Gravitational relics from topological defects

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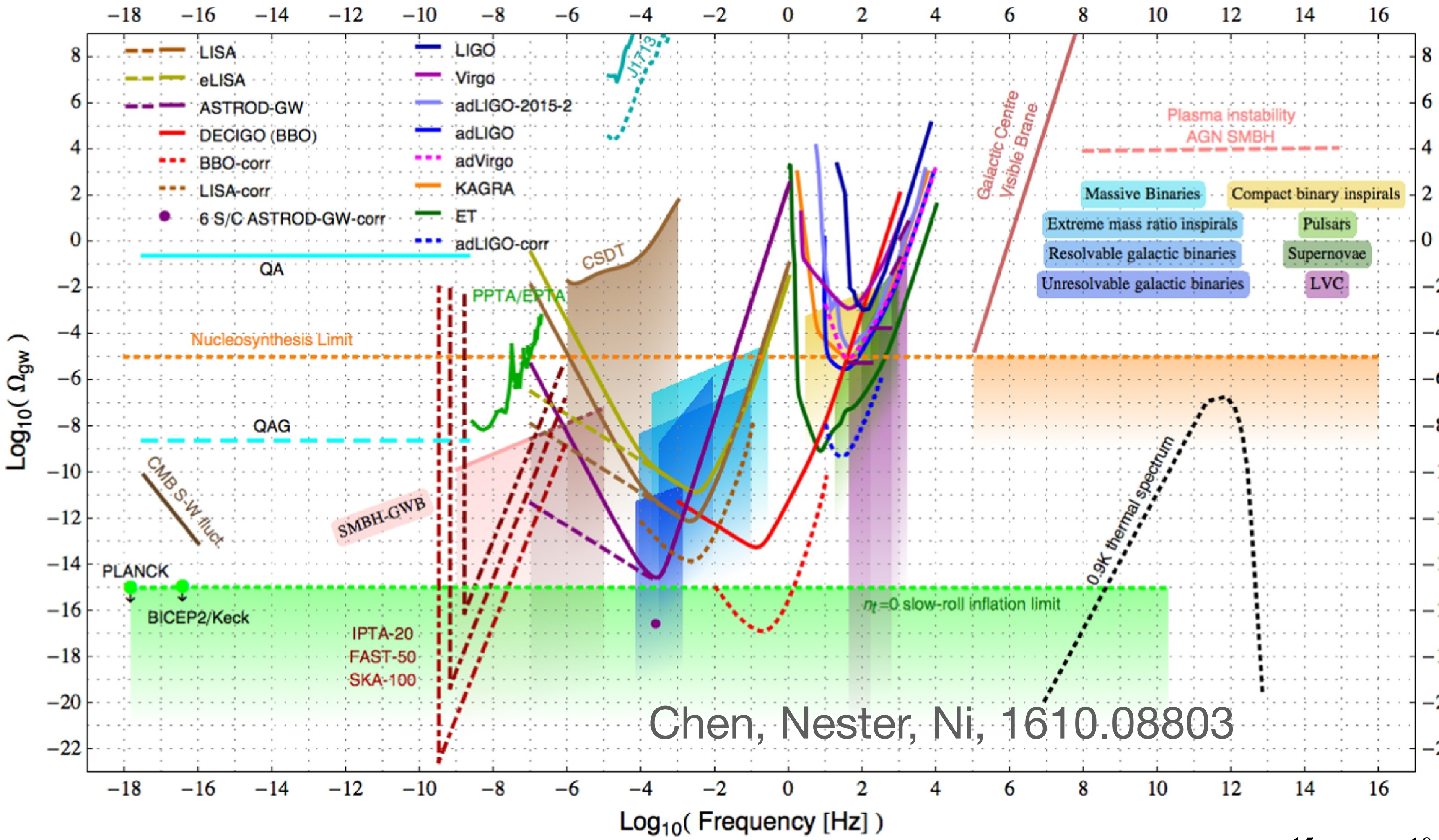
NK, Nakayama, 2212.13573, 2306.17390
NK, Lee, Murai, Takahashi, Yin, 2306.17146
NK, Lee, Takahashi, Yin, 2311.14590
+ ongoing work

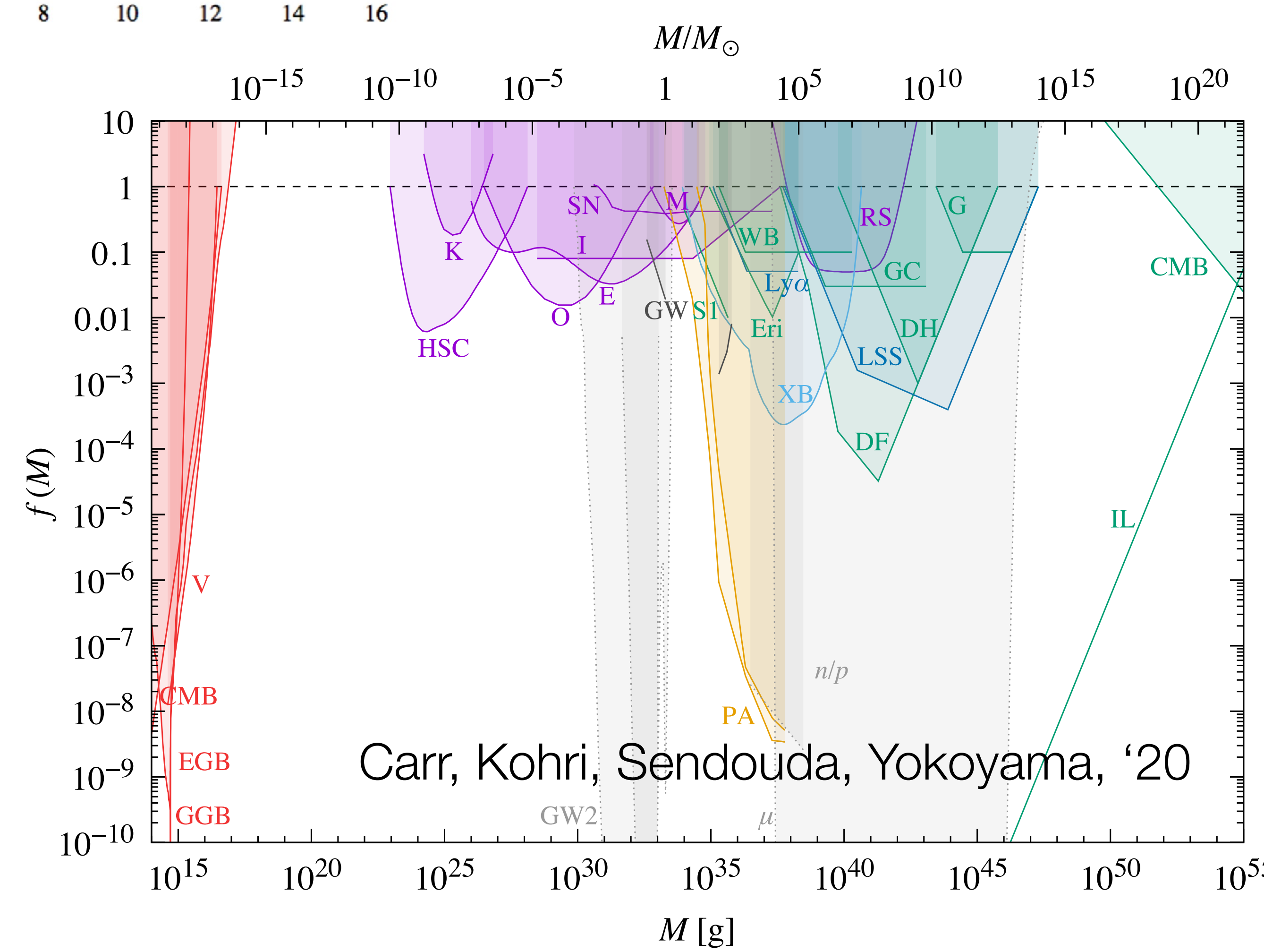
International Joint Workshop on Standard Model and Beyond, Sydney, 2024/12/9-13



<— gravitational wave

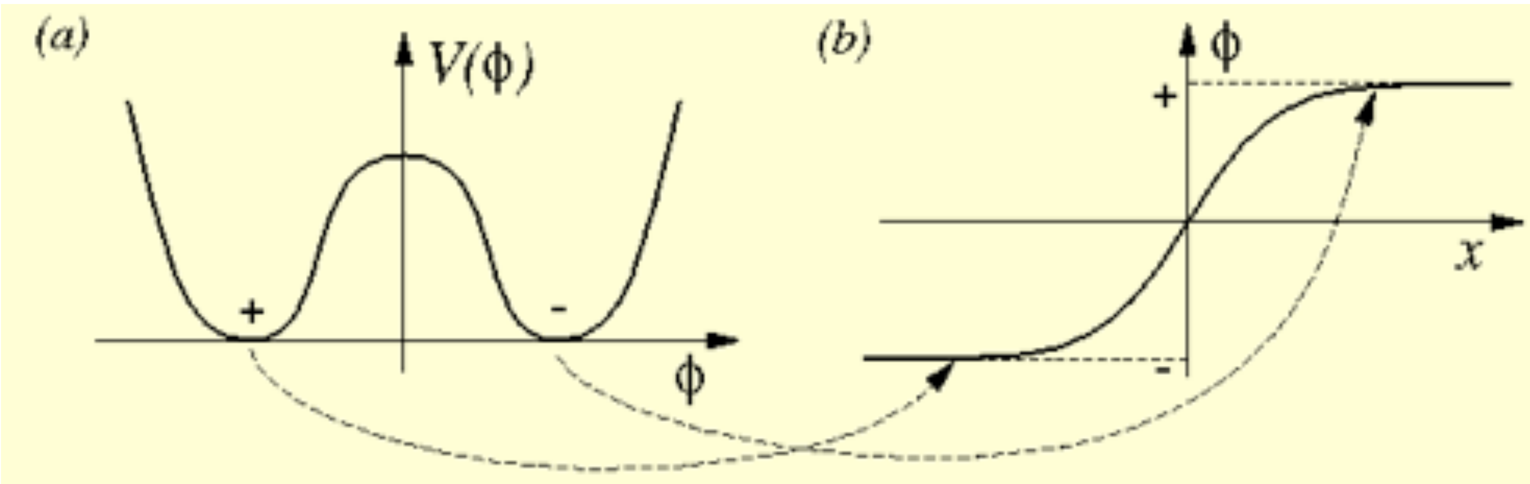
Primordial black hole —>

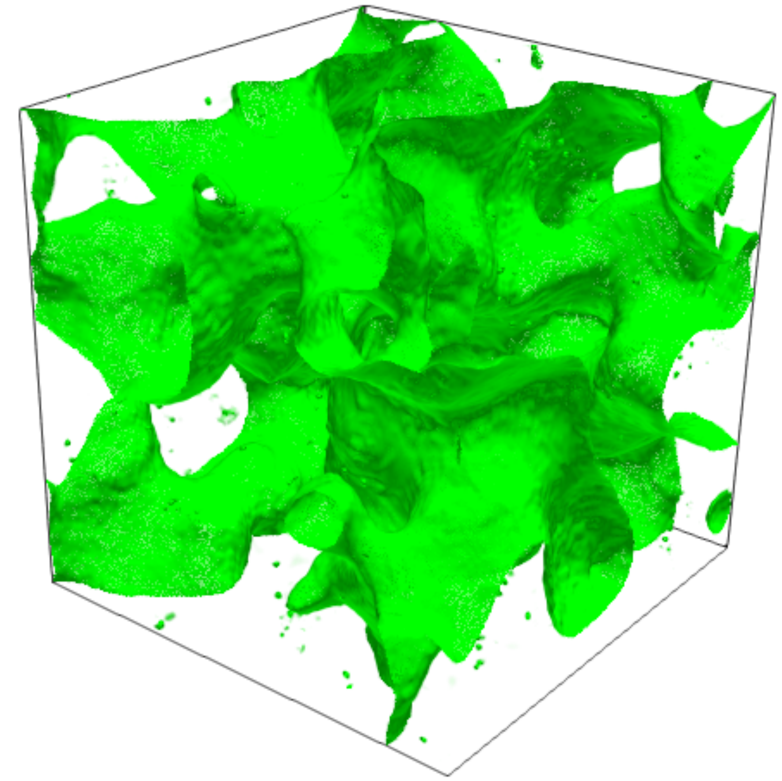
Indirect search for BSM through “gravitational relics”



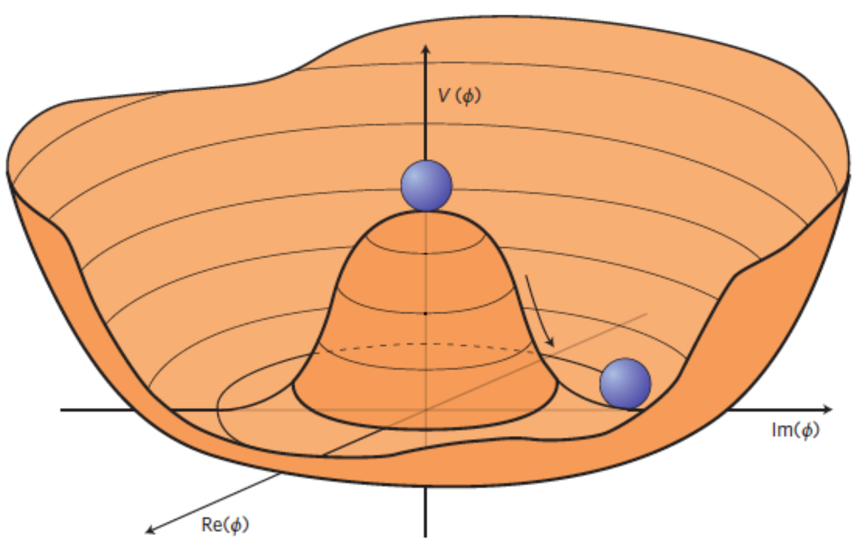
Gravitational waves from cosmic strings / domain walls

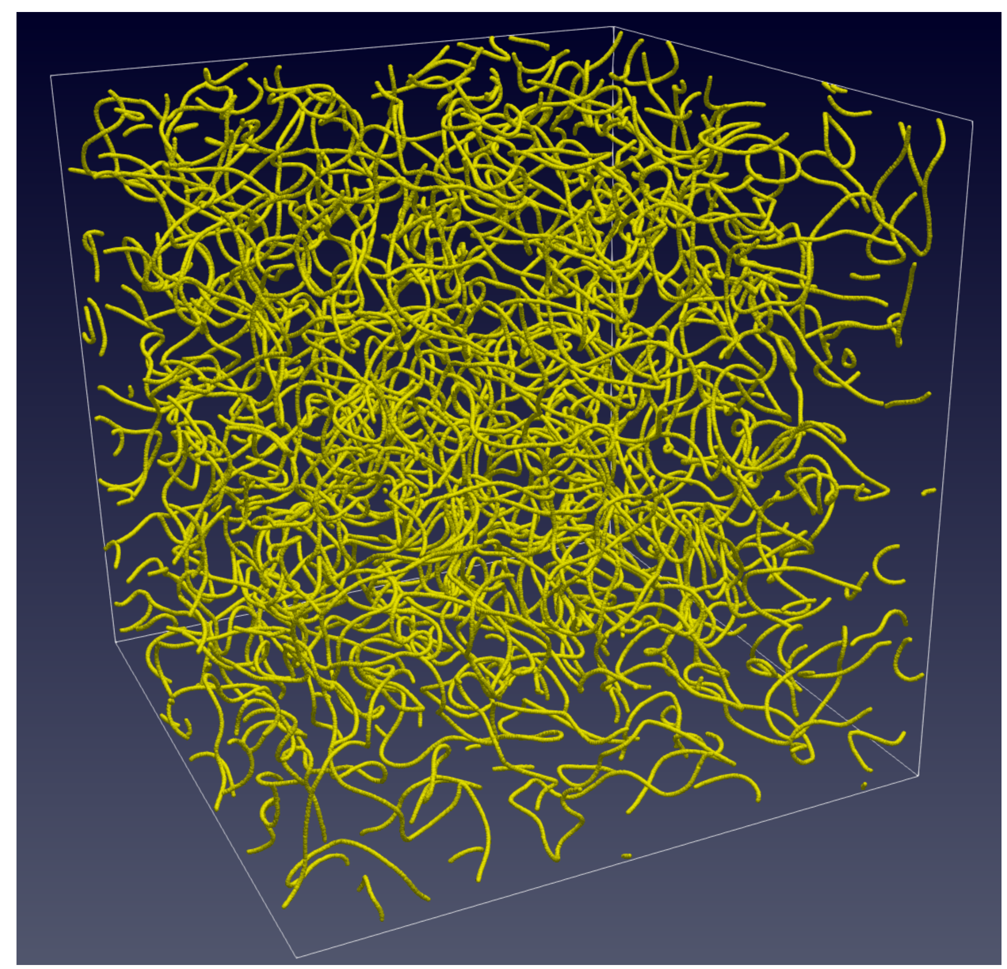
Topological defects in cosmology





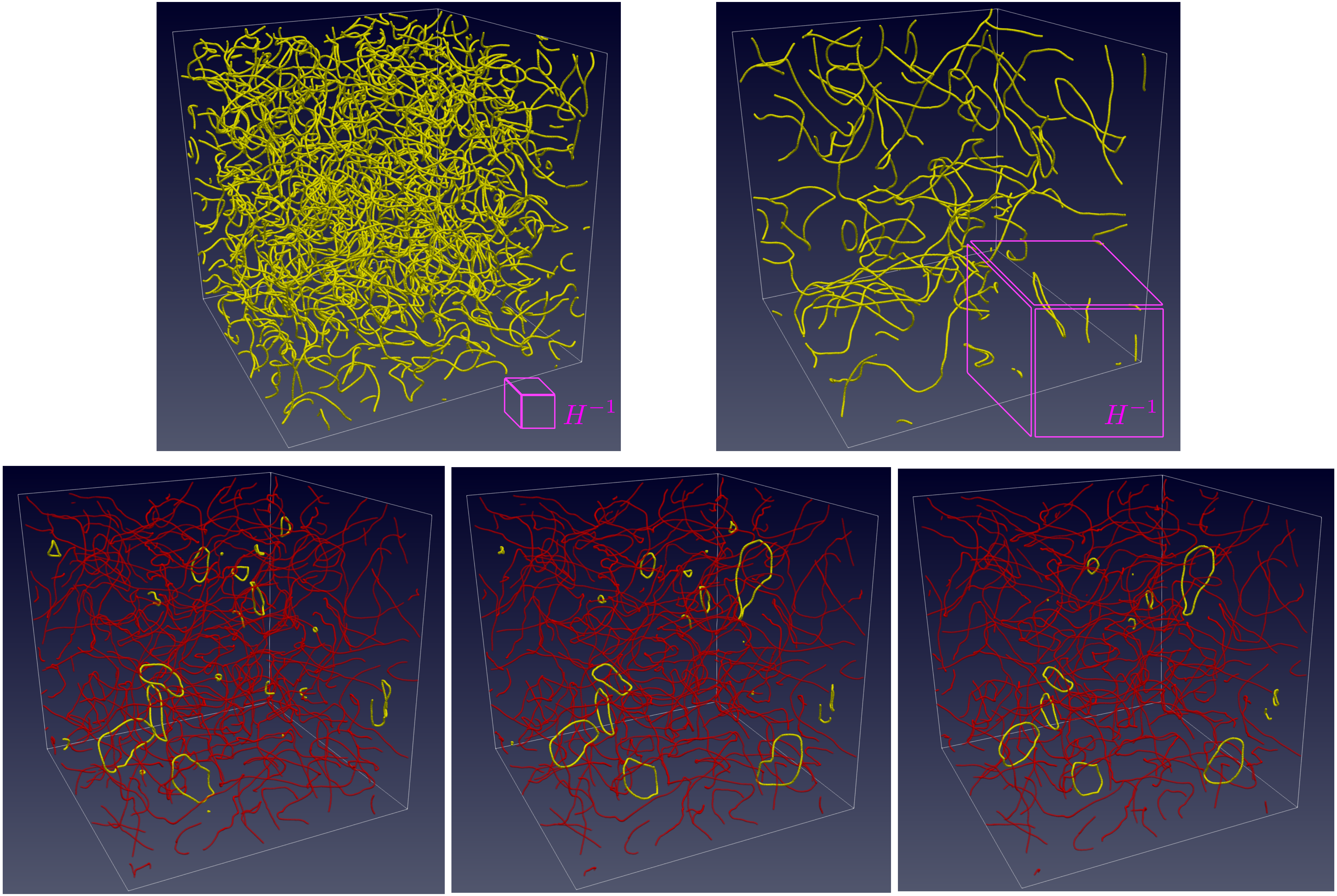
Domain wall





Cosmic string

Scaling law : O(1) (long) strings / Hubble volume



GW emission from cosmic strings

Credit: Daniel Dominguez/CERN

Quadrupole formula for GW emission: $\dot{E}_{\rm GW} \sim G(\dddot{D})^2$

quadrupole moment: $D \sim ML^2 \sim \mu L^3, \quad \dddot{D} \sim \omega^3 D \sim L^{-3} D$

μ : string tension, L : typical loop size ~ (typical oscillation frequency)$^{-1}$

GW emission rate: $\dot{E}_{\rm GW} \sim G\mu^2 \equiv \Gamma_{\rm GW} G\mu^2$

$$G\mu \sim (v/M_P)^2 \sim 10^{-7}(v/10^{15}{\rm GeV})^2$$

Abelian-Higgs model

$$\mathcal{L} = (\mathcal{D}_\mu \Phi)^* \mathcal{D}^\mu \Phi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - V(\Phi), \;\; V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$$

$$(\mathcal{D}_\mu = \partial_\mu - ieA_\mu, \;\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu)$$

Peccei-Quinn model

$$\mathcal{L} = (\partial_\mu \Phi)^* \partial^\mu \Phi - V(\Phi), \;\; V(\Phi) = \frac{\lambda}{4}(|\Phi|^2 - v^2)^2$$

Lattice simulation (with 4096^3 grids)

$$\Phi'' + 2\mathcal{H}\Phi' - D_i D_i \Phi + a^2 \frac{\partial V}{\partial \Phi^*} = 0,$$

$$F'_{0i} + \partial_j F_{ij} - 2ea^2 \, \mathrm{Im}(\Phi^* D_i \Phi) = 0,$$

$$\partial_i F_{0i} - 2ea^2 \, \mathrm{Im}(\Phi^* \Phi') = 0$$

AOBA supercomputing system
(SX-Aurora TUBASA) in Tohoku U.

