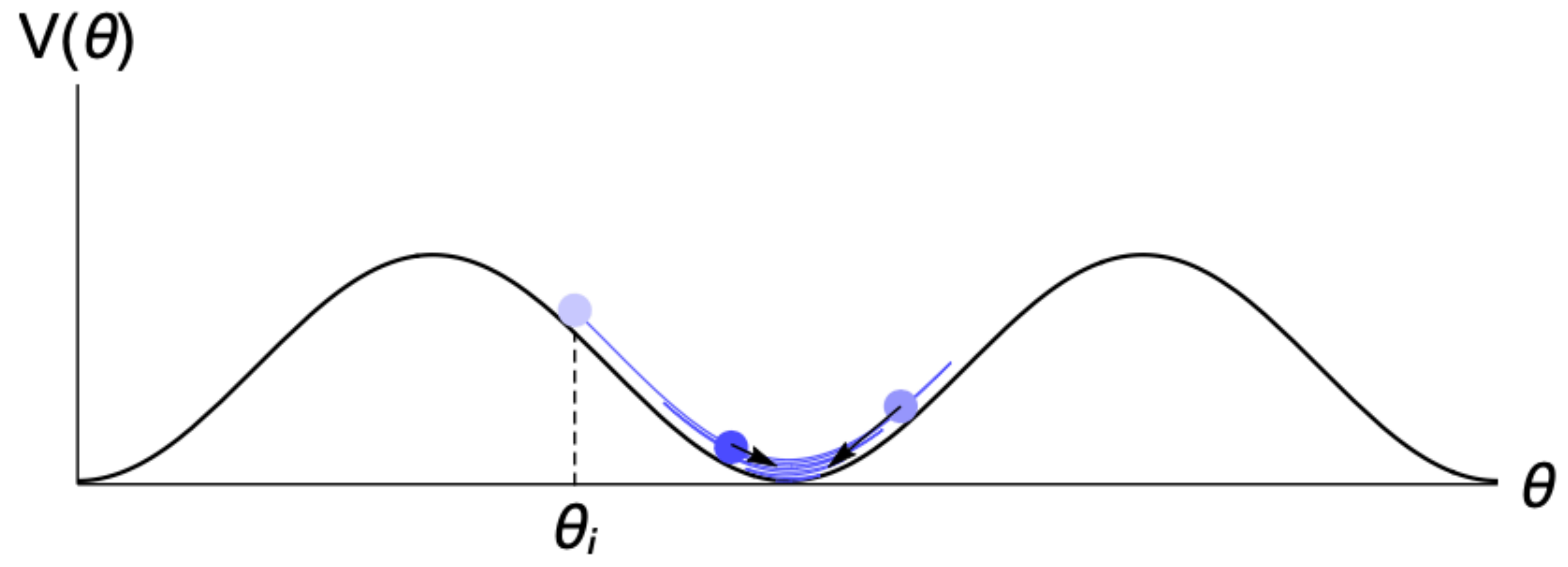

Initial value in inflationary patch displaced from minimum either because of quantum fluctuations when m<<H or because potential different than at T->0

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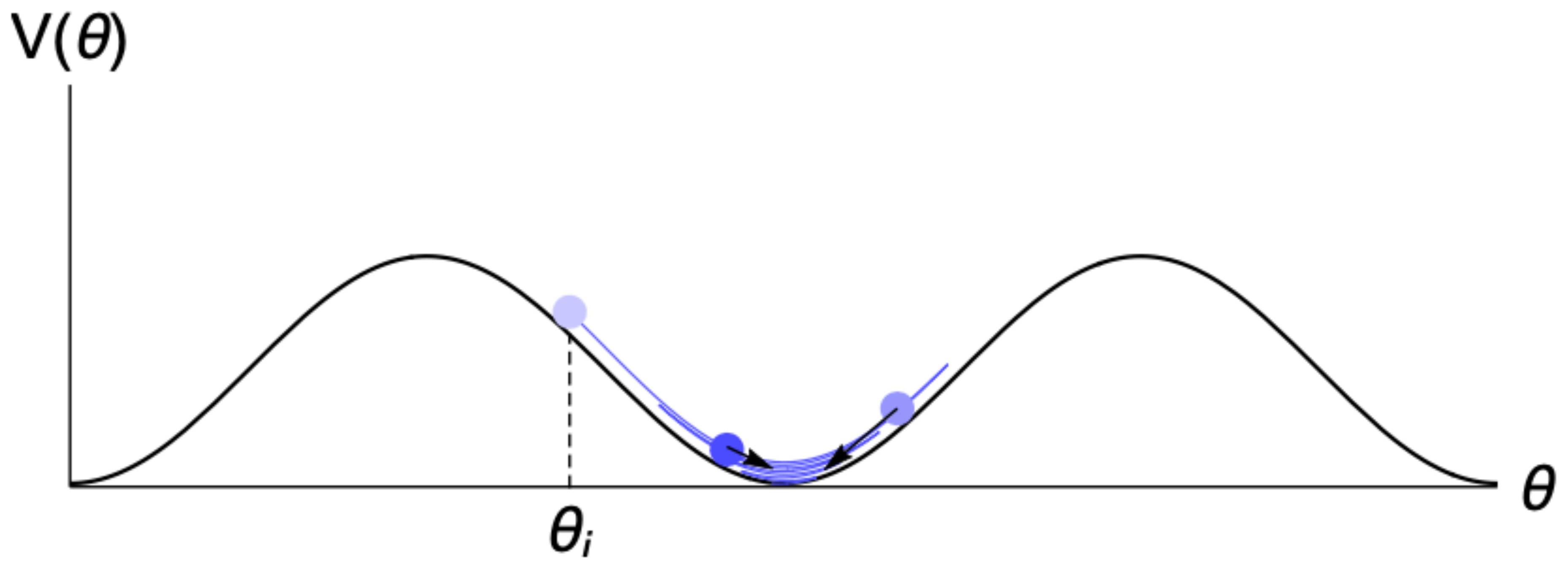

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phenomenological motivation for finding a new mechanism to reduce *f* for better experimental sensitivity

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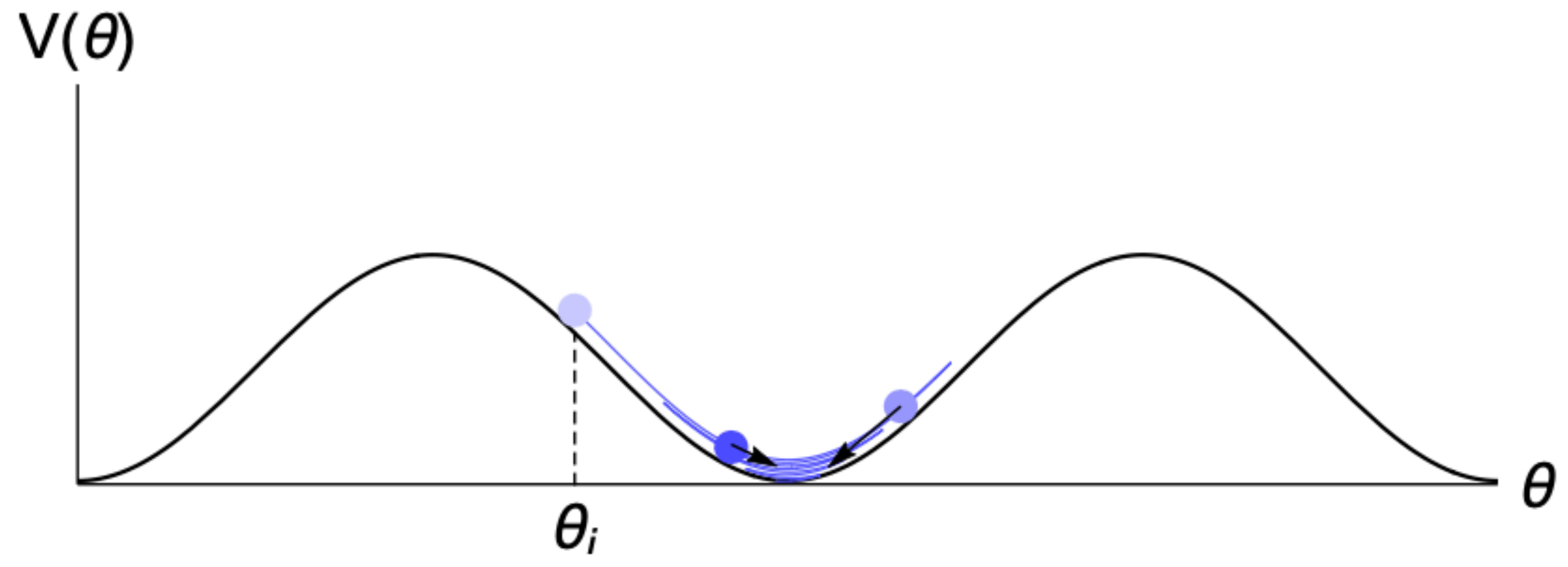

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V … Λ_η^2/f_η

ULDMs are very interesting, but their production is mostly assumed to be via misalignment mechanism.

Any alternative way to produce them?

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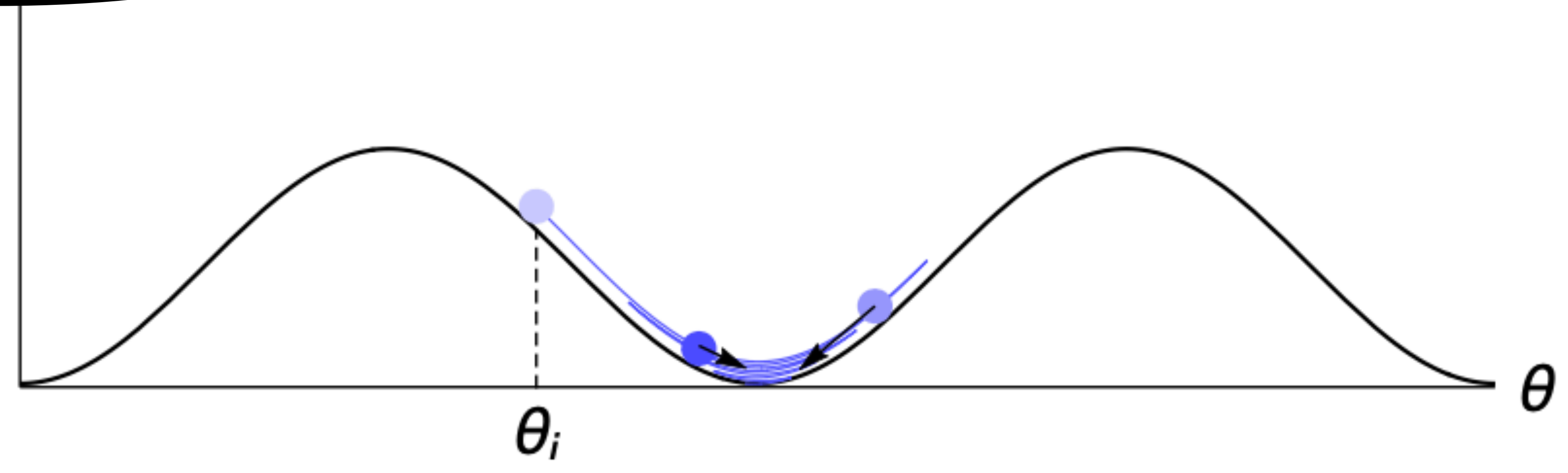

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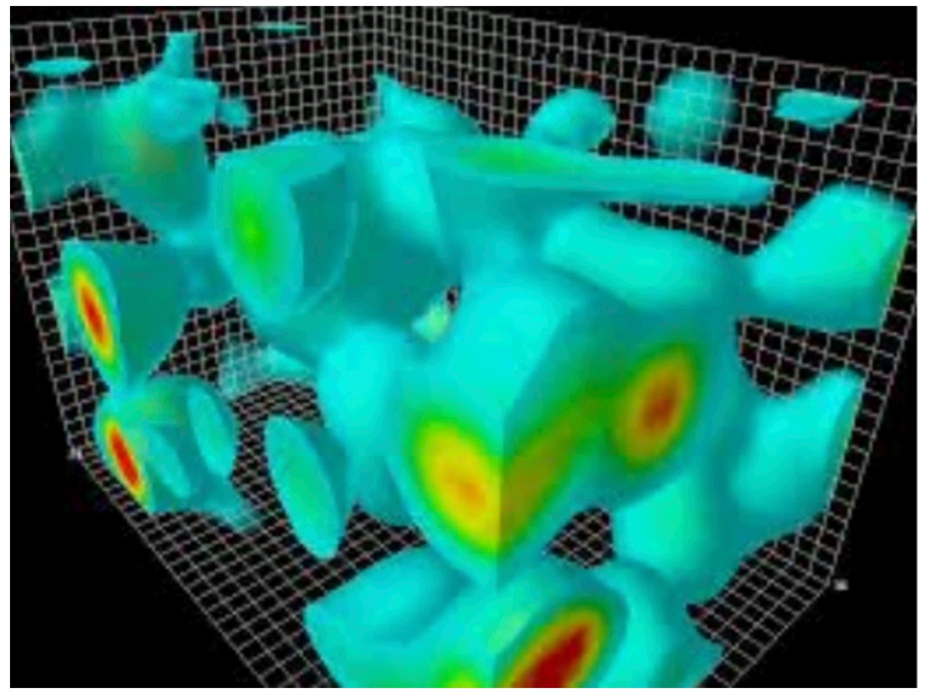
DM particle production from generic inflationary quantum fluctuations



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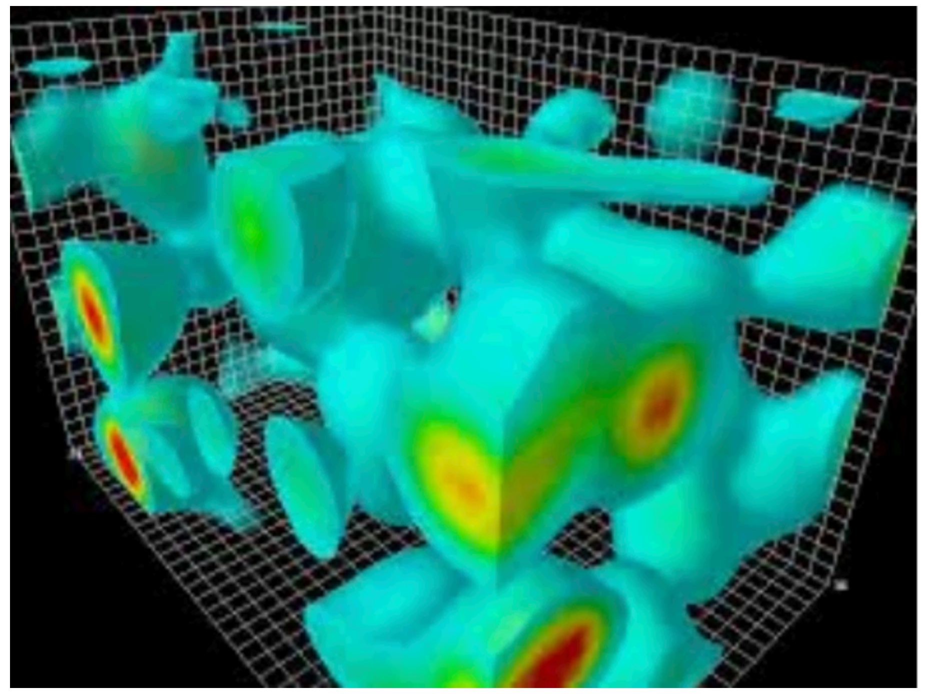

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$$H_{\rm inf}/2\pi$$ Gibbons-Hawking temperature

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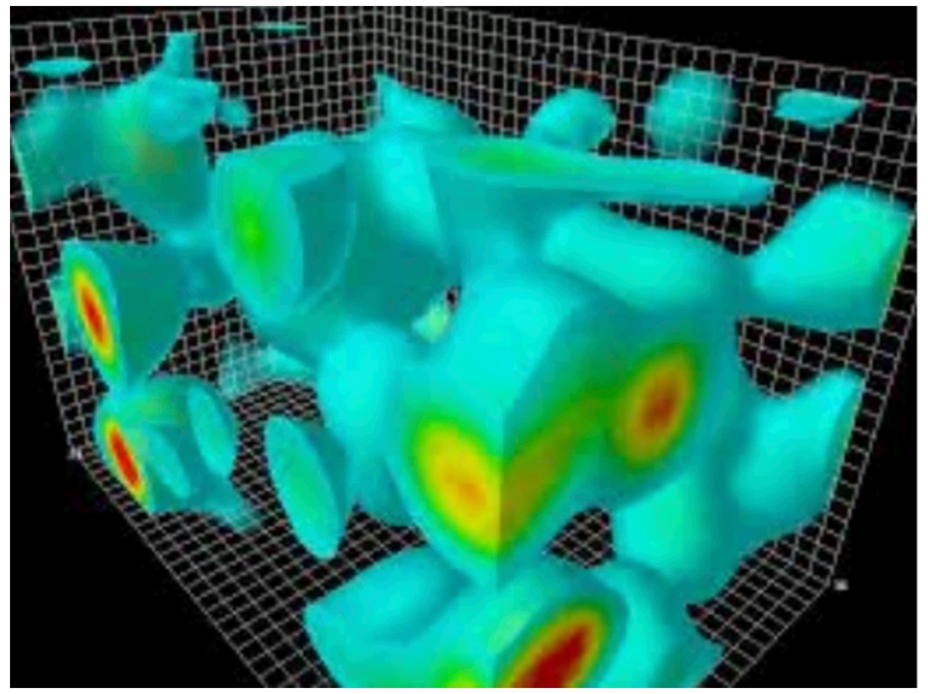

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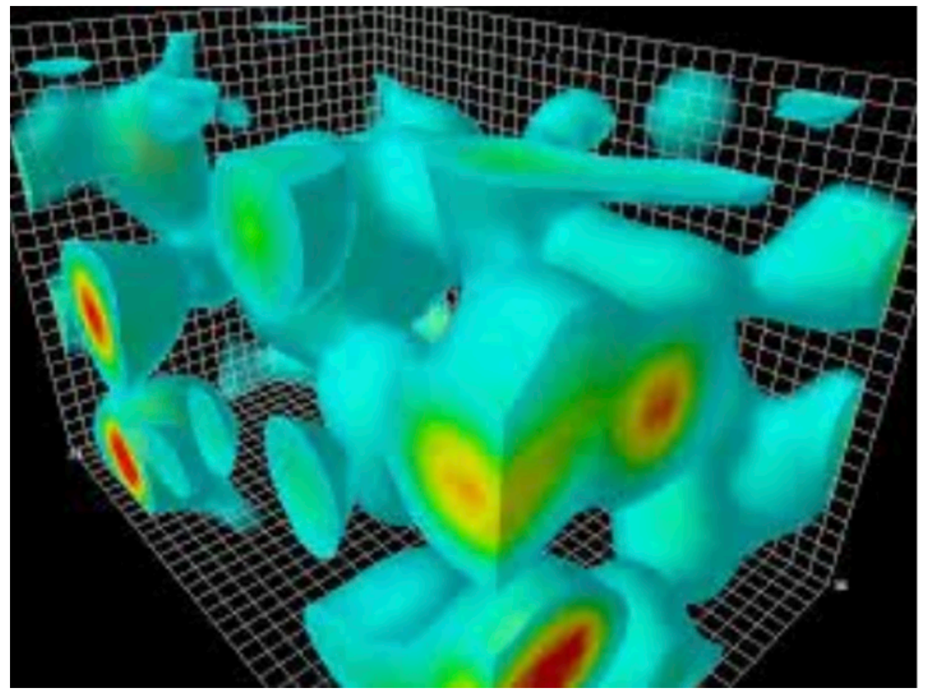

