θ-vacua, quantum gravity and particles spectrum[†]

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[†] with Gia Dvali and Archil Kobakhidze, [2406.18402] and [2408.07535]

GR vacua

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\,(R(g) - 2\Lambda)\,,$$

g - metric, Λ - cosmological term.
de Sitter $\Lambda > 0$, anti-de Sitter $\Lambda < 0$, Minkowski $\Lambda = 0$.

*Isolated Ads is fine, via Ads/CFT duality Maldacena '98

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Quantum = no de Sitter Dvali, Gomez '14,'16+Zell, '17. The ground state = no evolution in time. de Sitter space $T \propto H$ Gibbons, Hawking '77 . Temporary de Sitter as an exited state on Minkowski vacuum is fine Berezhiani, Dvali, Sakhelashvili '21, 24.

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Anti-de Sitter (AdS) cosmology leads to a big crunch. So, Cosmology = Minkowski vacua. *

S-matrix formulation singles out the Minkowski vacuum Dvali '20

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Can we fix an unique Minkowski vacuum?

Lets tune

$$\Lambda = 0.$$

We have Minkowski vacuum, and quantum gravity with cosmology.

Are we in a consistent theory?

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No, if we have super-selected vacua with different energies. We cannot pick one and discard others.

E.g. QCD θ-vacua, $\mathcal{E} \propto \theta^2$.

If $\theta = 0 =$ Minkowski, $\theta' \neq \theta =$ de Sitter.

In gravity, the strong CP puzzle = consistency problem Dvali '22

The QCD vacuum

Topology of the QCD vacuum,

$$\pi_3(SU(N_c)) = Z$$

with Instantons,

$$\mathcal{M} \sim e^{-\frac{8\pi^2}{g^2}},$$

making θ-angle physical

$$\mathcal{L}_\theta = \theta \frac{g^2}{16\pi^2} G\tilde{G},$$

and vacuum energy,

$$\mathcal{E} \propto \theta^2$$

Callan, Dashen, Gross '76, Jackiw, Rebbi '76
$\theta = 0$ is a minimum of energy Vafa, Witten '84

The (traditional) Strong CP puzzle

$\theta \leq 10^{-10}$ From EDMN e.g. C. Abel, et al. '20

Chiral quarks

$$\psi \to e^{i\gamma_5 \alpha}\psi,$$

$$\theta \to \theta + 2\alpha$$

Integral form of anomaly

$$Q_5(t=\infty) - Q_5(t=-\infty) = 2n,$$

Chiral massive quark needs Peccei, Quinn '77 symmetry

$$|\Phi| e^{-i\frac{a(x)}{f_a}} \bar{\psi}\psi$$

implies an axion Wilczek '78, Weinberg '78 with

$$a(x) \to a(x) - 2\alpha\, f_a$$

How does the axion work?

TSV correlator,

$$\mathrm{FT}\langle G\tilde{G}(x)\ G\tilde{G}(0)\rangle_{p\to 0} \propto \left.\frac{p^2}{p^2 - m^2}\right|_{p\to 0}$$

If $m = 0$, θ is physical, and

$$\theta \propto \langle \tilde{G}G\rangle$$

Axion makes θ unphysical, with $m \neq 0$. This effect alternatively can be understood as the 3-form Higgs effect (0+1) $\tilde{G}G = *dC$ Dvali '05

If $m_u = 0$, η' plays role of the axion. Also, In QCD η' gaps the correlator.