4,032 Vector Engine

16 cores, 256 vector length, 96GB / 1VE

Light Dark photon DM scenario

Long, Wang 1901.03312
NK, Nakayama 2212.13573

- "Light" dark photons can be produced by cosmic strings

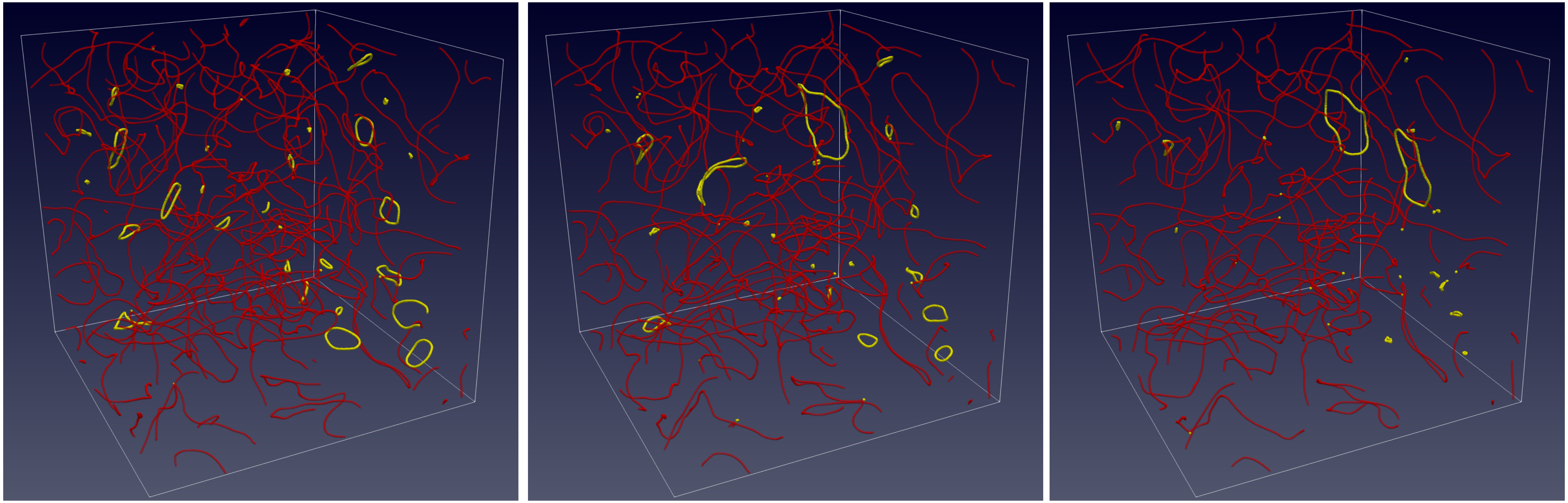
  e = 0 limit corresponds to the massless NG boson emission (global string case)

- Vector boson production becomes inefficient for $\ell_{\rm loop} \gtrsim m_A^{-1}$

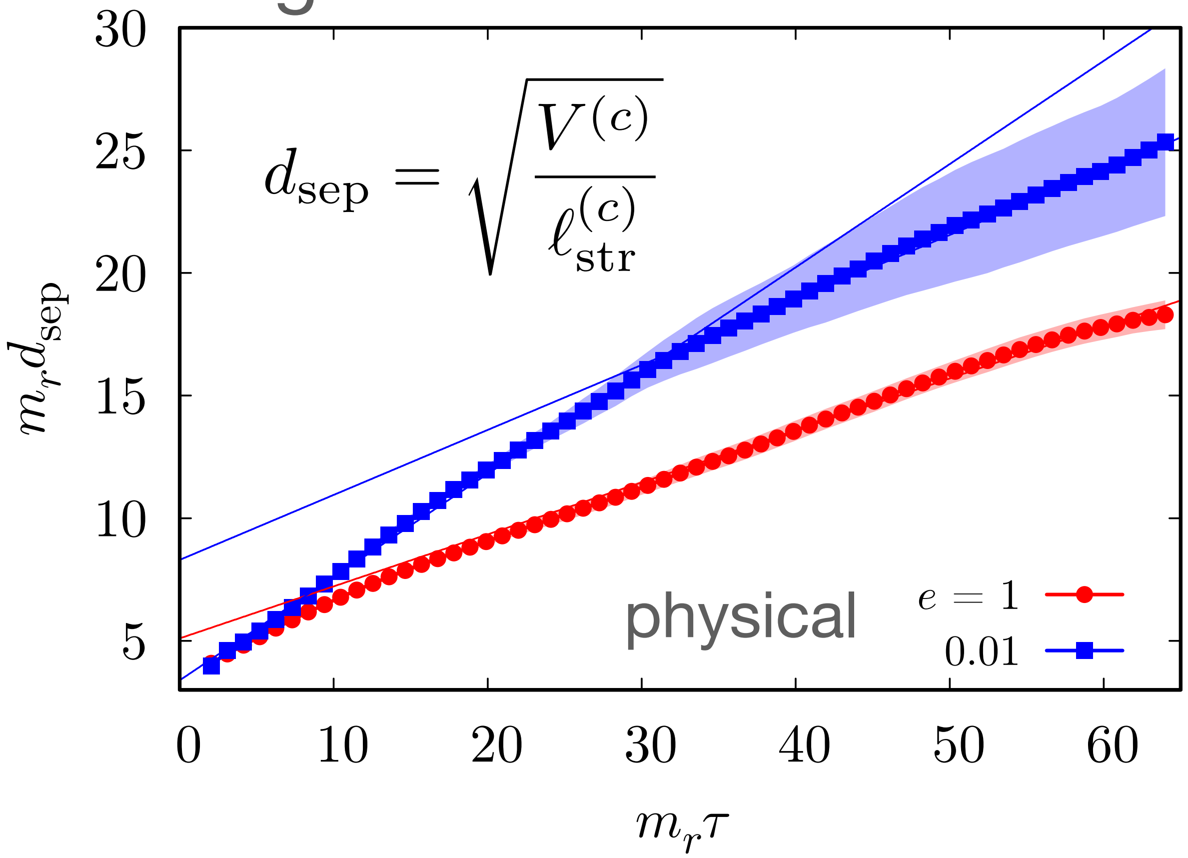
- After that, string evolves like "local" string

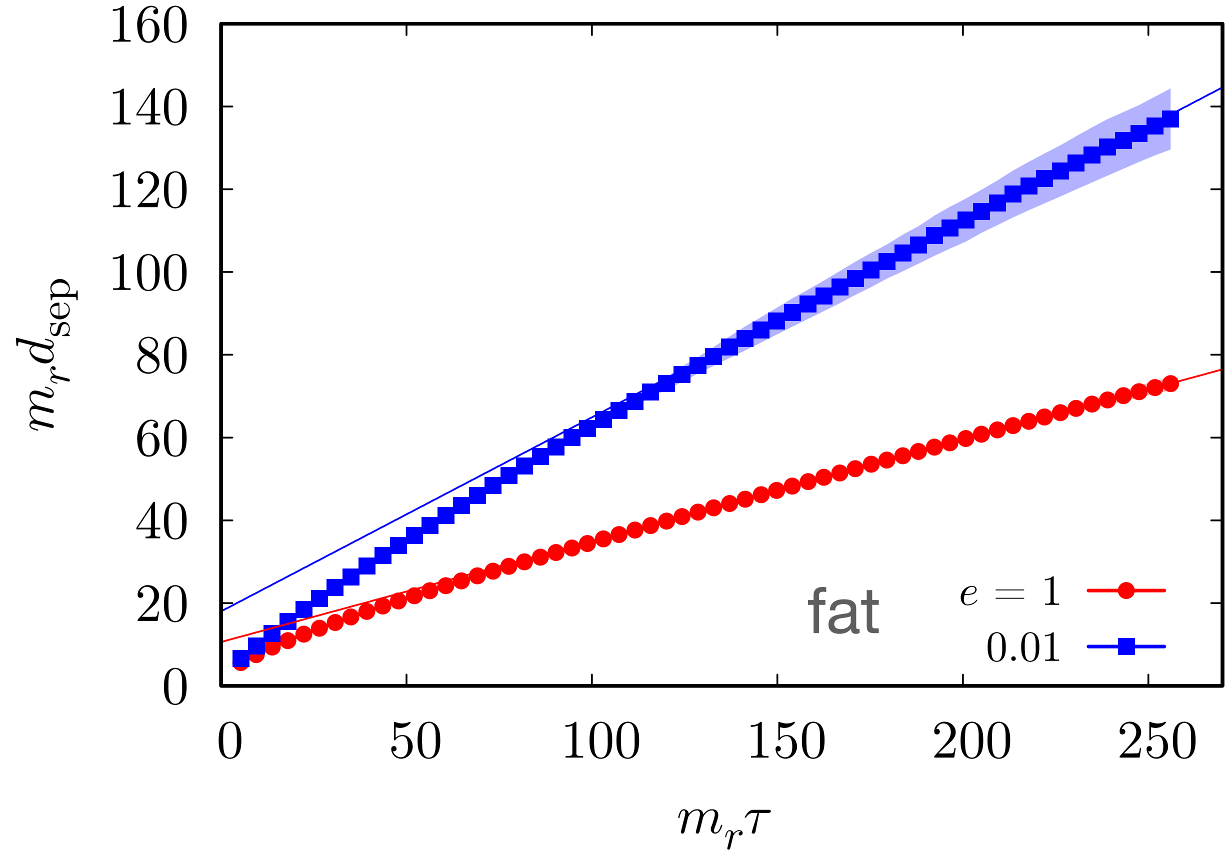
  network loses energy only through the GW emission (Nambu-Goto limit)

(near) global string —> local (gauge) string



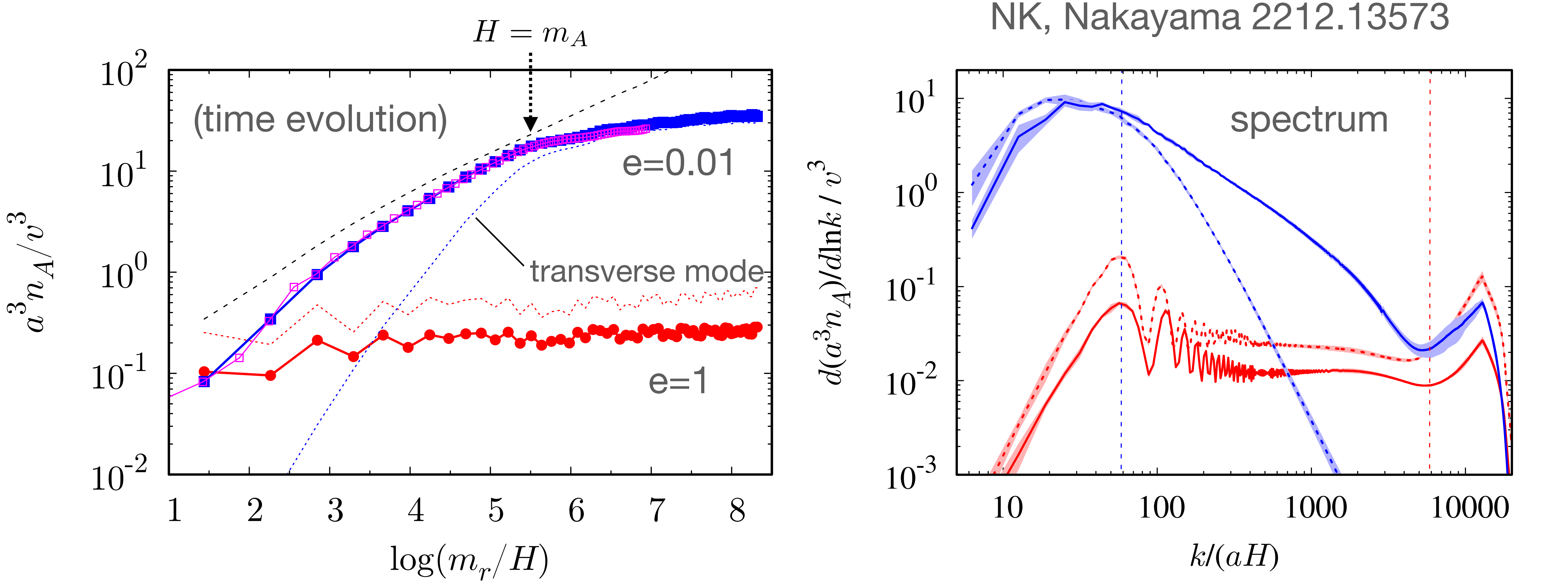
Mean separation NK, Nakayama 2212.13573





	grid	$m_r L$	$m_r \Delta x$	e	$m_r \tau$	a	b
physical string	4096^3	64	1/64	1	33.6 - 64.0	0.21 ± 0.0043	5.1 ± 0.21
physical string	4096^3	64	1/64	0.01	2.05 - 32.5	0.42 ± 0.0038	3.4 ± 0.073
physical string	4096^3	64	1/64	0.01	33.6 - 64.0	0.27 ± 0.019	8.4 ± 0.96
fat string	1024^3	512	1/2	1	133 - 256	0.24 ± 0.0013	11 ± 0.25
fat string	1024^3	512	1/2	0.01	133 - 256	0.47 ± 0.0081	18 ± 1.6

Table 1: Simulation setup and linear fitting parameters of the mean string separation in terms of the conformal time, defined by $m_r d_{\rm sep} = a m_r \tau + b$.



- Dark photon DM relic abundance:

$$\Omega_A h^2 = \frac{m_A (n_{A,0}/s_0) h^2}{\rho_{\mathrm{cr},0}/s_0} \simeq 0.091 \left(\frac{\xi}{12}\right)\left(\frac{m_A}{10^{-13}\,\mathrm{eV}}\right)^{1/2}\left(\frac{v}{10^{14}\,\mathrm{GeV}}\right)^2$$

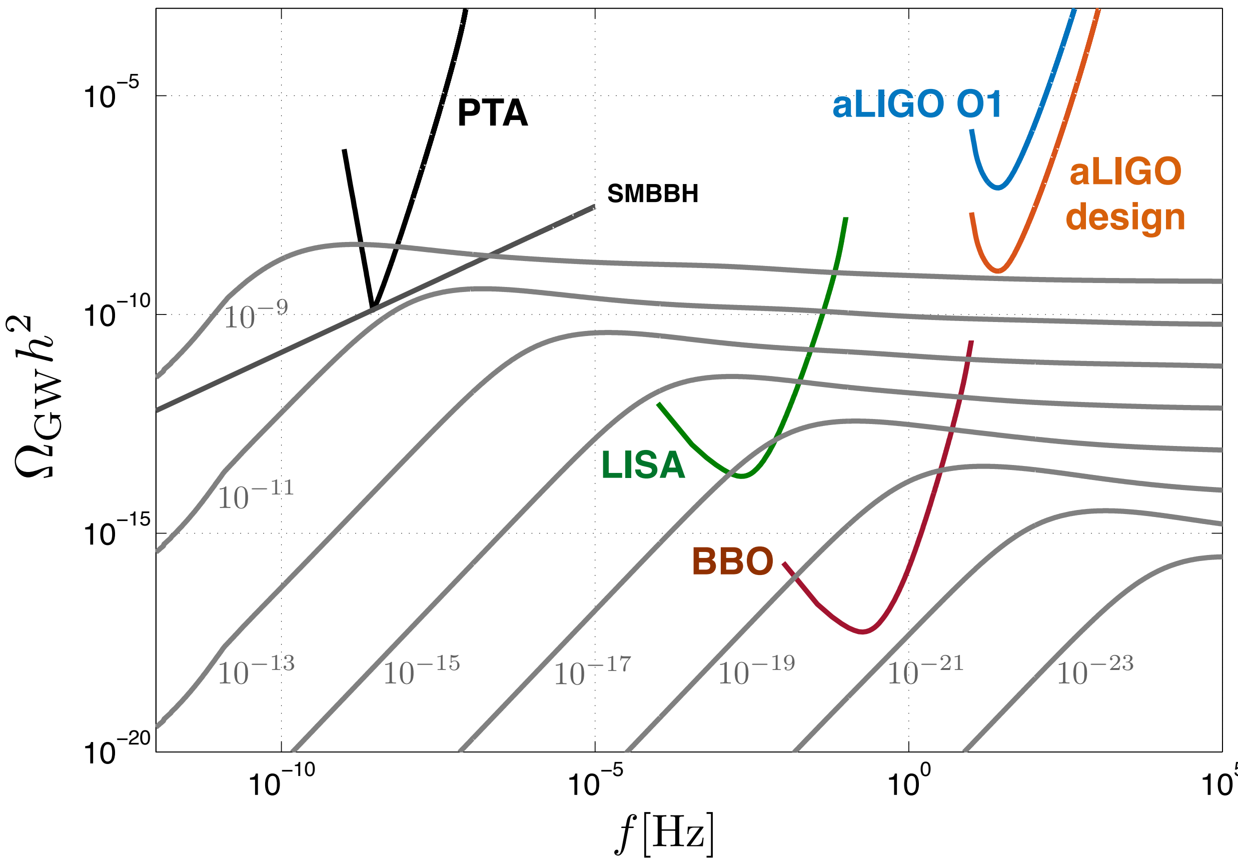
see also Long, Wang 1901.03312

$\xi = \mathrm{const}$ Hindmarsh et al, 1908.03522, Hindmarsh et al, 2102.07723

$$\xi = 0.15 \log\left(\frac{m_r}{m_A}\right) \simeq 12 + 0.15 \log\left[\left(\frac{m_r}{10^{14}\,\mathrm{GeV}}\right)\left(\frac{10^{-13}\,\mathrm{eV}}{m_A}\right)\right]$$