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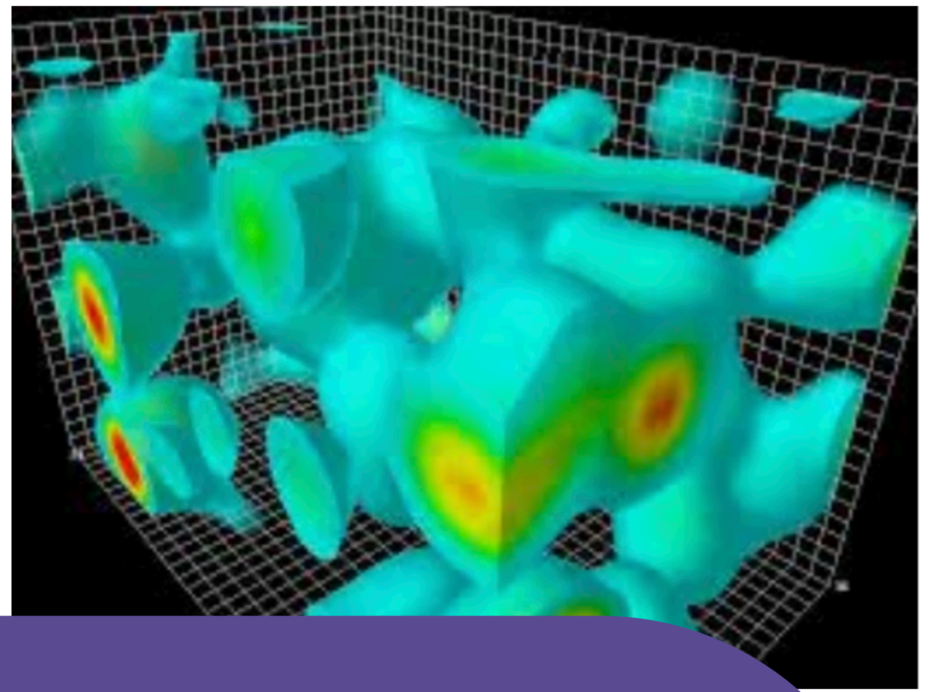

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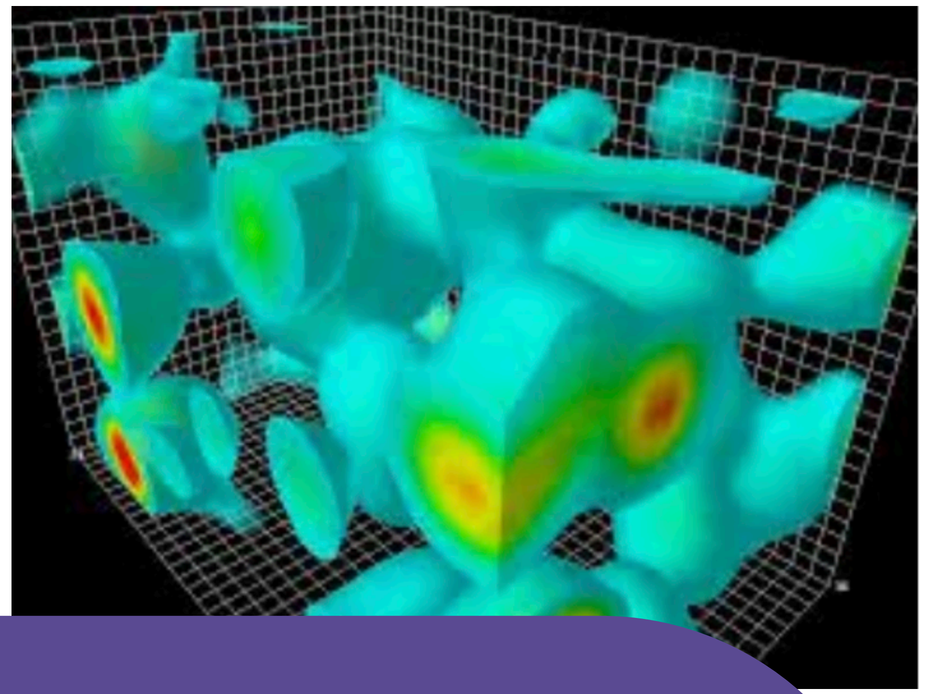

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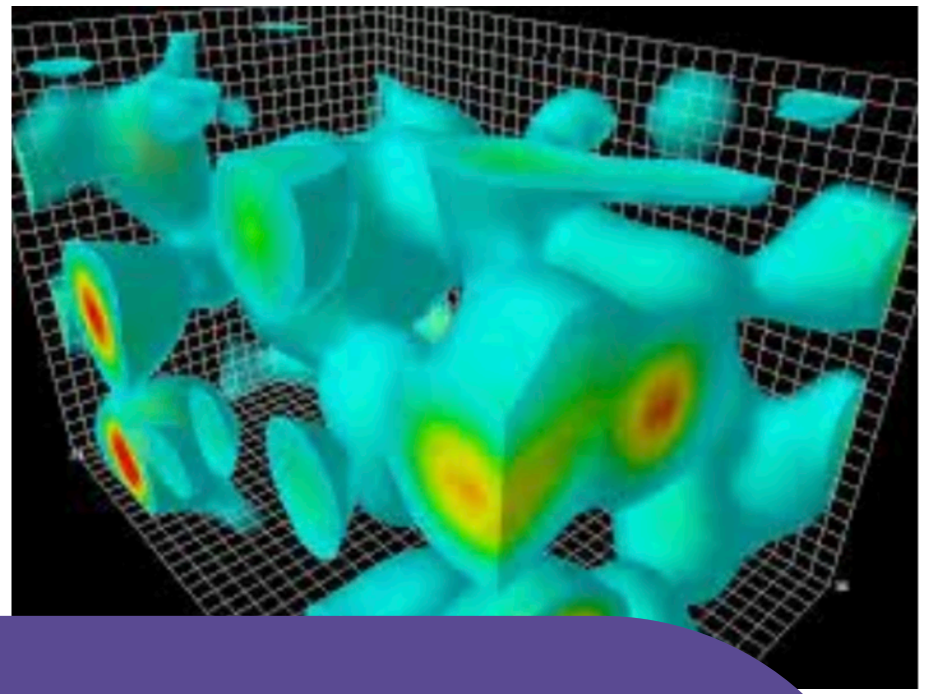

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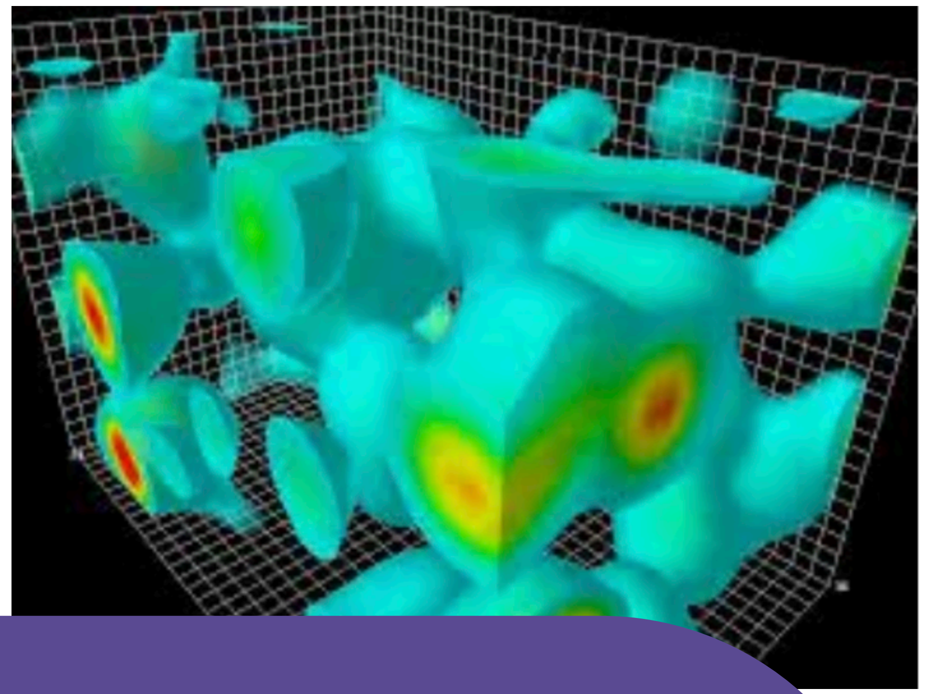

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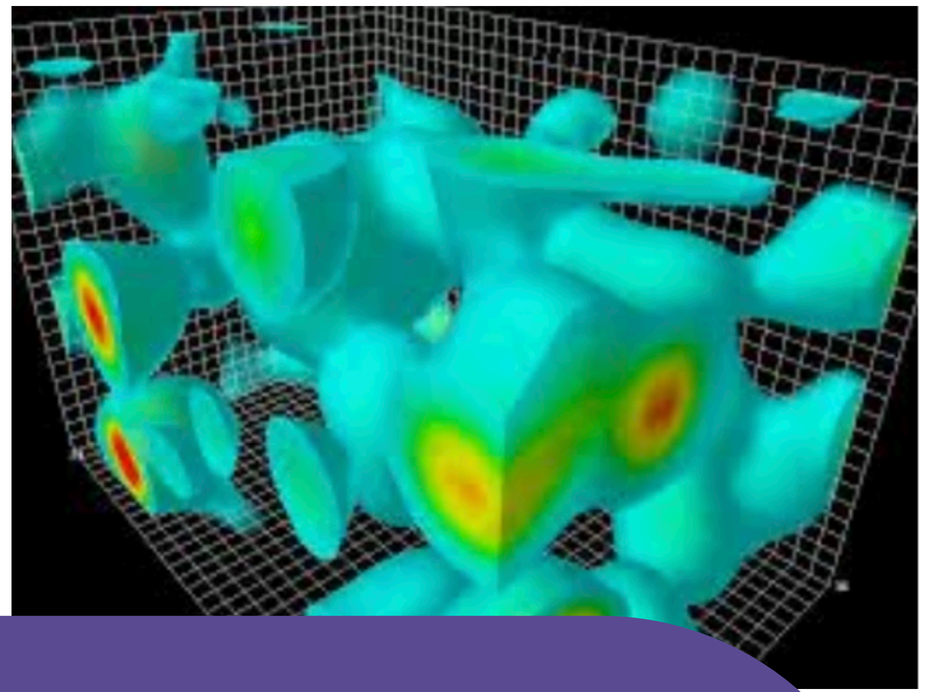

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Graham et al., 17’ for dark photon

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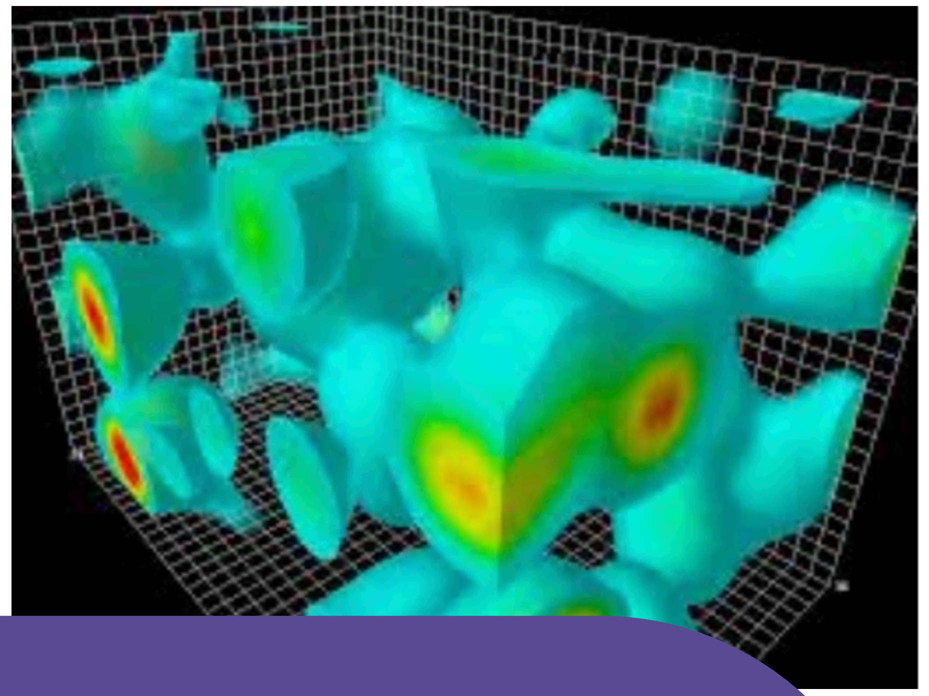

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$\rho_0 \sim (H$ ...

DM mass cannot go below 10^{-6} eV for generic particle production from inflationary quantum fluctuations.

Is there a way out?

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Ismail, SL, Yu. 24'

- Main idea (During inflation with PQ symmetry being broken)

Inflaton couple to axion Kinetic term

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Quantum phase transition is modulated by κ

$$P_k \sim \left(\frac{H_{\rm inf}}{2\pi}\right)^2 (-k\tau)^{3-2\nu} \sim \left(\frac{H_{\rm inf}}{2\pi}\right)^2 \left(\frac{1}{x}\right)^{2\kappa/3}$$

$$\nu \equiv \sqrt{9/4+\kappa} \qquad 1/x = aH_{\rm inf}/k,$$

$\kappa = 0$: critical point (scale invariant)

$\kappa > 0$: red tilt (exponential enhancement)

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→ Each mode grows after exiting horizon: $(aH_{\rm inf})^{-1} < k^{-1}$

comoving horizon shrinks during inflation:

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$\tau_{\rm i}/\tau_{\rm e} = e^N$

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At κ ≠ 0 : CFT is broken

κ > 0 : red spectrum (closed to k_{min}) dorminates
=> DM become non-relativistic either: at the end of inflation, or soon after the inflation

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Ising Model

$$H = -J \sum s(x)s(x+n)$$

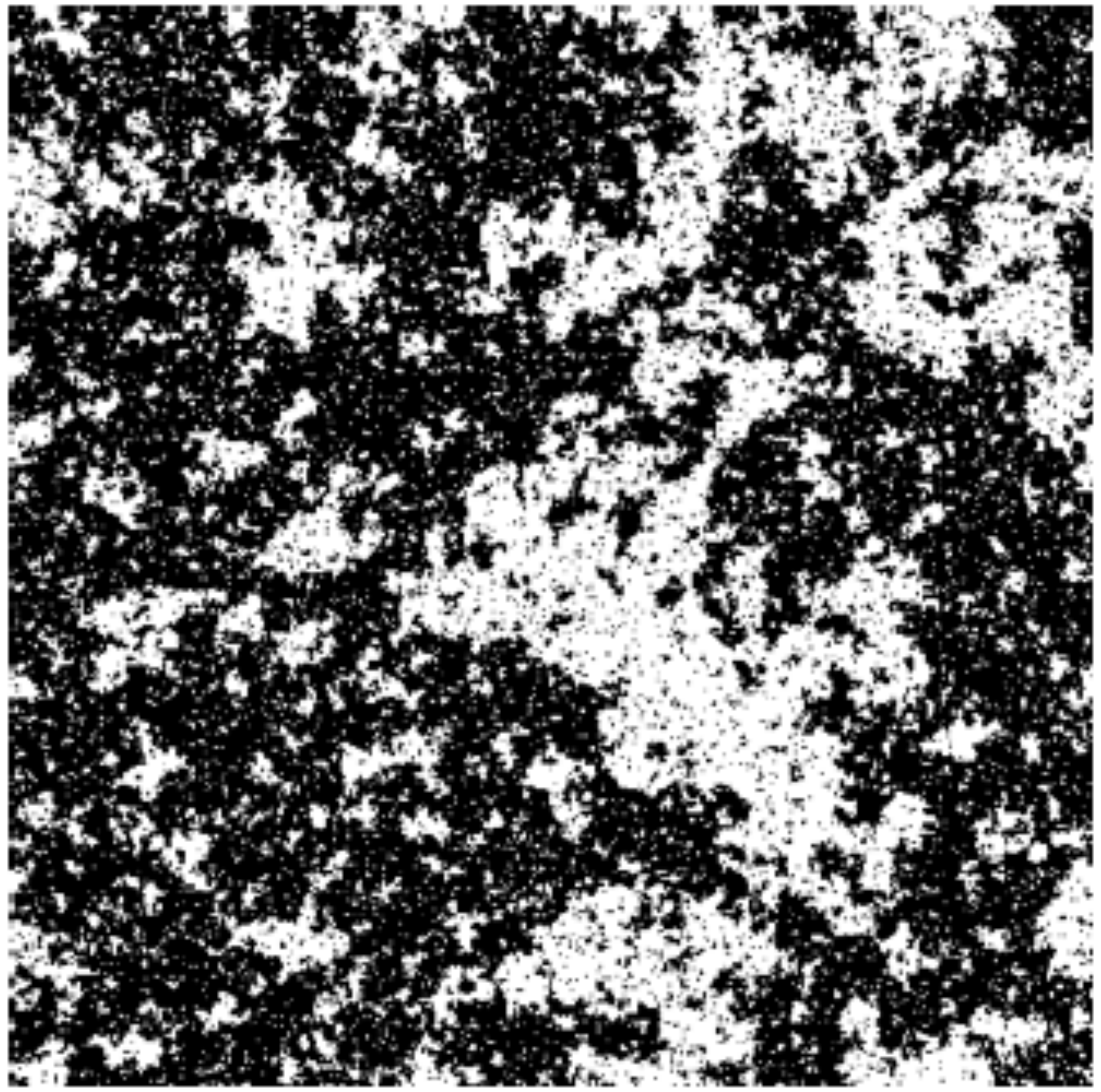
$$s(x) = \pm 1$$

$$\langle s(0)s(x) \rangle = e^{-|x|/\xi}$$

at T=Tc $\quad \xi \to \infty$

Courtesy of J. Terning

Critical Ising Model is Scale Invariant



http://bit.ly/2Dcrit

at T=T$_c$ $$\langle s(0)s(x)\rangle \propto \frac{1}{|x|^{2\Delta-1}}$$

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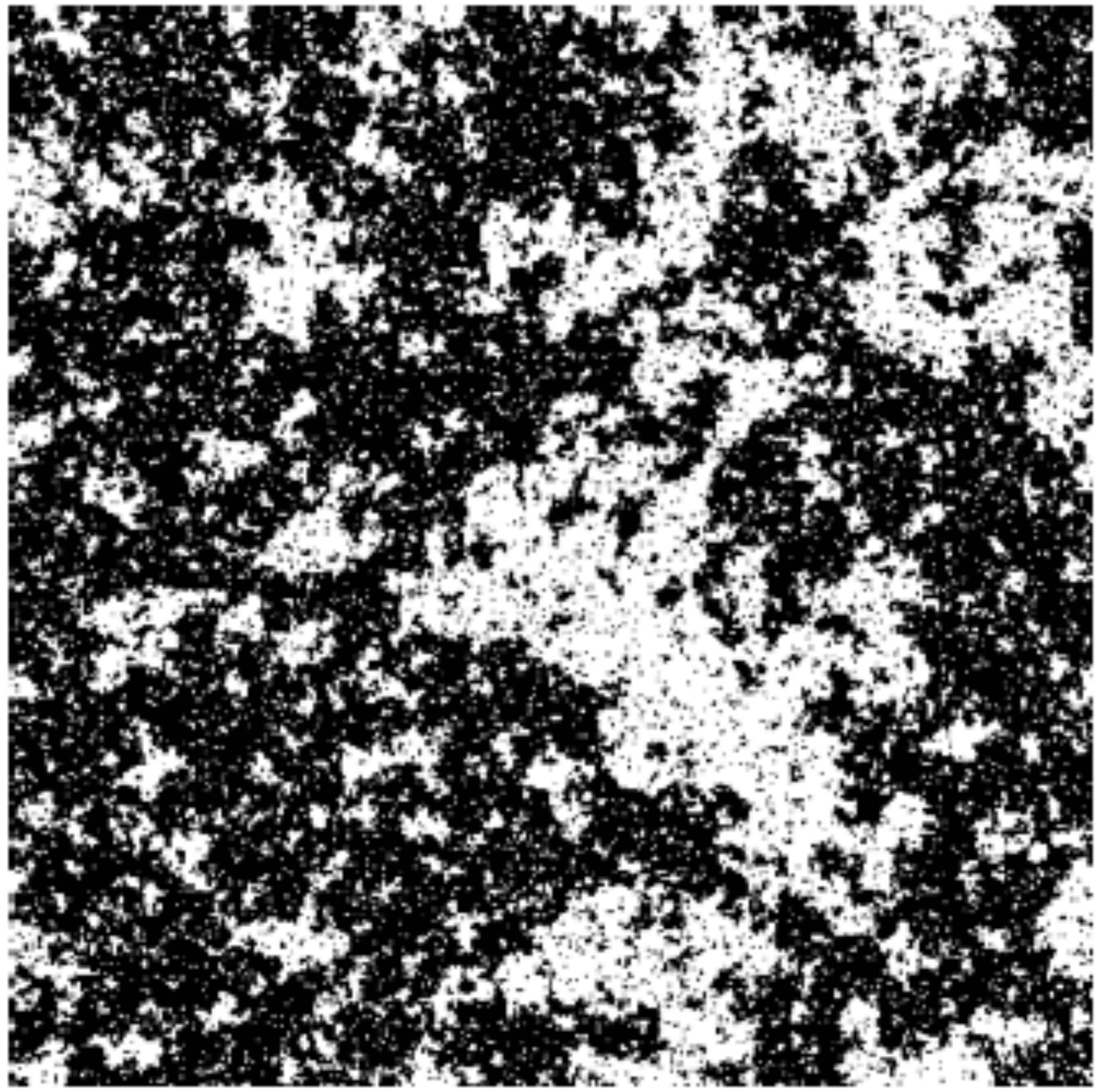