Axion in the context of the Gravity

$a \to a + c$ not exact means,

$$\mathrm{FT}\langle G\tilde{G}(x)\ G\tilde{G}(0)\rangle_{p\to 0} \neq 0$$

This is considered as a quality problem, and can not happen in the gravity. So we predict, $\bar{\theta} = 0$ '05 '22 Dvali, Sakhelashvili '21

Alternatively 2-form axion can solve the problem, which can not be undone via continues deformations.

$$\mathcal{L} \ = \ \frac{1}{f_a^2}(C - f_a dB)^2$$

Eguchi and Hanson '78 (EH) found euclidean solution of GR,

$$ds^2 = \left(1 - \frac{a^4}{r^4}\right)^{-1} dr^2 + r^2\left(\sigma_x^2 + \sigma_y^2\right) + r^2\left(1 - \frac{a^4}{r^4}\right)\sigma_z^2$$

σ's are $SU(2)$ elements (We have 3-angles ϕ, θ, ψ) $d\sigma_x = 2\sigma_y \wedge \sigma_z$.

Gravitational Instantons

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σ's are $SU(2)$ elements (We have 3-angles ϕ, θ, ψ) $d\sigma_x = 2\sigma_y \wedge \sigma_z$.

The boundary at infinity S^3/Z_2 and the boundary at $r = a$ (coordinate singularity) is S^2, corresponding invariants,

$$\chi = \tfrac{1}{8\pi^2}\int d^4x\sqrt{g}\left(R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\alpha\beta}^2\right) + \text{bound. terms} = 2$$
$$\tau = -\tfrac{1}{24\pi^2}\int d^4x\, R\tilde{R} = 1$$

The Gravity CP-problem

Vanilla GR has zero action. A term

$$\Delta S = c\,\frac{\chi}{2}$$

Makes action finite. $c >> 1$, EFT works $\mathcal{M} \sim e^{-c}$

c encodes the cut-off scale $c \sim \left(\frac{M_{pl}}{\Lambda_{gr}}\right)^2$

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We have θ-term in the theory

$$S = \frac{\theta_g}{24\pi^2} \int d^4x R\tilde{R}$$

EH-Instantons make

$$\mathrm{FT}\langle \tilde{R}R(x)\ \tilde{R}R(0)\rangle_{p\to 0} \neq 0$$

A new CP puzzle! The $\mathcal{S}$-matrix consistency = consistency problem!

Lets solve the Gravity-CP problem.

Solving the problem

The Grav. anomaly $\partial_\mu j_5^\mu \propto R\tilde{R}$ Delbourgo, Salam '72

But helicity 1/2 fermion does not have zero modes
$Q_5(t = \infty) - Q_5(t = -\infty) = 0$

Fermion with helicity 3/2 has 2 zero modes Eguchi, Hanson '78

$$\psi_\mu \to \mathrm{e}^{i\gamma_5 \alpha} \psi_\mu$$
$$\theta_g \to \theta_g + 2\alpha$$

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Consistency of chiral 3/2 = supergravity, a local (gauge) SUSY.
Freedman, Nieuwenhuizen, Ferrara, '76, see e.g. Freedman, Proeyen, Supergravity (book)

The solution of Gravity CP requires SUGRA

Breaking of SUSY

Instanton effects give effective t'Hooft vertex,

$$\frac{W^*_{3/2}}{M^2_{pl}}\,\bar{\psi}^\mu \sigma_{\mu\nu}\psi^\nu$$

It break R-symmetry, vacuum = AdS, energy $\propto -3|W_{3/2}|^2/M^2_{pl}$.

Uplift to Minkowski, with Superfield X and superpotential,

$$W \;=\; X\Lambda_X^2 + W_{3/2}$$

The Polonyi model (Generated by Instantons) with broken SUSY

We predict an ALP a_R (phase of X, $\langle X\rangle \sim M_{pl}$) with mass $\sim m_{3/2}$ and decay constant M_{pl} (maybe a good Dark matter)[†]

[†]Note: since 1/2 fermion does not deliver zero modes, their anomalies should cancel (making coefficient zero.) $\partial_\mu j_5^\mu \propto R\tilde{R}$

The Electroweak part of The Standard Model

Solution Gravity-CP, Strong-CP $= \eta'/$axion and η_R/a_R.