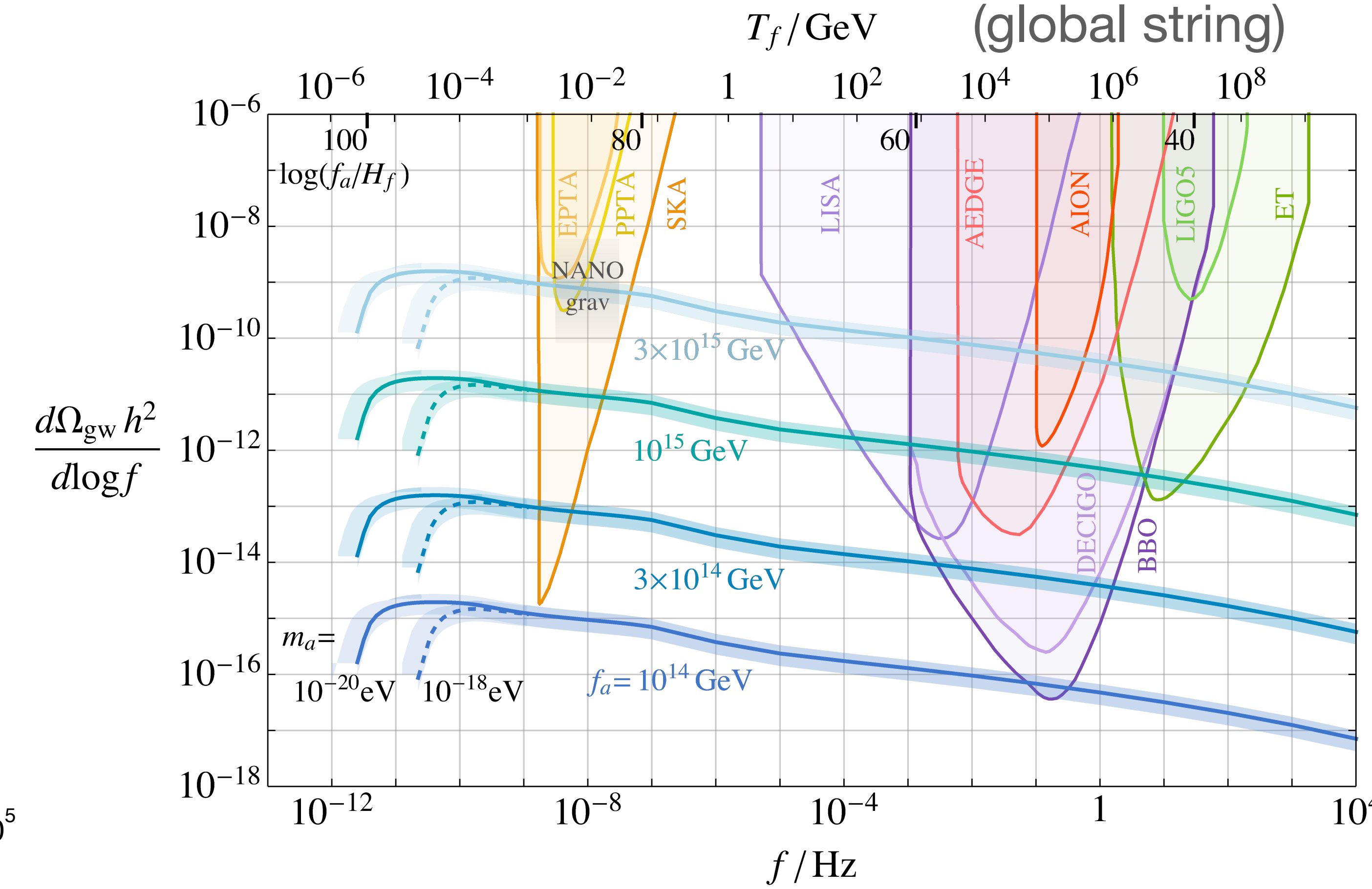
Gorghetto et al, 1806.04677
Kawasaki et al, 1806.05566
Buschmann et al, 2108.05368

(Abelian-Higgs string)

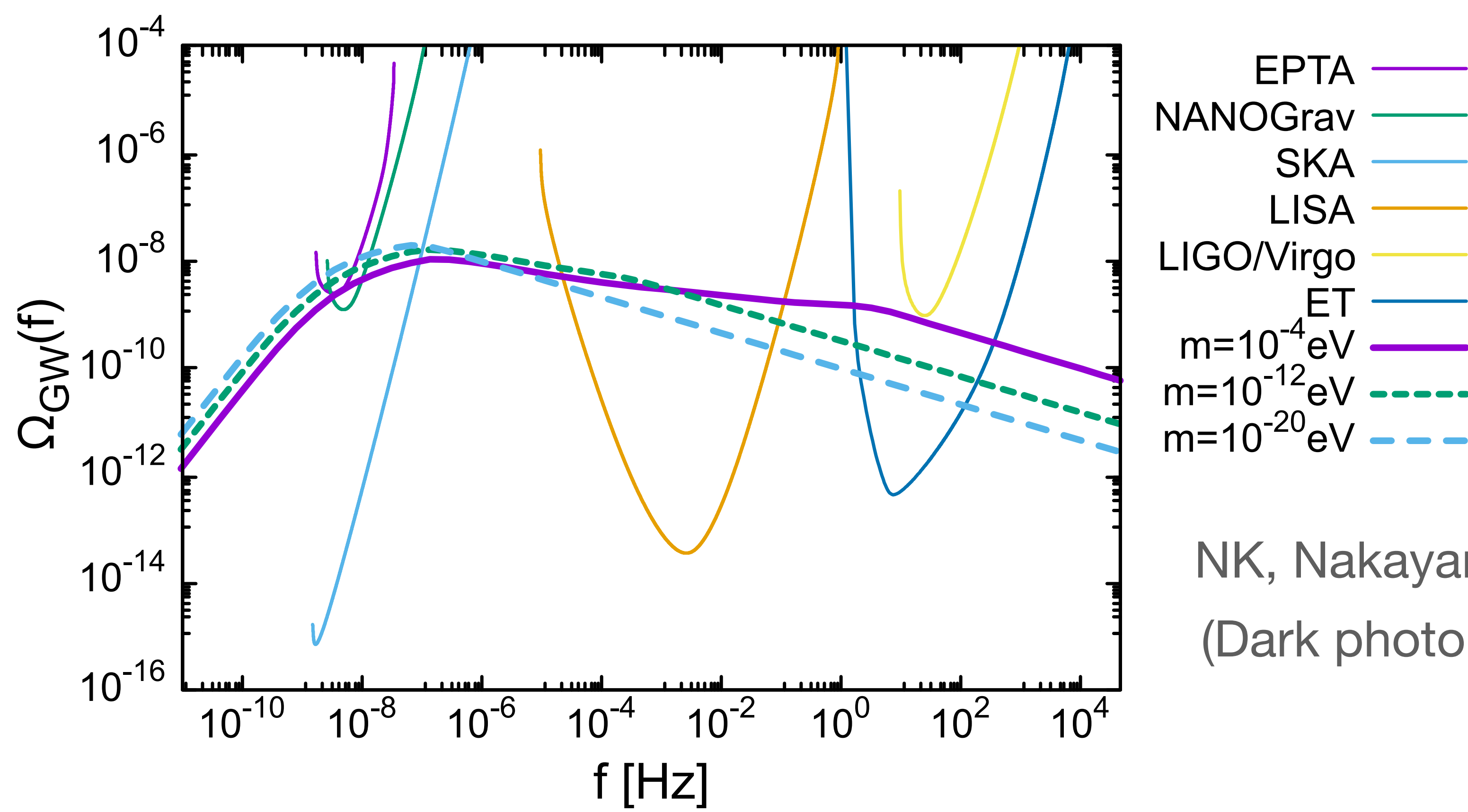


Blanco-Pillado+ 1709.02434

(global string)



Gorghetto et al, 2101.11007

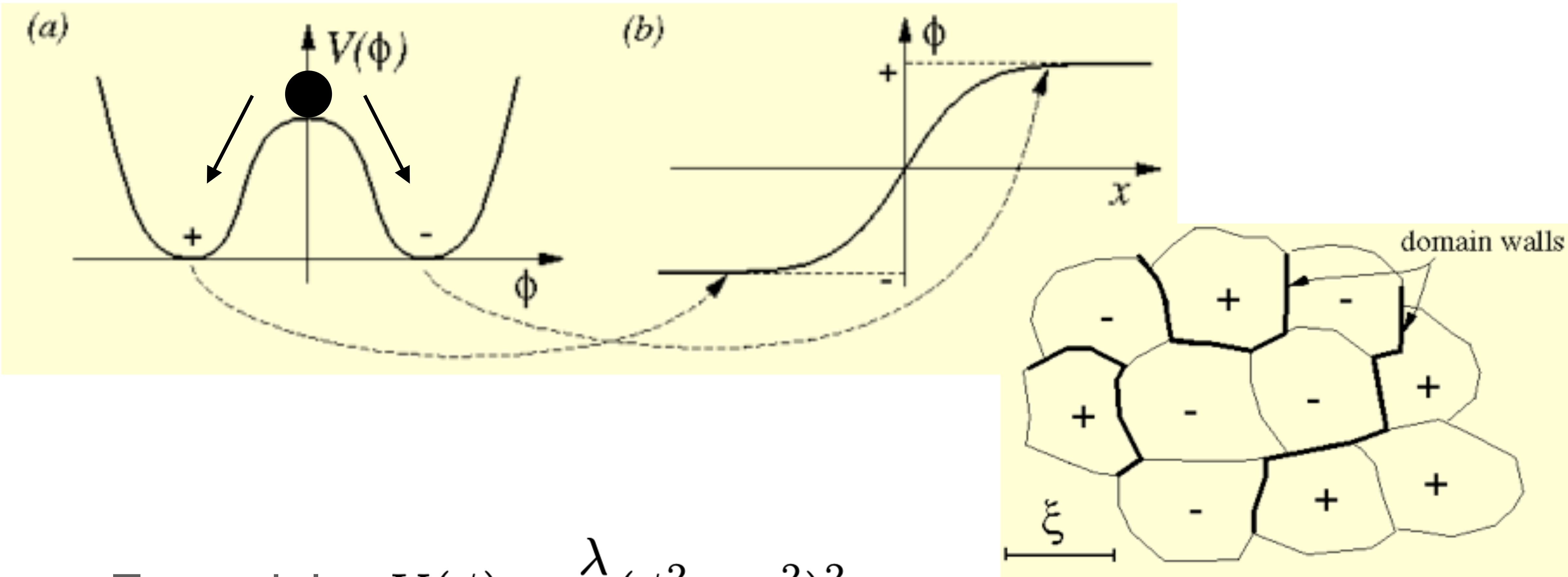


NK, Nakayama 2306.17390

(Dark photon DM scenario)

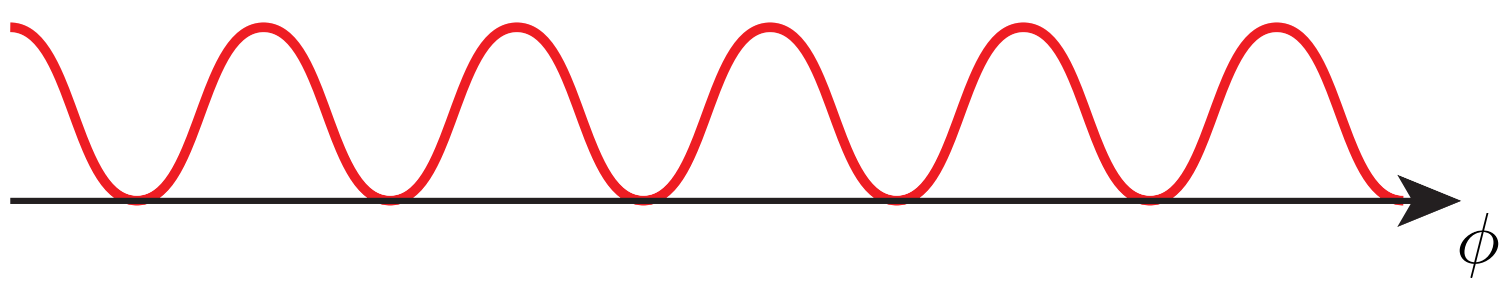
Domain walls

spontaneous breaking of discrete symmetry

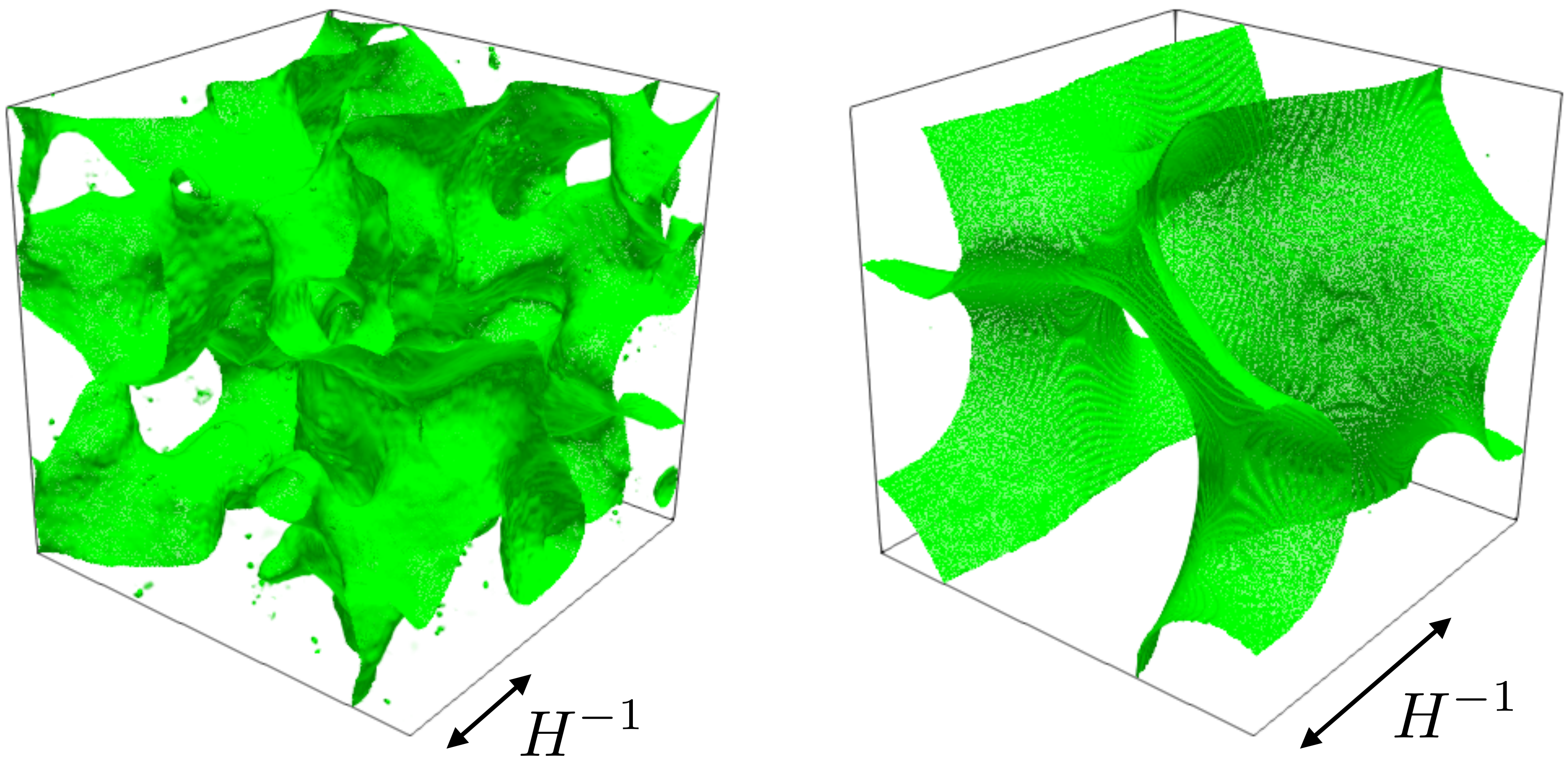


Z_2 model : $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$

Axion model : $V(\phi) = \Lambda^4 \left[1 - \cos\left(\frac{N_{\rm DW}\phi}{f}\right)\right]$



Scaling law of domain walls in the expanding Universe



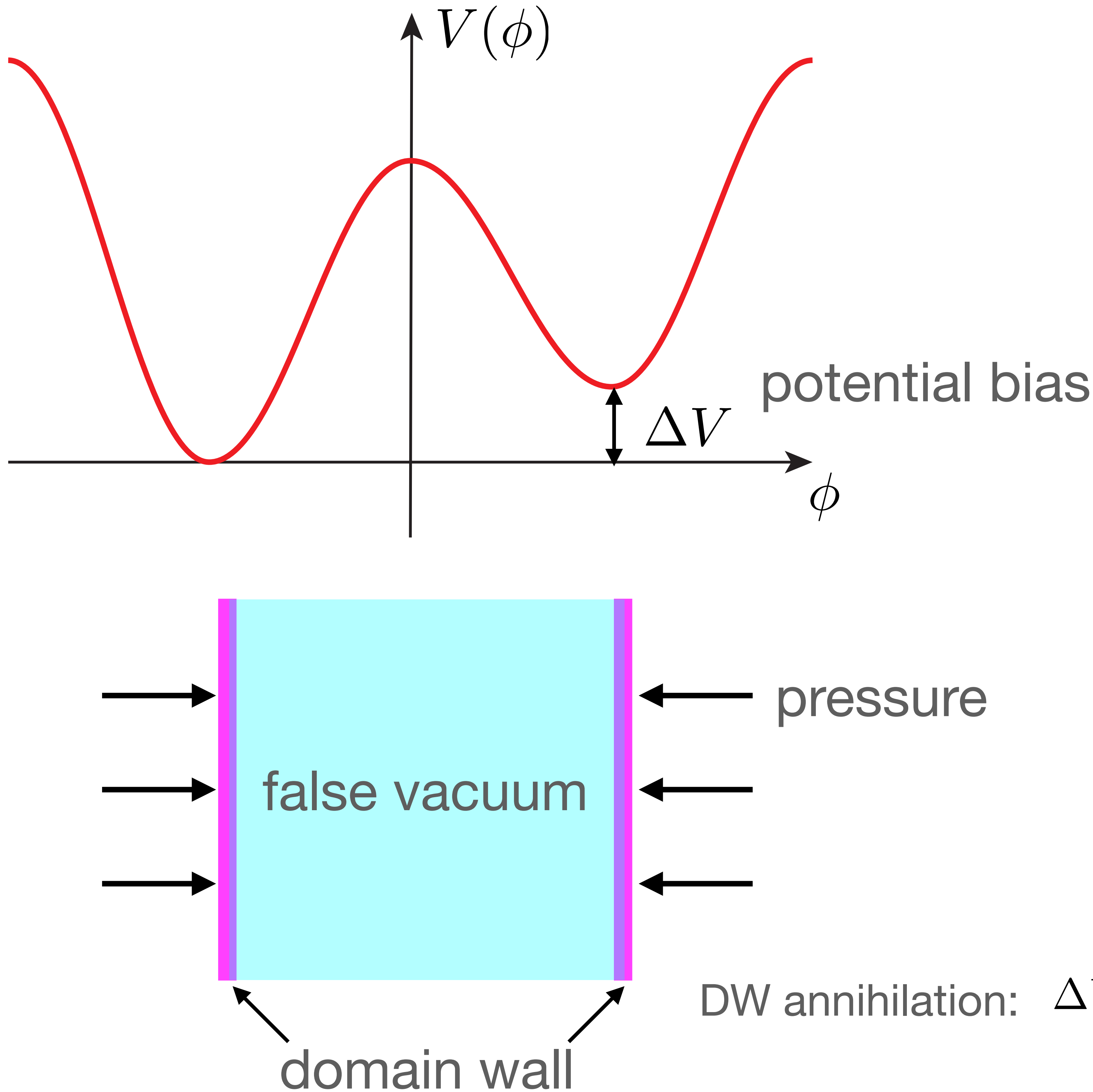
Energy density of domain wall

$$\rho_{\rm DW} = \sigma H \; (\propto H^{-2}/H^{-3} \propto a^{-2})$$

(in radiation dominated Universe)

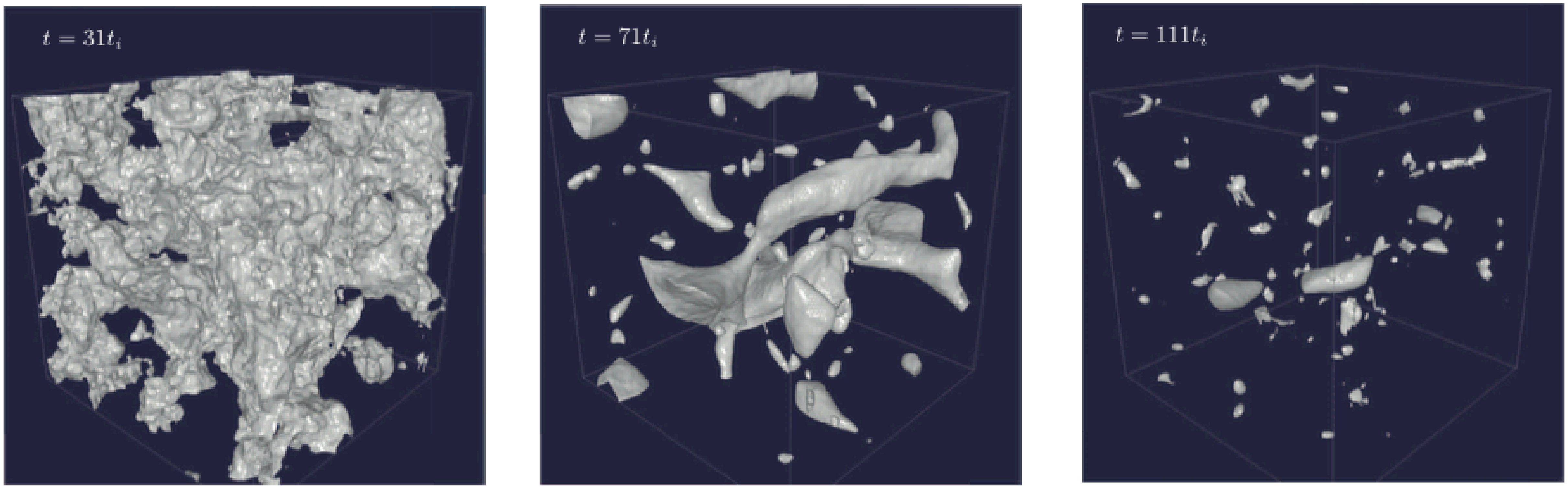
—> domain wall domination (domain wall problem)