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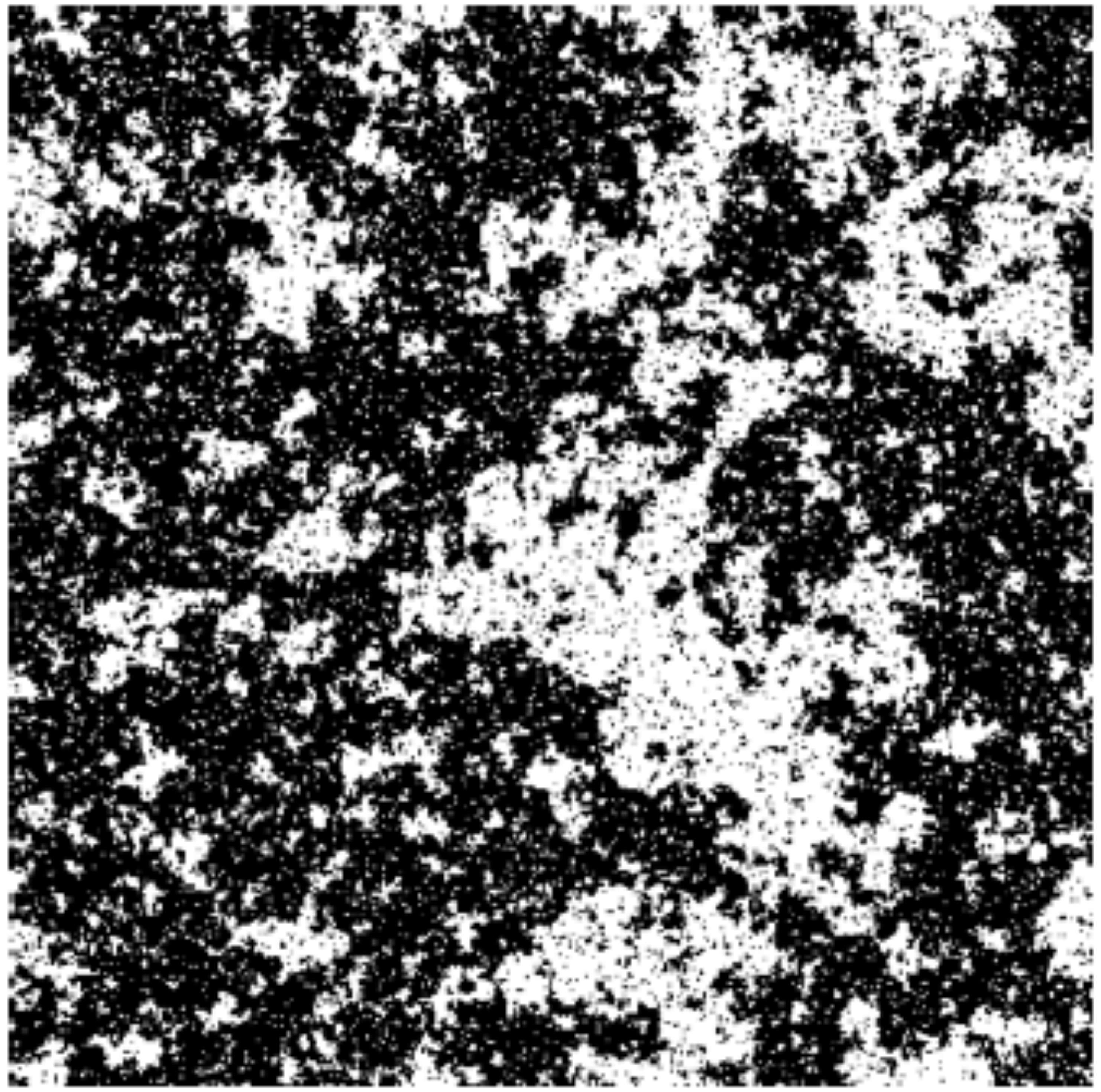

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critical exponent

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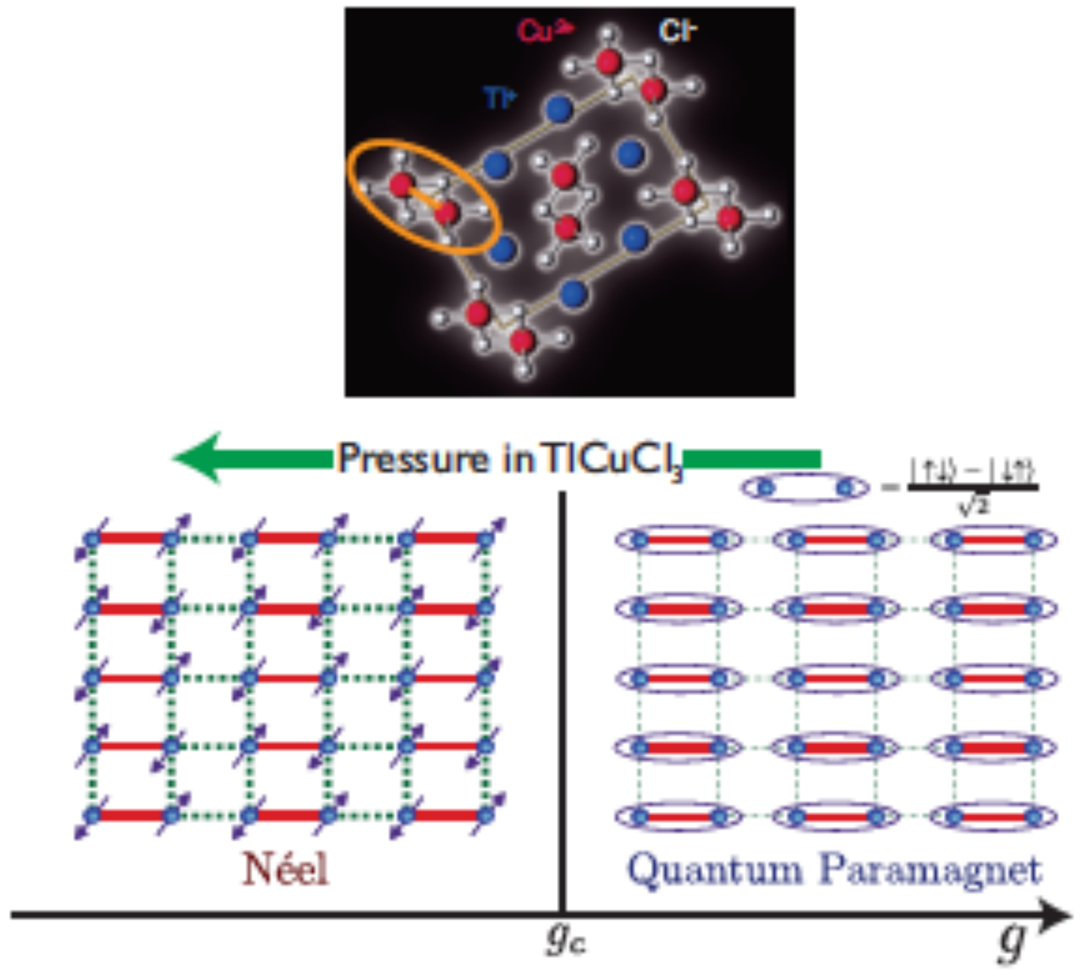
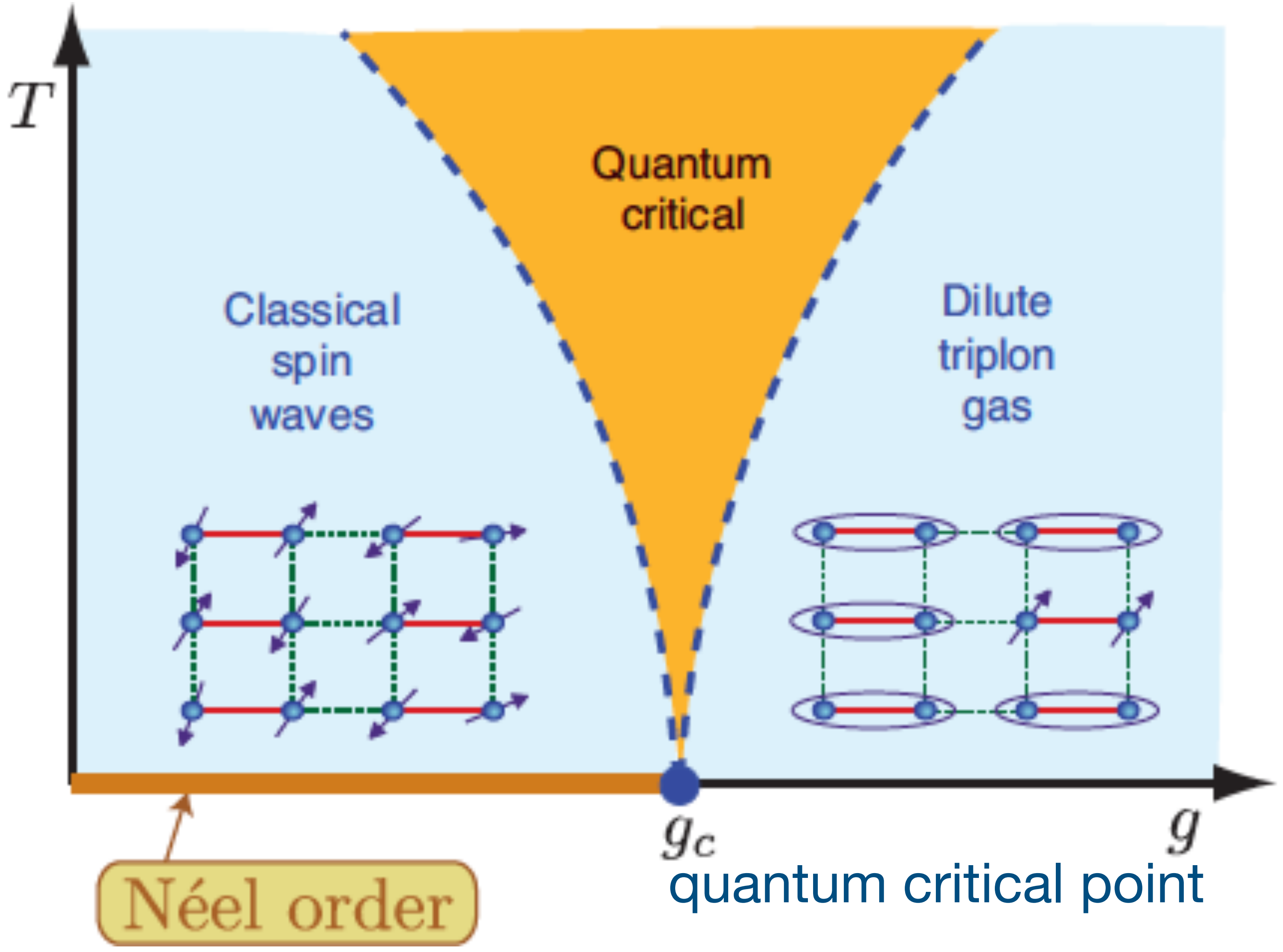
Axion & Quantum Phase Transition

Condensed matter systems can produce a scale invariant theory by tuning the parameters close to a critical value where a continuous phase transition occurs.



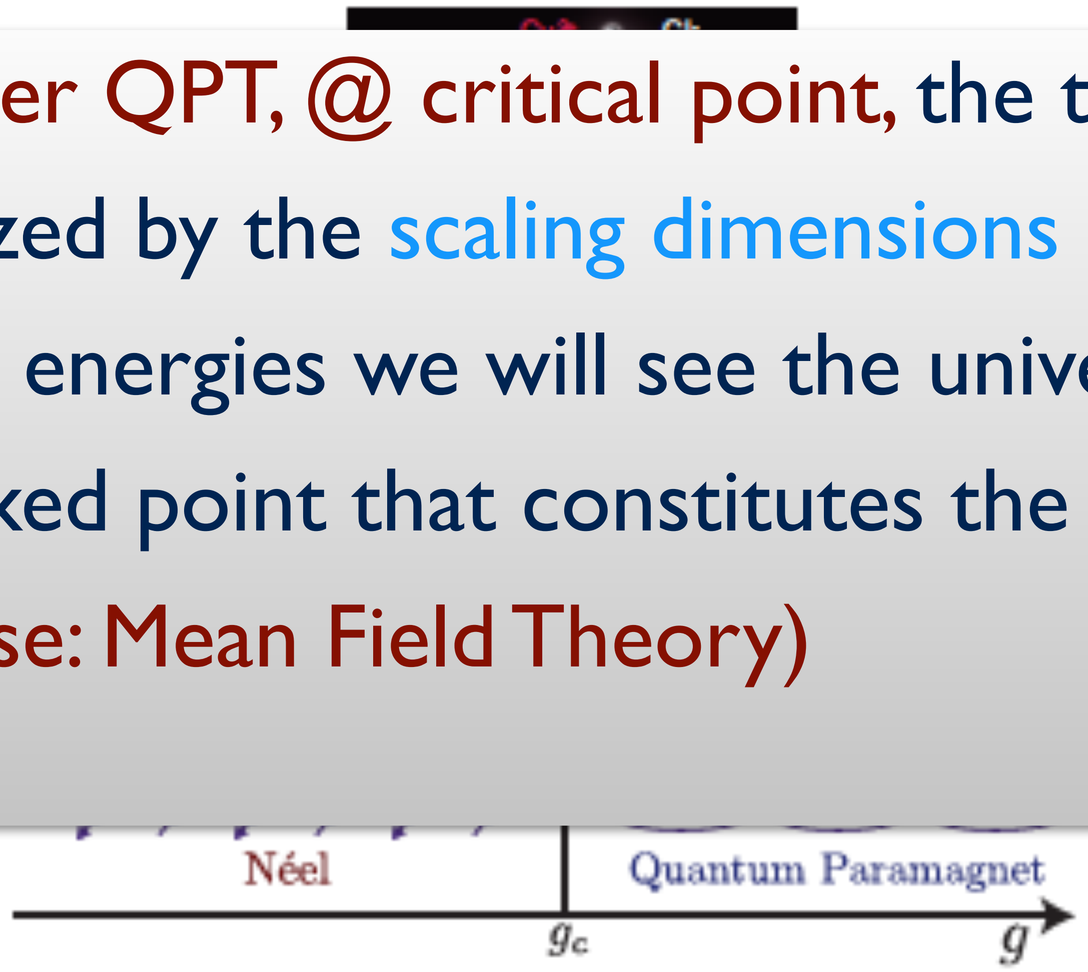
Sachdev, arXiv:1102.4268



Axion & Quantum Phase Transition

- Condensed matter systems can produce a scale invariant theory by tuning the parameters close to a critical value where a continuous phase transition occurs.

@2nd order QPT, @ critical point, the theory is scale invariant, characterized by the scaling dimensions of the field, and at low energies we will see the universal behavior of some fixed point that constitutes the low-energy EFT.
(Simple case: Mean Field Theory)



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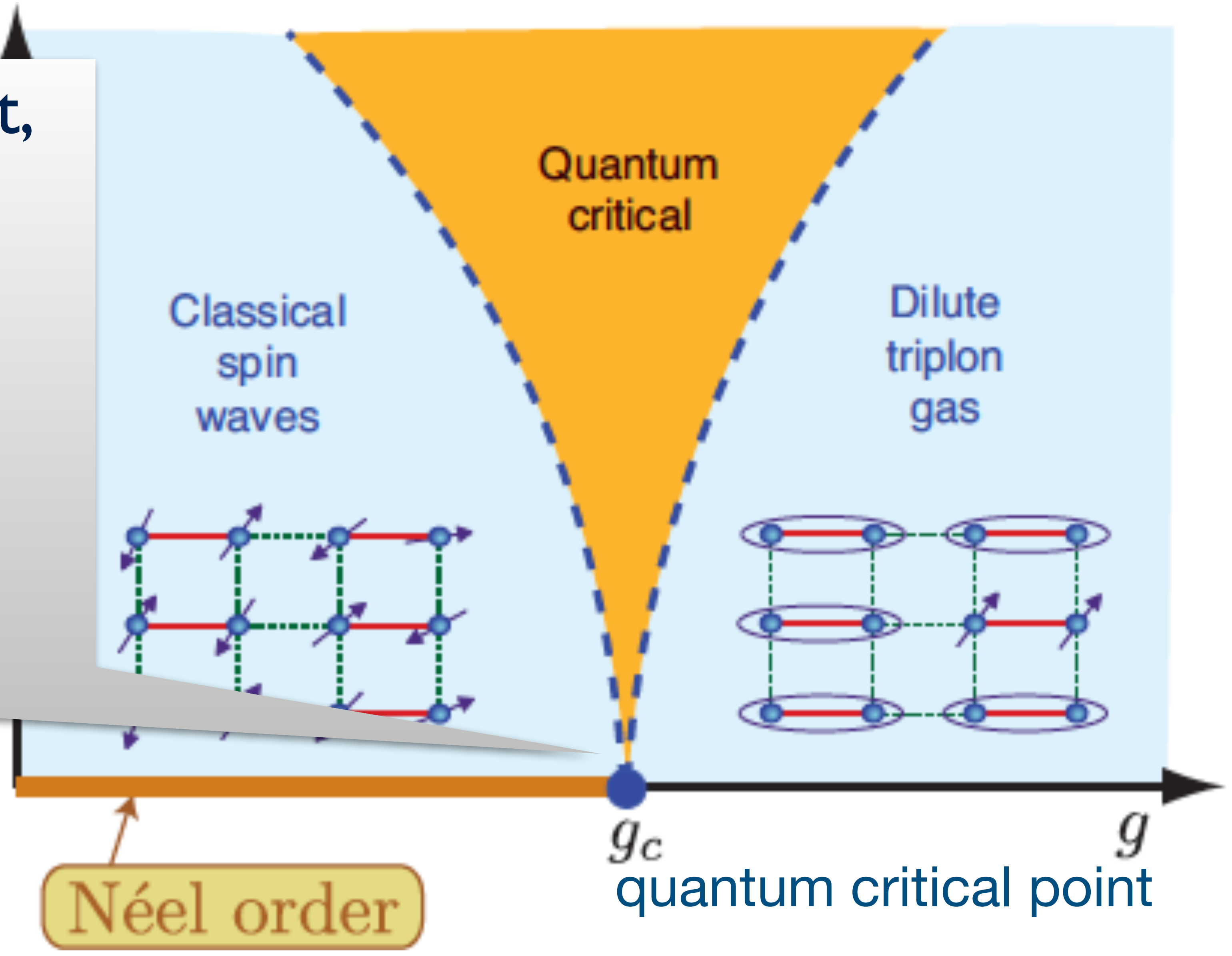