What about EW theory?

$$\mathcal{L} = -\frac{1}{4} W_{\mu\nu}^2 + \theta_W \frac{1}{16\pi^2} W\tilde{W} + |D\phi|^2 - V(\phi)$$

Constrained instantons Anselm, Johansen '93,'94, see e.g. Shifman's book AQFT '22 ,

$$\mathrm{FT}\langle W\tilde{W}(x)\ W\tilde{W}(0)\rangle_{p\to 0} \sim e^{-\frac{2\pi}{\alpha_W}} \neq 0$$

Gravitational framework we must have a scalar,

$$\frac{a}{f_a} \to \frac{a}{f_a} - \alpha$$
$$\theta_W \to \theta_W + \alpha$$

The matter content of the Standard Model

Add leptons and quarks. They have,

$$l \to e^{i\alpha} l$$
$$q \to e^{i\frac{\beta}{3}} q$$

Symmetry one $\alpha = -\beta$, $B - L$ symmetry, a good global symmetry.
Symmetry two $\alpha = \beta$, $B + L$ symmetry is anomalous, meaning

$$\theta_W \to \theta_W + \alpha$$

making,

$$\mathrm{FT}\langle W\tilde{W}(x)\ W\tilde{W}(0)\rangle_{p\to 0} = 0$$

A particle must gap it!
We predict a particle in the STANDARD MODEL!
We will call it η_w

Origin of the η_w

For simplicity: ONE generation, and ONE color.

$$ql$$

Carries an unit $B + L$ charge.
If the Standard Model delivers particle, the above must condense.
It is a t' Hooft vertex. At $p \to 0$ same point insertion, gives,

$$\langle | \, ql \, | \rangle \neq 0$$

This is in full agreement with the index theorem,

$$\Delta Q_{B+L} = \int \frac{1}{16\pi^2} \tilde{W} W$$

An explicit computation proves the condensate (see backup slides).

Good vs Bad quality B+L

We consider good quality $B + L = \theta_W$ unphysical
Explicit operators, break $B + L$

$$qqql$$

We can't rotate θ_W away. But gravity requires θ_W to be unphysical

Good vs Bad quality B+L

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We can't rotate θ_W away. But gravity requires θ_W to be unphysical

We must add

$$|\Phi|e^{i\frac{a}{f_a}}qqql,$$

ALP making $B+L$ symmetry good, or $B_{\mu\nu}=\theta_w$ unphysical.

With gravity we still predict of η_w, with (slightly) changed origin (mixture with an ALP).
This is like η' in the case of $m_u \neq 0$. It is mixed with axion.

Conclusions

- Quantum gravity works only on Minkowski space and on eternal AdS without cosmology
- All theta vacua should be exactly nullified
- QCD $\bar{\theta}$ is exactly zero
- Gravity has CP problem and solution requires SUGRA
- We predict ALP with mass, degenerated to gravitino mass
- We argue about existence of η_w
- The θ_W should be unphysical hence, η_w must exist, a good $B + L$ symmetry / $B_{\mu\nu}$
- Representations of the 1/2 fermion are constrained, perturbative gravitational anomaly must cancel.

Thank you

Backup slide (de Sitter in rigid limit)

To summarize,

$$t_Q \sim \frac{M_{pl}^2}{H^3}$$

Rigidity = double-scaling limit $M_{pl} \to \infty$, H fixed, but $2 \to 2$ Graviton interaction

$$\alpha_{gr} = P^2 / M_{pl}^2 \to 0,$$

is trivial.