Potential bias and domain wall annihilation

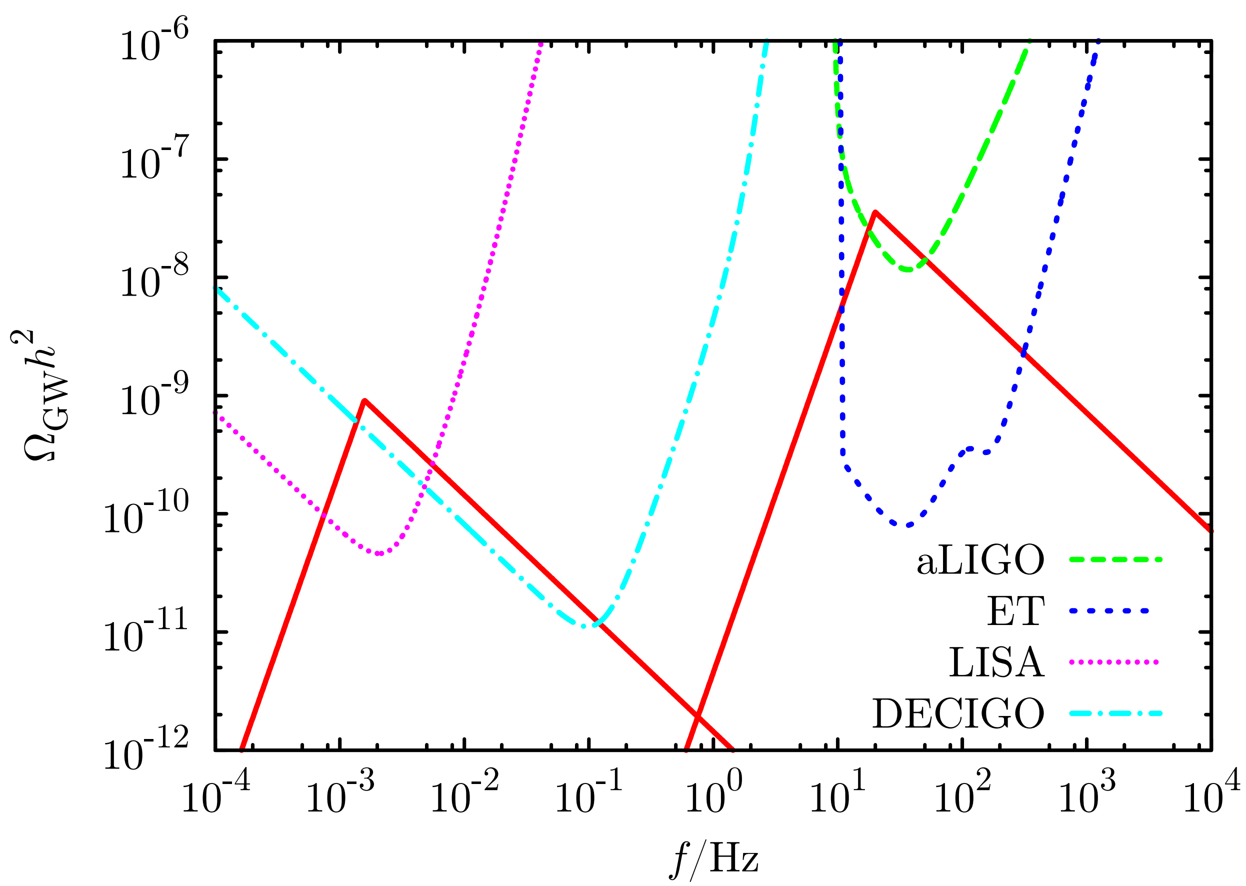


DW annihilation: $\Delta V = \sigma H(T_{\rm eq})$

Gravitational waves from domain wall decay



Hiramatsu, Kawasaki, Saikawa (2010)



DW tension: $\sigma = \dfrac{8 m_\phi f_\phi^2}{n^2}$

DW annihilation: $\Delta V = \sigma H(T_{\rm eq})$

GW Peak frequency: $f_{\rm peak} \sim H(T_{\rm eq})$

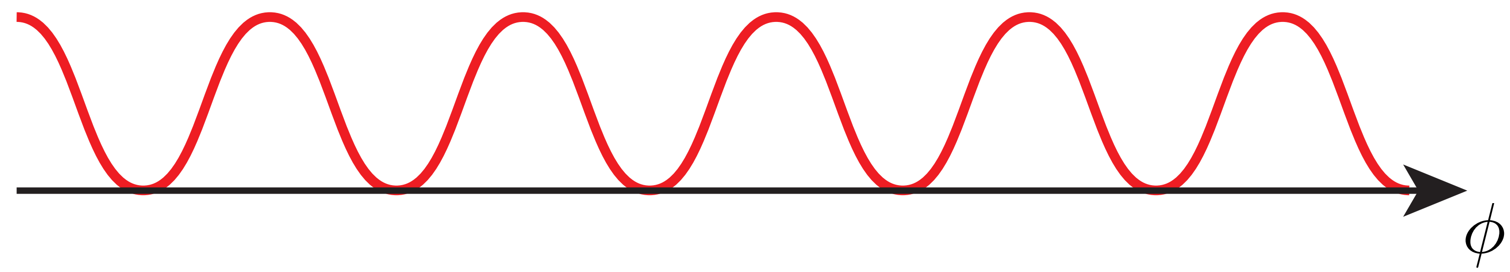
$$\Omega_{\rm GW,peak}(T_{\rm eq}) = \left.\frac{\epsilon_{\rm GW}\mathcal{A}^2\sigma^2}{24\pi M_{\rm pl}^4 H^2}\right|_{T=T_{\rm eq}}$$

Hiramatsu, Kawasaki, Saikawa (2014)

QCD axion domain wall

Axion potential:

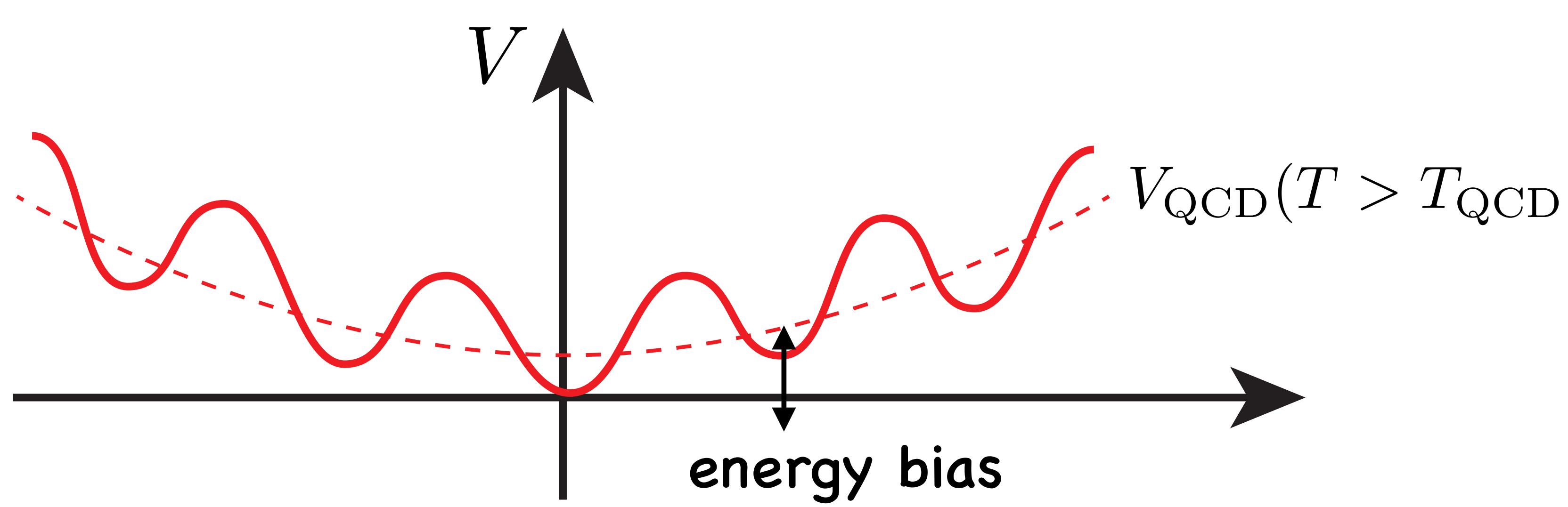
$$V(\phi) = \frac{m_\phi^2 f_\phi^2}{n^2}\left[1 - \cos\left(\frac{n\phi}{f_\phi}\right)\right]$$



$$\mathcal{L} \ni -\frac{\alpha_s}{8\pi}\left(\frac{n_g\phi}{f_\phi} + \theta\right) G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$ —> QCD axion potential (in the minimal scenario)

$$V_{\rm QCD}(\phi) = \chi(T)\left[1 - \cos\left(\frac{n_g\phi}{f_\phi} + \theta\right)\right], \quad \chi(T) = \begin{cases} \chi_0 & (T < T_{\rm QCD}) \\ \chi_0\left(\dfrac{T}{T_{\rm QCD}}\right)^{-c} & (T \geq T_{\rm QCD}) \end{cases}$$

$$c = 8.16,\ \chi_0 = (75.6\,{\rm MeV})^4,\ T_{\rm QCD} = 153\,{\rm MeV}$$ Borsanyi et al 1606.07494



- Field equation (Klein-Gordon equation) in flat FLRW universe

$$\ddot{\phi} + 3H\dot{\phi} - \frac{\nabla^2\phi}{a^2} + \frac{\partial V}{\partial \phi} = 0, \quad V(\phi) = V_0 - \frac{1}{2}m_0^2\phi^2 + \frac{\lambda}{4}\phi^4 + \Delta V$$

bias: $\Delta V = \lambda v^3 \phi \times b(\tau), \quad b(\tau) = \frac{\epsilon}{1 + e^{-2(\tau - \tau')/\delta\tau}}$

$\tau' = 4/m_0$

$\delta\tau = 0.2/m_0$

$\epsilon = 0.025, 0.05, 0.1$

- Gravitational wave (tensor metric perturbation)

$$ds^2 = dt^2 - a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j \longrightarrow \ddot{h}_{ij} + 3H\dot{h}_{ij} - \frac{\nabla^2 h_{ij}}{a^2} = -16\pi G \Lambda_{ij}^{kl}\Pi_{kl}$$

Λ_{ij}^{kl} : TT projection tensor

$\Pi_{ij} = -\partial_i\phi\partial_j\phi/a^2$: Energy momentum tensor

- GW density parameter:

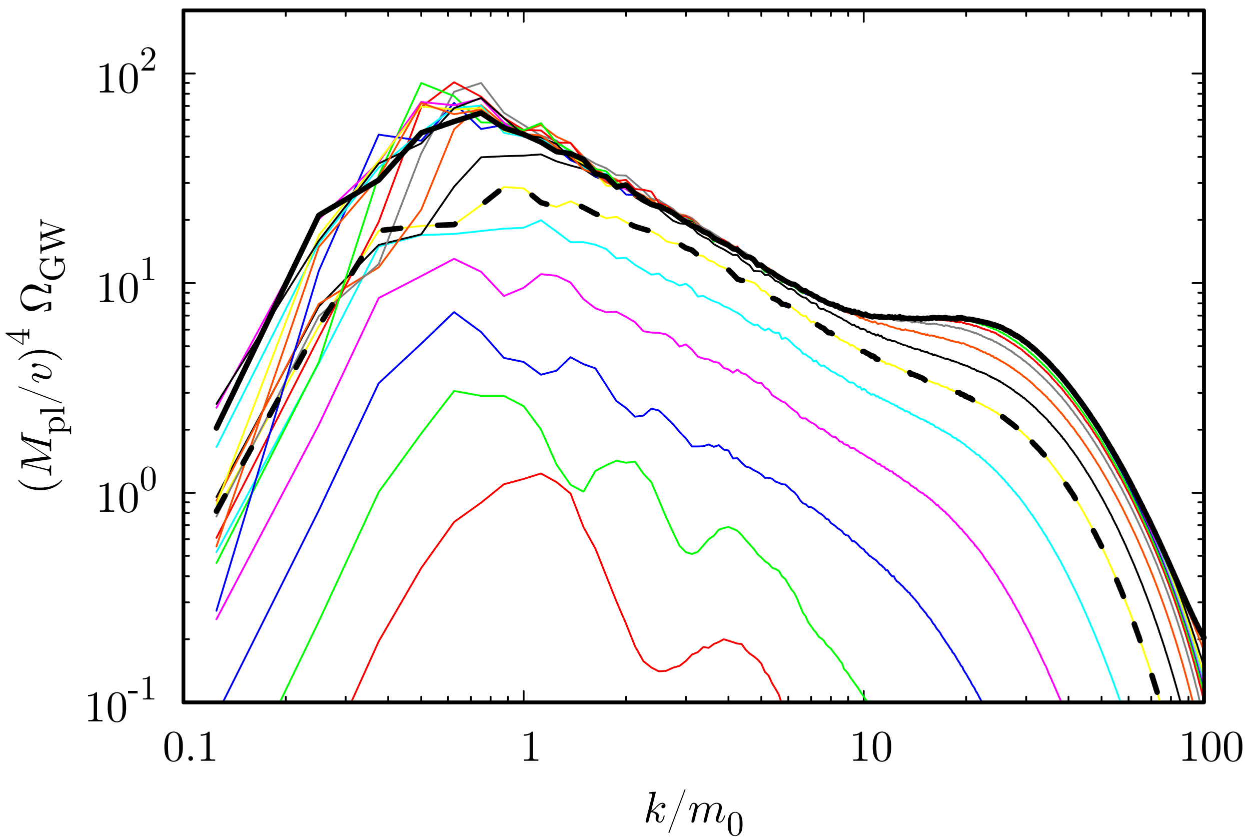
$$\rho_{\rm GW}(t) = \frac{1}{32\pi G}\langle \dot{h}_{ij}(\boldsymbol{x})\dot{h}_{ij}(\boldsymbol{x})\rangle \longrightarrow \Omega_{\rm GW}(f) = \frac{1}{\rho_{\rm cr}}\frac{d\rho_{\rm GW}}{d\ln f}$$

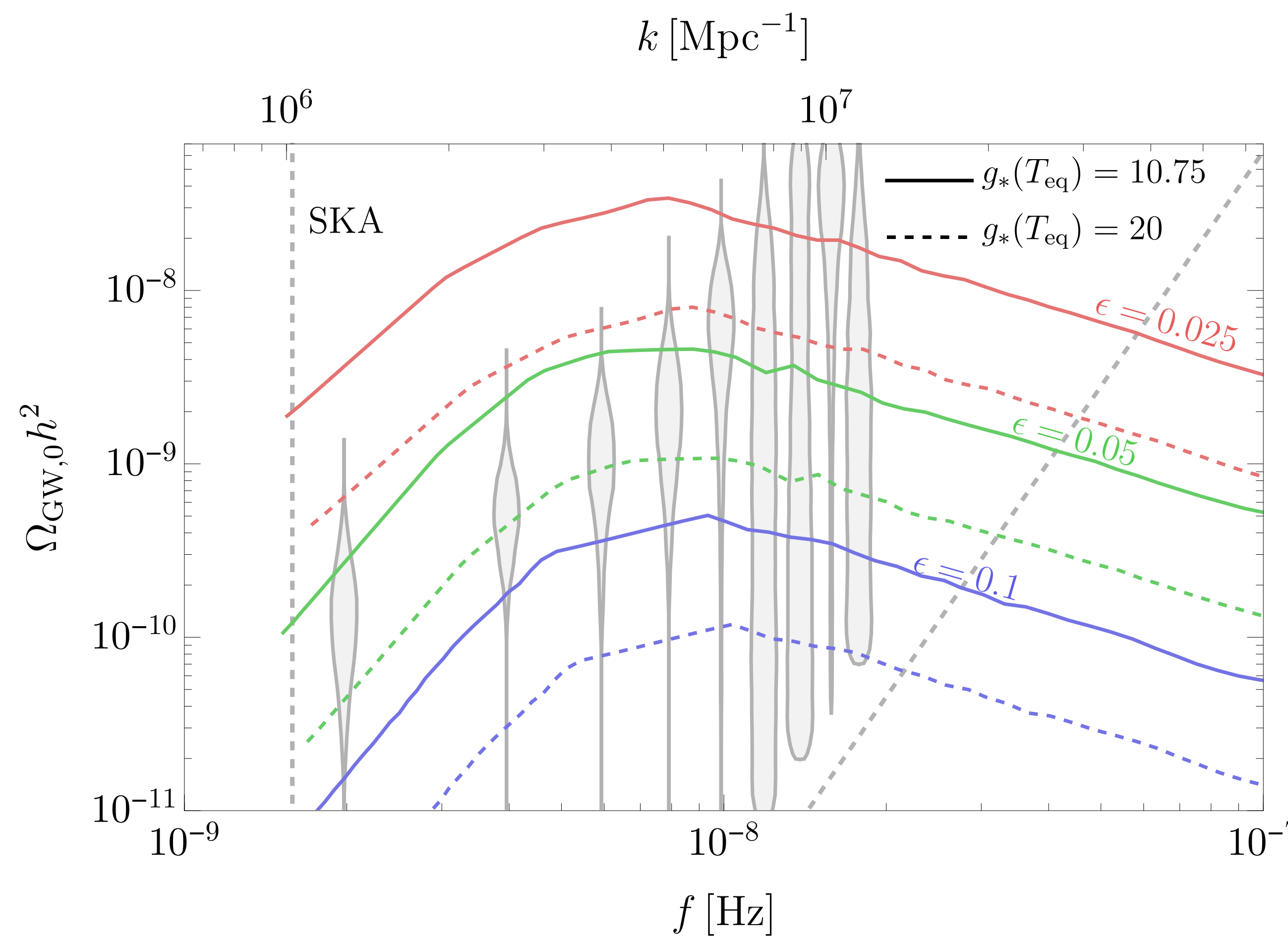
$\rho_{\rm cr} = 3H^2M_P^2$
(critical density)

We performed 3D lattice simulation ($4{,}096^3$ grids)

Gravitational wave spectrum

NK, Lee, Murai, Takahashi, Yin, 2306.17146





$$f_{\rm peak} \simeq 13\,{\rm nHz}\left(\frac{g_*(T_{\rm eq})}{20}\right)^{1/6}\left(\frac{T_{\rm eq}}{0.1\,{\rm GeV}}\right)$$

$$\Omega_{\rm GW,peak}(T_{\rm eq}) = \left.\frac{\epsilon_{\rm GW}\mathcal{A}^2\sigma^2}{24\pi M_{\rm pl}^4 H^2}\right|_{T=T_{\rm eq}}$$

QCD scale <—> nHz GW

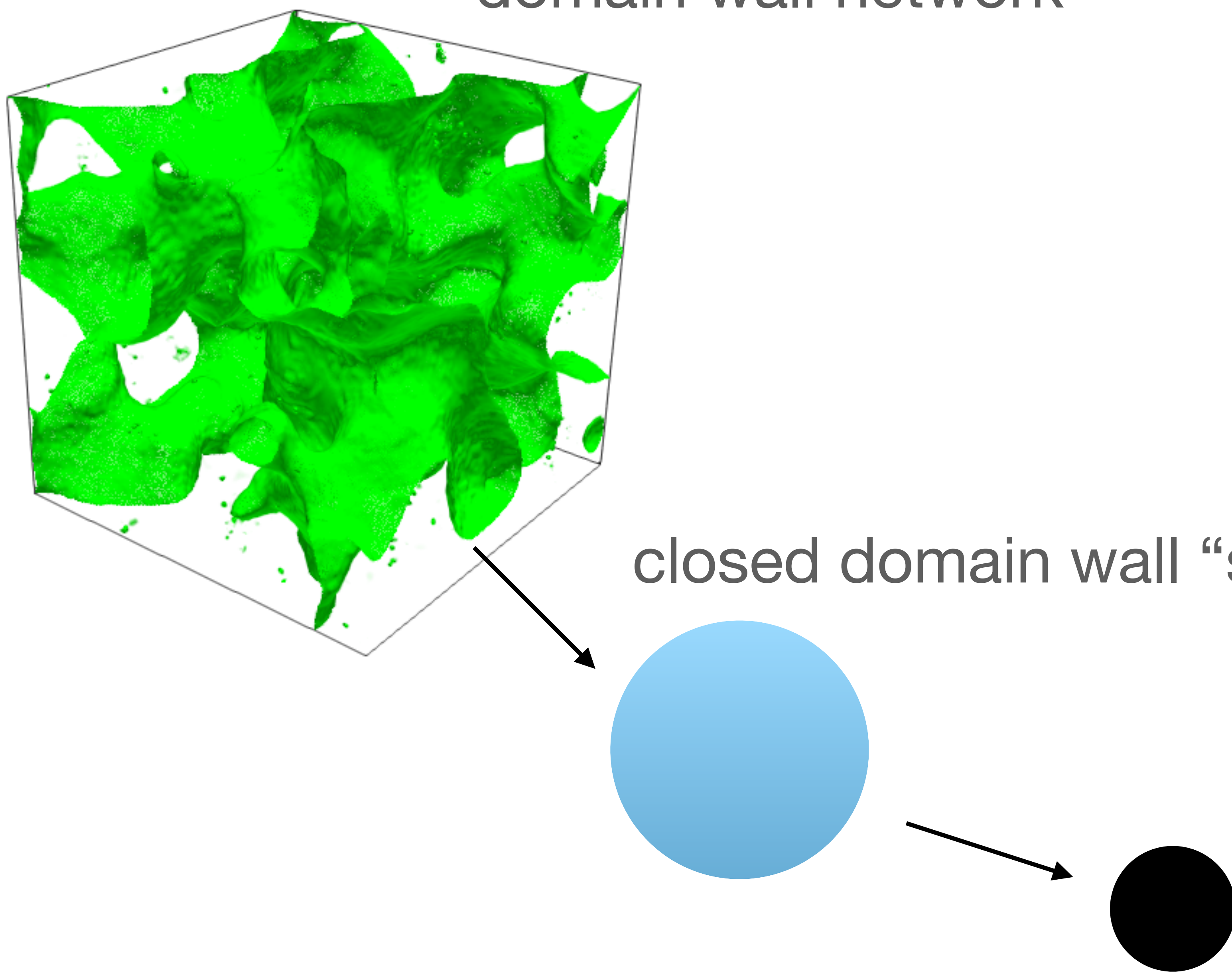
See also Ferreira, Gasparotto, Hiramatsu, Obata, Pujolas (2312.14104) for CMB-scale GW signal