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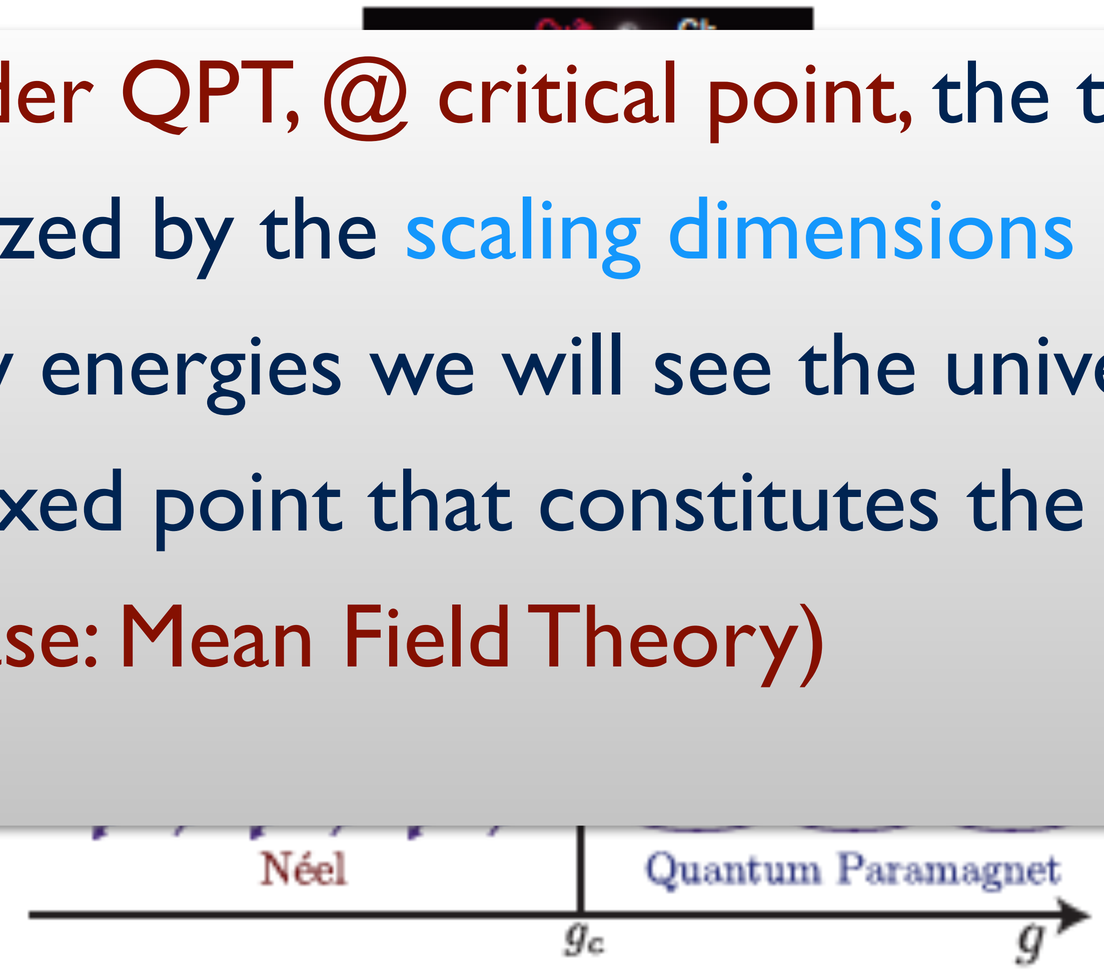

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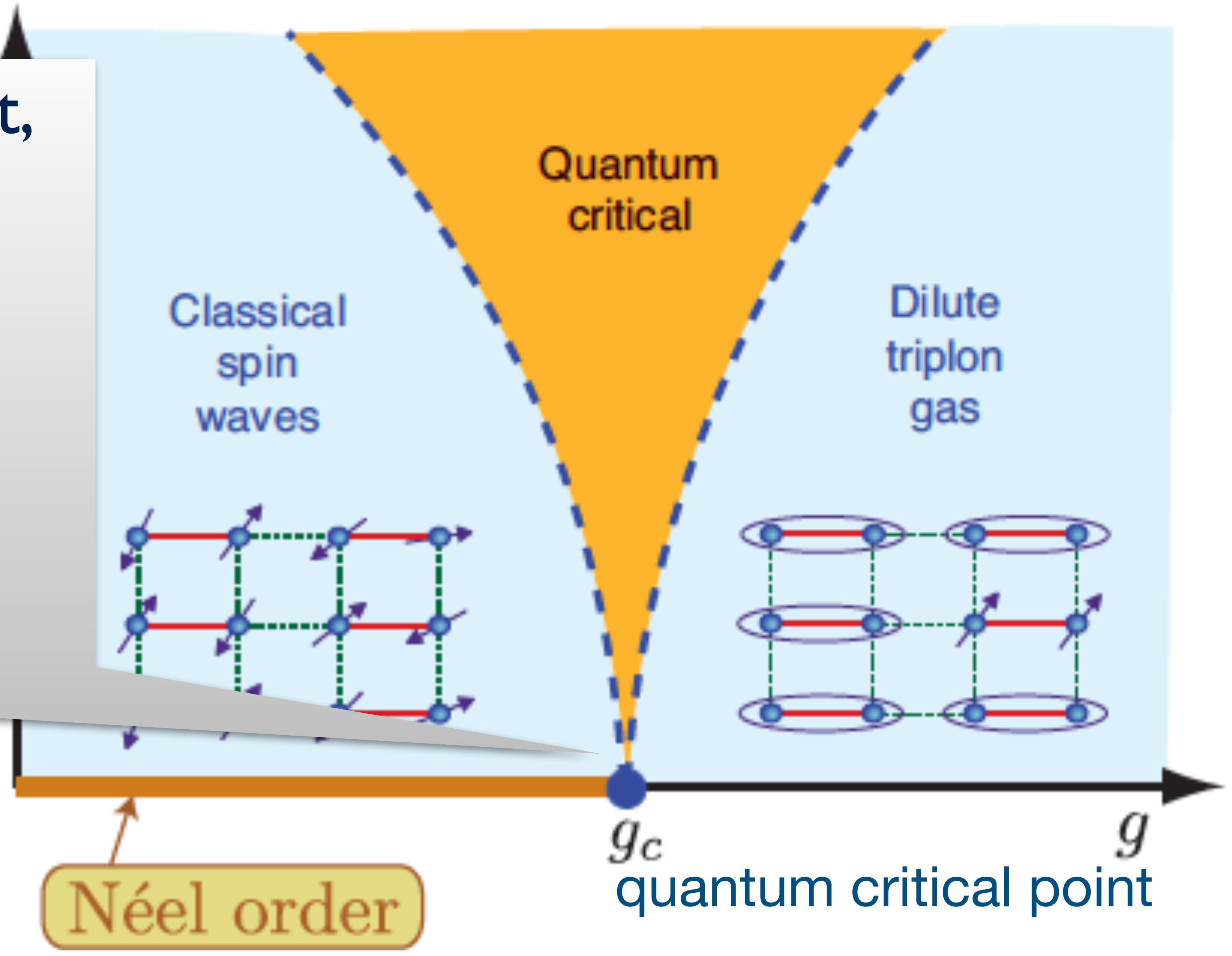

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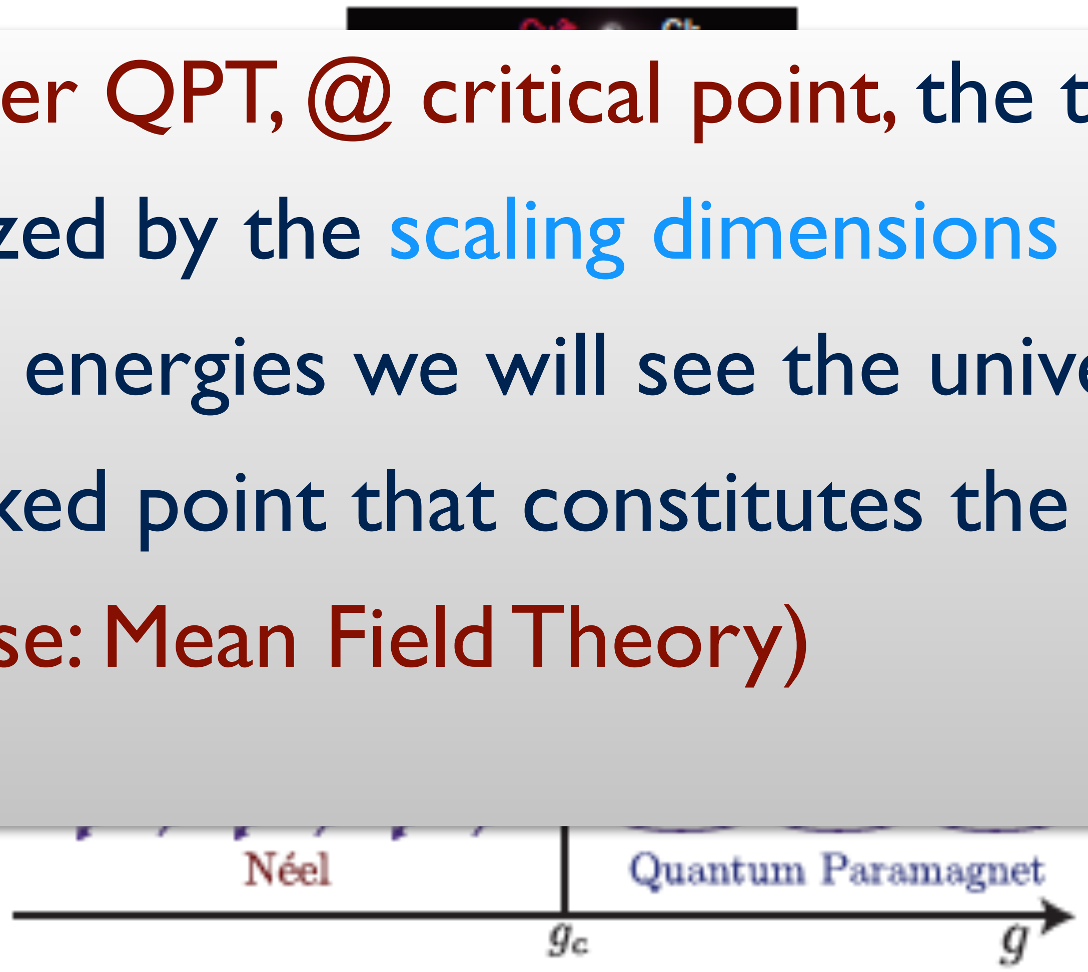
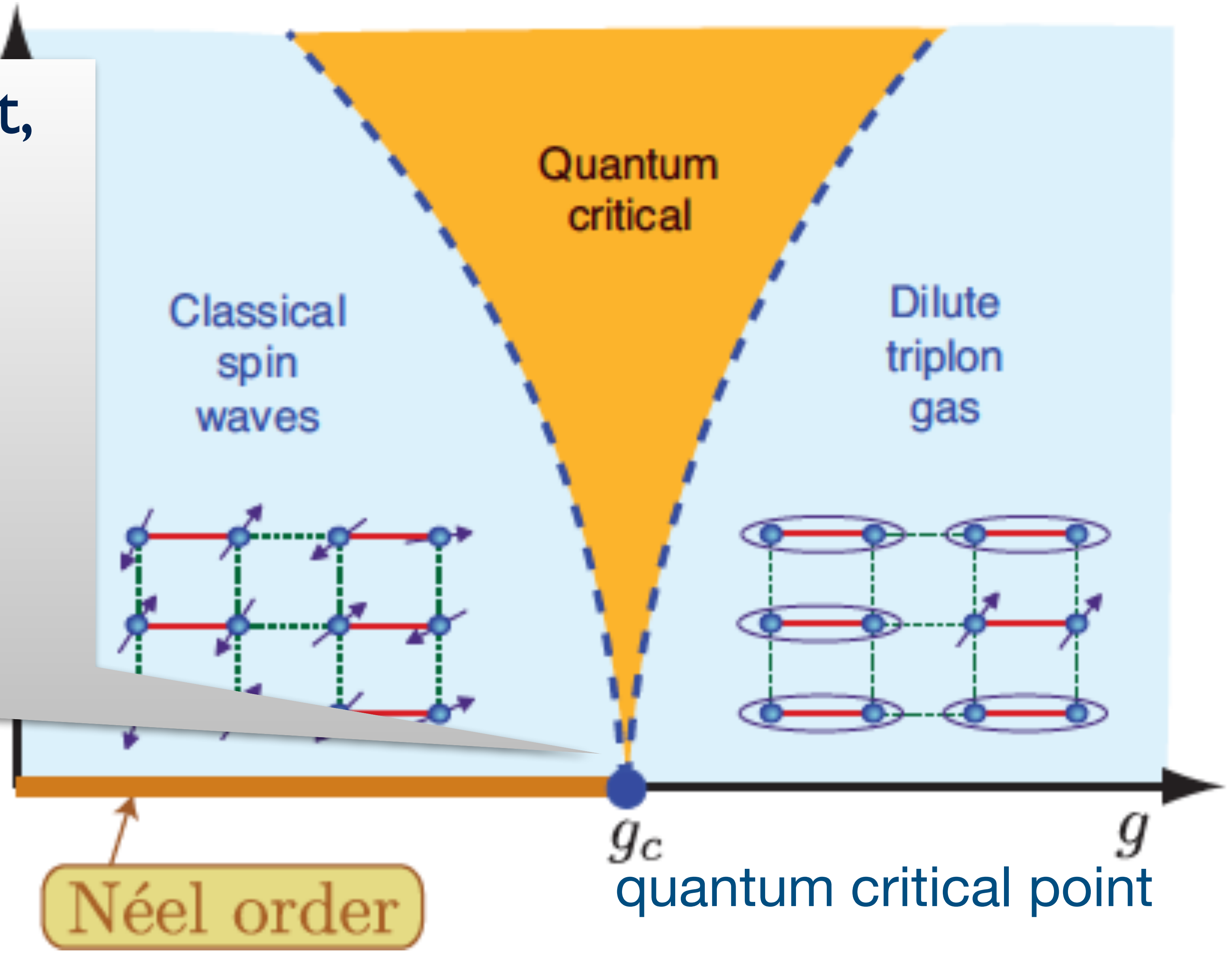
of some fixed point that constitutes the low-energy EFT.
(Simple case: Mean Field Theory)



Sachdev, arXiv:1102.4268



For us the quantum phase transition is modulated by κ

$$P_k \sim \left(\frac{H_{\rm inf}}{2\pi}\right)^2 (-k\tau)^{3-2\nu} \sim \left(\frac{H_{\rm inf}}{2\pi}\right)^2 \left(\frac{1}{x}\right)^{2\kappa/3}$$

$$1/x = aH_{\rm inf}/k,$$

Each mode grows after exiting horizon:

$$(aH_{\rm inf})^{-1} < k^{-1}$$

comoving horizon shrinks during inflation:

$$\langle \rho_\eta(\tau_e) \rangle \propto \kappa^3 \, e^{2\kappa N/3}$$

$$\tau_i/\tau_e = e^N$$

Axion from Quantum Phase Transition

Ismail, SL, Yu. 24′

- We assume the PQ symmetry has broken during inflation, $f_\eta > H_{\rm inf}$. Axion is effectively massless during inflation if $m_\eta/K < H_{\rm inf}$

ϕ: inflaton

η: axion

$\chi = \rho\, e^{i\eta/f_\eta}/\sqrt{2}$

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}K^2(\phi)g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta - \frac{1}{2}m_\eta^2\eta^2\right]$$

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K(ϕ) reduce to unity at the end of inflation (inflation decays away)

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- Flat FLRW metric

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j = a^2(\tau)\left(-d\tau^2 + \delta_{ij}dx^i dx^j\right)$$

conformal time: $d\tau \equiv \frac{dt}{a}$

de Sitter background: $a = -\frac{1}{H\tau}$

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- Abundance of axion is sufficiently produced through QPT induced by *K(ϕ)*

Axion from Quantum Phase Transition

Equation of motion

$$f'' - \nabla^2 f - \left(\frac{a''}{a} + \frac{K''}{K} + 2\frac{a'}{a}\frac{K'}{K} + \frac{a^2 m_\eta^2}{K^2}\right) f = 0$$

$$f \equiv aK\eta$$

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- Solution of axion field during inflation: $$f_k(\tau) = \frac{\sqrt{\pi}}{2}\sqrt{-\tau} H_\nu^{(1)}(-k\tau), \quad \nu \equiv \sqrt{9/4 + \kappa}$$

Axion from Quantum Phase Transition

- Power Spectrum $$P_k = \frac{1}{a^2}\frac{k^3}{2\pi^3}\langle |f_k|^2\rangle$$

$1/x = aH_{\rm inf}/k$

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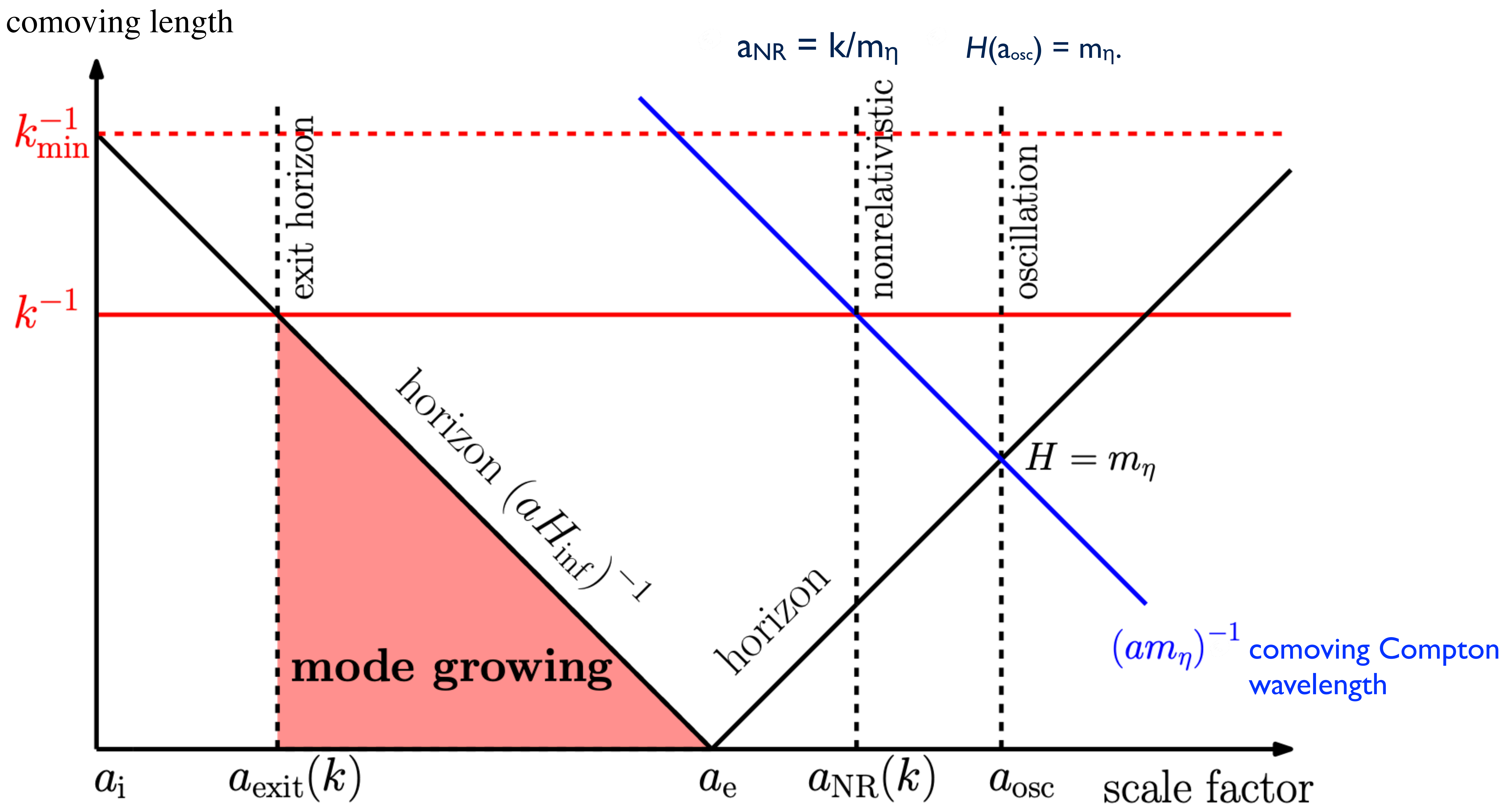
Axion Power Spectrum is Red-tilted



minimal mode receives largest enhancement

$$k_{min} \propto a_i H_{inf}$$

$$P_{k_{min}} \propto e^{2\kappa N/3}$$

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Axion from Quantum Phase Transition