Backup slide (An alternative approaches)

We could ask, what happens if we rely all the physics on gravitino condesate,

$$\langle \bar{\psi}^{\mu} \sigma_{\mu\nu} \psi^{\nu} \rangle \neq 0$$

In this scenario, role of the axion is played by η_R, which has mass $m_{3/2}$ and decay constant M_{pl}. We still study the mechanism of the SUSY breaking. A very similar mechanism, which we discuss in our paper "Electroweak η_w meson"

Why we do not use the two-form $B_{\mu\nu}$, like in QCD?
There are potential consistency issues Duff, Nieuwenhuizen '80

backup slide (SUSY no anomaly)

This has ramification in the SUSY framework, let us add an extra Y-fields,

$$W = \hat{X}\Lambda_X^2 - g\hat{X}\hat{Y}_j^2 + W_{3/2}$$

which sets the theory in AdS, and going back to Minkowski requires, extra fields $\bar{Y}$'s

$$W = \hat{X}\Lambda_X^2 - g\hat{X}\hat{Y}_j^2 + M\hat{\bar{Y}}_j\hat{Y}_j + W_{3/2}$$

The $1/2$-anomaly is cancelled.

Backup slide (Instanton)

For example,

$$\sum_{i=1}^{4} dx_i^2 = dr^2 + r^2(\sigma_x^2 + \sigma_y^2 + \sigma_z^2)$$

$$\sigma_z \sim d\psi + \cos\theta d\phi$$

$$u^2 = r^2(1 - \frac{a^4}{r^4})$$

$r = a,\ u = 0$

$$ds^2 \simeq \frac{1}{4}du^2 + \frac{1}{4}u^2(d\psi + \cos\theta d\phi)^2 + \frac{1}{4}a^2 d\Omega^2$$

Backup slide (SUSY)

$$X_0 = \pm M_{pl}(\sqrt{3}+1)$$

$$W_{3/2} = \mp \Lambda_X^2 M_{pl}(\sqrt{3}+2)$$

$$m_{3/2} = W/M_{pl}^2 = \Lambda_X^2/M_{pl}$$

$$gXY_j^2 \simeq \Lambda_Y^2$$

Backup slide (Condensate and zero modes)

$\Psi = (\psi, \phi)^{\mathrm{T}}$ Anselm, Johansen '93,94, where

$$\psi = q_L + \ell_R^c \ , \quad \phi = \begin{pmatrix} u_R \\ d_R \end{pmatrix} + \begin{pmatrix} e_L^c \\ -\nu_L^c \end{pmatrix} \ .$$

The Lagrangian

$$\mathcal{L} = \bar{\Psi}\hat{\mathcal{D}}\Psi$$

$$\Psi \to \mathrm{e}^{i\alpha\Gamma_5/2}\Psi \ , \quad \Psi^\dagger \to \Psi^\dagger \mathrm{e}^{i\alpha\Gamma_5/2} \ ,$$

$\Gamma_5 = \mathrm{diag}\left(\gamma_5, \ -\gamma_5\right)$. Lets add μ breaks $B+L$, Then,

$$\begin{aligned} \langle \Psi^\dagger(x)\Psi(x)\rangle &= \lim_{\mu\to 0}\int \frac{d^4 z d\rho}{\rho^5} \ D(\rho)\langle x| \left(\hat{\mathcal{D}} + i\mu\right)^{-1} |x\rangle \\ &\simeq -iv^3 \left(\frac{2\pi}{\alpha}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha}} \end{aligned}$$

$D(\rho) \sim \rho\mu$ and $\langle x|(\hat{\mathcal{D}} + i\mu)^{-1}|x\rangle = \frac{P_0(x-z)}{i\mu}$

Backup slide (D-operator)

$$\hat{\mathcal{D}} \equiv \begin{pmatrix} -i\not{D} & i\epsilon M_\ell^* \epsilon P_L - iM_q P_R \\ i\epsilon M_\ell^{\mathrm{T}} \epsilon P_R - iM_q^\dagger P_L & -i\not{\partial} \end{pmatrix}$$

$$\frac{1}{\hat{\mathcal{D}} + i\mu} = \frac{P_0}{i\mu} + \Delta - i\mu\Delta^2 + \mathcal{O}(\mu^2)$$

$$D(\rho) = \left(\frac{2\pi}{\alpha(\rho)}\right)^4 \mathrm{e}^{-\frac{2\pi}{\alpha(\rho)} - 2\pi^2 v^2 \rho^2} \ \rho\mu$$