Ferreira, Notari, Pujolas, Rompineve (2401.14331)

Primordial black holes from domain walls

(ongoing work)

PBH formation from domain wall collapse



Widrow (1989)
Rubin et al (2001)
Vachaspati (2017)
Liu, Guo, Cai (2019)
Ge (2020)
Ge, Guo, Liu (2024)
Dunsky, Kongsore (2024)
Ferreira et al (2024)

3+1 formalism

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(\beta^i dt + dx^i)(\beta^j dt + dx^j)$$

Einstein equations

$$\mathcal{H} = R + K^2 - K_{ij}K^{ij} - 16\pi\rho = 0$$ (Hamiltonian constraint)

$$\mathcal{M}^i = D_j(K^{ij} - \gamma^{ij}K) - 8\pi S^i = 0$$ (momentum constraint)

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_i\beta_j + D_j\beta_i$$

$$\partial_t K_{ij} = \alpha(R_{ij} - 2K_{ik}K^k_j + KK_{ij}) - D_iD_j\alpha - 8\pi\alpha\left[S_{ij} - \frac{1}{2}\gamma_{ij}(S-\rho)\right] + \beta^k\partial_k K_{ij} + K_{ik}\partial_j\beta^k + K_{jk}\partial_i\beta^k,$$

$$K_{ij} = -\gamma_i^\mu\gamma_j^\nu\nabla_\mu n_\nu, \quad K = \gamma^{ij}K_{ij} \qquad n^\mu = (\alpha^{-1}, -\alpha^{-1}\beta^i), \quad n_\mu = (-\alpha, 0, 0, 0)$$

(extrinsic curvature)

(Practically, we use the BSSN or CCZ4 formulations)

Gauge fixing

$$\partial_t\alpha = -2\eta\alpha(K - \langle K\rangle) + \beta^i\partial_i\alpha, \quad \partial_t\beta^i = \frac{3}{4}B^i, \quad \partial_t B^i = \partial_t\bar{\Gamma}^i - \eta B^i$$

(dynamical slice / moving puncture, Gamma-driver)

<u>α —> 0 indicates strong gravity / existence of black hole</u>

Scalar field evolution

$$S = -\int d^4x\sqrt{-g}\left[\frac{1}{2}\nabla^a\varphi\nabla_a\varphi - V(\varphi)\right], \quad T_{ab} = \nabla_a\varphi\nabla_b\varphi + g_{ab}\left(-\frac{1}{2}\partial_\alpha\varphi\partial^\alpha\varphi - V(\varphi)\right)$$

$$\partial_t\varphi = \alpha\Pi + \beta^i\partial_i\varphi, \quad \Pi = \frac{1}{\alpha}(\partial_t\varphi - \beta^i\partial_i\varphi),$$

$$\partial_t\Pi = \beta^i\partial_i\Pi + \gamma^{ij}(\alpha\partial_i\partial_j\varphi + \partial_j\varphi\partial_i\alpha) + \alpha\left(K\Pi - \Gamma^k\partial_k\varphi - \frac{\partial V}{\partial\varphi}\right)$$

Z_2 domain wall : $\quad V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$

Other cosmological applications : preheating, oscillon, cosmic string, axion star

See e.g. Helfer et al 1609.04724, Yoo et al 1811.00762; Giblin, Tishue 1907.10601; Nazari et al 2010.05933

PBH formation from domain wall collapse

$$\sigma = \frac{4}{3}\sqrt{\lambda}v^2, \quad \mathcal{A}_{\rm dw} = 4\pi R_0^2, \quad M = \sigma \mathcal{A}_{\rm dw}$$

(tension, surface area, and mass of spherically closed domain wall)

$$\frac{R_s}{\delta_{\rm dw}} = \frac{2GM}{(\sqrt{\lambda}v)^{-1}} = \frac{2}{3}(mR_0)^2\left(\frac{v}{M_P}\right)^2 > 1$$

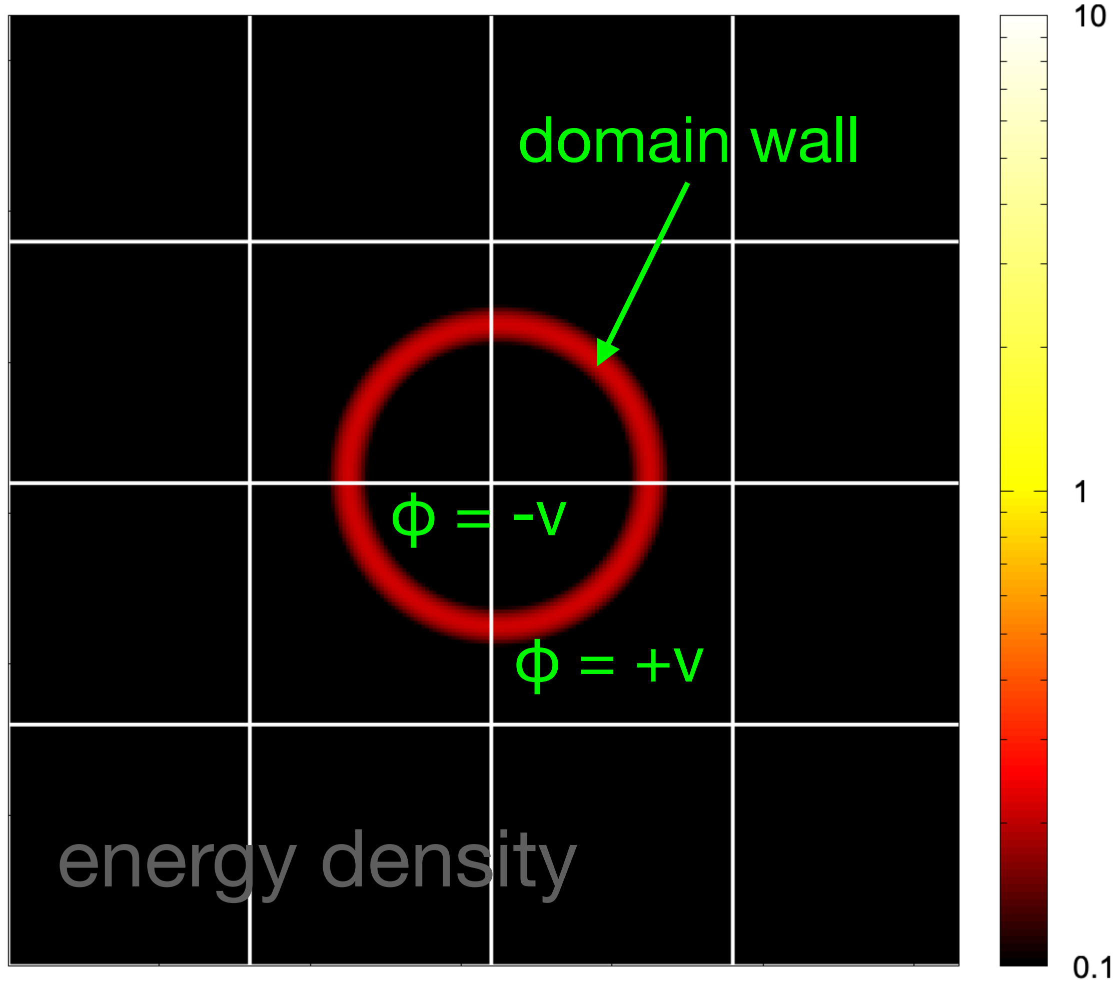
(PBH formation occurs when the Schwarzschild radius is larger than the domain wall width)

(i) $v = 0.3\ M_P$ & $R_0 = 10m^{-1}$ —> $R_s / \delta_{dw} = 6$ —> PBH formation

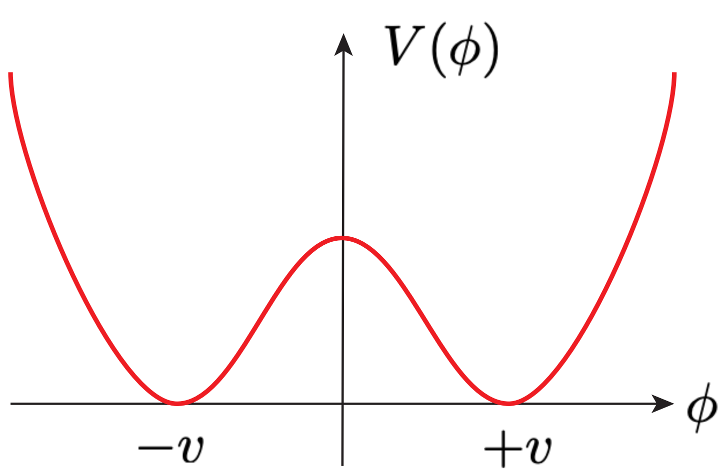
(ii) $v = 0.3\ M_P$ & $R_0 = 7m^{-1}$ —> $R_s / \delta_{dw} = 2.9$ —> PBH formation

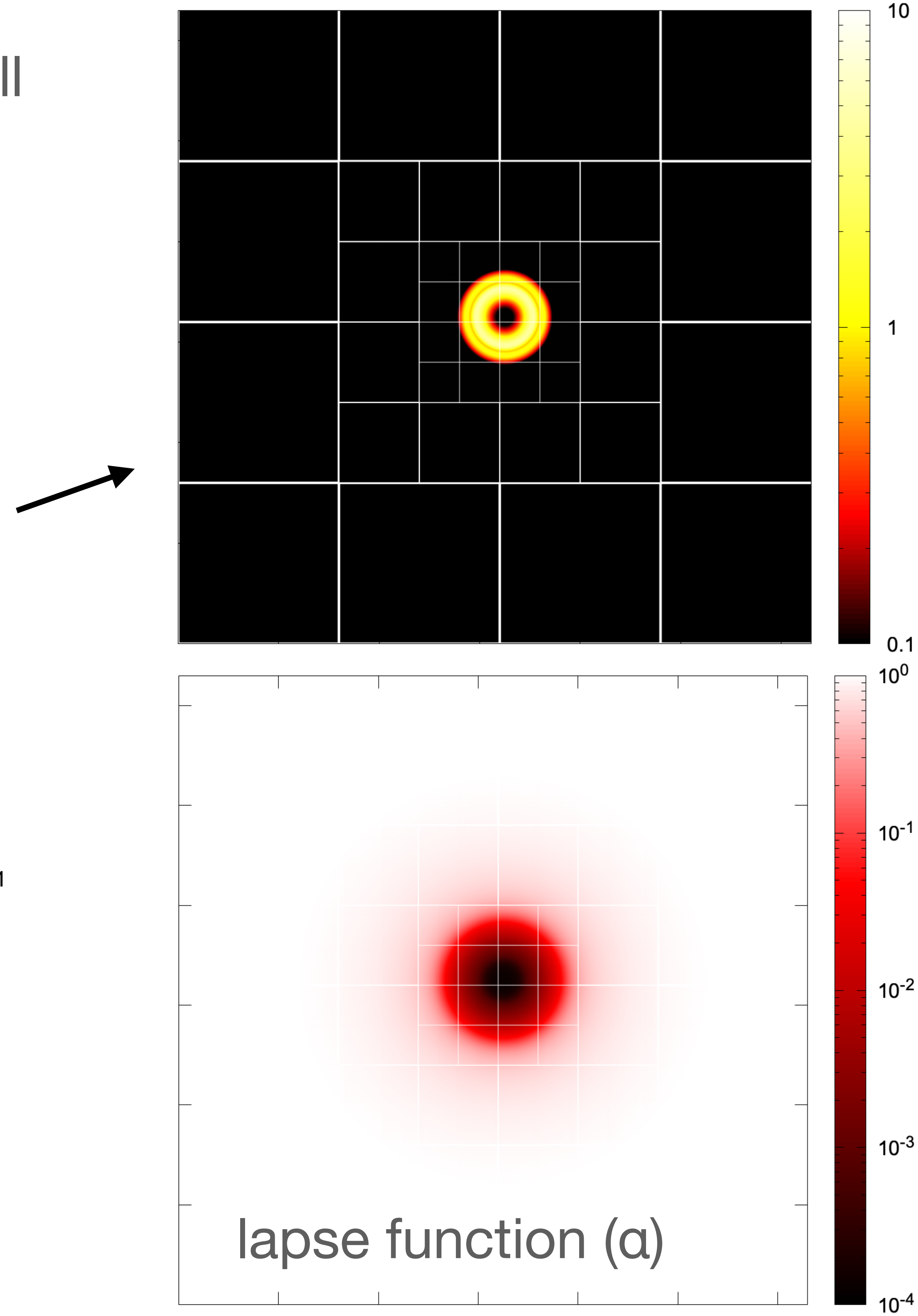
(iii) $v = 0.03\ M_P$ & $R_0 = 7m^{-1}$ —> $R_s / \delta_{dw} = 0.029$ —> no PBH

Initial profile of closed domain wall
(2D slice of 3D space)