Ismail, SL, Yu. 24'

- The axion energy density during inflation is given by (neglecting the tiny axion mass):

$$\langle \rho_\eta(\tau) \rangle = \frac{1}{2a^4} \int \frac{\mathrm{d}^3 k}{(2\pi)^3} \left(\underbrace{\left| f_k' + \frac{1+\kappa/3}{\tau} f_k \right|^2}_{\text{non-gradient term}} + \underbrace{k^2 \left| f_k \right|^2}_{\text{gradient term}} \right) . \tag{1}$$

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$$\frac{\mathrm{d}}{\mathrm{d}\log k} \langle \rho_\eta(\tau) \rangle_{\text{grad}} \simeq \frac{H_{\text{inf}}^2}{8\pi^2} \frac{k^2}{a^2} . \tag{2}$$

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- However, for a nonzero κ (which comes from the kinetic coupling in our mechanism), the whole story is changed. As one can easily check, the non-gradient term for superhorizon modes does not vanish in this case (for $0 < \kappa \ll 1$):

$$\frac{d}{d \log k} \langle \rho_\eta(\tau) \rangle_{\text{non-grad}} \simeq \frac{\kappa^4}{729} \frac{H_{\text{inf}}^4}{8\pi^2} . \quad (3)$$

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Axion from Quantum Phase Transition

♦ Energy Density

Integrating over all mode

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$$\approx \frac{H_{\rm inf}^4}{16\pi^3} 2^{2\nu} \left(\frac{\kappa}{3} + \frac{3}{2} - \nu \right)^2 \Gamma^2(\nu) \int_{-k_{\rm min}\tau}^{\mathcal{O}(1)} \mathrm{d}x x^{2-2\nu}$$

$$k_{\rm min} = \sqrt{\kappa} a_{\rm i} H_{\rm inf} = -\sqrt{\kappa}/\tau_{\rm i}$$

Axion from Quantum Phase Transition

♦ Energy Density

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♦ Energy Density

$$\left(\frac{\kappa}{3} + \frac{3}{2} - \nu\right) H_\nu^{(1)}(x)\Big|^2 + x^2 \left|H_\nu^{(1)}(x)\right|^2 \Big]$$

(1) $\mathrm{d}x x^{2-2\nu}$

$\mathrm{min}\,\tau$

$$k_{\mathrm{min}} = \sqrt{\kappa} a_{\mathrm{i}} H_{\mathrm{inf}} = -\sqrt{\kappa}/\tau_{\mathrm{i}}$$

Already Energy Density has the exponential enhancement for the magnitude of inflationary quantum fluctuations

But, if this factor is what compensate the smallness of ULDM mass, one should worry about cosmological constraints such as isocurvature bound (since PQ is broken during the inflation).

So, this exponential enhancement can be sizable, but cannot be the whole story for m ≲ 10^{-11} eV

$$= \frac{H_{\mathrm{inf}}^4}{16\pi^3} \frac{2^{2\nu} \left(\kappa/3 + 3/2 - \nu\right)^2 \Gamma^2(\nu)}{2\nu - 3} \left(\frac{1}{\kappa}\right)^{\nu - 3/2} e^{N(2\nu-3)}$$

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Axion Relic abundance

If $m_\eta < p_\mathrm{e}$,

$$T_\mathrm{NR} = \frac{m_\eta}{p_\mathrm{e}} T_\mathrm{reh} = \frac{1}{\sqrt{\kappa}} \frac{m_\eta}{H_\mathrm{inf}} T_\mathrm{reh} e^N$$

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=> Axion becomes NR before structure formation, T ~ keV

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The axion energy density today is given by $\langle \rho_\eta(\tau_0) \rangle = \langle \rho_\eta(\tau_{\rm e}) \rangle \left(a_{\rm e}/a_{\rm NR}\right)^4 \left(a_{\rm NR}/a_0\right)^3$

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The relic abundance today is given by

$$\Omega_\eta \equiv \langle \rho_\eta(\tau_0) \rangle / \rho_{\rm c} = \frac{g_{*0} g_{*\rm reh}^{-1/4}}{48\pi^3} \left(\frac{\pi^2}{90}\right)^{3/4} \frac{m_\eta T_0^3 H_{\rm inf}^{3/2}}{M_{\rm Pl}^{7/2} H_0^2} \frac{\mathcal{F}(\kappa)}{\sqrt{\kappa}} e^N e^{N(2\nu - 3)}$$

$$\rho_{\rm c} = 3H_0^2 M_{\rm Pl}^2$$

Axion Relic abundance

Subteltey:

For ultralight DM, the large enhancement is mostly from this e^N (~10^{26} for N=60) which comes from kinematics: positive κ leads to a red tilt => power spectrum is dominated by k_{min}, whose p_e is exponentially suppressed by the end of inflation

The other term $e^{N(2\nu-3)}$ is constrained by isocurvature bound, etc, and cannot be too large

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small mass compensated by exponential enhancement

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ULDM with m ~ 10^{-22}eV is easy to achieve!

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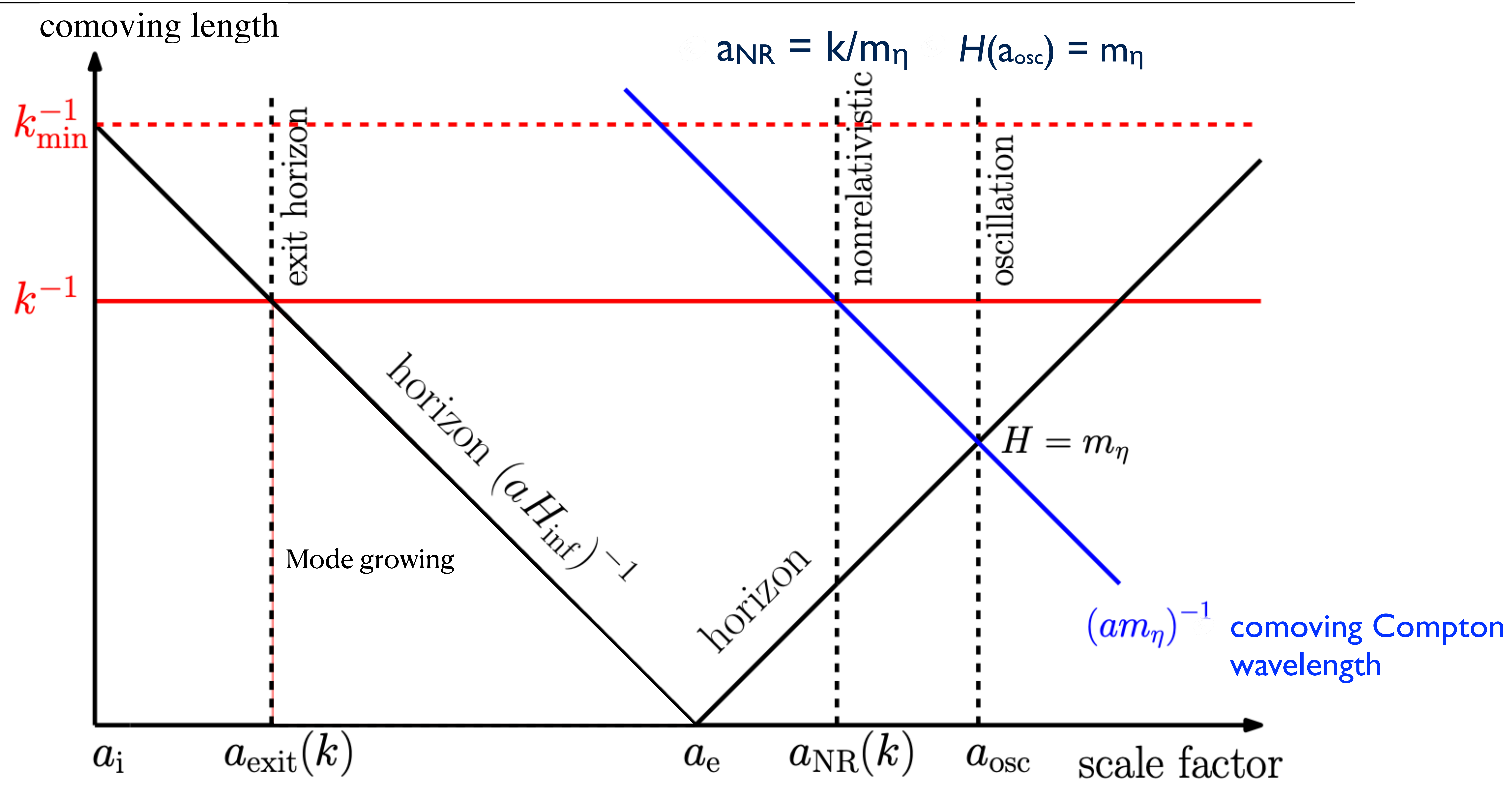
Axion evolution after inflation



a > a_{NR}: axion becomes nonrelativistic

a > a_{osc}: axion starts coherent oscillation

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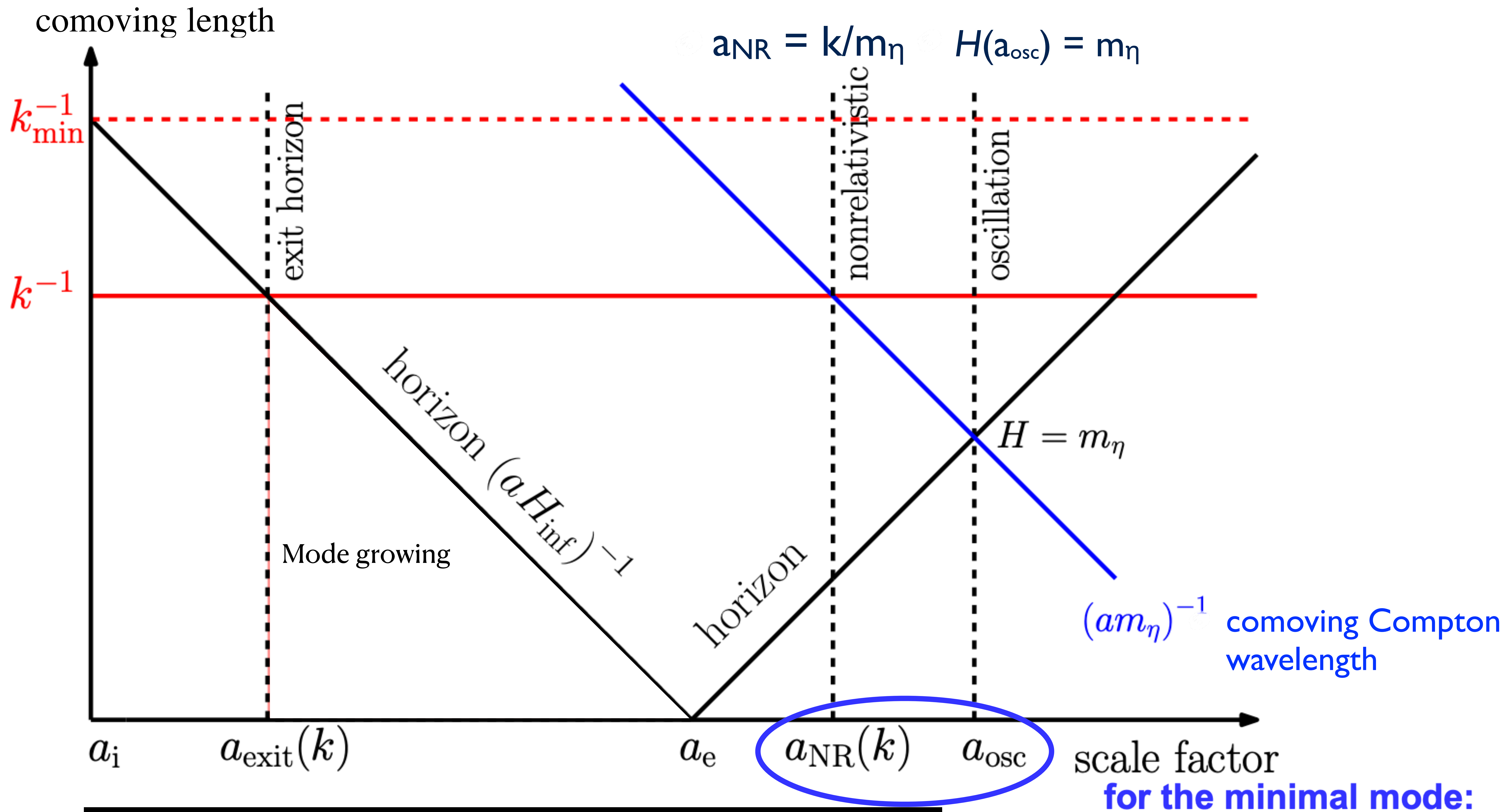

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for the minimal mode:

$$k_{min} \propto e^{-N} H_{inf}$$

$$a_{NR}(k_{min}) \ll a_{osc}$$

turn NR much earlier than oscillation

Origin of inflation coupling to axion kinetic term

1) EFT operators

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- Exponential enhancement could also be realized from effecive operator

$$K(\phi) = 1 + \frac{C_6}{M_{\rm Pl}^2}\ \phi^2 \qquad\qquad V(\phi) = m_\phi^2\phi^2/2$$

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$$\kappa_1 \approx M_{\rm Pl}^2 \left(2\epsilon \frac{K_{\phi\phi}}{K} - \frac{K_\phi}{K}\frac{V_\phi}{V} \right), \; \kappa_2 \approx -M_{\rm Pl}^2 \frac{K_\phi}{K}\frac{V_\phi}{V}$$

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effective Wilson coefficient plays the role of κ

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2) UV completion

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- Noncanonical kinetic term can be realized in the supergravity framework

Ellis et al, 2013, 1984

$$\mathcal{L}_{KE} = (\partial_\mu \phi^*, \partial_\mu T^*) \left(\frac{3}{(T + T^* - |\phi|^2/3)^2} \right) \begin{pmatrix} (T+T^*)/3 & -\phi/3 \\ -\phi^*/3 & 1 \end{pmatrix} \begin{pmatrix} \partial^\mu \phi \\ \partial^\mu T \end{pmatrix},$$

ϕ: inflaton $\quad T$: modulus

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Would it fit into a story of String Theory Axions? -ongoing discussion with Liam McAllister

Origin of inflation coupling to axion kinetic term

3) Radial mode as inflaton **Fairbairn, Hogan, and Marsh '15**

Origin of inflation coupling to axion kinetic term

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PQ scalar $\chi = \varrho\, e^{i\eta/f_\eta} / 2$ naturally leads to a coupling between inflaton ϱ and axion kinetic term:

$$|\partial_\mu \chi|^2 = \frac{1}{2}\left[(\partial_\mu \rho)^2 + \frac{\rho^2}{f_\eta^2}(\partial_\mu \eta)^2\right]$$

During inflation, we have $\varrho \gg f_\eta$ and the axion kinetic coupling is significant

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$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R\left(1+\xi\frac{\rho^2}{M_{\rm Pl}^2}\right) - \frac{1}{2}g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho \right.$$
$$\left. -\frac{1}{2}\frac{\rho^2}{f_\eta^2}g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta - \frac{\lambda}{4}\left(\rho^2 - f_\eta^2\right)^2\right]$$

$$K(\rho) = \frac{\rho}{f_\eta\sqrt{(1+\xi\rho^2/M_{\rm Pl}^2)}}\,,$$

$$V(\rho) = \frac{\lambda\left(\rho^2 - f_\eta^2\right)^2}{4\left(1+\xi\rho^2/M_{\rm Pl}^2\right)^2}\,.$$

Origin of inflation coupling to axion kinetic term

3) Radial mode as inflaton

Fairbairn, Hogan, and Marsh '15

PQ scalar $\chi = \varrho\, e^{i\eta/f_\eta} / 2$ naturally leads to a coupling between inflaton ϱ and axion kinetic term:

During inflation, we have $\varrho \gg f_\eta$ and the axion kinetic coupling is significant

$$|\partial_\mu \chi|^2 = \frac{1}{2}\left[(\partial_\mu \rho)^2 + \frac{\rho^2}{f_\eta^2}(\partial_\mu \eta)^2\right]$$

As ϱ rolls down along the potential and tends to the vacuum expectation value f_η, the inflation ends and the axion kinetic term reduces to the canonical form.

$$\mathcal{S} = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R\left(1+\xi\frac{\rho^2}{M_{\rm Pl}^2}\right) - \frac{1}{2}g^{\mu\nu}\partial_\mu\rho\partial_\nu\rho - \frac{1}{2}\frac{\rho^2}{f_\eta^2}g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta - \frac{\lambda}{4}\left(\rho^2 - f_\eta^2\right)^2\right]$$

$$K(\rho) = \frac{\rho}{f_\eta\sqrt{(1+\xi\rho^2/M_{\rm Pl}^2)}}\,,$$

$$V(\rho) = \frac{\lambda\left(\rho^2 - f_\eta^2\right)^2}{4\left(1+\xi\rho^2/M_{\rm Pl}^2\right)^2}\,.$$

$$\kappa \approx -4q^4\left[3\xi^2(6\xi+1)^2 + \left(24\xi^2+8\xi+3\right)q^4 + 2\xi\left(24\xi^2+22\xi+3\right)q^2\right]/\left(6\xi^2+\xi+q^2\right)^3$$

$$q \equiv M_{\rm Pl}/\rho.$$

→ ξ should satisfy $-1/6 < \xi < 0$

Theoretical Constraints

- Condition that we impose (also need to make sure slow roll potential is not spoiled)
  - Back-reaction constraint

$$\kappa\,\mathcal{F}(\kappa)\,e^{N(2\nu-3)} \ll 18\pi/A_{\rm s}$$

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Inflation mass get loop corrosion from axion coupling of order $f_\eta^4/(16\pi^2 m_\phi^2)$.

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- The isocurvature (entropy) mode measures the deviation from the adiabatic mode of single-field inflation, parametrized by:

$$\beta_{\mathrm{iso}} \equiv A_{\mathrm{iso}} / \left(A_{\mathrm{s}} + A_{\mathrm{iso}}\right) \approx A_{\mathrm{iso}} / A_{\mathrm{s}}$$

A_{s} = scalar amplitude

A_{iso} = isocurvature perturbation

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- The isocurvature mode is determined by the axion perturbation during inflation:

$$\langle \delta\eta^2(\tau, k)\rangle = \frac{k^3}{2\pi^2} \int \mathrm{d}^3x \, e^{-\mathrm{i}\vec{k}\cdot\vec{x}} \eta\left(\tau, \vec{x}\right) \eta\left(\tau, 0\right)$$

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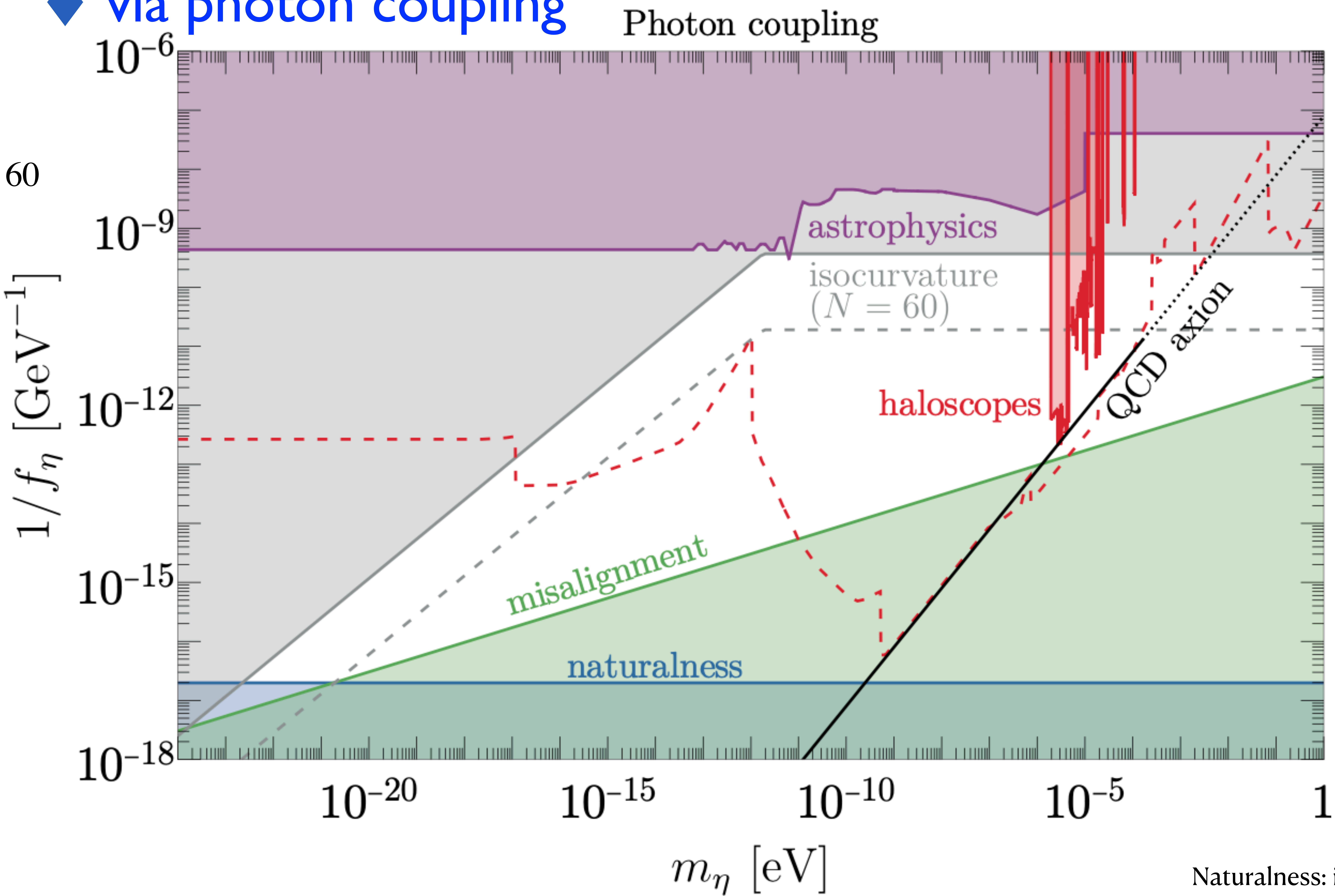
$$\tau_* = -1/k_* \qquad \left\langle \delta\eta^2(\tau_*, k_{\text{min}}) \right\rangle = \frac{2^{2\nu}}{2\pi} \Gamma^2(\nu) \left(\frac{H_{\text{inf}}}{2\pi}\right)^2 \left(\frac{k_*}{k_{\text{min}}}\right)^{2\nu-3}$$

Results

Axion from Quantum Phase Transition

- via photon coupling

N = 60



$$\frac{\alpha_{EM}}{8\pi f_\eta}\eta F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Future haloscopes (dashed line):

DANCE, SRF-m3, DMRadio, etc.

Future CMB, 21cm (dashed line):

CMB-S4

SKA2

Naturalness: inflation mass correction from axion is small

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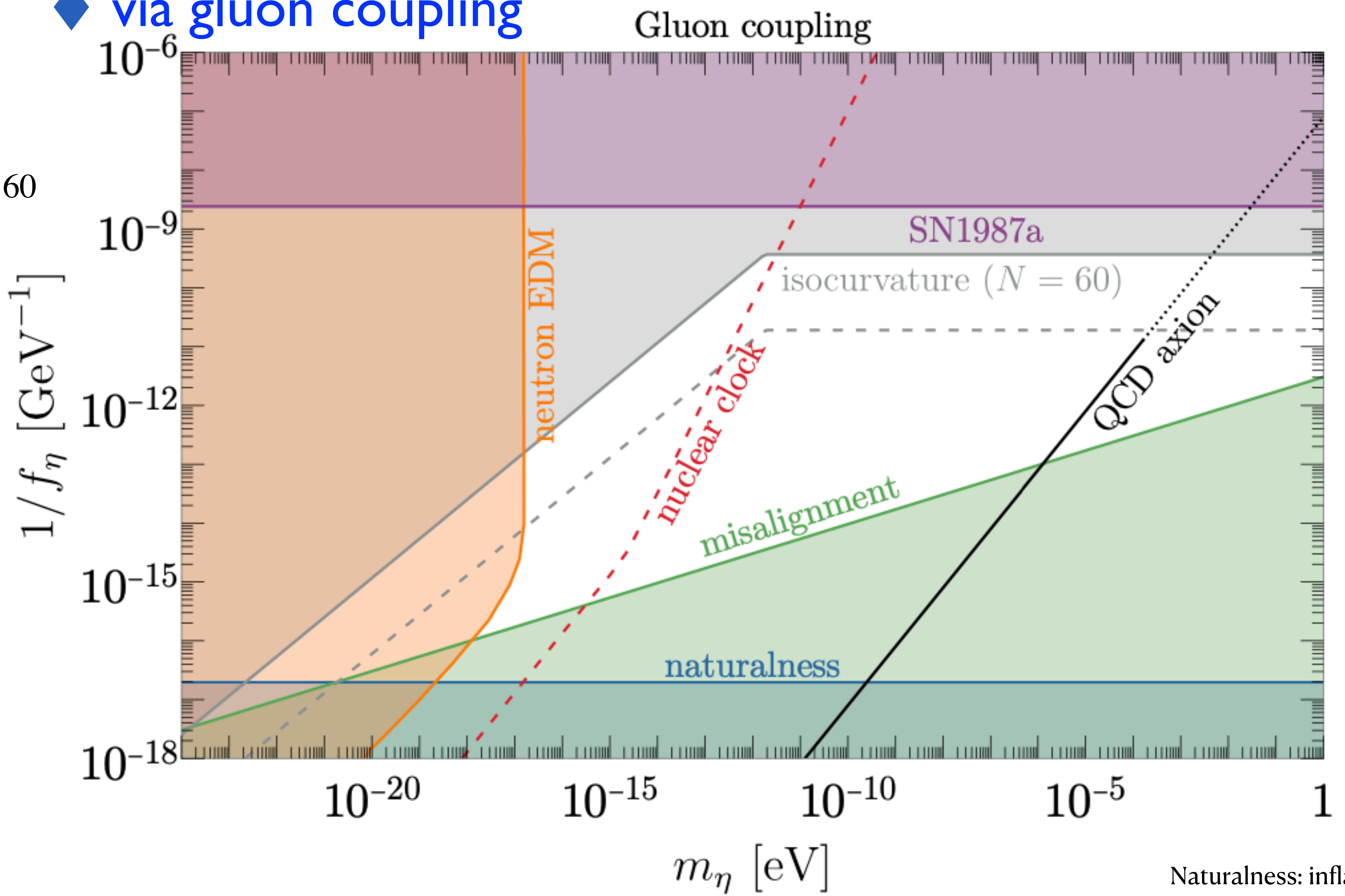

$$\frac{\alpha_s}{8\pi f_\eta}\eta G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a$$

Future nuclear clock (dashed line):

^{229}Th

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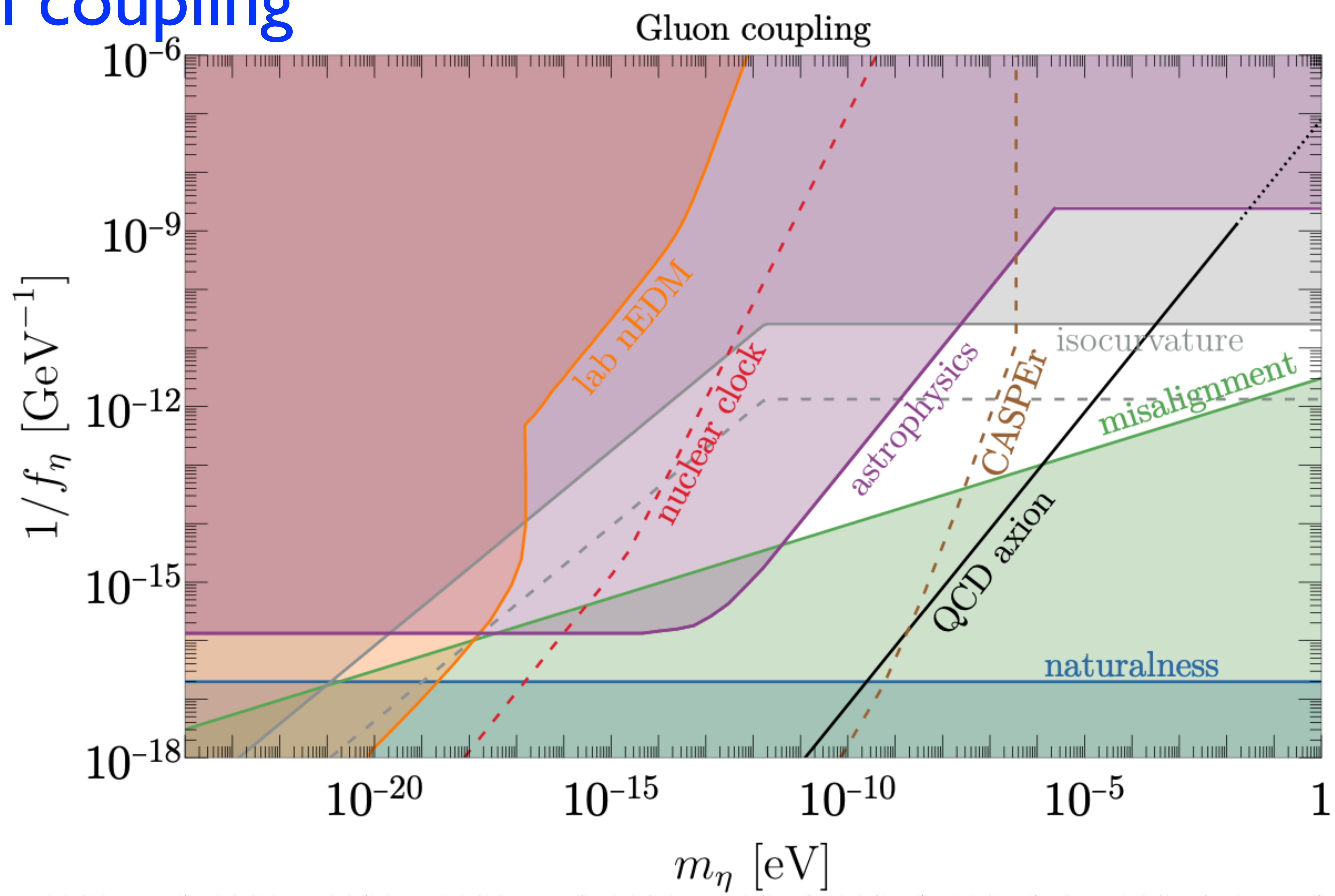

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CASPEr (brown dashed line)

$\eta G\tilde{G}$ is heavily constrained by experiments for ALPs lighter than 10^{-10} eV

But this operator is not predicted in our mechanism and can simply be turned off

The generic axion-gluon coupling can be induced by gravity

Summary: Comparison with Misalignment mechanism

Axion dark matter	QPT by Inflation	Misalignment
Production mechanism	kinetic coupling to inflaton	oscillation due to Hubble friction
Production era	during inflation	much later, when $H \sim m_\eta$
Kinematics	relativistic when produced Non-relativistic much earlier than keV (For heavy mass, NR by the end of inflation)	Non-relativistic when produced
Power spectrum	red spectrum, peaked at super-horizon scale	nearly scale-invariant spectrum
Relic abundance	insensitive to breaking scale	depend on breaking scale
Parameter space (ALP DM)	for m < 10^{-12} GeV, f_η > 10^{10} GeV (can be lowered with N>60)	for m < 10^{-12} GeV, f_η > 10^{14} GeV
Parameter space (QCD axion DM)	Maximum QCD axion mass of order 0.05 eV (can be heavier with N>60)	Maximum QCD axion mass of order 10^{-5} eV

Summary

- Inflationary quantum fluctuations + Quantum Phase Transition
  ➡ sufficient production of axion as ultralight DM

- This new mechanism predicts much larger couplings to SM particles and a wider range of allowed couplings than misalignment mechanism

- Much of the parameter space will be probed by near-future axion experiments

- It covers a large range of DM masses, from sub-eV down to fuzzy DM range

- It works for both QCD axion and ALPs. We expect it can also be applicable to other bosonic ultralight DM scenarios (e.g., dilaton, majoron, dark photon)

Thank You!

Back-up

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$$|KK_\phi\langle g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta\rangle| \ll \left|3H_{\rm inf}\dot{\phi}\right| \qquad \langle\rho_\eta\rangle \ll 3M_{\rm Pl}^2H_{\rm inf}^2$$

$$\boxed{\kappa\,\mathcal{F}(\kappa)\,e^{N(2\nu-3)} \ll 18\pi/A_{\rm s}}$$

In addition, the requirement of

$$|KK_\phi\langle g^{\mu\nu}\partial_\mu\eta\partial_\nu\eta\rangle| \ll V_\phi$$

gives a model-dependent constraint

$$\boxed{\kappa\mathcal{F}(\kappa)e^{N(2\nu-3)} \ll \frac{12\pi^2}{\sqrt{A_{\rm s}}}\frac{V_\phi}{H_{\rm inf}^3}}$$

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Axion from Quantum Phase Transition

♦ When Axion becomes NR

$$T_{\rm NR} = \frac{m_\eta}{p_{\rm e}} T_{\rm reh} = \frac{1}{\sqrt{\kappa}} \ldots$$

In addition, the requirement of

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$$\kappa \mathcal{F}(\kappa) e^{N(2\nu-3)} \ll \frac{12\pi^2}{\sqrt{A_{\rm s}}} \frac{V_\phi}{H_{\rm inf}^3}$$

it is easy for the axion to become nonrelativistic before structure formation, T ~ keV

QCD Axion Relic abundance

mass is induced by nonperturbative QCD effects

$$m_\eta(T) = \beta m_\eta \left(\frac{\Lambda_{\rm QCD}}{T}\right)^\gamma \ , \quad T \gg \Lambda_{\rm QCD}$$

- Effectively massless at the end of inflation (assuming $T_{reh} \gg \Lambda_{QCD}$), and becomes NR when $T \lesssim \Lambda QCD$

$$\langle \rho_\eta(\tau_0) \rangle = \langle \rho_\eta(\tau_{\rm e}) \rangle \left(a_{\rm e}/a_{\rm NR}\right)^4 \left(a_{\rm NR}/a_0\right)^3$$

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$$\frac{\Omega_\eta}{\Omega_{\rm cdm}} = 10^{-3} \mathcal{F}(\kappa) e^{N(2\nu-3)} \left(\frac{H_{\rm inf}}{10^{13} \text{ GeV}}\right)^2 \left(\frac{T_{\rm NR}}{10^2 \text{ GeV}}\right)$$

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- Upper bound on QCD axion mass: $f_\eta m_\eta \approx \Lambda_{\rm QCD}^2$ $\quad \gamma \approx 4$ and $\beta \sim \mathcal{O}(10^{-2})$

  PQ symmetry broken during inflation: $f_\eta > H_{\rm inf}/2\pi \Rightarrow m_\eta < 2\pi \Lambda_{\rm QCD}^2 / H_{\rm inf}$

  For $N = 60$ e-folds, we have $m_\eta <$ 0.05 eV

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can be further relaxed with a larger number of e-folds

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Numerically:

$$\frac{\Omega_\eta}{\Omega_{\rm cdm}} = 2.6\, \mathcal{F}(\kappa) e^{N(2\nu - 3)} \left(\frac{H_{\rm inf}}{10^9 \ {\rm GeV}} \right)^{5/2}$$

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Doesn’t need a large enhancement

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at the pivot scale $k_* = 0.05$

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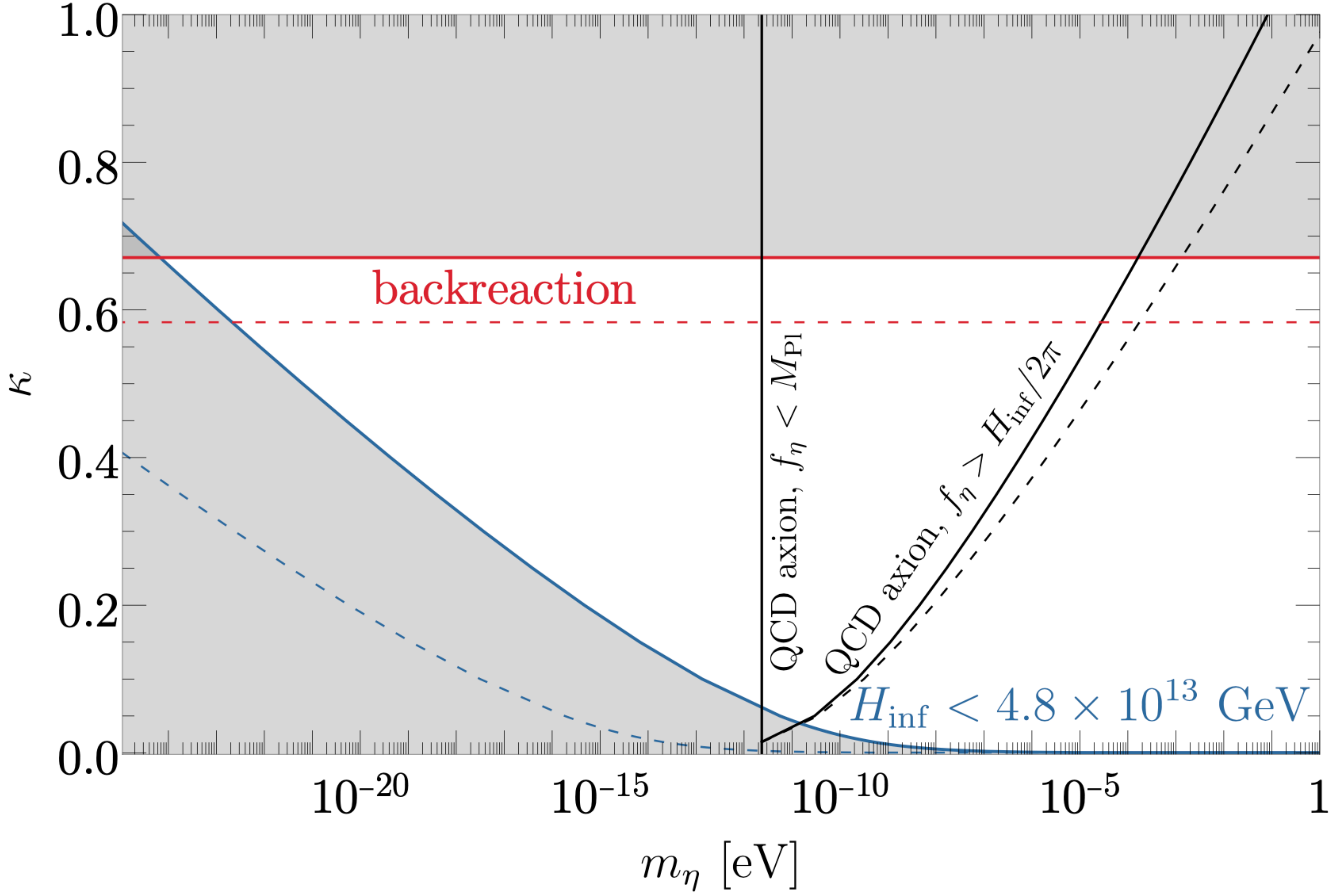
$$\kappa < 0.79\,, \quad \text{for } N = 50$$
$$\kappa < 0.67\,, \quad \text{for } N = 60$$
$$\kappa < 0.58\,, \quad \text{for } N = 70$$

$$\mathcal{F}(\kappa) \equiv \frac{2^{2\nu}\left(\kappa/3 + 3/2 - \nu\right)^2\Gamma^2(\nu)}{2\nu-3}\left(\frac{1}{\kappa}\right)^{\nu-3/2}$$