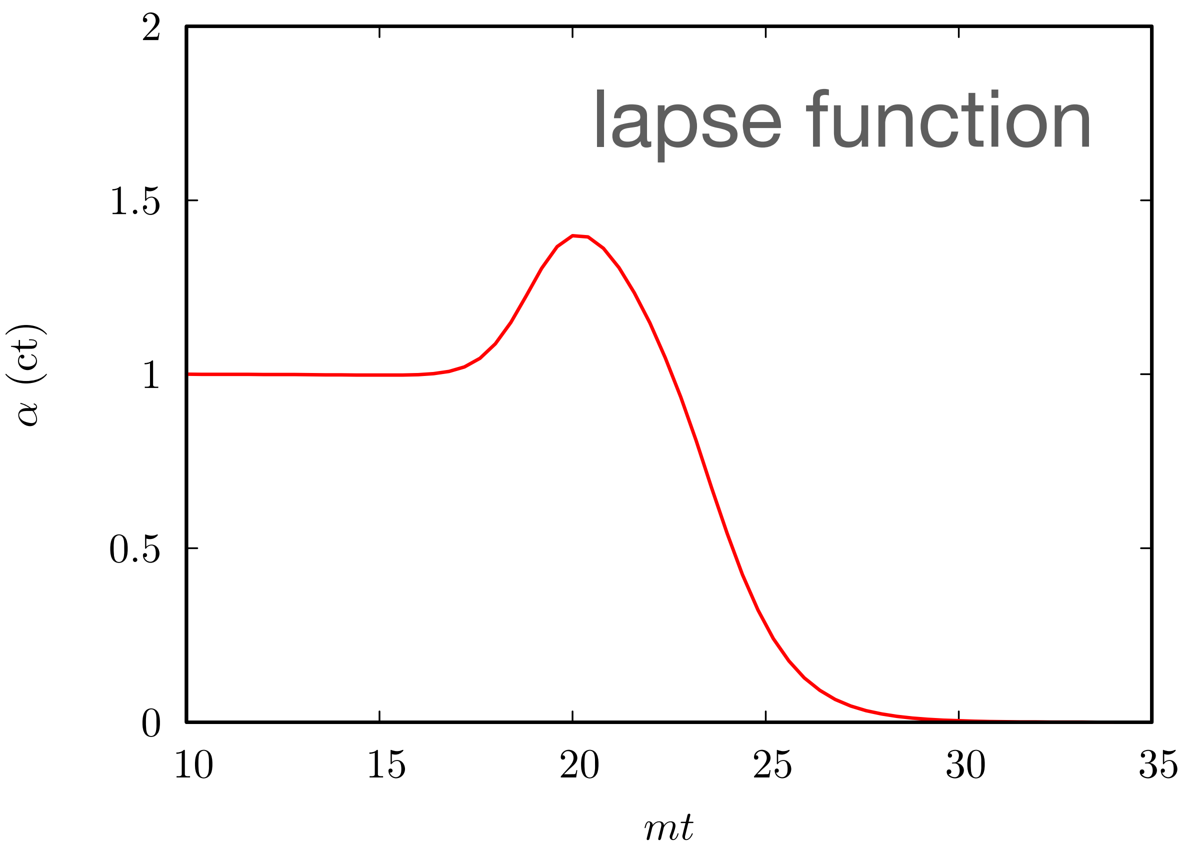


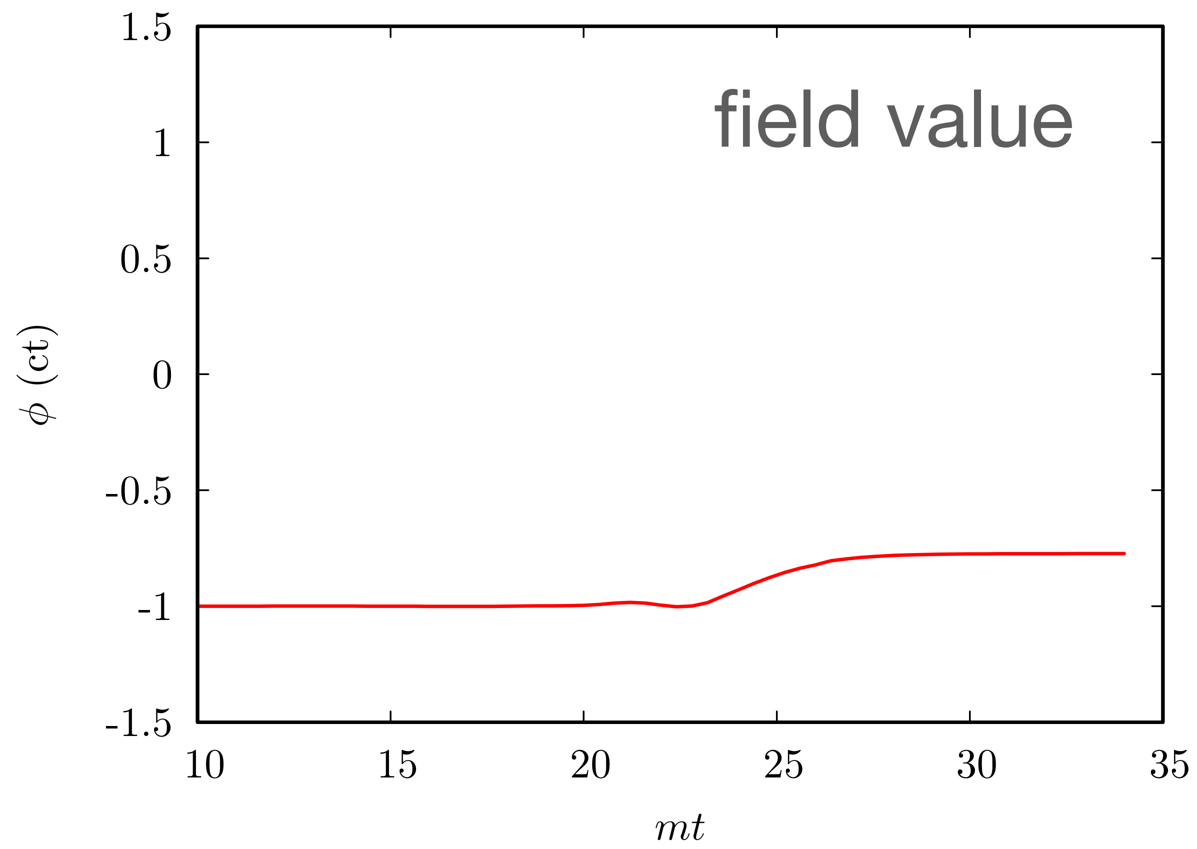
$v = 0.3\ M_P$ & $R_0 = 10m^{-1}$

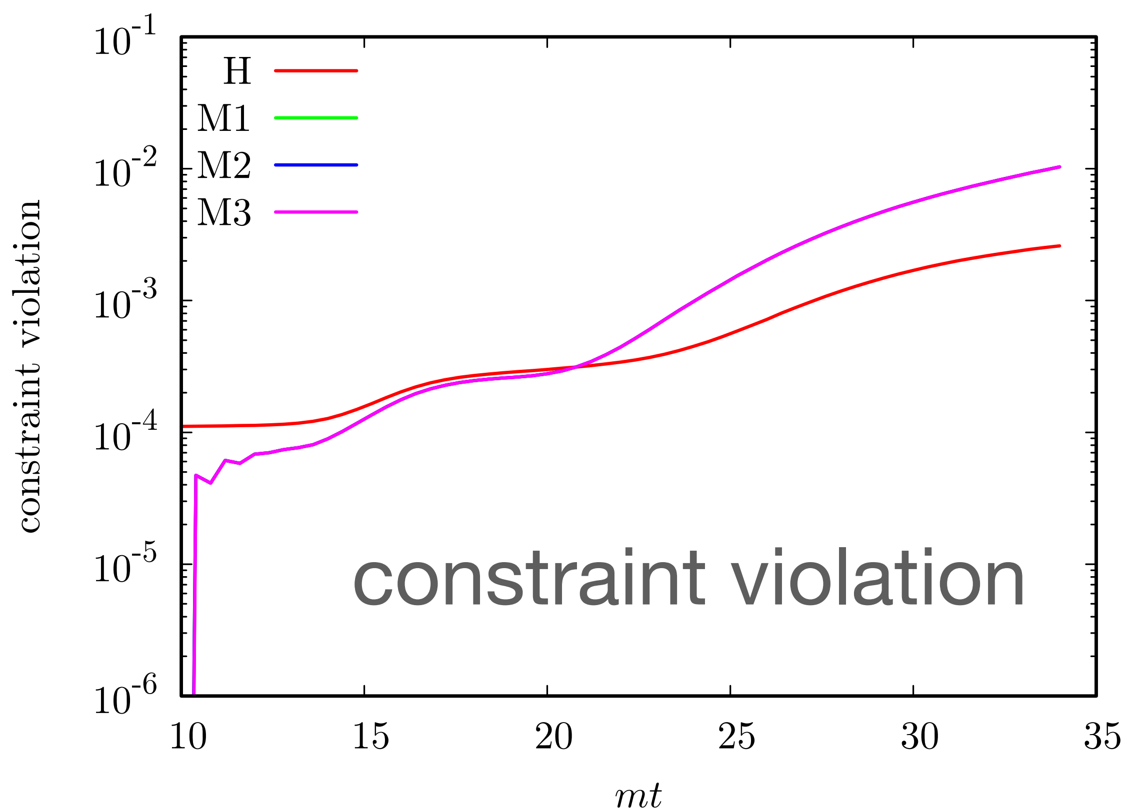




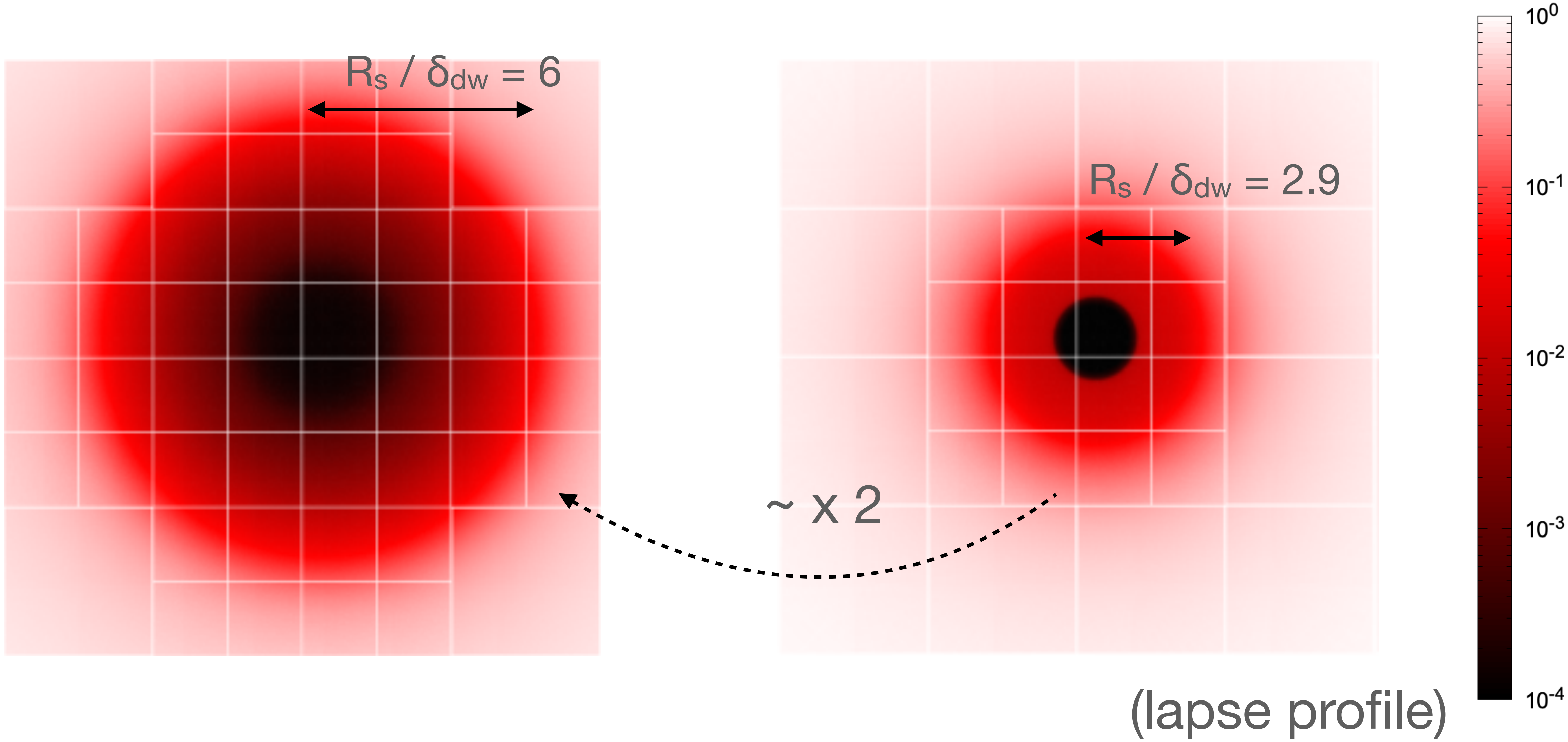
Time evolution of central values





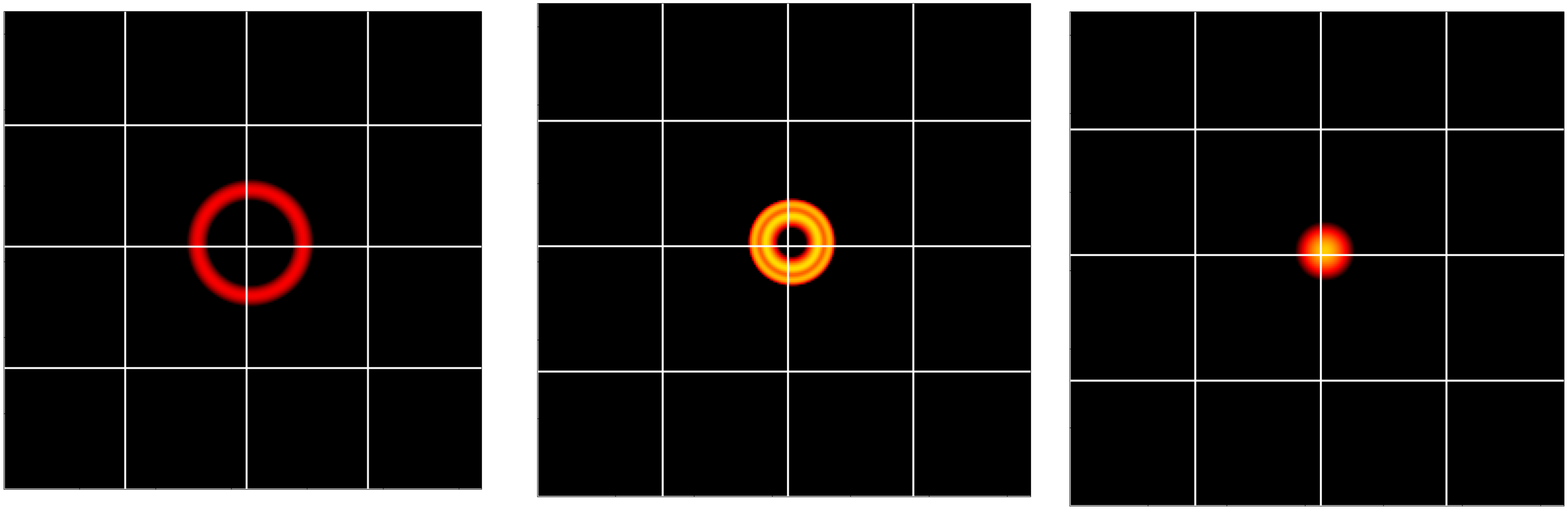


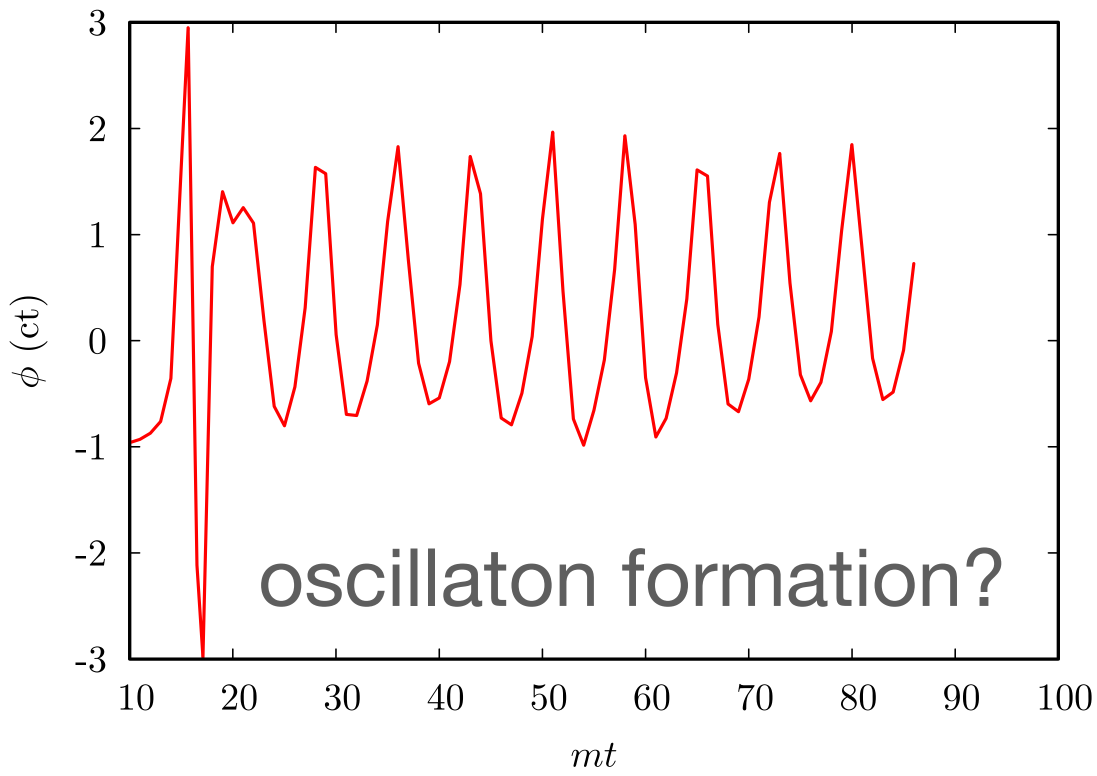
(i) $v = 0.3\ M_P$, $R_0 = 10m^{-1}$ (ii) $v = 0.3\ M_P$, $R_0 = 7m^{-1}$

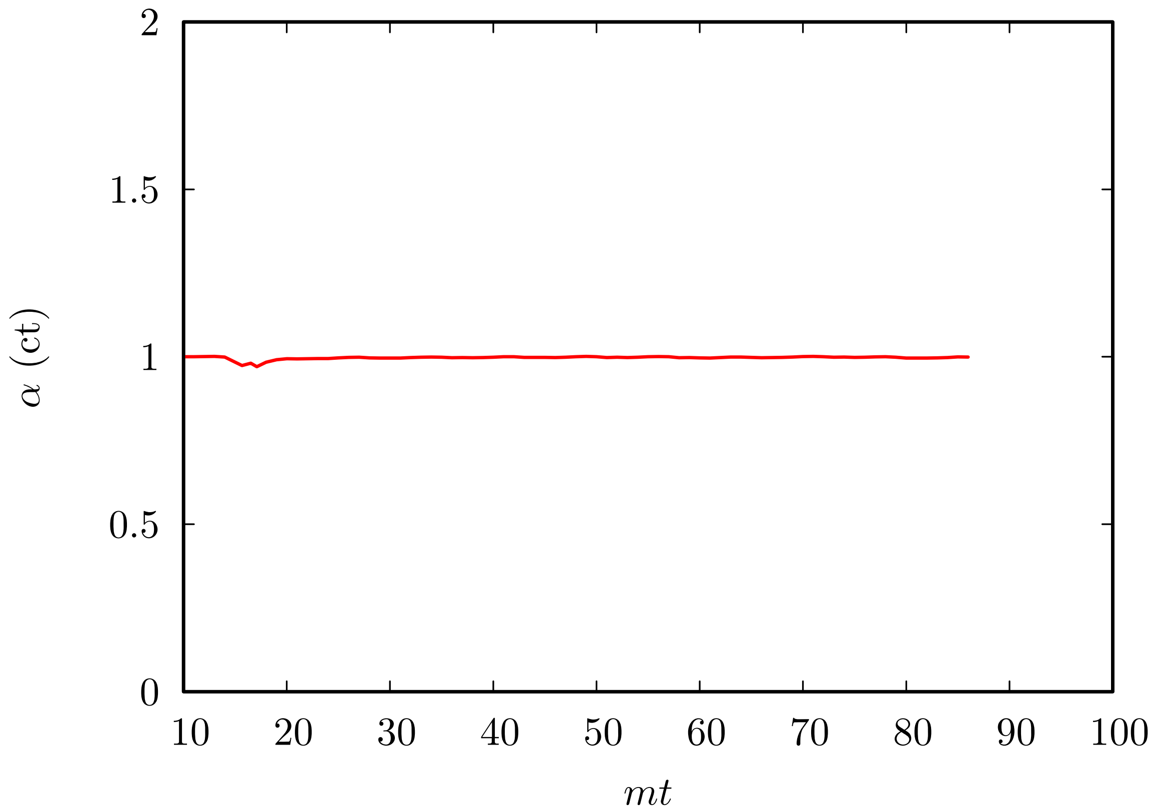


consistent with the rough estimate for Schwarzschild radius

(iii) v = 0.03 M_P & $R_0 = 7m^{-1}$ —> $R_s / \delta_{dw} = 0.029$







Summary and discussion

- Light dark photons can be produced from the string loop collapse

- Gravitational waves are emitted as a signal of this scenario

Spectrum is different from both local and global one
It can be tested by combining pulsar timing and direct detection

- We have numerically followed the DW annihilation process induced by the potential bias

- QCD axion domain walls naturally predict the GW with nHz band

It can be tested by pulsar timing observations

Summary and discussion

- We have shown numerically the PBH formation from DW collapse

(using numerical relativity)

- non-spherical collapse —> gravitational wave emission

(oscillaton case seems more interesting)

- Estimation of PBH abundance

    - percolation theory is necessary
    - domain walls from superhorizon-scale fluctuation is interesting (ruled-out?)

Gonzalez, NK, Takahashi, Yin 2211.06849
NK, Lee, Takahashi, Yin, 2311.14590

- Inclusion of the potential bias

BSSN formalism

Nakamura, Oohara, Kojima (1987), Shibata, Nakamura (1995), Baumgarte, Shapiro (1998)

$$\gamma_{ij} = e^{4\phi}\bar{\gamma}_{ij} \quad \text{with} \quad \det(\bar{\gamma}_{ij}) = 1, \quad K_{ij} = A_{ij} + \frac{1}{3}\gamma_{ij}K, \quad \tilde{A}_{ij} = e^{-4\phi}A_{ij}$$

Evolution equations

$$\partial_t \phi = -\frac{1}{6}\alpha K + \beta^i \partial_i \phi + \frac{1}{6}\partial_i \beta^i$$

$$\partial_t \bar{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \bar{\gamma}_{ij} + \bar{\gamma}_{ik}\partial_j \beta^k + \bar{\gamma}_{jk}\partial_i \beta^k - \frac{2}{3}\bar{\gamma}_{ij}\partial_k \beta^k$$

$$\partial_t K = -\gamma^{ij} D_i D_j \alpha + \alpha \left(\tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 \right) + 4\pi\alpha(\rho + S) + \beta^i \partial_i K$$

$$\partial_t \tilde{A}_{ij} = e^{-4\phi}[-D_i D_j \alpha + \alpha(R_{ij} - 8\pi S_{ij})]^{TF} + \alpha(K\tilde{A}_{ij} - 2\tilde{A}_{il}\tilde{A}^l_j) + \beta^k \partial_k \tilde{A}_{ij} + \tilde{A}_{ik}\partial_j \beta^k + \tilde{A}_{jk}\partial_i \beta^k - \frac{2}{3}\tilde{A}_{ij}\partial_k \beta^k$$

$$\partial_t \bar{\Gamma}^i = -2\tilde{A}^{ij}\partial_j \alpha + 2\alpha \left(\bar{\Gamma}^i_{jk}\tilde{A}^{jk} - \frac{2}{3}\bar{\gamma}^{ij}\partial_j K - 8\pi\bar{\gamma}^{ij}S_j + 6\tilde{A}^{ij}\partial_j \phi \right) + \beta^j \partial_j \bar{\Gamma}^i - \bar{\Gamma}^j \partial_j \beta^i + \frac{2}{3}\bar{\Gamma}^i \partial_j \beta^j + \frac{1}{3}\bar{\gamma}^{il}\partial_l \partial_j \beta^j + \bar{\gamma}^{lj}\partial_j \partial_l \beta^i$$

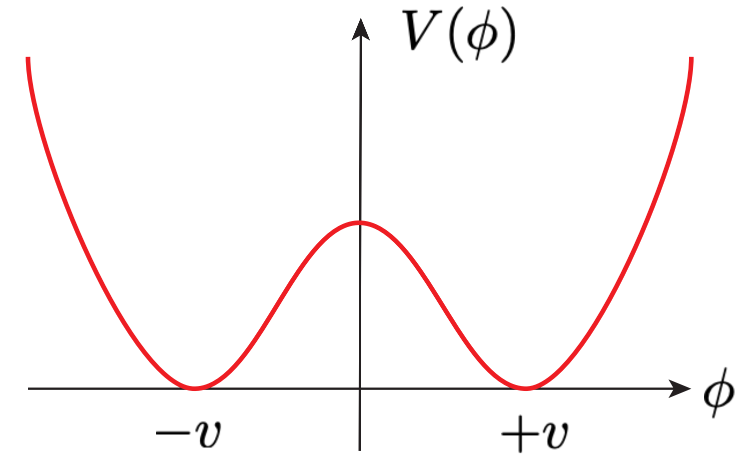
$\bar{\Gamma}^i = \bar{\gamma}^{jk}\bar{\Gamma}^i_{jk}$ are regarded as dynamical degrees of freedom

Hamiltonian & momentum constraints:

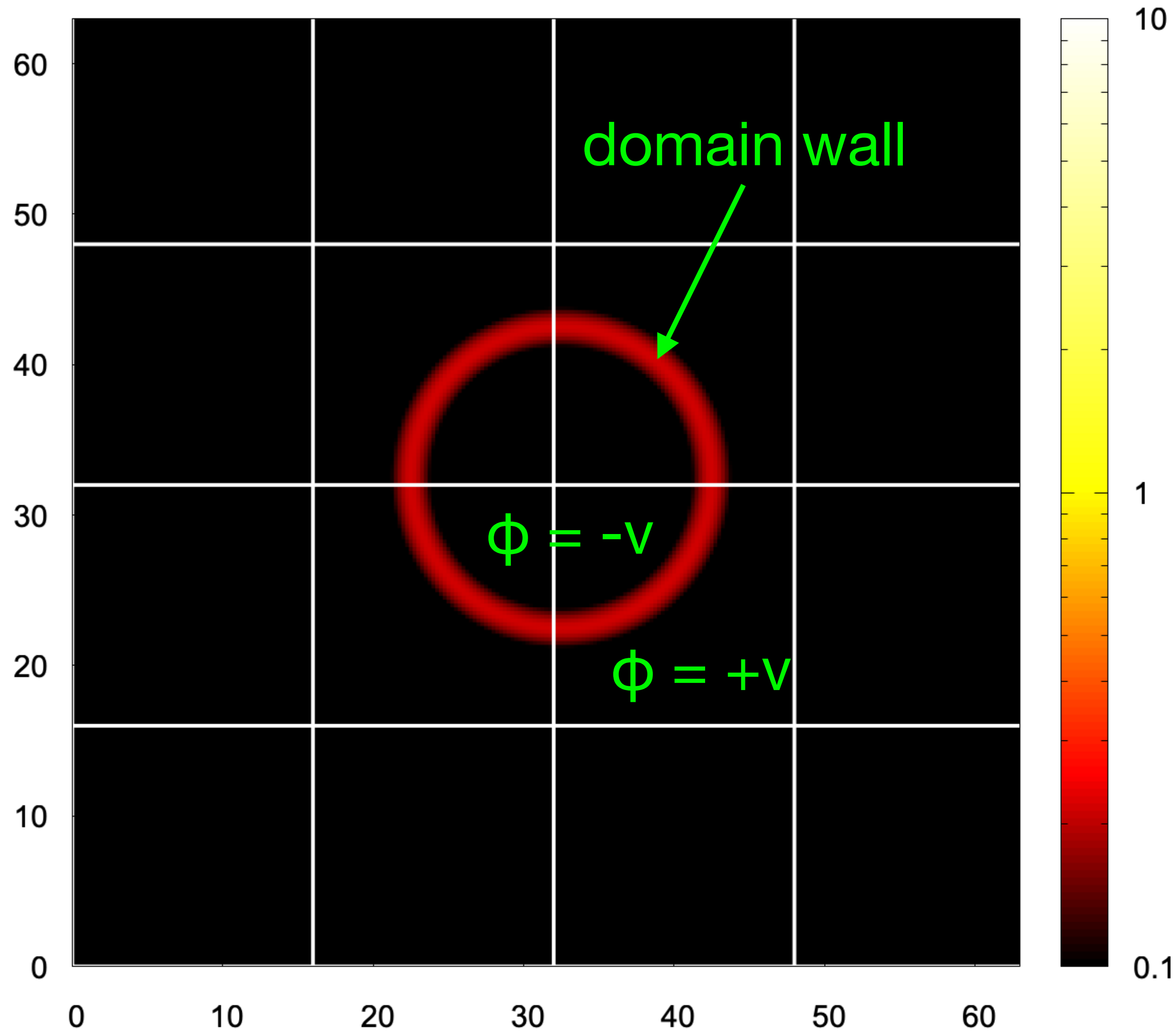
$$\mathcal{H} = -\frac{1}{8}e^{5\phi}R + \frac{1}{8}e^{5\phi}\tilde{A}^{ij}\tilde{A}_{ij} - \frac{1}{12}e^{5\phi}K^2 + 2\pi e^{5\phi}\rho = 0$$

$$\mathcal{M}^i = \bar{D}_j(e^{6\phi}\tilde{A}^{ij}) - \frac{2}{3}e^{6\phi}\bar{D}^iK - 8\pi e^{10\phi}S^i = 0$$

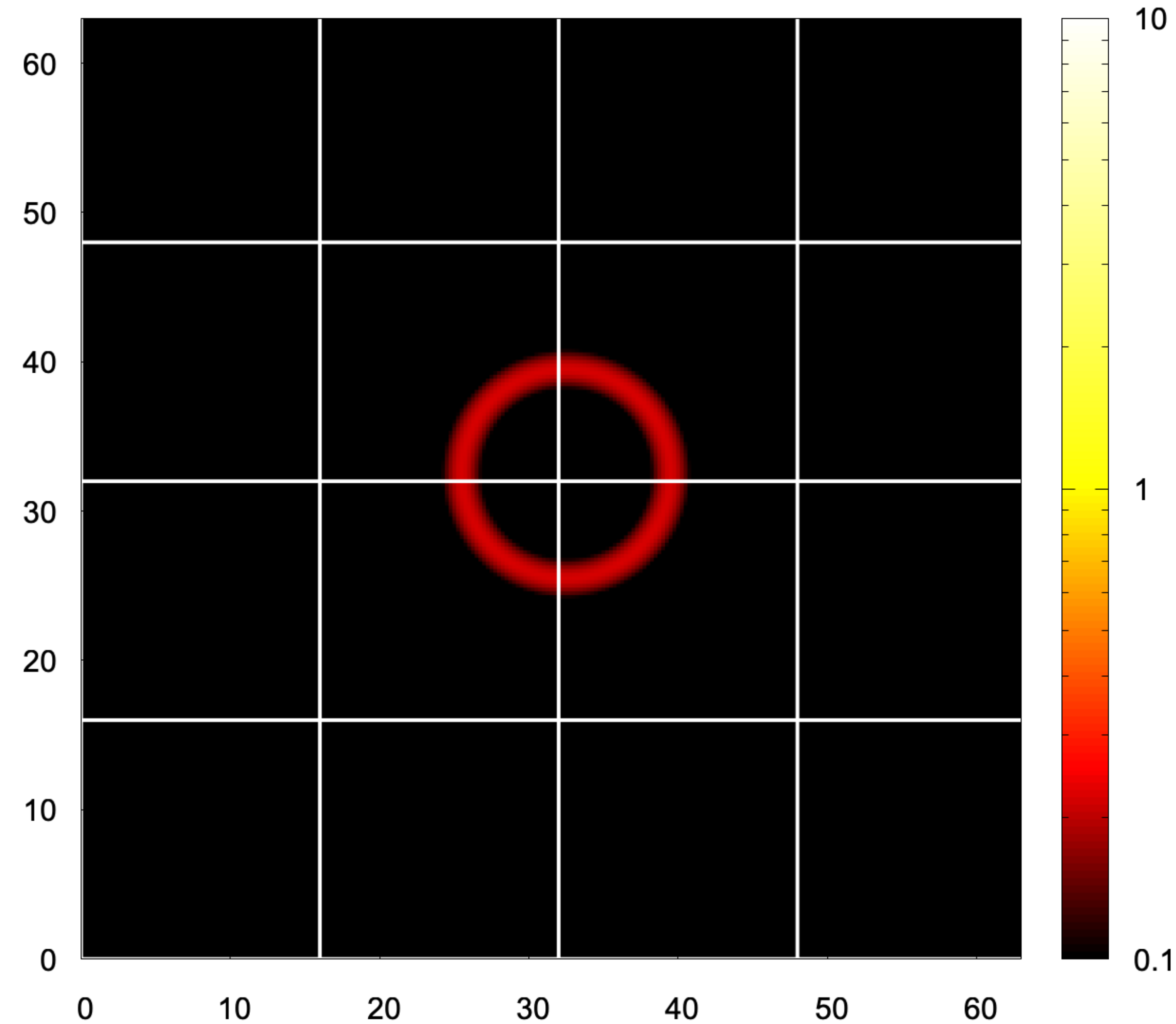
Initial profile of closed domain wall
(2D slice of 3D space)