Theoretical Constraints

♦ Parameter Space

$N = 60, 70$ e-folds   correct DM abundance



For $N = 60$ e-folds:

m_η can reach 10^{-24} eV

QCD axion is further bounded below 0.05 eV

relaxed with larger e-folds

$H_{\rm inf} = 2\pi M_{\rm pl}\sqrt{A_s r_T/8}$

$A_s = 2.2 \times 10^{-9}$ $r_T < 0.036$

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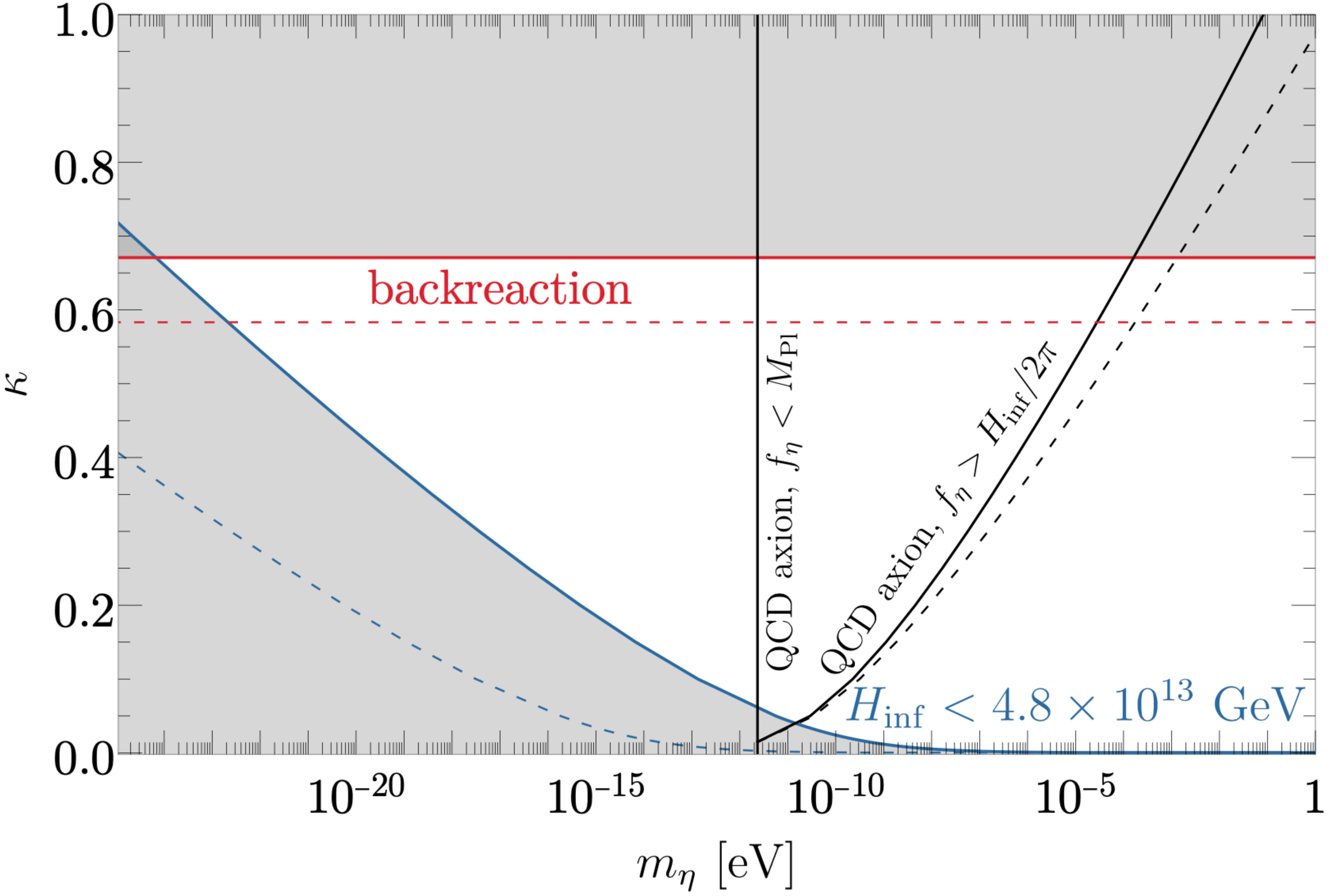

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DM relic abundance does not depend on the breaking scale directly

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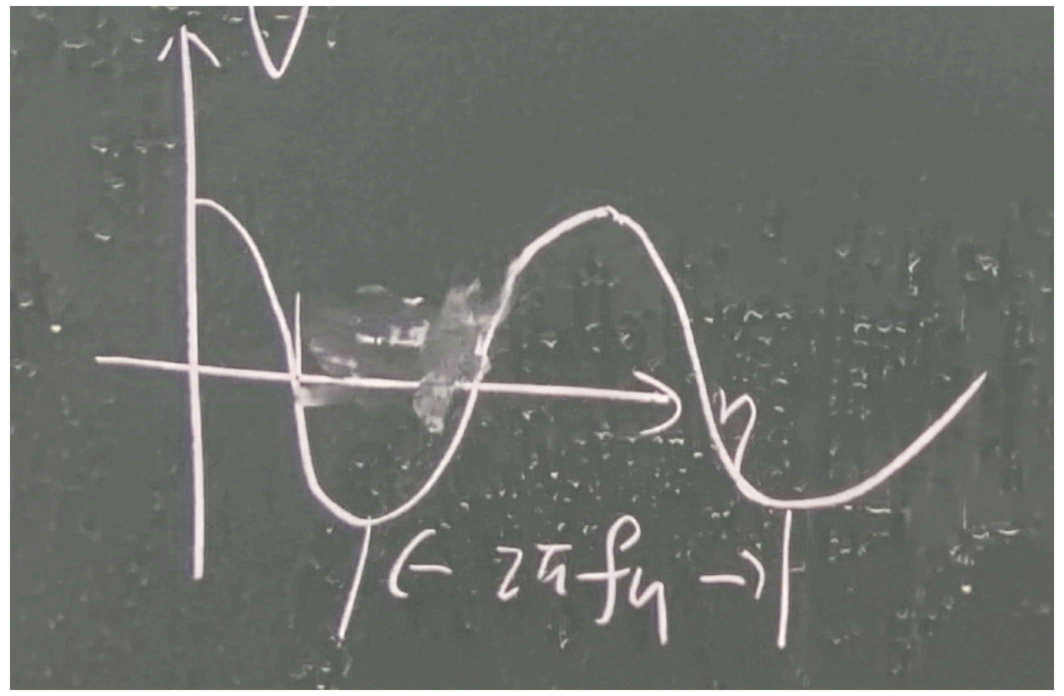

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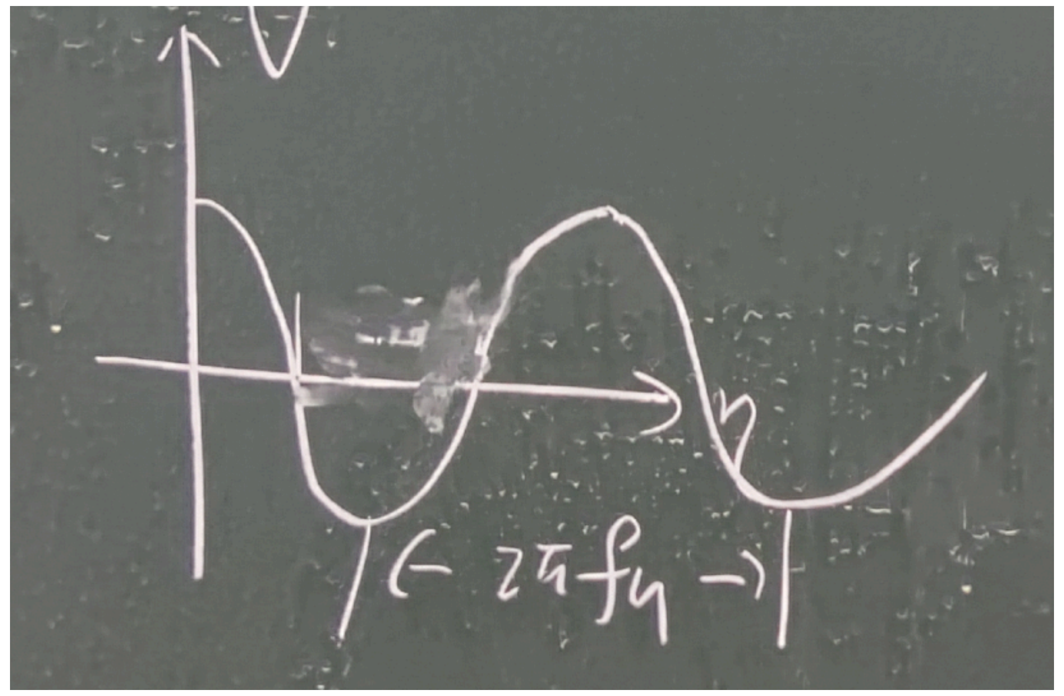

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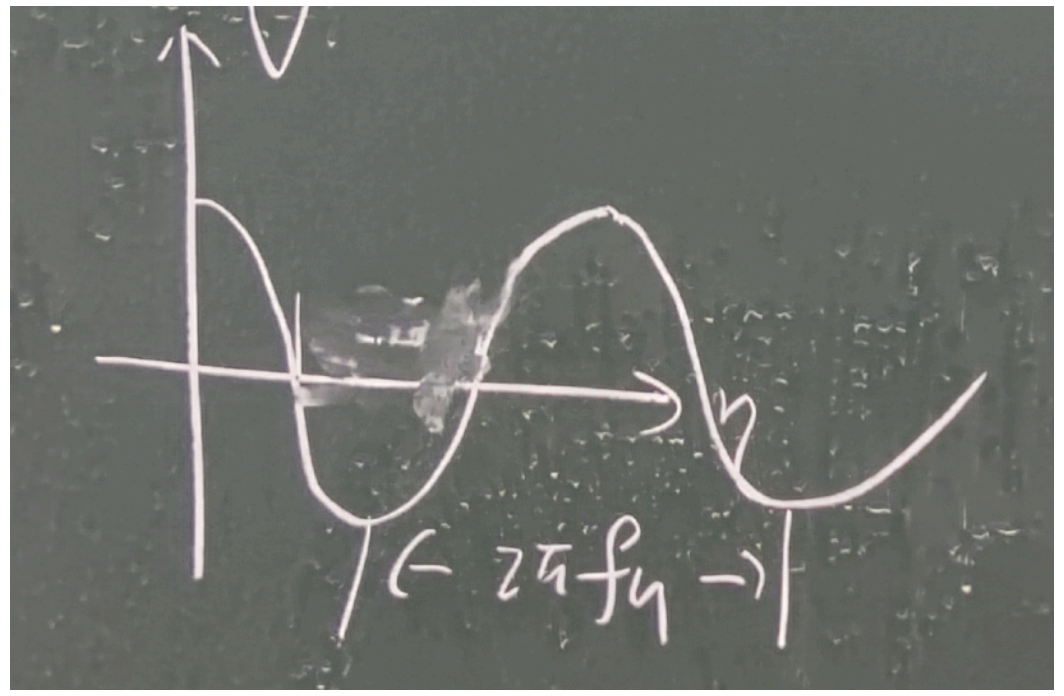

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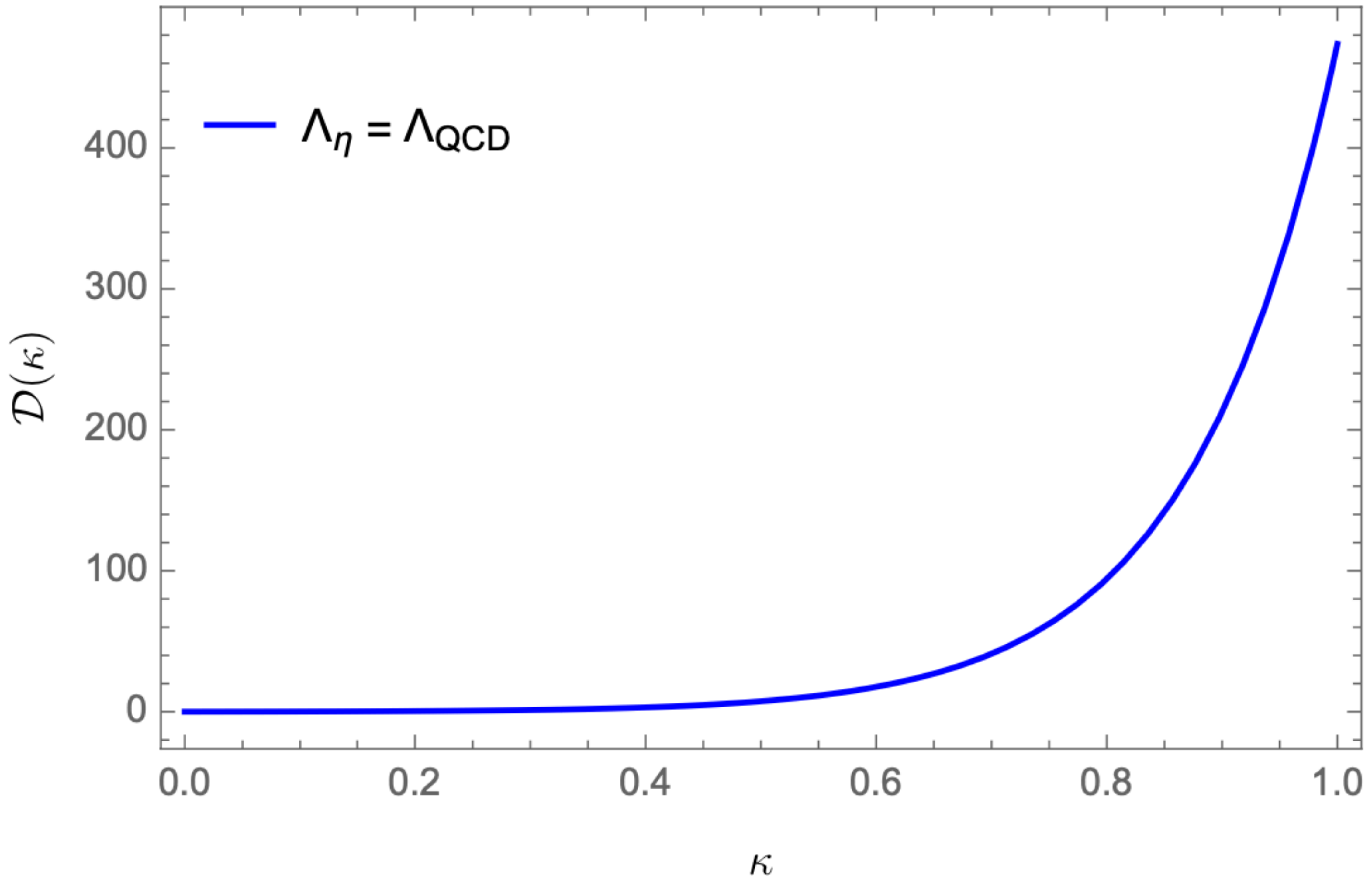

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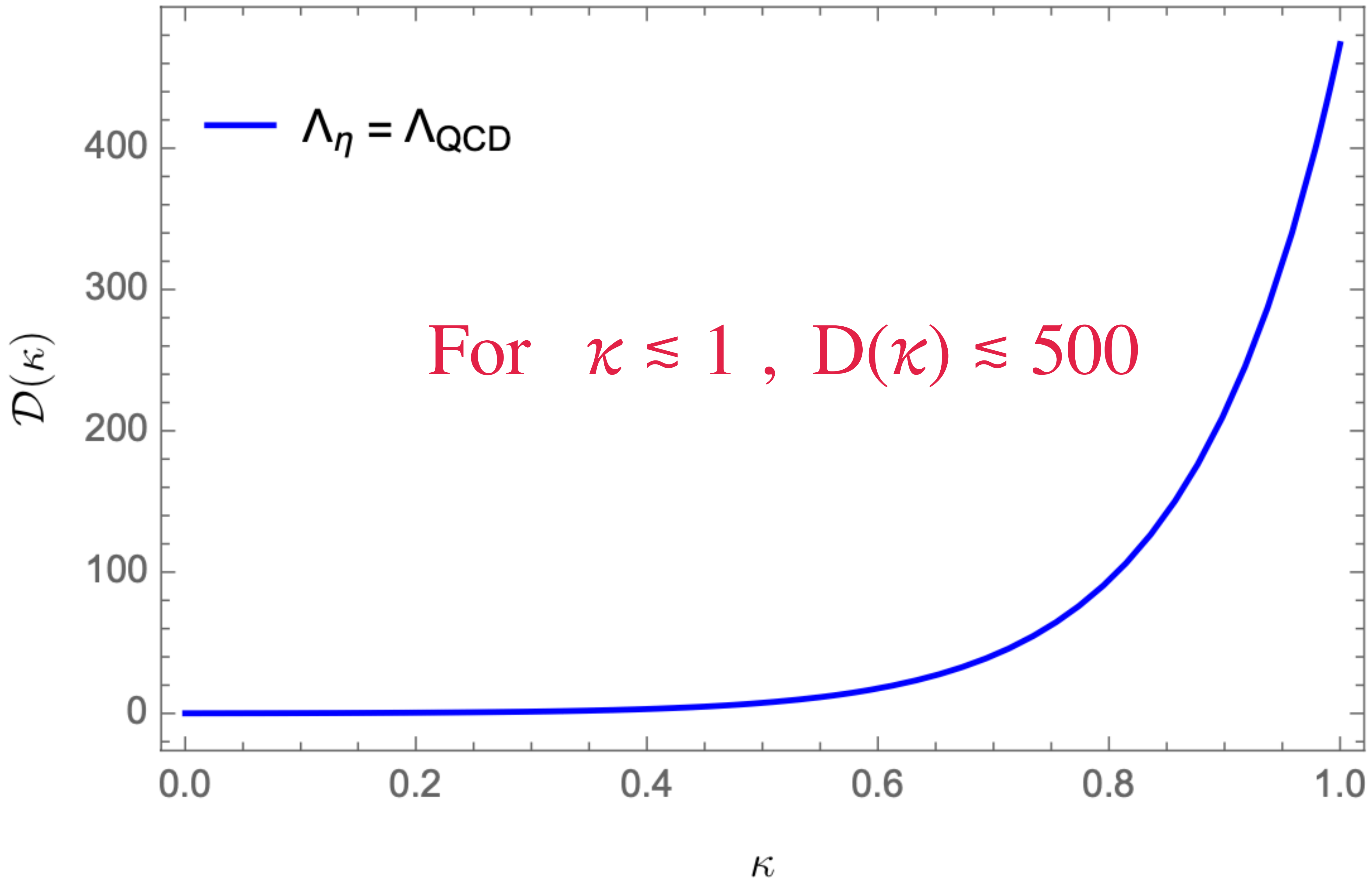

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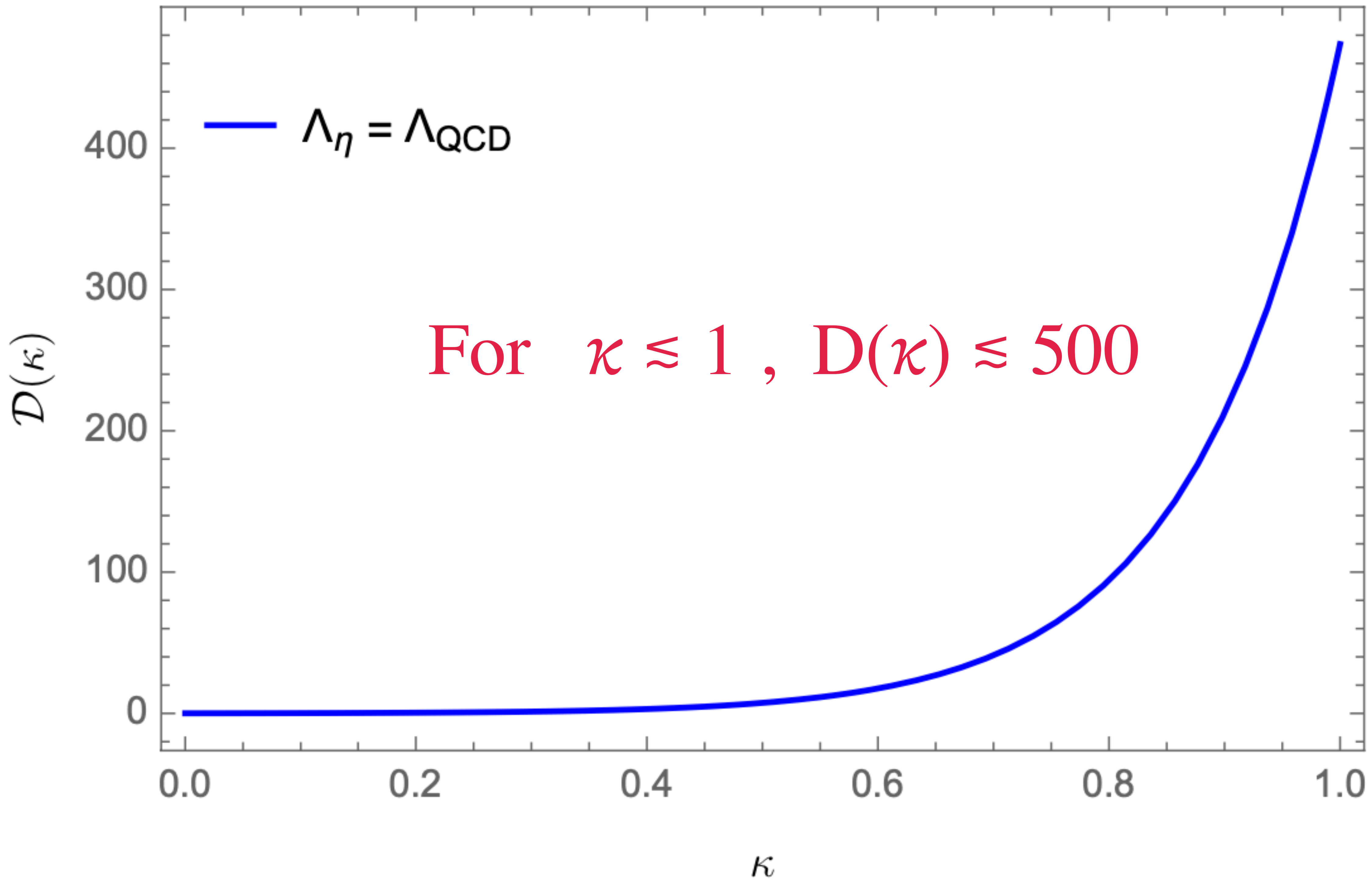

For $\kappa \lesssim 1$, $D(\kappa) \lesssim 500$

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Domain Wall constraint is less strict than the isocurvature bound for QCD axion if the backreaction constraint is satisfied

Experimental Constraints: Isocurvature bound

- For our mechanism, isocurvature perturbation magnitude is:

$$
\begin{aligned}
A_{\text{iso}}\left(k_*, k_{\min}\right) &= \frac{2^{2\nu}}{2\pi}\Gamma^2(\nu)\left(\frac{1}{\kappa}\right)^{\nu-3/2} e^{(N-N_*)(2\nu-3)}\left(\frac{H_{\text{inf}}}{\pi f_\eta \theta_{\text{i}}}\right)^2 \\
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upper bound on $\beta_{\rm iso}$	pivot scale k_*/Mpc^{-1}	effective e-folds $N - N_*$
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With our back reaction constraint
For single field inflation:

$$0 < \kappa < 1,\ \text{we have}\ 0.19 < \left(\sqrt{\kappa} H_0/k_*\right)^{\nu-3/2} < 1.$$

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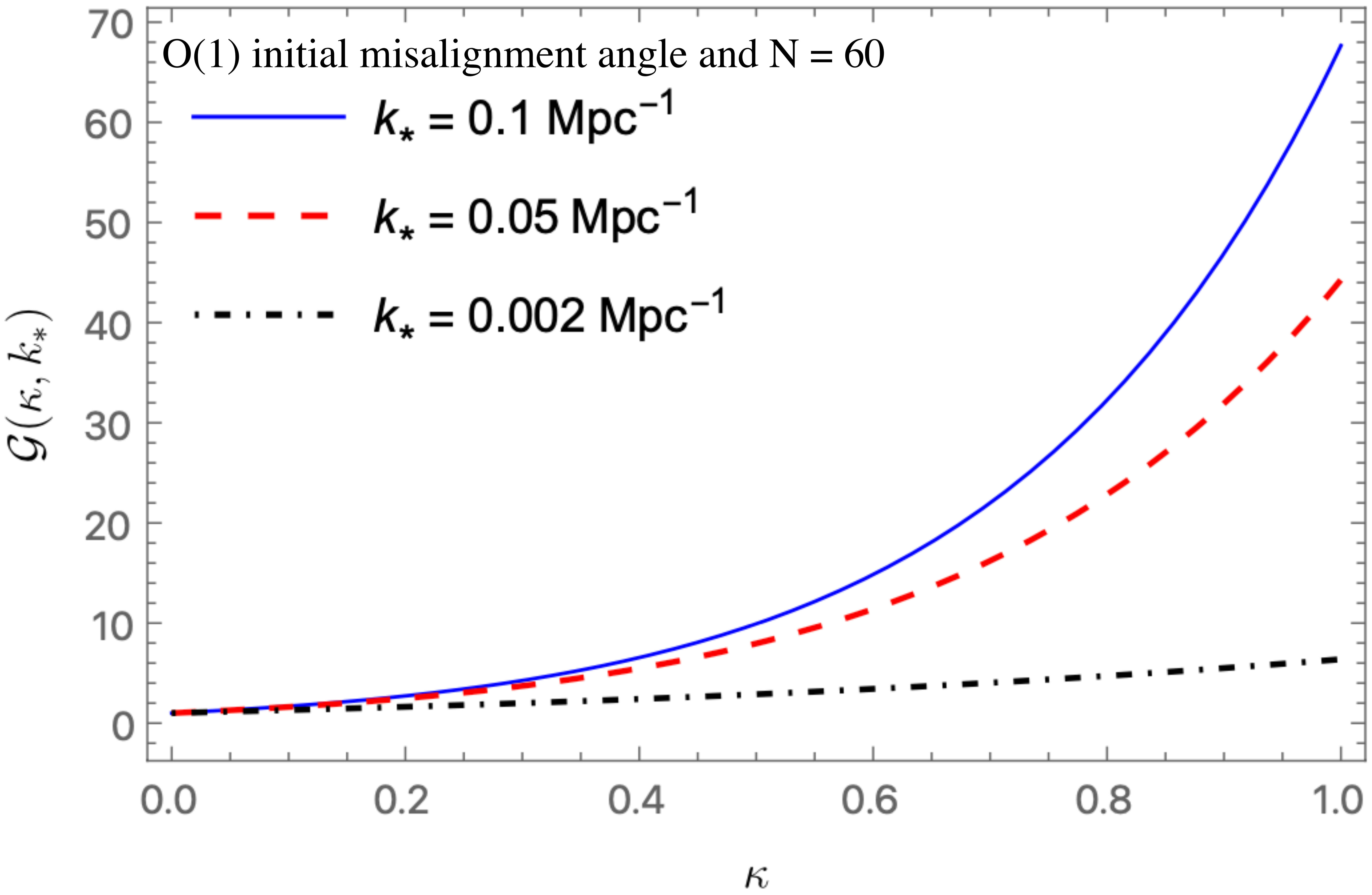

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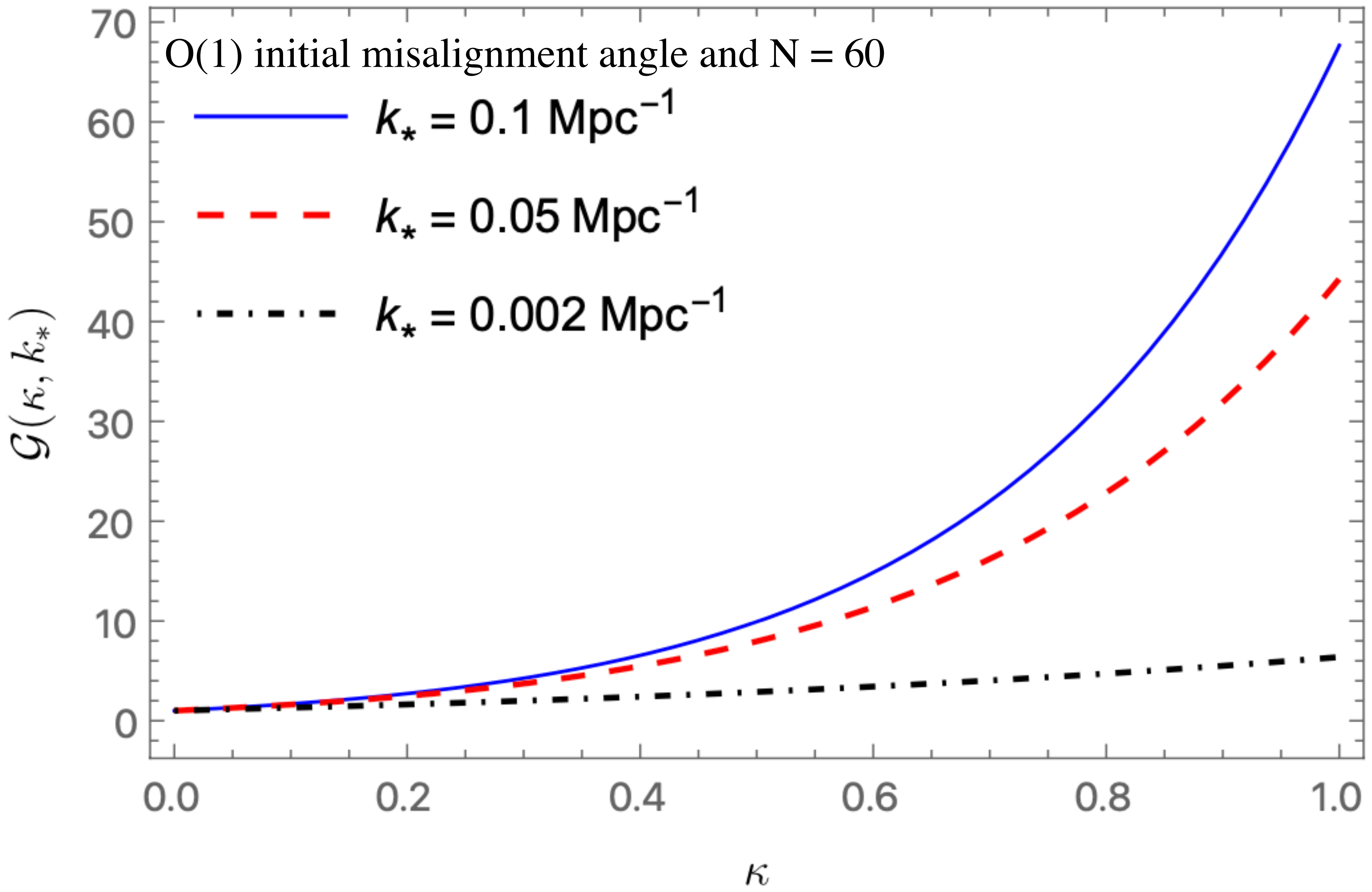

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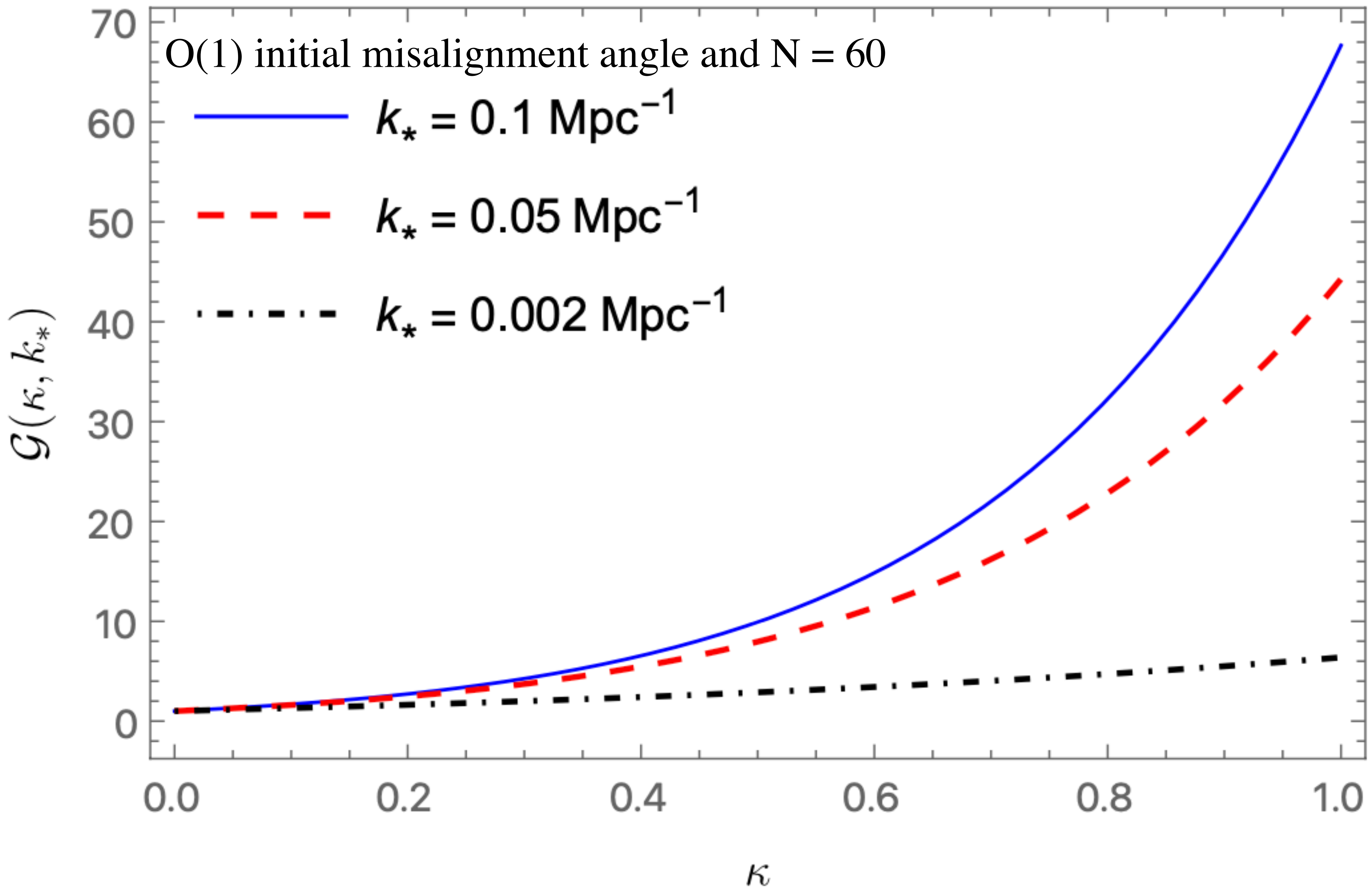

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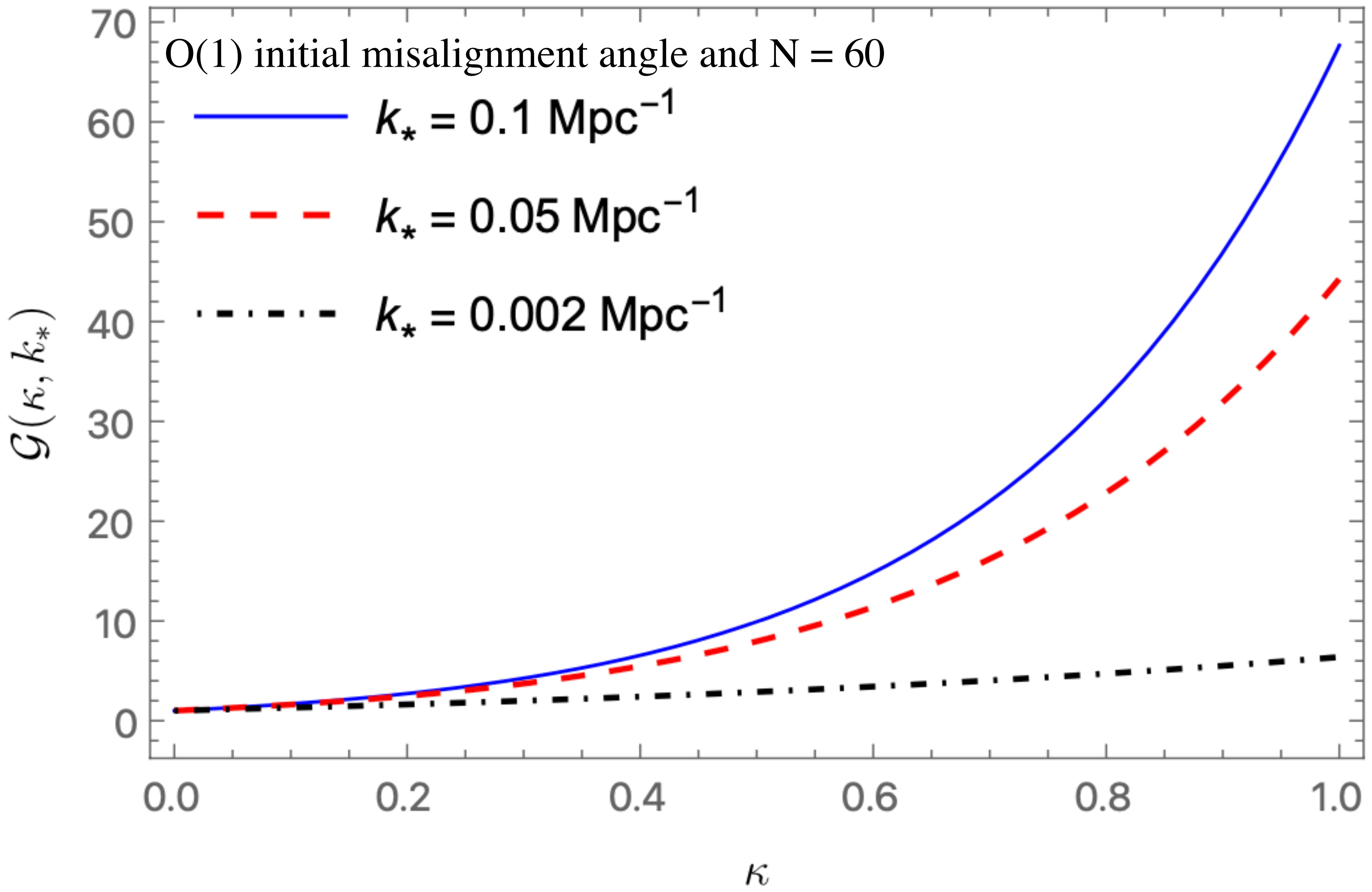

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For ALP with very small mass, which need a huge enhancement, recall that we have additional e^N enhancement allows us to achieve the right relic abundance

Experimental Constraints: Isocurvature bound

- numerical values of exponential enhancement with different κ :

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$$\Omega_\eta = \frac{g_{*0} g_{*\rm reh}^{-1/4}}{48\pi^3}\left(\frac{\pi^2}{90}\right)^{3/4} \frac{T_0^3 H_{\rm inf}^{5/2}}{M_{\rm Pl}^{7/2} H_0^2} \mathcal{F}(\kappa)\, e^{N(2\nu-3)}$$

$$g_{*0} = 2$$
$$g_{*\rm reh} = 106.75$$

κ	$e^{N(2\nu-2)}$ (enhancement to ultralight DM relic abundance)	$e^{N(2\nu-3)}$ (enhancement to inflationary quantum fluctuation)	$e^{(N-N_*)(2\nu-3)}$ (enhancement to axion isocurvature perturbation)
0.1	6.0×10^{27}	52	1.5
0.2	2.9×10^{29}	2.5×10^3	2.2
0.5	2.0×10^{34}	1.8×10^8	6.9
1.0	6.9×10^{41}	6.0×10^{15}	40

N = 60 and N − N* = 6.1 are fixed