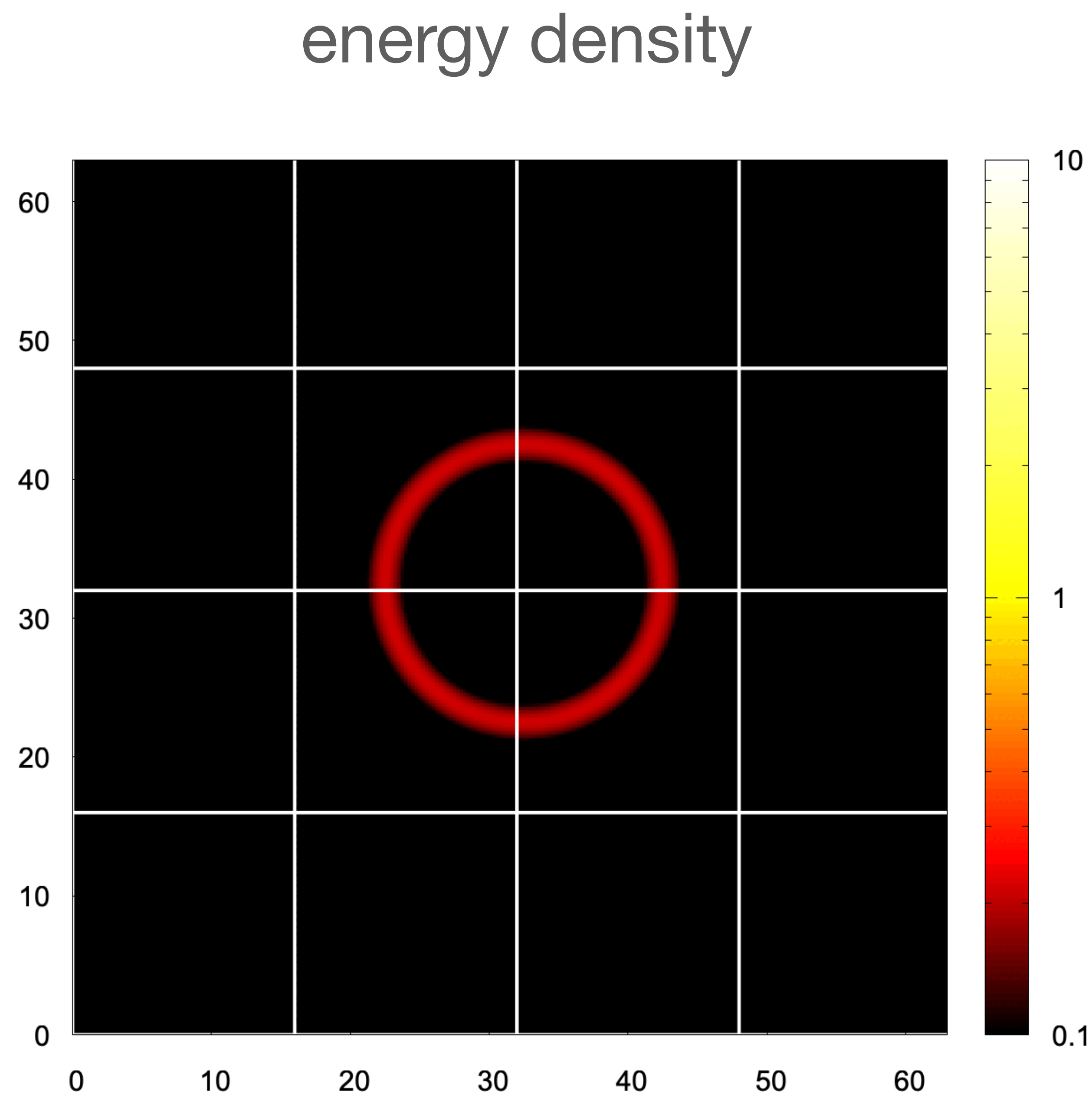
(i) $R_0 = 10m^{-1}$

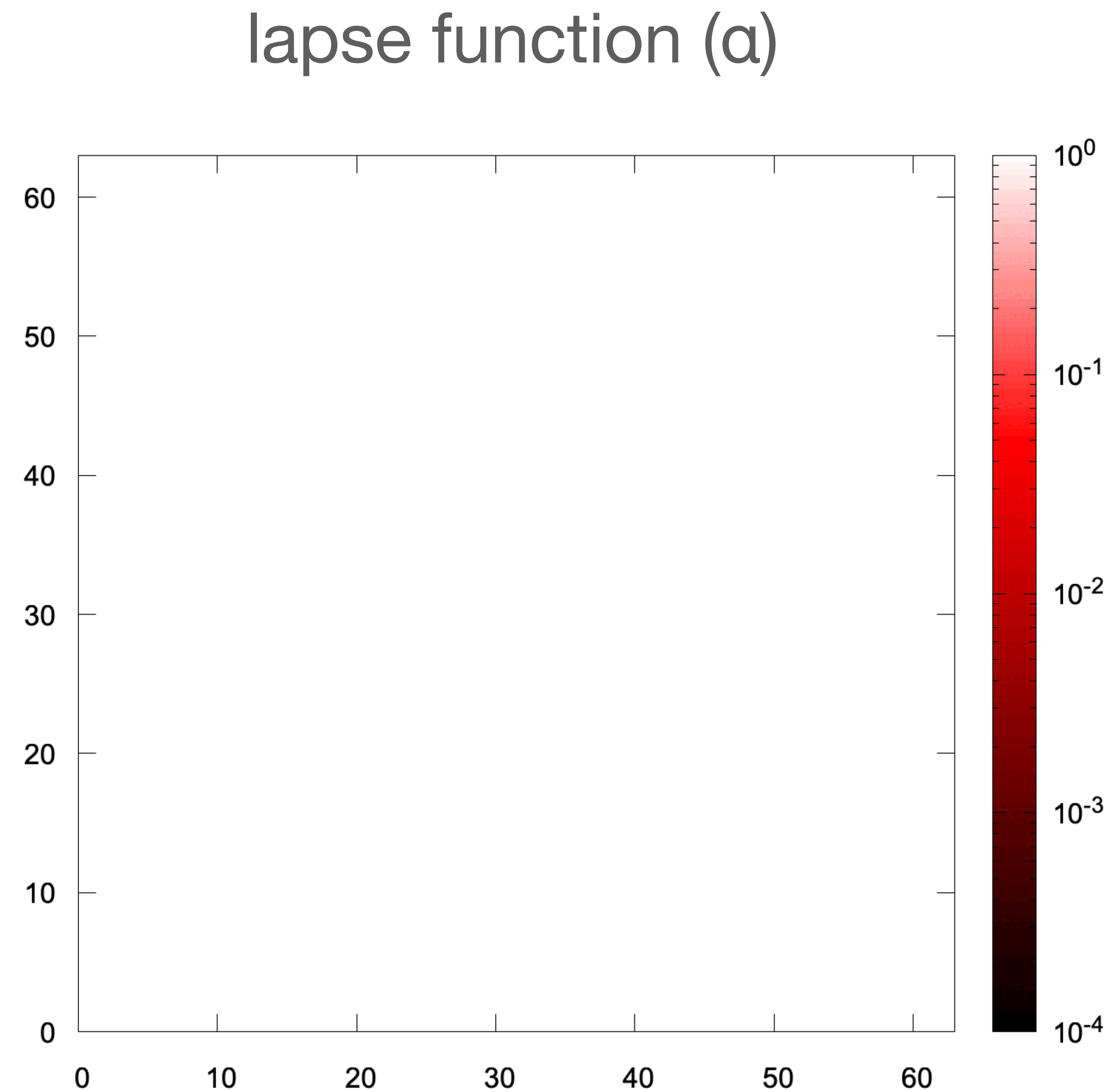


(ii) $R_0 = 7m^{-1}$



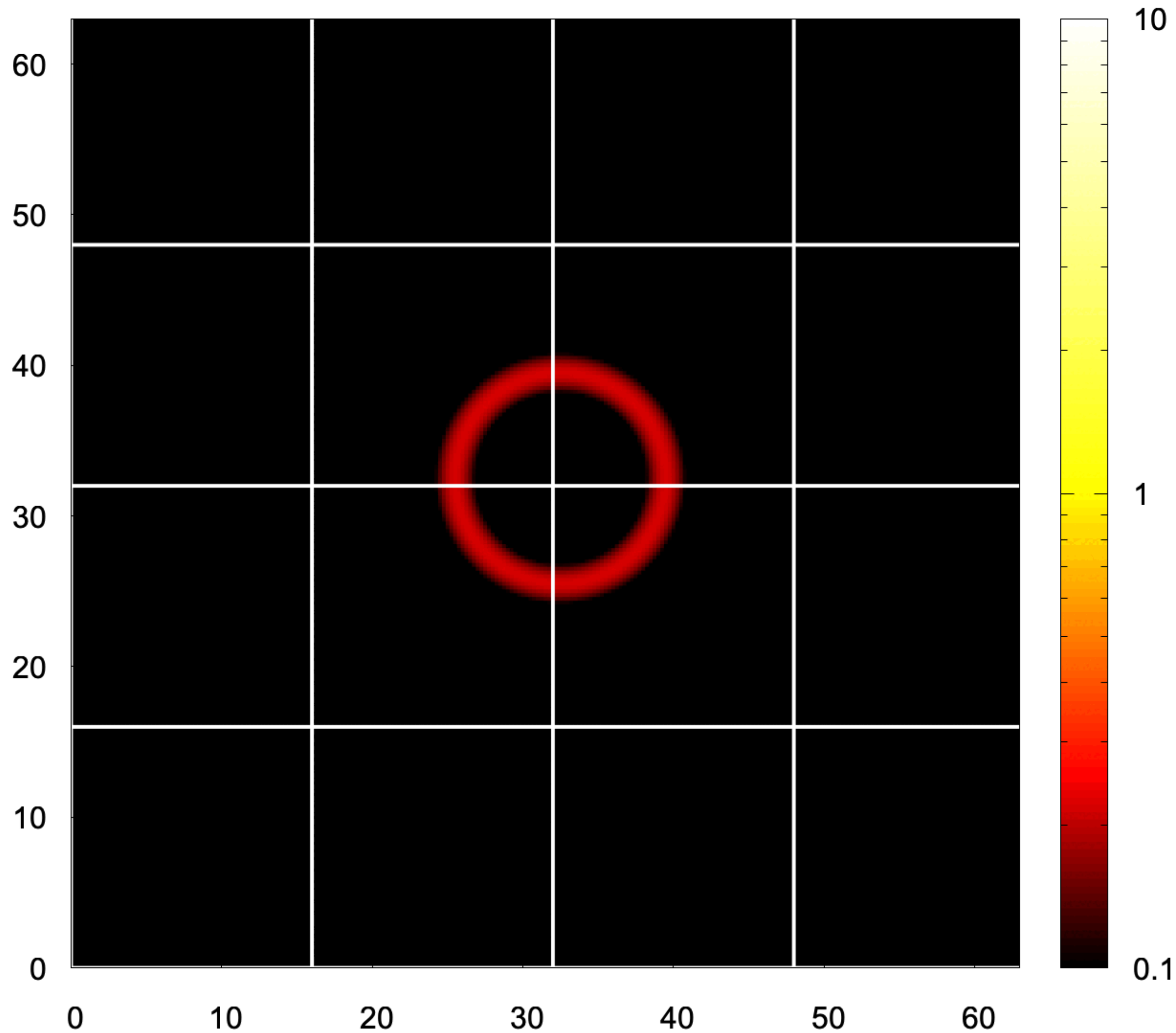
(i) $v = 0.3\,M_P$ & $R_0 = 10m^{-1}$



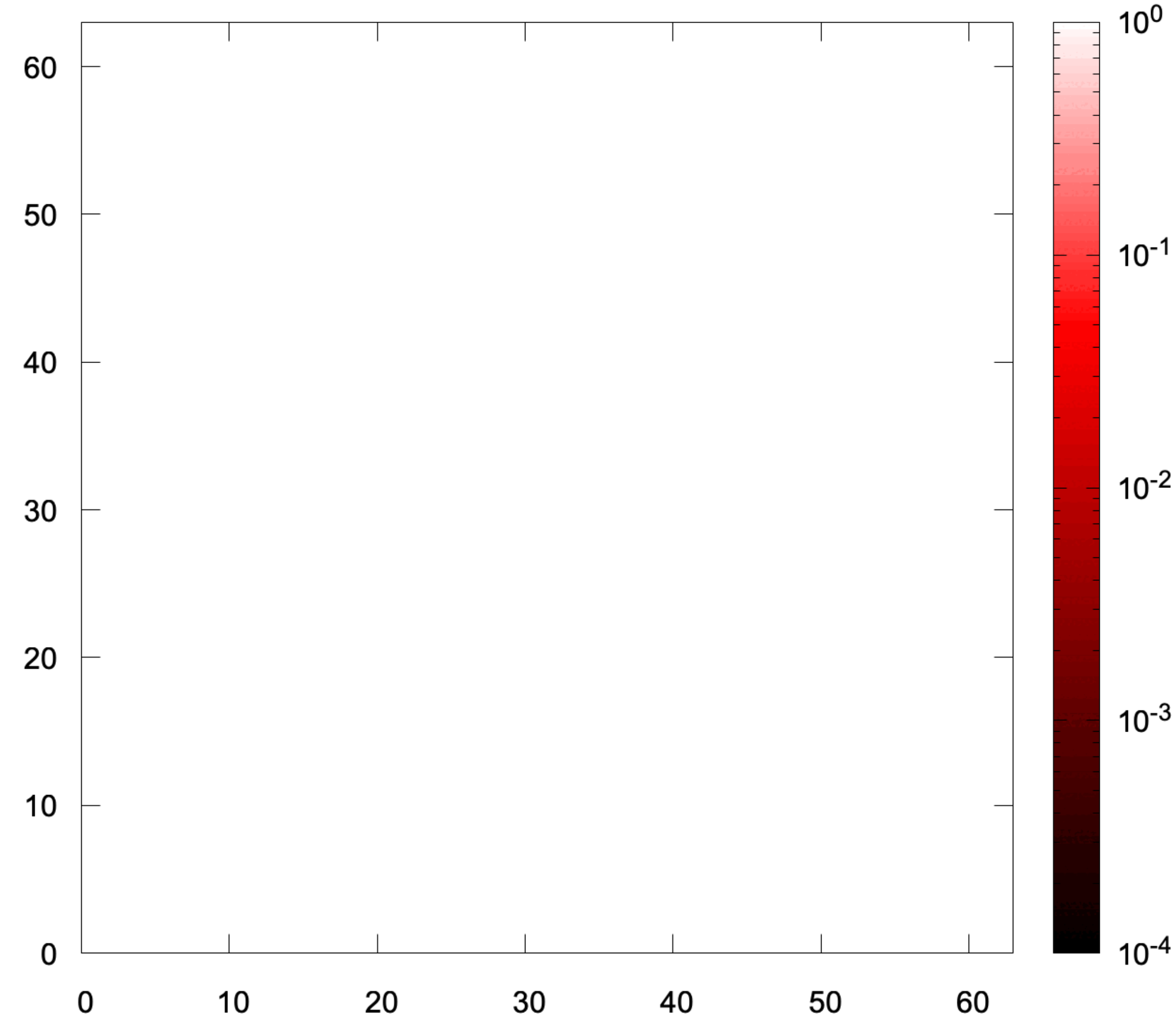


(ii) $v = 0.3\ M_P$ & $R_0 = 7m^{-1}$

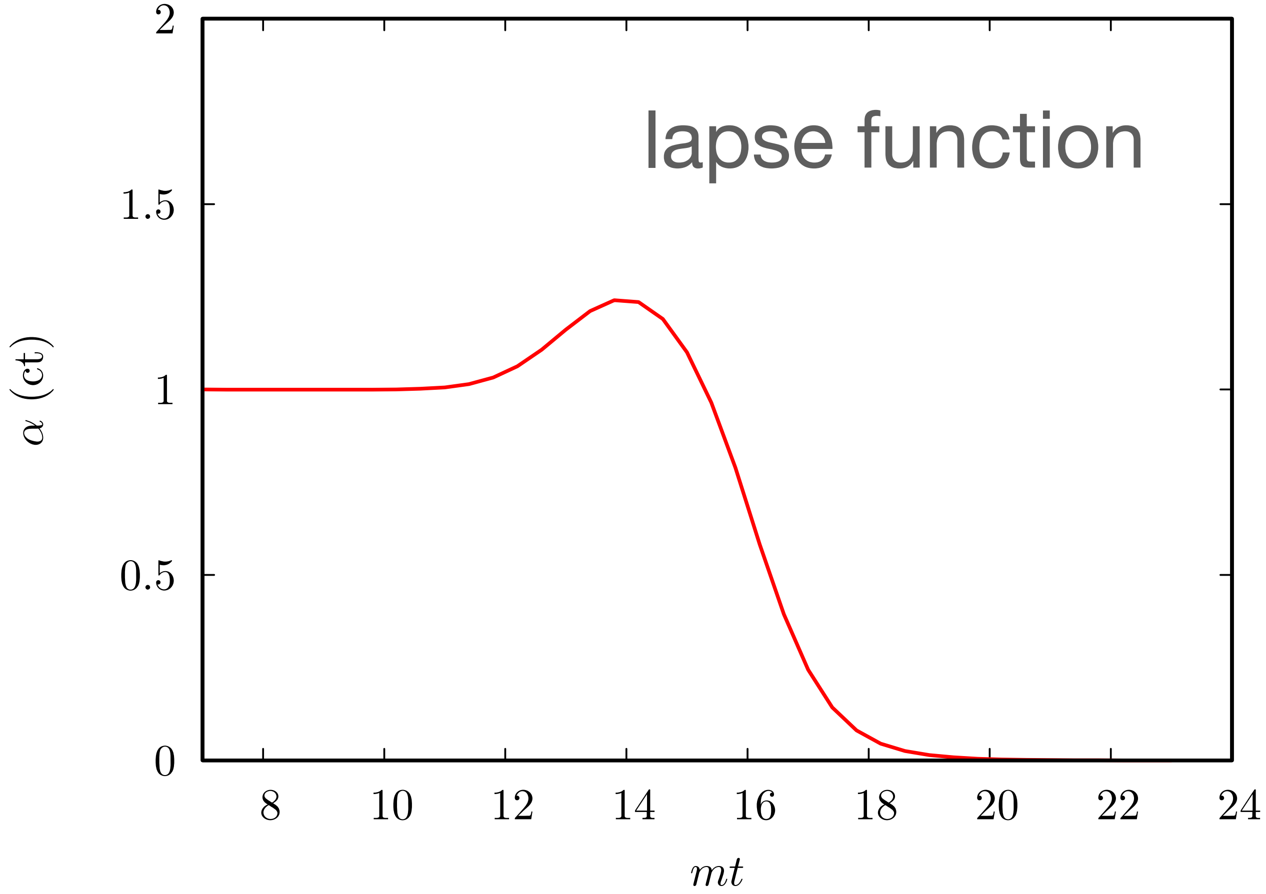
energy density

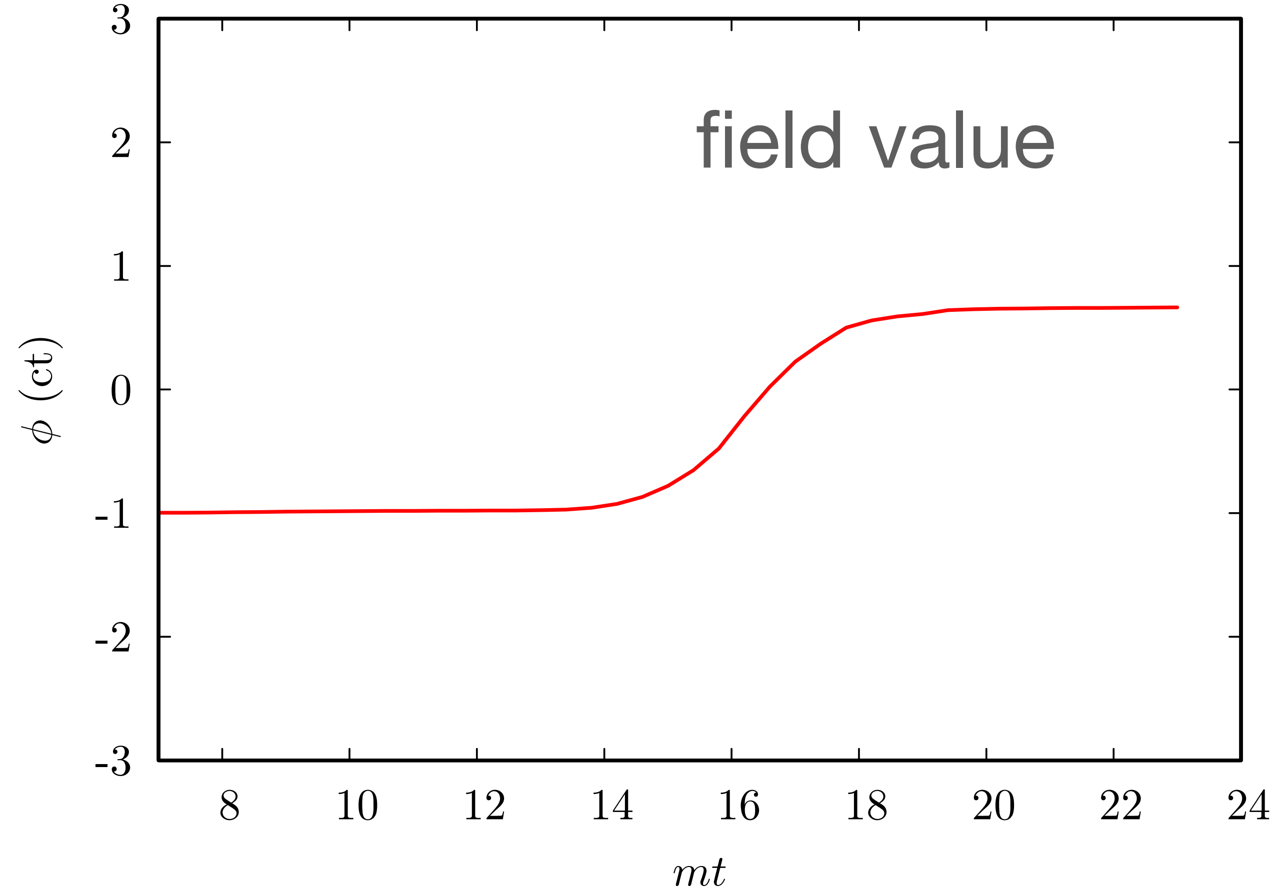


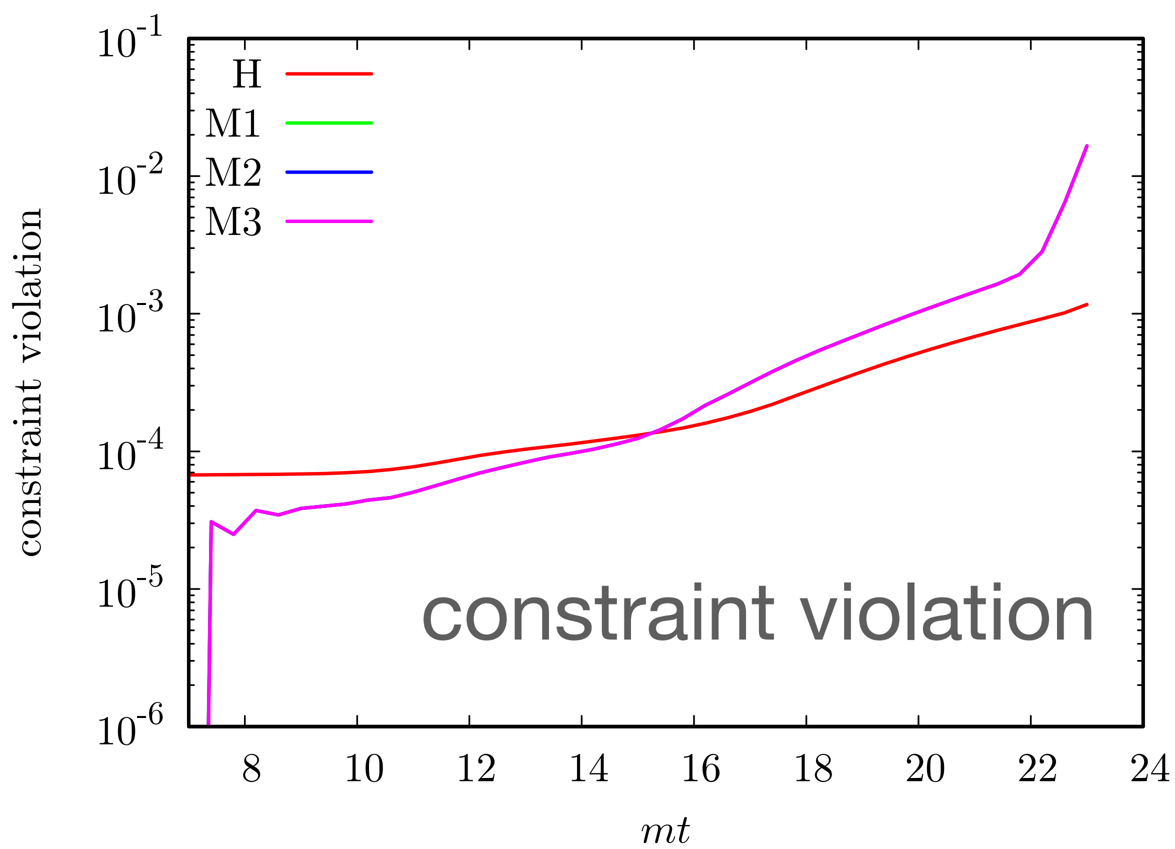
lapse function (α)



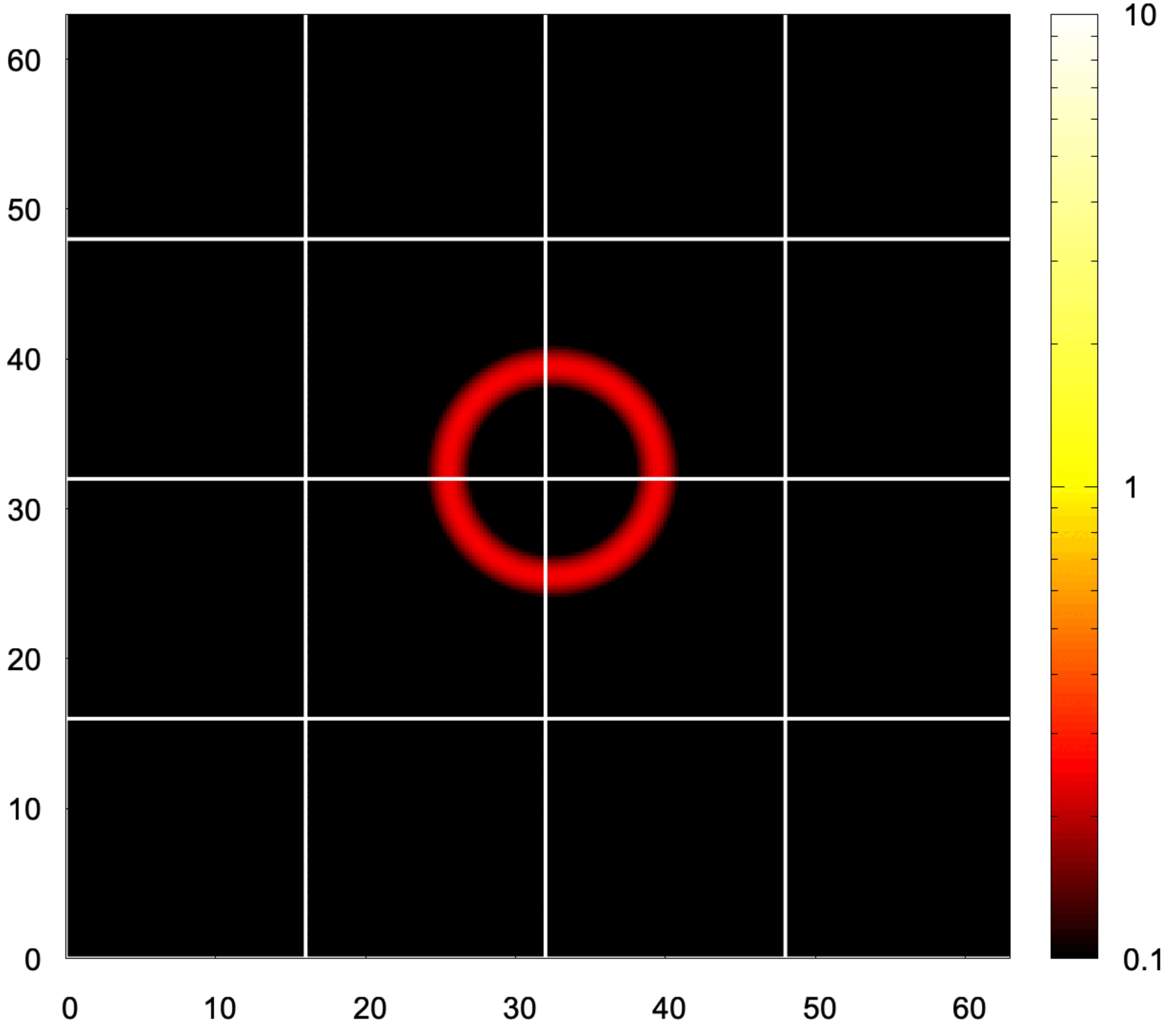
Central values

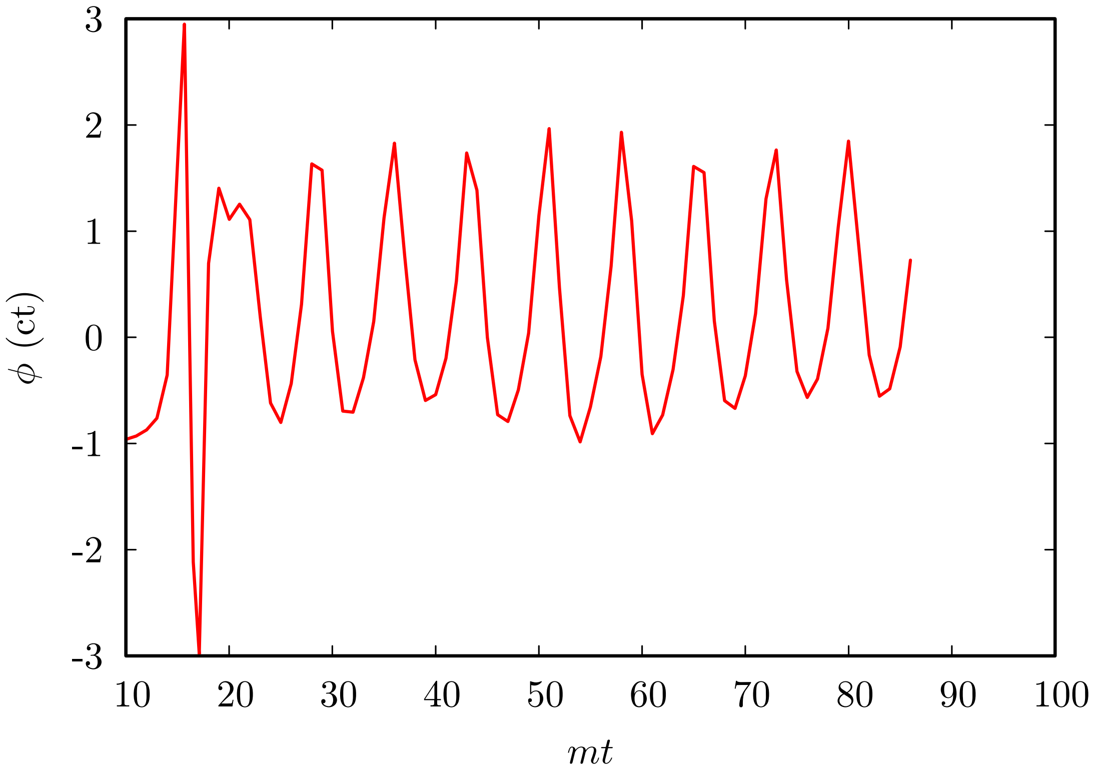


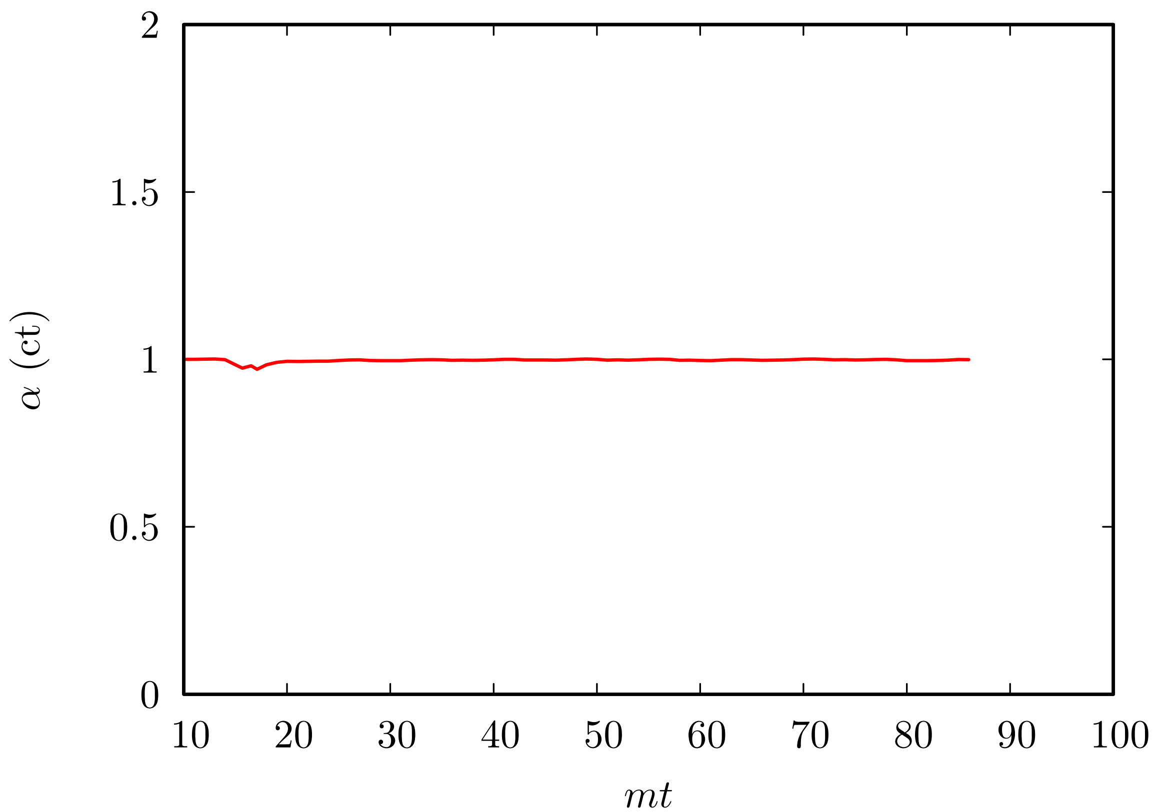




(iii) $v = 0.03\ M_P$ & $R_0 = 7m^{-1}$ —> $R_s / \delta_{dw} = 0.029$







oscillaton formation?